Quantum scale symmetry via constrained dimensional regularization

Paweł Olszewski

Based on:

D.M. Ghilencea, Z. Lalak, PO / Phys.Rev. D96 (2017) no.5, 055034
Standard Model with spontaneously broken quantum scale invariance

D.M. Ghilencea, Z. Lalak, PO / Eur.Phys.J. C76 (2016) no.12, 656
Two-loop scale-invariant potential and quantum effective operators

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References

S. Deser, “Scale invariance and gravitational coupling,” Annals Phys. 59 (1970)

F. Englert, C. Truffin and R. Gastmans, “Conformal Invariance in Quantum Gravity,” Nucl. Phys. B 117 (1976)

M. Shaposhnikov and D. Zenhausern, “Quantum scale invariance, cosmological constant and hierarchy problem,” Phys. Lett. B 671 (2009) 162 [arXiv:0809.3406 [hep-th]]

M. E. Shaposhnikov and F. V. Tkachov, “Quantum scale-invariant models as effective field theories,” arXiv:0905.4857 [hep-th]

- C. Tamarit, “Running couplings with a vanishing scale anomaly,” JHEP 1312 (2013) 098 [arXiv:1309.0913 [hep-th]].

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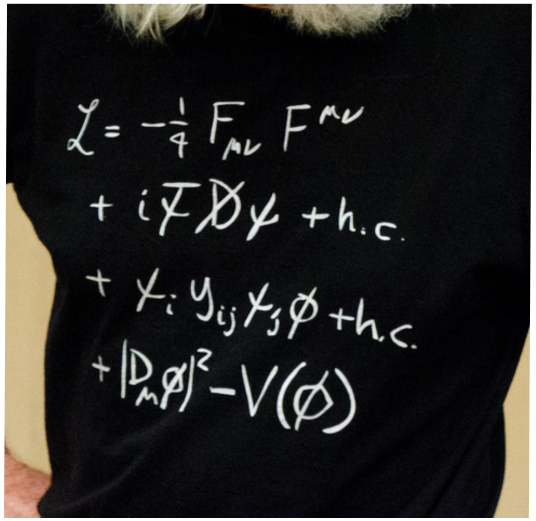
Plan

1) Anomaly of the scale symmetry
2) Un-doing Coleman-Weinberg dimensional transmutation
3) Quantum scale symmetry
4) Scale symmetric Standard Model

Mass parameter in the SM



$$V(\phi) = m^2 H^\dagger H + \lambda (H^\dagger H)^2$$

„Unnaturally” small.

Forbid dimensionful parameters altogether.

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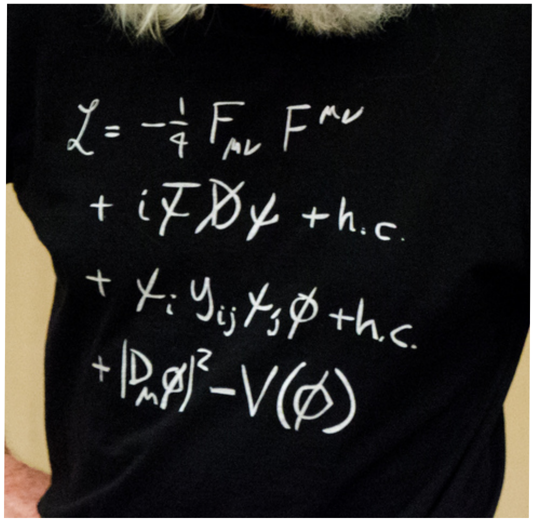

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Higgs mass via Higgs portal: new scalar field σ with a VEV

$$-\lambda_m \sigma^2 H^\dagger H \ , \quad \langle \sigma \rangle \neq 0$$

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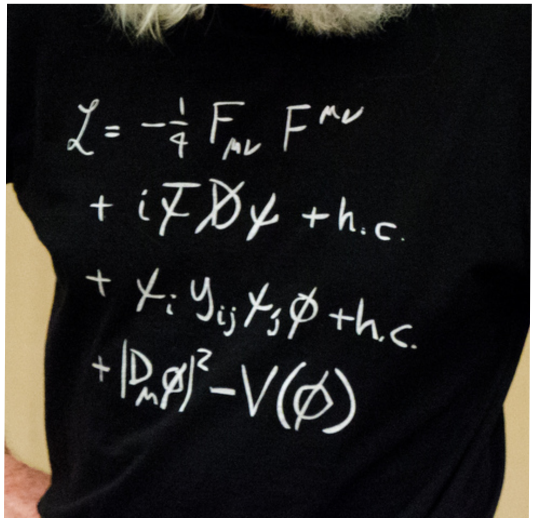

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Global symmetry wrt. dilatations:

$$[D, P_\mu] = iP_\mu$$

$$\begin{aligned} U(a) &= e^{iaD} \\ \Phi(x) \to U(a)\Phi(x)U^{-1}(a) &= e^{ad_\Phi}\Phi(e^a x) \\ \delta\Phi(x) = i[D,\Phi] &= (d_\Phi + x^\nu \partial_\nu)\,\Phi(x) \end{aligned}$$

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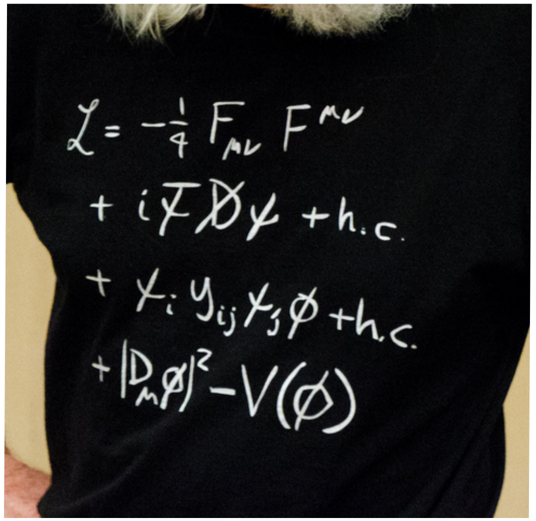

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The symmetry is anomalous!

Anomaly of the scale symmetry

A symmetric action $$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{4!}\phi^4$$

Noether current $$D^\mu = \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_j)}(x^\nu\partial_\nu\phi_j + d_\phi\phi_j) - x^\mu\mathcal{L}$$

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$$T^\mu{}_\mu = \partial_\mu D^\mu = d_\phi\,\phi_j\frac{\partial V}{\partial\phi_j} - d\,V \;=\; \frac{d}{d\,\rho}\left[V(\rho^{d_\phi}\phi_j) - \rho^d V(\phi_j)\right]\Big|_{\rho=1}$$

trace of the E-M tensor

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Homogeneity of the potential?

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$d = 4$

Quantum corrections require a dimensionfull regulator $$V = \frac{\lambda\phi^4}{4!} + \frac{1}{4(4\pi)^2}\left(\frac{\lambda\varphi^2}{2}\right)^2\left(\log\frac{\lambda\varphi^2}{2\,\overline{\mu}^2} - \frac{3}{2}\right)$$

$$T^\mu{}_\mu = \left(\phi\frac{\delta}{\delta\phi} - 4\right)V$$

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$$T^\mu{}_\mu = \left(\phi\frac{\delta}{\delta\phi} - 4\right)V = \frac{3\lambda^2}{(4\pi)^2}\frac{\phi^4}{4!} = \beta_\lambda{}^{(\text{1-loop})}\frac{\phi^4}{4!} \neq 0$$

Anomaly of the scale symmetry

$d=4$

$$T^{\mu}_{\mu} \sim \sum_g \beta_g \frac{\partial}{\partial g}\mathcal{L}_{\rm matter} + \frac{-a}{(4\pi)^2}\tilde{R}_{\alpha\beta\gamma\delta}\tilde{R}^{\alpha\beta\gamma\delta} + \frac{c}{(4\pi)^2}C_{\alpha\beta\gamma\delta}C^{\alpha\beta\gamma\delta}$$

(matter-related) scale anomaly

$g_{\mu\nu} = g_{\mu\nu}(x)$, gravitational anomaly

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→ *not discussed here ...*

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Insertion of composite operators

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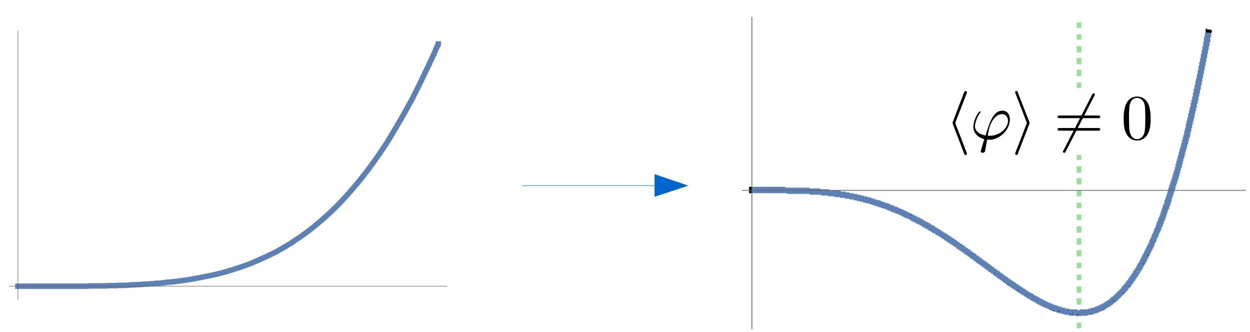
Insertion of composite operators

Dimensional transmutation by Coleman and Weinberg:

$$\varphi^4 \longrightarrow \varphi^4 + \alpha\, \varphi^4 \log \varphi^2$$



Un-doing the CW dim. transmutation with nonrenorm. interactions

$d = 4$

$$V^{(1)} \sim \phi^4 \log \frac{\phi^2}{\mu^2} + \overbrace{\left(-2\phi^4 \frac{\sigma'}{\Lambda} + \phi^4 \frac{\sigma'^2}{\Lambda^2} + \dots \right)}^{\overset{!}{=} \phi^4 \log \left(\frac{\Lambda}{\Lambda + \sigma'} \right)^2}$$

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$$= \boxed{\phi^4 \log \frac{\phi^2}{\sigma^2 \, (e^t)^2}}$$

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$t \in \mathbb{R}$

$\mu = e^t \mu_0 = e^t \Lambda$

$\sigma \equiv \Lambda + \sigma'$

$\langle \sigma \rangle \equiv \Lambda$

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$$i[D, \Lambda] = \Lambda$$

$$i[D, \mu] = \mu$$

ren. scale charged under dilatations !

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For consistency add also:

$$V^{(0)} += \sum_{n=1}^{\infty} \lambda_n \frac{\phi^{4+2n}}{(\Lambda + \sigma')^{2n}}$$

Un-doing the CW dim. transmutation with nonrenorm. interactions

$d = 4$

$$V_{\text{eff}}(\phi, \sigma) = V^{(0)} + V^{(1)} + \ldots$$

- regularized $\mu \sim \langle\sigma\rangle$
- nonrenormalizable $\Lambda \sim \langle\sigma\rangle$
- homogenous:

$$\left(\phi\frac{\delta}{\delta\phi} + \sigma\frac{\delta}{\delta\sigma} - 4\right) \begin{matrix} V^{(0)} \\ V^{(1)} \\ \vdots \end{matrix} = 0$$

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scale symmetry

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← ***scale symmetry***

→ *Does adding*
nonrenormalizable interactions
work this way at
all loop orders
*and **beyond the effective potential**?*

Yes! *Here's why...*

Un-doing the CW dim. transmutation with nonrenorm. interactions

Dim-Reg $d=4-2\varepsilon$

Simple solution

$$\mu = \mu(\sigma) = e^t\, f(\sigma)\,,$$

where $\left\{ \begin{array}{l} [f(\sigma)] = 1 \\ f \text{ analytic near } f(\langle\sigma\rangle) \equiv \mu_0 \equiv \Lambda \\ t \in \mathbb{R} \end{array} \right\} \Rightarrow \boxed{\mu(\sigma) = e^t\, \sigma^{\frac{1}{1-\varepsilon}}}$

is perfectly allowed.

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is perfectly allowed. The *broken phase* succesfully mimicks an ordinary model. E.g.:

$$\mu^{2\varepsilon} \frac{\phi^{4+2n}}{\sigma^{2n}} = (e^t)^{2\varepsilon} \, (\mu_0^\varepsilon)^{(2+2n)} \, \frac{\phi^{4+2n}}{\Lambda^{2n}} \left(1 + \mu_0^\varepsilon \frac{\sigma'}{\Lambda}\right)^{-2n+\frac{2\varepsilon}{1-\varepsilon}}$$

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nonrenormalizable series with some ***evanescent coefficients*** *(conjecture 1.)*

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n-independent
(conjecture 2.)

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similarly for gauge theories: $-\dfrac{1}{4g^2} \, \mu(\sigma)^{-2\varepsilon} \, F_{\mu\nu} F^{\mu\nu}$

Quantum scale symmetric effective lagrangian

No scale anomaly in

$$\mathcal{L}^{(0)}(\phi,\sigma) = \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(\partial\sigma)^2 - \overbrace{\mu^{2\varepsilon}(\sigma)}^{\text{„dynamical” regulator}} \left[\underbrace{V(\phi,\sigma)}_{\text{renormalizable, classicaly scale-invariant}} + \sum_{n=0} \lambda_n \frac{\phi^{4+2n}}{\sigma^{2n}} \right]$$

„dynamical” regulator

renormalizable, classicaly scale-invariant

$\mathbb{Z}^2 \times \mathbb{Z}^2$

$\phi \to -\phi$

$\sigma \to -\sigma$

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Insertion of $[\partial_\mu D^\mu]$ → unbroken scale transformations to all orders

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renormalizable, classicaly scale-invariant

$\mathbb{Z}^2 \times \mathbb{Z}^2$
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go to *broken phase*

$$\mathcal{L}^{(0)}(\phi_0 + \phi', \sigma_0 + \phi')$$

compute loop corrections (in momentum expansion) & RGE functions *β, γ*

$$\mathcal{L}_{\text{eff}}(\phi,\sigma) = -V_{\text{eff}}(\phi,\sigma) + \ldots$$

Insertion of $[\partial_\mu D^\mu]$ → unbroken scale transformations to all orders

- Homogenous function (no mass-parameters, only vev's)
- $\mathbb{Z}^2 \times \mathbb{Z}^2$ sym.
- Satisfies Callan-Symanzik eq.

Quantum scale symmetric effective lagrangian

RG-improvement:

$$\mu = e^{t}\mu_0\,, \qquad \lambda(t)\,\phi^4 + \frac{\lambda^2(t)\phi^4}{64\pi^2}\,\log\left(\frac{\phi}{e^{t}\sigma}\right)^2 + \dots$$

Choose

$$t = t(\phi,\sigma) \sim \log\frac{\phi}{\sigma}$$

to avoid large logs.

Quantum scale symmetric effective lagrangian

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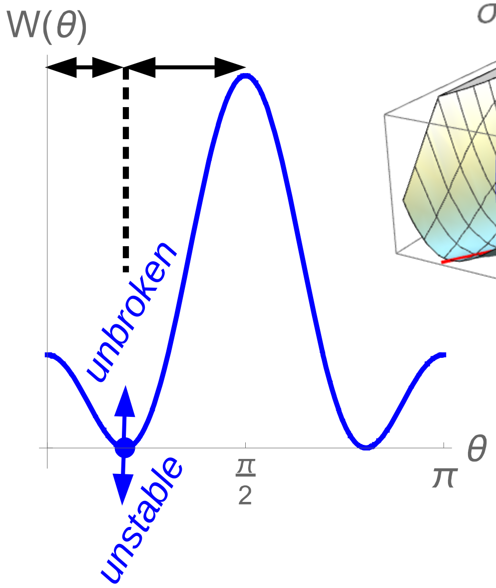
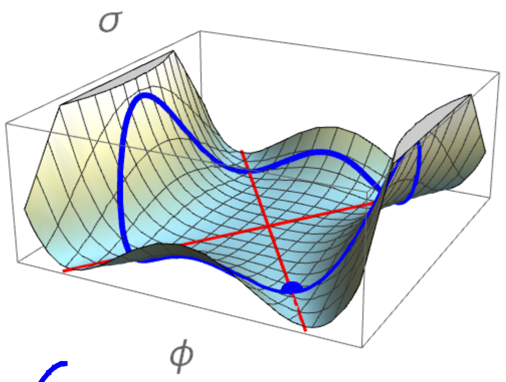
Spontanous scale-symmetry breaking:

$$\begin{pmatrix}\phi\\ \sigma\end{pmatrix} = M\begin{pmatrix}\sin\theta\\ \cos\theta\end{pmatrix}\,, \quad V_{\text{eff}} = M^4\,W(\theta)\,,$$





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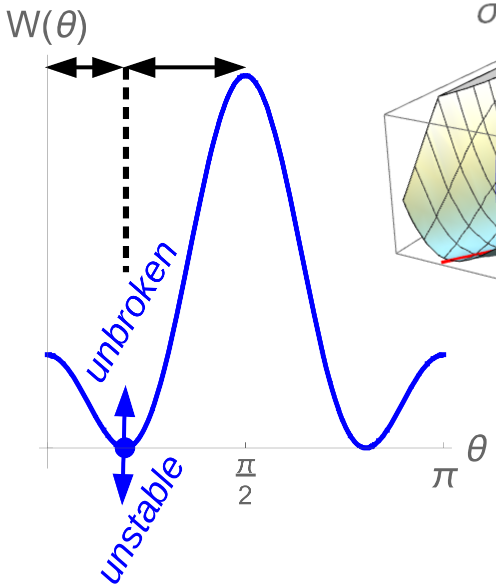
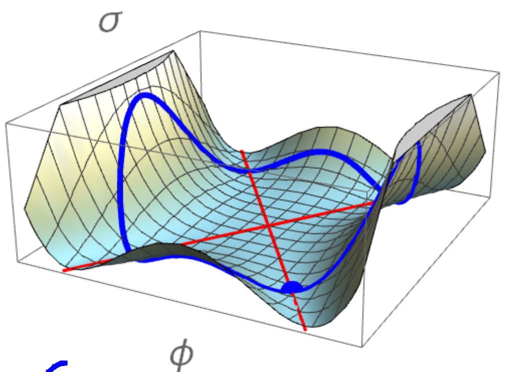
► flat direction in V_{eff} $\Rightarrow$

$$\exists_{\theta=\theta_0}\, W(\theta_0) = W'(\theta_0) = 0$$

renormalization condition, similar to choosing C.C.





Quantum scale symmetric effective lagrangian

RG-improvement:

$$\mu = e^t \mu_0\ , \qquad \lambda(t)\,\phi^4 + \frac{\lambda^2(t)\phi^4}{64\pi^2}\log\left(\frac{\phi}{e^t\sigma}\right)^2 + \ldots$$

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Spontanous scale-symmetry breaking:

$$\begin{pmatrix}\phi\\ \sigma\end{pmatrix} = M\begin{pmatrix}\sin\theta\\ \cos\theta\end{pmatrix}\ , \quad V_{\text{eff}} = M^4\, W(\theta)\,,$$

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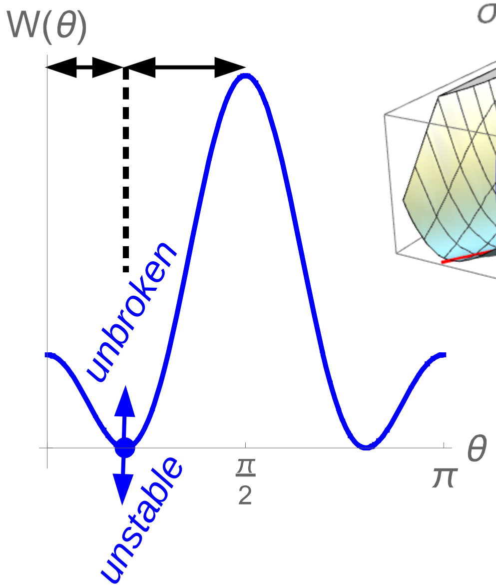

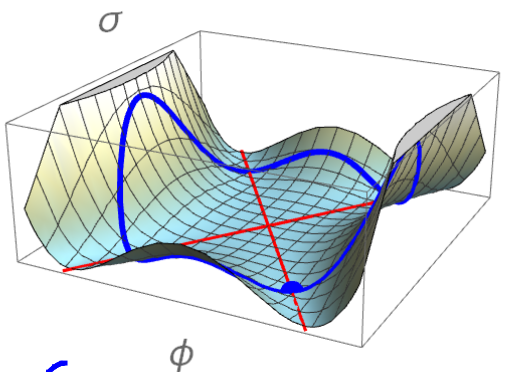

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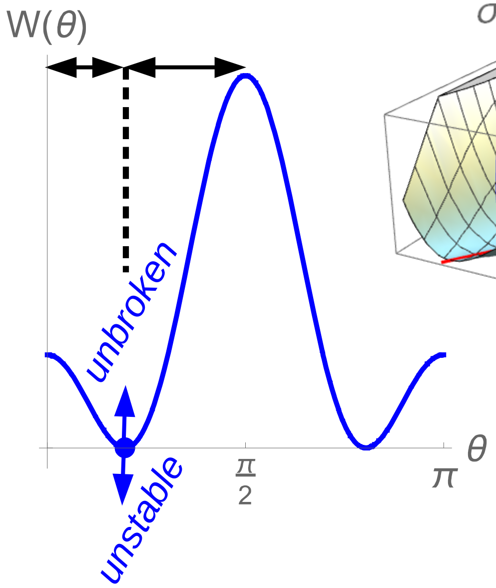

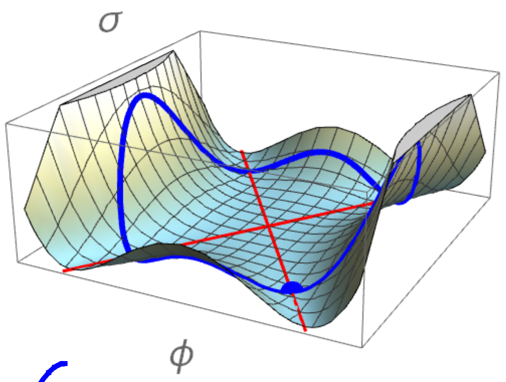

- **Hierarchy** of scales via **aligning** the flat direction $\perp\,\phi$ → $\theta_0 \approx \dfrac{\phi_0}{\sigma_0} \ll 1$
- New perspective on **naturalness**: is this alignement stable wrt. embedding in a UV completion?

Quantum scale symmetric SM + σ

$$H = \begin{pmatrix} 0 \\ \frac{\phi}{\sqrt{2}} \end{pmatrix}$$ (electroweak vacuum → electroweak flat direction)

$$\mathcal{L}_{SM}\Big|_{\substack{m^2=0 \\ \mu=\mu(\sigma)}} + \frac{1}{2}(\partial\sigma)^2 - \lambda_m |H|^2\sigma^2 - \frac{\lambda_\sigma}{4}\sigma^4 + \sum_{n=0} \lambda_n \frac{|H|^{4+2n}}{\sigma^{2n}}$$

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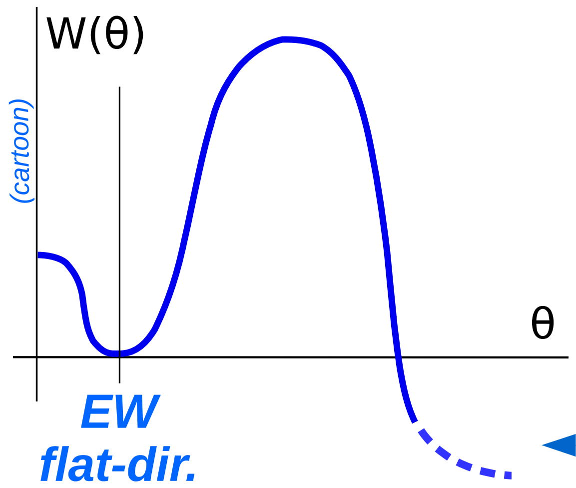

$V_{\rm eff}$ *is:*

- ***unstable***
- ***unbounded below***

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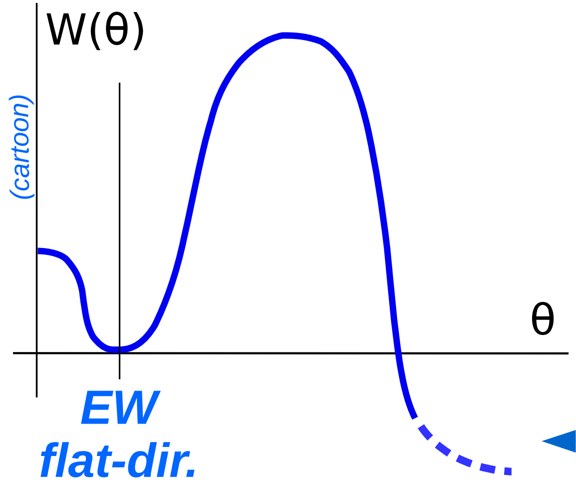

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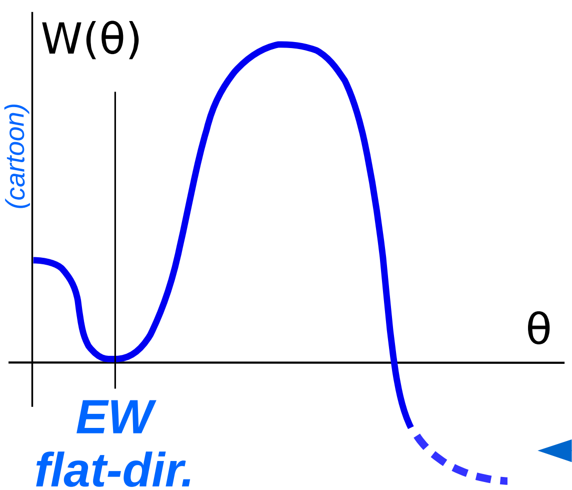

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(Even stronger) motivation to stabilise the $V_{\rm eff}$ completely: $\lambda_{\rm eff} \overset{!}{>} 0$

Summary

1) You may use **a field as the scale *μ*** in Dim-Reg to preserve scale symmetry at the quantum level.

2) The price to pay: infinitely many nonpolynomial ϕ/σ operators and corresponding couplings: **nonrenormalizability**.

3) Minimal subtraction scheme involves **evanescent interactions**.

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5) **Naturalness: aligning** the flat direction perpendicular to Higgs

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Thank You!

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Dim-Reg $d=4-2\varepsilon$

Consider *evanescent* interactions, $\lim_{\varepsilon\to 0} = 0$:

e.g. $$V^{(0)}_{\rm new} = \varepsilon\left(\phi^2\sigma'^2 + \frac{\phi^2\sigma'^3}{\Lambda} + \frac{\phi^4\sigma'}{\Lambda} + \frac{\phi^4\sigma'^2}{\Lambda^2} + \ldots\right) + \varepsilon^2\ldots$$

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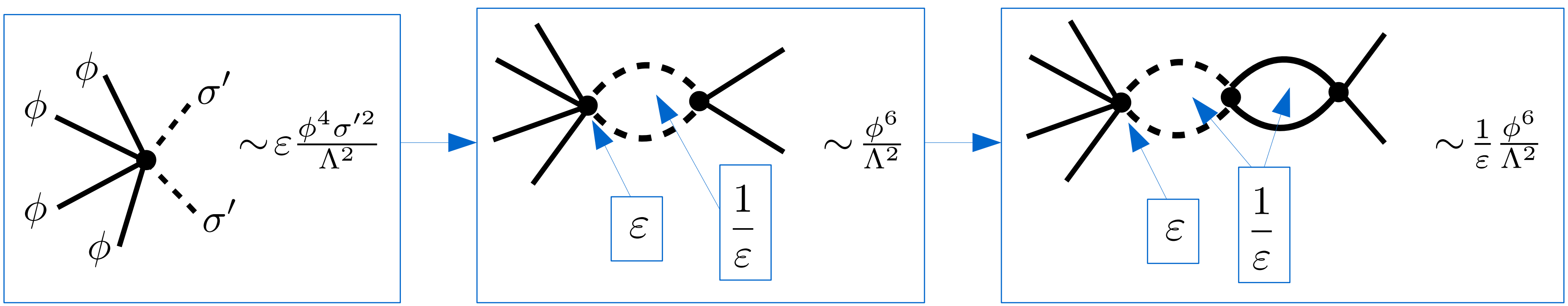

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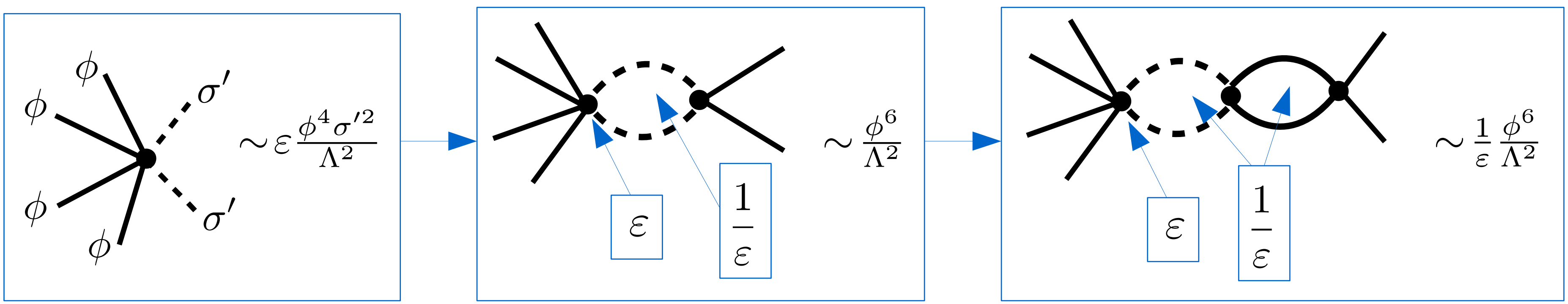

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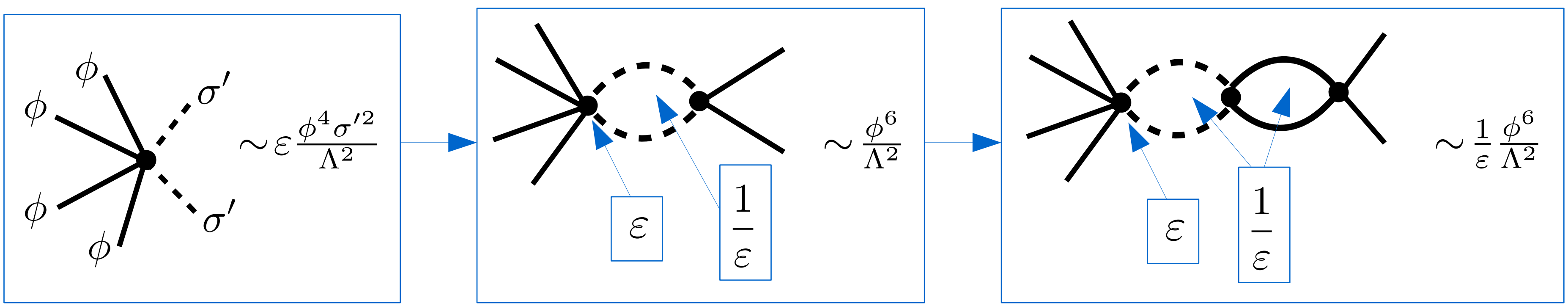

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evanescent interactions **=** change of the renormalization scheme
$\sim \varepsilon$

(MS with evanescent interactions **=** new corrections and running wrt MS without evanescent int.)

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schematically: $\lambda\,\Phi^4 \longrightarrow \mu^{2\varepsilon} \underbrace{Z}_{1+\delta(\lambda)} \lambda\,\Phi^4\,, \quad [\Phi] = 1-\varepsilon$

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