Determination of the resonant spin precession frequency using free precession approach

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Introduction.

At the W threshold in FCC-ee the standard resonant depolarization technique meets few serious difficulties.

The first one is that due to large absolute value of the energy spread σ_E=52 MeV at beam energy E=80 GeV it is very difficult to make beam polarized. It was shown that a reasonable self-polarization degree in the order of 5% – 10 % could be expected only near half-integer values of the spin tune ν_0 (see October's 2017 talks of Eliana Gianfelice-Wendt and myself at the EPOL-workshop, CERN).

The second problem arises from the completely unexpected side: polarization does not resonate! My spin tracking simulations have shown that there is no sharp drop of the vertical polarization component when the depolarizer's frequency crosses the resonant value. Such phenomena was observed when the synchrotron tune Q_s was set below 0.075, while the nominal value Q_s =0.023.

Most natural explanation is that in situations with very large average value of the synchrotron modulation index $\xi=\nu_0\sigma_\delta/Q_s$, a beam is subjected to influence of many high order side band synchrotron resonances, which overlap each other and depolarize a beam at any frequency. For reference: ξ=5.22, if ν_0=182, σ_δ=0.00066, Q_s=0.023. Then the strength of the main resonance is proportional to the zero-order Bessel function $J_0(\xi)$= -0.1, while the strengths of few first satellite resonances are proportional to much larger factors: $J_1(\xi)$=-0.35, $J_2(\xi)$=-0.03, $J_3(\xi)$=0.32, $J_4(\xi)$=0.40….

In this note I investigate an alternative approach for determination of the spin precession frequency. An idea is that after a beam became polarized via the Sokolov-Ternov mechanism, say to 10 % -15 % polarization level, the strong AC-magnet named "Flipper" is switched on for few dozens of turns at frequency, which is roughly the expected resonant one. So, flipper will rotate coherently the whole spin ensemble into the horizontal plane or at least to large angle off from the vertical direction. Then flipper is switched off and the polarization vector rotates freely around the vertical axis.

The Compton back-scattering super-polarimeter (discussed during EPOL meeting) will measure the precession frequency by counting the lost energy electrons. My simulations show that coherence of spin precession do not disappear completely until the synchrotron tune is equal or higher than Q_s =0.05.

Compton Polarimeter response at 80 GeV

Polarization asymmetry of the Compton scattering of the circularly polarized laser light (λ=532 nm) at 80 GeV by the longitudinally polarized electrons is very large, reaching 87.4 % at the edge of a spectrum and changes the sign below of 80% of maximal energy loss by an electron, which is ΔEmax=59.25 GeV, see Fig.1.

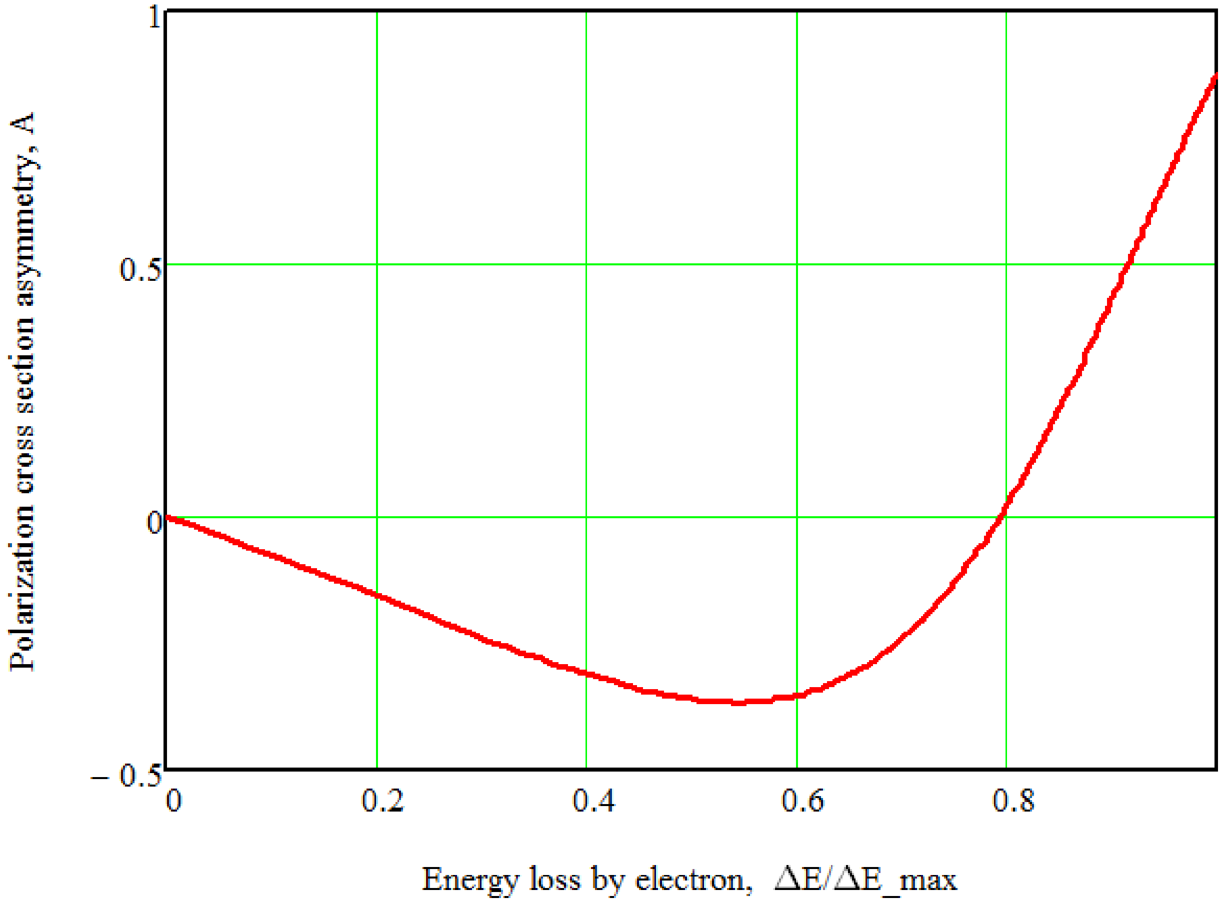

Fig.1 Cross-section asymmetry of the Compton scattering of the circularly polarized laser light on the longitudinally polarized electrons. Beam energy E=80 GeV, laser photon energy ω_0=2.33 eV, $\Delta Emax$=59.25 GeV, $Amax$=0.874, $Amin$= -0.367.

According to Nikolai Muchnoi estimations one can expect 700 scattering events per turn, assuming 1 mJ laser pulse energy and N_e=10^{10} electrons in a bunch. My analysis shows that statistical noise in determination of the one turn value of longitudinal polarization is about σ_{noise}=0.12. This is just comparable with what one can expect for beam polarization in the beginning of free precession run after bringing the polarization vector roughly into the horizontal plane. But due to fast spin decoherence the wanted beam signal will drop down quickly, if the synchrotron modulation index is too large, and this makes very difficult to find its spectrum peak among the noise spectrum continuum. This will be illustrated in the presented below results of spin tracking simulations. But first we shall discuss in some details the spin flipping technique.

Spin deflection by a flipper

Due to extremely large spin tune spread $\Delta\nu=\nu_0\cdot\sigma_\delta=182.5\cdot0.00066=0.12$ a spread of spin directions became very large after their rotation by a flipper into the horizontal plane. So, the precession angle difference $\Delta\phi$ between the nominal energy particle and a particle, whose energy is shifted by $\sigma_\delta=0.00066$, became $\Delta\phi=0.75$ radian, just in one turn! I would like to remind, that the adapted nominal synchrotron period is as large, as 40 turns! So, to rotate spins effectively into the horizontal plane, one should try to do this very quickly, preferably in one turn.

We suggest here to apply somewhere in the arc a pair of vertical kicks, spaced by half-integer tune FODO lattice, interleaved by normal horizontal bends. Such single quasi-local spin-rotator can produce about 6 degrees polarization deflection, not disturbing the betatron motion outside of that short arc section. About 10 such elementary spin-rotators are needed to provide fully coherent rotation of all spins to 60 degrees. Rearranging the sequence of bends and quads one can get few times stronger spin rotation if needed.

Spin tracking results

My code performs the Monte-Carlo simulation of a spin motion of ensemble of particles in presence of fluctuating synchrotron oscillations excited by the energy jumps, which each particle is subjected once per turn. The amplitude of synchrotron oscillations vanishes exponentially during a turn according to the nominal damping time.

In the beginning of each turn all particles are subjected to influence of a spin perturbation. This perturbation could be of any kind – stable rotation around the radial or the longitudinal axis (this we use for simulation of the depolarization due to close orbit distortions), or a rotation by the RF-field at some fractional tune of the revolution frequency (simulates the resonant depolarization). Along a turn all spins are rotated around the vertical axis - each particle according to its average energy during a turn. The betatron motion effects are not considered in the presented here approach, because they are too small in such a machine as FCC-ee.

After bringing the polarization vector into the horizontal plane by the described above strong RF-flipper device, spins ensemble began freely rotate around the vertical axis and only dephasing of their rotation due to large spread of spin precession frequencies will destroy the coherence of this rotation.

Minimal transverse polarization degree will be reached after the odd number of synchrotron half-periods, while all maximums are located at multiples of the synchrotron period. An example of such spins ensemble behavior is presented at Fig.2 for beam energy E=45 GeV.

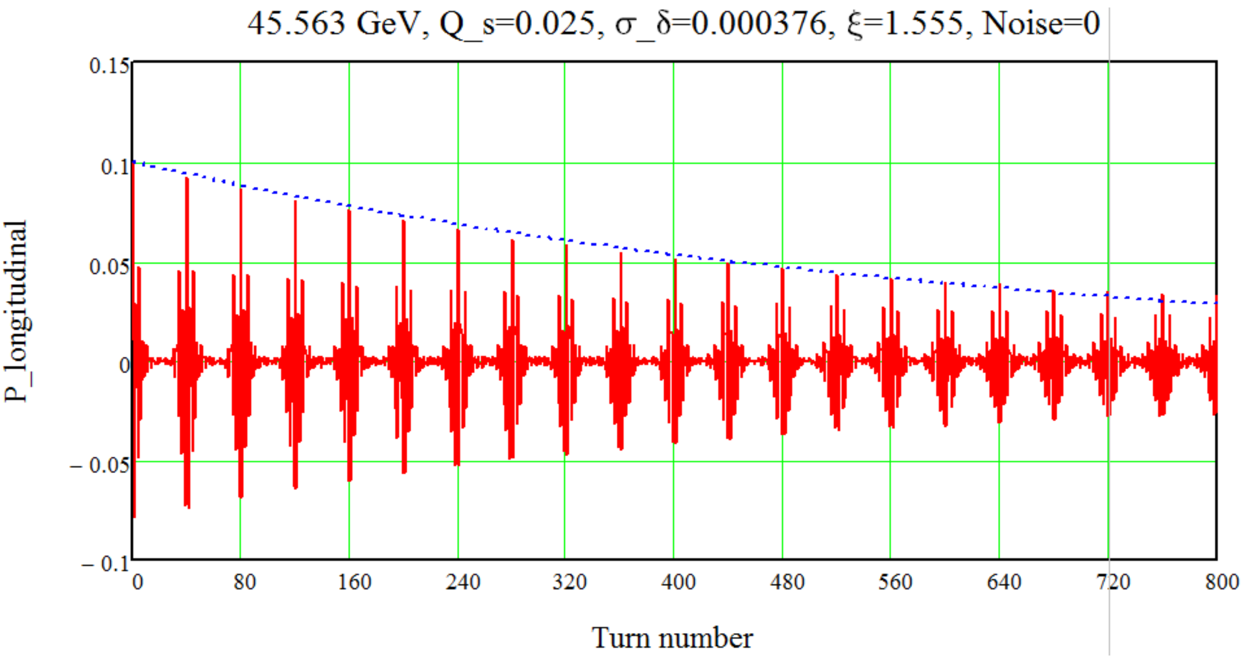

Fig.2 Turn by turn plot for the longitudinal polarization component for beam energy E=45.563 GeV. One can see polarization echoes at integer numbers of the synchrotron periods – each 40 turns. The dotted line is the exponential fit with τ=640 turns, which describes the long time decoherence.

Both, the short term and the long term decoherence, became extremely fast, if the average value of the synchrotron modulation index $\xi=\nu_0\cdot\sigma_\delta/Q_s$ is too high ($\xi > 1$). One can see on the Fig.2 a sequence of short pulses and dips in between repeated with the synchrotron period. Also my simulations show that amplitude of polarization echoes drops down with a rate proportional to ξ^2 (dotted line) and τ=2550 turns for Q_s=0.05. This is four times longer decay, than which was observed for the case with Q_s=0.025!

The corresponding FFT spectral plot for a sampling length of N=2048 turns and Np=4000 particles is presented on the Fig.3 as it would be obtained by the ideal longitudinal polarimeter, without statistical noise.

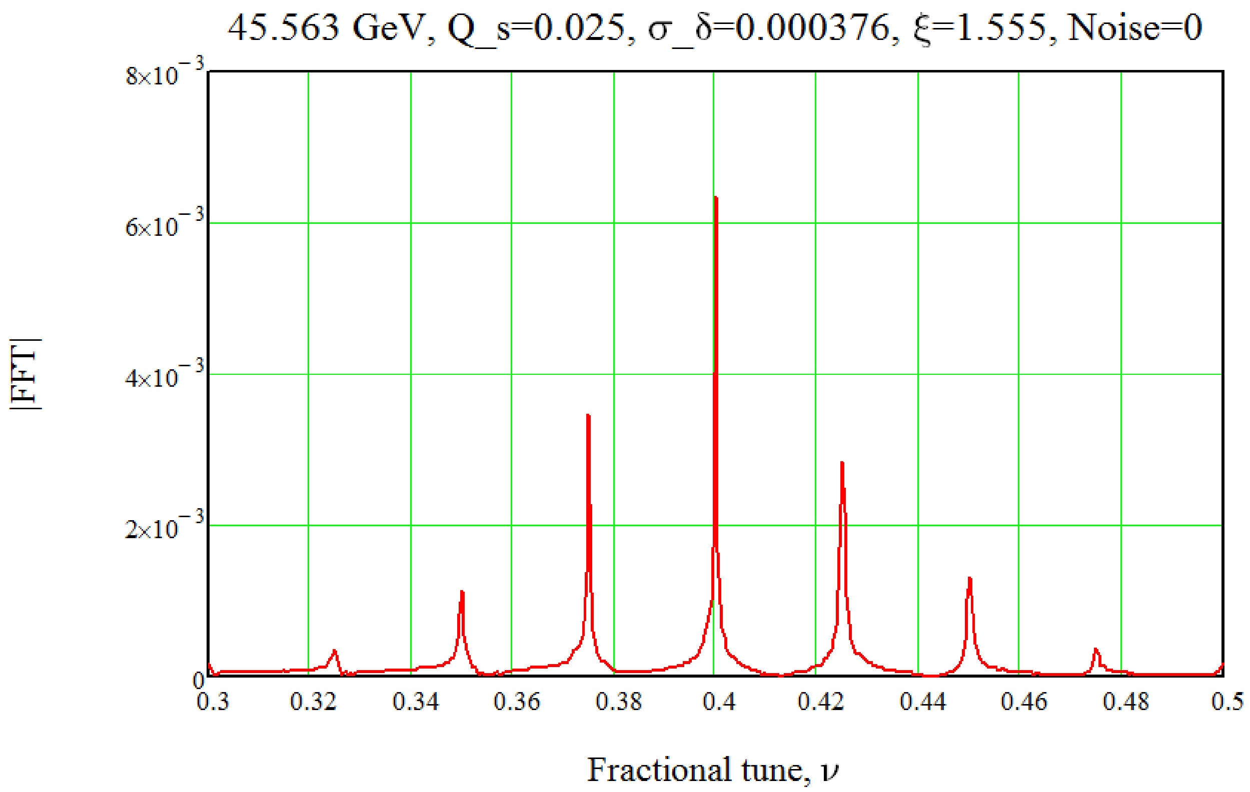

Fig.3 Fast Fourier Transform (FFT) for beam energy E=45 GeV, Q_s=0.025, N=2048 turns, Np=4000 particles, as it will be recorded by the ideal longitudinal polarimeter (means - no statistical noise in measuring of the polarization level).

After adding of statistical noise with σ_{noise}=0.08 to a plot on the Fig.2 one sees some chaotic picture (see Fig.4), but the spectral analysis presented on the Fig.5 reveals the presence of a wanted spectral line ν=0.400.

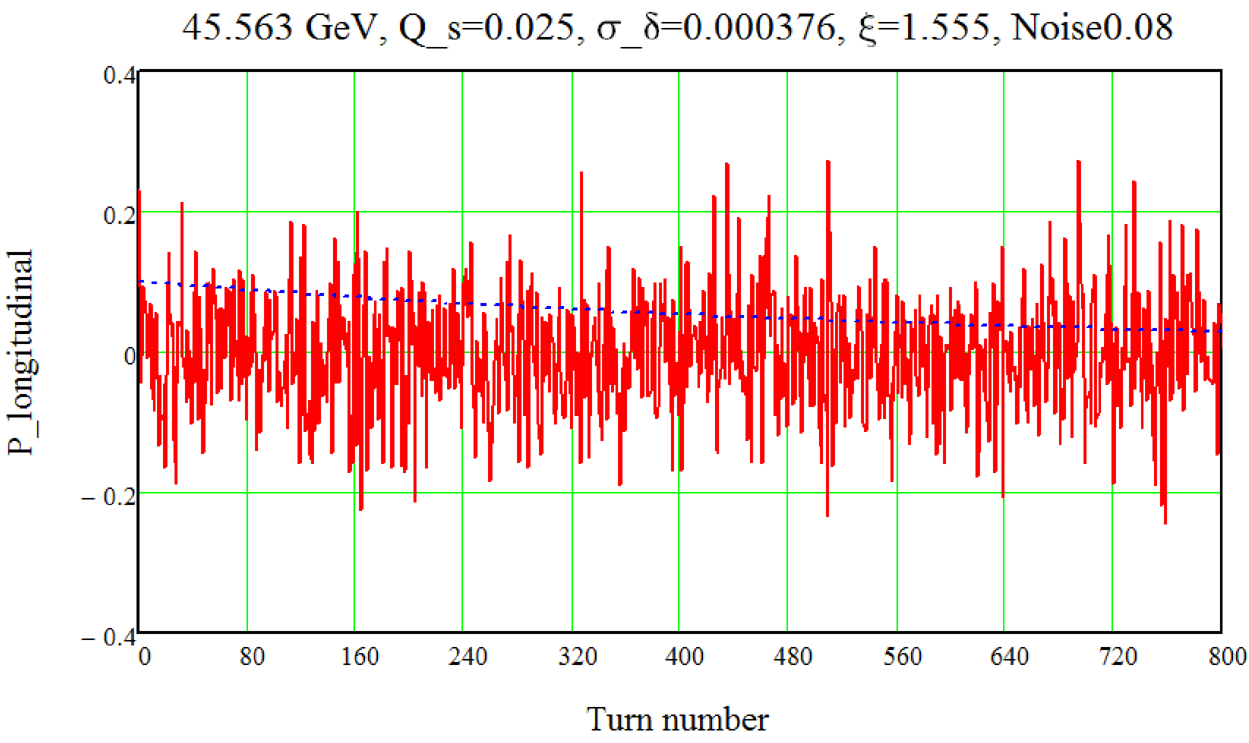

Fig.4 Turn by turn plot of the longitudinal polarization component for a beam energy E=45 GeV with polarimeter statistical noise σ_{noise}=0.08 added to a signal.

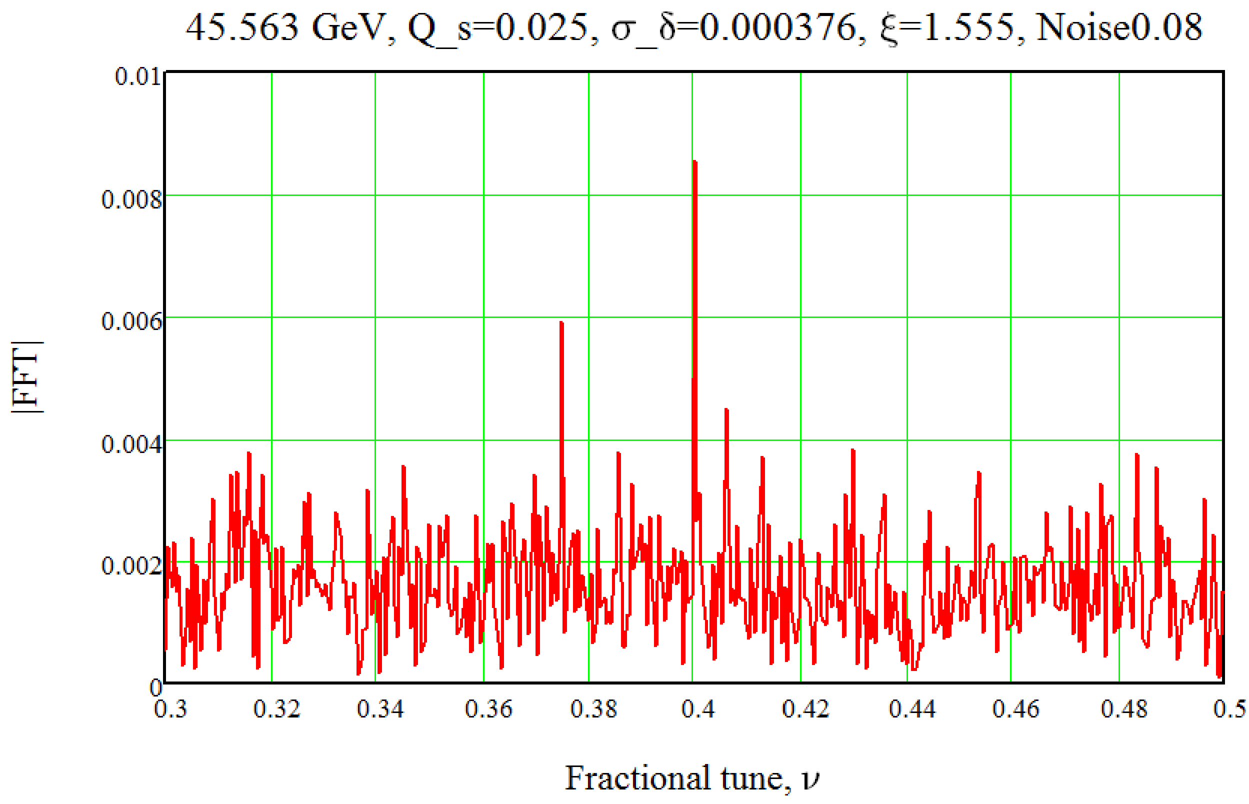

Fig.5 Fast Fourier Transform for a beam energy E=45.563 GeV, Q_s=0.025, N=2048 turns, Np=4000 particles. A polarimeter statistical noise σ_{noise}=0.08 is added to a signal. Still the wanted signal peak ν=0.400 is seen here almost at a right position.

On the Fig.6 is presented the so called NAF scan - the correlation between the signal shown on the Fig.4 and the harmonic reference signal with different probing frequency. From this plot one can extract the precession frequency with high enough precision.

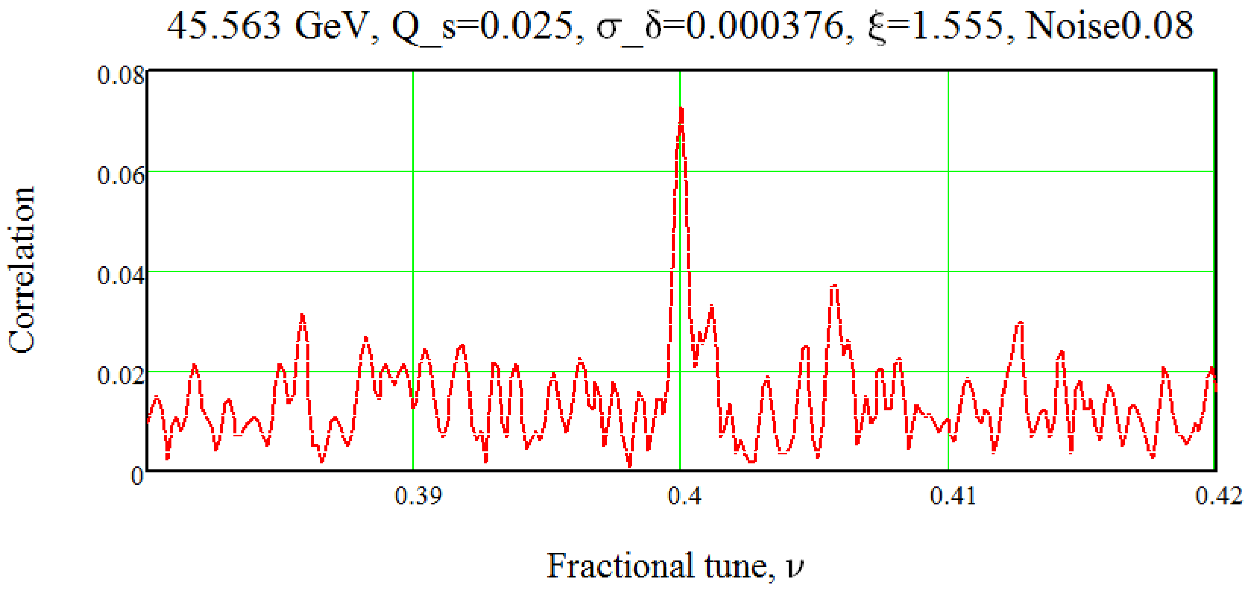

Fig.6 NAF scan with much smaller frequency step than given by the FFT analysis.

Spin tracking results for beam energy E=80 GeV.

For beam energy E=80 GeV decoherence of the spin ensemble becomes very fast. So, the long term decrease of the polarization echoes for Qs=0.075 proceeds with τ=100 turns, only! See turn by turn the longitudinal polarization dependence on the Fig.7.

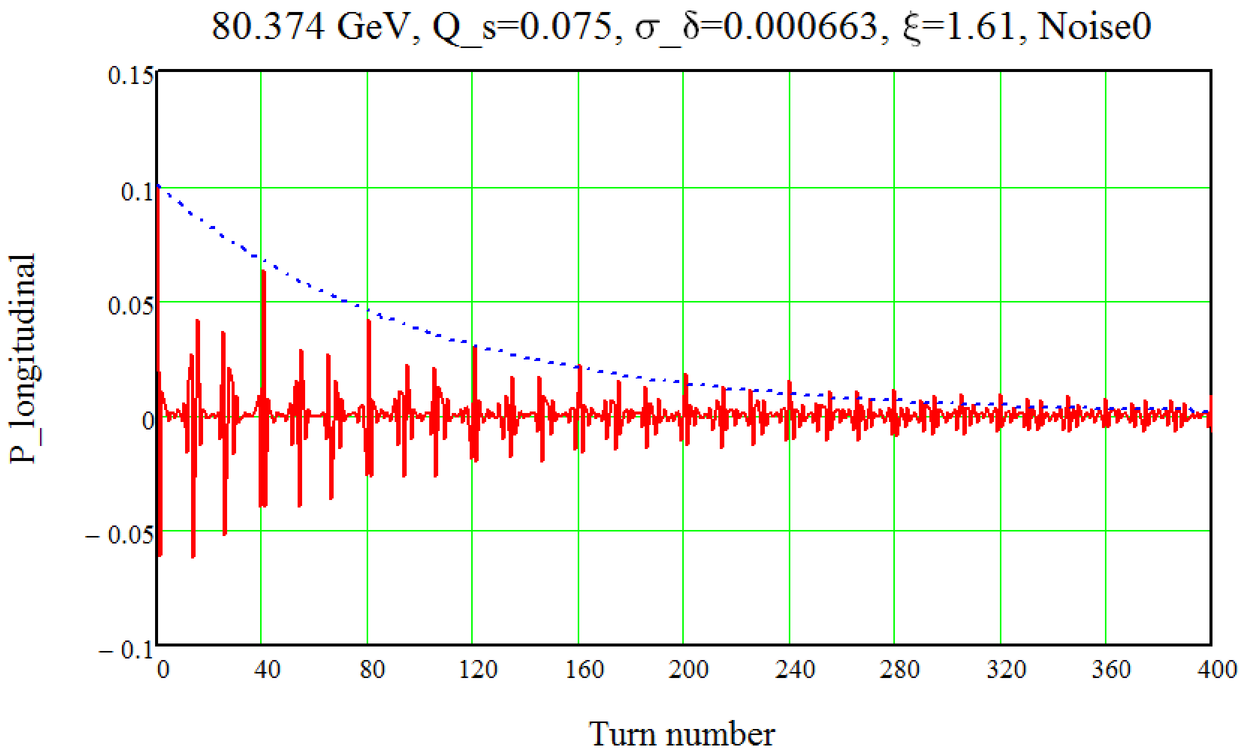

Fig.7 Spin precession echoes for the case E=80 GeV, Q_s=0.075 and σ_δ=0.000663. No any statistical noise is added here. Free precession starts from a level P=0.1. The long decoherence time is equal to τ=100 turns, only.

Not to obscure fully by a white noise the precession frequency signal at ν=0.400 one should use a polarimeter with extremely small statistical noise, with a sigma σ_{noise}=0.02, see Fig.8.

Somewhat better results one can achieve with the increased synchrotron tune, say to a level Q_s=0.1. Then the long decoherence time increases to τ=145 turns and the allowed statistical noise could be as high as σ_{noise}=0.05. And, conversely, the decoherence time decreases to τ=40 turns, if Q_s=0.05. The last case I consider as completely unacceptable – too low quality factor of that oscillator!

One also can make FFT transform of a shorter record. This helps but not too much, because the frequency step of FFT spectrum becomes bigger.

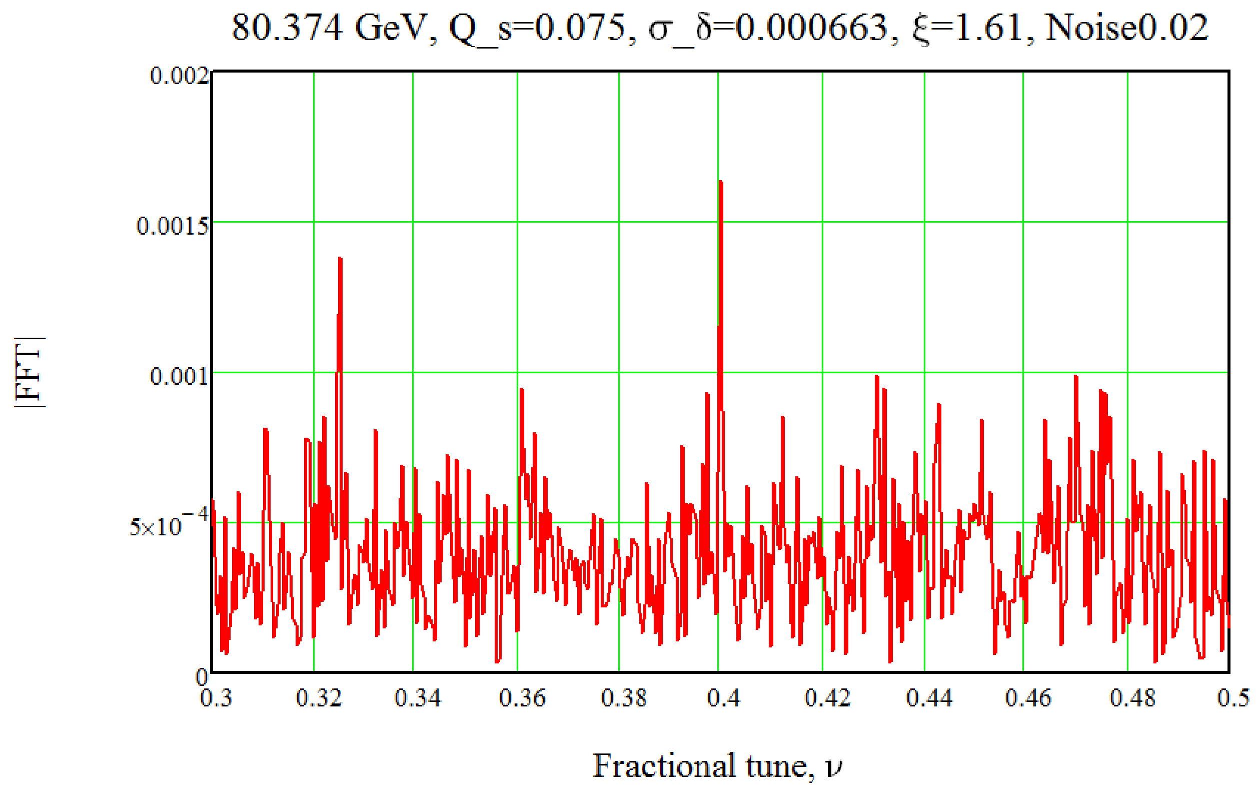

Fig.8 5 Fast Fourier Transform for a beam energy E=80.374 GeV, Q_s=0.075, N=2048 turns, Np=4000 particles, σ_{noise}=0.02. The precession peak is located at ν=0.4. A synchrotron side band at ν=0.325 is clearly visible, but not so at ν=0.475.

Discussion

Below I had combined in one Table all the results discussed above. The last column describes the statistical requirements to a polarimeter, which should be satisfied to be able to extract the signal frequency from a noise. In all cases I had assumed that spins free precession starts from a value P=0.1, which we hope to achieve after fast spin rotation by a powerful flipper.

At 45 GeV I do not see any problem, even with the extremely low Qs=0.025. Still Nikolai Muchnoi should confirm my very rough estimations for the analyzing power of a polarimeter. But at 80 GeV the situation looks not so good. The statistical noise of the longitudinal polarization measurements can completely obscure the wanted signal and the lowest acceptable value of the synchrotron tune is Qs=0.075. Here I would like remind that according to my previous simulations the lowest acceptable Qs value at 80 GeV for standard Resonance Depolarization method is the same. So, no essential advantage of the free precession technique in comparison with RD? Both approaches suffer from the same problem – too high synchrotron modulation index! Still, free precession method is much faster, it takes only 1 second to perform a measurement.

Beam ene2rgy, E (GeV)	Spin tune, ν_0	Relative energy spread, σ_δ	Synchro-tron tune, Q_s	Modulation index $\xi = \nu_0\,\sigma_\delta / Q_s$	Spin De-coherence time, τ (turns)	Statistical noise limit, σ_{noise}
45.563	103.4	0.000376	0.025	1.555	640	0.08
45.563	103.4	0.000376	0.050	0.777	2560	0.15
45.563	103.4	0.000376	0.075	0.518	5760	0.50
80.374	182.4	0.000663	0.050	2.419	40	0.01
80.374	182.4	0.000663	0.075	1.613	100	0.02
80.374	182.4	0.000663	0.100	1.210	145	0.05

Final remarks and conclusion.

On my opinion polarization is a complementary tool for beam energy control. Main job should be performed by few super-polarimeters-spectrometers, placed nearby detectors and may be near RF-sections. We discussed these devises based on Compton scattering at the EPOL meeting in October 2017. Potentially such spectrometers can provide the needed accuracy of energy measurement with a systematics at the required level $\Delta E/E=10^{-6}$. If so, the polarization is needed only to strengthen our understanding of the global saw-tooth picture. This can be done during special commissioning runs with the relaxed FF-optics and arcs tuned to much higher values of momentum compaction α_p.

The calibration of these local spectrometers could be done at 30 GeV or even at 20 GeV, where SR losses are small and saw-tooth uncertainties are negligible. At such low energies the polarizing wigglers should be powered to much higher fields, of course. And the laser wave length should be chosen as shorter as possible. Then the spread of x-coordinates on the detector plate of a Compton spectrometer will became almost the same as at higher beam energy but with longer wave length of a laser. So, possible geometrical distortions of the detected particles distribution (ideally ellipse) will become the same.

In any case, we can say that two approaches – the resonant depolarization method and the free precession technique – both can be used for beam average energy measurement up to 80 GeV, but with some difficulties. Hope, that very fast progress in the laser technique will remove these difficulties at the time when FCC-ee will start the commissioning.