Higher-order corrections to scalar couplings in BSM models with extended Higgs sectors

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based on
JB, Goodsell, Krauss, Opferkuch, Staub, 1711.08460
JB, Goodsell, Slavich, 180x.xxxxx

SUSY 2018

July 23, 2018

The Higgs boson as a probe for New Physics

Indirect search for New Physics

- Strong motivation for going beyond the SM, however **no sign** yet of BSM Physics or its nature
- Investigate properties of Higgs boson to access New Physics **indirectly**
- Among most studied properties of the Higgs: its mass m_h

$$m_h^{\text{exp.}} = 125.09 \pm 0.21(\text{stat.}) \pm 0.11(\text{syst.}) \text{ GeV}$$

[ATLAS & CMS combined, Moriond 2015]

In this talk:

2 approaches to the relation between Higgs quartic couplings and masses, for general renormalisable theories

$\rightarrow$ loop-corrected extraction of quartics from physical spectrum ($i.e.$ scalar masses and mixing angles)

$\rightarrow$ EFT matching of quartics between generic theories

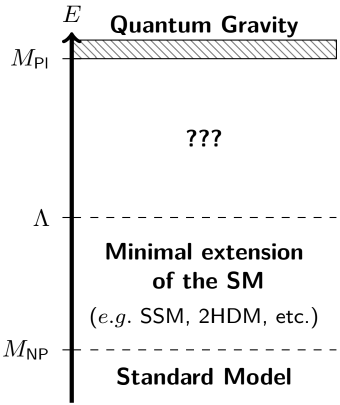
Extracting scalar quartic couplings from the physical spectrum

BSM models in the bottom-up approach

- Bottom-up approach $\rightarrow$ minimal extensions of SM, $e.g.$ with enlarged Higgs sectors
- **Up to what scale Λ can such a BSM model be extended?**
- Study behaviour of scalar quartic couplings at high scales:
  - $\rightarrow$ presence of Landau poles?
  - $\rightarrow$ loss of unitarity or perturbativity?
  - $\rightarrow$ EW vacuum becoming unstable?

$\Rightarrow$ would signal the need for additional states at the scale where the issue appears $\rightarrow$ cut-off Λ of the considered BSM extension

- Typical input in non-SUSY models: masses and mixing angles, from which quartics are **extracted** at a low scale
- **Problem:** most studies use only tree-level extraction of quartics $+$ 1ℓ (2ℓ) RGEs for running to high scales



$\Rightarrow$ We want to include **radiative corrections** in the extraction of couplings in generic BSM models

Extracting scalar quartics from the physical spectrum

In the absence of mixing

Higgs pole mass m_h related to Lagrangian coupling λ by

$$\underbrace{m_h^2}_{\text{pole mass}} = \underbrace{2\lambda v^2}_{\text{tree-level}} + \underbrace{\sum_\ell \frac{1}{(16\pi^2)^\ell}\Delta^{(\ell)} m_h^2(\lambda)}_{\text{loop corrections}}$$

$\rightarrow$ highly non-linear equation in λ...
but can be inverted analytically with a perturbative expansion of λ as

$$\lambda = \lambda^{(0)} + \frac{1}{16\pi^2}\delta^{(1)}\lambda + \frac{1}{(16\pi^2)^2}\delta^{(2)}\lambda + \cdots$$

$\rightarrow$ one obtains

$$\lambda^{(0)} = \frac{m_h^2}{2v^2}$$

$$\delta^{(1)}\lambda = -\frac{1}{2v^2}\Delta^{(1)} m_h^2 \,|_{\lambda=\lambda^{(0)}}$$

$$\delta^{(2)}\lambda = -\frac{1}{2v^2}\left[\delta^{(1)}\lambda \frac{\partial}{\partial\lambda}\Delta^{(1)} m_h^2 + \Delta^{(2)} m_h^2\right]_{\lambda=\lambda^{(0)}}$$

Extracting scalar quartics from the physical spectrum

An example without scalar mixing: the $\mathbb{Z}_2$SSM

$\mathbb{Z}_2$SSM $\equiv$ SM + real singlet S + $\mathbb{Z}_2$ symmetry under which $S \to -S$
$\Rightarrow$ no mixing between S and h

$$\boxed{V^{(0)} = \mu^2|\mathbf{H}|^2 + \tfrac{1}{2}M_S^2 S^2 + \tfrac{1}{2}\lambda|\mathbf{H}|^4 + \tfrac{1}{2}\lambda_{SH}S^2|\mathbf{H}|^2 + \tfrac{1}{2}\lambda_S S^4}$$

Extract λ from m_h at two loops

$$\begin{aligned}
\delta^{(1)}\lambda &= \delta^{(1)}_{\text{SM}}\lambda - \frac{1}{2}\lambda_{SH}^2\,\overline{\log}\, m_S^2\\
\delta^{(2)}\lambda &\simeq \delta^{(2)}_{\text{SM}}\lambda + \frac{9}{4v^2}\lambda_{SH}\lambda m_S^2(\overline{\log}\, m_S^2 - 1)\\
&\quad + \lambda_{SH}^3\Big[1 - 2\,\overline{\log}\, m_S^2 + \overline{\log}^2\, m_S^2\Big]\\
&\quad + \frac{1}{4}\lambda_{SH}^2\lambda\Big[-18 - 6\,\overline{\log}^2\, m_S^2\\
&\qquad\qquad + (36\,\overline{\log}\, m_h^2 - 12)\,\overline{\log}\, m_S^2\Big]\\
&\quad + 3\lambda_{SH}^2\lambda_S\Big[-1 + \overline{\log}\, m_S^2 + \overline{\log}^2\, m_S^2\Big] + \mathcal{O}(v^2/m_S^2)
\end{aligned}$$

Using $\overline{\log}\, m^2 \equiv \log m^2/Q^2$

Extracting scalar quartics from the physical spectrum

In presence of mixing

- Several relations between loop-corrected masses m_ϕ^2 and parameters/couplings λ_i

$$m_\phi^2 = \text{eig}[(\mathcal{M}^2)^{\text{loop}}(\{\lambda_i\})]_\phi$$

ϕ: list of fields

i: list of couplings

$\Rightarrow$ very difficult to invert analytically

- Instead proceed by numerical iterations: compute $m_\phi^2(\{\lambda_i, \cdots\})$ at given loop-order, for **varying** **Lagrangian parameters**, until results correspond to desired physical spectrum

Numerical iterations with SARAH/SPheno

- SARAH: spectrum generator generator by F. Staub $\rightarrow$ creates automatically for the desired model a spectrum generator, based on SPheno (by W. Porod)
- Currently available with SARAH/SPheno:
  - two-loop mass calculations* for neutral scalars in general renormalisable models, based on [Martin '01,'03,'05], [Goodsell, Nickel, Staub '15], and free of IR divergences from Goldstone bosons [JB, Goodsell '16], [JB, Goodsell, Staub '17] *(* in the gaugeless limit)*
  - two-loop RGEs for general QFTs [Machacek, Vaughn '83, '84, '85], [Luo, Wang, Xiao '02]

Extracting scalar quartics from the physical spectrum: an example in the 2HDM

CP-conserving 2HDM

$$V^{(0)} = m_{11}^2 \Phi_1^\dagger \cdot \Phi_1 + m_{22}^2 \Phi_2^\dagger \cdot \Phi_2 + m_{12}^2 \left[\Phi_1^\dagger \cdot \Phi_2 + \Phi_2^\dagger \cdot \Phi_1\right] + \lambda_1 \left(\Phi_1^\dagger \cdot \Phi_1\right)^2 + \lambda_2 \left(\Phi_2^\dagger \cdot \Phi_2\right)^2 + \lambda_3 \left(\Phi_1^\dagger \cdot \Phi_1\right)\left(\Phi_2^\dagger \cdot \Phi_2\right) + \lambda_4 \left(\Phi_1^\dagger \cdot \Phi_2\right)\left(\Phi_2^\dagger \cdot \Phi_1\right) + \frac{1}{2}\lambda_5 \left[\left(\Phi_1^\dagger \cdot \Phi_2\right)^2 + \left(\Phi_2^\dagger \cdot \Phi_1\right)^2\right]$$

- 2 $SU(2)_L$ doublets $\Phi_1,\, \Phi_2 \longrightarrow$ 5 physical d.o.f. $\underbrace{h,\, H}_{\text{CP-even}},\ \underbrace{A}_{\text{CP-odd}},\ \underbrace{H^\pm}_{\text{charged}}$
- Mixing among h and H $\rightarrow$ mixing angle α
- 7 free parameters in the Higgs sector

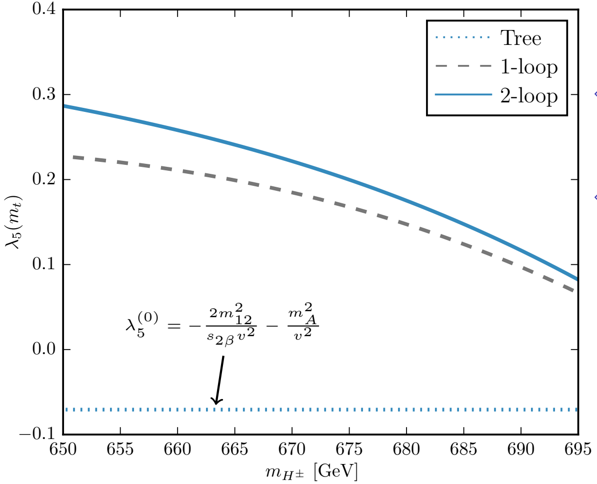
$$\lambda_1,\, \lambda_2,\, \lambda_3,\, \lambda_4,\, \lambda_5,\, m_{12}^2,\, \tan\beta \equiv v_2/v_1$$

(m_{11}^2, m_{22}^2 eliminated with tadpole equations)

- Common choice is to trade the quartics λ_i for m_h, m_H, m_A, $m_{H^\pm}$, $\tan\alpha$ $e.g.$ for λ_5 at tree level:

$$\lambda_5^{(0)} = -\frac{2m_{12}^2}{\sin 2\beta v^2} - \frac{m_A^2}{v^2}$$

Extracting scalar quartics from the physical spectrum: an example in the 2HDM



- Take $\tan\beta = 1.4$; $m_{12}^2 = -(500\text{ GeV})^2$
- Fix $m_h = 125$ GeV; $m_H = 750$ GeV; $m_A = 730$ GeV; $m_{H^\pm} \in [650, 695]$ GeV $\tan\alpha = -0.71$ at desired order

$\rightarrow$ **Can greatly modify the running of couplings to high-scales!**

(examples in backup)

Effective Field Theories and the matching of scalar couplings

Scalar couplings and Effective Field Theories

- Scale of New Physics M_{NP} driven higher by experimental searches $\rightarrow$ fixed-order calculations become plagued by large logs $\propto \log M_{\text{NP}}/m_{\text{EW}}$
- These logs need to be resummed $\rightarrow$ **EFT calculation** increasingly necessary
  - integrate out heavy fields and work in low energy EFT
  - couplings in the EFT receive threshold corrections at the matching scale
- In the context of Higgs mass calculations in SUSY models, heavy SUSY scenarios have been extensively investigated

  see $e.g$ [Bernal, Djouadi, Slavich '07], [Bagnaschi, Giudice, Slavich, Strumia '14], [Pardo Vega, Villadoro '15] [Bagnaschi, Pardo Vega, Slavich '17] + results of FlexibleEFTHiggs and FeynHiggs collaborations

  $\rightarrow$ Important matching conditions: **scalar quartic couplings** needed to compute m_h in the EFT!

  $\rightarrow$ EFTs typically considered to be SM or 2HDM, and UV theory is MSSM

- **Our objective**: automate the calculation of **threshold corrections** to scalar quartic couplings, when matching any high-energy model A onto any low-energy model B

$\rightarrow$ however there are challenges to address already from **one-loop order**!

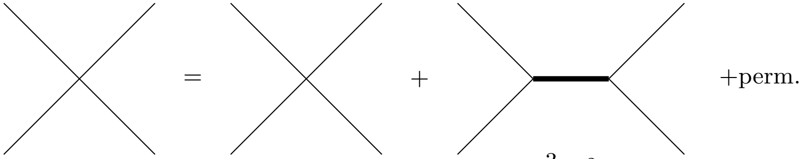
Matching of scalar couplings in a toy model at tree-level

- Consider a simple toy model: 2 scalars, a light L and a heavy H, with a $\mathbb{Z}_2$ symmetry under which $L \to -L \Rightarrow$ no mixing between L and H

High-energy model
$$\mathcal{L}_{\text{HE}} \supset -\frac{1}{2}m_L^2 L^2 - \frac{1}{2}m_H^2 H^2 - \frac{1}{2}a_{LLH}L^2 H - \frac{1}{6}a_{HHH}H^3 - \frac{1}{24}\tilde{\lambda}_{LLLL}L^4 - \frac{1}{4}\tilde{\lambda}_{LLHH}L^2H^2 - \frac{1}{24}\tilde{\lambda}_{HHHH}H^4$$

Low-energy model
$$\mathcal{L}_{\text{LE}} \supset -\frac{1}{2}m_L^2 L^2 - \frac{1}{24}\lambda_{LLLL}L^4$$

- Integrating out H, one finds at **tree-level**

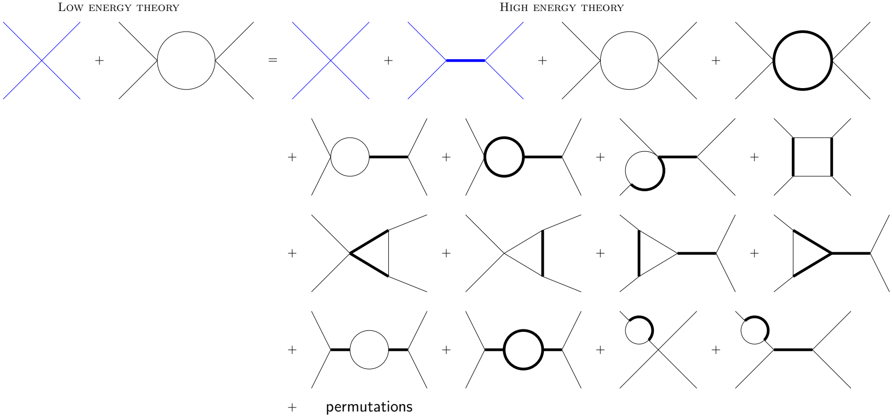


$$\lambda_{LLLL} = \tilde{\lambda}_{LLLL} + \frac{-3}{m_{HH}^2}a_{HLL}^2$$

thin line: light state; *thick line*: heavy state

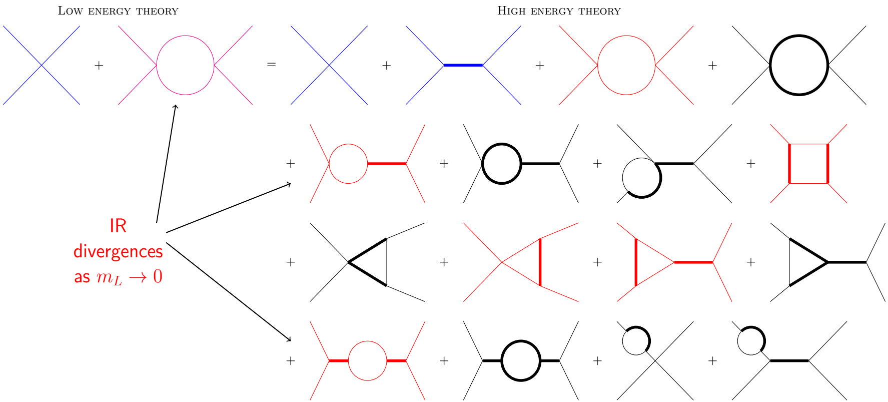
Matching of scalar couplings in a toy model at one loop

- Considering now the **one-loop** matching → many diagrams contribute!



thin line: light state; *thick line*: heavy state

Matching of scalar couplings in a toy model at one loop



- Several diagrams are IR divergent in limit $m_L \to 0$!
- IR parts in low and high energy theory **must** exactly cancel out, but automation impossible as is because of potentially large terms $\propto \log m_H/m_L$

$\rightarrow$ We have derived complete expressions for the matching of scalar couplings, at **one-loop** order, between two **generic** models

Matching quartic couplings between generic theories

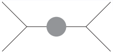
$$
\begin{aligned}
\lambda_{ijkl} + \delta\lambda_{ijkl} = {} & \tilde{\lambda}_{ijkl} + \delta\tilde{\lambda}_{ijkl} \\
& + \Bigg[-\frac{1}{8} m_{IJ}^{-2} a_{Iij} a_{Jkl} - \frac{1}{4} m_{IJ}^{-2} a_{Iij} \delta a_{Jkl} + \frac{1}{8 m_{IK}^2} \delta m_{KL}^2 \frac{1}{m_{LJ}^2} a_{Iij} a_{Jkl} \\
& \quad + \frac{1}{6} \frac{\delta m_{iK}^2}{m_K^2} \left(\tilde{\lambda}_{Kjkl} - \frac{3}{m_{IJ}^2} a_{IKj} a_{Jkl} \right) \\
& \quad + \frac{1}{12} \Pi'_{ii'}(0) \left(\tilde{\lambda}_{i'jkl} - \frac{3}{m_{IJ}^2} a_{Ii'j} a_{Jkl} \right) + (ijkl \text{ perms}) \Bigg]
\end{aligned}
$$

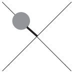
$i, j, k, l, \cdots$: indices for light states; $I, J, K, \cdots$: indices for heavy states,
$\delta\lambda_{ijkl}$, $\delta\tilde{\lambda}_{ijkl}$: corrections to the quartic couplings in the low and high-energy theories,
δa_{Ijk}: corrections to the trilinear coupling between one heavy and two light scalars,
δm^2: corrections to masses, Π': derivative of self-energies w.r.t. external momentum

- No IR divergence from diagrams with fermions or gauge bosons
- Expressions can be regularised by using modified (Passarino-Veltmann) loop functions

$$
B_0(0,0) \to 0, \quad C_0(0,0,X) \to -\frac{1}{X} B_0(0,X) = \frac{1}{X^2} A(X),
$$
$$
D_0(0,0,X,Y) \to -\frac{1}{X-Y} \left(\frac{1}{X^2} A(X) - \frac{1}{Y^2} A(Y) \right)
$$

Matching quartic couplings between generic theories





$$\lambda_{ijkl} + \delta\lambda_{ijkl} = \tilde{\lambda}_{ijkl} + \delta\tilde{\lambda}_{ijkl}$$
$$+ \Big[- \frac{1}{8m_{IJ}^2} a_{Iij} a_{Jkl} - \frac{1}{4m_{IJ}^2} a_{Iij} \delta a_{Jkl} + \frac{1}{8m_{IK}^2} \delta m_{KL}^2 \frac{1}{m_{LJ}^2} a_{Iij} a_{Jkl}$$
$$+ \frac{1}{6} \frac{\delta m_{iK}^2}{m_K^2} \left(\tilde{\lambda}_{Kjkl} - \frac{3}{m_{IJ}^2} a_{IKj} a_{Jkl} \right)$$
$$+ \frac{1}{12} \Pi'_{ii'}(0) \left(\tilde{\lambda}_{i'jkl} - \frac{3}{m_{IJ}^2} a_{Ii'j} a_{Jkl} \right) + (ijkl \text{ perms}) \Big]$$

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$$B_0(0,0) \to 0, \quad C_0(0,0,X) \to -\frac{1}{X} B_0(0,X) = \frac{1}{X^2} A(X),$$
$$D_0(0,0,X,Y) \to -\frac{1}{X-Y} \left(\frac{1}{X^2} A(X) - \frac{1}{Y^2} A(Y) \right)$$

- Redefinition of mass counter terms can allow eliminating δm_{KL}^2 and δm_{iK}^2

Summary

- For the extraction of scalar quartic couplings from the physical spectrum
  - Loop-corrected extraction is possible, analytically or by numerical iterations with `SARAH`/`SPheno`
  - Threshold corrections to quartics can be sizeable at the matching scale (low scale)
  - Conclusions on high-scale behaviour of models can be significantly modified w.r.t. using only a tree-level extraction of quartics
- For the matching of scalar quartic couplings in EFTs
  - matching of quartics between any two renormalisable theories can now be performed at one-loop order
  - IR divergences (in limit $m_L \to 0$) eliminated $\to$ no numerical instabilities
  - change of renormalisation scheme (from standard $\overline{\mathrm{MS}}/\overline{\mathrm{DR}}'$) allows further simplifying the expressions
  - results easily applicable to particular models: checks w.r.t. existing results in MSSM; applications for SUSY models beyond (N)MSSM, $e.g.$ Dirac gaugino models; etc.

Thank you for your attention!

Generic calculations: m_h with SARAH/SPheno

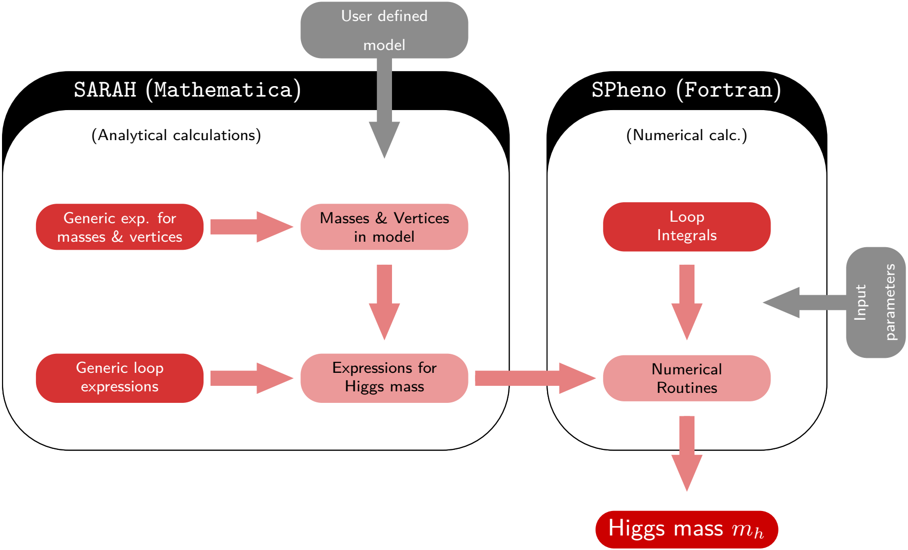


Figure by Florian Staub

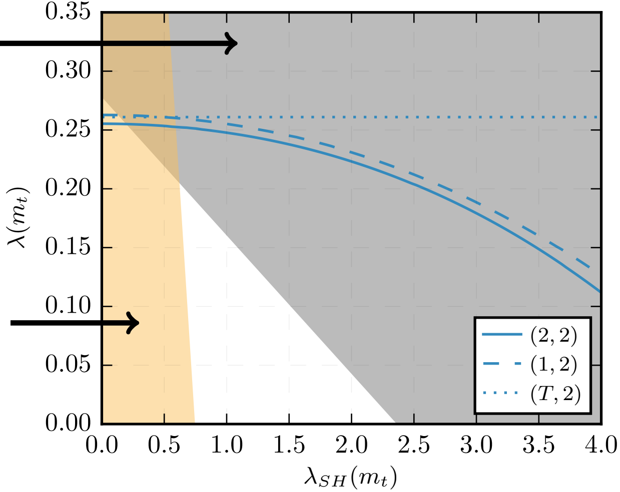
Impact of threshold corrections to scalar couplings: a phase diagram of the $\mathbb{Z}_2$SSM

EW vacuum is stable

perturbativity and unitarity not violated up to Planck scale

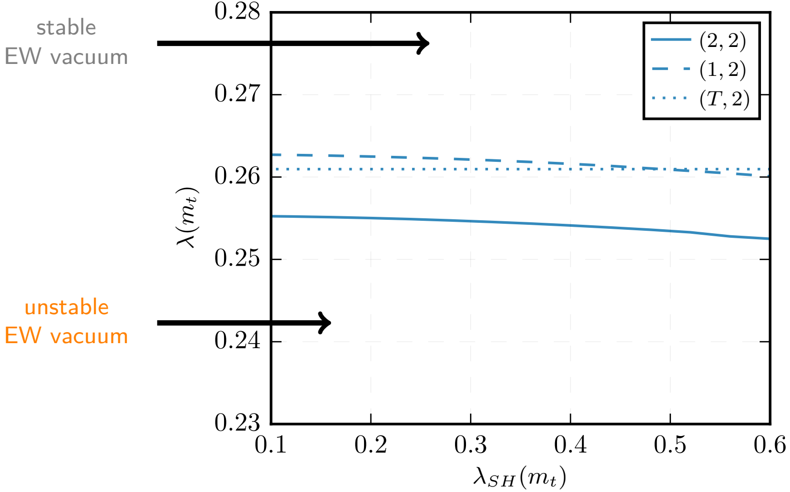
$\underbrace{\qquad\qquad}$

2ℓ RGEs used to run couplings



$\lambda_S = 0.1$, $M_S = 500$ GeV, $\lambda(m_t)$ extracted by requiring that $m_h = 125.15$ GeV

Impact of threshold corrections to scalar couplings: a phase diagram of the $\mathbb{Z}_2$SSM – zoomed



$\lambda_S = 0.1$, $M_S = 500$ GeV, $\lambda(m_t)$ extracted by requiring that $m_h = 125.15$ GeV

Singlet extension of the Standard Model

- Singlet extension of the SM (SSM) $\equiv$ SM + real singlet S, with scalar potential

$$V = \mu^2|\mathbf{H}|^2 + \frac{1}{2}M_S^2 S^2 + \kappa_1|\mathbf{H}|^2 S + \frac{1}{3}\kappa_2 S^3 + \frac{1}{2}\lambda|\mathbf{H}|^4 + \frac{1}{2}\lambda_{SH}S^2|\mathbf{H}|^2 + \frac{1}{2}\lambda_S S^4$$

  and S has a vev v_S.

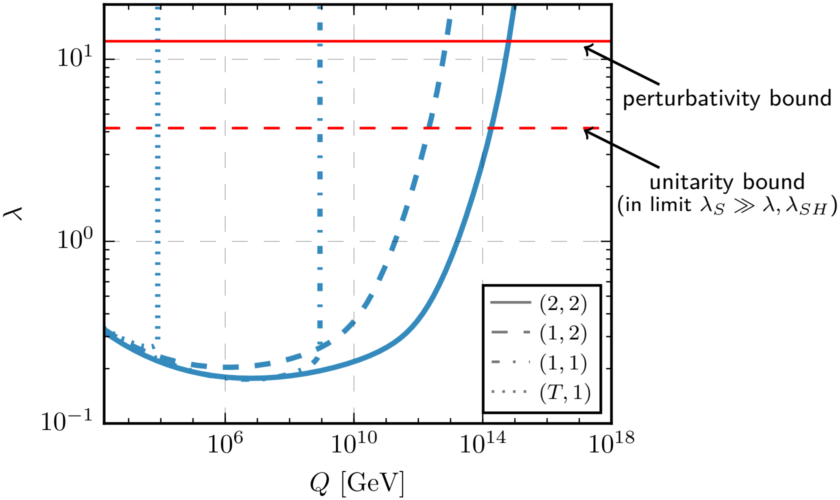
  $\rightarrow$ $\exists$ mixing in Higgs sector!
  $\rightarrow$ 2 mass eigenstates – h, H – and a mixing angle α

- $\mathbb{Z}_2$SSM $\equiv$ SSM + additional $\mathbb{Z}_2$ sym. under which S charged – *i.e.* $S \xrightarrow{\mathbb{Z}_2} -S$

  $\rightarrow$ $\kappa_1 = \kappa_2 = 0$, $v_S = 0$
  $\rightarrow$ no mixing in Higgs sector!
  $\Rightarrow$ two-loop analytic calculation of λ from m_h possible in $\mathbb{Z}_2$SSM!

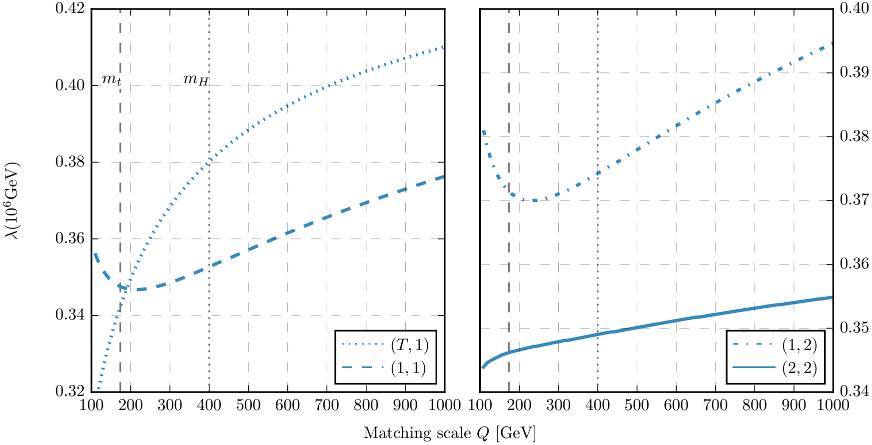
- Both SSM and $\mathbb{Z}_2$SSM can be found *e.g.* as low-energy theories from the NMSSM (without or with an R-parity)!

Impact of threshold corrections to scalar couplings in the SSM



λ, λ_S, λ_{SH} extracted, at $Q = m_t$, at n-loop order from the requirement that $m_h = 125$ GeV, $m_H = 700$ GeV, $\tan\alpha = 0.1$, and fixing $\kappa_1 = 0$ GeV, $\kappa_2 = 2$ TeV, $v_S = 175$ GeV.

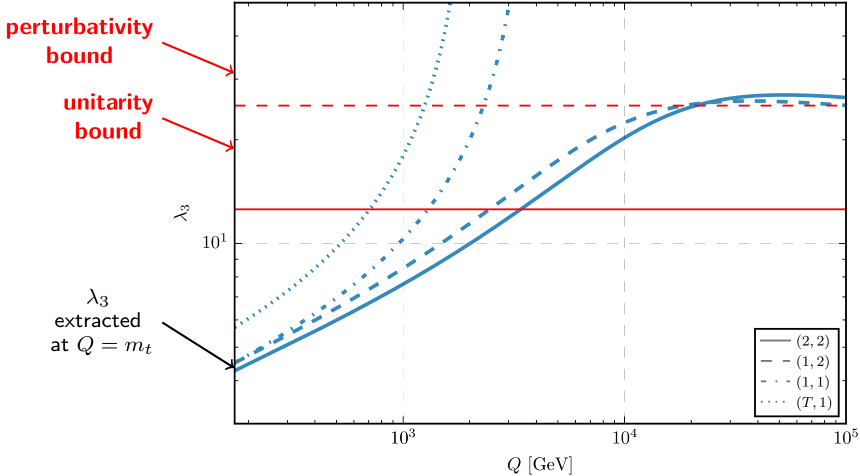
Varying the matching scale in the SSM



SSM parameters are taken at scale $Q \in [100, 1000]$ GeV to be: $\kappa_1 = \kappa_2 = 0$ and $v_S = 300$ GeV; the scalar quartics are extracted at each loop level by requiring that $m_h = 125$ GeV, $m_H = 400$ GeV and $t_\alpha = 0.3$.

Running of scalar couplings in 2HDMs

Impose at each order the scalar spectrum: $m_h = 125$ GeV, $m_H = 511$ GeV, $m_A = 607$ GeV, $m_{H\pm} = 605$ GeV, $t_\alpha = -0.82$ + we fix $t_\beta = 1.45$, $M_{12}^2 = -250^2$ GeV^2.



→ **Large differences in the cut-off scale of the theory obtained depending on the order at which the quartics are extracted !**