The Top-Quark Mass:
Uncertainties due to Bottom-Quark Fragmentation

Gennaro Corcella

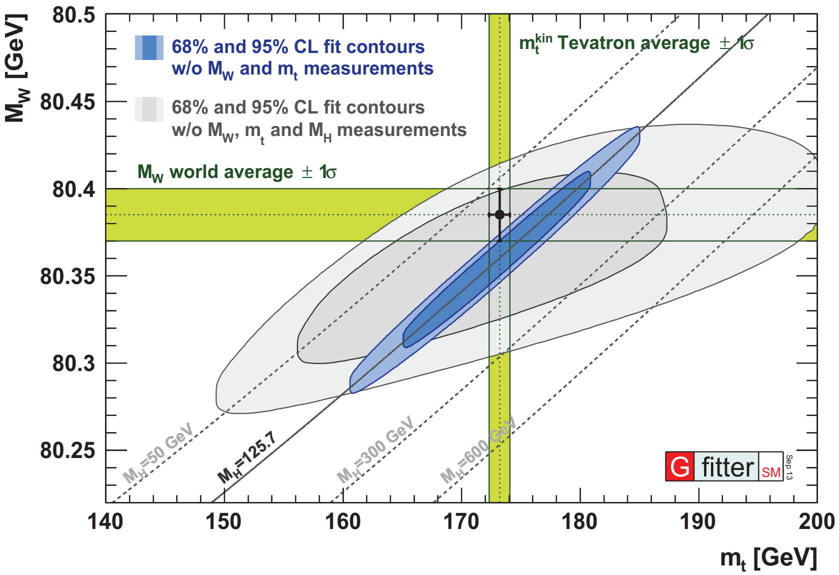
INFN - Laboratori Nazionali di Frascati

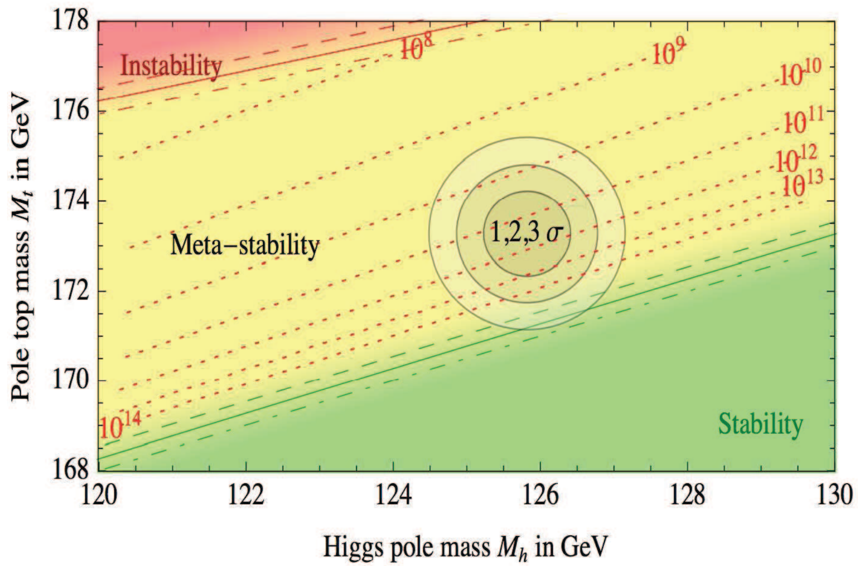
1. **Introduction**
2. **Monte Carlo codes and QCD calculations for b-fragmentation in top decays**
3. **Sensitivity of top mass to shower and hadronization parameters**
4. **Conclusions**

G.C., R. Franceschini and D. Kim, Nucl. Phys. B929 (2018) 485

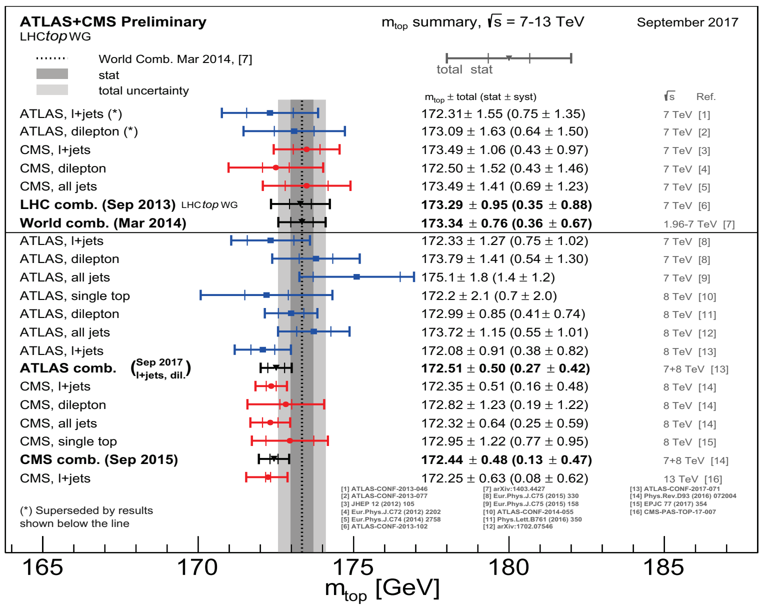
G.C., PoS(EPS-HEP2017) (2018) 437

Top mass plays crucial role in EW symmetry breaking and vacuum stability





Latest combinations by ATLAS and CMS and world average



All values in GeV	CDF	D0	ATLAS	CMS	Tevatron	LHC	WA
m_{top}	173.19	174.85	172.65	173.58	173.58	173.28	173.34
Stat	0.52	0.78	0.31	0.29	0.44	0.22	0.27
iJES	0.44	0.48	0.41	0.28	0.36	0.26	0.24
stdJES	0.30	0.62	0.78	0.33	0.27	0.31	0.20
flavourJES	0.08	0.27	0.21	0.19	0.09	0.16	0.12
bJES	0.15	0.08	0.35	0.57	0.13	0.44	0.25
MC	0.56	0.62	0.48	0.19	0.57	0.25	0.38
Rad	0.09	0.26	0.42	0.28	0.13	0.32	0.21
CR	0.21	0.31	0.31	0.48	0.23	0.43	0.31
PDF	0.09	0.22	0.15	0.07	0.12	0.09	0.09
DetMod	<0.01	0.37	0.22	0.25	0.09	0.20	0.10
b-tag	0.04	0.09	0.66	0.11	0.04	0.22	0.11
LepPt	<0.01	0.20	0.07	<0.01	0.05	0.01	0.02
BGMC	0.10	0.16	0.06	0.11	0.11	0.08	0.10
BGData	0.15	0.19	0.06	0.03	0.12	0.04	0.07
Meth	0.07	0.15	0.08	0.07	0.06	0.06	0.05
MHI	0.08	0.05	0.02	0.06	0.06	0.05	0.04
Total Syst	0.85	1.25	1.40	0.99	0.82	0.92	0.71
Total	1.00	1.48	1.44	1.03	0.94	0.94	0.76
χ^2/ndf	1.09 / 3	0.13 / 1	0.34 / 1	1.15 / 2	2.45 / 5	1.81 / 4	4.33 / 10
χ^2 probability [%]	78	72	56	56	78	77	93

World average: $m_t = [173.34 \pm 0.27(\mathrm{stat}) \pm 0.71(\mathrm{syst})]$ GeV (TeV+LHC)

Most precise measurements $\Delta m_t \simeq 0.5$ GeV: accurate calculations and MC's mandatory

Uncertainty on identification as pole mass $\mathcal{O}(\Gamma_t \alpha_S(Q_0)) \sim \mathcal{O}(200\ \mathrm{MeV})$ (Hoang et al,'08-'18)

Renormalons and pole mass: $\Sigma \sim \alpha_S^n n!$, $\Delta m_t^{\rm ren} \simeq$ 110-250 MeV (Beneke et al'16, Hoang et al'17)

Both pole-mass and renormalon uncertainties still below the current error on m_t

This talk: b-fragmentation in t-decays entering JES and MC uncertainties

Several calculations and tools are available for bottom fragmentation in top decays

Perturbative-fragmentation approach: massless hard scattering ($m_b \ll m_t$), universal perturbative fragmentation functions, DGLAP evolution like in pdfs (Mele-Nason,'91)

NLO+NLL calculation for b and B-hadron energy spectra in top decays (NWA), including threshold resummation; hadron corrections from e^+e^- data (G.C., M.Cacciari and A.Mitov '02)

Could be extended to NNLO+NNLL thanks to NNLO coefficient functions (Bruchenseifer, Caola, Melnikov '13), perturbative fragmentation function (Melnikov, Mitov '14) and time-like splitting functions (Moch, Mitov, Vogt '06)

NNLO+NNLL fragmentation in SCET for $e^+e^- \to b\bar{b}$ Fickinger, Fleming, Kim and Mereghetti '16

This talk: mostly MC generators, NLO/resummed calculations for validation/comparison

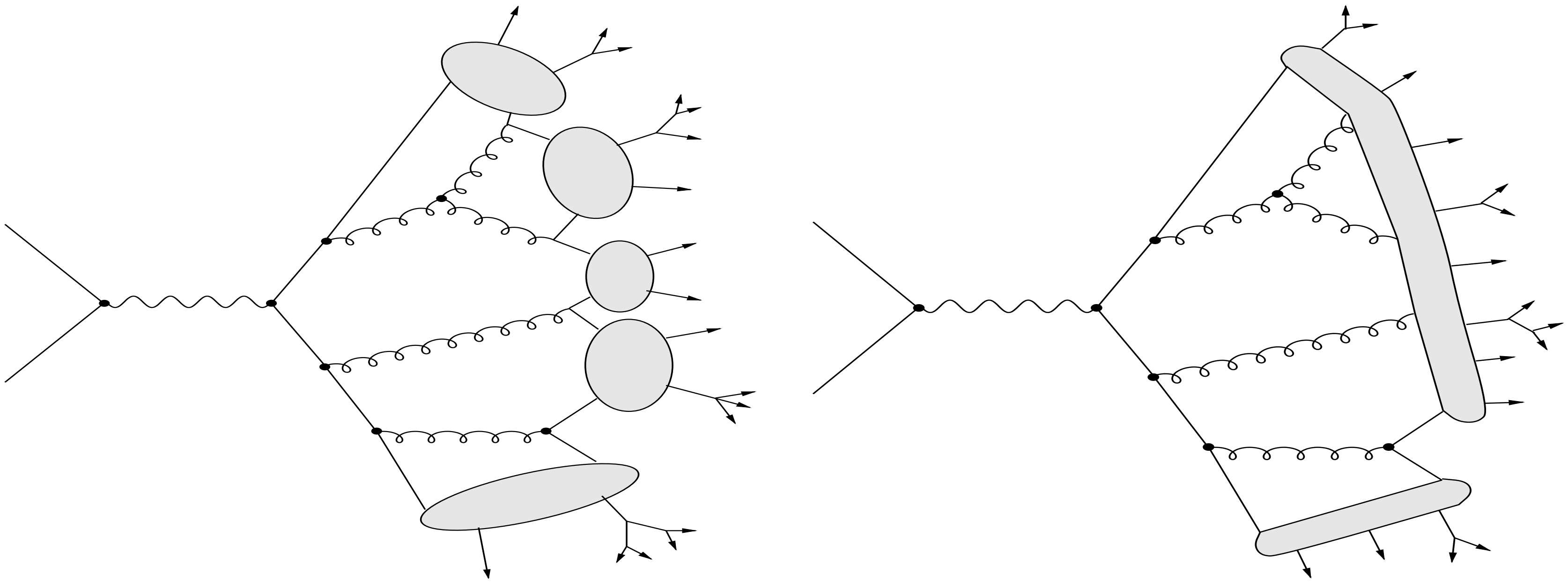
Standard generators (HERWIG, PYTHIA): LO matrix element, parton showers LL+(N)LL, tree-level matrix-element matching, string/cluster models for hadronization

Late progress in NLO+shower generators:

aMC@NLO: NLO single top, not yet $t\bar{t}$, though MadSpin includes some off-shell effects

POWHEG: NLO+PS for $t\bar{t}$ production and decay using OpenLoops, including non-resonant diagrams and interference top production/decay

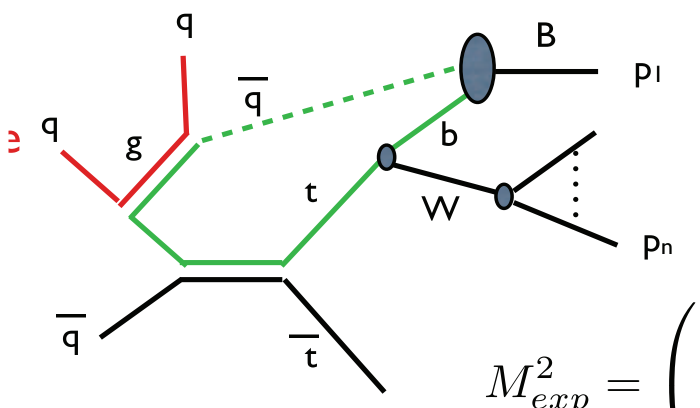
aMC@NLO and POWHEG rely on HERWIG and PYTHIA for shower and hadronization



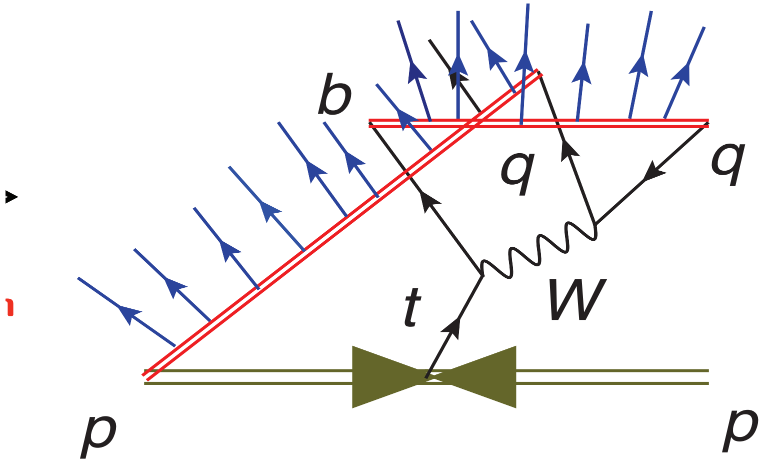
Left: HERWIG cluster model;   Right: PYTHIA string model

Figure from Ellis, Stirling, Webber, 'QCD and Collider Physics'

b-fragmentation and colour reconnection (uncertainty on $m_t^{\rm pole}$) $\Delta m({\rm CR}) \simeq 0.3$ GeV

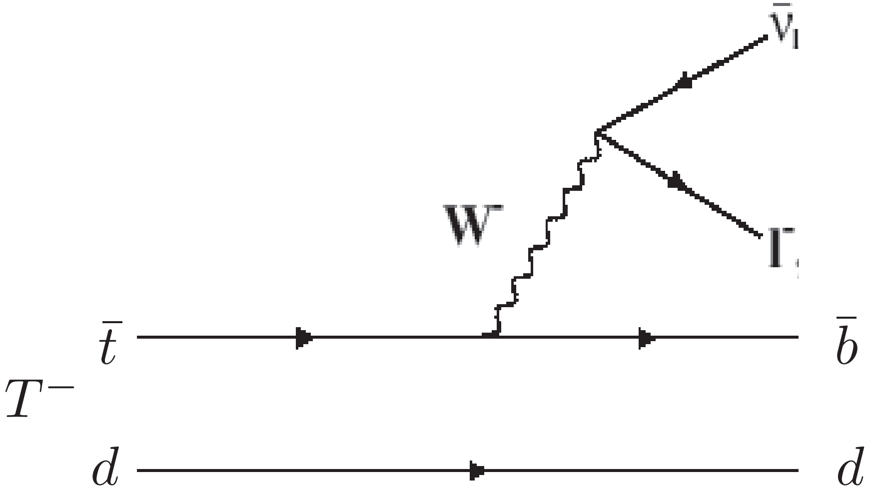


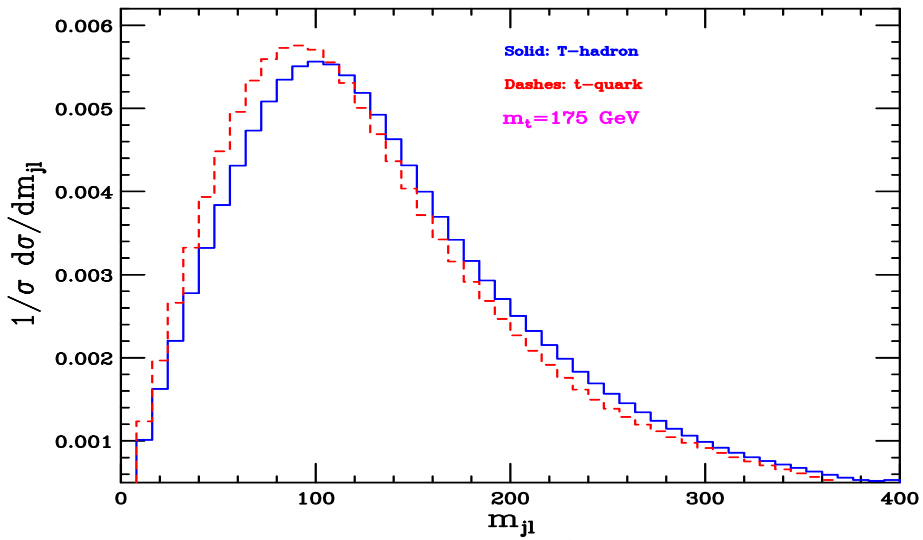
$$M^2_{exp} = \left(\sum_{i=1,\dots,n} p_i \right)^2$$



Left: M.Mangano, TOP'13 (HERWIG cluster model), Right: S.Argyropoulos, LNF'15 m_t workshop (PYTHIA string model)

Simulation of fictitious T-hadrons (HERWIG 6, preliminary) (G.C. and M.Mangano, in progress)





Useful to study colour reconnection and uncertainty on measured m_t vs pole mass

$m_T^{\rm reco} = m_t^{\rm reco} + \Delta m^{\rm reco}$ with $m_T = m_t^{\rm pole} + m_q + \Delta m_{t,T}$ (lattice, NRQCD, etc.)

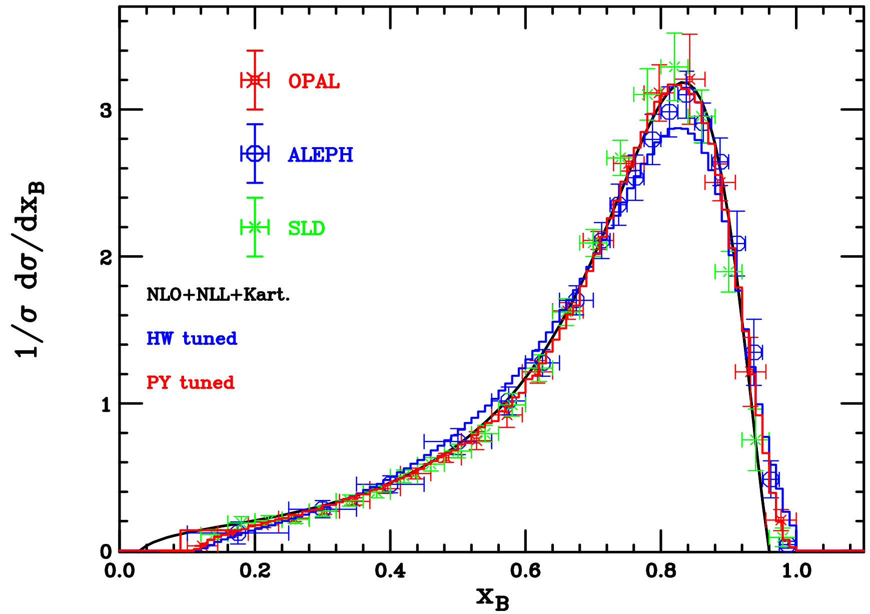
Tuning HERWIG and PYTHIA to e^+e^- data (G.C. and Drollinger '05, G.C. and F.Mescia '10)

HERWIG	PYTHIA
CLSMR(2) = 0.3 (0.0)	PARJ(41) = 0.85 (0.30)
DECWT = 0.7 (1.0)	PARJ(42) = 1.03 (0.58)
CLPOW = 2.1 (2.0)	PARJ(46) = 0.85 (1.00)
PSPLT(2) = 0.33 (1.00)	
χ^2/dof = 222.4/61 (739.4/61)	χ^2/dof = 45.7/61 (467.9/61)

Lund/Bowler fragmentation function (PYTHIA): $f_B(z) \sim (1-z)^a \exp(-bm_T^2/z)/z^{1+brm_b^2}$

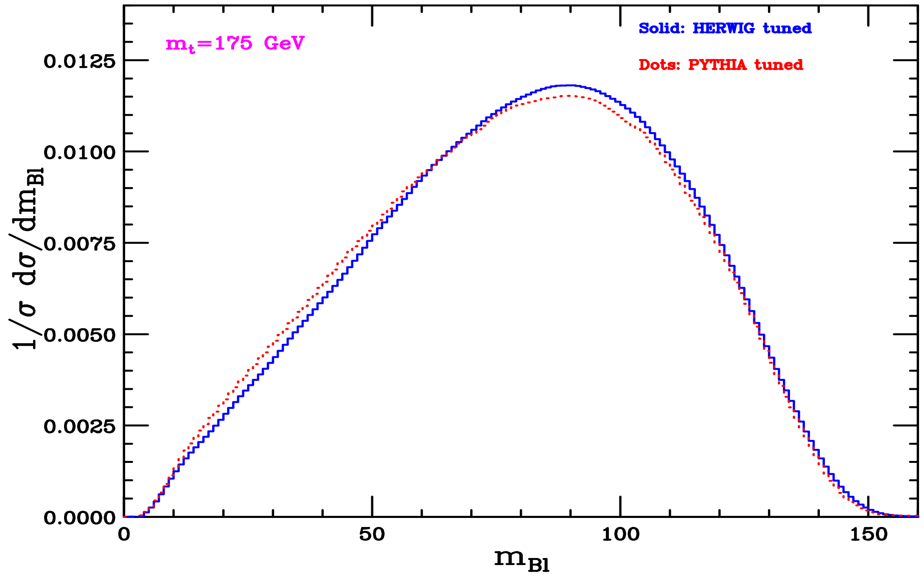
HERWIG parameters describe gaussian smearing (CLSMR), baryon/meson (CLPOW) and decuplet/octet (DECWT) ratios, mass spectrum of b-like clusters (PSPLT)

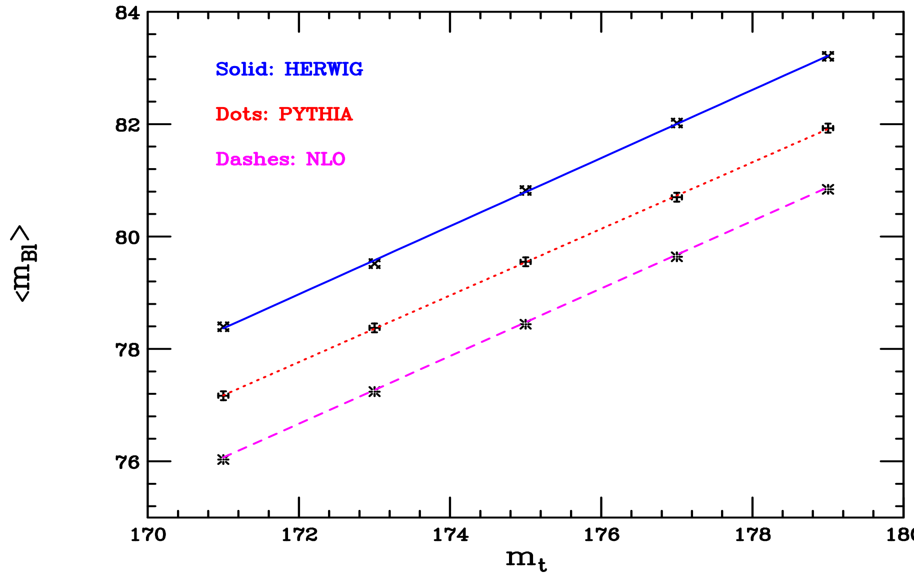
Our PYTHIA tuning in ATLAS jet-energy measurement and m_t world average



NLO+NLL calculation uses: $D(x) = Nx^{\alpha}(1-x)$ and fits α to data

$m_{B\ell}$ according to tuned HERWIG and PYTHIA and NLO (Biswas, Melnikov, Schulze, '10)





Linear fits to extract m_t from $m_{B\ell}$: large uncertainty on m_t

HERWIG: $\langle m_{B\ell}\rangle_{\rm H} \simeq -25.31 \text{ GeV} + 0.61\ m_t$

PYTHIA: $\langle m_{B\ell}\rangle_{\rm P} \simeq -24.11 \text{ GeV} + 0.59\ m_t$

NLO: $\langle m_{B\ell}\rangle_{\rm NLO} \simeq -26.7 \text{ GeV} + 0.60\ m_t$

$\Delta\langle m_{B\ell}\rangle_{\rm H,P} \simeq 1.2$ GeV ; $\Delta\langle m_{B\ell}\rangle_{\rm H,NLO} \simeq 2.2$ GeV ; $\Delta\langle m_{B\ell}\rangle_{\rm P,NLO} \simeq 1.1$ GeV

NLO+showers for top decays or re-tuning C++ codes to shed light on this discrepancy

In progress (G.C.+ATLAS): m_t from $m_{\mu\ell}$ in $B \to \mu X$ improving fragmentation models/fits

Novel investigation on fragmentation error and *in-situ* calibration of hadronization at LHC for top analyses (beyond LEP fits) (G.C., R.Franceschini and D.Kim, NPB'18)

Most m_t methods (template, $m_{b\ell}$, E_b-peak, endpoint, etc.) rely on b-jets: JES error

Confronting b-jets with B-hadrons: JES $\Leftrightarrow$ hadronization uncertainty:
$(m_{j\ell}, E_j, p_{T,j}) \Leftrightarrow (m_{B\ell}, E_B, p_{T,B})$ in dilepton channel

Sensitivities of observables O and top mass to Monte Carlo parameters θ:

$$\frac{dm_t}{m_t} = \Delta_O^m \frac{d\langle O\rangle}{\langle O\rangle} \;\; ; \;\; \frac{d\langle O\rangle}{\langle O\rangle} = \Delta_\theta^O \frac{d\theta}{\theta}$$

Precision of 0.3% on m_t ($\Delta m_t \leq 500$ MeV) from measurement of $\langle O\rangle$:

$$\frac{dm_t}{m_t} < 0.003 \;\Rightarrow\; \Delta_O^m \frac{d\langle O\rangle}{\langle O\rangle} < 0.003 \;\Rightarrow\; \Delta_O^m \, \Delta_\theta^O \frac{d\theta}{\theta} < 0.003$$

HERWIG 6: vary CLPOW, PSPLT, CLMSR, CLMAX, QCDLAM $\simeq \exp(K/4\pi\beta_0)\Lambda_{\overline{\rm MS}}$ [CMW scheme for NLLs in the shower at large x]; shower cutoffs [VGCUT and VQCUT]; bottom mass and gluon effective mass [RMASS(5) and RMASS(13)]

PYTHIA 8: string model a, b, r_B parameters, $p_{T,\rm min}$, $\alpha_{S,\rm FSR}$

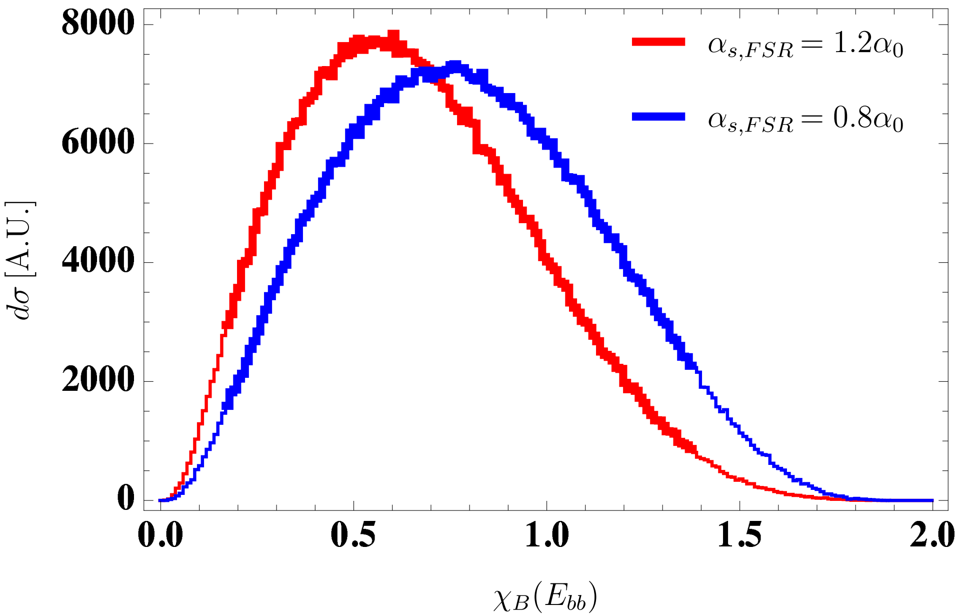
All parameters varied by $\sim 10\%$ around default values

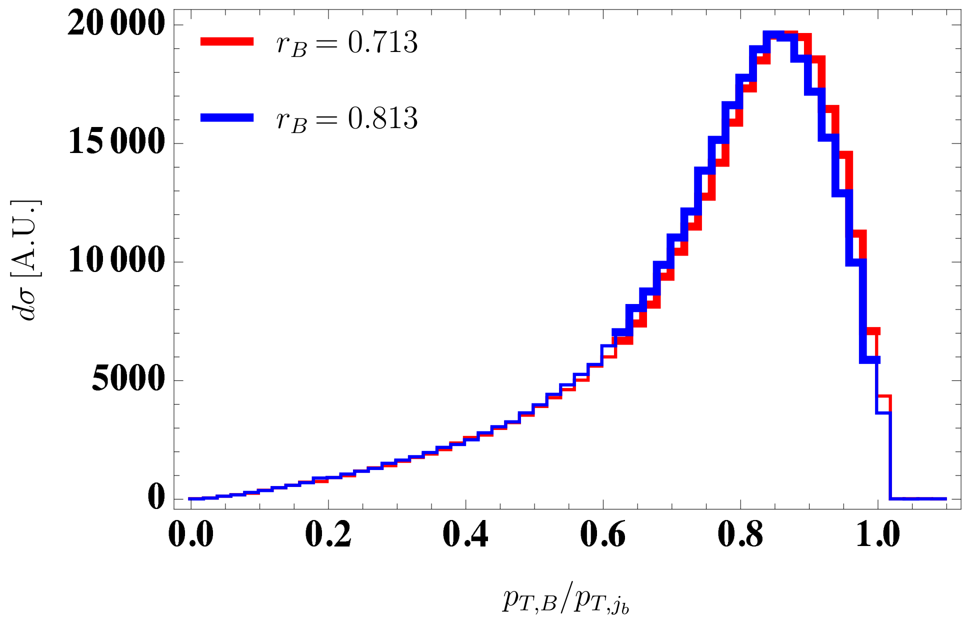
Calibration observables: quantities which exhibit mild dependence on m_t can be used to tune fragmentation parameters ($j = b$-jet)

$p_{T,B}/p_{T,j}$; $\rho(r) = \frac{1}{E_j \times \Delta R} \sum_{\text{tracks}} E(\text{tracks})\Theta(|r - \Delta R|)$ (radial energy density);

$\Delta\phi(j\bar{j})$; $\Delta R(j\bar{j})$; $\Delta\phi(B\bar{B})$; $\Delta\phi(j\bar{j})$; $m_{B\bar{B}}/m_{j\bar{j}}$

$\chi_B(X_B) = 2E_B/X_B$, with $X_B = m_{j\bar{j}}$, $|p_{T,j}| + |p_{T,\bar{j}}|$ $E_j + E_{\bar{j}}$





Left: Dependence of $\chi_B = E_B/(E_j + E_{\bar{j}})$ on $\alpha_{S,\text{FSR}}$

Right: Dependence of $p_{T,B}/p_{T,j}$ on r_B

Mellin moments: $\mathcal{M}_N = \frac{1}{x_{\max} - x_{\min}} \int_{x_{\min}}^{x_{\max}} dx\ x^{N-1} f(x)$; $\mathcal{M}_1 = \langle f \rangle$

$x_{\min}$ and $x_{\max}$ such that full width at half maximum (FWHM)

Sensitivity of calibration observables to top mass and PYTHIA parameters

$\mathcal{O}$	Range	$\Delta_{m_t}^{(\mathcal{M}_\mathcal{O})}$	$\Delta_\theta^{(\mathcal{M}_\mathcal{O})}$					
			$\alpha_{s,FSR}$	m_b	$p_{T,\text{min}}$	a	b	r_B
$\rho(r)$	0-0.04	-0.007	0.78	0.204	-0.1286	0.029	-0.043	0.056
$p_{T,B}/p_{T,j_b}$	0.6-0.998	-0.053	-0.220	-0.1397	0.0353	-0.0187	0.0451	-0.0518
E_B/E_{j_b}	0.6-0.998	-0.049	-0.220	-0.1381	0.0360	-0.0186	0.0447	-0.052
E_B/E_ℓ	0.05-1.5	-0.155	-0.156	-0.053	0.0149	-0.007	0.016	-0.016
$E_B/(E_\ell + E_{\bar\ell})$	0.05-1.0	0.021	-0.231	-0.082	0.0228	-0.011	0.026	-0.028
$m(j_{\bar b})$/GeV	8-20	0.229	0.218	0.022	-0.0219	0.000	-0.001	0.001
$\chi_B(\sqrt{s_{\text{min},bb}})$	0.07-0.87	-0.177	-0.262	-0.086	0.0255	-0.0105	0.027	-0.031
$\chi_B\left(E_{j_b} + E_{\bar j_b}\right)$	0.17-1.37	-0.109	-0.357	-0.134	0.0373	-0.016	0.040	-0.045
$\chi_B(m_{j_b j_{\bar b}})$	0.17-1.37	-0.089	-0.252	-0.080	0.0248	-0.010	0.024	-0.028
$\chi_B(\|p_{T,j_b}\| + \|p_{T,\bar j_b}\|)$	0.46-1.38	-0.15	-0.47	-0.189	0.054	-0.023	0.06	-0.07
$m_{BB}/m_{j_b j_{\bar b}}$	0.8-0.95	-0.0191	-0.0623	-0.0464	0.0146	-0.0093	0.0180	-0.0212
$\Delta\phi(j_b j_{\bar b})$	0.28-3.	-0.210	0.027	0.001	-0.0014	-0.000	-0.000	-0.003
$\Delta R(j_b j_{\bar b})$	1.4-3.3	-0.071	0.010	0.0005	-0.0004	-0.000	0.0004	0.001
$\Delta\phi(BB)$	0.28-3.	-0.207	0.026	0.001	-0.0008	0.000	0.000	-0.000
$\Delta R(BB)$	1.4-3.3	-0.070	0.009	0.000	-0.0003	-0.0003	0.0002	-0.000
$\Delta\phi(BB) - \Delta\phi(j_b j_{\bar b})$	0-0.0488	0.06	0.734	0.099	-0.088	0.006	-0.004	0.01
$\Delta R(BB) - \Delta R(j_b j_{\bar b})$	0-0.0992	0.10	0.920	0.079	-0.075	-0.000	0.005	-0.00

With the exception of m_j and χ_B, all observables have $\Delta_m^{\langle O\rangle} \leq \mathcal{O}(10^{-2})$

$\rho(r)$ and χ_B largest parameter dependence; mild dependence of ΔR, $\Delta\phi$ and B/j ratios

Sensitivity of m_t to PYTHIA parameters using m_t-dependent observables

Mellin moments of E_B, $E_B + E_{\bar{B}}$, $p_{T,B}$, $p_{T,B} + p_{T,\bar{B}}$, $m_{B\ell}$, m_{T2}

$$m_{T2} = \min_{\not p_1 + \not p_2 = \not p_T} \left\{ \max \left[m_T^2(p_{T,\ell^-}, \not p_1), m_T^2(p_{T,\ell^+}, \not p_2) \right] \right\}$$

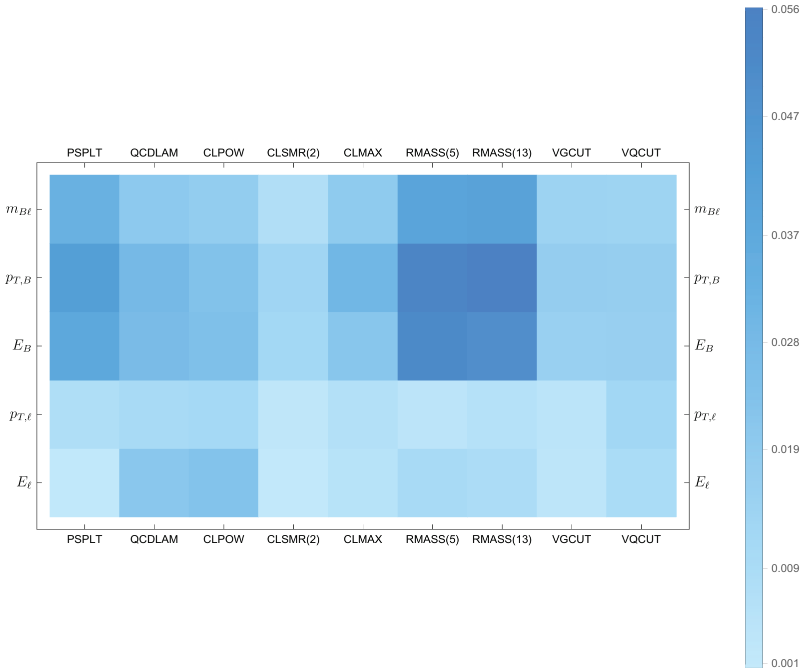
$\mathcal{O}$	Range	$\Delta_{m_t}^{\mathcal{M}_\mathcal{O}}$	$\Delta_\theta^{(m_t)}$						
			$\alpha_{s,FSR}$	m_b	$p_{T,\text{min}}$	a	b	r_B	recoil
E_B	28-110	0.92	-0.52	-0.21	0.057	-0.02	0.06	-0.10	-0.022
$p_{T,B}$	24-72	0.92	-0.54	-0.21	0.056	-0.03	0.07	-0.09	-0.023
$m_{B\ell,\text{true}}$	47-125	1.30	-0.241	-0.072	0.022	-0.007	0.023	-0.02	-0.008
$m_{B\ell^+,\text{min}}$	30-115	1.16	-0.282	-0.078	0.024	-0.011	0.021	-0.04	-0.010
$E_B + E_B$	83-244	0.92	-0.50	-0.21	0.056	-0.02	0.07	-0.08	-0.020
$m_{BB\ell\ell}$	172-329	0.96	-0.25	-0.10	0.028	-0.01	0.026	-0.03	-0.008
$m_{T2,B\ell,\text{true}}^{(\text{mET})}$	73-148	0.95	-0.27	-0.09	0.029	-0.009	0.03	-0.03	-0.010
$m_{T2,B\ell,\text{min}}^{(\text{mET})}$	73-148	0.95	-0.27	-0.09	0.029	-0.009	0.03	-0.03	-0.010
$m_{T2}^{(\ell\nu)}$	0.5-80	-0.118	-0.03	0.00	0.002	0.00	-0.01	0.00	0.004
$m_{\ell\ell}$	37.5-145	0.40	-0.03	-0.01	0.00	0.01	0.01	0.0	0.00
$E_\ell + E_\ell$	75-230	0.54	-0.03	0.00	0.003	0.01	-0.00	0.06	0.003
E_ℓ	23-100	0.48	-0.02	0.00	0.004	0.01	-0.01	-0.06	0.003

Final-state strong coupling constant needed at 1%, other parameters at 10%

Comparable parameter dependence of m_t observables in PYTHIA and HERWIG on m_t

HERWIG: FHWM Mellin moments of $m_{B\ell}$, $p_{T,B}$, E_B, $p_{T,\ell}$, E_ℓ

$\mathcal{O}$	$\Delta_{m_t}^{\mathcal{M}_\mathcal{O}}$	$\Delta_\theta^{(m_t)}$								
		PSPLT	QCDLAM	CLPOW	CLSMR	CLMAX	MASS(5)	MASS(13)	VGCUT	VQCUT
$m_{B\ell}$	0.52	0.036	-0.008	-0.007	0.002	-0.007	0.058	0.06	0.003	-0.003
$p_{T,B}$	0.47	0.072	-0.03	-0.02	0.0035	-0.03	0.11	0.12	0.0066	-0.006
E_B	0.43	0.069	-0.026	-0.017	0.0038	-0.01	0.12	0.12	0.006	-0.007
E_ℓ	0.13	0.0005	-0.04	0.04	-0.0002	-0.004	0.008	0.008	-0.002	0.008



PSPLT, m_b and m_g to be determined at $\mathcal{O}(10\%)$ for a 300 MeV accuracy on m_t

$\mathrm{QCDLAM} = \Lambda_{\overline{\mathrm{MS}}} \exp\left[K/(4\pi\beta_0)\right]$: 10% on QCDLAM yields 1-2% precision on $\alpha_S(m_Z)$

Other parameters: order of magnitude is enough to meet precision goals

Shape analysis beyond Mellin moments (average values): peaks and endpoints ($\breve{O}$)

$\mathcal{O}$	Range	$\Delta_{m_t}^{(\mathcal{O})}$	$\Delta_\theta^{(m_t)}$						
			$\alpha_{s,FSR}$	m_b	$p_{T,\text{min}}$	a	b	r_B	recoil
$E_{B,\text{peak}}$	35-85	0.8	-0.74	-0.26	0.05	-0.04	0.08	-0.07	-0.031
$\breve{m}_{B\ell,\text{true}}$	127-150	1.26	0.017	0.003	-0.006	-0.008	0.008	-0.016	-0.00042
$\breve{m}_{B\ell,\text{min}}$	127-150	1.28	-0.023	-0.022	0.006	-0.008	0.008	-0.02	-0.0001
$\breve{m}_{T2,B\ell,\text{true}}^{(\text{mET})}$	150-170	0.98	-0.01	-0.023	0.007	-0.006	0.010	-0.011	-0.0002
$\breve{m}_{T2,B\ell,\text{min}}^{(\text{mET})}$	150-170	0.97	-0.02	-0.021	0.006	-0.006	0.009	-0.01	-0.0001
$\breve{m}_{T2,B\ell,\text{min},\perp}^{(\text{mET})}$	138-170	0.89	-0.071	-0.046	0.012	-0.011	0.010	-0.01	-0.002
$\breve{m}_{T2,B}^{(\text{mET})}$	142-170	0.95	-0.089	-0.064	0.018	-0.017	0.031	-0.04	-0.0028
$\breve{m}_{T2,B,\perp}^{(\text{mET})}$	126-170	0.94	-0.07	-0.04	0.011	-0.009	0.02	-0.03	-0.001

E_B-peak exhibits substantial dependence on $\alpha_{S,FSR}$ and m_b, milder on other parameters

Mild dependence of endpoints on all parameters: $\Delta_\theta \leq 10^{-3} - 10^{-2}$

All observables pretty sensitive to top-quark mass: $\Delta_{m_t} \sim \mathcal{O}(1)$

Conclusions and outlook

Top mass of paramount importance in the Standard Model: reliable description of b-fragmentation is crucial for accurate m_t measurements

Predictions on top-decay observables yielded by HERWIG and PYTHIA exhibit discrepancies, mostly driven by the different quality of the tunings

Trading b-jets with B-hadrons: from JES to b-quark hadronization uncertainties

Novel investigation on dependence of top-mass observables on MC parameters: determination may be needed to 1% accuracy to meet 0.3% precision on m_t

Calibration observables (angular variables, hadron/jet ratios, etc.) exhibiting mild m_t-dependence to constrain *in situ* fragmentation parameters

Moments, endpoints, peaks of m_t-dependent observables to measure m_t after calibration

Tuning fragmentation parameters directly to LHC data ($t\bar{t}$, $b\bar{b}$, $Z/\gamma + b$)

Calibration and sensitivity analysis with POWHEG/aMC@NLO+PY8/HW7

Thorough investigation of colour reconnection effects

Monte Carlo parameters with errors (like pdfs) would be ultimately desirable