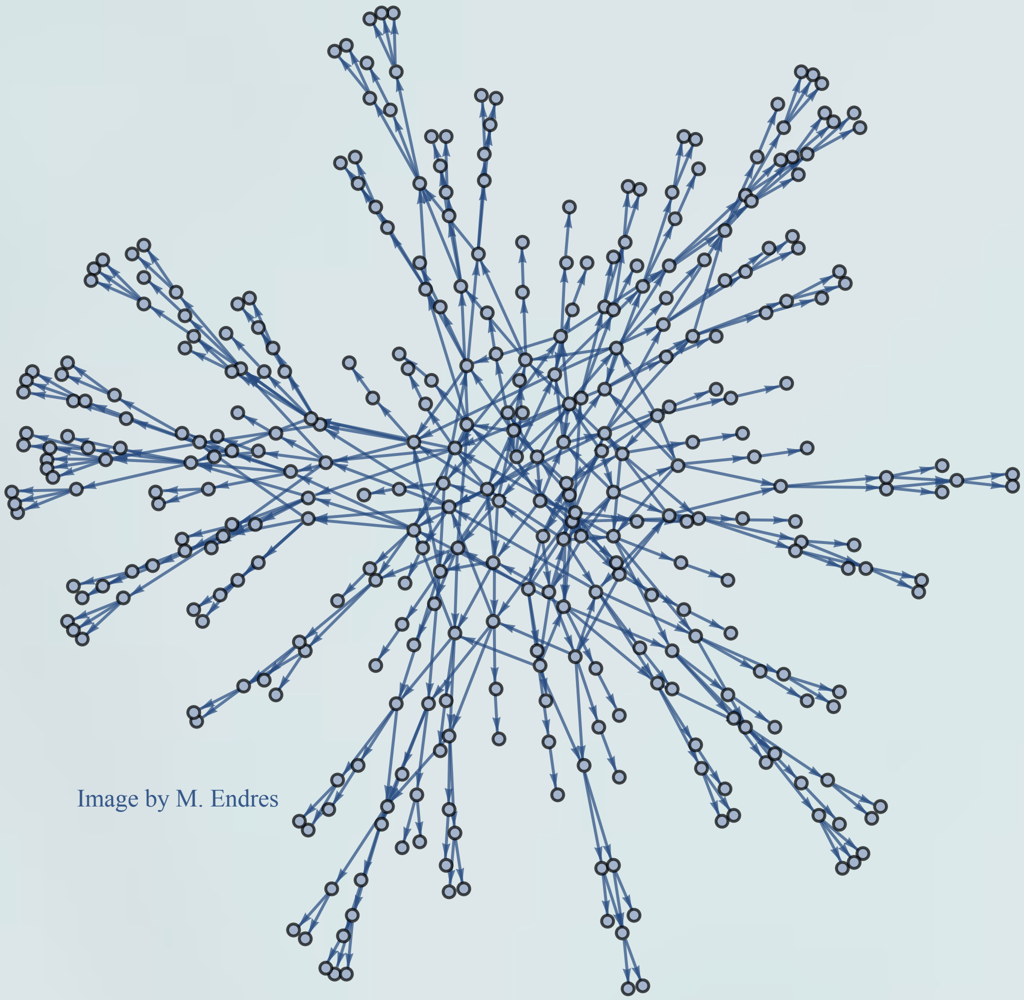


Image by M. Endres

ZD and Savage, PRD 86, 054505 (2012).

ZD, Will Detmold, Mike Endres, Andrew Pochinsky and Phiala Shanahan, work in progress.

LATTICE PDF WORKSHOP, APRIL 6-9, COLLEGE PARK, MD

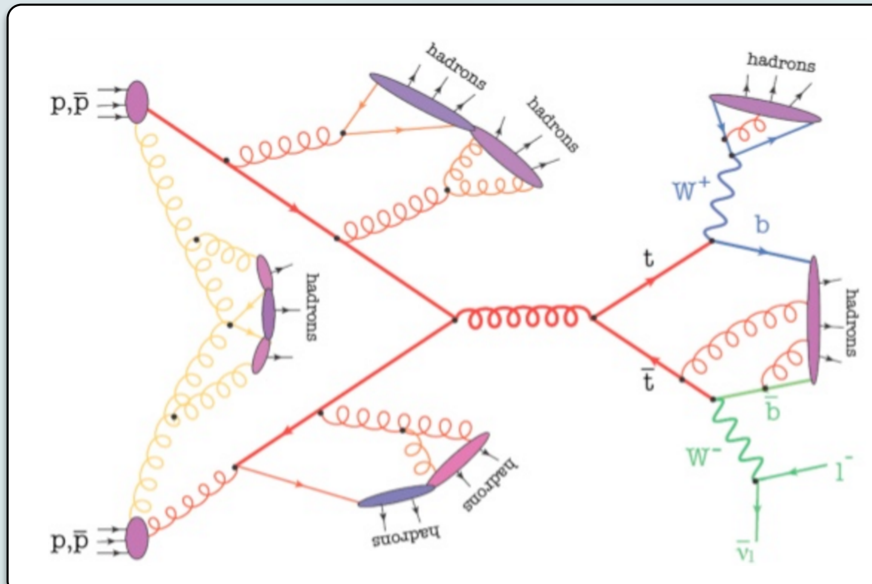
HIGHER MOMENTS OF PARTON DISTRIBUTION FUNCTIONS FROM LATTICE QCD

ZOHREH DAVOUDI
UNIVERSITY OF MARYLAND

CONSTRUCTING PDFS FROM MOMENTS:

$$\langle x^n \rangle_{q,\mu^2} = \int dx x^n q\left(x;\mu^2\right)$$

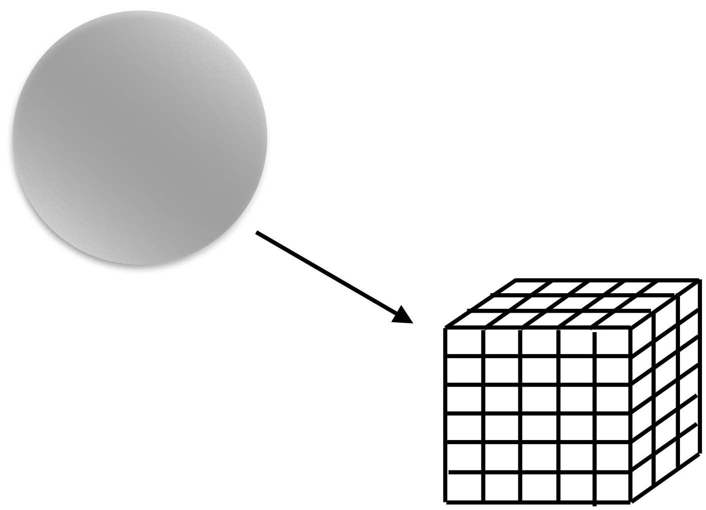
$$\langle p, s \left| \mathcal{O}_{\mu_1 \mu_2 \ldots \mu_n} \right| p, s \rangle \Big|_{\mu^2} = 2 \langle x^n \rangle_{q,\mu^2} p^{\{\mu_1} p^{\mu_2} \ldots p^{\mu_n\}}$$



LQCD IS IDEAL FOR EVALUATING SUCH MES.

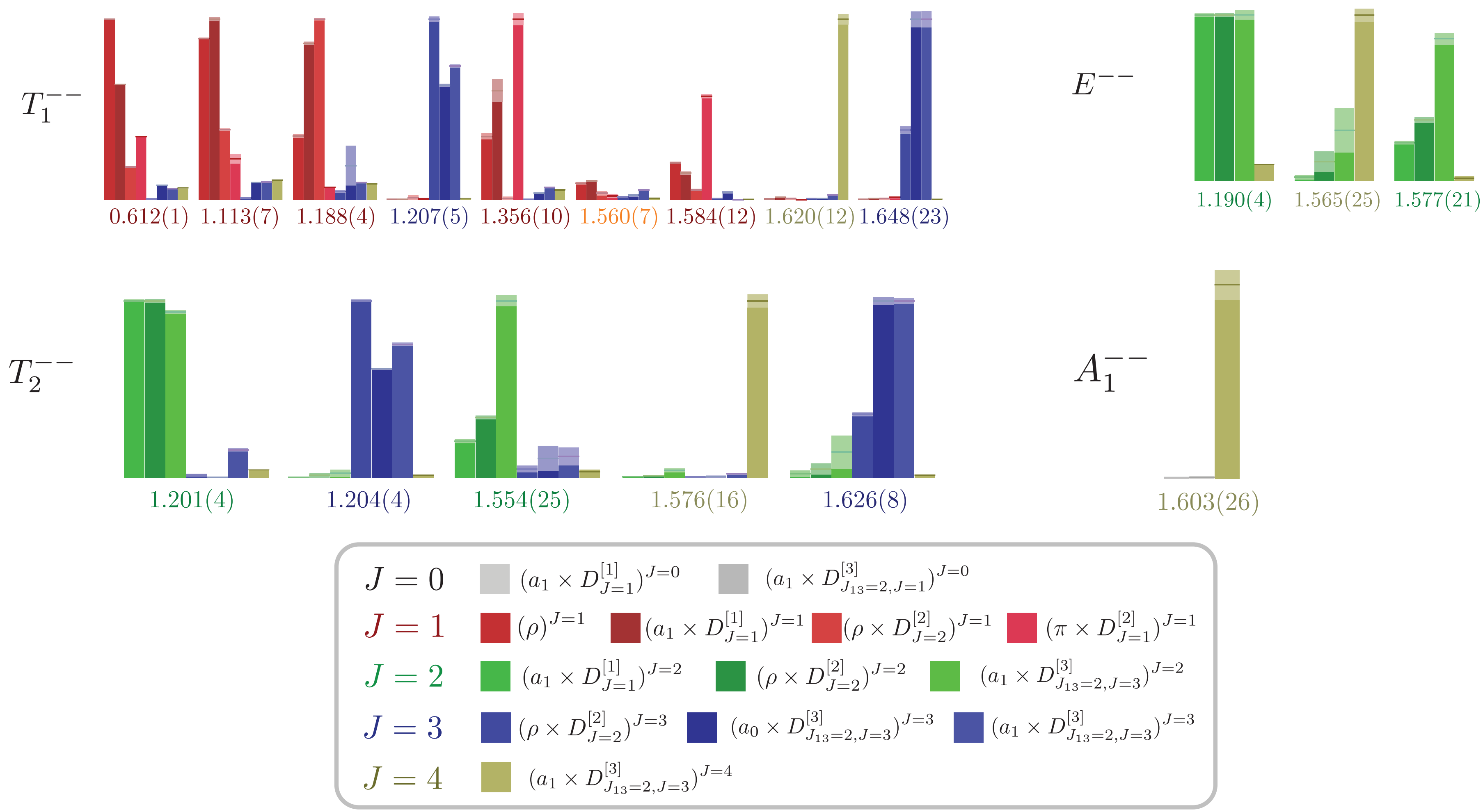
PHENOMENOLOGICALLY 6-8 MOMENTS APPEAR TO BE SUFFICIENT.

HOWEVER, ONLY UP TO THE FIRST THREE MOMENTS HAVE BEEN ACCESSIBLE WITH LQCD DUE TO A POWER-DIVERGENCE MIXING WITH LOWER DIMENSIONAL OPERATORS.



LESSON FROM MODERN LQCD SPECTROSCOPY STUDIES:

Dudek, Edwards, Peardon, Richards, and Thomas, Phys.Rev.Lett., 103, 262001 (2009); Phys. Rev. D82, 034508 (2010),



SMEARED OPERATORS FROM A CONTINUUM OPERATOR WITH A GIVEN J

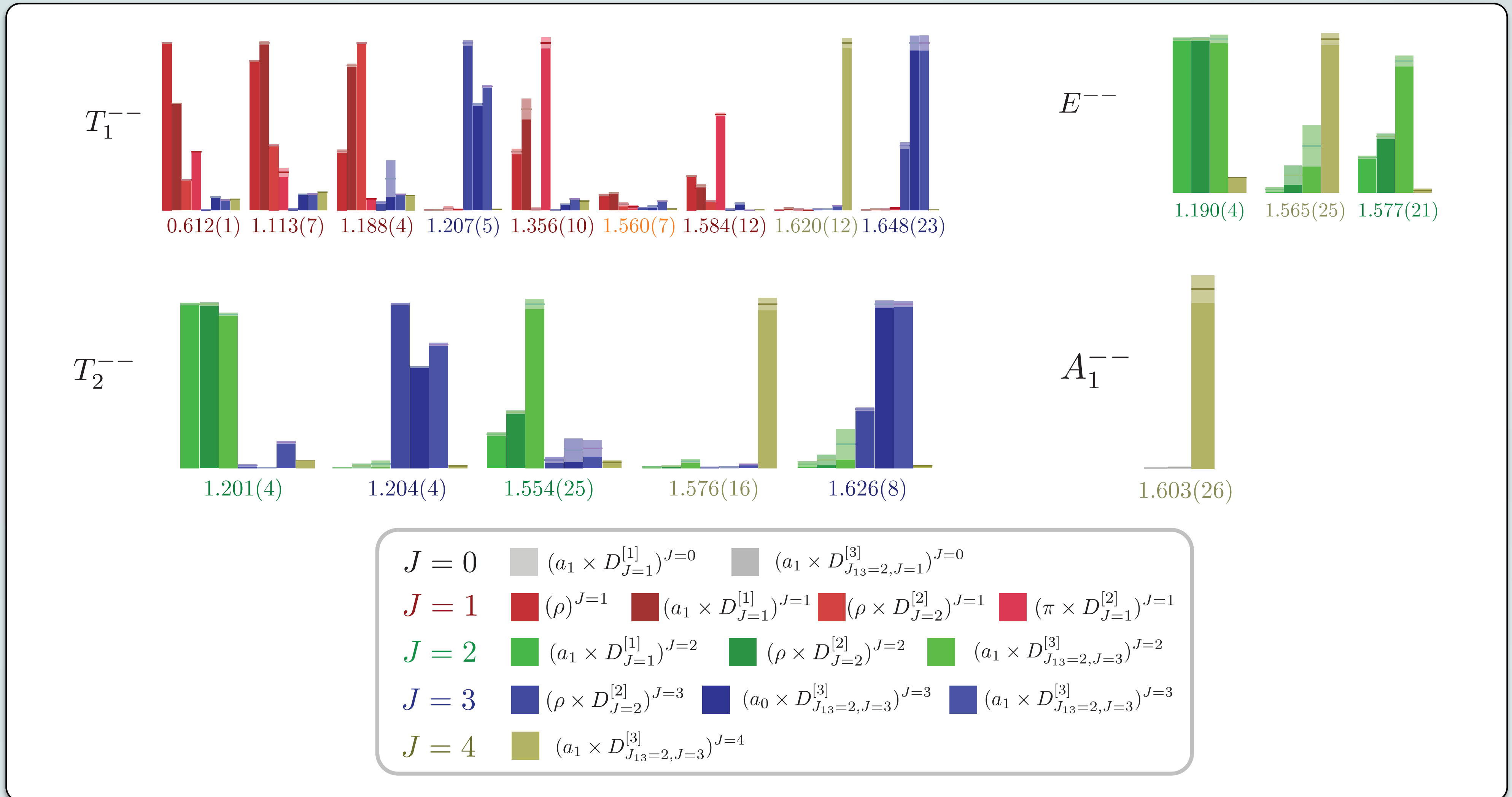
$$\mathcal{O}^{[J]}_{\Lambda,\lambda} \equiv \sum_M \mathcal{S}^{J,M}_{\Lambda,\lambda} \mathcal{O}^{J,M}$$

$$\mathcal{S}^{J,M}_{\Lambda,\lambda} = \langle \Lambda, \lambda | J, M \rangle$$

$$\mathcal{O}^{J,M} \equiv (\Gamma \times D^{n_D})^{J,M}$$

LESSON FROM MODERN LQCD SPECTROSCOPY STUDIES:

Dudek, Edwards, Peardon, Richards, and Thomas, Phys.Rev.Lett., 103, 262001 (2009); Phys. Rev. D82, 034508 (2010),



RELATED IDEAS:

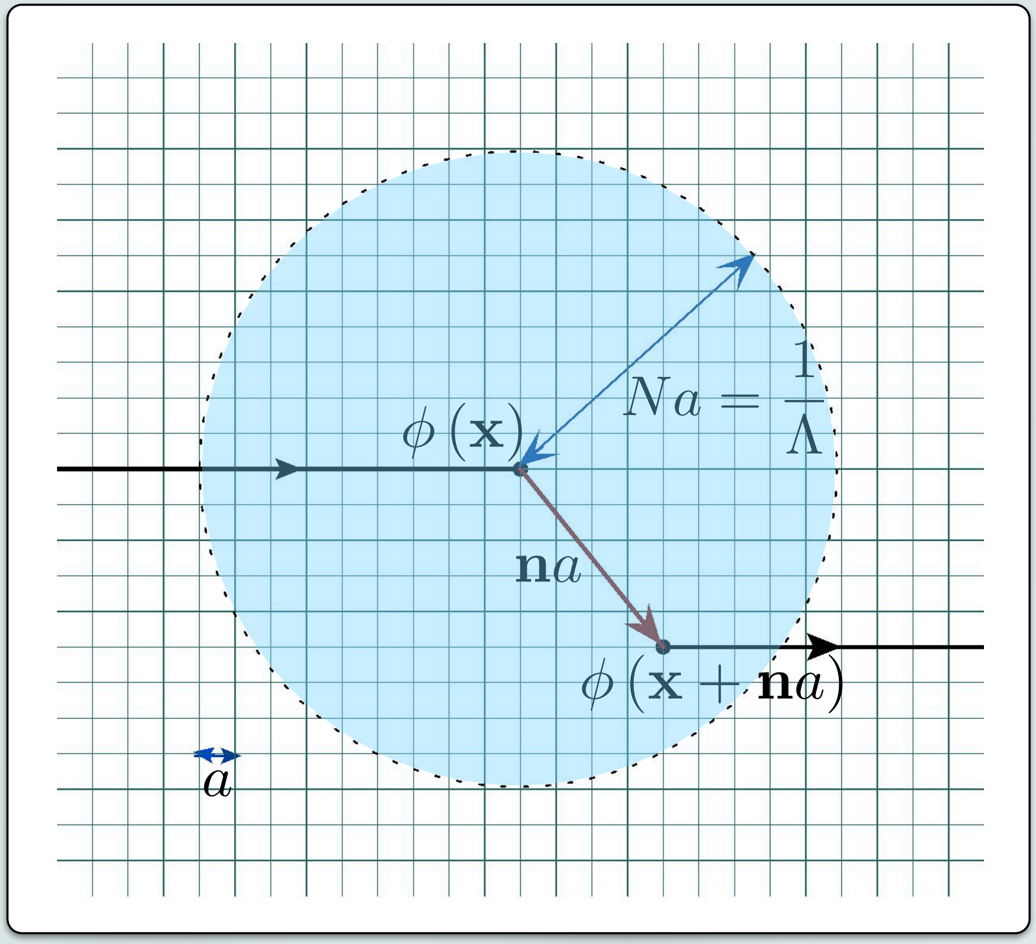
Detmold and Lin, Phys. Rev. D73, 014501 (2006).

Dawson, Martinelli, Rossi, Sachrajda, Sharpe, Talevi, and Testa, Nucl. Phys. B514, 313 (1998).

Monahan and Orginos, Phys. Rev. D91, 074513 (2015).

IN THAT SPIRIT, WE CONSIDERED A SIMPLE OPERATOR:

$$\hat{\theta}_{L,M}\left(\mathbf{x};a,N\right) = \frac{3}{4\pi N^3}\sum_{\mathbf{n}}^{|\mathbf{n}|\leq N}\phi\left(\mathbf{x}\right)\phi\left(\mathbf{x}+\mathbf{n}a\right)\ Y_{L,M}\left(\hat{\mathbf{n}}\right)$$



ZD and Savage, PRD 86, 054505 (2012).

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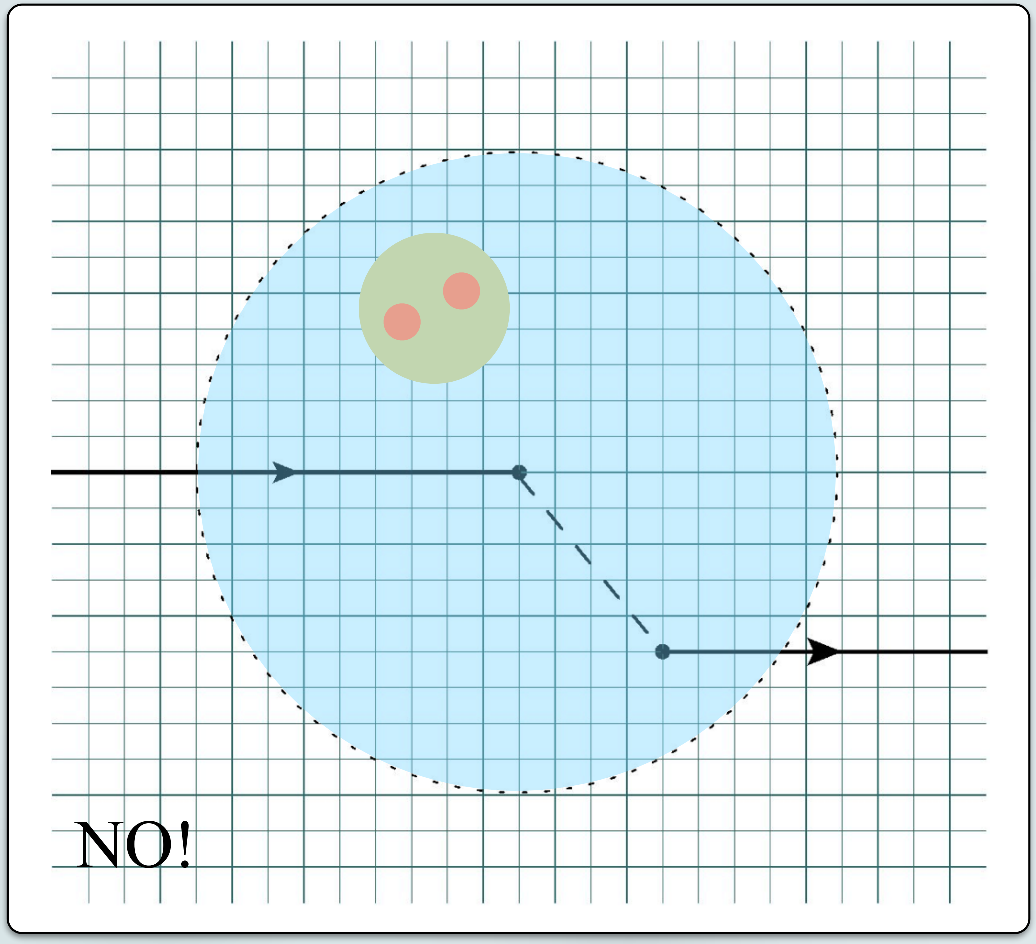

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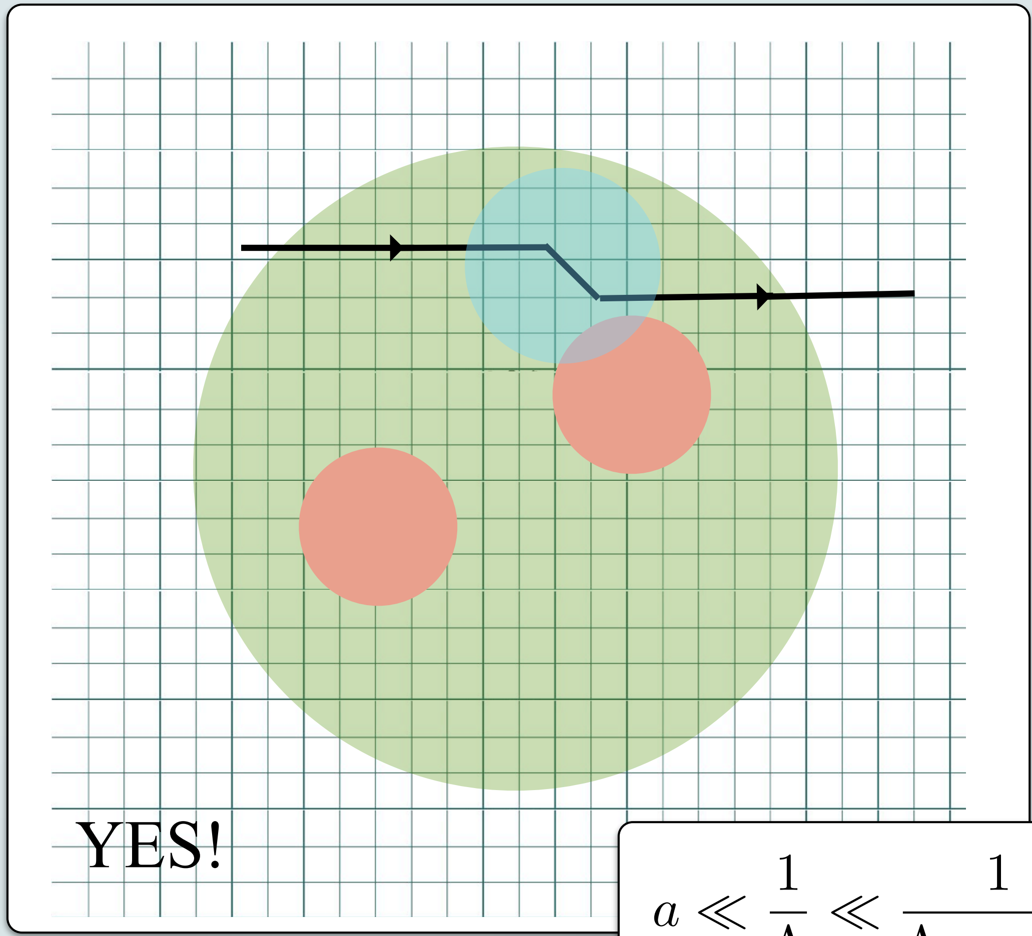
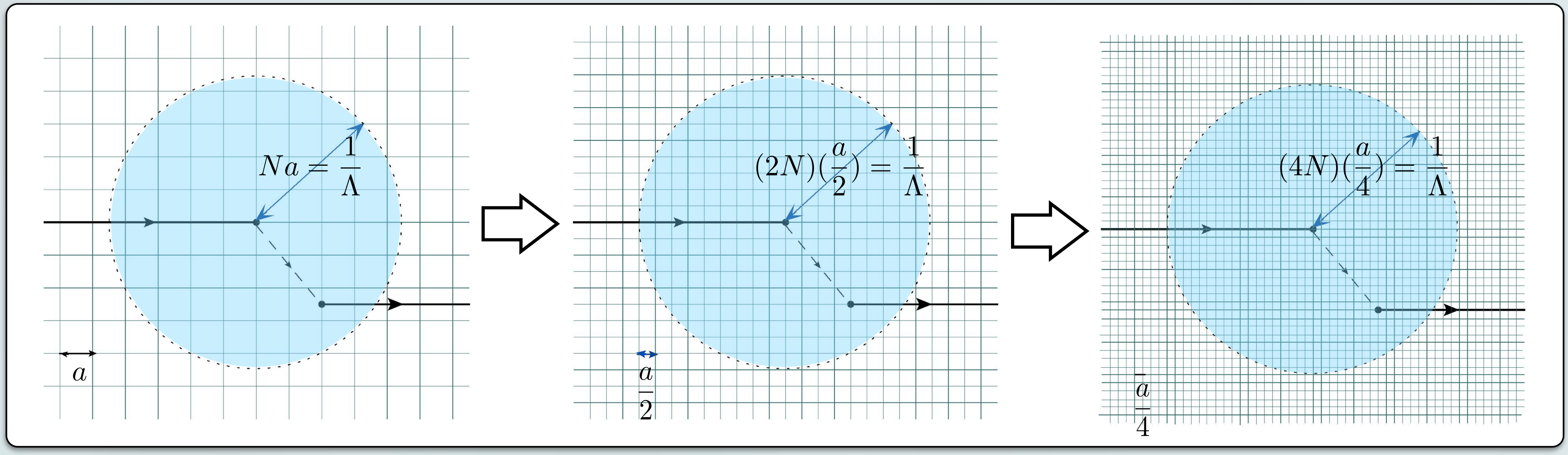


$$a \ll \frac{1}{\Lambda} \ll \frac{1}{\Lambda_{Hadron}}\left(\frac{1}{p}\right)$$

ZD and Savage, PRD 86, 054505 (2012).

CORRECT PROCEDURE:
KEEP THE PHYSICAL SIZE OF THE OPERATOR FIXED, THEN TAKE THE CONTINUUM LIMIT:



ZD and Savage, PRD 86, 054505 (2012).

AN EXAMPLE:

$$\hat{\theta}_{3,0}(\mathbf{x}; a, N) = \frac{C^{(1)}_{30;10}(N)}{\Lambda}\mathcal{O}^{(1)}_{z}(\mathbf{x}; a) + \frac{C^{(3)}_{30;10}(N)}{\Lambda^3}\mathcal{O}^{(3)}_{z}(\mathbf{x}; a) + \frac{C^{(5)}_{30;10}(N)}{\Lambda^5}\mathcal{O}^{(5)}_{z}(\mathbf{x}; a) +$$
$$\frac{C^{(5;RV)}_{30;10}(N)}{\Lambda^5}\mathcal{O}^{(5;RV)}_{z}(\mathbf{x}; a) + \frac{C^{(3)}_{30;30}(N)}{\Lambda^3}\mathcal{O}^{(3)}_{zzz}(\mathbf{x}; a) + \frac{C^{(5)}_{30;30}(N)}{\Lambda^5}\mathcal{O}^{(5)}_{zzz}(\mathbf{x}; a) +$$
$$\frac{C^{(5)}_{30;50}(N)}{\Lambda^5}\mathcal{O}^{(5)}_{zzzzz}(\mathbf{x}; a) + \mathcal{O}\left(\frac{\nabla^7_z}{\Lambda^7}\right)$$

DESIRED $L = 3$ OPERATOR

HOW DO THE COEFFICIENTS SCALE WITH N (a)? BETTER HAVE:

$C^{(d)}_{30;L'0}(N)$ IS FINITE FOR $L' = 3$

$C^{(d)}_{30;L'0}(N) \to 0$ FOR $L' \neq 3$

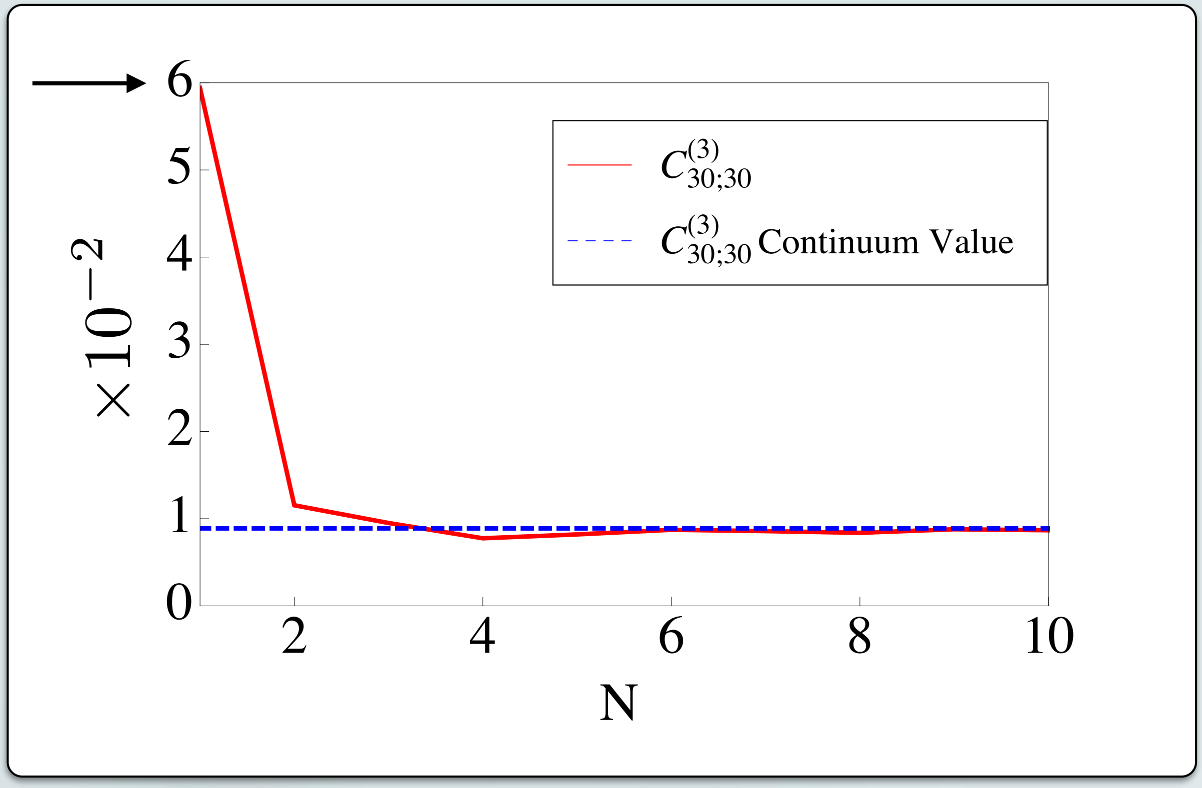
$C^{(d;RV)}_{30;L'0}(N) \to 0$

AS $N \to \infty$.

ZD and Savage, PRD 86, 054505 (2012).

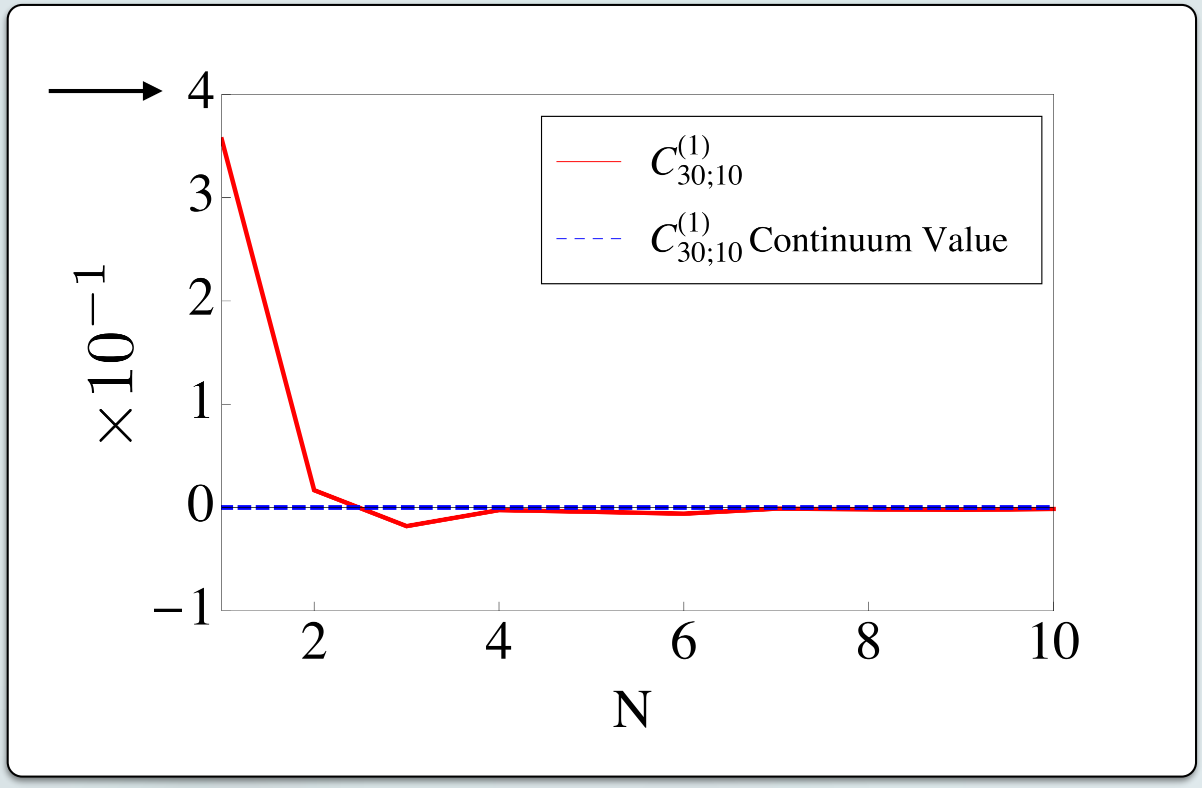
SEE ANALYTICAL RESULTS IN THE PAPER…

THE COEFFICIENT OF DESIRED OPERATOR:



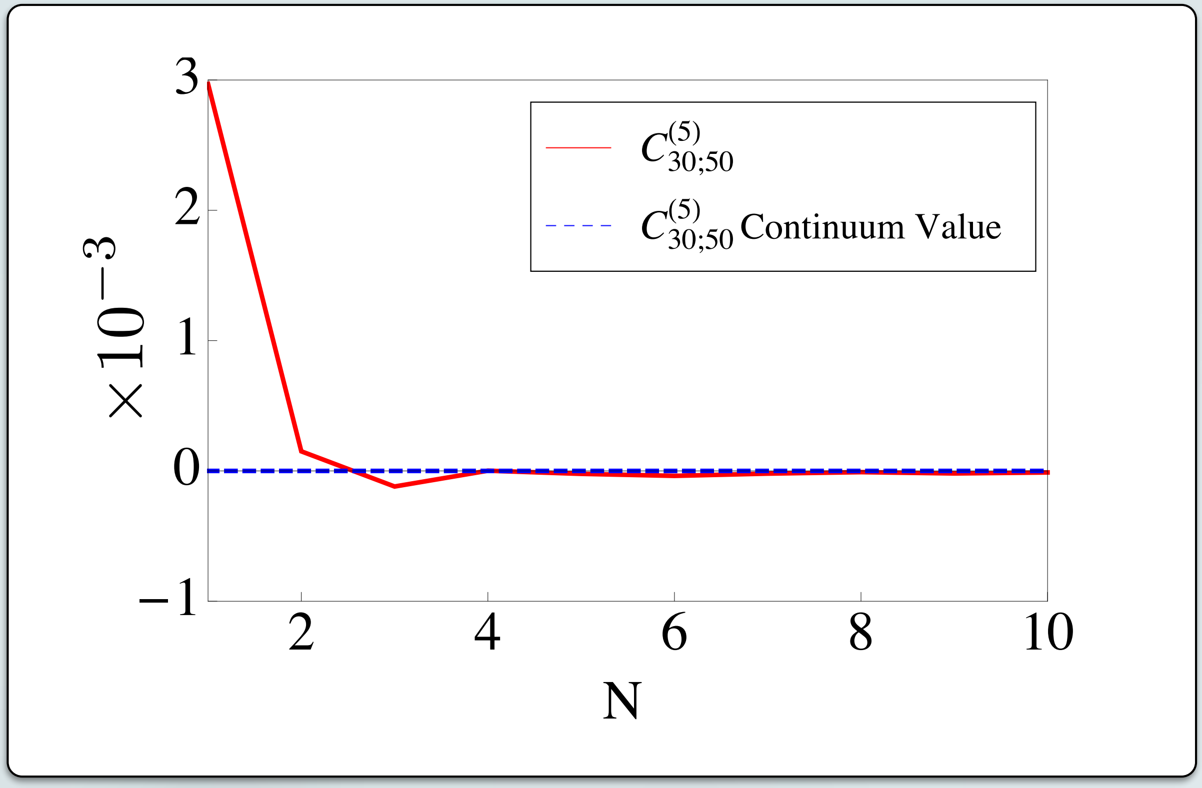
ZD and Savage, PRD 86, 054505 (2012).

THE COEFFICIENT OF LOWER-DIMENSIONAL OPERATOR:



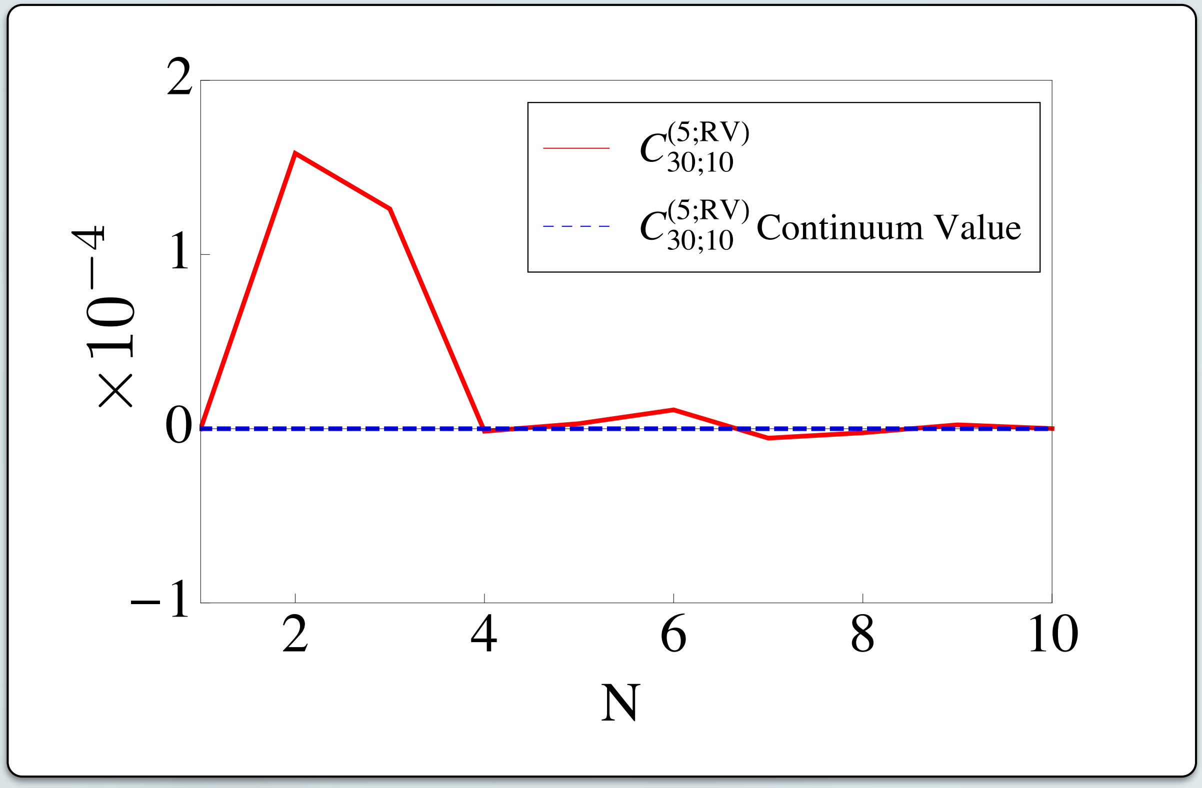
ZD and Savage, PRD 86, 054505 (2012).

THE COEFFICIENT OF HIGHER-DIMENSIONAL OPERATOR:



ZD and Savage, PRD 86, 054505 (2012).

THE COEFFICIENT OF LORENTZ-BREAKING OPERATOR:



ZD and Savage, PRD 86, 054505 (2012).

RECALLING THE EXPANSION OF OUR CHOSEN OPERATOR…

$$\hat{\theta}_{3,0}(\mathbf{x}; a, N) = \frac{C^{(1)}_{30;10}(N)}{\Lambda}\mathcal{O}^{(1)}_{z}(\mathbf{x}; a) + \frac{C^{(3)}_{30;10}(N)}{\Lambda^3}\mathcal{O}^{(3)}_{z}(\mathbf{x}; a) + \frac{C^{(5)}_{30;10}(N)}{\Lambda^5}\mathcal{O}^{(5)}_{z}(\mathbf{x}; a) + \frac{C^{(5;RV)}_{30;10}(N)}{\Lambda^5}\mathcal{O}^{(5;RV)}_{z}(\mathbf{x}; a) + \frac{C^{(3)}_{30;30}(N)}{\Lambda^3}\mathcal{O}^{(3)}_{zzz}(\mathbf{x}; a) + \frac{C^{(5)}_{30;30}(N)}{\Lambda^5}\mathcal{O}^{(5)}_{zzz}(\mathbf{x}; a) + \frac{C^{(5)}_{30;50}(N)}{\Lambda^5}\mathcal{O}^{(5)}_{zzzzz}(\mathbf{x}; a) + \mathcal{O}\left(\frac{\nabla^7_z}{\Lambda^7}\right)$$

ZD and Savage, PRD 86, 054505 (2012).

...WE HAVE NOW FOUND THAT THE OPERATOR IS DOMINATED BY THE DESIRED TERM:

$$\Lambda^3 \hat{\theta}_{3,0}(\mathbf{x}; a, N) = \alpha_1 \frac{\Lambda^2}{N^2} \mathcal{O}_z^{(1)}(\mathbf{x}) + \alpha_2 \frac{1}{N^2} \mathcal{O}_z^{(3)}(\mathbf{x}) + \alpha_3 \frac{1}{\Lambda^2 N^2} \mathcal{O}_z^{(5)}(\mathbf{x}) + \alpha_4 \frac{1}{\Lambda^2 N^2} \mathcal{O}_z^{(5;RV)}(\mathbf{x}) + \alpha_5 \mathcal{O}_{zzz}^{(3)}(\mathbf{x}) + \alpha_6 \frac{1}{\Lambda^2} \mathcal{O}_{zzz}^{(5)}(\mathbf{x}) + \alpha_7 \frac{1}{\Lambda^2 N^2} \mathcal{O}_{zzzzz}^{(5)}(\mathbf{x}) + \mathcal{O}\left(\frac{\nabla_z^7}{\Lambda^4}\right)$$

POWER DIVERGENCE OF THE NAIVE OPERATOR EVIDENT:

$$\alpha_1 \frac{1}{a^2} \mathcal{O}_z^{(1)} + \alpha_2 \mathcal{O}_z^{(3)} + \alpha_3 a^2 \mathcal{O}_z^{(5)} + \alpha_4 a^2 \mathcal{O}_z^{(5;RV)} + \alpha_5 \mathcal{O}_{zzz}^{(3)} + \alpha_6 a^2 \mathcal{O}_{zzz}^{(5)} + \alpha_7 a^2 \mathcal{O}_{zzzzz}^{(5)} + \mathcal{O}\left(a^4 \nabla_z^7\right)$$

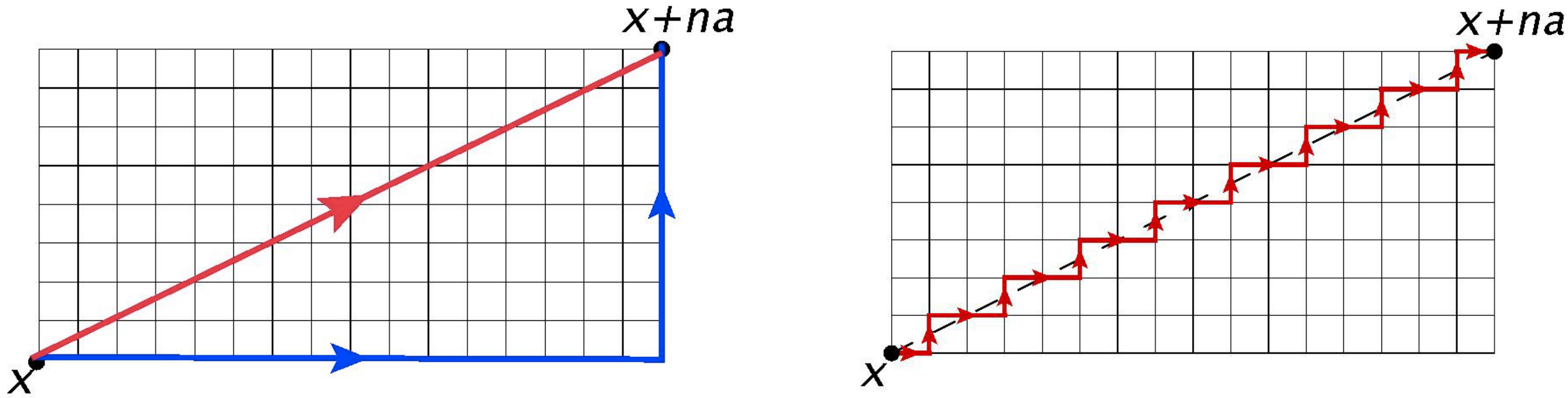
$N = 1$

ZD and Savage, PRD 86, 054505 (2012).

HOW ABOUT QCD AND BEYOND CLASSICAL EFFECTS?

$$\hat{\theta}_{L,M}(\mathbf{x}; a, N) = \frac{3}{4\pi N^3} \sum_{\mathbf{n}}^{|\mathbf{n}| \leq N} \overline{\psi}(\mathbf{x})\, U(\mathbf{x}, \mathbf{x}+\mathbf{n}a)\, \psi(\mathbf{x}+\mathbf{n}a)\; Y_{L,M}(\hat{\mathbf{n}})$$

FEATURE 1: SOME EXTENDED LINKS MAXIMALLY BREAK ROTATIONAL SYMMETRY



ZD and Savage, PRD 86, 054505 (2012).

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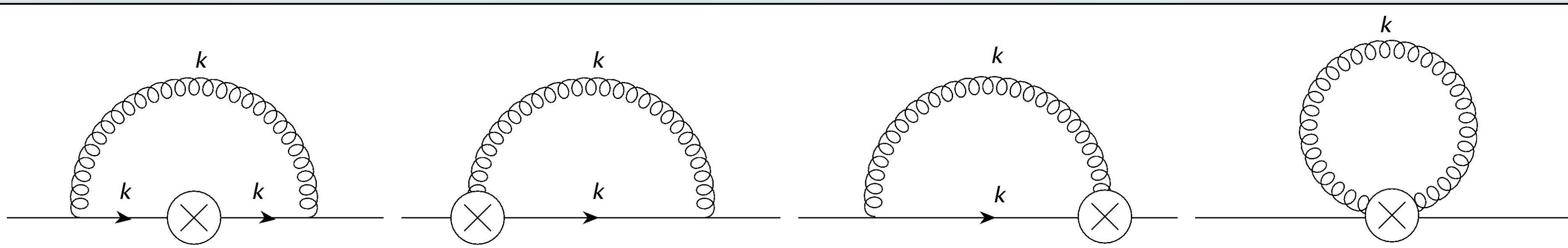
FEATURE 2: NONVANISHING TADPOLES WITH LATTICE REGULARIZATION



ZD and Savage, PRD 86, 054505 (2012).

SEE THE PAPER FOR CAREFUL TREATMENT OF THESE FEATURES IN LATTICE PERTURBATION THEORY. THE CONCLUSION IS THAT:

SCALING OF ROTATIONAL-INVARIANT CONTRIBUTIONS AT 1-LOOP LPT WITH WILSON FERMIONS:

$$\sim \alpha_s / N$$

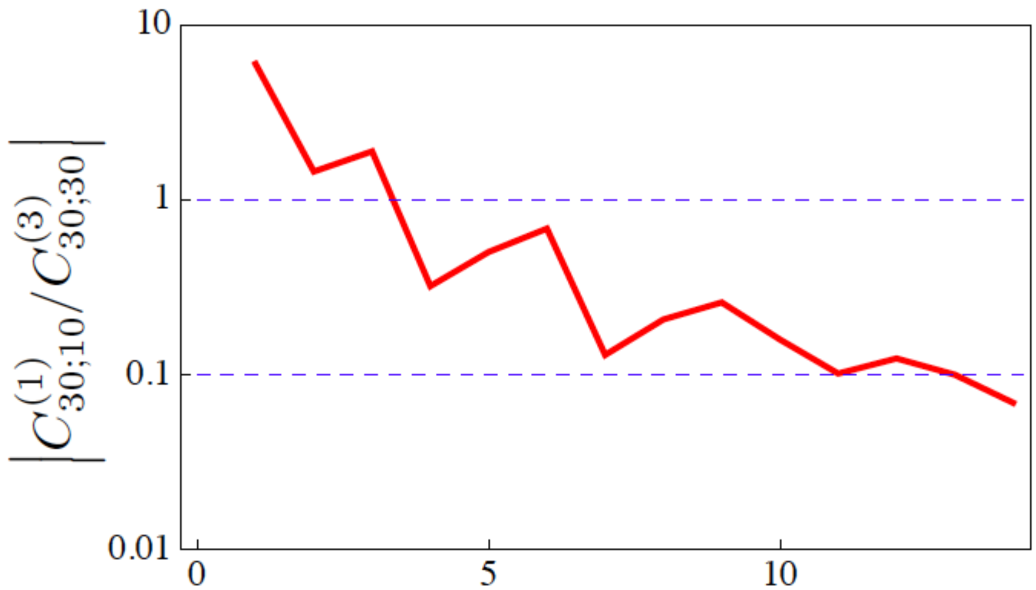
SCALING OF NON-ROTATIONAL-INVARIANT CONTRIBUTIONS AT 1-LOOP LPT WITH WILSON FERMIONS:

$$\sim \alpha_s a^2 \Lambda_g^2 \sim \frac{\alpha_s}{N_g^2}$$

DOES THIS WORK NON-PERTURBATIVELY?

ZD and Savage, PRD 86, 054505 (2012).

EVEN A SMALL SHELL LARGELY ELIMINATES THE CONTAMINATION:



A TREE LEVEL EXPECTATION

$$N = \frac{1}{\Lambda a}$$

ZD and Savage, PRD 86, 054505 (2012).

IN PRACTICE, HOW LARGE CAN THE OPERATOR BE?

a	μ	N^2
0.08 fm	– $\sim$ 2 GeV	– 1
0.06 fm	– $\sim$ 2 GeV	– 2
0.05 fm	– $\sim$ 2 GeV	– 4
0.04 fm	$\sim$ 5 GeV $\sim$ 2 GeV	1 6
0.03 fm	$\sim$ 5 GeV $\sim$ 2 GeV	2 11
0.02 fm	$\sim$ 5 GeV $\sim$ 2 GeV	4 25

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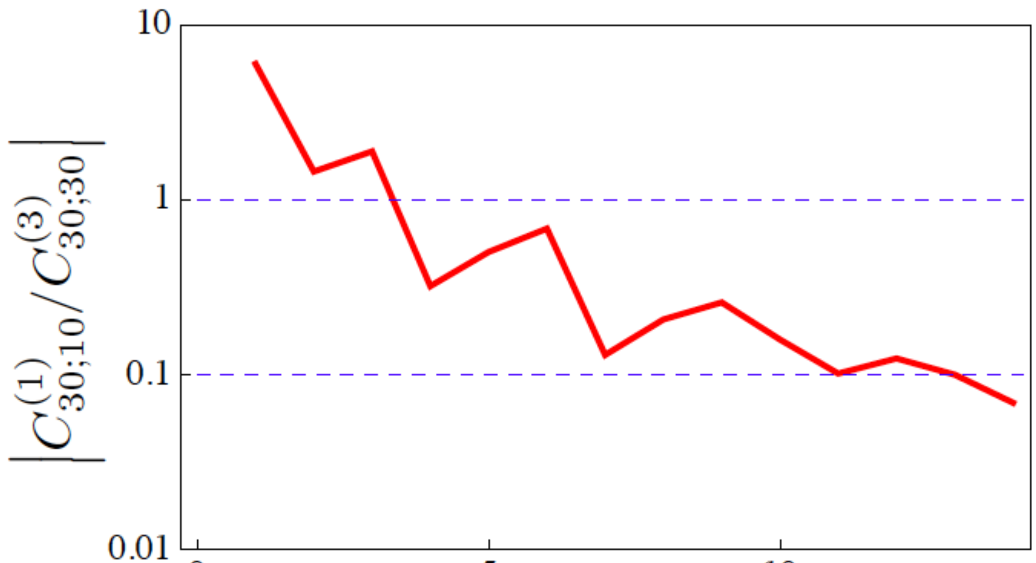

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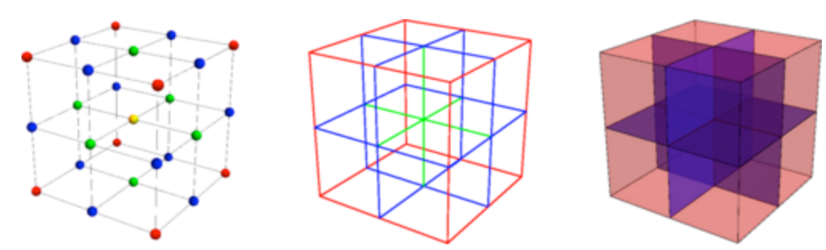
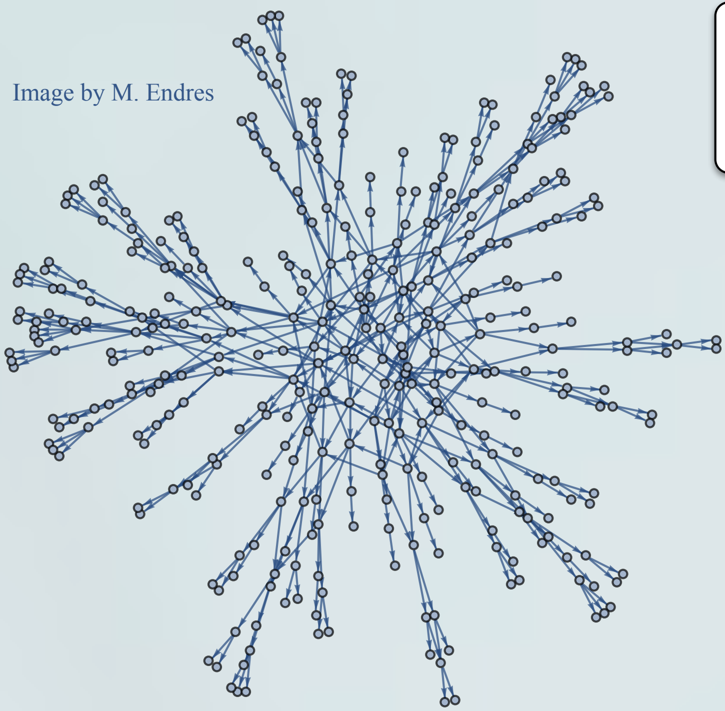


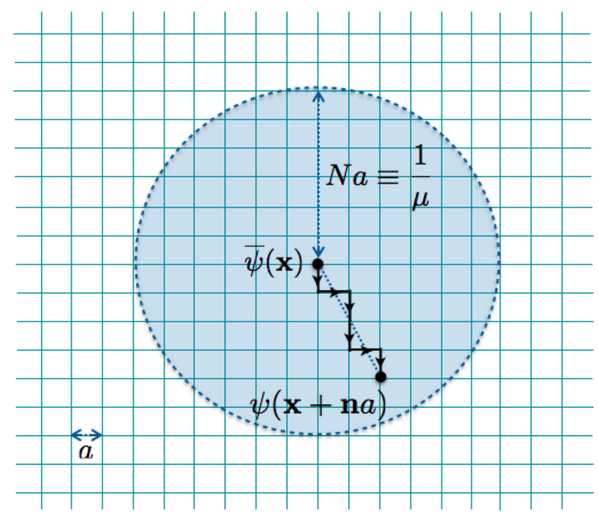
Endres, Brower, Detmold, Orginos, and Pochinsky, Phys. Rev. D 92, 114516, Endres and Detmold, Phys. Rev. D 94, 114502 (2016), and arXiv:1801.06132 [hep-lat].

GENERATING GAUGE CONFIGURATIONS WITH A MULTI SCALE ALGORITHM METHOD

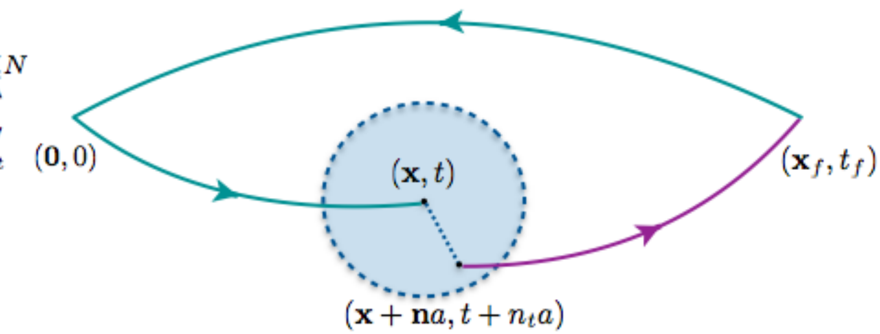
Image by M. Endres



$$\hat{\theta}_{n,l,m}\left(x_\mu;\mu\right) = \frac{2}{\pi^2 N^4}\sum_{n_\mu}^{|n_\mu|\leq N}\overline{\psi}(x_\mu)U^{(\mathcal{C})}_{x_\mu,x_\mu+n_\mu a}\psi\left(x_\mu+n_\mu a\right)\mathcal{Y}_{n,l,m}\left(\hat{n}_\mu\right)$$



$$C_{3p}(t_f,t) = \sum_{\mathbf{x}_f,\mathbf{x}}\sum_{\mathbf{n},n_t}^{|n_\mu|\leq N}$$

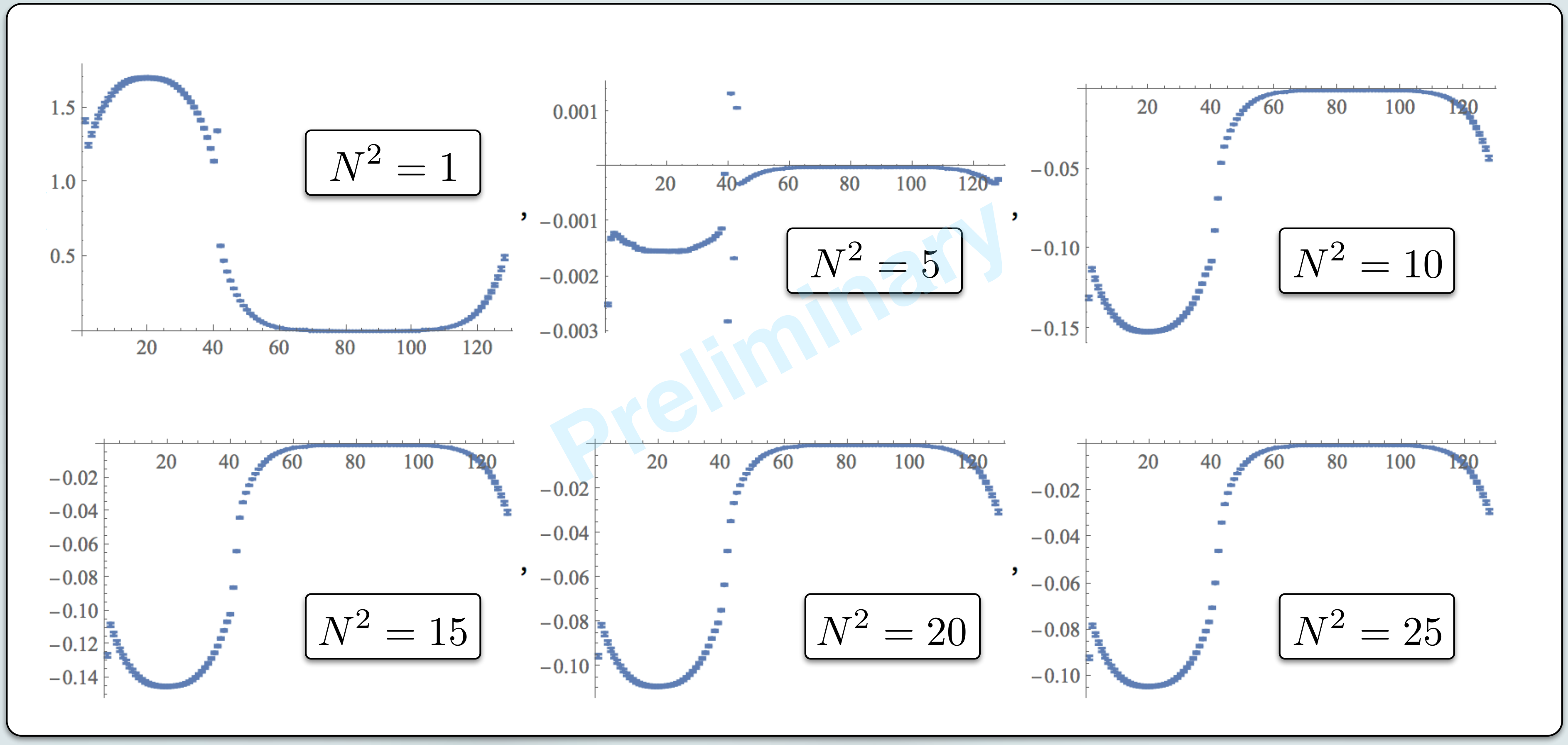


$$\times\ \mathcal{Y}_{n,l,m}(\hat{n}_\mu)$$

$$C_{3p}(t_f,t) = \sum_{\mathbf{x}_f,\mathbf{x}}\langle 0|\chi_\pi(\mathbf{x}_f,t_f)\hat{\theta}_{n,l,m}\left(\mathbf{x},t;\mu\right)\chi^\dagger_\pi(\mathbf{0},0)|0\rangle$$

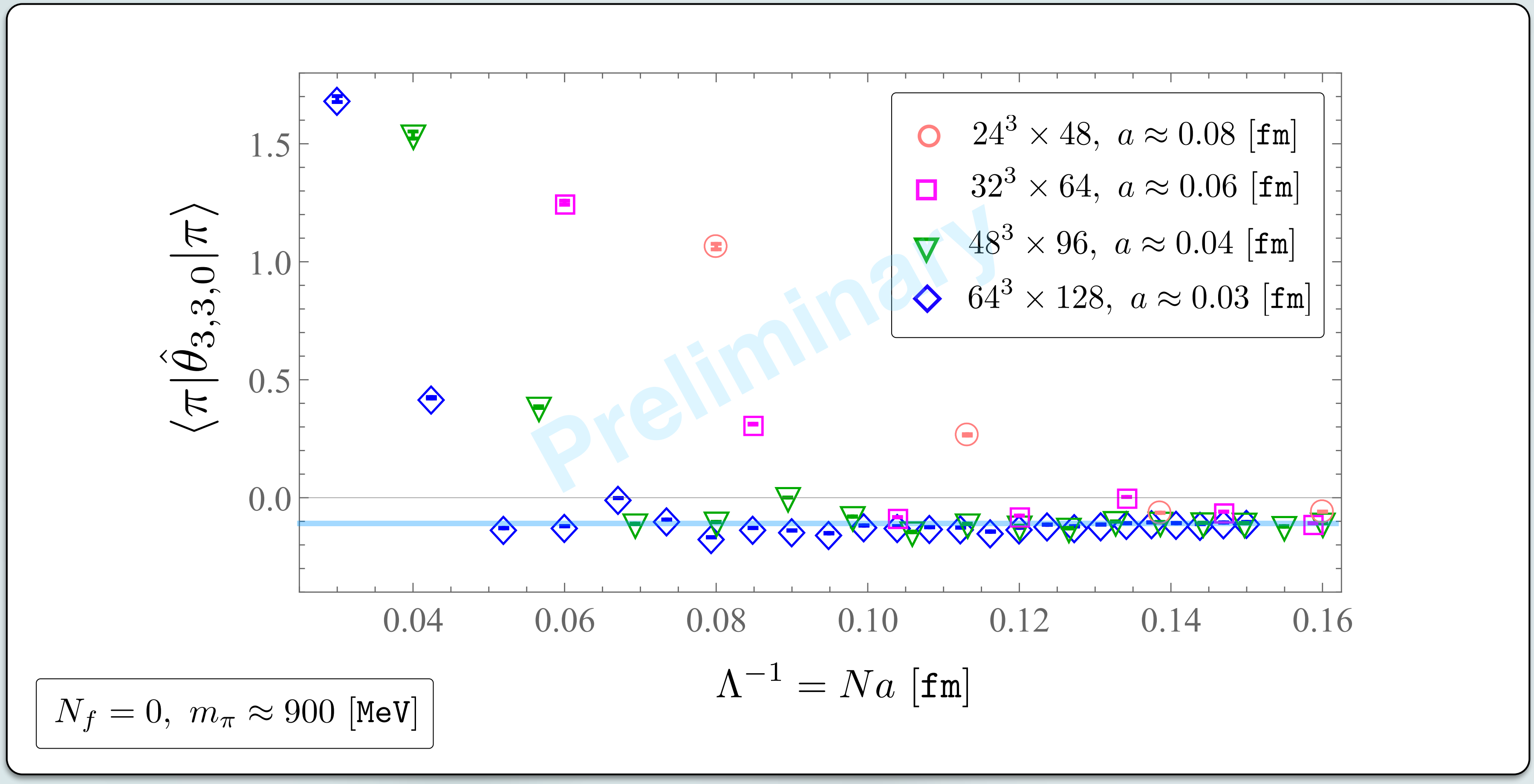
ZD, Will Detmold, Mike Endres, Andrew Pochinsky and Phiala Shanahan, work in progress.

$C_{3pt}(t_f; t)/C_{2pt}(t)$ AS A FUNCTION OF t AT A FIXED t_f



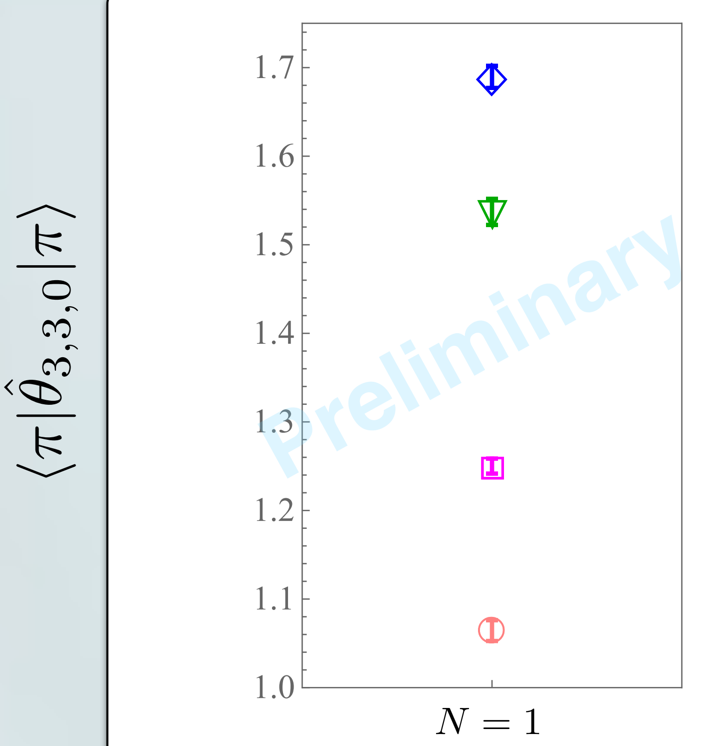
ZD, Will Detmold, Mike Endres, Andrew Pochinsky and Phiala Shanahan, work in progress.

THE MATRIX ELEMENT OF A HIGH “ANGULAR MOMENTUM” QUARK BILINEAR OPERATOR IN PION AT REST AS A FUNCTION OF THE OPERATOR SIZE:

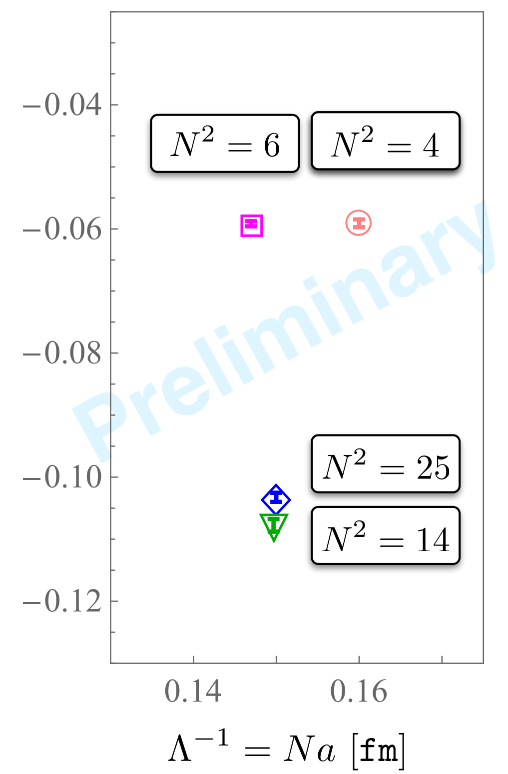


ZD, Will Detmold, Mike Endres, Andrew Pochinsky and Phiala Shanahan, work in progress.

CONTINUUM LIMIT OF THE NAIVE OPERATOR



EXTENDED OPERATOR WITH A FIXED SIZE



$N_f = 0,\ m_\pi \approx 900$ [MeV]

ZD, Will Detmold, Mike Endres, Andrew Pochinsky and Phiala Shanahan, work in progress.

IN SUMMARY

- THE PROPOSED OPERATOR ON THE LATTICE APPROACHES THE CONTINUUM OPERATOR IN A SMOOTH WAY WITH CORRECTIONS THAT SCALE AT MOST BY a^2. TADPOLE IMPROVEMENT AND GAUGE-FIELD SMEARING ARE ESSENTIAL FOR RECOVERING ROTATIONAL INVARIANCE IN LATTICE GAUGE THEORIES.

- NO POWER DIVERGENCE! THE SPECTRUM OF EXCITED STATES AND HIGHER MOMENTS OF HADRON DISTRIBUTION FUNCTIONS ARE CALCULABLE FROM LATTICE QCD.

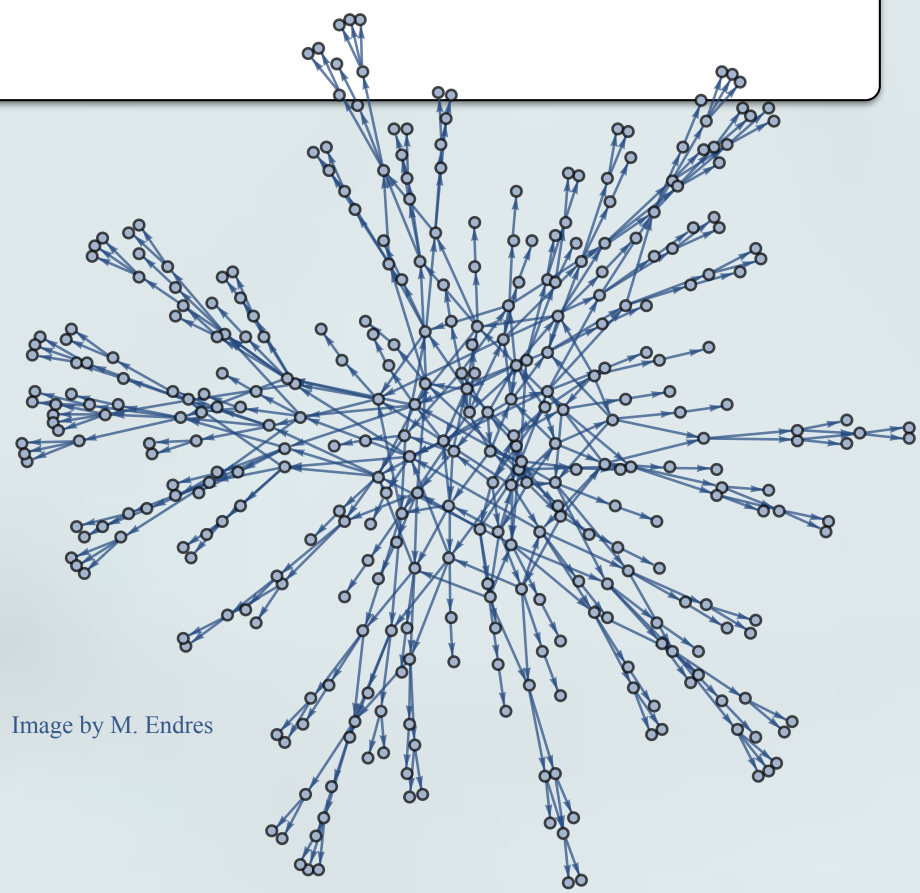
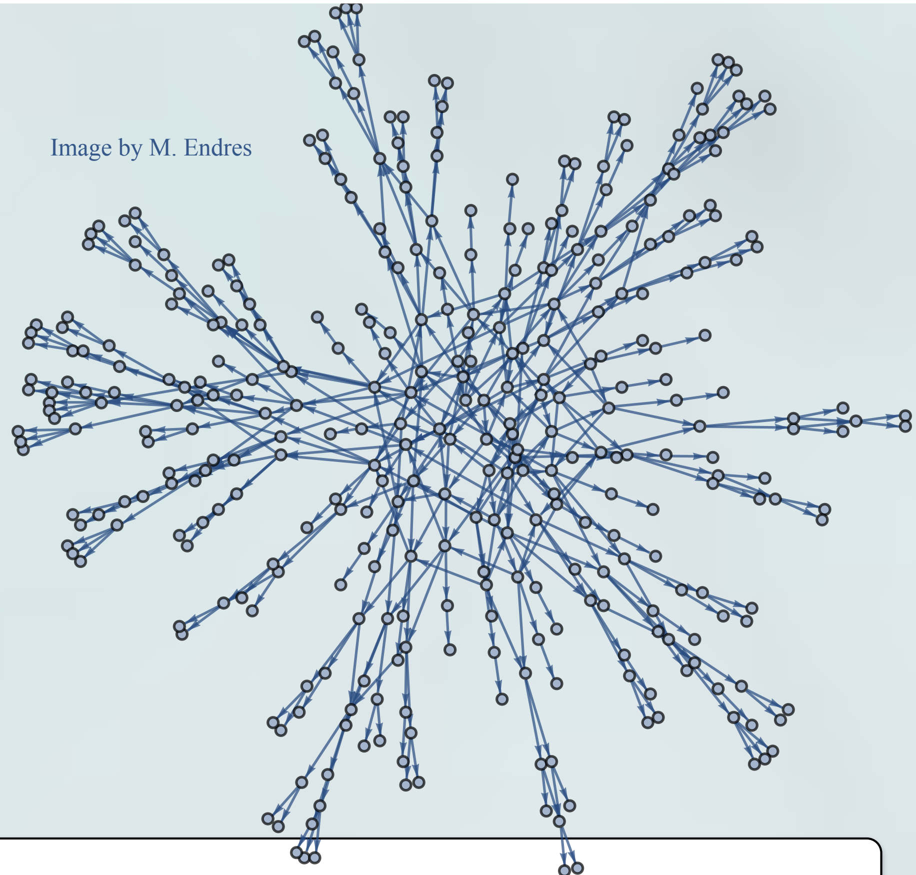


Image by M. Endres



OUTLOOK

- CAN THE OPERATOR BE FURTHER IMPROVED TOWARDS THE CONTINUUM LIMIT?
- RENORMALIZTION OF THE OPERATOR AND MATCHING.
- ARE OTHER SMEARING PROFILES POTENTIALLY MORE USEFUL?
- COMPARISON WITH OTHER METHODS AND PROPOSALS, e.g., DETMOLD AND LIN, JI, MONAHAN AND ORGINOS.

SUPPLEMENTARY SLIDES

AN EXAMPLE OF OPERATOR BASIS: L=1, m=0

$$\mathcal{O}_z^{(1)}(\mathbf{x}) = \phi(\mathbf{x})\,\nabla_z\phi(\mathbf{x})$$

$$\mathcal{O}_z^{(3)}(\mathbf{x}) = \phi(\mathbf{x})\,\nabla^2\nabla_z\phi(\mathbf{x})$$

$$\mathcal{O}_z^{(5)}(\mathbf{x}) = \phi(\mathbf{x})\left(\nabla^2\right)^2\nabla_z\phi(\mathbf{x})$$

$$\mathcal{O}_z^{(5,RV)}(\mathbf{x}) = \phi(\mathbf{x})\sum_j \nabla_j^4\nabla_z\phi(\mathbf{x})$$

LORENTZ-VIOLATING OPERATOR