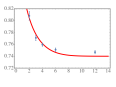

Structure of Pseudo- and Quasi-PDFs

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U. Maryland
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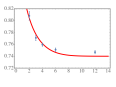
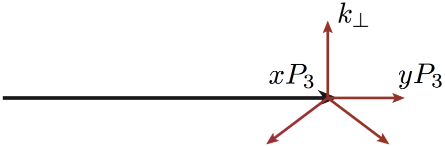
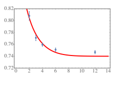


Parton Densities and Transverse Momentum Cut-Off

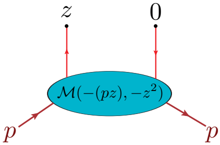
- Original Feynman approach to PDFs $f(x)$:
  infinite momentum $P_3 \to \infty$ limit of $k_3 = xP_3$ momentum distributions
  ($\sim$ quasi-PDFs $Q(x, P_3)$)
- $f(x)$ were treated as $k_\perp$-integrated $f(x, k_\perp)$ distributions
- Understood from the start: $Q(x, P_3 \to \infty) \to f(x)$ limit exists only if $f(x, k_\perp)$ rapidly decreases with $k_\perp$
- "Transverse momentum cut-off", $\langle k_\perp^2 \rangle \sim 1/R_{\rm hadr}^2$
- Question 1: why $Q(x, P_3)$ differs from $f(x)$?
- Question 2: how does $Q(x, P_3)$ convert into $f(x)$ when $P_3 \to \infty$?
- Qualitative answer: yP_3 comes from two sources:
  from the motion of the hadron (xP_3) and
  from Fermi motion of quarks inside the hadron $(y - x)P_3 \sim 1/R_{\rm hadr}$



- $(y - x)P_3 \sim 1/R_{\rm hadr}$ part has the same origin as transverse momentum
- $\Rightarrow$ One should be able to relate quasi-PDFs to TMDs



Parton Densities and Transverse Momentum Cut-Off



- Basic matrix element (ignoring spin)

$$\langle p|\phi(0)\phi(z)|p\rangle = \mathcal{M}(-(pz),-z^2)$$

- Lorentz invariance: $\mathcal{M}$ depends on z through (pz) and z^2
- Take $z=(0,0,0,z_3)$, then $-(pz)\equiv \nu = Pz_3$ and $-z^2 = z_3^2$
- Ioffe time ν: $\mathcal{M}(\nu, z_3^2)$ = Ioffe time pseudo-distribution (pseudo-ITD)
- Introduce quasi-PDF (Ji,2013)

$$Q(y,P) = \frac{P}{2\pi}\int_{-\infty}^{\infty} dz_3\, e^{-iyPz_3}\,\mathcal{M}(Pz_3, z_3^2) = \int_{-\infty}^{\infty}\frac{d\nu}{2\pi}\, e^{-iy\nu}\,\mathcal{M}(\nu,\nu^2/P^2)$$

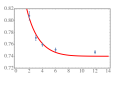
- Take $z=(z_+=0, z_-, z_1, z_2)$, then $\nu = -p^+z^-$ and $-z^2 = z_1^2+z_2^2$. TMD:

$$\mathcal{M}(\nu, z_1^2+z_2^2) = \int_{-1}^{1} dx\; e^{ix\nu}\int_{-\infty}^{\infty} dk_1 dk_2 e^{i(k_1z_1+k_2z_2)}\mathcal{F}(x, k_1^2+k_2^2)$$

- Take $z_1=0, z_2=\nu/P$ and use for qPDF

$$Q(y,P) = P\int_{-1}^{1} dx\int_{-\infty}^{\infty} dk_1 \mathcal{F}(x, k_1^2+(y-x)^2P^2)$$

- qPDF variable y has the $-\infty < y < \infty$ support, since $-\infty < k_2 < \infty$



- Pseudo-PDF $\mathcal{P}(x,-z^2)$: Fourier transform of pseudo-ITD with respect to ν

$$\mathcal{M}(\nu,-z^2) = \int_{-1}^{1} dx\, e^{-ix\nu}\, \mathcal{P}(x,-z^2)$$

- Limits $-1 \leq x \leq 1$ for any Feynman diagram. Relation to TMD

$$\mathcal{P}(x,z_\perp^2) = \int d^2\mathbf{k}_\perp e^{i(\mathbf{k}_\perp \mathbf{z}_\perp)} \mathcal{F}(x,k_\perp^2)$$

- When $\mathcal{F}(x,k_\perp^2)$ rapidly vanishes with $k_\perp$, pseudo-PDF and pseudo-ITD are regular for $z^2=0$, and $\mathcal{P}(x,0)=f(x)$
- Quasi-PDF to pseudo-PDF relation

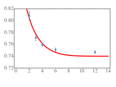
$$Q(y,P) = \frac{|P|}{2\pi} \int_{-1}^{1} dx \int_{-\infty}^{\infty} dz_3\, e^{-i(y-x)Pz_3}\, \mathcal{P}(x,z_3^2)$$

- Expand $\mathcal{P}(x,z_3^2)$ in z_3^2

$$\mathcal{P}(x,z_3^2) = \sum_{l=0}^{\infty} (z_3^2\Lambda^2)^l\, \mathcal{P}_l(x)$$

- $Q(y,P)$ approaches $f(y)$ like

$$Q(y,P) = f(y) + \sum_{l=1}^{\infty} \left(\frac{\Lambda^2}{P^2}\right)^l \frac{\partial^{2l}}{\partial y^{2l}} \mathcal{P}_l(y)$$



Quasi-PDFs and Pseudo-PDFs

$$Q(y,P) = f(y) + \sum_{l=1}^{\infty} \left(\frac{\Lambda^2}{P^2}\right)^l \frac{\partial^{2l}}{\partial y^{2l}} \mathcal{P}_l(y)$$

- Support mismatch: $-\infty < y < \infty$ for qPDF $Q(y,P)$, while $\mathcal{P}_l(y)$'s vanish outside $-1 \le y \le 1$
- Do not take this expansion too literally
- Innocently-looking derivatives of $\mathcal{P}_l(y)$ generate infinite tower of singular functions like $\delta(y)$, $\delta(y \pm 1)$ and their derivatives
- Recall: even if a function $f(y)$ has a nontrivial support Ω (say, $-1 \le y \le 1$), one may formally represent it by a series

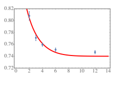
$$f(y) = \sum_{N=0}^{\infty} \frac{(-1)^N}{N!} M_N \, \delta^{(N)}(y)$$

over the functions $\delta^{(N)}(y)$ with an apparent support at one point $y = 0$ only

- M_N are moments of $f(y)$

$$M_N = \int_\Omega dy \, y^N f(y)$$

- While the difference between $Q(y,P)$ and $f(y)$ is formally given by a series in powers of $1/P^2$, its coefficients are not the ordinary functions of y



Moments of Quasi-PDFs

- In terms of TMDs:

$$Q(y,P) = f(y) + \sum_{l=1}^{\infty} \int d^2k_\perp \frac{k_\perp^{2l}}{4^l P^{2l} (l!)^2} \frac{\partial^{2l}}{\partial y^{2l}} \mathcal{F}(y, k_\perp^2)$$

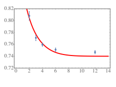
- To eliminate mismatch, take y^n moments $\langle y^n \rangle_Q$ of the quasi-PDFs

$$\langle y^n \rangle_Q \equiv \int_{-\infty}^{\infty} dy\, y^n Q(y,P) = \sum_{l=0}^{[n/2]} \frac{n!}{(n-2l)!(l!)^2} \frac{\langle x^{n-2l} k_\perp^{2l} \rangle_{\mathcal{F}}}{4^l P^{2l}}$$

- $\langle x^{n-2l} k_\perp^{2l} \rangle_{\mathcal{F}}$ are the combined moments of TMDs

$$\langle x^{n-2l} k_\perp^{2l} \rangle_{\mathcal{F}} \equiv \int_{-1}^{1} dx\, x^{n-2l} \int d^2k_\perp\, k_\perp^{2l}\, \mathcal{F}(x, k_\perp^2)$$

- Expansion makes sense only when $\mathcal{F}(x, k_\perp^2)$ vanishes faster than any power of $1/k_\perp^2$
- Is it possible to study the approach of $Q(y,P)$ to $f(y)$ for fixed y?



Relations between quasi-PDFs and TMDs

- z_3-dependence has the same origin as $k_\perp$ dependence of TMDs
- Quasi-PDFs can be obtained from TMDs (A.R., 2016)

$$Q(y,P)/P = \int_{-1}^{1} dx \int_{-\infty}^{\infty} dk_1 \mathcal{F}(x, k_1^2 + (y-x)^2 P^2)$$

- Or from pseudo-PDFs

$$Q(y,P) = \frac{P}{2\pi} \int_{-1}^{1} dx \int_{-\infty}^{\infty} dz_3 \;\; e^{i(x-y)(Pz_3)} \, \mathcal{P}(x, z_3^2)$$

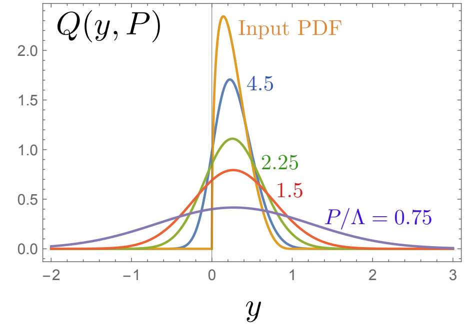
- Try factorized model

$$\mathcal{P}(x, z_3^2) = f(x) I(z_3^2)$$

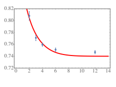
- Popular idea: Gaussian dependence $I(z_3^2) = e^{-z_3^2 \Lambda^2/4}$

$$Q_G^{\rm fact}(y,P) = \frac{P}{\Lambda\sqrt{\pi}} \int_{-1}^{1} dx \, f(x) \, e^{-(y-x)P^2/\Lambda^2}$$

- Take PDF $f(x) = u_v(x) - d_v(x) = \frac{315}{32}\sqrt{x}(1-x)^3\theta(0 \le x \le 1)$ obtained by pseudo-PDF method (Orginos et al. 2017)

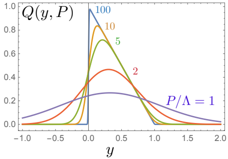


- Curves for $P/\Lambda = 0.75, 1.5, 2.25$ are close to qPDFs obtained by Lin et al (2016), upper momentum $P = 1.3$ GeV, effective $\Lambda \approx 600$ MeV
- Need $P \sim 4.5\,\Lambda \approx 2.7$ GeV to get reasonably close to input PDF
- Note a lot of dirt for negative y, even for $P/\Lambda = 4.5$



Rate of approach

- How do the quasi-PDF curves approach the limiting PDF curve point by point in y?
- Take a simple input PDF $f(x) = 1 - x$ (and Gaussian dependence on $k_\perp$)



Analytic form:

$$Q(y, P) = \frac{1}{2}(1-y)\Big[\text{erf}\,[(1-y)P/\Lambda] + \text{erf}\,[yP/\Lambda]\Big] + \frac{\Lambda}{2\sqrt{\pi}P}\Big[e^{-(1-y)^2P^2/\Lambda^2} - e^{-y^2P^2/\Lambda^2}\Big]$$

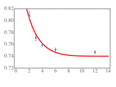
- P-dependence reflects the $k_\perp$-dependence of TMD
- In the middle of the $0 \le y \le 1$ interval

$$Q(1/2, P) = \frac{1}{2} - \frac{\Lambda e^{-P^2/4\Lambda^2}}{\sqrt{\pi}P}\left[1 - \frac{2\Lambda^2}{P^2} - \dots\right]$$

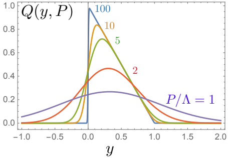
- The approach to the limiting value is $\sim e^{-P^2/4\Lambda^2}$ rather than a powerlike
- For $y = 1$, the approach is like $\sqrt{\Lambda^2/P^2}$

$$Q(1, P) = \frac{\Lambda}{2\sqrt{\pi}P}\left[1 - e^{-P^2/\Lambda^2}\right]$$

rather than like Λ^2/P^2



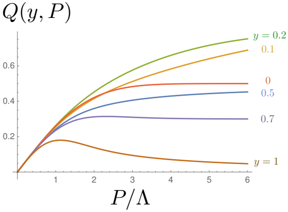
Rate of approach, cont.



Non-analytic behavior with respect to Λ^2/P^2 is present at another end-point as well

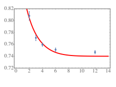
$$Q(0, P) = \frac{1}{2} + \frac{\Lambda}{2\sqrt{\pi}P}\left[1 - 2e^{-P^2/\Lambda^2}\left(1 - \frac{\Lambda^2}{4P^2} - \dots\right)\right]$$

- Quasi-PDF approaches 1/2, average of its 0_+ and 0_- limits of the input PDF



- Curves illustrating P-dependence of quasi-PDFs for particular values of y
- With just three points, at $P/\Lambda = 0.75, 1.5$ and 2.25, it is rather difficult to make an accurate extrapolation to correct $P = \infty$ values

- $k_\perp$ effects generate a very nontrivial TMD-dependent pattern of nonperturbative evolution of the quasi-PDFs $Q(y, P)$
- It cannot be described by a $\mathcal{O}(\Lambda^2/P^2)$ correction on the point-by-point basis in y-variable



Target mass corrections

- All $(\Lambda^2/P^2)^n$ corrections come from $\langle k_\perp^{2n}\rangle_{\mathcal{F}}$ moments of TMD $F(x, k_\perp^2)$
- Statement is based on the ordinary Taylor expansion. In scalar case

$$\phi(0)\phi(z) = \sum_{n=0}^{\infty} \phi(0)(z\partial)^N \phi(0)$$

- Usual statement: $(1/P^2)^N$ terms come from higher twists and target mass corrections (TMCs)
- Expand $(z\partial)^N$ over the combinations $\{z\partial\}^l$ involving traceless tensor $\{z_{\mu_1} \dots z_{\mu_n}\}$

$$\{z\partial\}^l \equiv \{z_{\mu_1} \dots z_{\mu_l}\}\, \partial^{\mu_1} \dots \partial^{\mu_l}$$

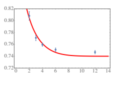
- Obtain twist expansion. In scalar case

$$\phi(0)\phi(z) = \sum_{l=0}^{\infty} \left(\frac{z^2}{4}\right)^l \sum_{N=0}^{\infty} \frac{N+1}{l!(N+l+1)!} \phi(0)\{z\partial\}^N (\partial^2)^l \phi(0)$$

- For matrix elements, combination $\{z\partial\}^N$ translates into

$$\{pz\}^N \equiv z_{\mu_1} \dots z_{\mu_N} \{p^{\mu_1} \dots p^{\mu_N}\}$$

- Take $n = 2$. Then $\{zp\}^2 = (zp)^2 + \frac{1}{4} z^2 M^2$
- Transformation to quasi-PDF converts z^2 into $1/P^2$ which gives M^2/P^2 TMC



Target mass corrections, cont.

- Evident conclusion: TMCs in qPDFs are created “by hand”
- Apply twist decomposition to simplest matrix element, and define $\langle p|\phi(0)\partial^2\phi(0)|p\rangle = \lambda^2\langle p|\phi(0)\phi(0)|p\rangle$

$$\langle p|\phi(0)(z\partial)^2\phi(0)|p\rangle = -\left[(zp)^2 + \frac{1}{4}z^2M^2\right]\langle x^2\rangle_f + \frac{z^2}{4}\lambda^2$$

- Using expression of ME in terms of the TMD

$$\langle p|\phi(0)(z\partial)^2\phi(0)|p\rangle = -(zp)^2\langle x^2\rangle_f + \frac{z^2}{2}\langle k_\perp^2\rangle_{\mathcal{F}}$$

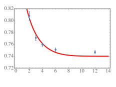
- This gives relation $M^2\langle x^2\rangle_f + \lambda^2 = 2\langle k_\perp^2\rangle_{\mathcal{F}}$
- In explicit form,

$$\langle p|\phi(0)\partial^2\phi(0)|p\rangle = -M^2\int_0^1 dx\, x^2 f(x) + 2\int_0^1 dx\int d^2k_\perp\, k_\perp^2\,\mathcal{F}(x,k_\perp^2)$$

- Simple estimate. Take $f(x) = 4(1-x)^3$, then

$$\frac{M^2}{2}\int_0^1 dx\, x^2 f(x) = \frac{M^2}{30} \approx 0.03\,\mathrm{GeV}^2$$

- More realistic valence PDFs $f(x)$ are singular for $x=0$, and integral is even smaller. For $f(x)\sim(1-x)^3/\sqrt{x}$, it equals to $M^2/66 \approx 0.013\,\mathrm{GeV}^2$
- For Gaussian TMD $\langle k_\perp^2\rangle_G = \Lambda^2 \sim 0.1$ GeV^2 for $\Lambda = 300$ MeV
- Target-mass corrections are much smaller than $k_\perp$ effects



Renormalizable theories and hard term

- In QCD $\mathcal{F}(x, k_\perp^2)$ has $1/k_\perp^2$ hard part and moments $\langle x^{n-2l} k_\perp^{2l} \rangle_{\mathcal{F}}$ diverge
- In the $l = 0$ case, the divergence is logarithmic
- Reflects the perturbative evolution of quasi-PDFs $Q(y, P)$ for large P
- Logarithmic singularity in z_3^2 in coordinate representation. At one loop,

$$\mathcal{M}^{\text{hard}}(\nu, z_3^2) = -\frac{\alpha_s}{2\pi} C_F \ln(z_3^2) \int_0^1 du\, B(u)\, \mathcal{M}^{\text{soft}}(u\nu, 0)$$

- Altarelli-Parisi (AP) evolution kernel

$$B(u) = \left[\frac{1+u^2}{1-u}\right]_+$$

- The function $\mathcal{M}(\nu, \nu^2/P^2)$ that generates the quasi-PDF gets

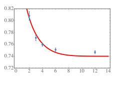
$$\mathcal{M}^{\text{hard}}(\nu, \nu^2/P^2) = -\frac{\alpha_s}{2\pi} C_F \ln(\nu^2/P^2) \int_0^1 du\, B(u) \int_{-1}^1 dx\, e^{-iux\nu} f^{\text{soft}}(x)$$

- Hard part of the quasi-PDF $Q(y, P)$ has a $\ln P^2$ term

$$Q^{\text{hard}}(y, P) = \ln(P^2)\, \Delta(y) + \ldots$$

- It is nonzero in the $-1 \leq y \leq 1$ region only

$$\Delta(y) = \frac{\alpha_s}{2\pi} C_F \int_0^1 \frac{du}{u} B(u) f^{\text{soft}}(y/u)$$



Hard part of quasi-PDF

- $\ln z_3^2$ singularity of the ITD leads to a logarithmic perturbative evolution of the quasi-PDF $Q(y, P)$ for large P
- For TMDs, the $\ln z^2$ behavior translates into large-$k_\perp$ hard tail

$$\mathcal{F}^{\rm hard}(x, k_\perp^2) = \frac{\Delta(x)}{\pi k_\perp^2}$$

- Regularizing $1/k_\perp^2 \to 1/(k_\perp^2 + m^2)$ gives

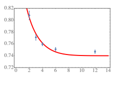
$$\int_{-\infty}^{\infty} \frac{dk_1}{k_1^2 + (x-y)^2 P^2 + m^2} = \frac{\pi}{\sqrt{(x-y)^2 P^2 + m^2}}$$

- Determines the hard part of a quasi-distribution

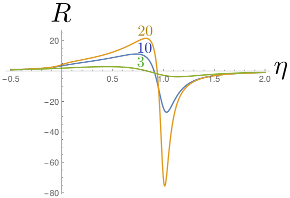
$$Q^{\rm hard}(y, P) = \int_{-1}^{1} dx \, \frac{\Delta(x)}{\sqrt{(x-y)^2 + m^2/P^2}}$$
$$= C_F \frac{\alpha_s}{2\pi} \int_{-1}^{1} \frac{d\xi}{|\xi|} R(y/\xi, m^2/\xi^2 P^2) \, f^{\rm soft}(\xi)$$

- Generating kernel $R(\eta, m^2/P^2)$

$$R(\eta; m^2/P^2) = \int_0^1 du \, \frac{B(u)}{\sqrt{(\eta-u)^2 + m^2/P^2}}$$



Structure of kernel



- Kernel for several values of P/m
- Understand m as IR cut-off $\sim 1/R_{\text{hadr}} \sim 0.5$ GeV
- In the $m/P \to 0$ limit

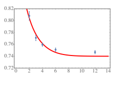
$$\left.\frac{1}{\sqrt{(x-y)^2 + m^2/P^2}}\right|_{m^2/P^2 \to 0} = \left(\frac{1}{|x-y|}\right)_+ + \delta(x-y) \ln\left[4y(1-y)\frac{P^2}{m^2}\right]$$

- $\delta(x-y)$ gives $\ln P^2$ evolution in $-1 \le y \le 1$ region
- Outside $|\eta| \le 1$ region, limit $m/P \to 0$ is finite

$$R(\eta; 0) = \int_0^1 \frac{du}{|\eta - u|} B(u)$$

- Kernel can be written as a series in $1/\eta$,

$$R(\eta;0)|_{\eta>1} = -\sum_{n=1}^{\infty} \frac{\gamma_n}{\eta^{n+1}} \quad , \quad R(\eta;0)|_{\eta<-1} = \sum_{n=1}^{\infty} \frac{\gamma_n}{\eta^{n+1}}$$



Kernel outside central region

$$R(\eta;0)|_{\eta>1} = -\sum_{n=1}^{\infty} \frac{\gamma_n}{\eta^{n+1}} \quad , \quad R(\eta;0)|_{\eta<-1} = \sum_{n=1}^{\infty} \frac{\gamma_n}{\eta^{n+1}}$$

- γ_n are proportional to anomalous dimensions of operators with n derivatives

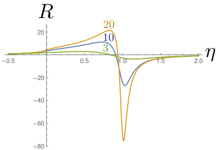
$$\gamma_n = \int_0^1 du\, u^n\, B(u)$$

- $\gamma_0 = 0$, hence the asymptotic behavior for large $|\eta|$ is

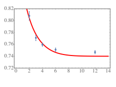
$$R(\eta;0)|_{|\eta|\gg 1} = -\frac{4}{3}\frac{\mathrm{sgn}(\eta)}{\eta^2} + \mathcal{O}(1/\eta^3)$$

- Explicit expression for $m/P = 0$

$$R(\eta;0)|_{\eta>1} = \frac{1+\eta^2}{\eta-1}\ln\left(\frac{\eta-1}{\eta}\right) + \frac{3}{2(\eta-1)} + 1$$



- Realistic value $P/m \sim 3$
- Curve is very far from asymptotic shape
- Neglecting α_s correction is a better approximation than using it in the $m/P = 0$ limit



Subtlety of $P \to \infty$ limit

- Recall the structure of the hard part

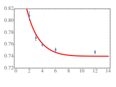
$$Q^{\text{hard}}(y, P) = \int_{-1}^{1} dx \, \frac{\Delta(x)}{\sqrt{(x-y)^2 + m^2/P^2}}$$

$$= C_F \frac{\alpha_s}{2\pi} \int_{-1}^{1} \frac{d\xi}{|\xi|} R(y/\xi, m^2/\xi^2 P^2) \, f^{\text{soft}}(\xi)$$

- Outside $|\eta| < 1$, the kernel has finite $P \to \infty$ limit

$$R(\eta; 0)|_{\eta > 1} = \frac{1+\eta^2}{\eta - 1} \ln\left(\frac{\eta - 1}{\eta}\right) + \frac{3}{2(\eta - 1)} + 1$$

- Even when powers of Λ^2/P^2 may be neglected, quasi-PDFs differ from PDFs
- Shape of $Q(y, P)$ for $y > 1$ is calculable (if PDF is known)
- One should see that lattice gives it, and subtract
- Only then one gets PDF with $|x| \leq 1$ support

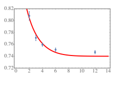


Gauge link complications

- Terms outside $|y| \leq 1$ are generated by $\ln z_3^2$ term
- In QCD, there is one more source of the z^2-dependence of pseudo-ITD: gauge link $\hat{E}(0, z; A)$
- It has specific ultraviolet divergences
- Use Polyakov regularization $1/z^2 \to 1/(z^2 - a^2)$ for gluon propagator in coordinate space
- Effect of the UV cut-off a is similar to that of the lattice spacing
- At one loop, link-related UV singular terms have the structure

$$\Gamma_{\mathrm{UV}}(z_3, a) \sim -\frac{\alpha_s}{2\pi} C_F \left[2\frac{|z_3|}{a} \tan^{-1}\left(\frac{|z_3|}{a}\right) - 2 \ln\left(1 + \frac{z_3^2}{a^2}\right)\right]$$

- For fixed a, these terms vanish when $z_3 \to 0$
- No violation of quark number conservation



Link contribution to quasi-PDFs

- Addition due to UV singular terms

$$Q^{\text{UV}}(y, P) = \int_{-1}^{1} dx\, R^{\text{UV}}(y - x; a)\, f(x)\ ,$$

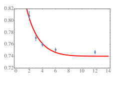
- Kernel $R_{\text{UV}}(y - x; a)$ is given by

$$R^{\text{UV}}(y - x; a) = \frac{P}{2\pi} \int_{-\infty}^{\infty} dz_3\, e^{-i(y-x)Pz_3}\, \Gamma_{\text{UV}}(z_3, a)$$

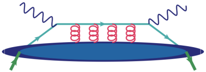
- Take $\ln(1 + z_3^2/a^2)$ "vertex" term. Its Fourier transform gives

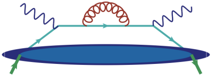
$$R_V(y, x; Pa) \sim -\frac{1}{|y - x|} e^{-|y-x|Pa} - \delta(y - x) \int_{-\infty}^{\infty} \frac{d\zeta}{|y - \zeta|} e^{-|y-\zeta|Pa}$$

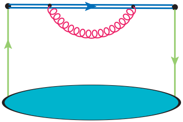
- Taking $a = 0$ gives $\sim 1/|y - x|$ term similar to that appearing in the evolution-related kernel
- However, for $a = 0$ the ζ-integral accompanying the $\delta(y - x)$ term diverges when $\zeta \to \pm\infty$
- Need to keep nonzero a to have the exponential suppression factor that guarantees that $R_V(y, x; Pa)$ is given by a mathematically well-defined expression



Renormalize or exterminate?





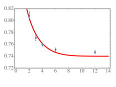


- Structure of factorization for DIS in Feynman gauge
- Gluon insertions generate gauge link $\hat{E}(0, z; A)$
- Quark self-energy diagram is not factorized as $S^c(z) \times \langle AA \rangle$
- Operator $\bar{\psi}(0)\hat{E}(0, z; A)\psi(z)$ should be accompanied by “no AA contractions”
- Link self-energy diagrams and UV-singular parts of vertex diagrams should be excluded together with associated z_3^2-dependence
- It is not sufficient just to subtract UV divergences

- Easy way out: consider reduced pseudo-ITD

$$\mathfrak{M}(\nu, z_3^2) \equiv \frac{\mathcal{M}(\nu, z_3^2)}{\mathcal{M}(0, z_3^2)}$$

- $\mathfrak{M}(\nu, z_3^2)$ has finite $a \to 0$ limit



Reduced Ioffe-time pseudo-distribution

- Reduced pseudo-ITD $\mathfrak{M}(\nu, z_3^2)$ is a physical observable (like, say, DIS structure functions)
- No need to specify renormalization scheme, scale, etc.
- $\mathfrak{M}(\nu, z_3^2)$ is singular in $z_3 \to 0$ limit, $\ln z_3^2$ terms reflect perturbative evolution
- At one loop (with mass-type IR regularization)

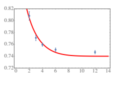
$$\mathfrak{M}(\nu, z_3^2) = \mathfrak{M}^{\rm soft}(\nu, 0) - \frac{\alpha_s}{2\pi} C_F \int_0^1 dw \left\{ \frac{1+w^2}{1-w} \left[\ln\left(z_3^2 m^2 \frac{e^{2\gamma_E}}{4} \right) + 1 \right] + 4 \frac{\ln(1-w)}{1-w} \right\} \left[\mathfrak{M}^{\rm soft}(w\nu, 0) - \mathfrak{M}^{\rm soft}(\nu, 0) \right]$$

- For light-cone PDF, one should take $z^2 = 0$ and use some scheme for resulting UV divergence, say, $\overline{\rm MS}$
- Ioffe-time distribution $\mathcal{I}(\nu, \mu^2)$ is UV scheme and scale dependent

$$\mathcal{I}(\nu, \mu^2) = \int_{-1}^1 dx \, e^{ix\nu} \, f(x, \mu^2)$$

- At one loop (with the same mass-type IR regularization)

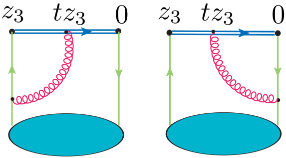
$$\mathcal{I}(\nu, \mu^2) = \mathfrak{M}^{\rm soft}(\nu, 0) - \frac{\alpha_s}{2\pi} C_F \int_0^1 dw \left[\mathfrak{M}^{\rm soft}(w\nu, 0) - \mathfrak{M}^{\rm soft}(\nu, 0) \right] \times \left\{ \frac{1+w^2}{1-w} \ln(m^2/\mu^2) + 2(1-w) \right\}$$



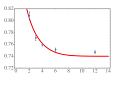
- Writing $\overline{\rm MS}$ ITD in terms of reduced pseudo-ITD

$$\mathcal{I}(\nu,\mu^2) = \mathfrak{M}(\nu, z_3^2) + \frac{\alpha_s}{2\pi} C_F \int_0^1 dw\, \mathfrak{M}(w\nu, z_3^2) \times \left\{ B(w) \left[\ln\left(z_3^2 \mu^2 \frac{e^{2\gamma_E}}{4} \right) + 1 \right] + \left[4 \frac{\ln(1-w)}{1-w} - 2(1-w) \right]_+ \right\}$$

- Altarelli-Parisi kernel $B(w) = \left[(1+w^2)/(1-w)\right]_+$
- Multiplicative scale difference between z^2 and $\overline{\rm MS}$ cut-offs $\mu^2 = 4e^{-2\gamma_E}/z_3^2$
- Simple rescaling relation is modified when all terms are taken into account

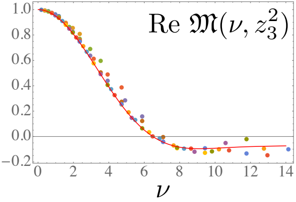


- Term with $[\ln(1-w)]/(1-w)$ produces large negative contribution
- In Feynman gauge, it comes from vertex diagrams
- Gluon is attached to running tz_3 position on the link
- z_3-dependence is generated then by effective scale smaller than z_3



- Exploratory lattice study of reduced pseudo-ITD $\mathfrak{M}(\nu, z_3^2)$ for the valence $u_v - d_v$ parton distribution in the nucleon [Orginos et al. 2017]
- When plotted as function of ν, data both for real and imaginary parts lie close to respective universal curves
- Data show no polynomial z_3-dependence for large z_3 though z_3^2/a^2 changes from 1 to ~ 200
- Apparently no higher-twist terms in the reduced pseudo-ITD
- Real part corresponds to the cosine Fourier transform of $q_v(x) = u_v(x) - d_v(x)$

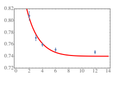
$$\mathfrak{R}(\nu) \equiv \mathrm{Re}\,\mathfrak{M}(\nu) = \int_0^1 dx\, \cos(\nu x)\, q_v(x)$$



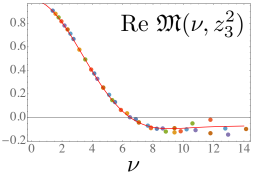
- Overall curve corresponds to the function

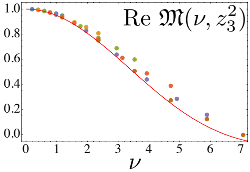
$$f(x) = \frac{315}{32}\sqrt{x}(1-x)^3$$

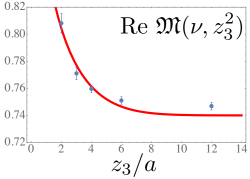
- Obtained by forming cosine Fourier transforms of $x^a(1-x)^b$-type functions and fitting a, b
- Shape is dominated by points with smaller values of Re $\mathfrak{M}(\nu, z_3^2)$



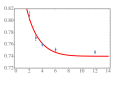
Evolution in lattice data, cont.







- Points corresponding to $7a \le z_3 \le 13a$ values
- Some scatter for points with $\nu \gtrsim 10$
- Otherwise, practically all the points lie on the universal curve based on $f(x)$.
- No z_3-evolution visible in large-z_3 data
- Points in $a \le z_3 \le 6a$ region
- All points lie higher than universal curve
- Perturbative evolution increases real part of the pseudo-ITD when z_3 decreases
- Conjecture that the observed higher values of $\mathfrak{R}$ for smaller-z_3 points may be a consequence of evolution
- z_3-dependence of the lattice points for "magic" Ioffe-time value $\nu = 3\pi/4$
- Shape of eye-ball fit line is $\Gamma(0, z_3^2/30a^2)$
- "Perturbative" $\ln(1/z_3^2)$ behavior for small z_3, rapidly vanishes for $z_3 > 6a$
- $\mathfrak{R}(\nu, z_3^2)$ decreases when z_3 increases



Building $\overline{\text{MS}}$ ITD

- Data show a logarithmic evolution behavior in small z_3 region
- Starts to visibly deviate from a pure logarithmic $\ln z_3^2$ pattern for $z_3 \gtrsim 5a$
- This sets the boundary $z_3 \leq 4a$ on the "logarithmic region"
- "Evolution" part of 1-loop correction

$$\mathcal{I}_R^{\text{ev}}(\nu, \mu^2) = \mathfrak{R}(\nu, z_3^2) + \frac{\alpha_s}{2\pi} C_F \int_0^1 dw\, \mathfrak{R}(w\nu, z_3^2) B(w) \ln\left(z_3^2 \mu^2 \frac{e^{2\gamma_E}}{4}\right)$$

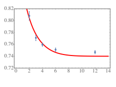
- For $z_3 = 2e^{-\gamma_E}/\mu$, the logarithm vanishes, and we have

$$\mathcal{I}_R^{\text{ev}}(\nu, \mu^2) = \mathfrak{R}(\nu, (2e^{-\gamma_E}/\mu)^2) = \mathfrak{R}(\nu, (1.12/\mu)^2)$$

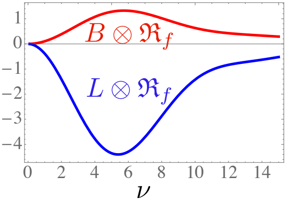
- This happens only if, for some α_s, the $\ln z_3^2$-dependence of the1-loop term cancels actual z_3^2-dependence of the data, visible as scatter in the data
- Fitted value: $\alpha_s/\pi \approx 0.1$
- Remaining part of $\mathcal{I}(\nu, \mu^2)$ is due to corrections beyond the leading log approximation

$$\mathcal{I}_R^{\text{NL}}(\nu) = \frac{\alpha_s}{2\pi} C_F \int_0^1 dw\, \mathfrak{R}_f(w\nu) \left\{ B(w) + \left[4\frac{\ln(1-w)}{1-w} - 2(1-w) \right]_+ \right\}$$

$$\equiv \frac{\alpha_s}{2\pi} C_F \left[B \otimes \mathfrak{R}_f + L \otimes \mathfrak{R}_f \right]$$

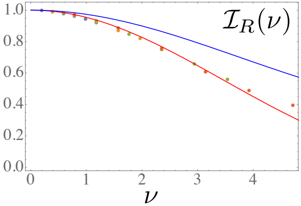


Numerical results



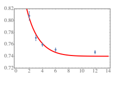
- $L \otimes \mathfrak{R}_f$ is negative and rather large
- In $\nu < 5$ region, $L \otimes \mathfrak{R}_f \approx -3.5 B \otimes \mathfrak{R}_v$
- Combined effect is close to LLA evolution with modified rescaling factor

$$\mathcal{I}_R(\nu, \mu^2) \approx \mathfrak{R}(\nu, (4/\mu)^2)$$

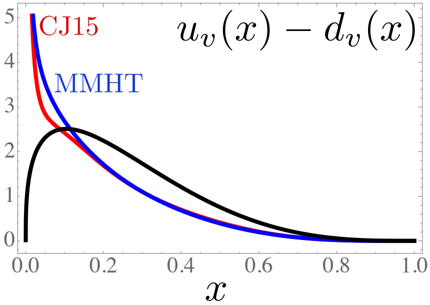


- Actual calculations should be done using “exact” formula
- We choose $\mu = 1/a$ which, at lattice spacing of 0.093 fm is $\approx$ 2.15 GeV
- Using $\alpha_s/\pi = 0.1$ and $z_3 \leq 4a$ data, we generate the points for $\mathcal{I}_R(\nu, (1/a)^2)$
- Upper curve corresponds to the ITD of the CJ15 global fit PDF for μ =2.15 GeV

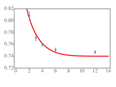
- Evolved points are close to some universal curve with a rather small scatter
- The curve itself corresponds to the cosine transform of a normalized $\sim x^a(1-x)^b$ distribution with $a = 0.35$ and $b = 3$



Numerical results, cont.



- $\sim x^{0.35}(1-x)^3$ PDF compared to CJ15 and MMHT global fits for $\mu = 2.15$ GeV
- Unable to reproduce $\sim x^{-0.5}$ Regge behavior
- Possible reasons: quenched approximation, large pion mass



Summary

- Analyzed nonperturbative structure of quasi-PDFs $Q(y, P)$ using their relation to pseudo-ITDs and TMDs
- Shown that $(\Lambda^2/P^2)^n$ expansion for $Q(y, P)$ involves generalized functions
- Using factorized models for TMDs, studied rate of approach of quasi-PDFs $Q(y, P)$ to PDFs $f(y)$ when $P \to \infty$
- Demonstrated that target-mass corrections are a small part of $k_\perp^2$ corrections artificially singled out from them
- Analyzed perturbative structure of quasi-PDFs using their relation to pseudo-ITDs and TMDs
- Shown that evolution log $\ln z_3^2$ gives $\sim 1/y^2$ behavior of qPDFs for large y
- $\sim 1/y$ terms come from UV singular link-related terms
- Argued that link-related terms should be "exterminated"
- Proposed to use reduced pseudo-ITD
- Studied evolution of exploratory lattice data for reduced pseudo-ITD