Weak Gravity Conjecture from Black Hole Entropy

Clifford Cheung

CC, Liu, Remmen (1801.08546)
CC, Remmen (1407.7865)
CC, Remmen (1402.2287)

weak gravity conjecture (WGC)

A U(1) gauge theory consistently coupled to gravity requires a charged state with

$$q \geq m/m_{\mathrm{Pl}}$$

paraphrased by the slogan

"Gravity is the weakest force."

Arkani-Hamed, Motl, Nicolis, Vafa (2006)

circumstantial evidence #1

The WGC is automatically satisfied in many examples in string theory.

The WGC is natural in QFT. For example,

$$g \geq m/m_{\rm Pl} \xrightarrow{(m_W = gv)} m_{\rm Pl} \geq v$$

in gauge theory with SU(2) → U(1) breaking.

circumstantial evidence #2

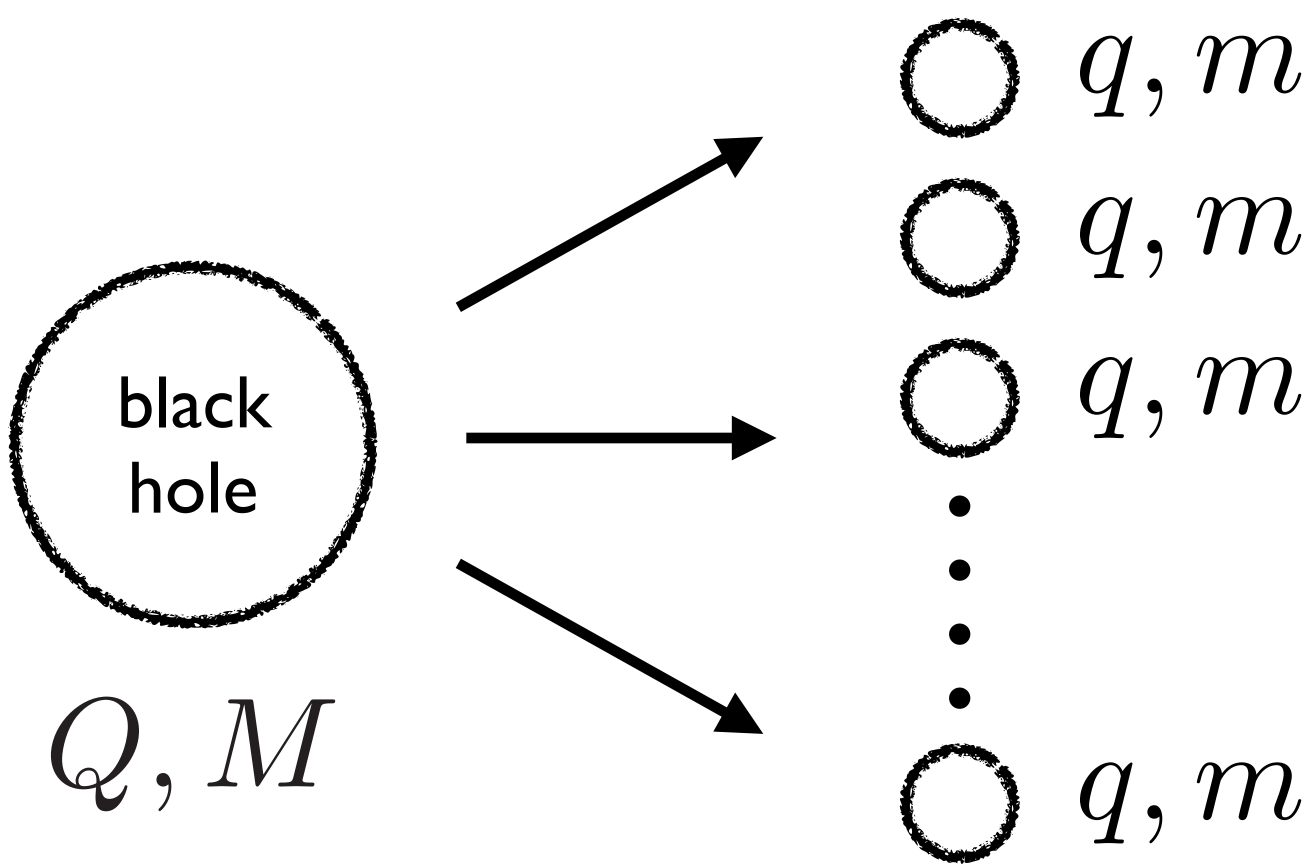
Without the WGC, the $q \to 0$ limit induces an exact global symmetry.

Exact global symmetries are suspect due to black hole no-hair theorems.

The WGC obstructs the global limit by placing a lower bound on q.

circumstantial evidence #3

Consider the decay of a charged black hole.



number of particles in final state = Q/q conservation of charge

total rest mass in final state = $mQ/q < M$ conservation of energy

For an extremal black hole, $Q = M/m_{\rm Pl}$, so

$$q > m/m_{\rm Pl}$$

Satisfaction of WGC implies that extremal black holes can decay.

Violation of WGC implies that extremal black holes are stable remnants.

These might be pathological:

- thermodynamic issues???
- tension with holography???

Question: is a theory that violates the WGC

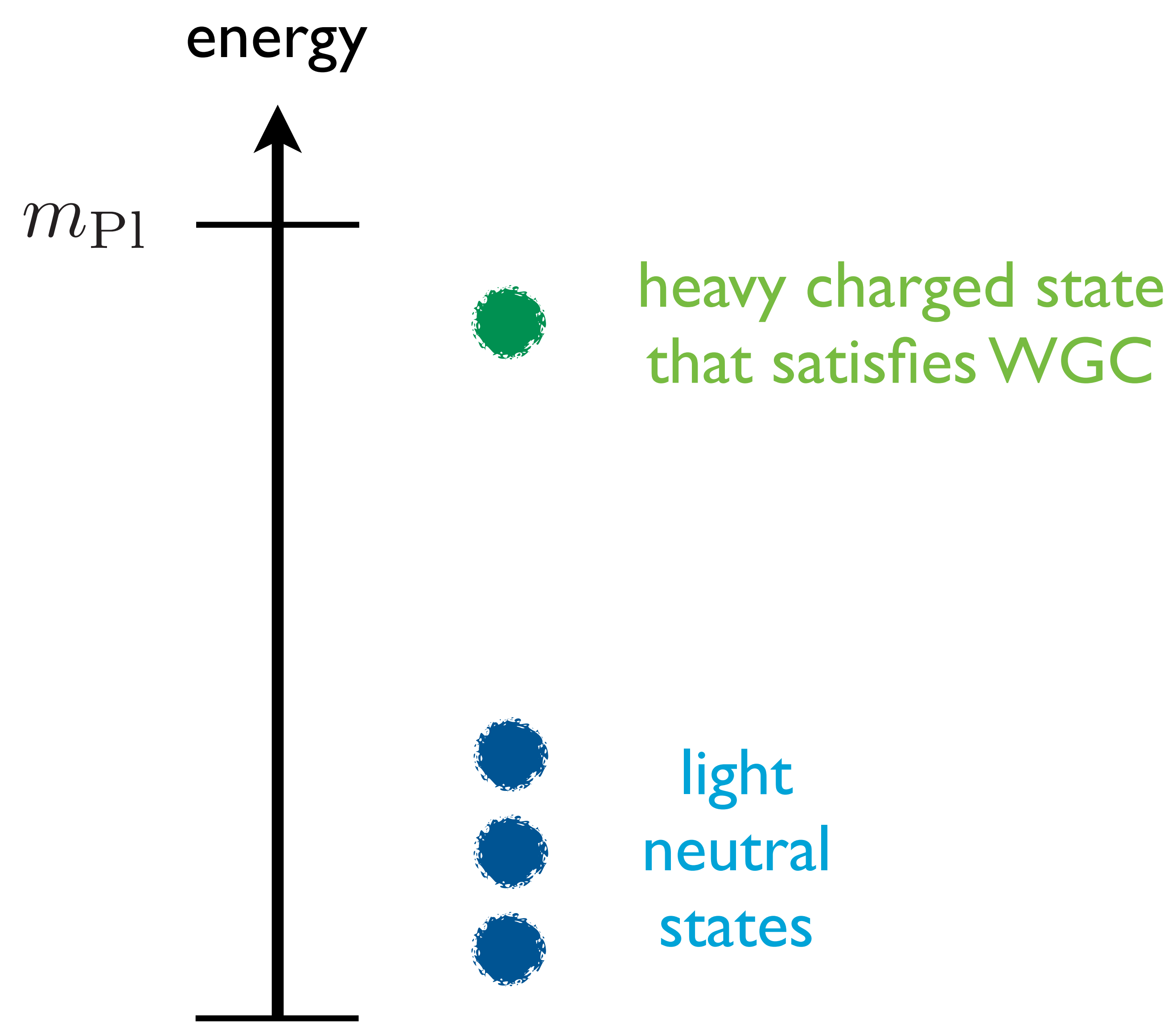
- in the swampland (diagnose in UV)

OR

- merely pathological? (diagnose in IR)

Many properties of EFTs that arise from consistent UV completion are in actuality dictated by IR constraints: anomaly cancellation, charge quantization, positivity.

Can WGC violation yield IR pathologies?



Studying the IR spectrum is not enough!

We consider Einstein-Maxwell theory plus higher dimension operator corrections.

$$\Delta\mathcal{L} = \sum_{I=1}^{8} c_i \mathcal{O}_i$$

$$\mathcal{O}_7 = F_{\mu\nu}F^{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}$$

$$\mathcal{O}_8 = F_{\mu\nu}F^{\nu\rho}F_{\rho\sigma}F^{\sigma\mu}$$

$$\mathcal{O}_1 = R^2$$

$$\mathcal{O}_4 = RF_{\mu\nu}F^{\mu\nu}$$

$$\mathcal{O}_2 = R_{\mu\nu}R^{\mu\nu}$$

$$\mathcal{O}_5 = R_{\mu\nu}F^{\mu\rho}F^{\nu}{}_{\rho}$$

$$\mathcal{O}_3 = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$$

$$\mathcal{O}_6 = R_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$$

A physical bound must be invariant under a change of field basis.

$$g_{\mu\nu} \to g_{\mu\nu} + r_1 g_{\mu\nu} R + r_2 R_{\mu\nu} + r_3 g_{\mu\nu} F^2 + r_4 F^2_{\mu\nu}$$

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This shifts the operator coefficients by

$$c_1 \to c_1 - r_2 - r_2/2$$

$$c_2 \to c_2 + r_2$$

$$c_3 \to c_3$$

$$c_4 \to c_4 + r_2/4 - r_3 - r_4/2$$

$$c_5 \to c_5 - r_2 + r_4$$

$$c_6 \to c_6$$

$$c_7 \to c_7 + r_4/4$$

$$c_8 \to c_8 - r_4$$

There are only four field basis invariants :

$$c_0 \equiv c_2 + 4c_3 + c_5 + c_6 + 4c_7 + 2c_8$$

$$c_3$$

$$c_6$$

$$c_9 \equiv c_2 + c_5 + c_8$$

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Any physical bound should be a function of these combinations of coefficients, e.g.

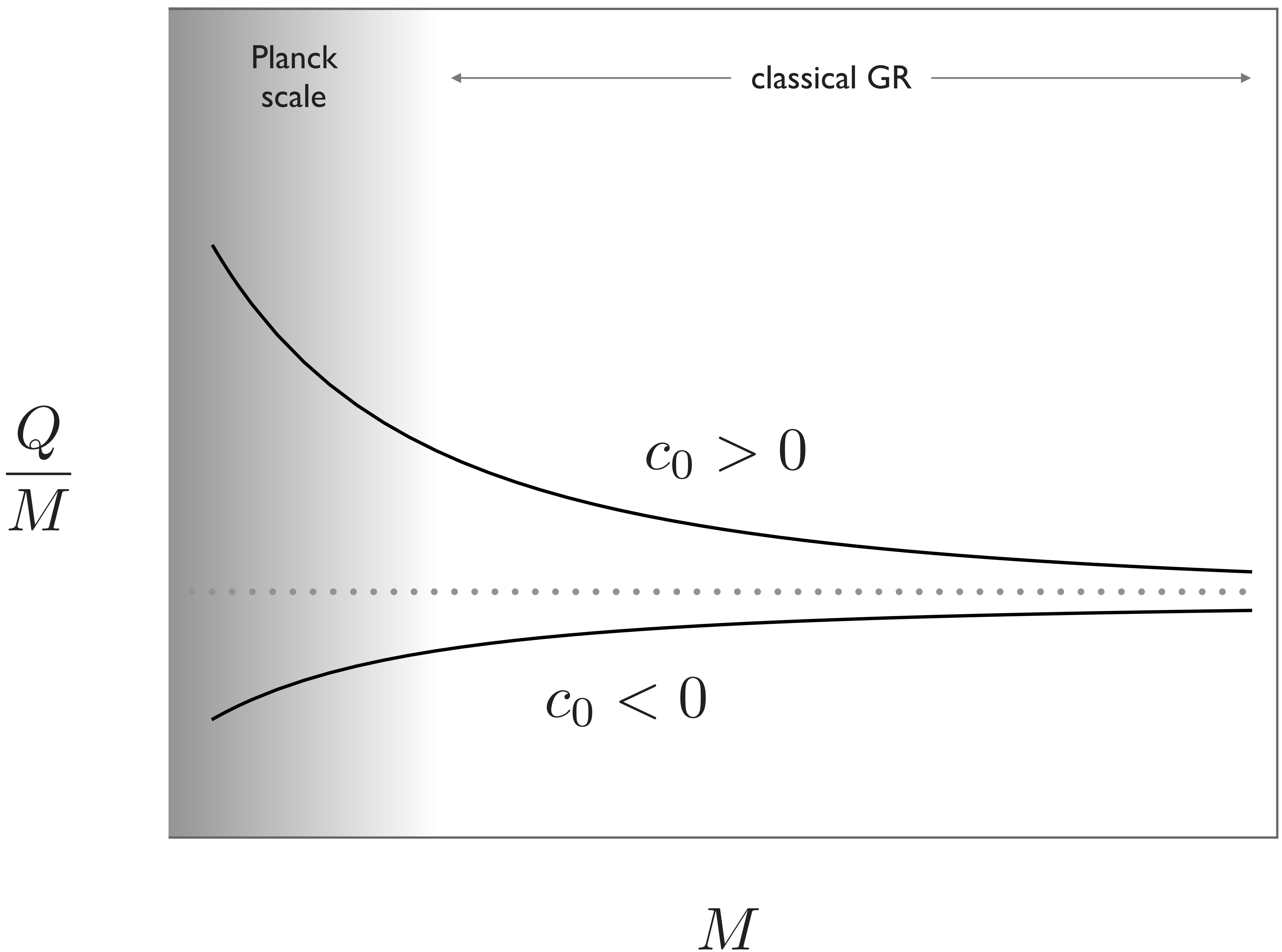
$$\alpha_0 c_0 + \alpha_3 c_3 + \alpha_6 c_6 + \alpha_9 c_9 > 0$$

can black holes ensure WGC?

It is possible that the WGC might be satisfied automatically by small black holes.

$$\frac{Q}{M} = 1 + \frac{128\pi^2 c_0}{5M^2} \cdots$$

The higher-dimension operator corrections decouple for large black holes.


Planck
scale
classical GR
$c_0 > 0$
$c_0 < 0$
$\frac{Q}{M}$
M

perturbed black hole system

Study a black hole of mass M and charge Q perturbed by higher-dimension operators.

$$\mathcal{L} = \underbrace{\tilde{\mathcal{L}}}_{\text{Einstein-Maxwell}} + \Delta\mathcal{L} \leftarrow \sum_{i=1}^{8} c_i \mathcal{O}_i$$

Compute first order perturbed metric.

$$g_{\mu\nu} = \underbrace{\tilde{g}_{\mu\nu}}_{\text{Reissner-Nordstrom}} + \Delta g_{\mu\nu} \leftarrow \propto c_i$$

Now compute the black hole Wald entropy.

$$S = -2\pi \int_\Sigma \frac{\delta \mathcal{L}}{\delta R_{\mu\nu\rho\sigma}} \epsilon_{\mu\nu}\epsilon_{\rho\sigma}$$

$$= -2\pi \left(\tilde{A} \frac{\delta \tilde{\mathcal{L}}}{\delta R_{\mu\nu\rho\sigma}} + \tilde{A} \frac{\delta \Delta \mathcal{L}}{\delta R_{\mu\nu\rho\sigma}} + \Delta A \frac{\delta \tilde{\mathcal{L}}}{\delta R_{\mu\nu\rho\sigma}} + \cdots \right) \epsilon_{\mu\nu}\epsilon_{\rho\sigma}$$

Bekenstein-Hawking entropy

correction from modified interactions

correction from shift of horizon

$$= \quad \tilde{S} \quad + \quad \Delta S$$

The entropy is corrected by two effects.

After some tedious calculation, we obtain

$$\Delta S \propto \tilde{S} \times \frac{\mathcal{B}}{M^2}$$

invariant under field redefinitions!

$$\mathcal{B} = (1-\xi)^2 c_0 + 20\xi c_3 - 5\xi(1-\xi)(2c_3 + c_6)$$

where the charge-to-mass parameter is

$$\xi = \sqrt{1 - \frac{Q^2}{M^2}}$$

so

$\xi = 0$ (extremal)

$\xi = 1$ (neutral)

positive entropy shift → WGC

For highly charged black holes, the entropy shift is proportional to the WGC bound.

$$\Delta S \propto c_0 \qquad (\xi \ll 1)$$

Entropy and WGC are intimately connected!

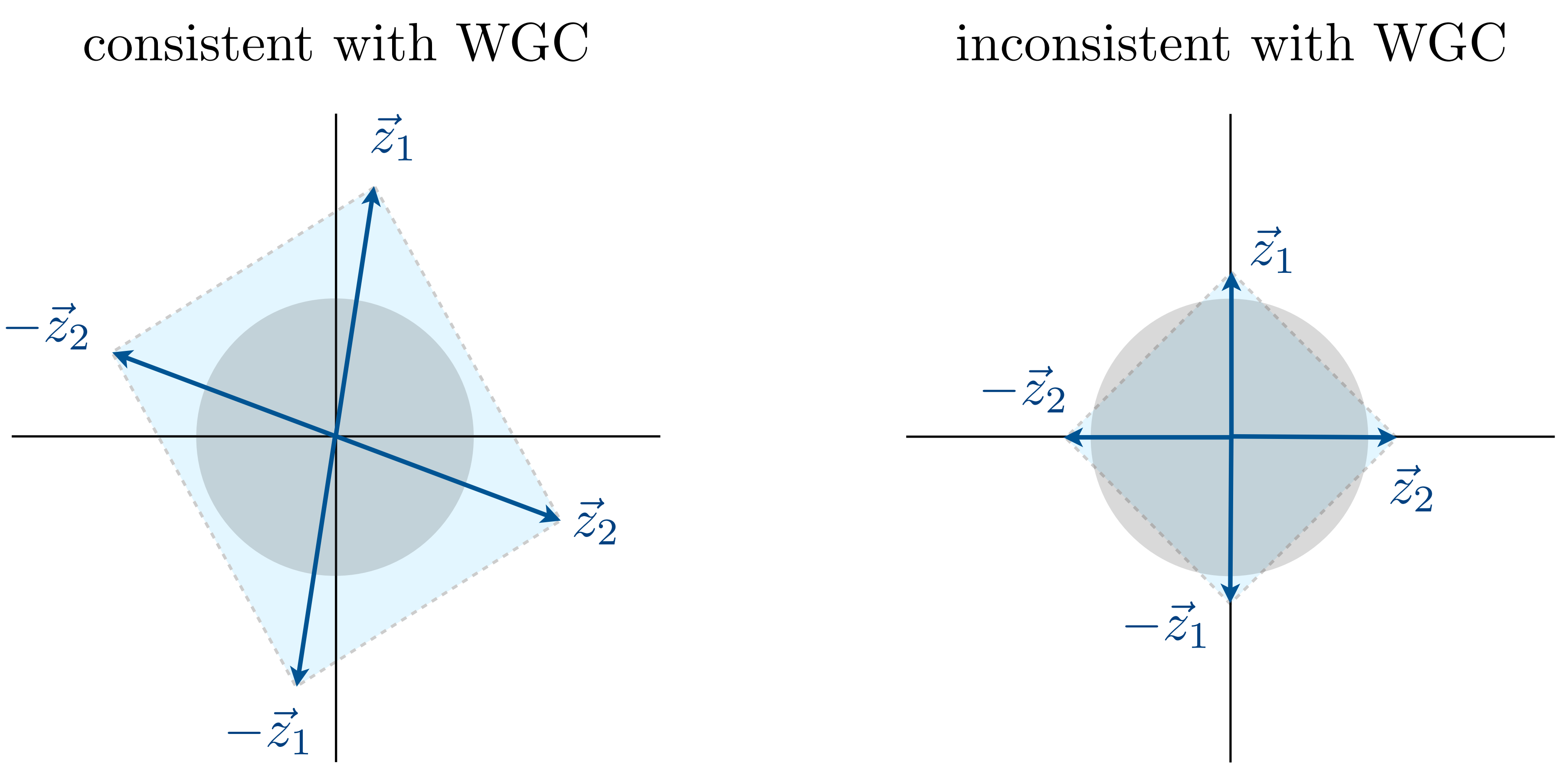
$\Delta S > 0$ implies $c_0 > 0$

positive entropy shift

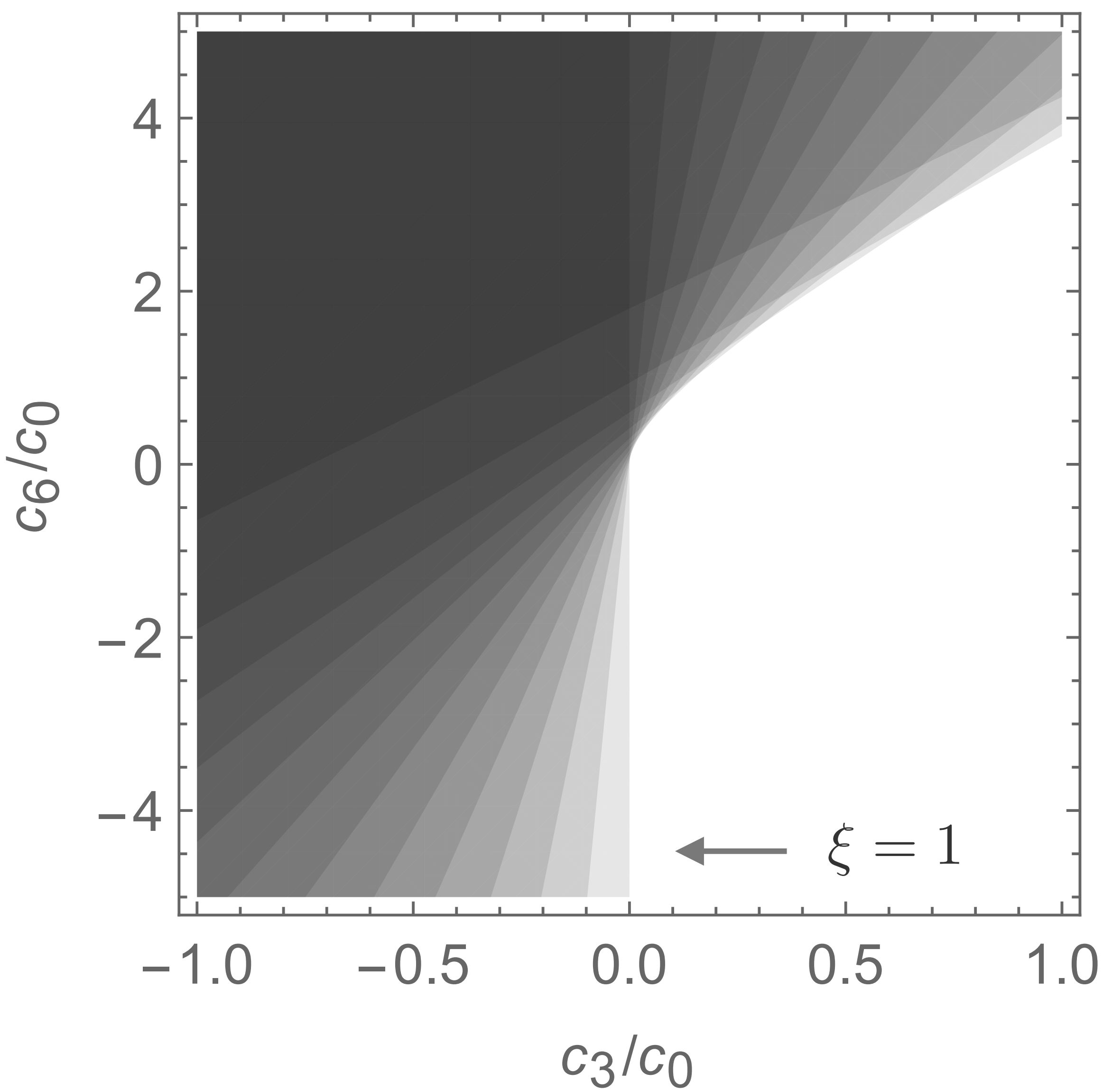
black holes ensure WGC

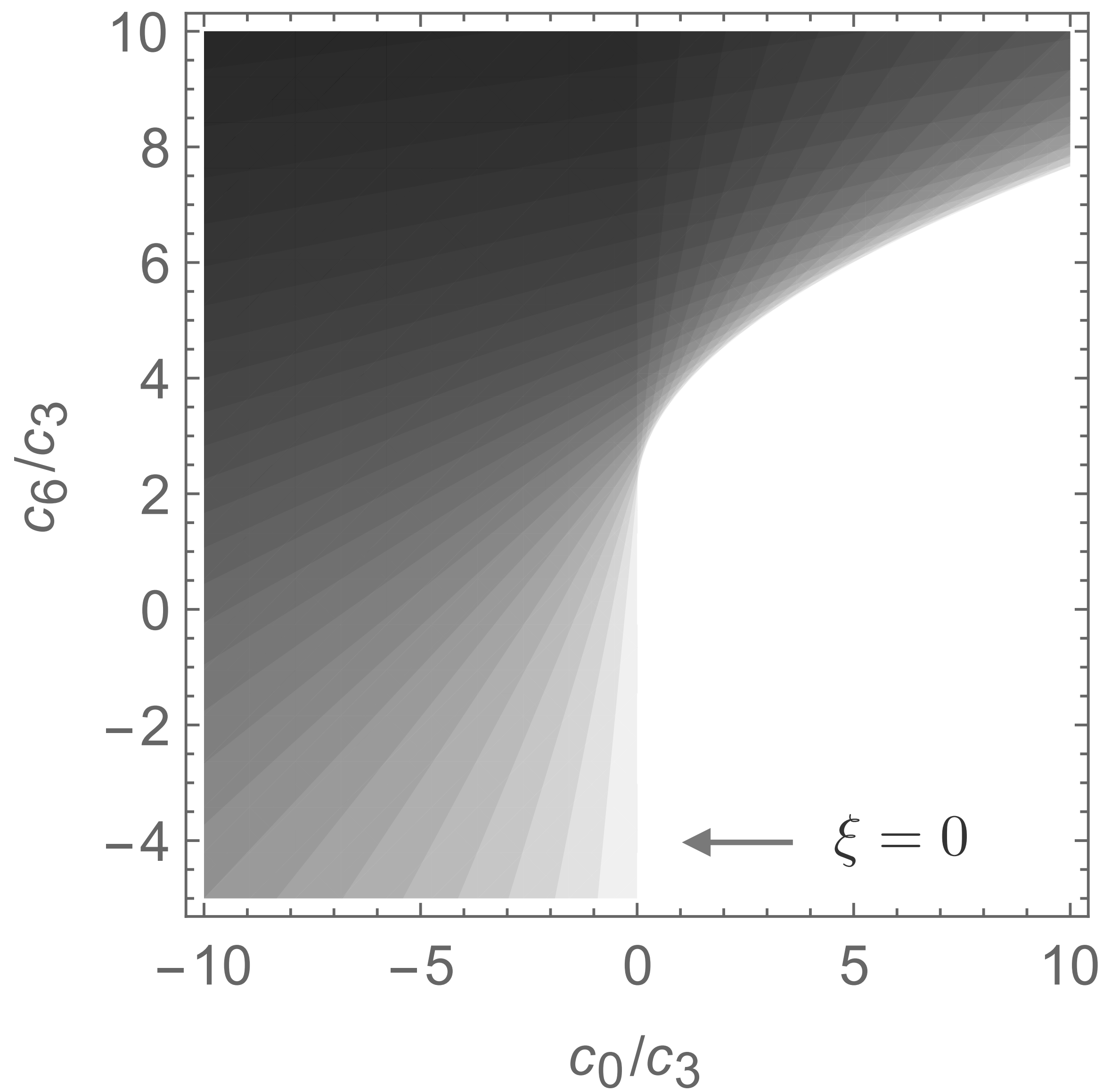
positive entropy shift → WGC

The WGC/entropy linkage is true in arbitrary dimensions and for multiple U(1) factors.



other implications





Which entropy?

From dimensional analysis, we know that the entropy receives contributions of the form

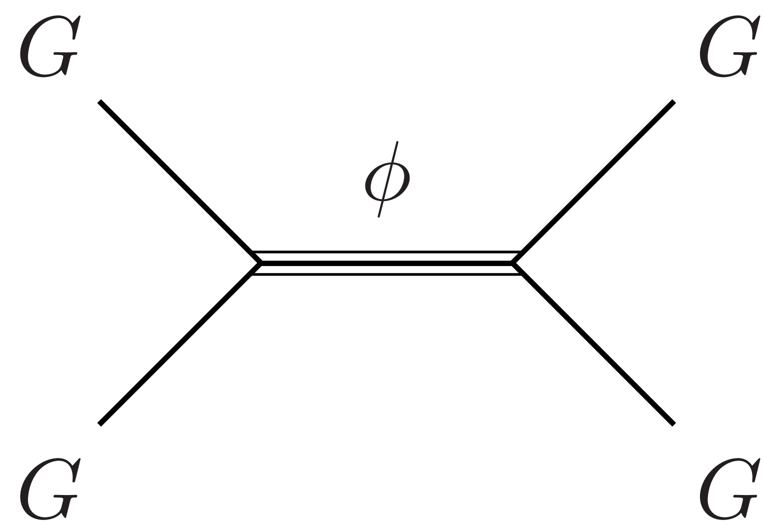
$$S \sim M^2 + \log M + 1 + \cdots$$

Local higher-dimension operators comprise the last term, which naively never dominates.

However, the actual parametric size of the dimensionless prefactors is crucial.

In a weakly coupled completion of gravity, the higher-dimensional operators are enhanced.

$$A(GGGG) \quad \sim$$



$$\sim \frac{s^2}{m_{\mathrm{Pl}}^2 m_\phi^2} + \cdots$$

E.g. the Virasoro-Shapiro amplitude implies

$$\Delta\mathcal{L} \quad \sim \quad \boxed{\frac{m_{\mathrm{Pl}}^2}{m_\phi^2}} \times R^2$$

Modulo signs, the entropy contributions are

$$S \sim \frac{M^2}{m_{\rm Pl}^2}\left(1+\frac{m_\phi^2}{m_{\rm Pl}^2}\right)+\log\left(\frac{M}{m_{\rm Pl}}\right)+\frac{m_{\rm Pl}^2}{m_\phi^2}+\cdots$$

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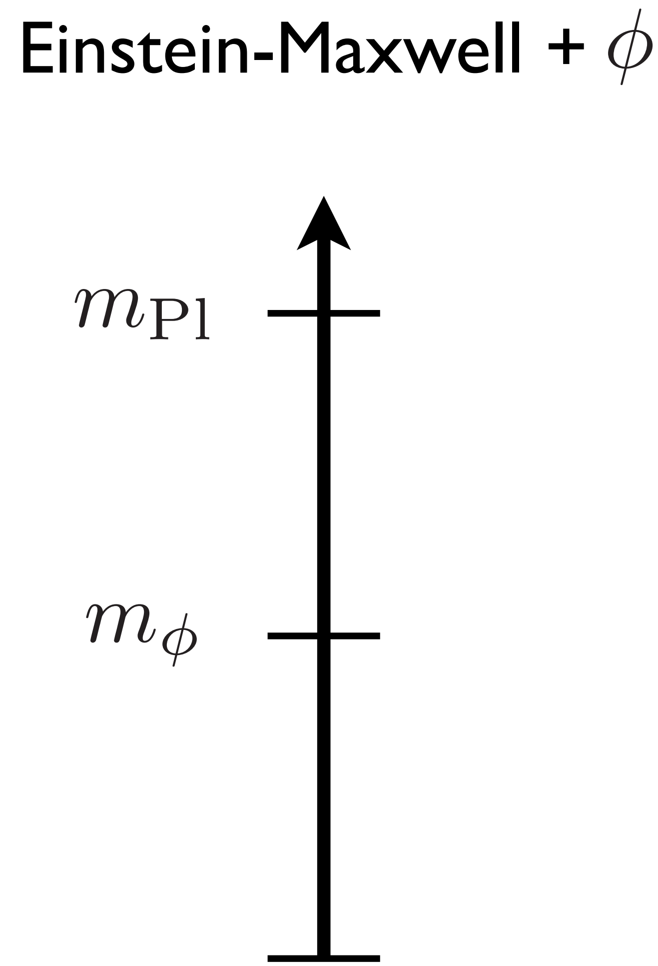
$\Downarrow$

Possible for small black holes still within the EFT: $\dfrac{m_{\text{Pl}}}{m_\phi^2} \gg \dfrac{M}{m_{\text{Pl}}^2} \sim R$

Why $\Delta S > 0$?

Consider a QFT analyzed at two scales.



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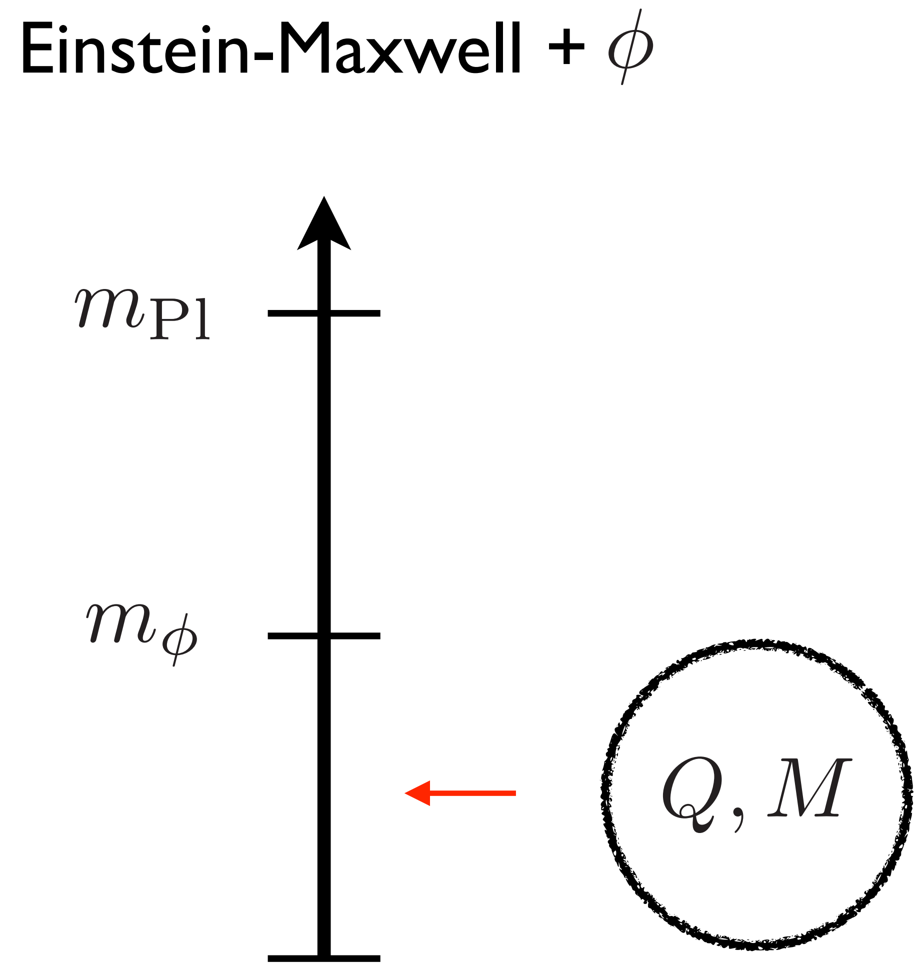

Compute the same entropy in two ways.

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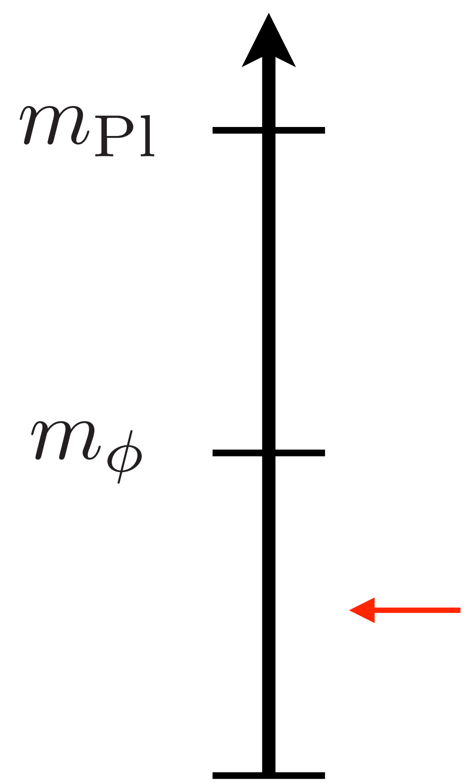
Einstein-Maxwell + ϕ



compute entropy
in low energy EFT

$\tilde{S} + \Delta S$

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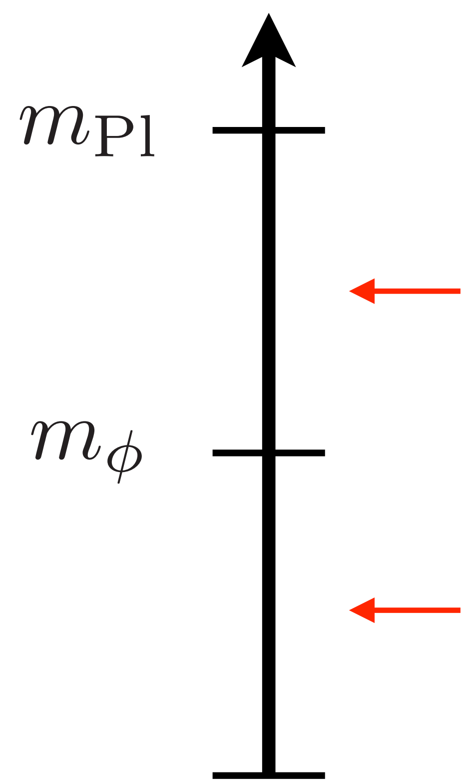

compute entropy in full theory $\quad \tilde{S} + S_\phi$

compute entropy in low energy EFT $\quad \tilde{S} + \Delta S$

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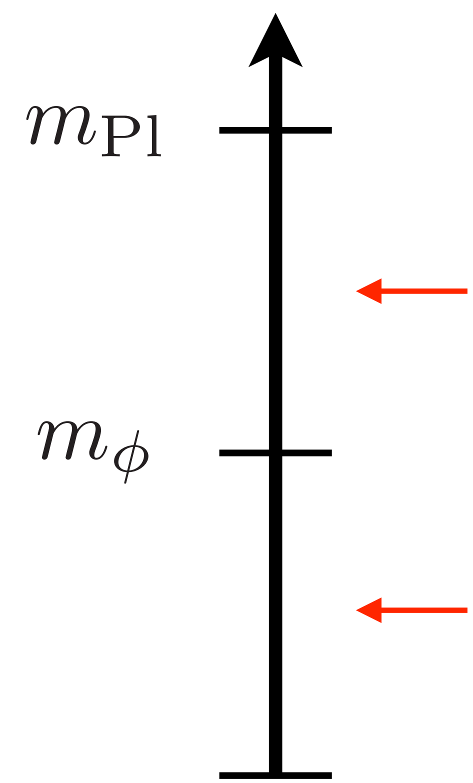

compute entropy
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$$\boxed{\begin{array}{c} \tilde{S} + S_\phi \\ \parallel \\ \tilde{S} + \Delta S \end{array}}$$

Compute the same entropy in two ways.

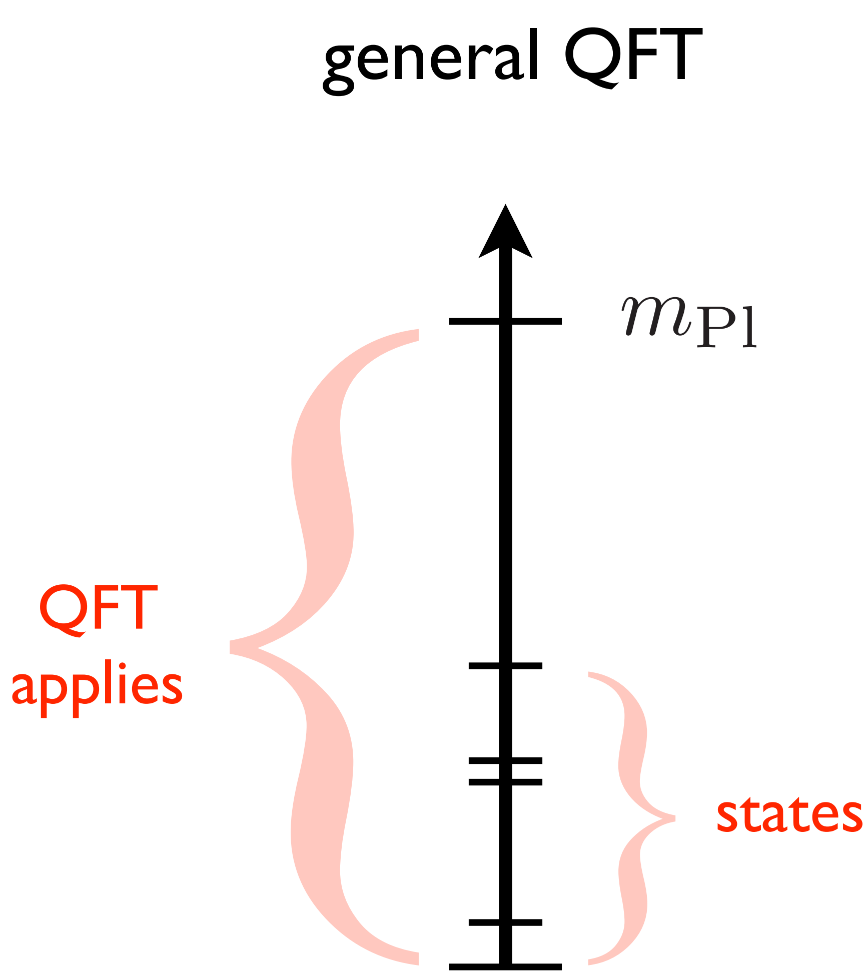
proof of $\Delta S > 0$

quantum field
theoretic

proof of $\Delta S > 0$

classical
entropy shift

Assume QFT applies in some energy range.



$$\Delta S > 0$$

$$c_0 > 0$$

Einstein-Maxwell is natural "baseline" theory.

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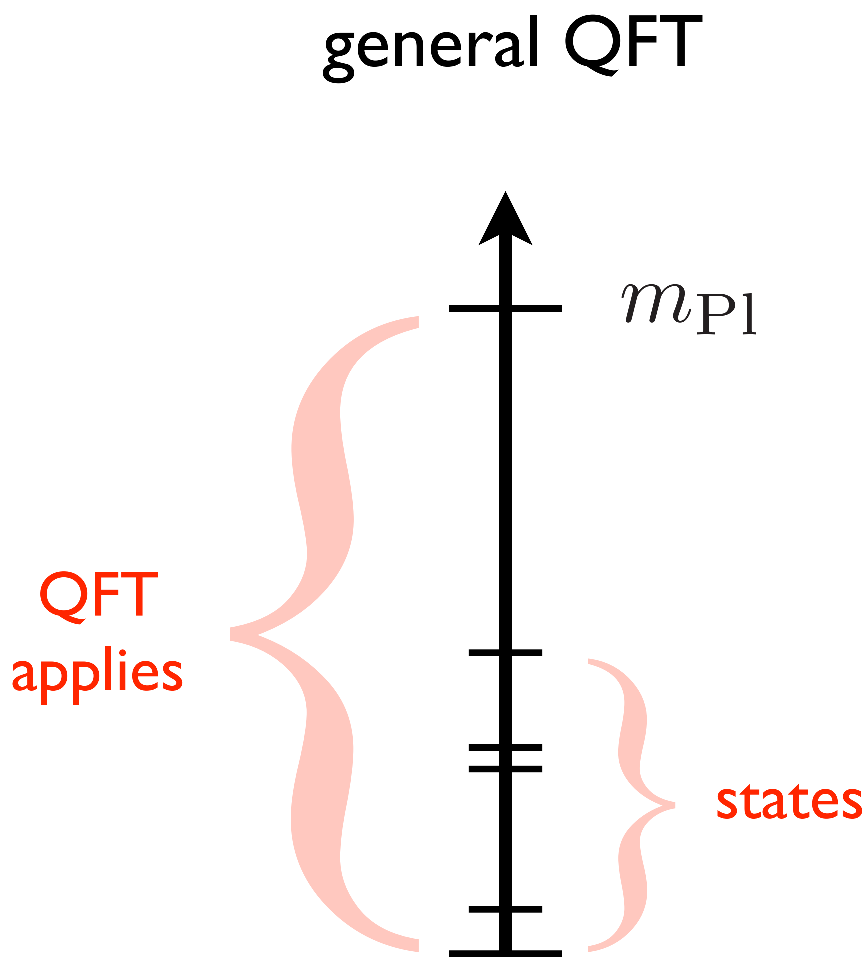

$$\Delta S > 0 + \mathcal{O}(1/m_{\mathrm{Pl}})$$

$$c_0 > 0 + \mathcal{O}(1/m_{\mathrm{Pl}})$$

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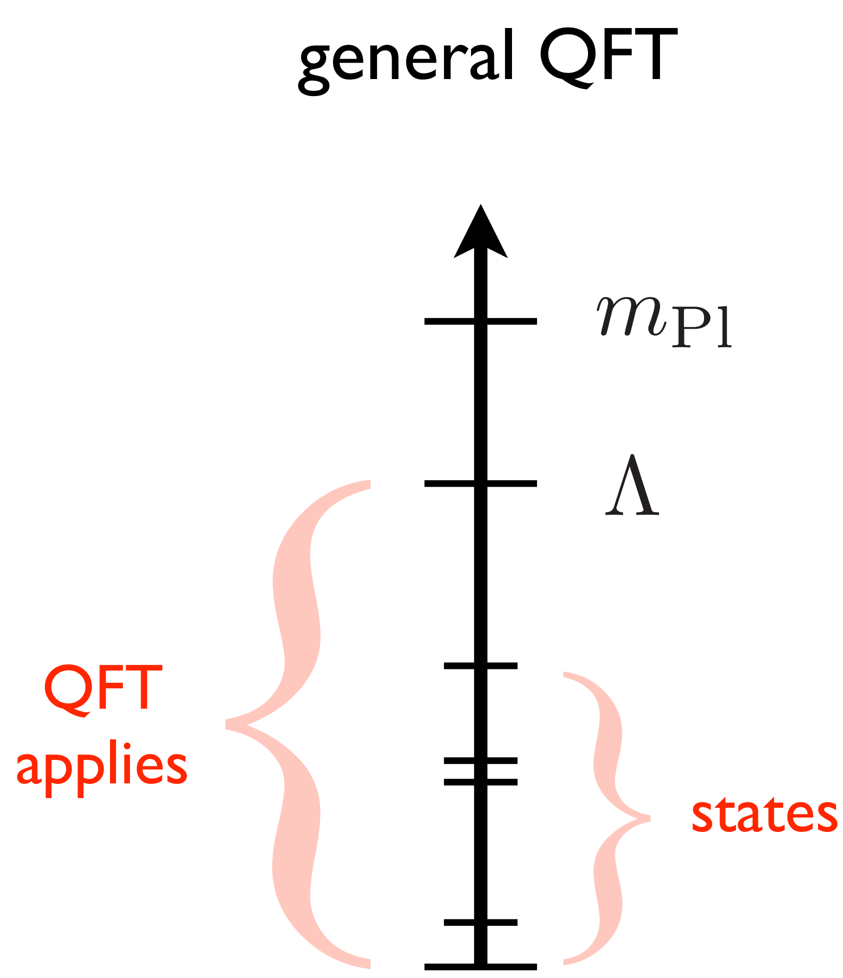

$$\Delta S > 0 + \mathcal{O}(1/\Lambda)$$

$$c_0 > 0 + \mathcal{O}(1/\Lambda)$$

Note we can make a weaker assumption.

Free energy from the Euclidean path integral:

$$Z(\beta) = \int_\beta [d\hat{g}] e^{-I[\hat{g}]}$$

$$I[\hat{g}] = \tilde{I}[\hat{g}] + \Delta I[\hat{g}]$$

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Black hole mass, temperature, entropy are:

$$M = -\partial_\beta \log Z(\beta) \qquad \beta = \partial_M S$$

$$S = \beta M + \log Z(\beta)$$

Classically, higher-dimensional operators are generated at tree-level by heavy states.

$$Z(\beta) = \int_\beta [d\hat{g}][d\hat{\phi}]e^{-I_{\mathrm{UV}}[\hat{g},\hat{\phi}]} = \int_\beta [d\hat{g}]e^{-I[\hat{g}]}$$

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Dropping heavy states reduces the theory to the “baseline”, e.g. Einstein-Maxwell.

$$I_{\mathrm{UV}}[\hat{g}, 0] = \tilde{I}[\hat{g}]$$

Compute the free energy via the saddle point approximation.

insert classical solutions

perturbed Reissner-Nordstrom

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$$\vee$$

definition of extremization

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$$\vee$$

definition of extremization

$$-I_{\mathrm{UV}}[\tilde{g}, 0]$$

$$||$$

truncating heavy states yields Einstein-Maxwell

pure Reissner-Nordstrom

$$\log \tilde{Z}(\beta) \sim -\tilde{I}[\tilde{g}]$$

We derive an inequality at fixed temperature.

The free energy inequality can be written as:

$$\log Z(\beta) > \log \tilde{Z}(\beta) = (1 + \Delta\beta\partial_{\tilde{\beta}}) \log \tilde{Z}(\tilde{\beta})$$

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$(1 - M\partial_M)S$

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$(1 - M\partial_M)\tilde{S}$ $M\partial_M \Delta S$

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$(1 - M\partial_M)\tilde{S}$ $\quad$ $M\partial_M \Delta S$

This implies that classical black hole entropy from heavy states at fixed M and Q satisfies:

$$\boxed{\Delta S > 0}$$

Integrating out states sequentially translates to extremizing over each field sequentially:

$$\int_{\Lambda}^{\mu} dc_0 = c_0(\mu) > 0$$

Corollary: each correction from a heavy field is positive, yielding a differential bound:

$$dc_0 > 0$$

conclusions

- WGC violation might be visible in low energy pathologies of the effective action for photons and gravitons.

- Small black holes automatically fulfill the WGC if the entropy shift from higher dimension operators is positive.

- The classical entropy shift from tree-level heavy particle exchange is positive via a simple thermodynamic proof.

thank you!