

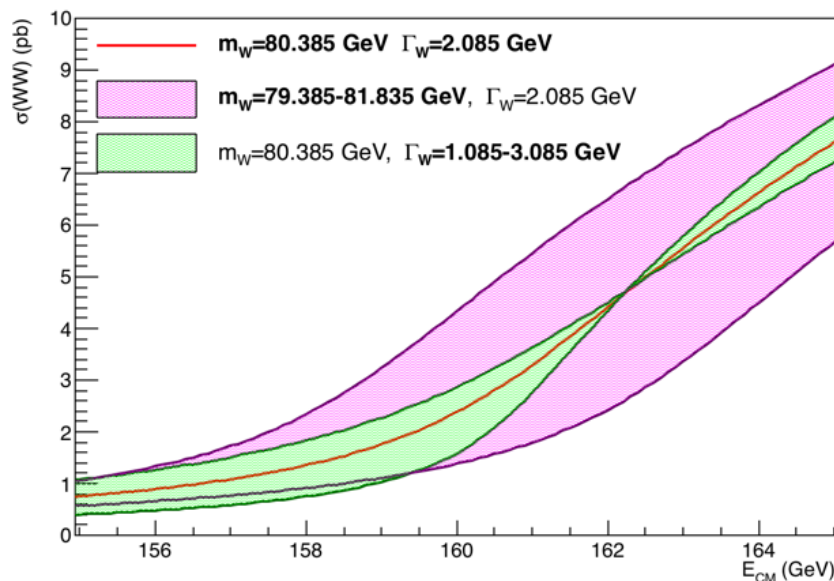
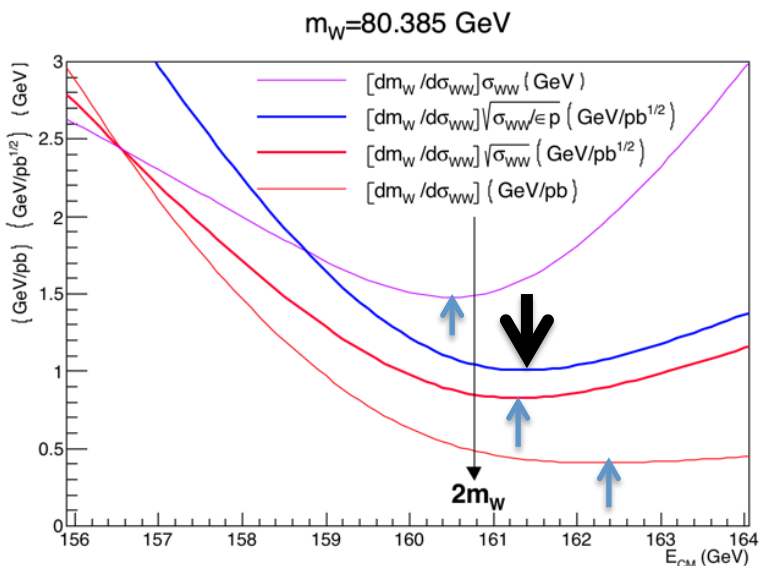
# Effect of energy spread on the W mass and width determination



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# $m_W$ and $\Gamma_W$ from $\sigma_{WW}$

$\sigma_{WW}$  with YFSWW3 1.18



**For  $m_W$  determination only**  
**max stat sensitivity at  $\sqrt{s} \sim 2m_W + 600$  MeV**  
*statistical precision*  
 with  $L = 8/ab \rightarrow \Delta m_W \approx 0.40$  MeV

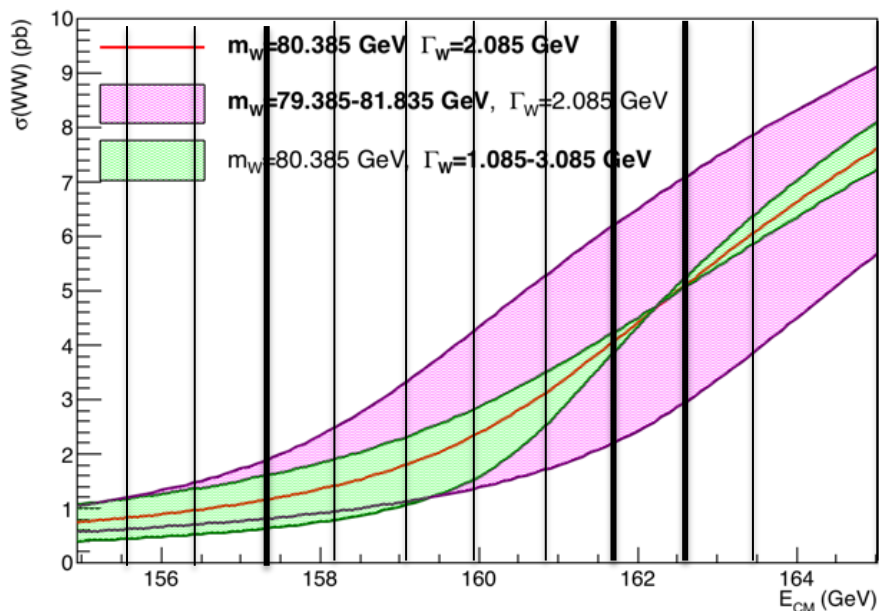
Measure  $\sigma_{WW}$  in two energy points  $E_1, E_2$   
 with a fraction  $f$  of lumi in  $E_1$   
 $\rightarrow$  determine both  $m_W$  &  $\Gamma_W$

$d\sigma_{WW}/d\Gamma_W = 0$  at  $E_{CM} \sim 162.3$  GeV  $\sim 2m_W + 1.5$  GeV

# optimal energy points

limiting data taking points to

$$E_{CM} = (2n+1) * 0.4406486 \text{ GeV}$$

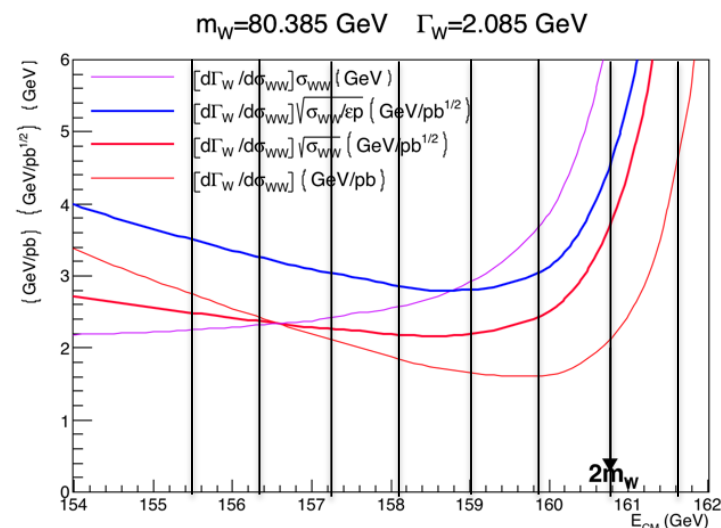
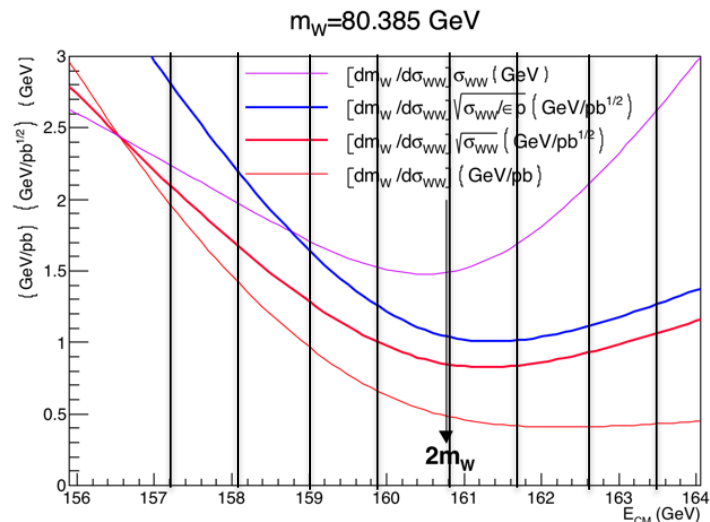


min  $\Delta m_W$  with  $E_0=161.7 \text{ GeV}$   $\Delta m_W=0.4 \text{ (MeV)}$

min  $\Delta m_W + \Delta \Gamma_W$

with  $E_1=157.3 \text{ GeV}$   $E_2=162.6 \text{ GeV}$   $f=0.4$

$\Delta m_W=0.65$   $\Delta \Gamma_W=1.6$   $\Delta m_W=0.55 \text{ (MeV)}$



# E spread effects on $\sigma_{WW}$

Gaussian Energy spread smearing

$$\sigma'_{WW}(E_0) = \int \sigma_{WW}(E) G(E - E_0) dE$$

$$G(E) = \frac{1}{\sigma_E \sqrt{2\pi}} \exp\left(-\frac{E^2}{2\sigma_E^2}\right)$$

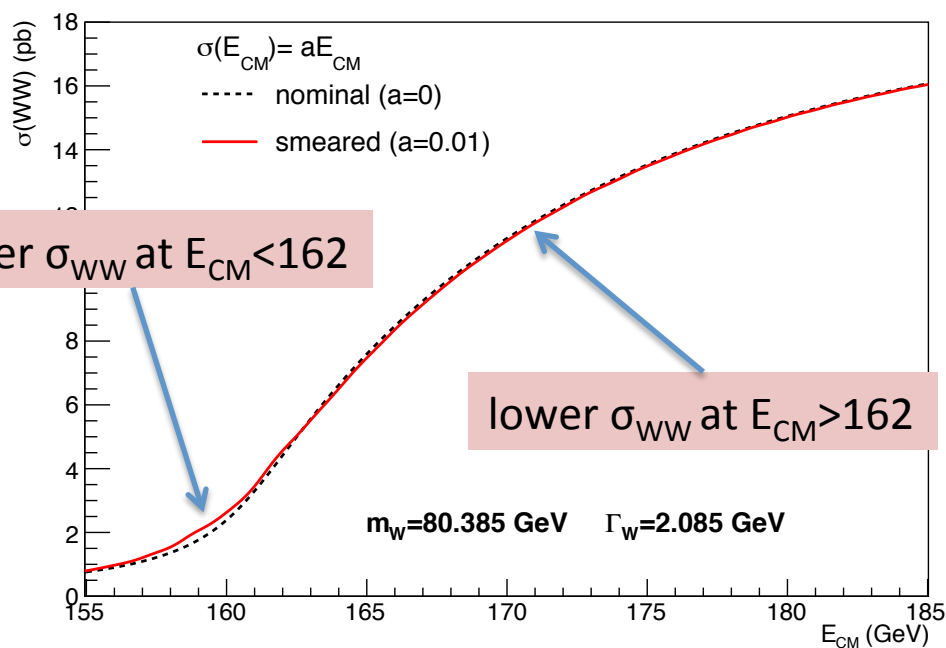
$$\sigma_E = aE$$

$$a = (0.47 - 1.10) \cdot 10^{-3}$$

$$\rightarrow \sigma_E = 75 - 180 \text{ MeV @ } E_{CM} = 161 \text{ GeV}$$

example with  $a = 10^{-2}$

effect of  $E_{CM}$  spread on  $\sigma_{WW}$



crossing point near the  $d\sigma_{WW}/\Gamma_W = 0$  point

# E spread effects on $\sigma_{WW}$

Gaussian Energy spread smearing : Taylor expansion

$$\sigma_{WW}(E) = \sigma_{WW}(E_0) + \frac{d\sigma_{WW}}{dE}(E - E_0) + \frac{1}{2!} \frac{d^2\sigma_{WW}}{dE^2}(E - E_0)^2 + \frac{1}{3!} \frac{d^3\sigma_{WW}}{dE^3}(E - E_0)^3 + \frac{1}{4!} \frac{d^4\sigma_{WW}}{dE^4}(E - E_0)^4 + \dots$$

$$\sigma'_{WW}(E_0) = \int \sigma_{WW}(E) G(E - E_0) dE = \sigma_{WW}(E_0) + \frac{1}{2} \frac{d^2\sigma_{WW}}{dE^2} \sigma_E^2 + \frac{1}{8} \frac{d^4\sigma_{WW}}{dE^4} \sigma_E^4 + \dots$$

$$\int E^2 G(E) dE = \sigma_E^2$$

$$\int E^4 G(E) dE = 3\sigma_E^4$$

For  $E_{CM} = 161.7$  GeV and  $a = 10^{-3}$  spread

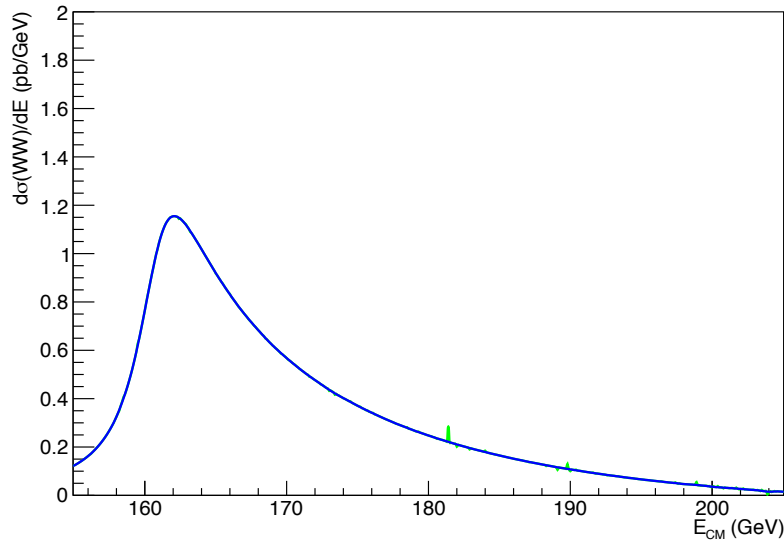
$$d^2\sigma_{WW}/dE^2 \approx +0.07 \text{ pb/GeV}^2 \rightarrow \Delta\sigma_{WW} \approx +0.9 \text{ fb}$$

$$d^4\sigma_{WW}/dE^4 \approx +0.09 \text{ pb/GeV}^4 \rightarrow \Delta\sigma_{WW} \approx +0.008 \text{ fb (negligible)}$$

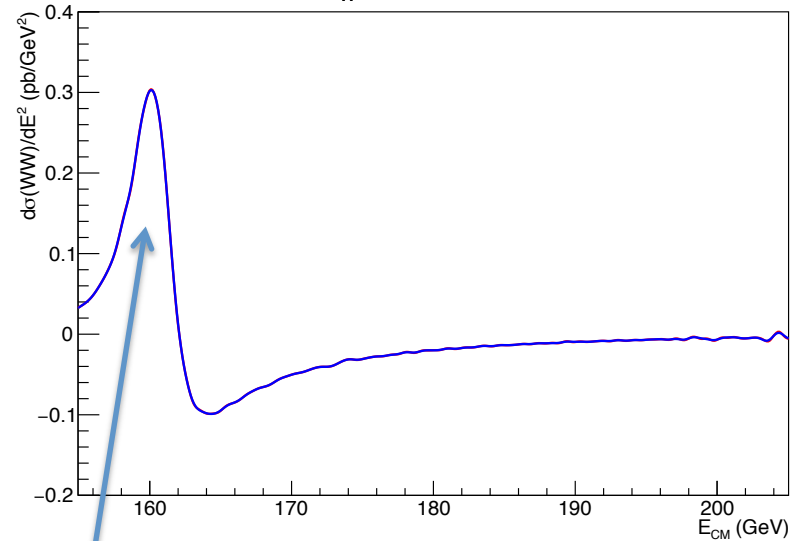
effect on  $\sigma_{WW}$  is dominated by value of second derivative  $d^2\sigma_{WW}/dE^2$

# E spread effects on $\sigma_{WW}$

$d\sigma(WW)/dE$  ( $m_W=80.385$  GeV  $\Gamma_W=2.085$  GeV)



$d^2\sigma(WW)/dE^2$  ( $m_W=80.385$  GeV  $\Gamma_W=2.085$  GeV)



maximum effect at  $E_{CM}=160.1$  GeV

# E spread effect on $m_W$ & $\Gamma_W$

$$\sigma(E_{CM}) = (0.47-1.10) 10^{-3} E_{CM}$$

Single point @  $E_{CM} = 161.7$  GeV

$$\rightarrow \Delta\sigma_{WW} = +(0.2-1.1) \text{ fb}$$

$$\rightarrow \Delta m_W = -(0.08-0.46) \text{ MeV}$$

Optimal  $m_W$  &  $\Gamma_W$  points @  $E_{CM} = 157.3$  &  $162.6$  GeV

$$\rightarrow \Delta\sigma_{WW} = +(0.24-1.3) \text{ fb} \text{ \& } = -(0.18-1.0) \text{ fb}$$

$$\rightarrow \Delta m_W = -(0.09-0.48) \text{ MeV}$$

$$\rightarrow \Delta\Gamma_W = +(0.6-3.3) \text{ MeV}$$

# Conclusions

- The WW threshold  $\sigma$  lineshape is a great opportunity to measure both  $m_W$  and  $\Gamma_W$  at the sub-MeV level with FCCee
  - optimal points to take data are  $\sqrt{s}=2m_W+1.5$  GeV ( **$\Gamma$ -insensitive**) and  $\sqrt{s}=2m_W-2-3$  GeV ( **$-\Gamma$ off shell**)
  - limiting data taking ECM points to half-integer spin tunes will bring a limited degradation to the optimal stat sensitivity.
- Beam energy spread effects at the level of  $\Delta E_{CM}/E_{CM} \approx 10^{-3}$  i.e.  $\Delta E_{CM} \approx 160$  MeV on the  $\sigma_{WW}$  lineshape are very small, yet they yield sizeable effects on the extracted  $m_W$  and  $\Gamma_W$  avalues, at the level of the expected statistical precision with  $L=8/\text{ab}$  of data.
- Maximum effects are at the level of  $\Delta m_W(\text{stat})$  and  $2x \Delta \Gamma_W(\text{stat})$  so that control on the beam energy RMS  $<50\%$  is required to avoid additional syst contributions from this source



# $m_W$ & $\Gamma_W$ from $\sigma_{WW}$

Uncertainty propagation

$$\begin{cases} \Delta\sigma_1 = a_1\Delta m + b_1\Delta\Gamma \\ \Delta\sigma_2 = a_2\Delta m + b_2\Delta\Gamma \end{cases}$$

$$\begin{aligned} a_1 &= \frac{d\sigma_1}{dm} & b_1 &= \frac{d\sigma_1}{d\Gamma} \\ a_2 &= \frac{d\sigma_2}{dm} & b_2 &= \frac{d\sigma_2}{d\Gamma} \end{aligned}$$

$$\Delta m = -\frac{b_2\Delta\sigma_1 - b_1\Delta\sigma_2}{a_2b_1 - a_1b_2}$$

$$\Delta\Gamma = \frac{a_2\Delta\sigma_1 - a_1\Delta\sigma_2}{a_2b_1 - a_1b_2}$$

$\Delta m, \Delta\Gamma$  linear correlation with uncorrelated  $\Delta\sigma_1, \Delta\sigma_2$

$$r = -\frac{1}{\Delta m \Delta\Gamma} \frac{a_2b_2\Delta\sigma_1^2 + a_1b_1\Delta\sigma_2^2}{(a_2b_1 - a_1b_2)^2}$$