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Slepton non-universality in the flavor effective MSSM

Based on : JHEP 1711 (2017) 162 & arXiv:1807.00860

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Outlook

- Show results of two representative models with discrete flavor symmetries. Analysis of FV effects in leptonic sector.
- Application to a complete flavor model.

Motivations

Froggatt-Nielsen and flavor symmetries nice way yo explain SM flavor parameters

but…

- Flavor scale Λ_f **arbitrarily heavy**
- **Many possible choices** for flavor symmetry

  Abelian: $U(1)\,, SU(3)\,, \ldots$

  Non-abelian: $A_4\,, S_3\,, \Delta(27)\,, \ldots$

How to choose?
New flavor observables needed!

New flavor couplings generic feature of many NP models, in SUSY soft breaking terms:

$\left.\begin{array}{l}\textbf{trilinears interactions}\\ \textbf{sfermion soft masses}\end{array}\right\}$ fixed by m_0

but…

- If $\mathcal{O}(m_0)$ entries
  $\rightarrow$ **severe Flavor Violating problems**
- LHC won't give stronger mass limits over SUSY sparticles

New ways
to restrict parameter space are needed!

What about Flavor symmetries in SUSY ?

Flavor symmetry relates the structure in Yukawa matrices to the non-universality in Soft breaking terms

FV effects still present but controlled

Phenomenology of flavor symmetries

Constrain the MSSM parameter space

Review of the mechanism

SUSY breaking Mediation scale — $\Lambda_{\cancel{\mathrm{SUSY}}} \sim M_{Pl} \sim 10^{19}\,\mathrm{GeV}$

$M_{\mathrm{GUT}} \sim 10^{16}\,\mathrm{GeV}$

Λ_f?

SUSY Breaking Scale — $\Lambda_{\cancel{\mathrm{SUSY}}} \simeq \sqrt{\langle F_X \rangle} \sim 10^{11}\,\mathrm{GeV}$

Sparticles scale — $m_0 \simeq \dfrac{\langle F_X \rangle}{M_{Pl}} \sim 1\,\mathrm{TeV}$

M_{EW}

We need : $\Lambda_{\cancel{\mathrm{SUSY}}} \gg \Lambda_f$

for example gravity mediation : $\Lambda_{\cancel{\mathrm{SUSY}}} \sim M_{Pl}$

X: hidden sector spurion field
interacts gravitationally with visible sector
let's consider it single and universal

$$\mathcal{L}_{\mathrm{int}} = \frac{s}{M_{Pl}} X\, W_a^\alpha W_\alpha^a + \frac{b}{M_{Pl}} X^\dagger H_u H_d + \frac{a_{ij}}{M_{Pl}} X \psi_i \overline{\psi}_j H_{u,d} + \frac{c_{ij}}{M_{Pl}^2} X^\dagger X \psi_i^\dagger \psi_j + \mathrm{h.c.}$$

soft breaking interactions must respect G_f
different ways to couple the spurion field

$\rightarrow$ **mismatch coefficients :** c_{ij} a_{ij}**!**

SUSY broken in an Hidden sector by

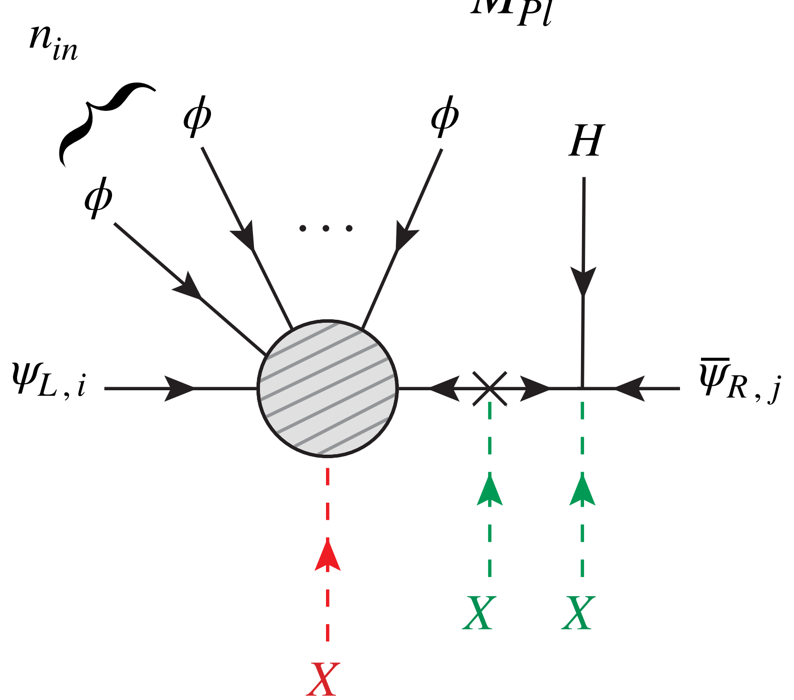
X getting $\langle F_X \rangle \neq 0$

$$\mathcal{L}_{\mathrm{int}} \rightarrow \mathcal{L}_{\mathrm{soft}}$$

Review of the mechanism

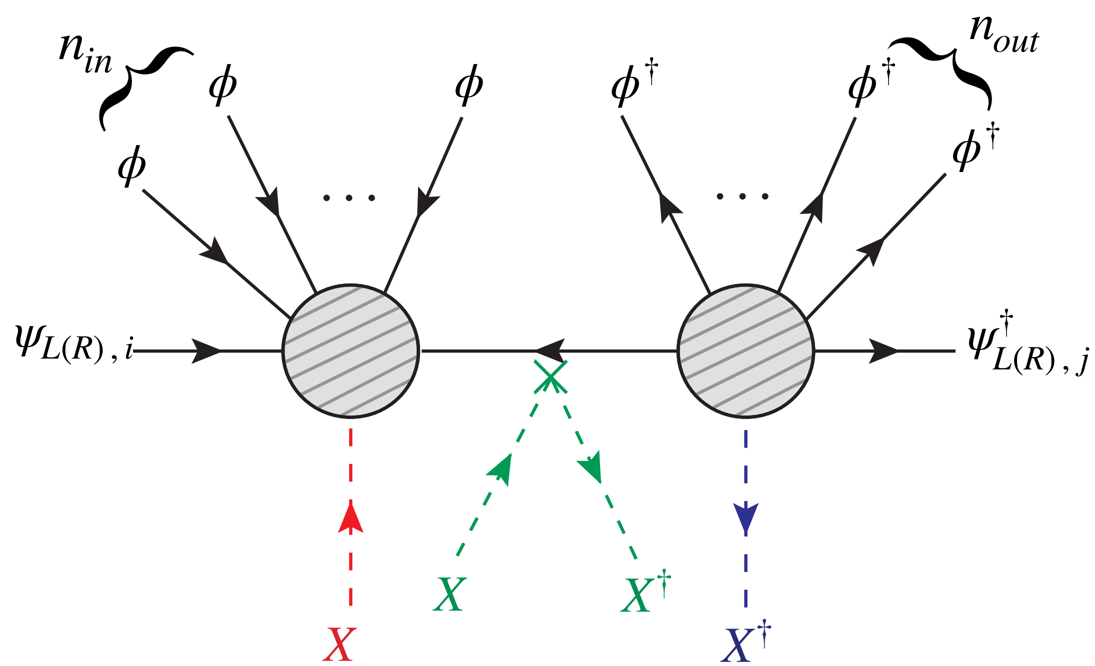
Mismatch coefficients * in $\mathscr{L}_{\text{soft}}$ are given by the number of flavon insertions in each diagram

Trilinear terms : $\dfrac{a_{ij}}{M_{Pl}} X \psi_i \overline{\psi}_j H_{u,d}$



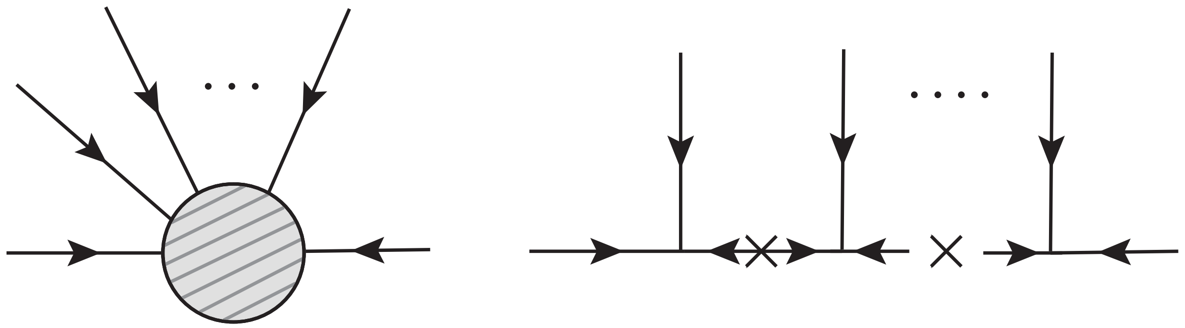
$$A_{ij} = a_0 \left[(2\, n_{in} - 1) + 2 \right] Y_{ij}$$

Soft mass terms : $\dfrac{c_{ij}}{M_{Pl}^2} X^\dagger X \psi_i^\dagger \psi_j$ + h.c.



$$m_{ij}^2 = m_0^2 \left[(2\, n_{in} - 1)(2\, n_{out} - 1) + 1 \right] K_{ij}$$

where each bubble is given by :



(*) Phys. Rev. D 95 (2017) no.3 [arXiv:1607.06827 [hep-ph]]

Bounds on FV processes *

- Lepton FV transitions would be a clear signal of New Physics!
- Variety of channels, most sensitive involving the muon.
- Next decade: **several experiments are planned to pursue the search for $\mu \to e\gamma$, $\mu \to eee$, $\mu \to e$** conversion in nuclei, as well as processes involving the τ, to an **unprecedented level of precision**.

Table 1: Relevant Flavor Violating (FV) processes considered in our analysis.

FV process	Current Bounds	Future Bounds
$\mathrm{BR}(\mu \to e\gamma)$	$\mathbf{4.2 \times 10^{-13}}$ (MEG at PSI)	4×10^{-14} (MEG II)
$\mathrm{BR}(\mu \to eee)$	1.0×10^{-12} (SINDRUM)	$\mathbf{10^{-16}}$ (Mu3e)
$\mathrm{CR}(\mu - e)_{A_l}$	-	$\mathbf{10^{-17}}$ (Mu2e , COMET)
$\mathrm{BR}(\tau \to e\gamma)$	3.3×10^{-8} (BaBar)	5×10^{-9} (Belle II)
$\mathrm{BR}(\tau \to \mu\gamma)$	4.4×10^{-8} (BaBar)	10^{-9} (Belle II)
$\mathrm{BR}(\tau \to eee)$	2.7×10^{-8} (Belle)	5×10^{-10} (Belle II)
$\mathrm{BR}(\tau \to \mu\mu\mu)$	2.1×10^{-8} (Belle)	5×10^{-10} (Belle II)
ΔM_K	$(52.89 \pm 0.09) \times 10^8\, \hbar\, s^{-1}$ (PDG)	
ϵ_K	$(2.228 \pm 0.011) \times 10^{-3}$ (PDG)	

(*) Riv. Nuovo Cim. 41 (2018) no.2 [arXiv:1709.00294 [hep-ph]]

$\ell_i \to \ell_j \gamma$ in the MIA approximation

$$\frac{BR(\ell_i \to \ell_j \gamma)}{BR(\ell_i \to \ell_j \nu_i \overline{\nu}_j)} = \frac{48\pi^3 \alpha}{G_F^2}\left(|\mathcal{A}_{ij}^L|^2 + |\mathcal{A}_{ij}^R|^2\right) \sim \frac{\alpha^3}{G_F^2}\frac{\delta_{ij}^2}{m_0^4}\tan\beta^2$$

$$\begin{aligned}
\mathcal{A}_{ij}^L &= \frac{\alpha_2}{4\pi}\frac{\overset{\sim\, m^{LL}_{\ell,ij}}{\delta^{LL}_{\ell,ij}}}{m_{\tilde{\ell}}^2}\Bigg[f_{1n}(a_2) + f_{1c}(a_2) + \frac{\mu M_2 \tan\beta}{M_2^2 - \mu^2}\left(f_{2n}(a_2, b) + f_{2c}(a_2, b)\right) \\
&\qquad + \tan\theta_W^2\left(f_{1n}(a_1) + \mu\, M_1 \tan\beta\left(\frac{f_{3n}(a_1)}{\tilde{m}_\ell^2} + \frac{f_{2n}(a_1, b)}{\mu^2 - M_1^2}\right)\right)\Bigg] \\
&+ \frac{\alpha_1}{4\pi}\frac{\overset{\sim\, A_{\ell,ij}}{\delta^{RL}_{\ell,ij}}}{m_{\tilde{\ell}}^2}\left(\frac{M_1}{m_{\ell_i}}\right) 2\, f_{2n}(a_1)
\end{aligned}$$

$$\begin{aligned}
\mathcal{A}_{ij}^R &= \frac{\alpha_1}{4\pi}\frac{\overset{\sim\, m^{RR}_{\ell,ij}}{\delta^{RR}_{\ell,ij}}}{m_{\tilde{\ell}}^2}\left[4\, f_{1n}(a_1) + \mu\, M_1 \tan\beta\left(\underbrace{\frac{f_{3n}(a_1)}{\tilde{m}_\ell^2} - \frac{2 f_{2n}(a_1, b)}{\mu^2 - M_1^2}}_{\textbf{Cancellation}}\right)\right] \\
&+ \frac{\alpha_1}{4\pi}\frac{\overset{\sim\, A_{\ell,ij}}{\delta^{LR}_{\ell,ij}}}{m_{\tilde{\ell}}^2}\left(\frac{M_1}{m_{\ell_i}}\right) 2\, f_{2n}(a_1)
\end{aligned}$$

(*) JHEP 10 (2005) 006, [hep-ph/0505046]

An A_4 model example * : Superpotential

Field	ν^c	ℓ	e^c	μ^c	τ^c	H_d	H_u	ϕ_S	ϕ_T	ξ	ξ'	$\xi'^\dagger$
A_4	**3**	**3**	**1**	**1**	**1**	**1**	**1**	**3**	**3**	**1**	**1′**	**1″**
Z_4	-1	i	1	i	-1	1	i	1	i	1	i	-i
$U(1)_R$	1	1	1	1	1	0	0	0	0	0	0	0

Table 1: Transformation of the matter and flavon superfields under $\mathcal{G}_f = A_4 \times Z_4$

Reproduces the **lepton** hierarchies and mixings TBM + $\boldsymbol{\theta_{13}}$**!** $\sim \mathcal{O}(\varepsilon')$

Alignment: $\frac{\langle\phi_T\rangle}{M} \propto \varepsilon \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, $\frac{\langle\phi_S\rangle}{M} \propto \varepsilon' \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, $\frac{\langle\xi\rangle}{M} \propto \varepsilon'$, $\frac{\langle\xi'\rangle}{M} \propto \varepsilon$

$$A_{ij} = a_0\,[(2\,\mathrm{n}_{in} - 1) + 2\,]\,Y_{ij}$$

Superpotential

$$\begin{aligned}
\textbf{LO: } \mathcal{W}_\ell \;&=\; \frac{1}{M}\,\tau^c(\ell\phi_T)\,H_d \\
&+\; \frac{1}{M^2}\,\mu^c\left[(\ell\phi_T^2) \;+\; (\ell\phi_T)''\xi'\right] H_d \\
&+\; \frac{1}{M^3}\,e^c\left[(\ell\phi_T^3) \;+\; (\ell\phi_T^2)''\xi' \;+\; (\ell\phi_T)'\xi'^2\right] H_d \\
\textbf{NLO: } \delta\mathcal{W}_\ell \;&=\; \frac{1}{M^2}\,\tau^c\left[(\ell\phi_T\phi_S) \;+\; (\ell\phi_S)''\xi'\right] H_d \\
&+\; \frac{1}{M^3}\,\mu^c\left[(\ell\phi_T^2\phi_S) \;+\; (\ell\phi_T\phi_S)''\xi' \;+\; (\ell\phi_S)'\xi'^2\right] H_d \\
&+\; \frac{1}{M^4}\,e^c\left[(\ell\phi_T^3\phi_S) \;+\; (\ell\phi_T^2\phi_S)''\xi' \;+\; (\ell\phi_T\phi_S)'\xi'^2 \;+\; (\ell\phi_S)\xi'^3\right] H_d
\end{aligned}$$

$$Y_\ell \sim \begin{pmatrix} x_1\,\varepsilon^3 & x_2\,\varepsilon^3\varepsilon' & x_3\,\varepsilon^3\varepsilon' \\ x_4\,\varepsilon^2\varepsilon' & x_5\,\varepsilon^2 & x_6\,\varepsilon^2\varepsilon' \\ x_7\,\varepsilon\,\varepsilon' & x_8\,\varepsilon\,\varepsilon' & x_9\,\varepsilon \end{pmatrix}$$

$$A_\ell \sim a_0 \begin{pmatrix} \mathbf{7}\,x_1\,\varepsilon^3 & \mathbf{9}\,x_2\,\varepsilon^3\varepsilon' & \mathbf{9}\,x_3\,\varepsilon^3\varepsilon' \\ \mathbf{7}\,x_4\,\varepsilon^2\varepsilon' & \mathbf{5}\,x_5\,\varepsilon^2 & \mathbf{7}\,x_6\,\varepsilon^2\varepsilon' \\ \mathbf{5}\,x_7\,\varepsilon\,\varepsilon' & \mathbf{5}\,x_8\,\varepsilon\,\varepsilon' & \mathbf{3}\,x_9\,\varepsilon \end{pmatrix}$$

(*) J. Phys. G36 (2009) 085005, [0905.0620].

An A_4 model example : Kähler potential

Field	ν^c	ℓ	e^c	μ^c	τ^c	H_d	H_u	ϕ_S	ϕ_T	ξ	ξ'	$\xi'^\dagger$
A_4	**3**	**3**	**1**	**1**	**1**	**1**	**1**	**3**	**3**	**1**	**1′**	**1″**
Z_4	-1	i	1	i	-1	1	i	1	i	1	i	-i
$U(1)_R$	1	1	1	1	1	0	0	0	0	0	0	0

Reproduces the **lepton** hierarchies and mixings TBM + $\boldsymbol{\theta_{13}}$**!** $\sim \mathcal{O}(\varepsilon')$

Table 1: Transformation of the matter and flavon superfields under $\mathcal{G}_f = A_4 \times Z_4$

Alignment: $\frac{\langle\phi_T\rangle}{M} \propto \varepsilon \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, $\frac{\langle\phi_S\rangle}{M} \propto \varepsilon' \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, $\frac{\langle\xi\rangle}{M} \propto \varepsilon'$, $\frac{\langle\xi'\rangle}{M} \propto \varepsilon$

$$\boldsymbol{m^2_{ij} = m^2_0\,[(2\,n_{in}-1)(2\,n_{out}-1)+1]\,K_{ij}}$$

(LH) Kähler potential

$$K_{\ell,L} = \ell\,\ell^\dagger + \frac{1}{M^2}\left[(\ell\,\ell^\dagger\,\phi_S\,\phi_S^\dagger) + (\ell\,\ell^\dagger\,\phi_S)\,\xi^\dagger\right] + \text{h.c.}$$

$$K_{\ell,L} \sim \mathbb{1} + \begin{pmatrix} \varepsilon^2+\varepsilon'^2 & \varepsilon'^2 & \varepsilon'^2 \\ \varepsilon'^2 & \varepsilon^2+\varepsilon'^2 & \varepsilon'^2 \\ \varepsilon'^2 & \varepsilon'^2 & \varepsilon^2+\varepsilon'^2 \end{pmatrix}$$

$$m^2_{\ell,L} \sim m_0^2\,\mathbb{1} + \mathbf{2}\,m_0^2 \begin{pmatrix} \varepsilon^2+\varepsilon'^2 & \varepsilon'^2 & \varepsilon'^2 \\ \varepsilon'^2 & \varepsilon^2+\varepsilon'^2 & \varepsilon'^2 \\ \varepsilon'^2 & \varepsilon'^2 & \varepsilon^2+\varepsilon'^2 \end{pmatrix}$$

(RH) Kähler potential

$$K_{\ell,R} = e^c e^{c\dagger} + \mu^c\mu^{c\dagger} + \tau^c\tau^{c\dagger} + \frac{1}{M^2}\left[e^c(\phi_T\phi_S^\dagger)\mu^{c\dagger} + \mu^c(\phi_T\phi_S^\dagger)\tau^{c\dagger}\right] + \frac{1}{M^3}\,e^c\left[(\phi_S\phi_T^{\dagger\,2}) + (\phi_S\phi_T^\dagger)'\xi'^\dagger + \text{h.c.}\right]\tau^{c\dagger} + \text{h.c.}$$

$$K_{\ell,R} \sim \mathbb{1} + \begin{pmatrix} \varepsilon^2+\varepsilon'^2 & \varepsilon\,\varepsilon' & \varepsilon^2\varepsilon' \\ \varepsilon\,\varepsilon' & \varepsilon^2+\varepsilon'^2 & \varepsilon\,\varepsilon' \\ \varepsilon^2\varepsilon' & \varepsilon\,\varepsilon' & \varepsilon^2+\varepsilon'^2 \end{pmatrix}$$

$$m^2_{\ell,R} \sim m_0^2\,\mathbb{1} + m_0^2 \begin{pmatrix} \mathbf{2}\,(\varepsilon^2+\varepsilon'^2) & \mathbf{2}\,\varepsilon\,\varepsilon' & \mathbf{4}\,\varepsilon^2\varepsilon' \\ \mathbf{2}\,\varepsilon\,\varepsilon' & \mathbf{2}\,(\varepsilon^2+\varepsilon'^2) & \mathbf{2}\,\varepsilon\,\varepsilon' \\ \mathbf{4}\,\varepsilon^2\varepsilon' & \mathbf{2}\,\varepsilon\,\varepsilon' & \mathbf{2}\,(\varepsilon^2+\varepsilon'^2) \end{pmatrix}$$

An A_4 model example : Soft terms in physical basis

2 rotations to go to the physical basis

- **Canonical rotation:** Kähler is the identity
- **Mass basis rotation:** Yukawas are diagonal

$$\boldsymbol{K_{\ell,L} \longrightarrow U^\dagger_{K_L}\, K_{\ell,L}\, U_{K_L}} = \mathbb{1} \;\; , \;\; \boldsymbol{K_{\ell,R} \longrightarrow U^\dagger_{K_R}\, K_{\ell,R}\, U_{K_R}} = \mathbb{1} \;\; , \;\; \boldsymbol{Y_\ell \longrightarrow V^\dagger_Y\, U^\dagger_{K_L}\, Y_\ell\, U_{K_R}\, U_Y = Y_\ell^{(diag)}}$$

$$\boldsymbol{A_\ell} \;\longrightarrow\; V_Y^{-1}\, U^\dagger_{K_L}\, A_\ell\, U_{K_R}\, U_Y \;=\; a_0 \begin{pmatrix} 7\,x_1\,\varepsilon^3 & \left(4\,x_2 + 2\dfrac{x_1 x_4}{x_5}\right)\varepsilon^3\varepsilon' & \left(6\,x_3 + 4\dfrac{x_1 x_7}{x_9}\right)\varepsilon^3\varepsilon' \\ 2\,x_4\,\varepsilon^2\varepsilon' & 5\,x_5\,\varepsilon^2 & \left(4\,x_6 + 2\dfrac{x_5 x_8}{x_9}\right)\varepsilon^2\varepsilon' \\ 2\,x_7\,\varepsilon\,\varepsilon' & 2\,x_8\,\varepsilon\,\varepsilon' & 3\,x_9\,\varepsilon \end{pmatrix}$$

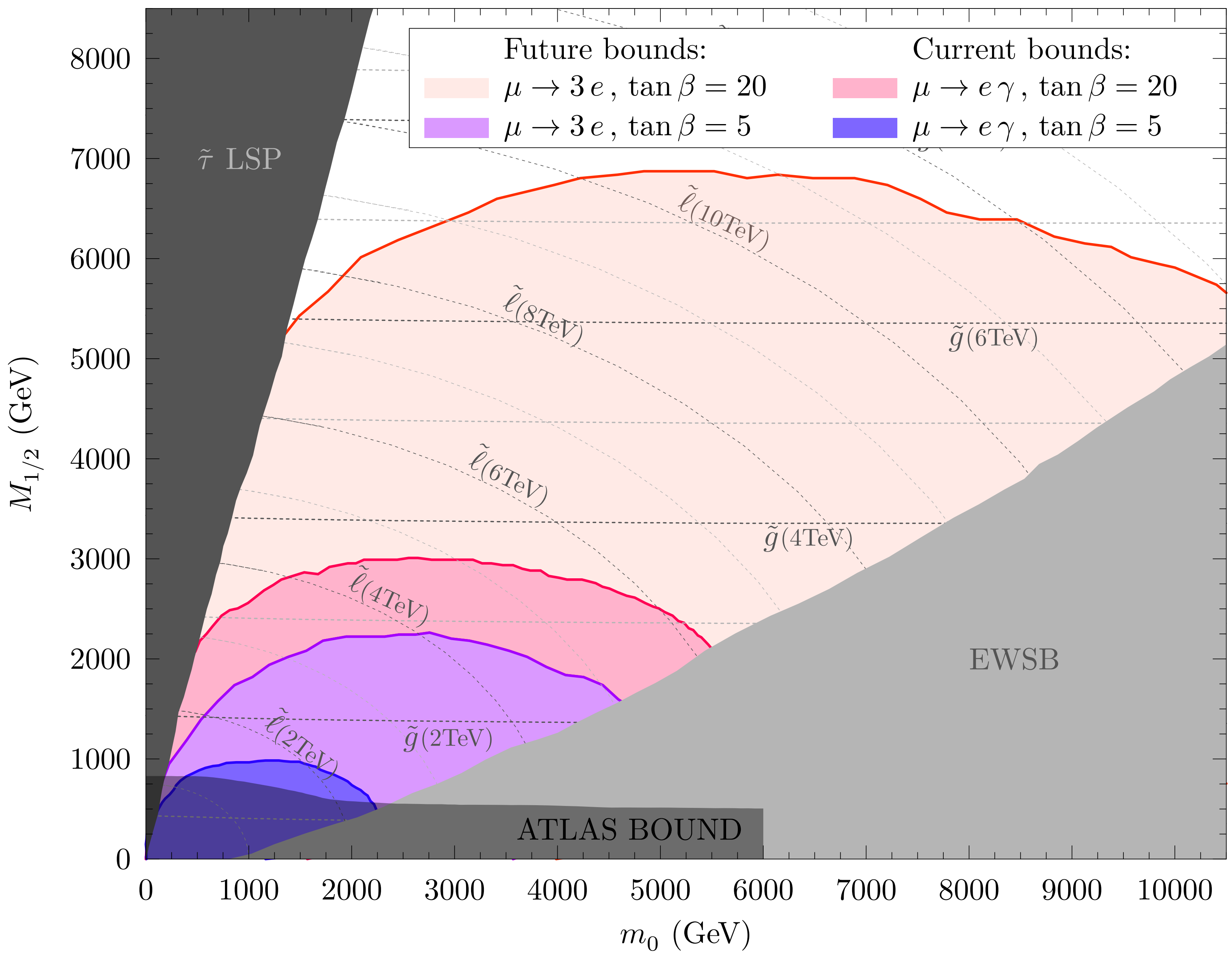
$$\boldsymbol{m^2_{\ell,L}} \;\longrightarrow\; V_Y^{-1}\, U^\dagger_{K_L} m^2_{\ell,L}\, U_{K_L} V_Y \;=\; m_0^2\,\mathbb{1} + m_0^2 \begin{pmatrix} \varepsilon^2 + \varepsilon'^2 & \varepsilon'^2 & \varepsilon'^2 \\ \varepsilon'^2 & \varepsilon^2 + \varepsilon'^2 & \varepsilon'^2 \\ \varepsilon'^2 & \varepsilon'^2 & \varepsilon^2 + \varepsilon'^2 \end{pmatrix}$$

Do not get diagonalized!

$$\boldsymbol{m^2_{\ell,R}} \;\longrightarrow\; U_Y^{-1}\, U^\dagger_{K_R} m^2_{\ell,R}\, U_{K_R} U_Y \;=\; m_0^2\,\mathbb{1} + m_0^2 \begin{pmatrix} \varepsilon^2 + \varepsilon'^2 & \varepsilon\,\varepsilon' & 3\,\varepsilon^2\varepsilon' + \left(\dfrac{x_4}{x_5} - \dfrac{x_8}{x_9}\right)\varepsilon\,\varepsilon'^2 \\ \varepsilon\,\varepsilon' & \varepsilon^2 + \varepsilon'^2 & \varepsilon\,\varepsilon' \\ 3\,\varepsilon^2\varepsilon' + \left(\dfrac{x_4}{x_5} - \dfrac{x_8}{x_9}\right)\varepsilon\,\varepsilon'^2 & \varepsilon\,\varepsilon' & \varepsilon^2 + \varepsilon'^2 \end{pmatrix}$$

An A_4 model example : FV effects

FIGURE 1: **Excluded regions due to $\mu \to e\gamma$ and $\mu \to 3\,e$ in A_4**



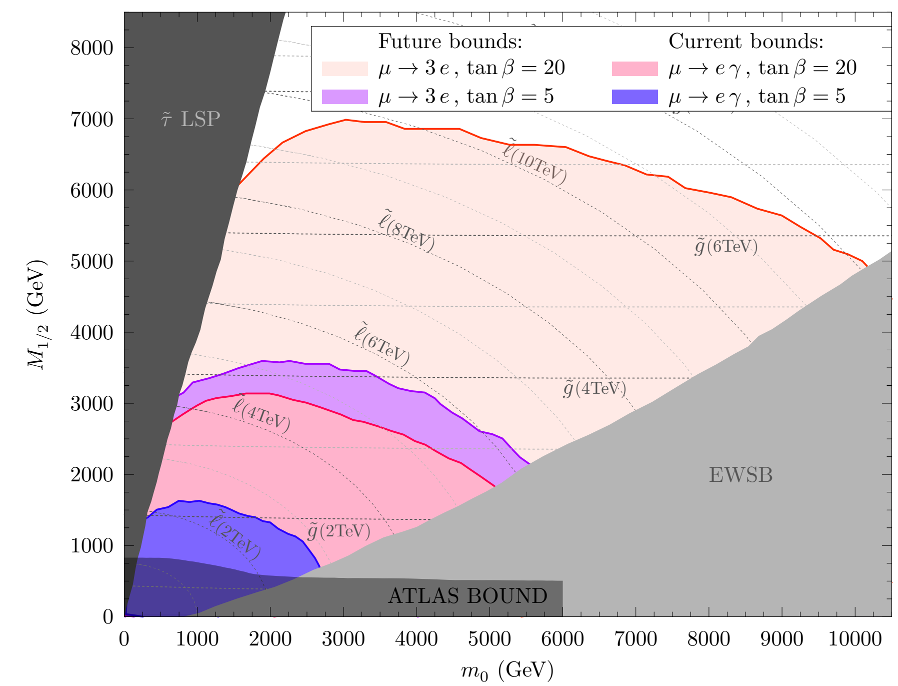


FIGURE 2 : Results of an S_3 Model

Running to the EW scale with the SPheno package

Dominant contribution comes from LL - mass insertion

$\tan\beta$ – enhanced

(*) JHEP 1711 (2017) 162 [arXiv:1710.02593 [hep-ph]]

An $\Delta(27)$ model example * : Superpotential

Field	ℓ, ν	ℓ^c, ν^c	$H_{u,d}$	Σ	ϕ_{123}	ϕ_1	$\bar{\phi}_3$	$\bar{\phi}_{23}$	$\bar{\phi}_{123}$
$\Delta(27)$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$
Z_2	1	1	1	1	1	-1	-1	-1	-1
$U(1)_{FN}$	0	0	0	2	-1	-4	0	-1	1
$U(1)_R$	1	1	0	0	0	0	0	0	0

Reproduces the charged **lepton and quark** hierarchies and mixings CKM + TBM

Table 1: Transformation of matter superfields under $\mathcal{G}_f = \Delta(27) \times Z_2 \times U(1)_{FN}$

Alignment: $\frac{\langle \phi_3 \rangle}{M} = \sqrt{y_\tau} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, $\frac{\langle \phi_{23} \rangle}{M} = \sqrt{y_\tau}\, \varepsilon \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$, $\frac{\langle \phi_{123} \rangle}{M} = \sqrt{y_\tau}\, \varepsilon^2 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, $\langle \phi_1 \rangle \propto \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\frac{\langle \Sigma \rangle}{M} = -3$

Superpotential

$$\textbf{LO:}\ \mathcal{W}_\ell \;=\; \frac{1}{M^2} (\ell\, \bar{\phi}_3)(\ell^c\, \bar{\phi}_3)\, H_d \;+\; \frac{1}{M^2} (\ell\, \bar{\phi}_{23})(\ell^c\, \bar{\phi}_{123})\, H_d \;+\; \frac{1}{M^2} (\ell\, \bar{\phi}_{123})(\ell^c \bar{\phi}_{23})\, H_d + \frac{1}{M^3} (\ell\, \bar{\phi}_{23})\, (\ell^c\, \bar{\phi}_{23})\, \Sigma\, H_d$$

$$Y_\ell \sim y_\tau \begin{pmatrix} 0 & -x_2\, \varepsilon^3 & x_2\, \varepsilon^3 \\ -x_3\, \varepsilon^3 & 3\, x_1\, \varepsilon^2 & -3\, x_1 \varepsilon^2 \\ x_3\, \varepsilon^3 & -3\, x_1\, \varepsilon^2 & 1 \end{pmatrix} \qquad A_\ell \sim y_\tau\, a_0 \begin{pmatrix} 0 & -\mathbf{5}\, x_2\, \varepsilon^3 & \mathbf{5}\, x_2\, \varepsilon^3 \\ -\mathbf{5}\, x_3\, \varepsilon^3 & \mathbf{21}\, x_1\, \varepsilon^2 & -\mathbf{21}\, x_1\, \varepsilon^2 \\ \mathbf{5}\, x_3\, \varepsilon^3 & -\mathbf{21}\, x_1\, \varepsilon^2 & \mathbf{5} \end{pmatrix}$$

$$\frac{\langle \phi_3 \rangle}{M_3} \gg \frac{\langle \phi_{23} \rangle}{M_{23}} \gg \frac{\langle \phi_{123} \rangle}{M_{123}}$$

(*) Phys. Lett. B648 (2007) 201 206, [hep-ph/0607045]

An $\Delta(27)$ model example : Kähler potential

Field	ℓ, ν	ℓ^c, ν^c	$H_{u,d}$	Σ	ϕ_{123}	ϕ_1	$\bar{\phi}_3$	$\bar{\phi}_{23}$	$\bar{\phi}_{123}$
$\Delta(27)$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$
Z_2	1	1	1	1	1	-1	-1	-1	-1
$U(1)_{FN}$	0	0	0	2	-1	-4	0	-1	1
$U(1)_R$	1	1	0	0	0	0	0	0	0

Reproduces the charged **lepton and quark** hierarchies and mixings CKM + TBM

Table 1: Transformation of matter superfields under $\mathcal{G}_f = \Delta(27) \times Z_2 \times U(1)_{FN}$

Alignment: $\frac{\langle \phi_3 \rangle}{M} = \sqrt{y_\tau} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, $\frac{\langle \phi_{23} \rangle}{M} = \sqrt{y_\tau}\, \varepsilon \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$, $\frac{\langle \phi_{123} \rangle}{M} = \sqrt{y_\tau}\, \varepsilon^2 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, $\langle \phi_1 \rangle \propto \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\frac{\langle \Sigma \rangle}{M} = -3$

(RH) Kähler potential

$$\begin{aligned} K_{\ell,R} \;=\;& \ell^c \ell^{c\dagger} + \frac{1}{M^2}\Big[(\ell^c \bar{\phi}_3)(\bar{\phi}_3^\dagger \ell^{c\dagger}) + (\ell^c \bar{\phi}_{23})(\bar{\phi}_{23}^\dagger \ell^{c\dagger}) + (\ell^c \bar{\phi}_{123})(\bar{\phi}_{123}^\dagger \ell^{c\dagger})\Big] \\ &+ \frac{1}{M^3}\Big[(\ell^c \bar{\phi}_{23})(\bar{\phi}_{123}^\dagger \ell^{c\dagger})\,\Sigma + \text{h.c.}\Big] + \frac{1}{M^5}\Big[(\ell^c \bar{\phi}_{123})\,(\bar{\phi}_{23}^\dagger \ell^{c\dagger})\,(\bar{\phi}_3 \phi_1)\,\Sigma + \text{h.c.}\Big] \end{aligned}$$

$\boldsymbol{\varepsilon_u \neq \varepsilon_d}$

mediators : LH $\gg$ RH

$$K_{\ell,R} = \mathbb{1} + y_\tau \begin{pmatrix} \varepsilon^4 & -3\,(1+ y_\tau)\,\varepsilon^3 & 3\,(1+ y_\tau)\,\varepsilon^3 \\ -3\,(1+ y_\tau)\,\varepsilon^3 & \varepsilon^2 & -\varepsilon^2 \\ 3\,(1+ y_\tau)\,\varepsilon^3 & -\varepsilon^2 & 1 \end{pmatrix}$$

$$m^2_{\ell,R} = m_0^2 \mathbb{1} + m_0^2\, y_\tau \begin{pmatrix} \mathbf{2}\,\varepsilon^4 & -3\,(\mathbf{4}+\mathbf{8}\,y_\tau)\,\varepsilon^3 & 3\,(\mathbf{4}+\mathbf{8}\,y_\tau)\,\varepsilon^3 \\ -3\,(\mathbf{4}+\mathbf{8}\,y_\tau)\,\varepsilon^3 & \mathbf{2}\,\varepsilon^2 & -\mathbf{2}\,\varepsilon^2 \\ 3\,(\mathbf{4}+\mathbf{8}\,y_\tau)\,\varepsilon^3 & -\mathbf{2}\,\varepsilon^2 & \mathbf{2} \end{pmatrix}$$

An $\Delta(27)$ model example : Soft terms

2 rotations to go to the physical basis

• **Canonical rotation:** Kähler is the identity • **Mass basis rotation:** Yukawas are diagonal

$$\boldsymbol{K_{\ell,R}} \longrightarrow \boldsymbol{U^\dagger_{K_R}}\,\boldsymbol{K_{\ell,R}}\,\boldsymbol{U_{K_R}} = \mathbb{1} \quad , \quad \boldsymbol{Y_\ell} \longrightarrow \boldsymbol{V^\dagger_Y}\,\boldsymbol{Y_\ell}\,\boldsymbol{U_{K_R}}\,\boldsymbol{U_Y} = \boldsymbol{Y_\ell^{(diag)}}$$

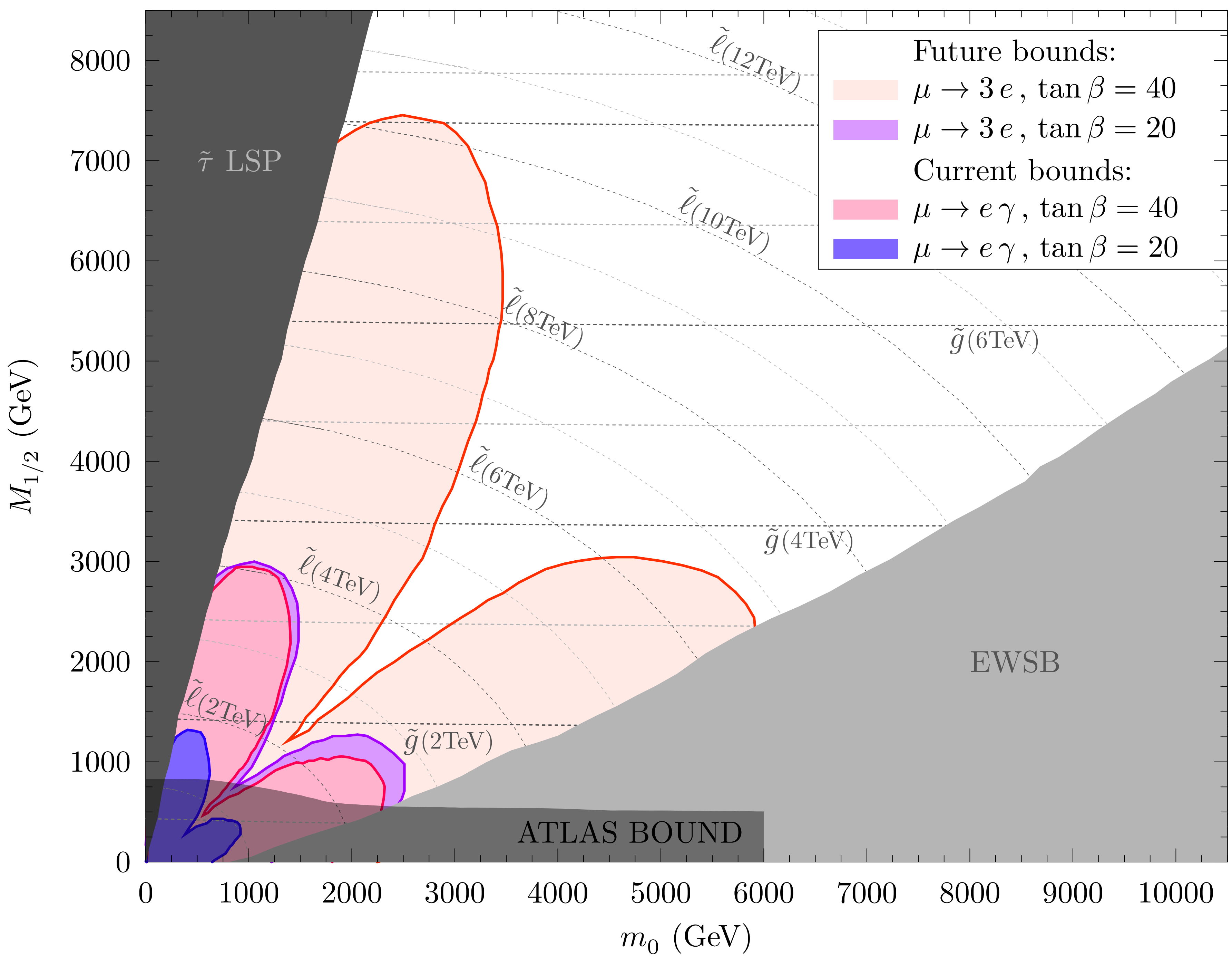
$$\boldsymbol{A_\ell} \longrightarrow a_0\, y_\tau \begin{pmatrix} \frac{x_2 x_3}{x_4}\varepsilon^4 & 2\,x_2\,\varepsilon^3 & -2\,\frac{x_2}{x_5}\,\varepsilon^3 \\ 2\,x_2\,\varepsilon^3 & 24\,x_4\,\varepsilon^2 & -6\,x_4\,\varepsilon^2 \\ -2\,x_2\,\varepsilon^3 & -6\,x_4\,\varepsilon^2 & 5 \end{pmatrix}$$

Do not get diagonalized !

$$\boldsymbol{m^2_{\ell,R}} \longrightarrow m_0^2\,\mathbb{1} + m_0^2\, y_\tau \begin{pmatrix} 0 & -3\,(3+7y_\tau)\,\varepsilon^3 & 3\left(3+\frac{11}{2}y_\tau - \frac{x_2}{3\,x_4}\right)\varepsilon^3 \\ -3\,(3+7y_\tau)\,\varepsilon^3 & \varepsilon^2 & -\,(1-3\,x_4)\,\varepsilon^2 \\ 3\left(3+\frac{11}{2}y_\tau - \frac{x_2}{3\,x_4}\right)\varepsilon^3 & -\,(1-3\,x_4)\,\varepsilon^2 & 1 \end{pmatrix}$$

An $\Delta(27)$ model example : FV effects

FIGURE 3: **Excluded regions due to $\mu \to e\gamma$ and $\mu \to 3\,e$ in $\Delta(27)$**



JHEP 1711 (2017) 162 [arXiv:1710.02593 [hep-ph]]

A $\Delta(27)$ unified model with a Universal Texture Zero

Field	$\psi_{q,\,e,\,\nu}$	$\psi^c_{q,\,e,\,\nu}$	H_5	Σ	S	θ_3	θ_{23}	θ_{123}	θ	θ_X
$\Delta(27)$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}_{00}$	$\mathbf{1}_{00}$	$\mathbf{1}_{00}$	$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$	$\mathbf{3}$
Z_N	0	0	0	2	-1	0	-1	2	0	x

Table 1: Transformation of the matter superfields under $\mathcal{G}_f = \Delta(27) \times Z_N$

Appealing flavor model *

small group with $\mathbf{3}, \bar{\mathbf{3}}$: consistent with underlying $SO(10)$ grand unification

accommodates **quark and lepton** mass hierarchies, mixing angles and CP phases

Dirac and Majorana mass matrices have a nice unified texture zero in (1,1)

$$Y_a = y_{3,a} \begin{pmatrix} \mathbf{0} & x_{1,a}\, e^{i\,\gamma_a}\, \varepsilon_a^3 & x_{1,a}\, e^{i\,\gamma_a}\, \varepsilon_a^3 \\ x_{1,a}\, e^{i\,\gamma_a}\, \varepsilon_a^3 & x_{2,a}\, r_a\, e^{i\delta_a}\, \varepsilon_a^2 & x_{2,a}\, r_a\, e^{i\delta_a}\, \varepsilon_a^2 \\ x_{1,a}\, e^{i\,\gamma_a}\, \varepsilon_a^3 & x_{2,a}\, r_a\, e^{i\delta_a}\, \varepsilon_a^2 & 1 \end{pmatrix}$$

$a = e, u, d$

$y_3 = \{y_\tau, y_t, y_b\}$

- Gatto - Sartori - Tonin relation $\sin_{\theta_c} = |\sqrt{\frac{m_d}{m_s}} - e^{i\delta}\sqrt{\frac{m_u}{m_c}}|$
- natural departure of θ^ℓ_{13} angle

(*) JHEP 1803 (2018) 007 [arXiv:1710.01741 [hep-ph]]

A Δ(27) unified model with a Universal Texture Zero

Superpotential: $$\mathcal{W}_\psi = \frac{1}{M^2}(\psi\,\theta_3)(\psi^c\,\theta_3)\,H_5 + \frac{1}{M^3}(\psi\,\theta_{23})(\psi^c\,\theta_{23})\,\Sigma\,H_5 + \frac{1}{M^3}(\psi\,\theta_{23})(\psi^c\,\theta_{123})\,S\,H_5 + \frac{1}{M^3}(\psi\,\theta_{123})(\psi^c\theta_{23})\,S\,H_5$$

Kahler potential: $$\mathcal{K}_{\psi^c} = \psi^c\psi^{c\dagger} + \frac{1}{M^2}\Big[(\psi^c\theta_3)(\theta_3^\dagger\psi^{c\dagger}) + (\psi^c\theta_{23})(\theta_{23}^\dagger\psi^{c\dagger}) + (\psi^c\theta_{123})(\theta_{123}^\dagger\psi^{c\dagger})\Big] + \frac{1}{M^3}\Big[(\psi^c\theta_3)(\theta_{23}^\dagger\psi^{c\dagger})\,S + \text{h.c.}\Big] + \frac{1}{M^3}\Big[(\psi^c\theta_3)\,(\theta_{123}^\dagger\psi^{c\dagger})\,\Sigma + \text{h.c.}\Big]$$

Typical Alignment: $$\langle\theta_3\rangle \propto \begin{pmatrix}0\\0\\1\end{pmatrix}, \quad \langle\theta_{23}\rangle \propto \begin{pmatrix}0\\1\\1\end{pmatrix}, \quad \langle\theta_{123}\rangle \propto \begin{pmatrix}1\\1\\1\end{pmatrix}$$

$$Y_a = y_{3,a}\begin{pmatrix} \mathbf{0} & x_{1,a}\,e^{i\,\gamma_a}\,\varepsilon_a^3 & x_{1,a}\,e^{i\,\gamma_a}\,\varepsilon_a^3 \\ x_{1,a}\,e^{i\,\gamma_a}\,\varepsilon_a^3 & x_{2,a}\,r_a\,e^{i\delta_a}\,\varepsilon_a^2 & x_{2,a}\,r_a\,e^{i\delta_a}\,\varepsilon_a^2 \\ x_{1,a}\,e^{i\,\gamma_a}\,\varepsilon_a^3 & x_{2,a}\,r_a\,e^{i\delta_a}\,\varepsilon_a^2 & 1 \end{pmatrix}$$

$$K_{R,a} = \mathbb{1} + y_{3,a}\begin{pmatrix} \varepsilon_a^{2\alpha} & \varepsilon_a^{2\alpha} & e^{i(\gamma_a-\frac{\delta_a}{2})}\,r_a\,\varepsilon_a^\alpha + \varepsilon_a^{2\alpha} \\ \text{c.c.} & \varepsilon_a^{2\alpha} & e^{i(\gamma_a-\frac{\delta_a}{2})}\,r_a\,\varepsilon_a^\alpha + \varepsilon_a^{2\alpha} \\ \text{c.c.} & \text{c.c.} & 1 \end{pmatrix}$$

VEV alignment prefers small values of α

Some freedom in VEV $\dfrac{\langle\theta_{23}\rangle\,\langle\theta_{123}\rangle\,\langle S\rangle}{M_{123,a}^3}\,\dfrac{M_{3,a}^2}{\langle\theta_3\rangle^2} \propto e^{i\,\gamma_a}\varepsilon_a^3$ **:** $\dfrac{\langle\theta_{123}\rangle}{M_a} = \sqrt{y_{3,a}}\,e^{i\,(\gamma_a-\delta_a/2)}\,\varepsilon_a^\alpha$ with $\alpha\in[0,1]$

A Δ(27) unified model with a Universal Texture Zero

Soft matrices in the physical basis

Kahler + Yukawa diagonalization + re-phasing of the CKM + re-phasing for real Yukawas

Leptonic sector

$$A_e \longrightarrow a_0\, y_\tau \begin{pmatrix} -7\,\dfrac{x_{1,e}^2}{r_e\, x_{2,e}}\,\varepsilon_e^4 & 0 & 0 \\ 0 & -7\; r_e\, x_{2,e}\,\varepsilon_e^2 & 2\,e^{i\,\delta_e}\; r_e\, x_{2,e}\,\varepsilon_e^2 \\ 0 & -2\; r_e\, x_{2,e}\,\varepsilon_e^2 & 5 \end{pmatrix}$$

$$m_{R,e}^2 \longrightarrow m_0^2\,\mathbb{1} + m_0^2\, y_\tau \begin{pmatrix} \varepsilon_e^{2\alpha} & -e^{2\,i\,(\gamma_e-\delta_e)}\,\varepsilon_e^{2\alpha} & 3\,e^{3\,i(\gamma_e-\frac{\delta_e}{2})}\; r_e\,\varepsilon_e^{\alpha} + \varepsilon_e^{2\alpha} \\ \text{c.c.} & \varepsilon_e^{2\alpha} & 3\,e^{i(\gamma_e-\frac{\delta_e}{2})}\; r_e\,\varepsilon_e^{\alpha} + \varepsilon_e^{2\alpha} \\ \text{c.c.} & \text{c.c.} & 1 \end{pmatrix}$$

Down quark sector

$$m_{R,d}^2 \longrightarrow m_0^2\,\mathbb{1} + m_0^2\, y_b \begin{pmatrix} \varepsilon_d^{2\alpha} & -e^{i\,(\gamma_d-\delta_d)}\,\varepsilon_d^{2\alpha} & 3\,e^{i(2\,\gamma_d-\frac{3\,\delta_d}{2})}\; r_d\,\varepsilon_d^{\alpha} + \varepsilon_d^{2\alpha} \\ \text{c.c.} & \varepsilon_d^{2\alpha} & 3\,e^{i(\gamma_d-\frac{\delta_d}{2})}\; r_d\,\varepsilon_d^{\alpha} + \varepsilon_d^{2\alpha} \\ \text{c.c.} & \text{c.c.} & 1 \end{pmatrix}$$

$\boldsymbol{\delta^{RL}_{e,12} \sim \delta^{RL}_{e,13} \sim 0}$

Trilinears block diagonalized

(CCB : $a_0 \leq \sqrt{3}\, m_0/7$)

$\boldsymbol{\mu \to e :}$

$\boldsymbol{\delta^{RR}_{e,12} \sim \varepsilon^{2\alpha}}$

$\sim \mathrm{y}_\tau\,[0.02 \div 0.15]$

$\boldsymbol{\tau \to e \;\&\; \tau \to \mu :}$

$\boldsymbol{\delta^{RR}_{e,13} \sim \delta^{RR}_{e,23} \sim \varepsilon^{\alpha}}$

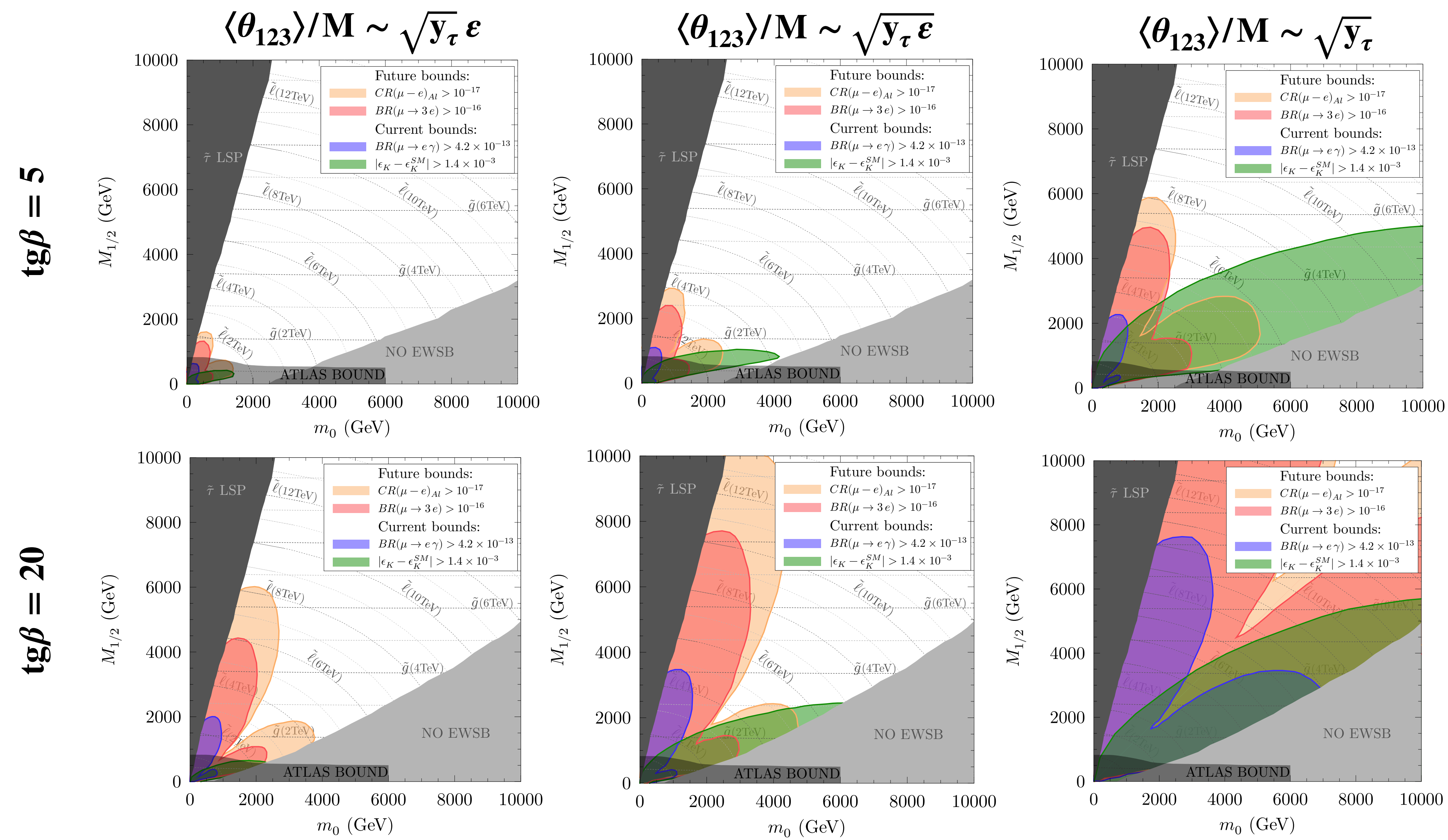
$\sim \mathrm{y}_\tau\,[0.15 \div 1]$

$\boldsymbol{\epsilon_K :}$

$\boldsymbol{\Im[\delta^{RR}_{d,12}] \sim e^{i\,(\gamma_d-\delta_d)}}$

A $\Delta(27)$ unified model with a Universal Texture Zero

FIGURE 4: **Excluded regions of the MSSM parameter space due to LFV constraints.**
Blue shape: current bound on $BR(\mu \to e\,\gamma)$. Green shape: current bound on ϵ_K.
Red shape: future sensitivity on $BR(\mu \to 3e)$. Orange shape: future sensitivity on $CR(\mu - e)_{Al}$.
Future sensitivity on $BR(\mu \to e\gamma)$ excludes a region similar to $BR(\mu \to 3e)$.



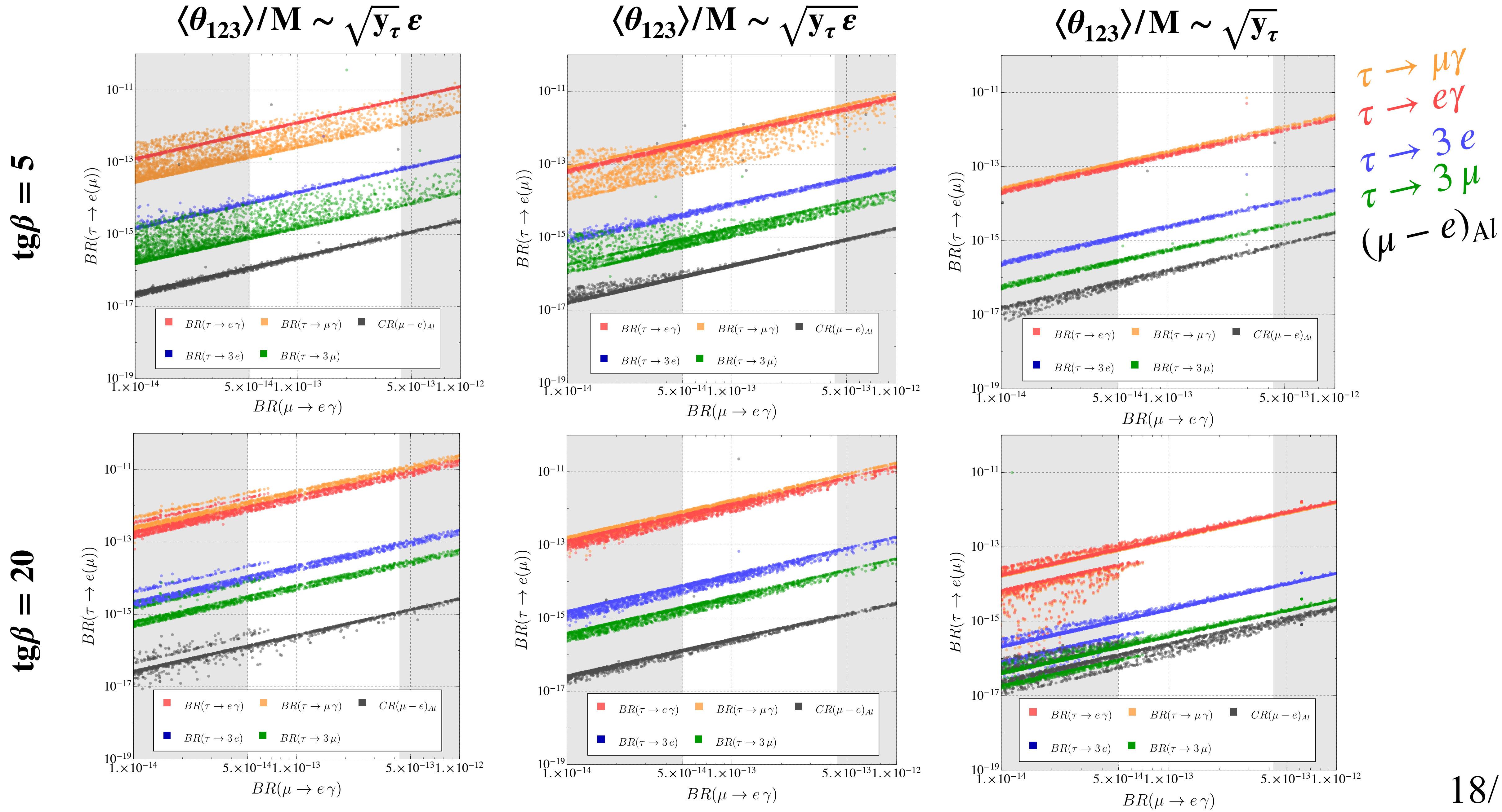
A $\Delta(27)$ unified model with a Universal Texture Zero

FIGURE 5: **FV τ-decays as a function of $BR(\mu \to e\gamma)$.**

White region: future accessible sensitivity for $BR(\mu \to e\gamma)$ (between blue - red shapes in Figure 4).

Gray region: future accessible sensitivity for $CR(\mu - e)_{Al}$ (yellow shape in Figure 4).

Predictions out of reach for the near future experiments (future limits on τ-decays are $\sim 10^{-10}$)



Conclusions

We have

- performed an analysis of lepton and quark FV-processes in the **MSSM** enlarged **with a flavor symmetry**
- shown that **non-universality** of soft breaking matrices (trilinears & soft masses) is generally present ⟵ **easily calculable**
- shown the predictivity of flavor models in SUSY
- demonstrated that non-universality remembers the details of the flavor model and its breaking ⟶ **easy to (dis)prove the model: correlation** between observables in different sectors!

This analysis allow to

- **constrain** sparticle masses well above the LHC reach, strongest bounds from $\mu \to e$ and ϵ_K
- even **distinguish** flavor models!

Backup slides

A Δ(27) unified model : fit results

Uncertainties on UV Mixing Observables								
$(\mu = M_X)$	$\sin\theta^q_{12}$	$\sin\theta^q_{23}$	$\sin\theta^q_{13}$	$\sin\delta^q_{CP}$	$\sin\theta^l_{12}$	$\sin\theta^l_{23}$	$\sin\theta^l_{13}$	$\sin\delta^l_{CP}$
Upper	.228	.0468	.00508	1.000	.588	.800	.155	-
Lower	.226	.0220	.00169	.186	.520	.620	.139	-

Universal Texture Zero Mixing Predictions								
$(\mu = M_X)$	$\sin\theta^q_{12}$	$\sin\theta^q_{23}$	$\sin\theta^q_{13}$	$\sin\delta^q_{CP}$	$\sin\theta^l_{12}$	$\sin\theta^l_{23}$	$\sin\theta^l_{13}$	$\sin\delta^l_{CP}$
L.O. Prediction	.226	.0191	.0042	.561	.554	.778	.152	-.905
H.O. Prediction	.226	.0313	.00307	.788	.543	.751	.153	-.925

Uncertainties on UV Mass Ratios							
$(\mu = M_X)$	m_e/m_τ	m_μ/m_τ	m_u/m_t	m_c/m_t	m_d/m_b	m_s/m_b	$\Delta m^2_{\text{sol}}/\Delta m^2_{\text{atm}}$
Upper	.00031	.061	8.91×10^{-6}	.0027	.0012	.021	.0336
Lower	.00022	.048	1.68×10^{-6}	.00084	.00035	.008	.021

Universal Texture Zero Mass Predictions							
$(\mu = M_X)$	m_e/m_τ	m_μ/m_τ	m_u/m_t	m_c/m_t	m_d/m_b	m_s/m_b	$\Delta m^2_{\text{sol}}/\Delta m^2_{\text{atm}}$
L.O. Prediction	.00031	.055	7.16×10^{-6}	.0027	.00090	.020	.0213
H.O. Prediction	.00026	.049	7.89×10^{-6}	.0025	.0010	.020	.0213

$$|V_{\text{CKM}}|^{\text{HO}} = \begin{pmatrix} .974 & .226 & .00307 \\ .226 & .974 & .0313 \\ .00574 & .0309 & .9995 \end{pmatrix}$$

$$\mathcal{J}^{\text{HO}}_{\text{CKM}} = 1.665 \times 10^{-5}$$

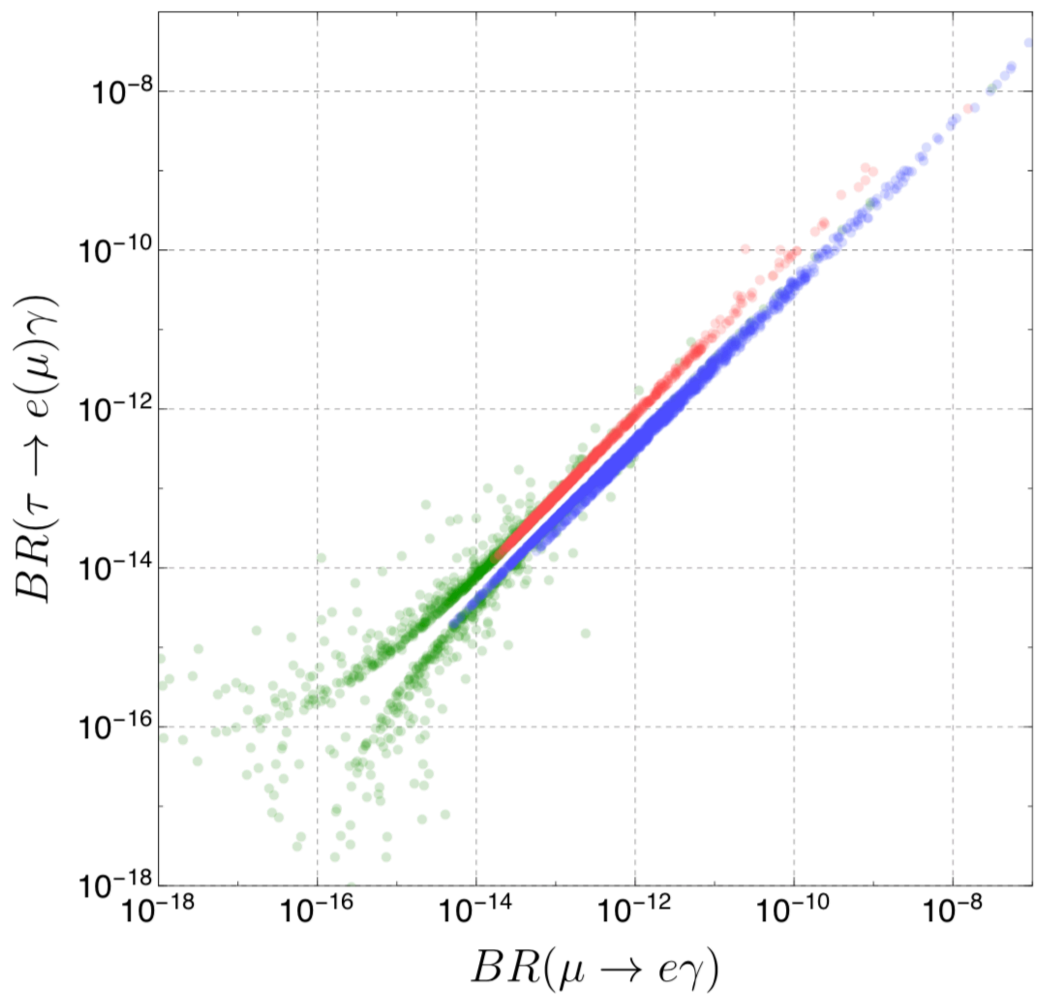
$$|V_{\text{PMNS}}|^{\text{HO}} = \begin{pmatrix} .830 & .536 & .153 \\ .405 & .534 & .742 \\ .384 & .654 & .652 \end{pmatrix}$$

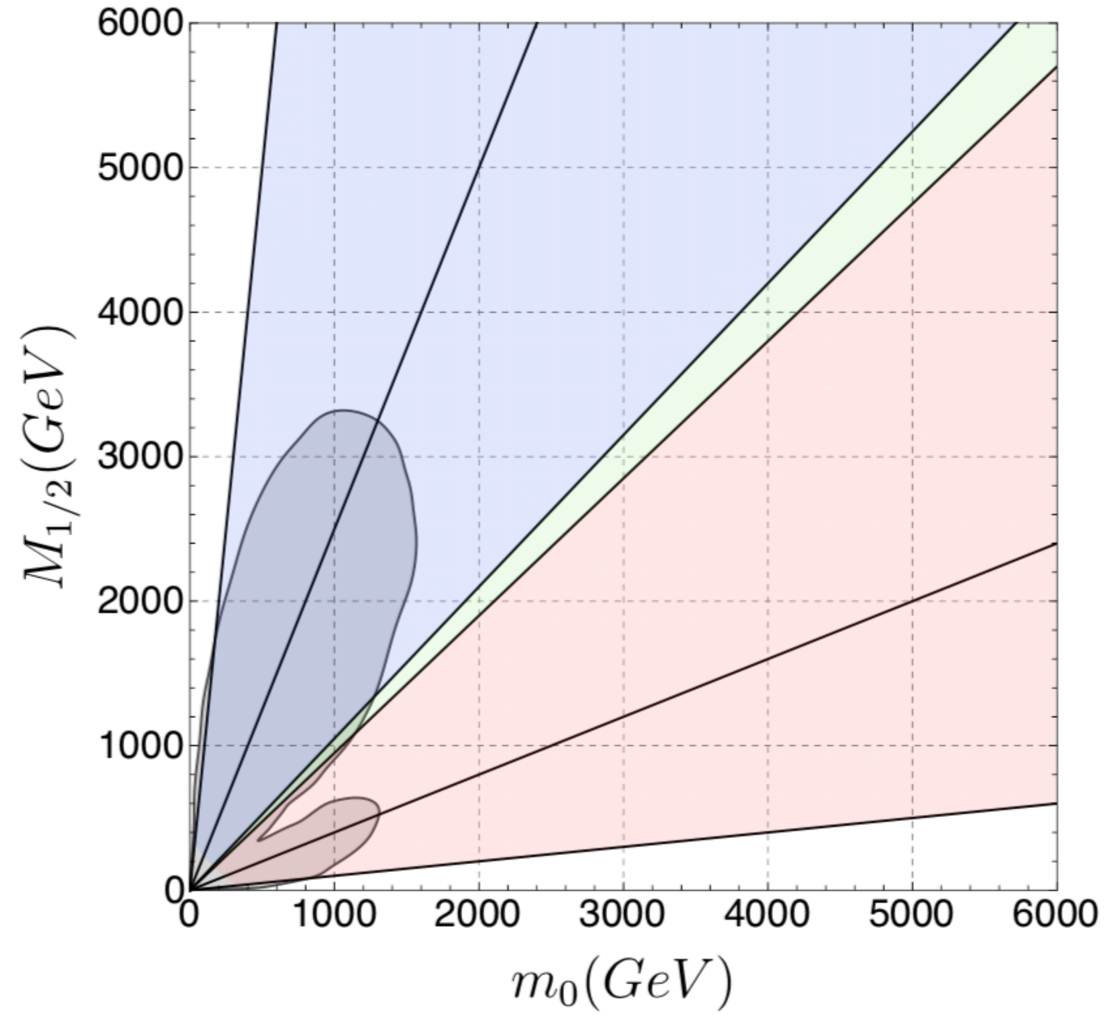
$$\mathcal{J}^{\text{HO}}_{\text{PMNS}} = -.0311$$

✓ The H.O. predictions are within the 3σ - uncertainty bounds

(*) JHEP 1803 (2018) 007 [arXiv:1710.01741 [hep-ph]]

A Δ(27) unified model : understanding the results





In some cases, particularly in the $\tan\beta = 20$ panels, for each branching ratio a second line becomes visible, and the two lines correspond to the maximum directions of growth in the $\{m_0, M_{1/2}\}$ planes of Fig. 5. This is caused by a misalignment of the cancellation region with respect to the one of $\mu \to e\gamma$, which results in two distinct directions of growth. The misalignment stems from additional contributions, deriving mainly from the inclusion of the two mass insertions $\delta^{RR}_{ik}\delta^{RR}_{kj}$

(*) JHEP 1803 (2018) 007 [arXiv:1710.01741 [hep-ph]]

An S_3 model example

Field	ν^c	ν_3^c	e	e^c	ℓ	ℓ^c	$H_{u,d}$	ϕ	χ	ξ	χ'	$\chi'^{\dagger}$
S_3	$\mathbf{2}$	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}'$	$\mathbf{1}'$
Z_6	ω	ω	1	ω^3	ω^5	ω^3	1	ω^4	ω^4	ω^4	ω^5	ω^{-5}
Z_3	1	1	1	ω	1	ω^2	1	ω	ω	1	1	1
$U(1)_R$	1	1	1	1	1	1	0	0	0	0	0	0

Reproduces the charged **lepton & quark** hierarchies and mixings CKM + TBM + $\boldsymbol{\theta_{13}}$**!**

Table 1: Transformation of the matter superfields under the $\mathcal{G}_f = S_3 \times Z_6 \times Z_3$.

$$\ell = \begin{pmatrix} \tau \\ \mu \end{pmatrix}, \; \ell^c = \begin{pmatrix} \mu^c \\ \tau^c \end{pmatrix}$$

Alignment: $$\frac{\langle\phi\rangle}{M} \propto \varepsilon \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \; \frac{\langle\xi\rangle}{M} \propto \varepsilon \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \; \frac{\langle\chi\rangle}{M} \propto \varepsilon, \; \frac{\langle\chi'\rangle}{M} \propto \varepsilon'$$

Superpotential

$$\textbf{LO}\;\; \mathcal{W}_\ell = \frac{1}{M}\left[(\ell^c\ell\,\phi) + (\ell^c\ell)\,\chi\right] H_d + \frac{1}{M^2}(\ell^c\ell\phi)'\,\chi'\,H_d$$

$$Y_\ell \sim \begin{pmatrix} x_1\,\varepsilon^2\,\varepsilon'^3 & x_2\,\varepsilon\,\varepsilon' & -x_2\,\varepsilon\,\varepsilon' \\ x_3\,\varepsilon^2\,\varepsilon'^2 & x_4\,\varepsilon & x_5\,\varepsilon \\ x_6\,\varepsilon^2\,\varepsilon'^2 & x_5\,\varepsilon & x_4\,\varepsilon \end{pmatrix}$$

$$\textbf{NLO}\;\; \delta\mathcal{W}_\ell = \frac{1}{M^4}\,e^c\left[(\ell\,\xi^2)\chi^2 + (\ell\,\phi\xi^2)\chi + (\ell\,\phi^2\xi^2)\right] H_d + \frac{1}{M^5}\,e^c e\left[(\phi\,\xi^2)'\chi'\chi + (\phi^2\xi^2)'\chi'\right] H_d$$

$$A_\ell \sim a_0 \begin{pmatrix} \mathbf{11}\,x_1\,\varepsilon^2\,\varepsilon'^3 & \mathbf{5}\,x_2\,\varepsilon\,\varepsilon' & -\mathbf{5}\,x_2\,\varepsilon\,\varepsilon' \\ \mathbf{9}\,x_3\,\varepsilon^2\,\varepsilon'^2 & \mathbf{3}\,x_4\,\varepsilon & \mathbf{3}\,x_5\,\varepsilon \\ \mathbf{9}\,x_6\,\varepsilon^2\,\varepsilon'^2 & \mathbf{3}\,x_5\,\varepsilon & \mathbf{3}\,x_4\,\varepsilon \end{pmatrix}$$

(*) JHEP 05 (2012) 124, [1203.3126]

An S_3 model example

Field	ν^c	ν_3^c	e	e^c	ℓ	ℓ^c	$H_{u,d}$	ϕ	χ	ξ	χ'	$\chi'^\dagger$
S_3	$\mathbf{2}$	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}'$	$\mathbf{1}'$
Z_6	ω	ω	1	ω^3	ω^5	ω^3	1	ω^4	ω^4	ω^4	ω^5	ω^{-5}
Z_3	1	1	1	ω	1	ω^2	1	ω	ω	1	1	1
$U(1)_R$	1	1	1	1	1	1	0	0	0	0	0	0

Reproduces the charged lepton and quark hierarchies and mixings CKM + TBM + $\boldsymbol{\theta_{13}}$**!**

Table 1: Transformation of the matter superfields under the $\mathcal{G}_f = S_3 \times Z_6 \times Z_3$.

Alignment: $\dfrac{\langle\phi\rangle}{M} \propto \varepsilon \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \dfrac{\langle\xi\rangle}{M} \propto \varepsilon \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \dfrac{\langle\chi\rangle}{M} \propto \varepsilon, \dfrac{\langle\chi'\rangle}{M} \propto \varepsilon'$

(LH) Kähler potential

$$K_{\ell,L} = \ell\,\ell^\dagger + e\,e^\dagger + \frac{1}{M^2}\left[(\ell\,\ell^\dagger\phi\phi^\dagger) + (\ell\,\ell^\dagger\phi)\,\chi^\dagger + \chi'\,(\ell\,\xi^\dagger)'\,e^\dagger + \text{h.c.}\right] + \text{h.c.}$$

$$K_{\ell,L} \sim \mathbb{1} + \begin{pmatrix} \varepsilon^2+\varepsilon'^2 & \varepsilon'^2 & \varepsilon^2\varepsilon' \\ \varepsilon'^2 & \varepsilon^2+\varepsilon'^2 & \varepsilon^2 \\ \varepsilon^2\varepsilon' & \varepsilon^2 & \varepsilon^2+\varepsilon'^2 \end{pmatrix}$$

$$m^2_{\ell,L} \sim m_0^2\,\mathbb{1} + m_0^2 \begin{pmatrix} \mathbf{2}\,(\varepsilon^2+\varepsilon'^2) & \mathbf{2}\,\varepsilon'^2 & \mathbf{4}\,\varepsilon^2\varepsilon' \\ \mathbf{2}\,\varepsilon'^2 & \mathbf{2}\,(\varepsilon^2+\varepsilon'^2) & \mathbf{2}\,\varepsilon^2 \\ \mathbf{4}\,\varepsilon^2\varepsilon' & \mathbf{2}\,\varepsilon^2 & \mathbf{2}\,(\varepsilon^2+\varepsilon'^2)2 \end{pmatrix}$$

(RH) Kähler potential

$$K_{\ell,R} = \ell^c\ell^{c\dagger} + e^c e^{c\dagger} + \frac{1}{M^2}\left[(\ell^c\ell^{c\dagger}\phi\phi^\dagger) + (\ell^c\ell^{c\dagger}\phi)\,\chi^\dagger + (\ell^c\xi\phi^\dagger)\,e^{c\dagger} + \text{h.c.}\right] + \text{h.c.}$$

$$K_{\ell,R} \sim \mathbb{1} + \begin{pmatrix} \varepsilon^2+\varepsilon'^2 & \varepsilon\varepsilon' & \varepsilon\varepsilon' \\ \varepsilon\varepsilon' & \varepsilon^2+\varepsilon'^2 & \varepsilon^2 \\ \varepsilon\varepsilon' & \varepsilon^2 & \varepsilon^2+\varepsilon'^2 \end{pmatrix}$$

$$m^2_{\ell,R} \sim m_0^2\,\mathbb{1} + \mathbf{2}\,m_0^2 \begin{pmatrix} \varepsilon^2+\varepsilon'^2 & \varepsilon\varepsilon' & \varepsilon\varepsilon' \\ \varepsilon\varepsilon' & \varepsilon^2+\varepsilon'^2 & \varepsilon^2 \\ \varepsilon\varepsilon' & \varepsilon^2 & \varepsilon^2+\varepsilon'^2 \end{pmatrix}$$

An S_3 model example

2 rotations to go to the physical basis

- **Canonical rotation:** Kähler is the identity
- **Mass basis rotation:** Yukawas are diagonal

$$\boldsymbol{K_{\ell,L}} \longrightarrow \boldsymbol{U^\dagger_{K_L}\, K_{\ell,L}\, U_{K_L}} = \mathbb{1} \;\; , \;\; \boldsymbol{K_{\ell,R}} \longrightarrow \boldsymbol{U^\dagger_{K_R}\, K_{\ell,R}\, U_{K_R}} = \mathbb{1} \;\; , \;\; \boldsymbol{Y_\ell} \longrightarrow \boldsymbol{V^\dagger_Y\, U^\dagger_{K_L}\, Y_\ell\, U_{K_R}\, U_Y} = \boldsymbol{Y_\ell^{(diag)}}$$

$$\boldsymbol{A_\ell} \longrightarrow V_Y^{-1}\, U^\dagger_{K_L}\, A_\ell\, U_{K_R}\, U_Y = a_0 \begin{pmatrix} 11\, x_1\, \varepsilon^2 \varepsilon'^3 & \left(-\frac{5}{\sqrt{2}} x_2 + \frac{3\sqrt{2} x_2 x_5}{x_4 + x_5}\right) \varepsilon\, \varepsilon' & -2\sqrt{2}\, x_2\, \varepsilon\, \varepsilon' \\ \frac{9}{\sqrt{2}}(x_6 + x_3)\, \varepsilon^2 \varepsilon'^2 & 3\,(x_5 - x_4)\varepsilon & -3\, x_5\, \varepsilon^3 \\ \frac{9}{\sqrt{2}}(x_6 - x_3)\, \varepsilon^2 \varepsilon'^2 & -3\, x_5\, \varepsilon^3 & -3\,(x_5 + x_4)\varepsilon \end{pmatrix}$$

$$\boldsymbol{m^2_{\ell,L}} \longrightarrow V_Y^{-1}\, U^\dagger_{K_L} m^2_{\ell,L}\, U_{K_L} V_Y = m_0^2\, \mathbb{1} + m_0^2 \begin{pmatrix} \varepsilon^2 + \varepsilon'^2 & \frac{1}{\sqrt{2}}\varepsilon'^2 & -\frac{1}{\sqrt{2}}\varepsilon'^2 \\ \frac{1}{\sqrt{2}}\varepsilon'^2 & 2\,\varepsilon^2 + \varepsilon'^2 & 3\,\varepsilon^2 \varepsilon'^2 \\ -\frac{1}{\sqrt{2}}\varepsilon'^2 & 3\,\varepsilon^2 \varepsilon'^2 & \varepsilon'^2 \end{pmatrix}$$

Do not get diagonalized!

$$\boldsymbol{m^2_{\ell,R}} \longrightarrow U_Y^{-1}\, U^\dagger_{K_R} m^2_{\ell,R}\, U_{K_R} U_Y = m_0^2\, \mathbb{1} + m_0^2 \begin{pmatrix} \varepsilon^2 + \varepsilon'^2 & \sqrt{2}\, \varepsilon\, \varepsilon' & 0 \\ \sqrt{2}\, \varepsilon\, \varepsilon' & 2\,\varepsilon^2 + \varepsilon'^2 & 0 \\ 0 & 0 & \varepsilon'^2 \end{pmatrix}$$