Revisiting the Dark Matter Relic Abundance Calculation:

The Case of Early Kinetic Decoupling

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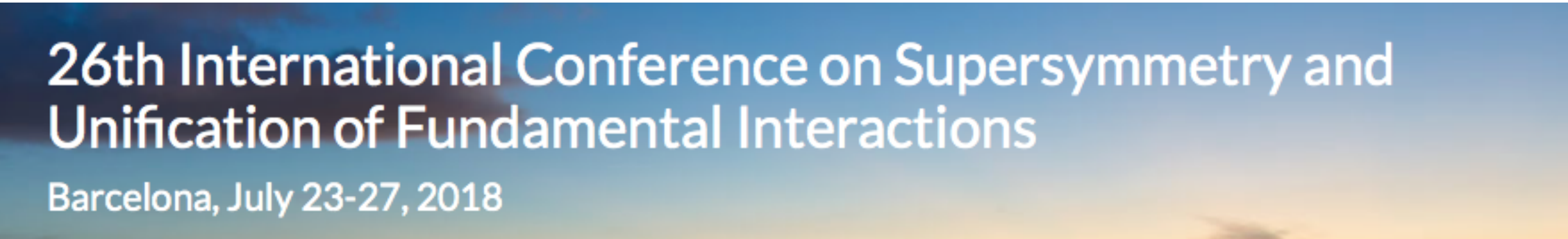


Outline

1. The standard freeze-out formalism

2. A refined treatment

Coupled 0th and 2nd moment of Boltzmann Equation

Full phase-space Boltzmann Equation

3. Applied to the Singlet Scalar DM model

   —Order of magnitude impact on relic abundance Ω_{DM}

Standard formalism

Boltzmann Equation

The Dynamics of the DM's phase-space density $f_\chi(t,p)$ is govern by the BE:

energy

Hubble

momentum

$$E\left(\partial_t - H\mathbf{p}\cdot\nabla_{\mathbf{p}}\right) f_\chi = C[f_\chi]\,.$$

Liouville operator **=** **Collision terms**

Boltzmann Equation

The Dynamics of the DM's phase-space density $f_\chi(t,p)$ is govern by the BE:

energy, Hubble, momentum

$$E\left(\partial_t - H\mathbf{p}\cdot\nabla_{\mathbf{p}}\right) f_\chi = C[f_\chi]\,.$$

Liouville operator = **Collision terms**

Collision terms for CP invariant 2 ↔ 2 annihilations and scattering with SM

$C[f_\chi]$:

$$C_{\rm ann} = E\int \frac{d^3\tilde{p}}{(2\pi)^3}\, v\sigma_{\bar{\chi}\chi\to\bar{f}f}\left[f_{\chi,{\rm eq}}(E)f_{\chi,{\rm eq}}(\tilde{E}) - f_\chi(E)f_\chi(\tilde{E})\right]\,,$$

$$C_{\rm el} \simeq \frac{m_\chi}{2}\gamma(T)\left[Tm_\chi\partial_p^2 + \left(p + 2T\frac{m_\chi}{p}\right)\partial_p + 3\right] f_\chi$$

Non relativistic **Fokker-Planck Eq.** (Bringmann & Hofmann '06, Binder et al.'16)

DM Particle Model Dependent

A Stiff Non-linear Integro Partial differential equation → **Challenging!**

Taking a moment…

$$E\left(\partial_t - H\mathbf{p}\cdot\nabla_{\mathbf{p}}\right) f_\chi = C[f_\chi]\,.$$

Taking momentum moments of BE …

0th moment:
(number density)

$$n_\chi(t) = \int \frac{\mathrm{d}^3 p}{(2\pi)^3}\, f_\chi(t,\mathbf{p})$$

2nd moment:
(velocity dispersion
aka "temperature")

$$T_\chi(t) = \frac{1}{n_\chi}\int \frac{d^3 p}{(2\pi)^3}\,\frac{\mathbf{p}^2}{3E} f_\chi(t,\mathbf{p})$$

… results in a coupled system of non-closed ODE

⟶ **assumption** on f_χ or the collision terms necessary in order to close the system of Eqs.

2nd moment: Kinetic decoupling

- Integrate BE over $\int \frac{d^3p}{(2\pi)^3} \frac{\mathbf{p^2}}{E^2}$ but

  without annihilation terms ($\sigma_{\bar{\chi}\chi\to\bar{f}f} = 0$ **or** $f_\chi \lll f_{\rm SM}$)

- This gives the standard **Kinetic decoupling Eq.**

$$\boxed{\dot{T}_\chi + 2HT_\chi = \gamma(T)\,(T - T_\chi)\,.}$$

(non-rel. DM)

Momentum transfer rate with the thermal bkg.

$$\gamma(T) = \int d\omega \, \underbrace{\frac{8k^4 f_{\rm q}^{\pm}(1 \mp f_{\rm q}^{\pm})}{3\pi^2 m_\chi T}}_{f_q \,=\, \text{bkg. particle's phase-sp distribution}} \underbrace{\int d\Omega\,(1-\cos\theta)\frac{d\sigma_{\chi q\to\chi q}}{d\Omega}}_{\text{DM's transfer x-section}}$$

0th **moment:** chemical decoupling

➡ Integrate BE over $\int \frac{d^3p}{(2\pi)^3 E}$ and

assume kinetic equilibrium with SM background

$$f_\chi = \frac{n_\chi}{n_\chi^{\rm eq}} f_\chi^{\rm eq} = \frac{n_\chi}{n_\chi^{\rm eq}} e^{-E/T_{\rm SM}}$$

➡ This gives the **standard** chemical **freeze-out Eq.**

$$\frac{dn_\chi}{dt} + 3Hn_\chi = \langle \sigma v \rangle_{\rm eq} \left(n_{\chi,\rm eq}^2 - n_\chi^2 \right),$$

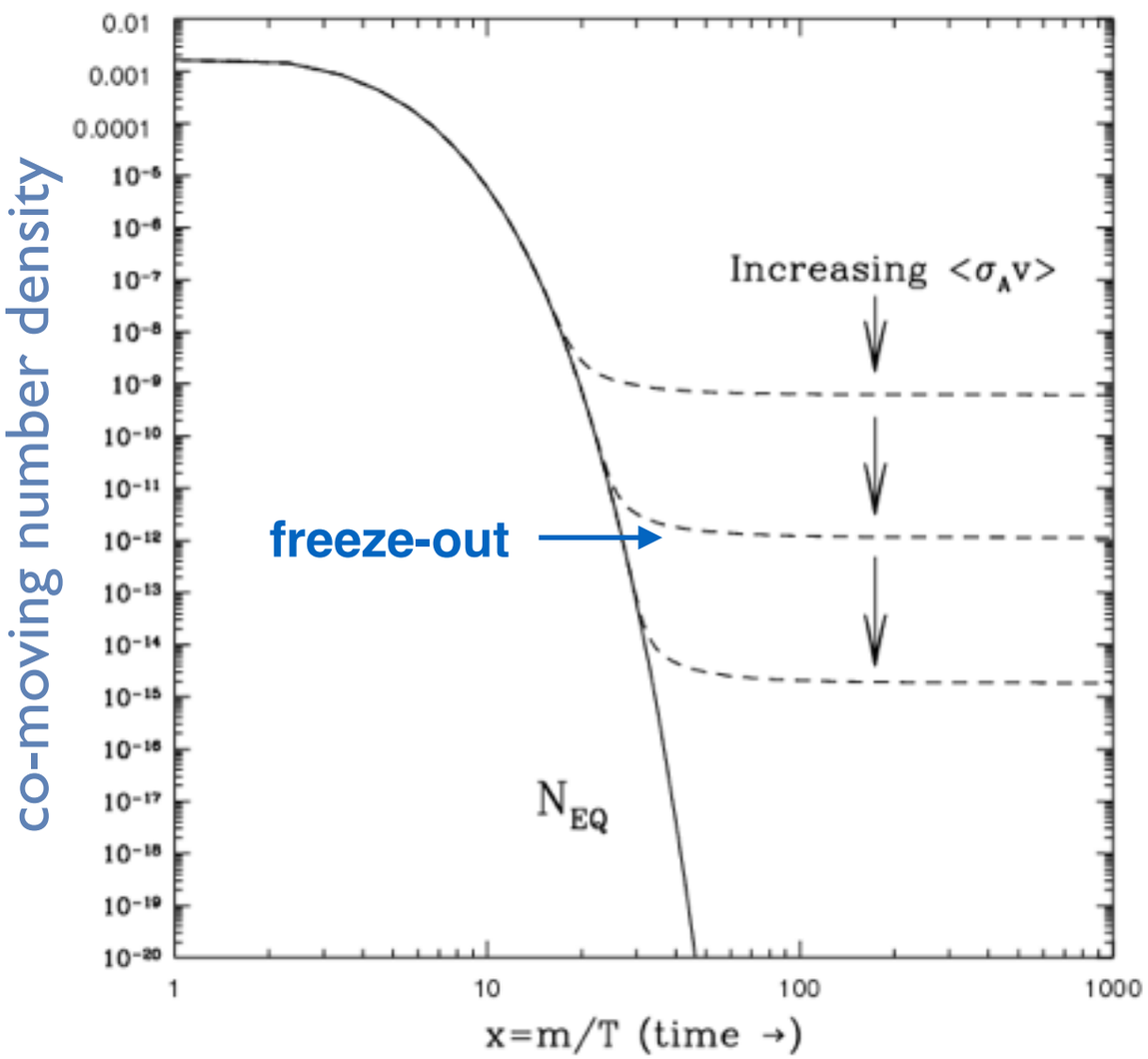
Thermal averaged **annihilation x-section**

$$\langle \sigma v \rangle_{\rm eq} \equiv \frac{1}{n_{\chi,\rm eq}^2} \int \frac{d^3p}{(2\pi)^3} \frac{d^3\tilde{p}}{(2\pi)^3} \sigma v_{\bar{\chi}\chi \to \bar{f} f} f_\chi^{\rm eq}(\mathbf{p}) f_\chi^{\rm eq}(\tilde{\mathbf{p}})$$

Under the assumption reducible to 1D integral

Gondolo & Gelmini '91

Standard chemical freeze-out



➡ **Numerical codes** guarantee sub-percent accuracy

$$\Omega_\chi h^2 = 0.1198 \pm 0.0012$$

Planck 2018

even for sophisticated DM models (e.g. **DarkSuSy**, **micrOMEGAs**,…)

and account for many exceptions:

- ✓ Co-annihilations
- ✓ Kinematic Threshold effects
- ✓ Near resonance annihilations
- ✓ …

Griest & Seckel '91

➡ **but what if** the DM is **not** in **kinetic equilibrium** during chemical decoupling?

→ Do results change?

→ How to compute Ω_{DM} ?

→ **A refined treatment required**

A refined treatment

Solving the Boltzmann Equation

$$E\left(\partial_t - H\mathbf{p}\cdot\nabla_{\mathbf{p}}\right) f_\chi = C[f_\chi]\,.$$

Two approaches

Solving the Boltzmann Equation

$$E\left(\partial_t - H\mathbf{p}\cdot\nabla_{\mathbf{p}}\right) f_\chi = C[f_\chi]\,.$$

Two approaches

1st Method:

Combine 0th and 2nd momentum moments
… with an assumption on $f_\chi(p)$

Analytical insights/
numerically easier.

Finite range of validity.
No information on underlying
phase-space distribution.

See also van den Aarssen, Bringmann
& Goedecke, PRD '12

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Analytical insights/
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No information on underlying
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2nd Method:

Numerically solve for the full phase-space distribution $f_\chi(p)$

Full information on the
phase-space distribution.

Numerically challenging
(sometimes an overkill).

See also van den Aarssen, Bringmann
& Goedecke, PRD '12

1st Method

1st Method — momentum moments of BE

➡ In dimensionless DM **density** $Y \equiv n_\chi / s$ and **temperature** $y \equiv \frac{m_\chi T_\chi}{s^{2/3}}$:

0th moment:

$$\frac{Y'}{Y} = \frac{sY}{x\tilde{H}} \left[\frac{Y_{\rm eq}^2}{Y^2} \langle \sigma v \rangle_{\rm eq} - \langle \sigma v \rangle_{\rm neq} \right] ,$$

2nd moment:

$$\frac{y'}{y} = \frac{\gamma(T)}{x\tilde{H}} \left[\frac{y_{\rm eq}}{y} - 1 \right] + \frac{Y'}{Y} \left[\frac{\langle \sigma v \rangle_{2,\rm neq}}{\langle \sigma v \rangle_{\rm neq}} - 1 \right] + \frac{H}{x\tilde{H}} \frac{\langle p^4/E^3 \rangle_{\rm neq}}{3T_\chi}$$
$$+ \frac{sY}{x\tilde{H}} \frac{Y_{\rm eq}^2}{Y^2} \left[\frac{y_{\rm eq}}{y} \langle \sigma v \rangle_{2,\rm eq} - \frac{\langle \sigma v \rangle_{\rm eq}}{\langle \sigma v \rangle_{\rm neq}} \langle \sigma v \rangle_{2,\rm neq} \right]$$

+ update to semi-relativistic

functions of $x = m/T_{\rm SM}$

with

$$\langle \sigma v \rangle_{\rm neq} \equiv \frac{1}{n_{\chi,\rm eq}^2} \int \frac{d^3p}{(2\pi)^3} \frac{d^3\tilde{p}}{(2\pi)^3} \sigma v_{\bar{\chi}\chi \to \bar{f}f} f_{\chi,}(\mathbf{p}) f_{\chi,}(\tilde{\mathbf{p}})$$

x-section

$$\langle \sigma v \rangle_{2,\rm neq} \equiv \frac{1}{T_\chi n_\chi^2} \int \frac{d^3p\, d^3\tilde{p}}{(2\pi)^6} \frac{p^2}{3E} \sigma v_{\bar{\chi}\chi \to \bar{f}f} f_\chi(\mathbf{p}) f_\chi(\tilde{\mathbf{p}})$$

***p^2* weighted**

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***p^2* weighted**

$\langle\ldots\rangle_{\rm neq}$ are with $f_\chi \neq f_{\chi,\rm eq}$

and to close the equations we <u>need an assumption on</u> $f_\chi(p)$

1st Method — momentum moments of BE

Now close the system by a (more general) assumption:

$$f_\chi = \frac{n_\chi}{n_\chi^{\rm eq}} e^{-E/T_\chi}$$

← motivated **if** strong self-scattering

or more precise $\langle \ldots \rangle_{\rm neq} \to \langle \ldots \rangle_{T_\chi = y\, s^{2/3}/m_\chi}$

$$\frac{Y'}{Y} = \frac{sY}{x\tilde{H}} \left[\frac{Y_{\rm eq}^2}{Y^2} \langle \sigma v \rangle_{\rm eq} - \langle \sigma v \rangle_{\rm neq} \right],$$

production and annihilation averaged x-sections are now with different f_χ

$$\frac{y'}{y} = \underbrace{\frac{\gamma(T)}{x\tilde{H}} \left[\frac{y_{\rm eq}}{y} - 1 \right]}_{\text{Elastic scatterings}} + \underbrace{\frac{Y'}{Y} \left[\frac{\langle \sigma v \rangle_{2,\rm neq}}{\langle \sigma v \rangle_{\rm neq}} - 1 \right]}_{\text{Annihilation feedback}} +$$

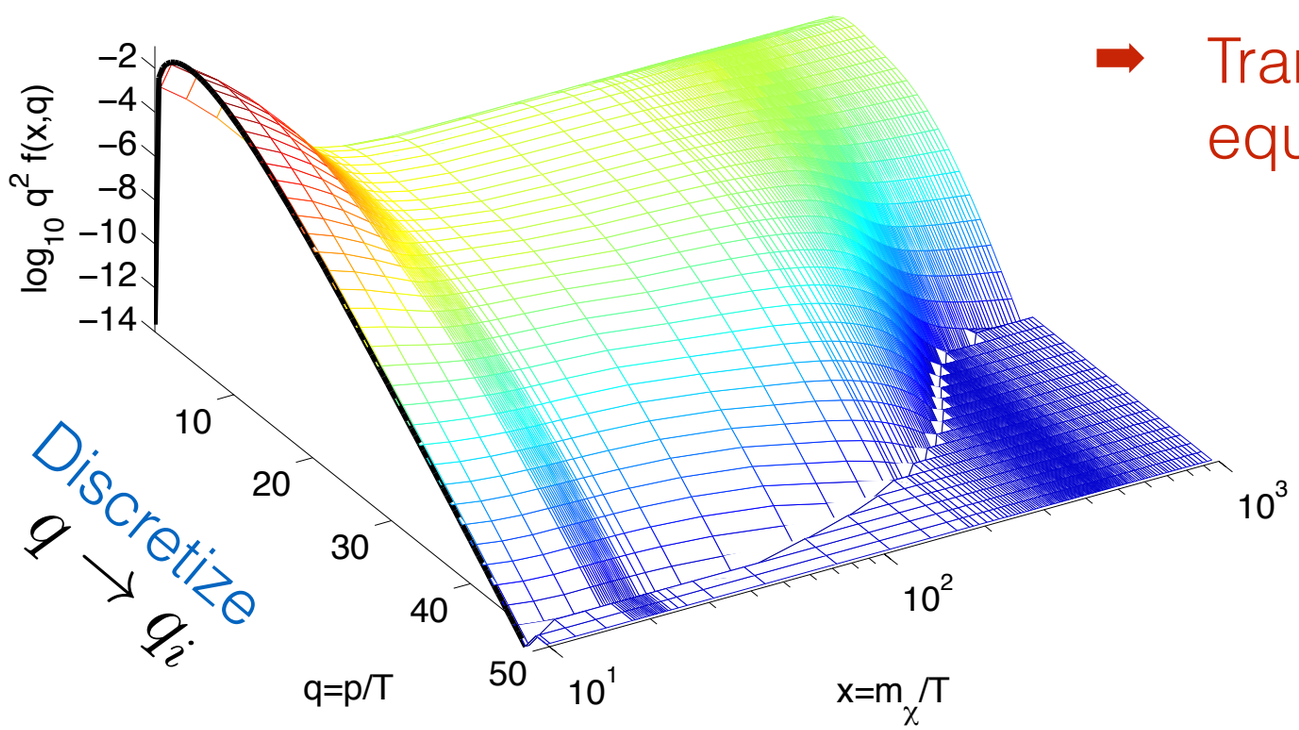
For brevity, less relevant terms left out

2nd Method

2nd Method — full phase space solution

- Rewrite BE in $x \equiv m_\chi/T$ and $q \equiv p/T$
- Discretize $q \to q_i$ and impose boundary conditions at $q_{\min}$ and $q_{\max}$

$$\frac{d}{dx} f_i = \frac{m_\chi^3}{\tilde{H} x^4} \frac{1}{2\pi^2} \sum_{j=1}^{N-1} \frac{\Delta q_j}{2} \left[\sum_{k=j}^{j+1} q_k^2 \langle v_{\text{Møl}} \sigma_{\bar{\chi}\chi \to \bar{f} f} \rangle_{i,k}^{\theta} \left(f_i^{\text{eq}} f_k^{\text{eq}} - f_i f_k \right) \right]$$
$$+ \frac{\gamma(x)}{2\tilde{H} x} \left[x_{q,i} \partial_q^2 f_i + \left(q_i + \frac{2 x_{q,i}}{q_i} + \frac{q_i}{x_{q,i}} \right) \partial_q f_i + 3 f_i \right]$$
$$+ \tilde{g} \frac{q_i}{x} \partial_q f_i \qquad x_q \equiv \sqrt{x^2 + q^2}$$



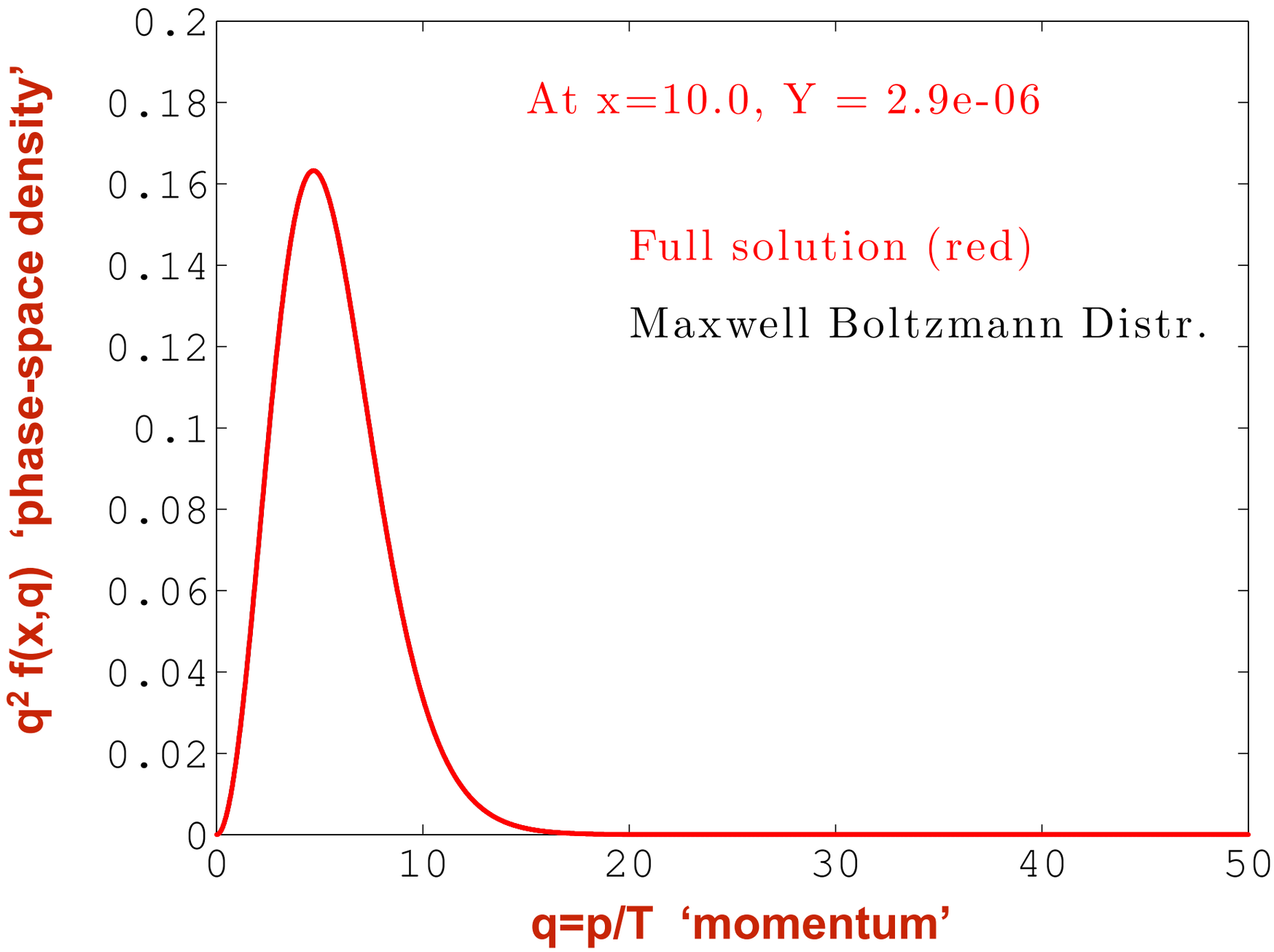
- Transformed the partial integro differential equation into ***N* coupled ODEs !**

$$f_i \equiv f_\chi(x, q_i) \to f_\chi(x, q)$$

- *N* ~1000 fixed steps in *q*
- Dynamically adjusted steps in *x*
- Boundary conditions:
  @ $q_{\min}$: forward derivates
  @ $q_{\max}$: zero out-flow

DM phase-space evolution during freeze-out

An example of a very non-trivial evolution of f(x,q)



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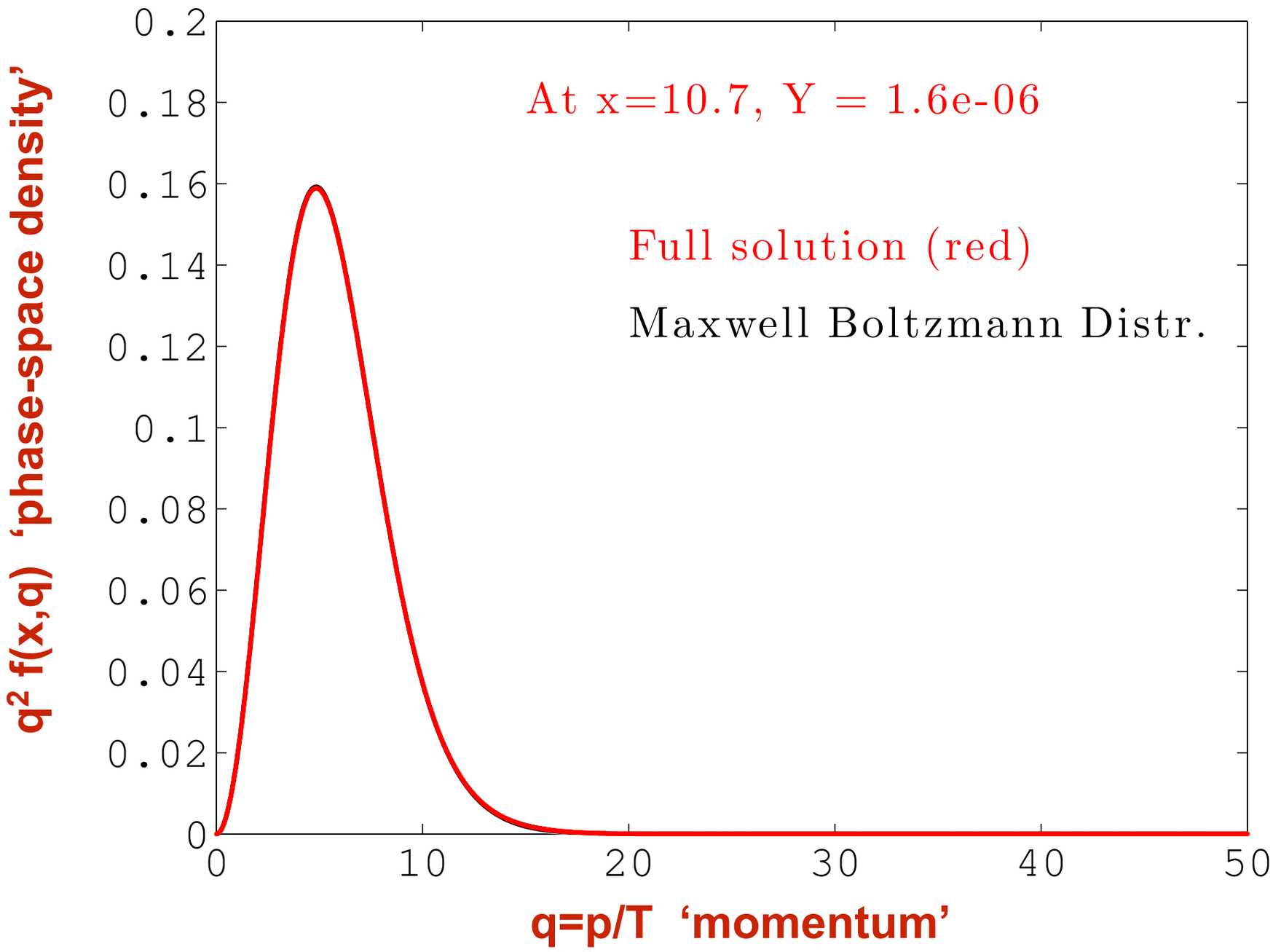

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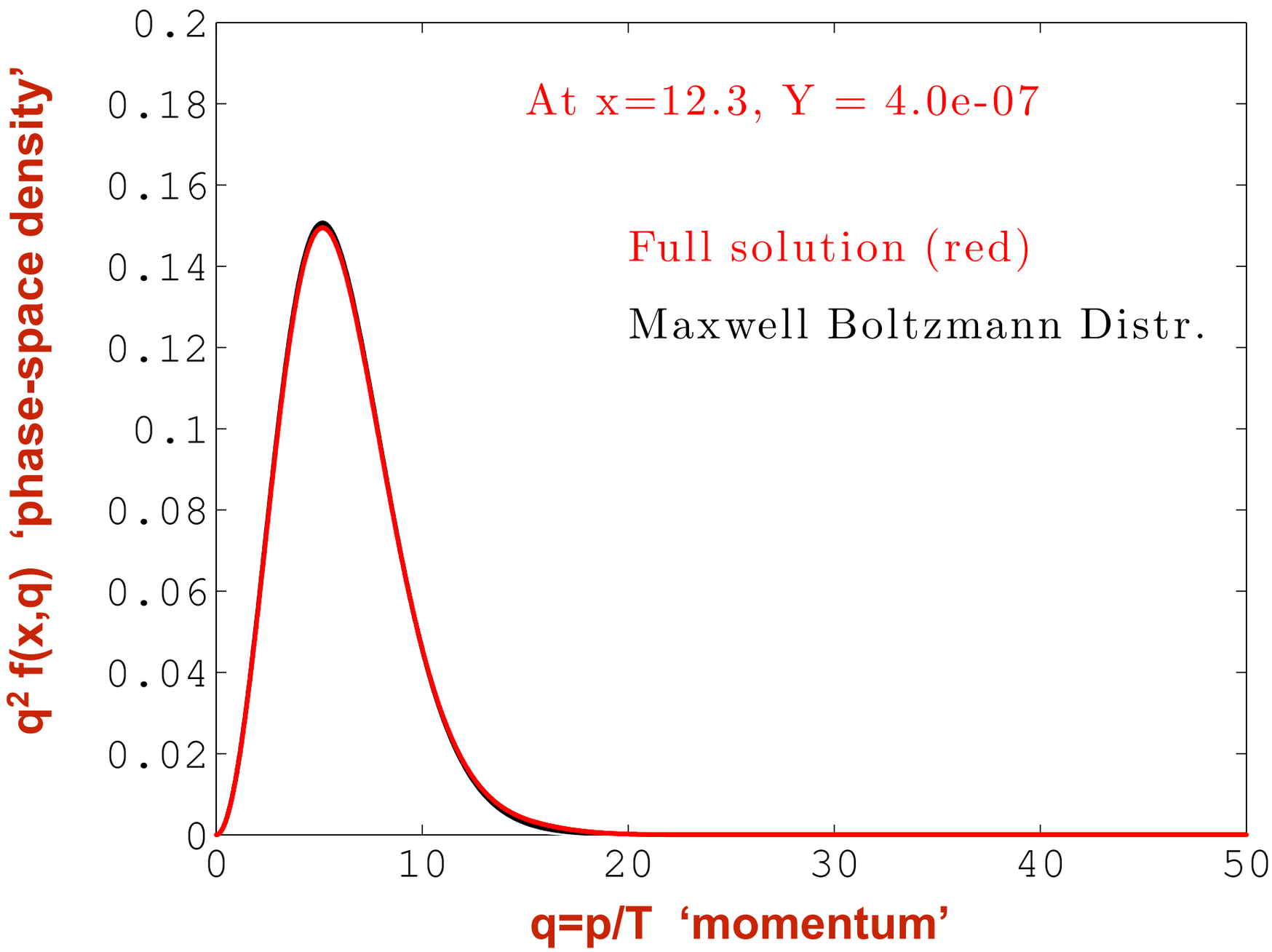

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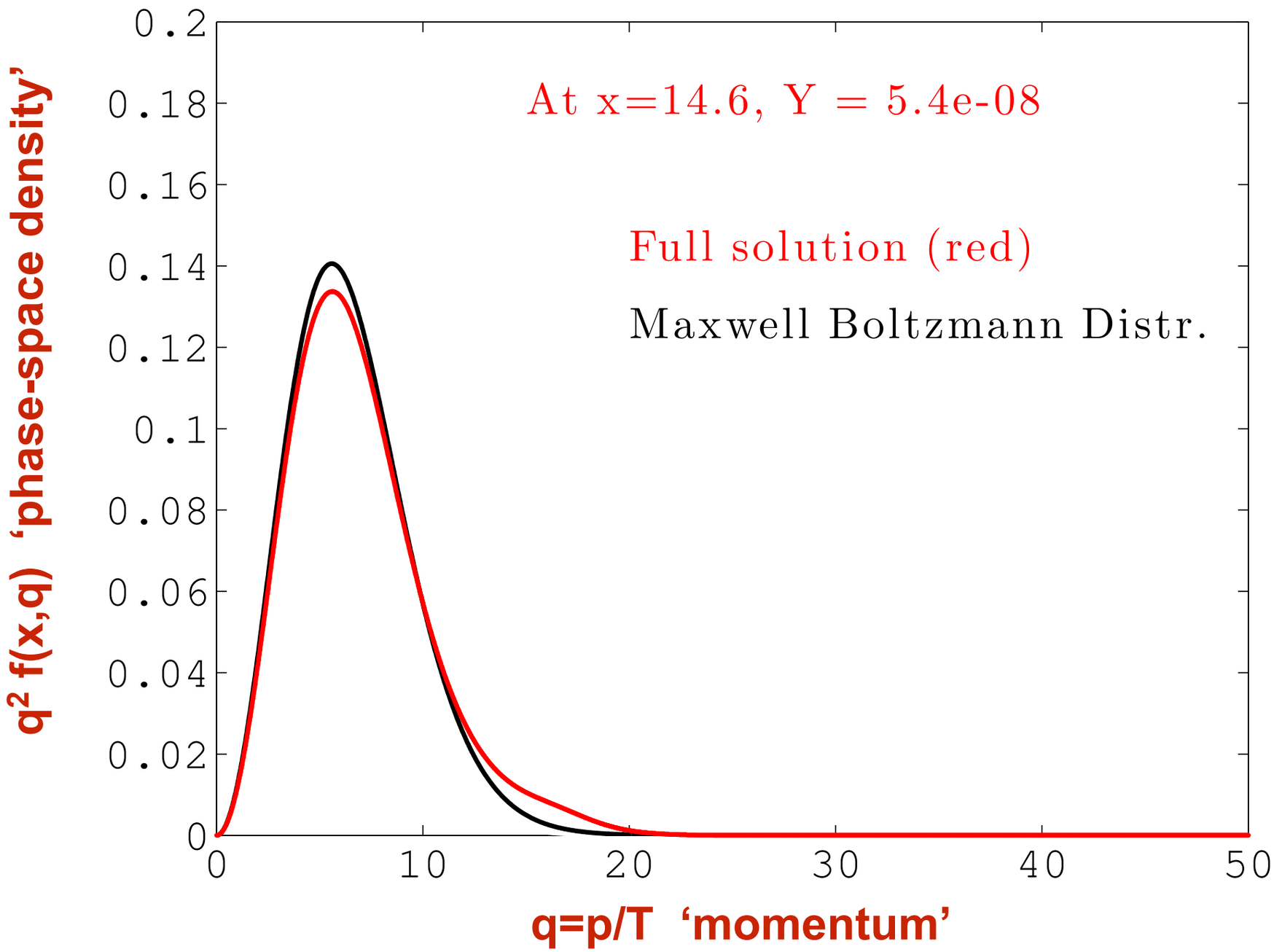

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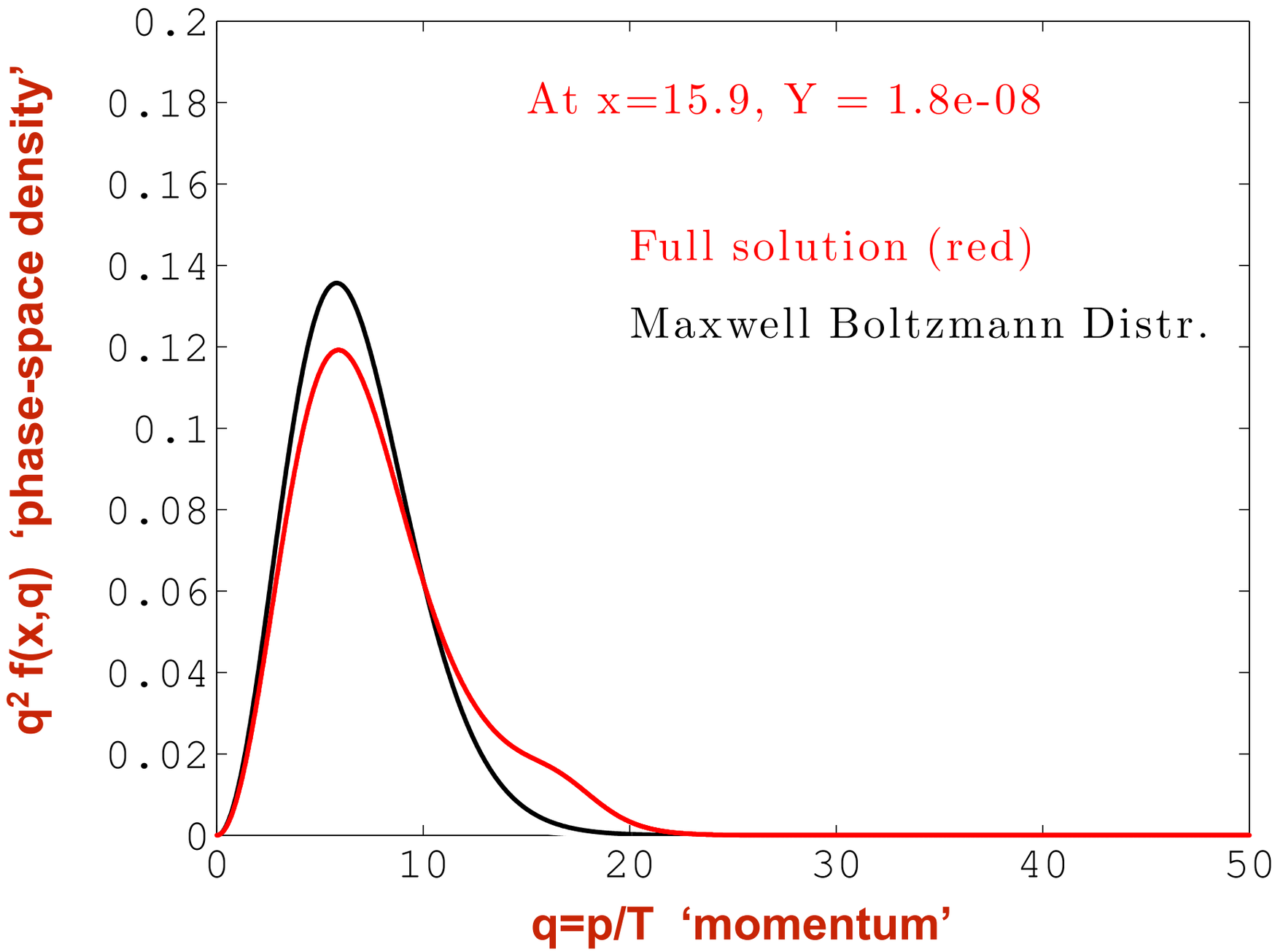

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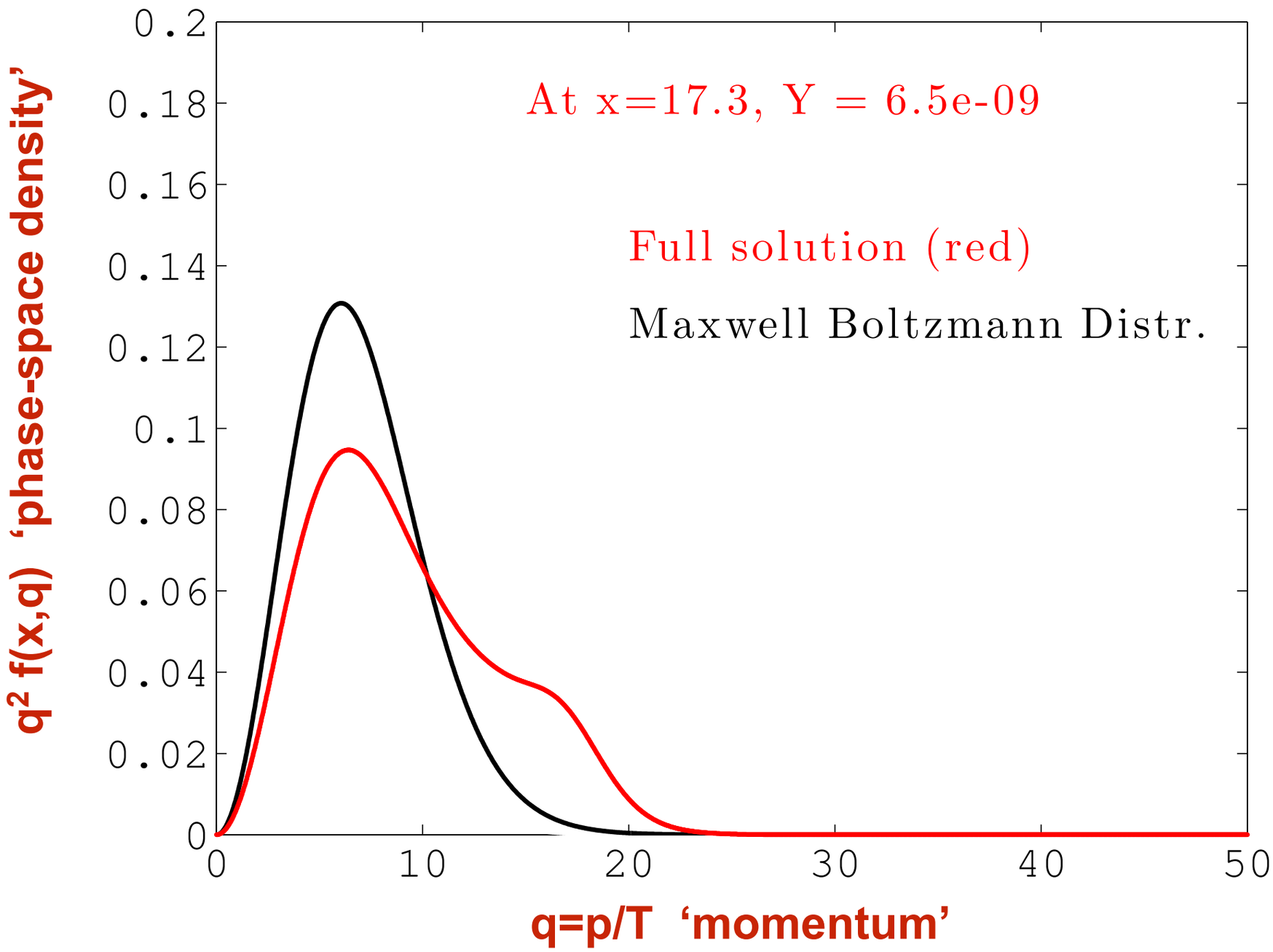

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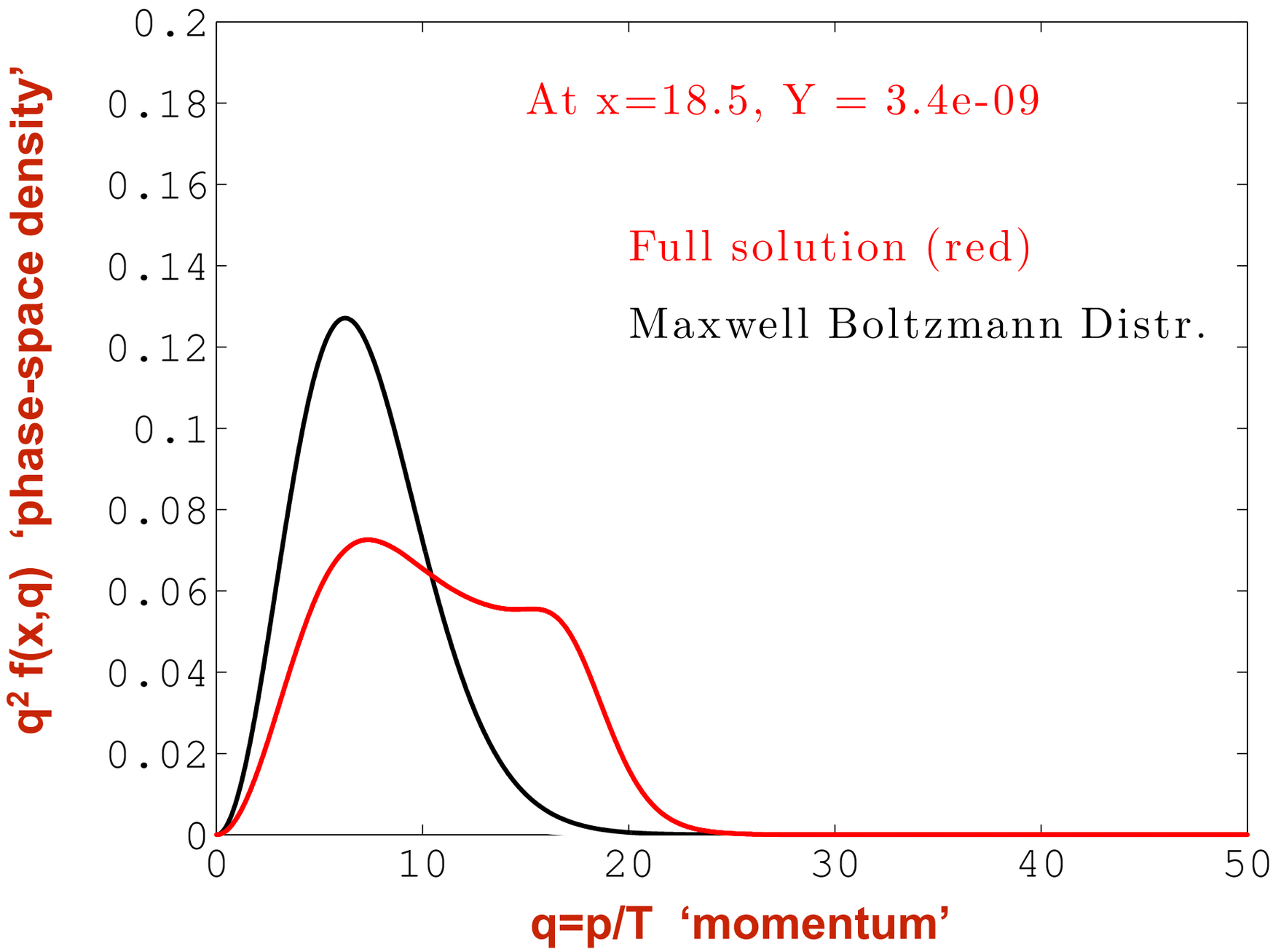

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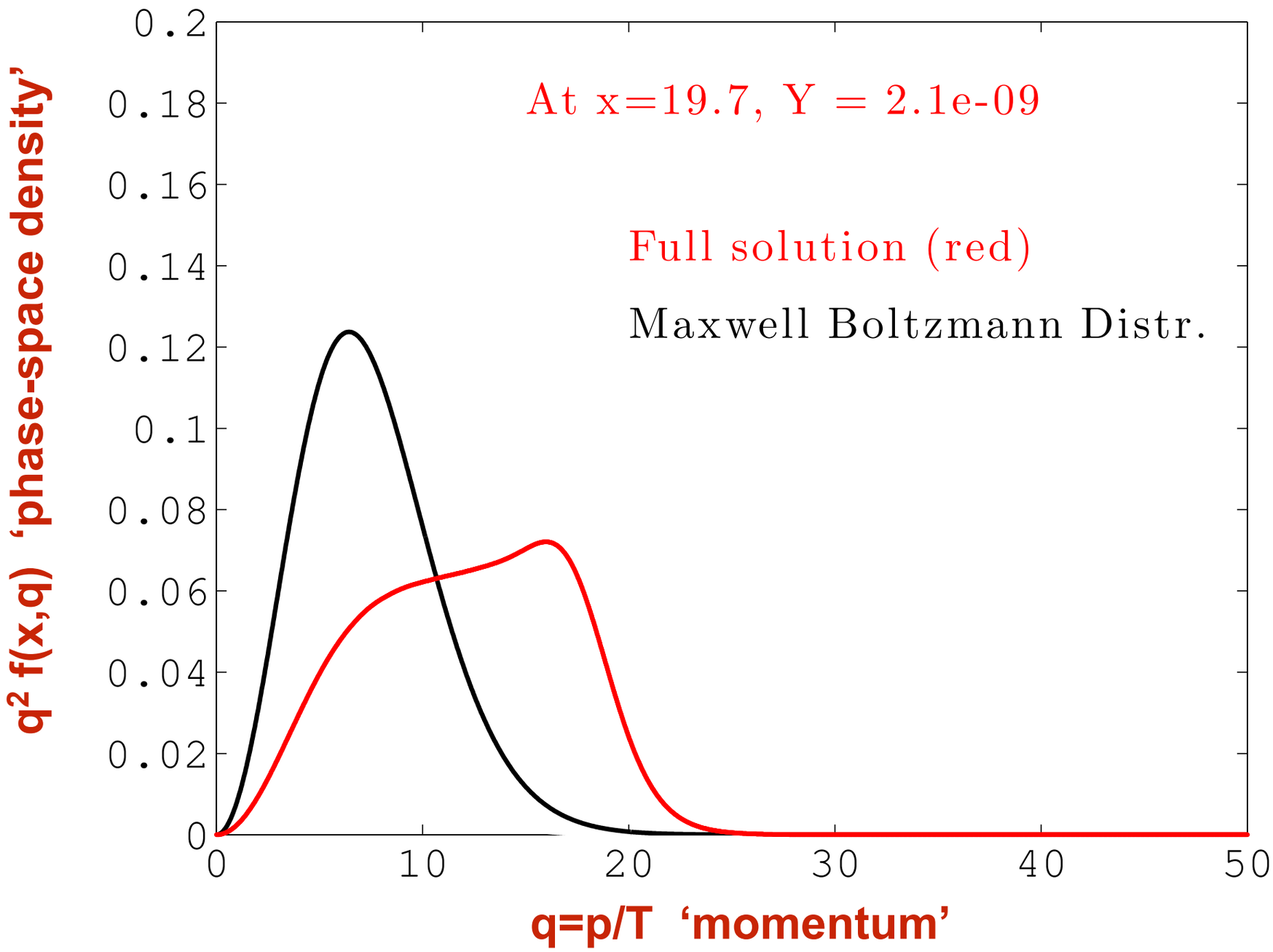

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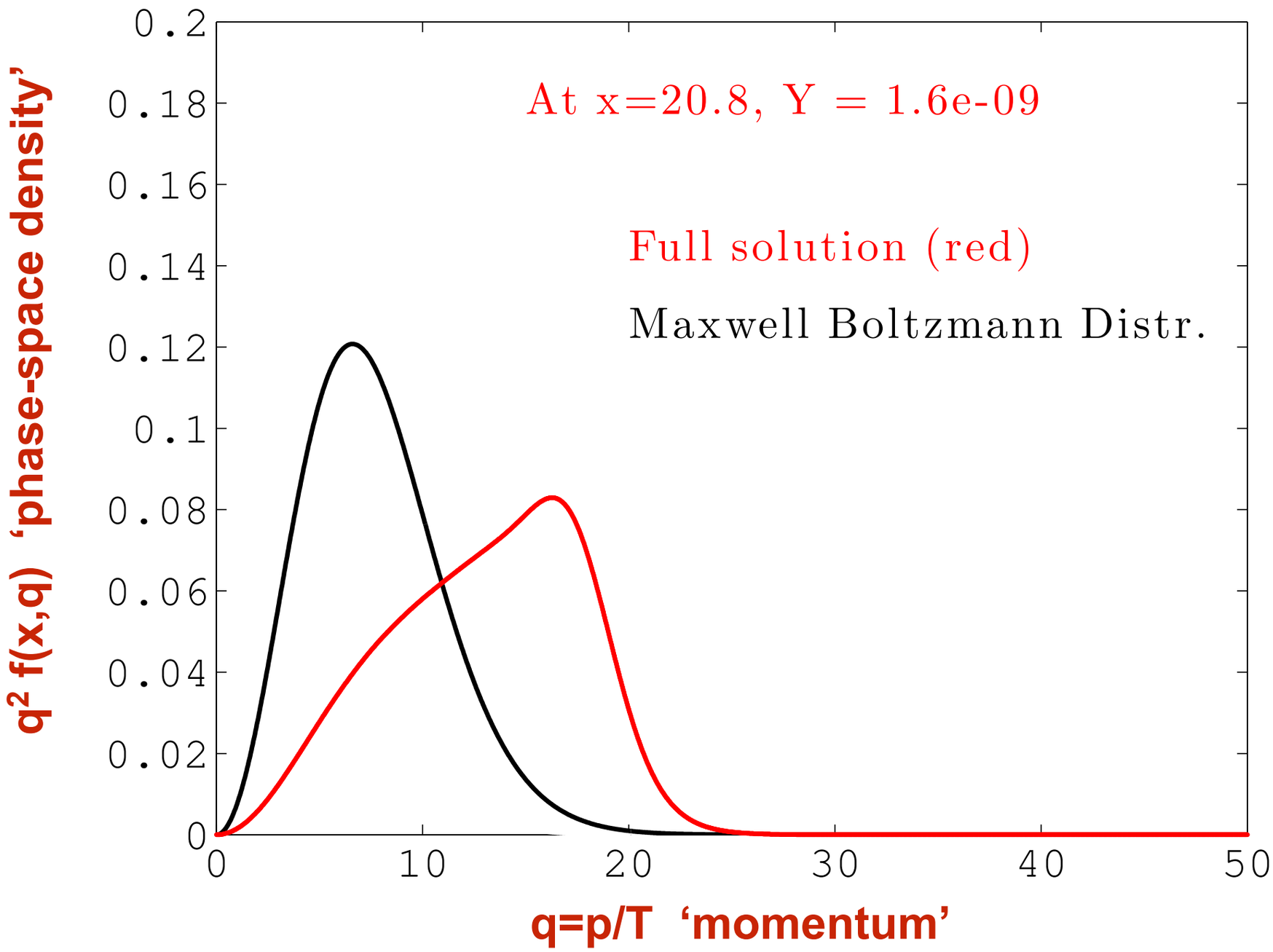

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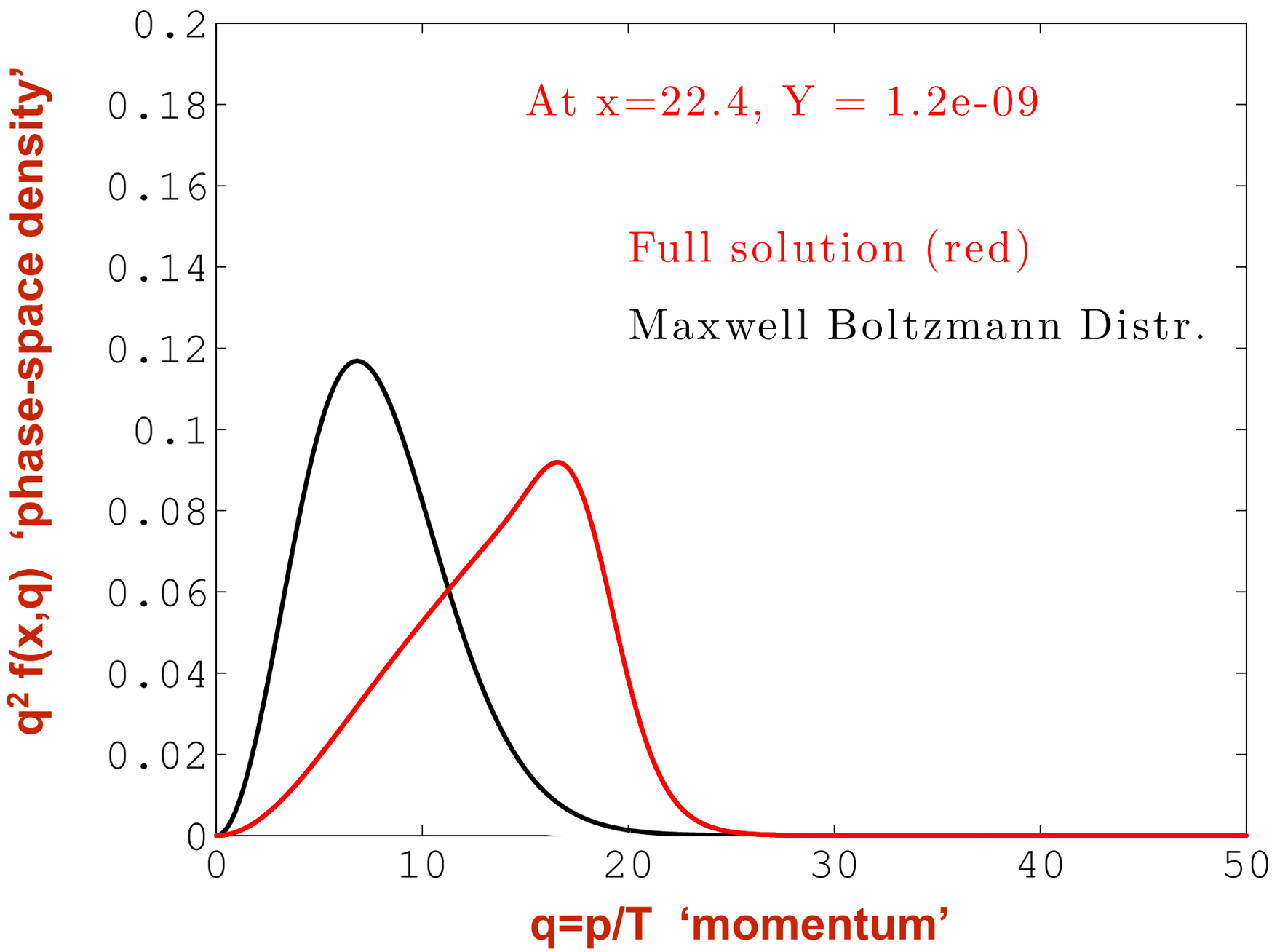

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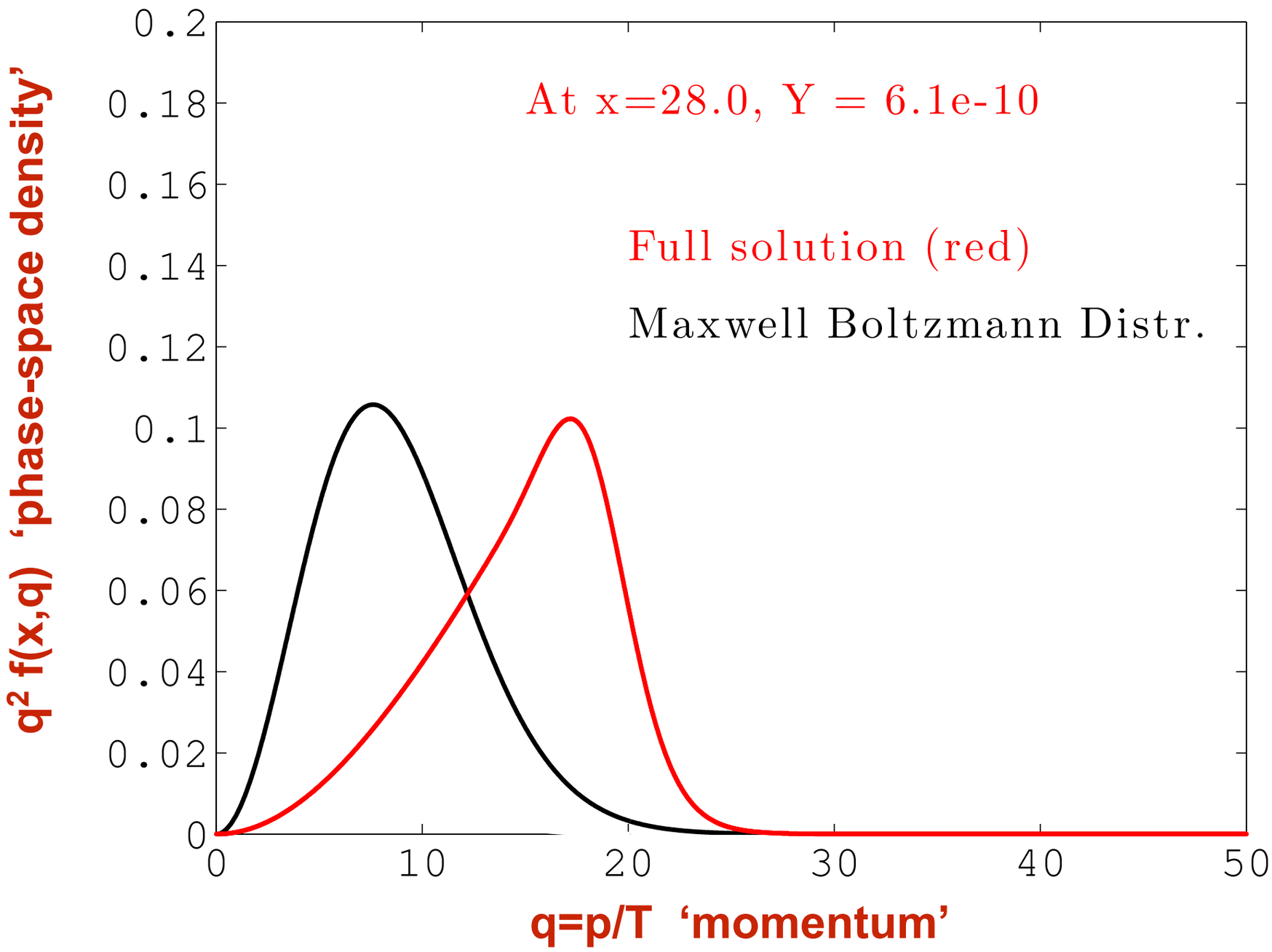

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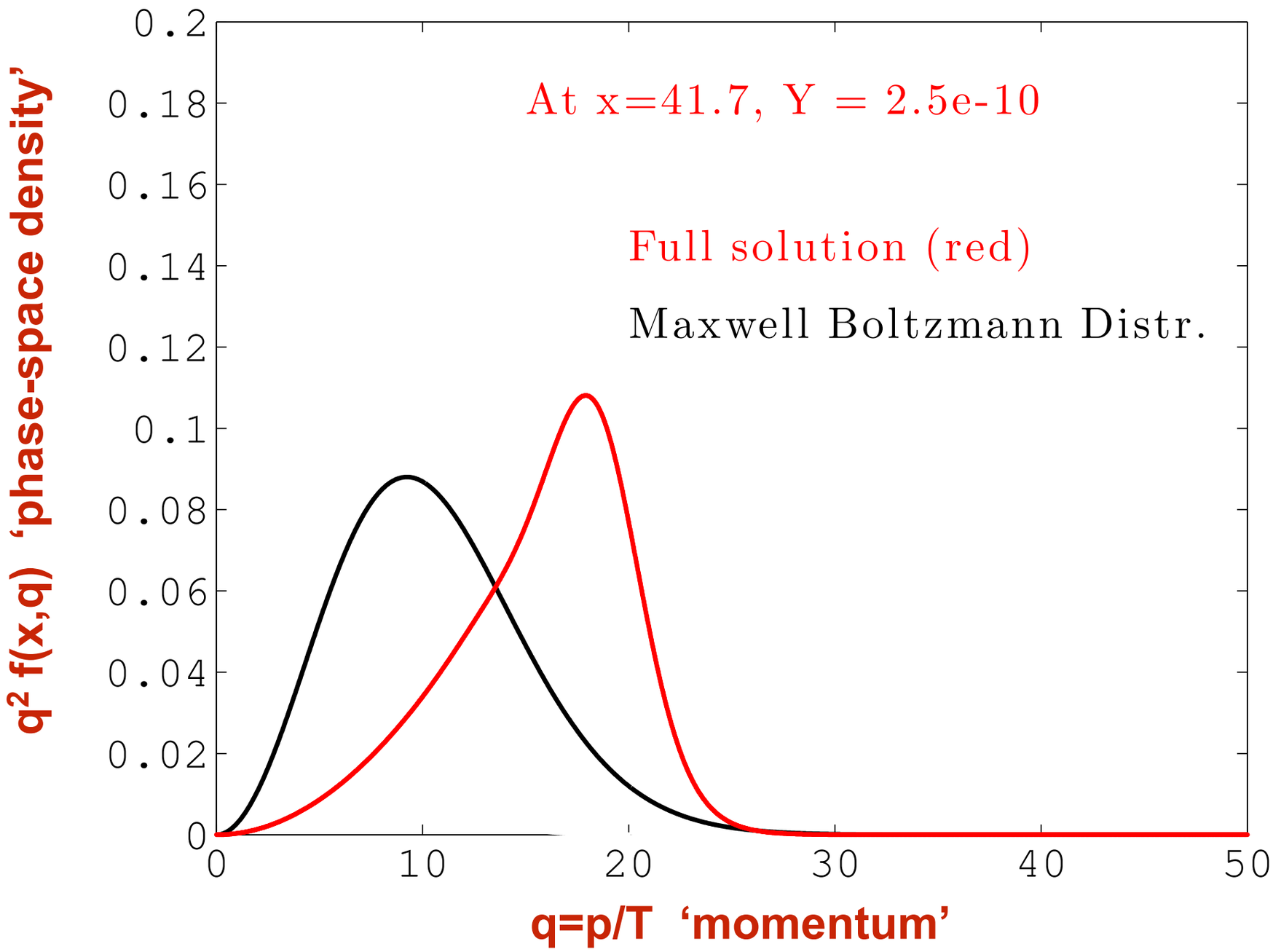

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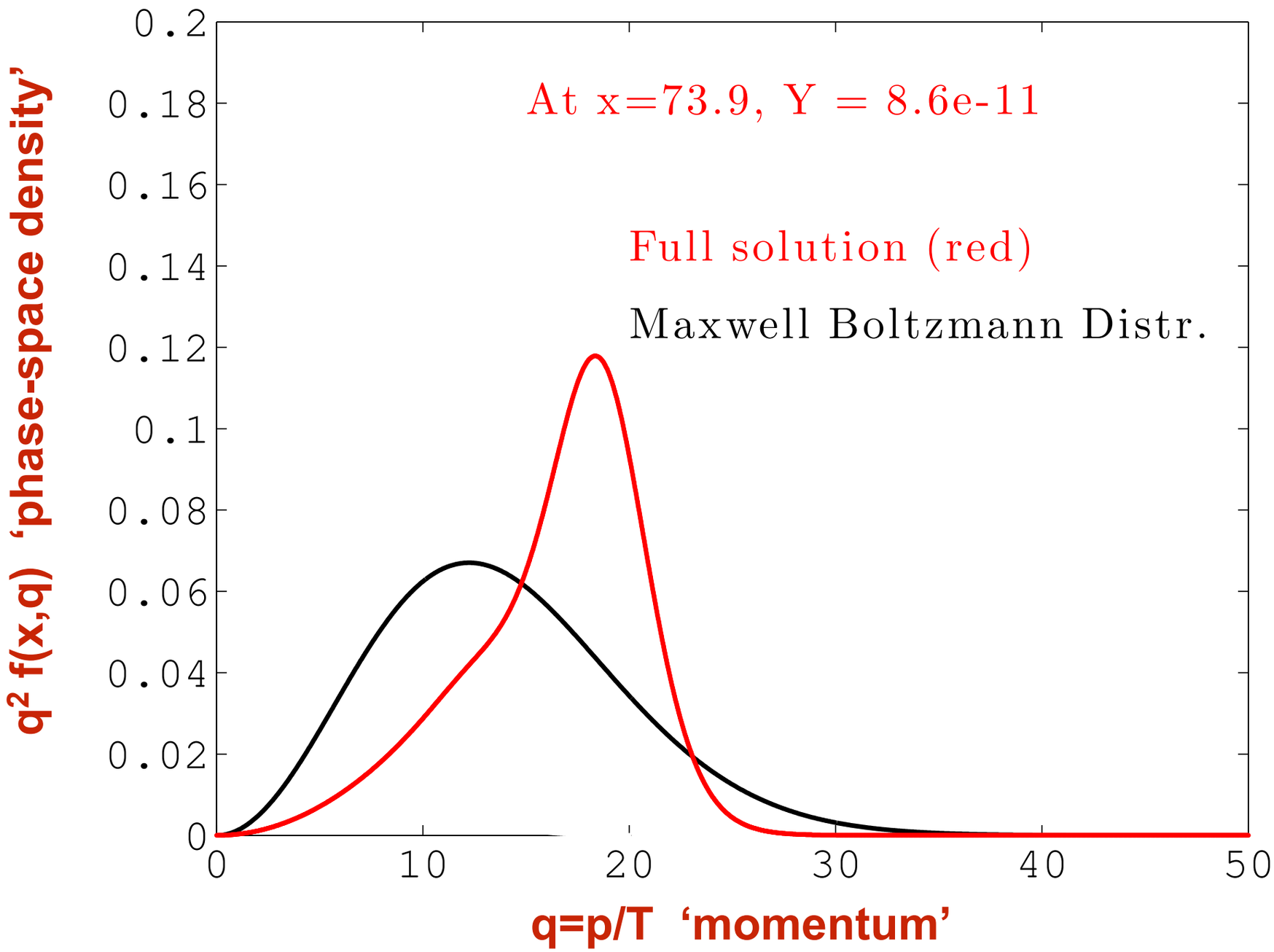

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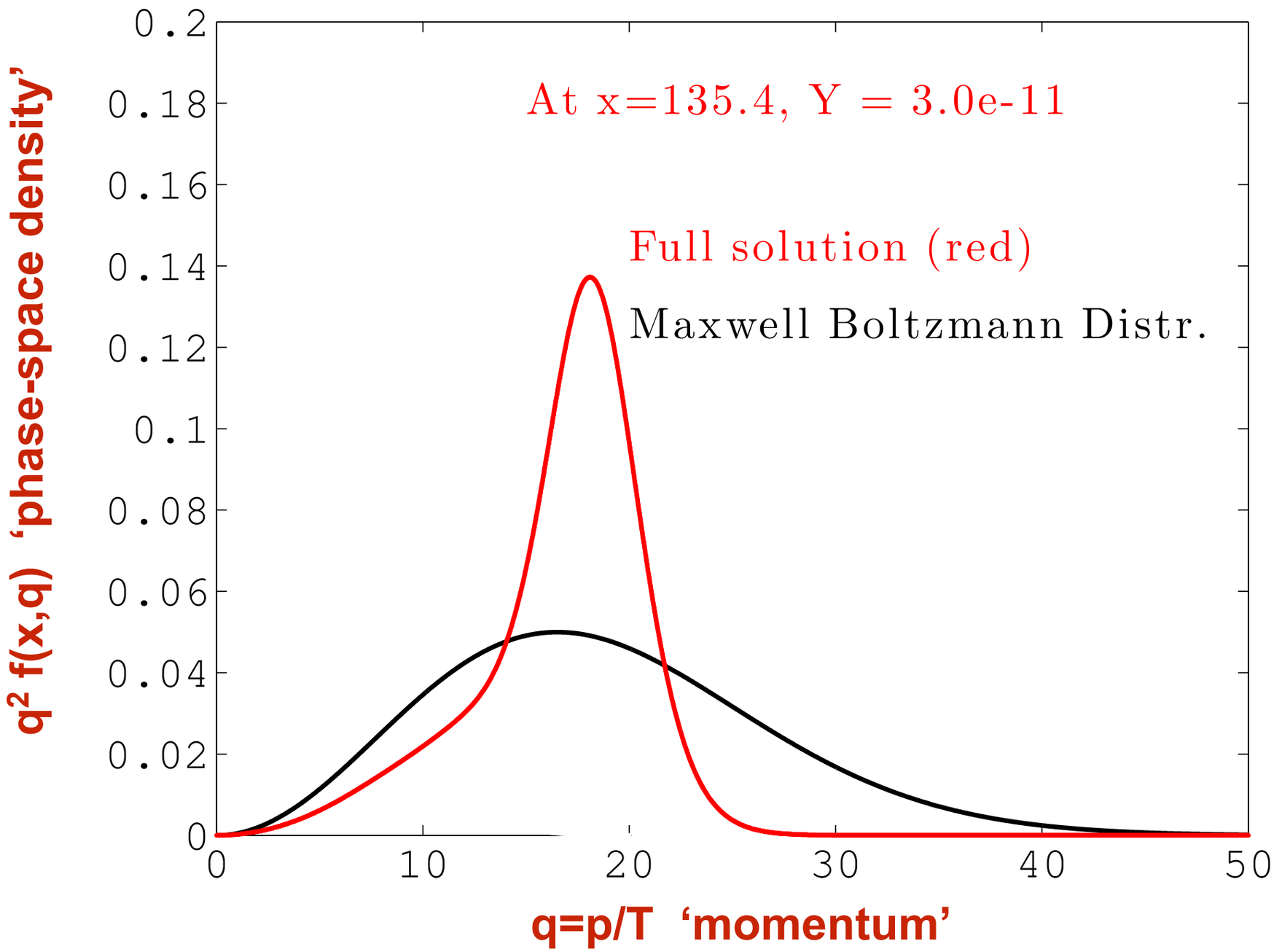

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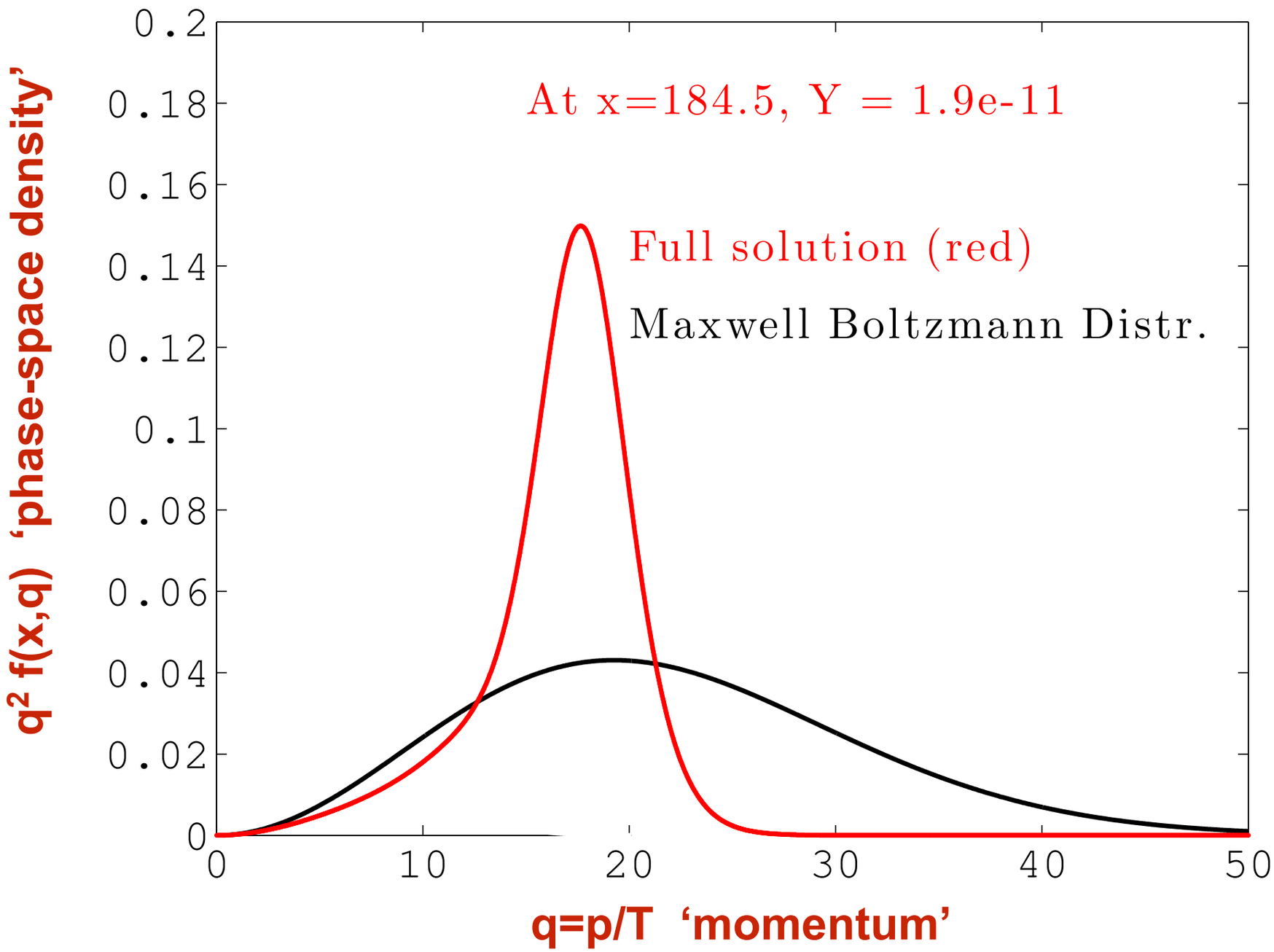

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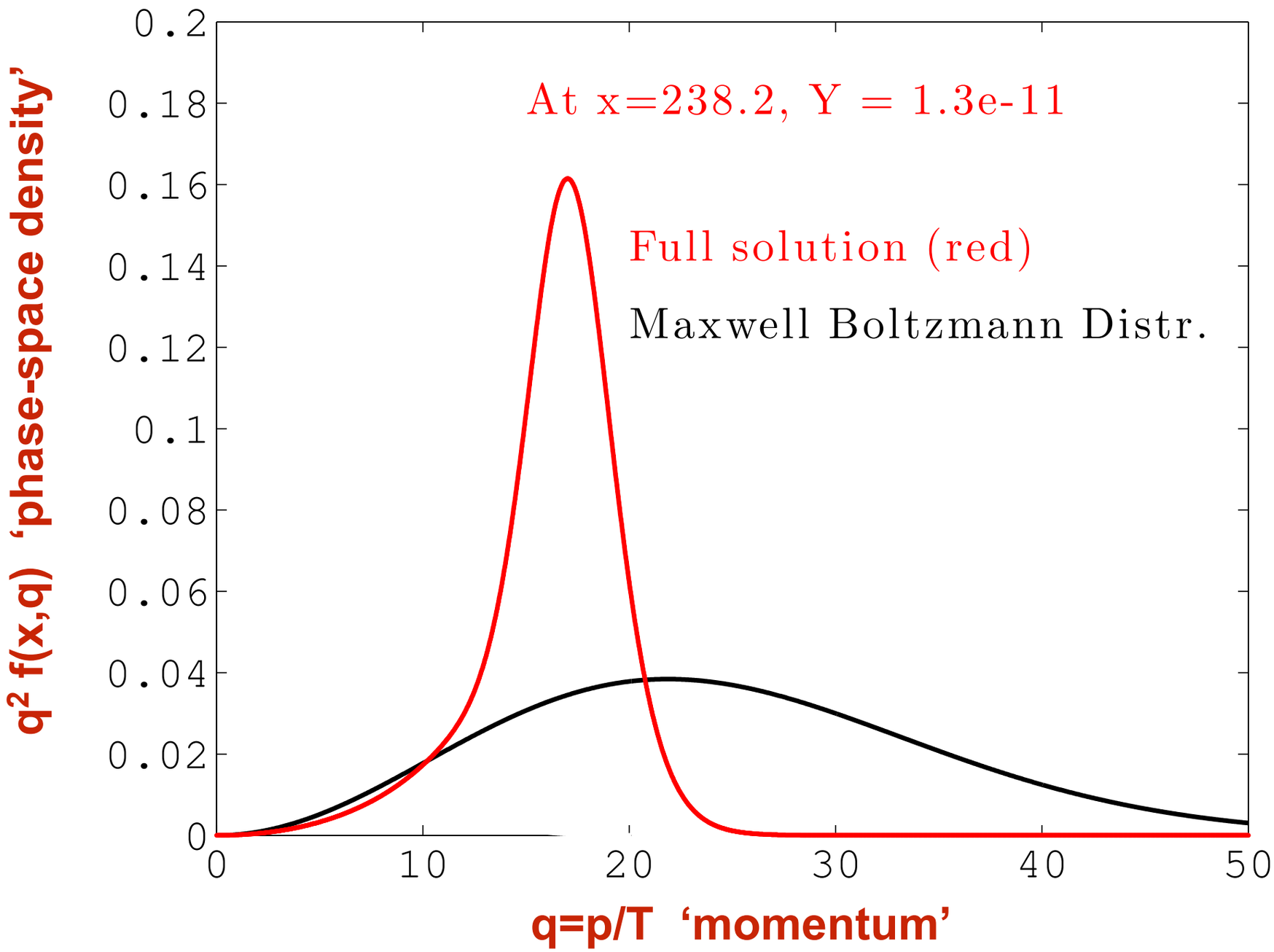

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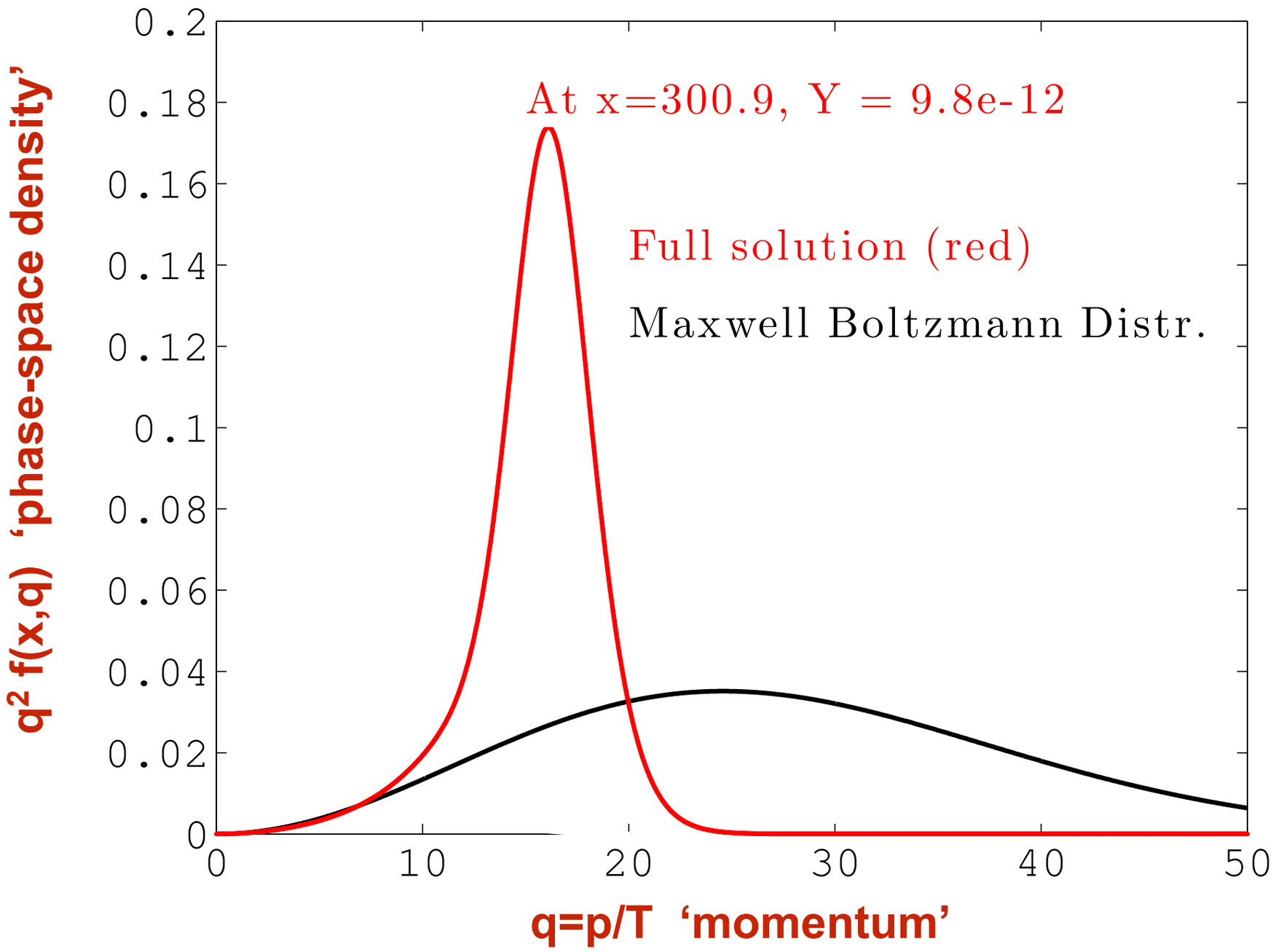

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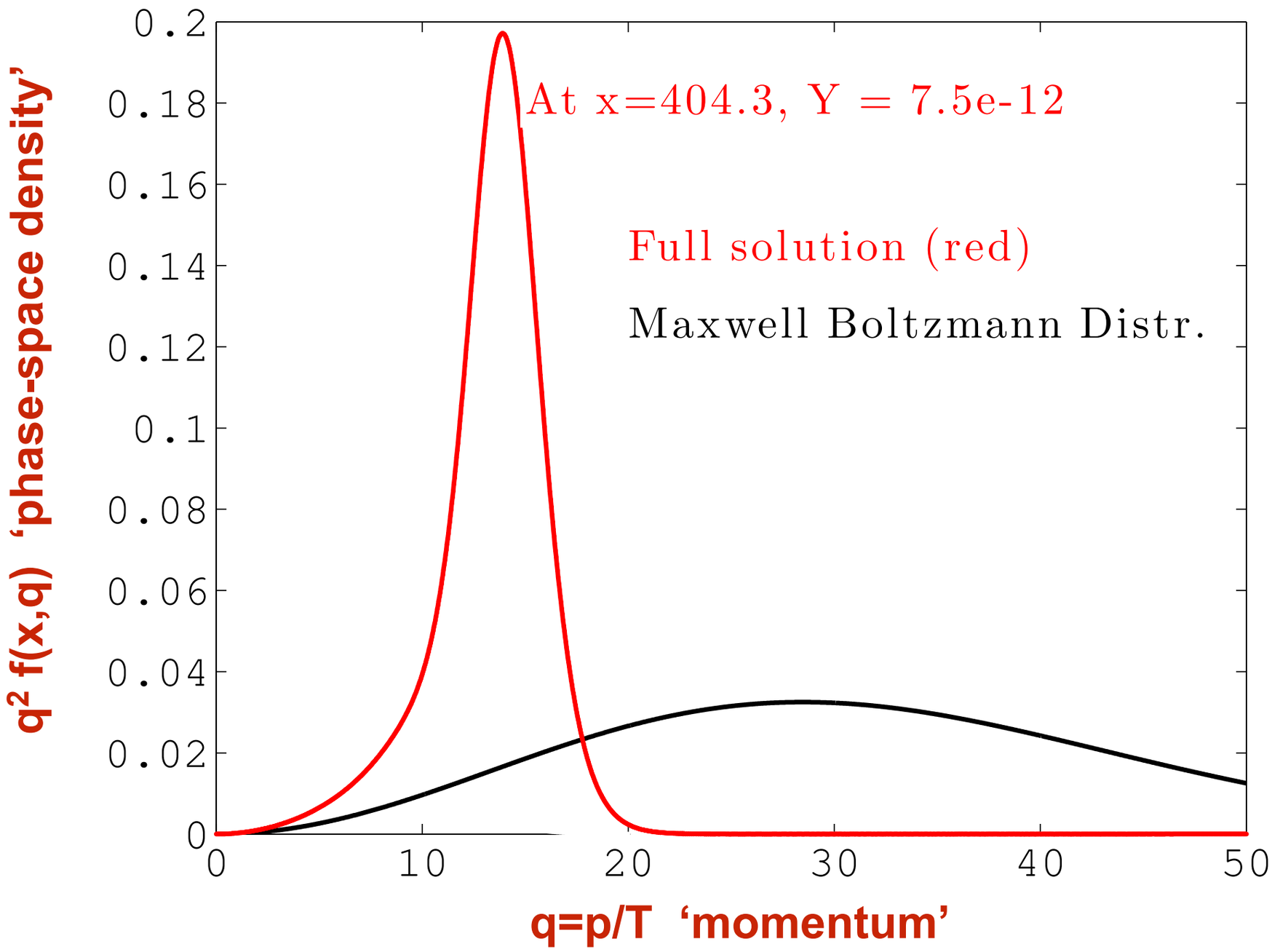

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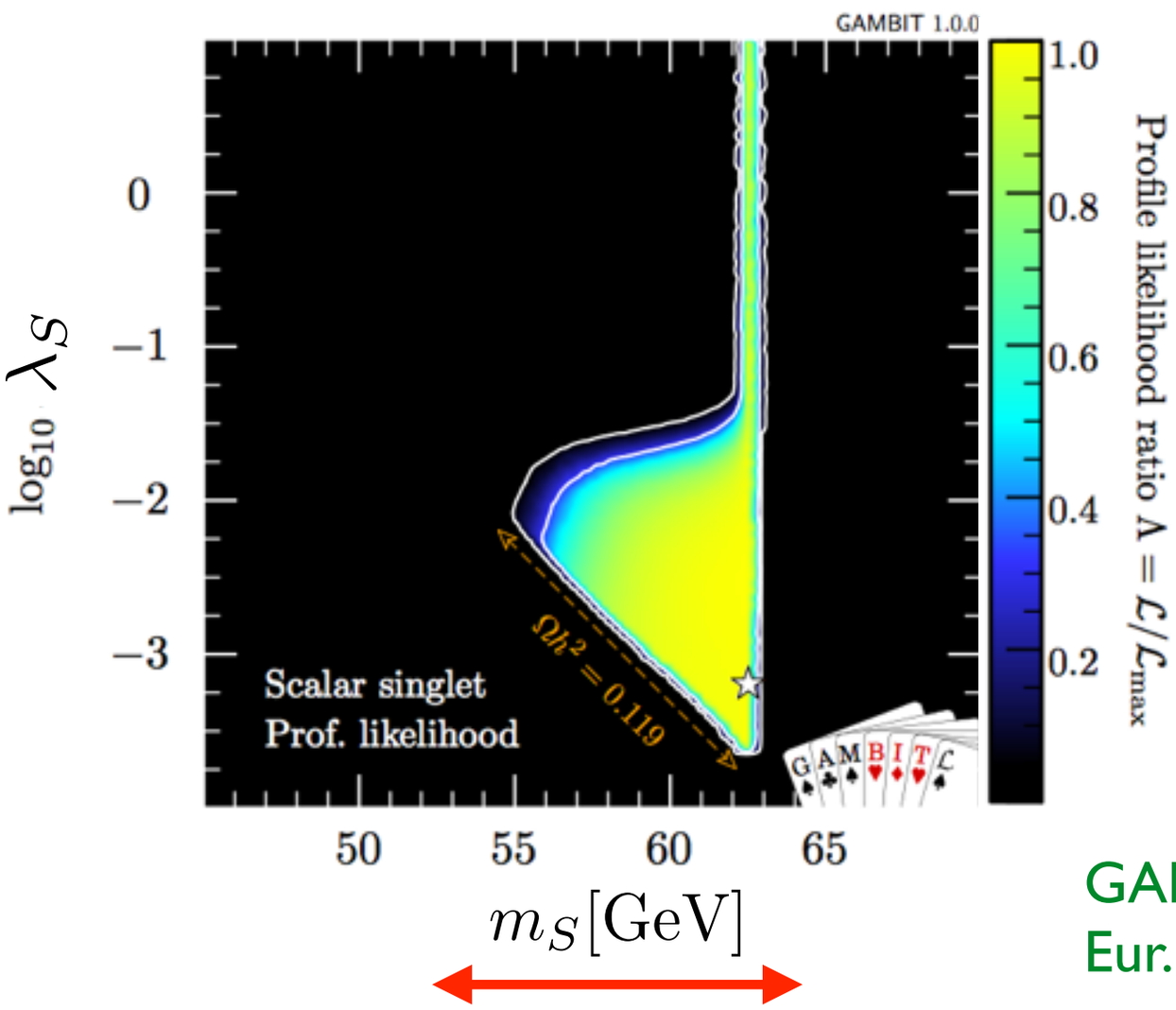
Example:
The Singlet Scalar model

The Singlet Scalar Model

Extend SM with a Dark Scalar Singlet S coupled to the higgs:

$$\mathcal{L}_{\rm SS} = \frac{1}{2}\partial_\mu S \partial^\mu S - \frac{1}{2}\mu_S^2 S^2 - \frac{1}{2}\lambda_S S^2 H^\dagger H$$

$$(m_h = 125.1\text{GeV})$$



Essentially a two-parameter model

$$m_s = \sqrt{\mu_S^2 + \frac{1}{2}\lambda_s v_0^2}$$

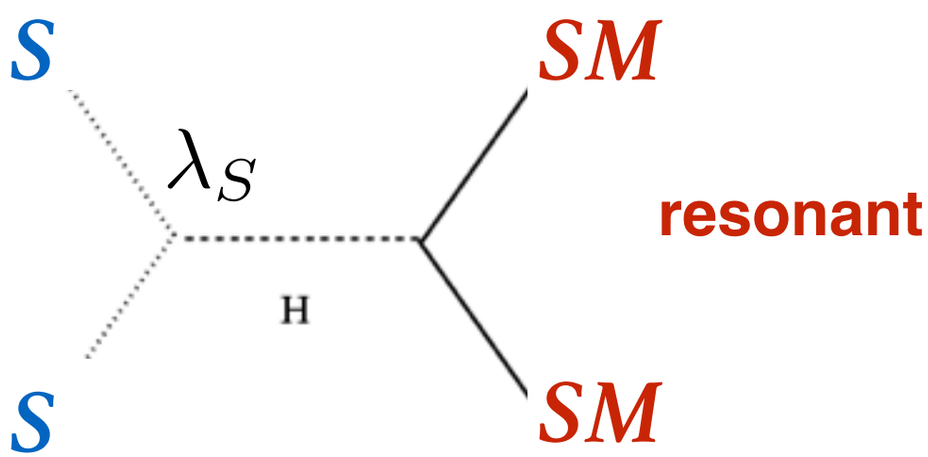
… with a surviving region around

***m_S* ~ 55 to 65 GeV**

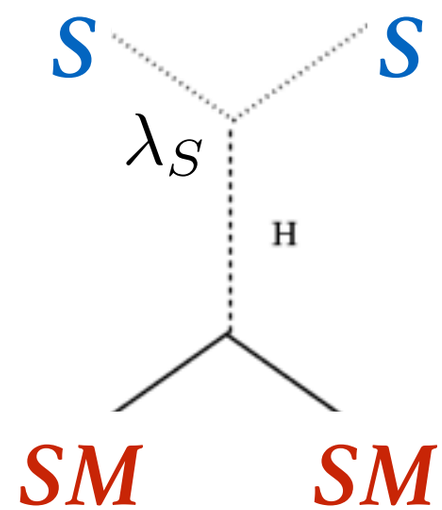
GAMBIT collaboration,
Eur. Phys. J. C 77, 568 (2017)

Why this Model ?

Annihilation **VS** **Scattering**



resonant



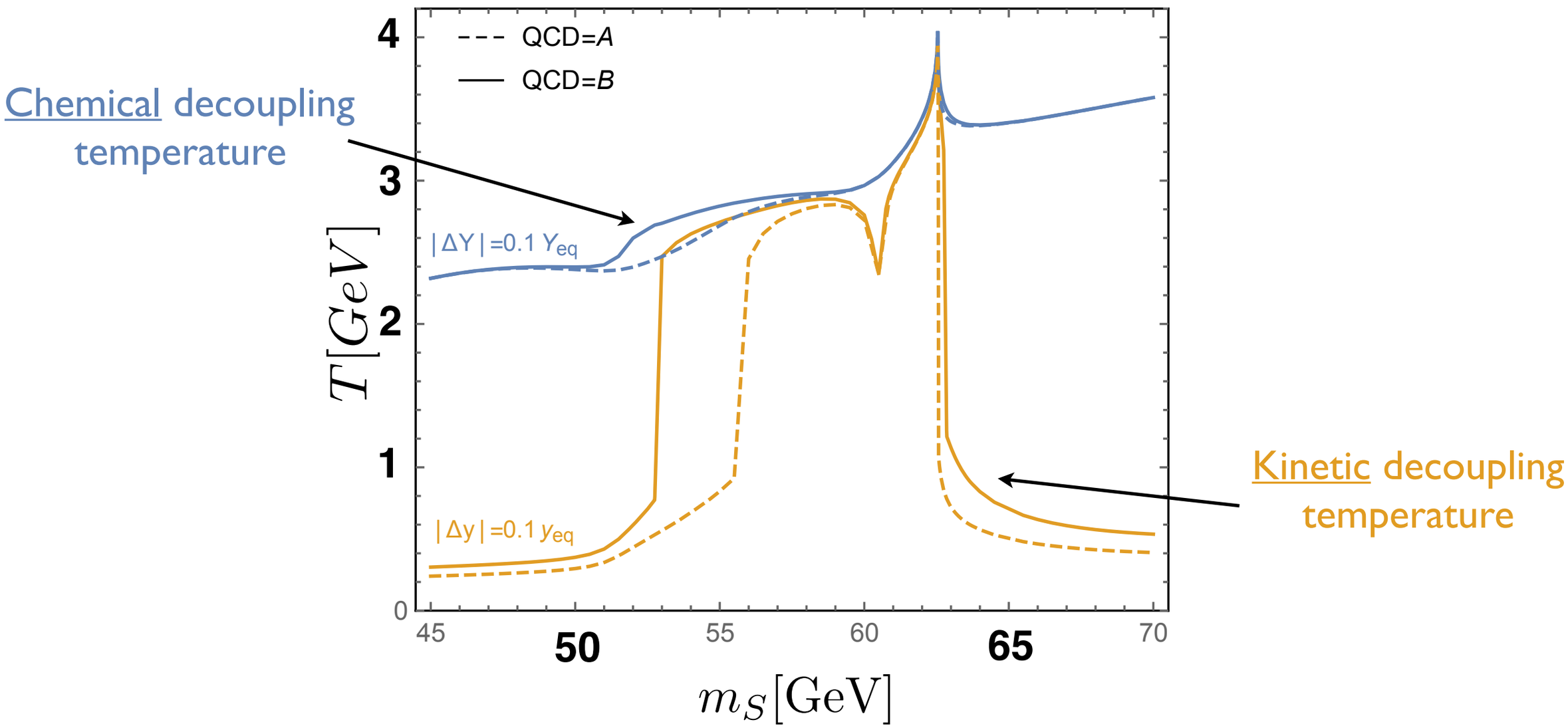
non-resonant

➡ No crossing symmetry: **large annihilation & weak scattering x-section**

➡ **Hierarchical Yukawa couplings** $\propto m_{\rm SM}$ **:**
Strongest coupling to Boltzmann suppressed quarks

↳ **a DM scenario with expected Early Kinetic Decoupling**

Decoupling temperatures — Chemical & Kinetic



→ **Kinetic decoupling around chemical freeze-out !**

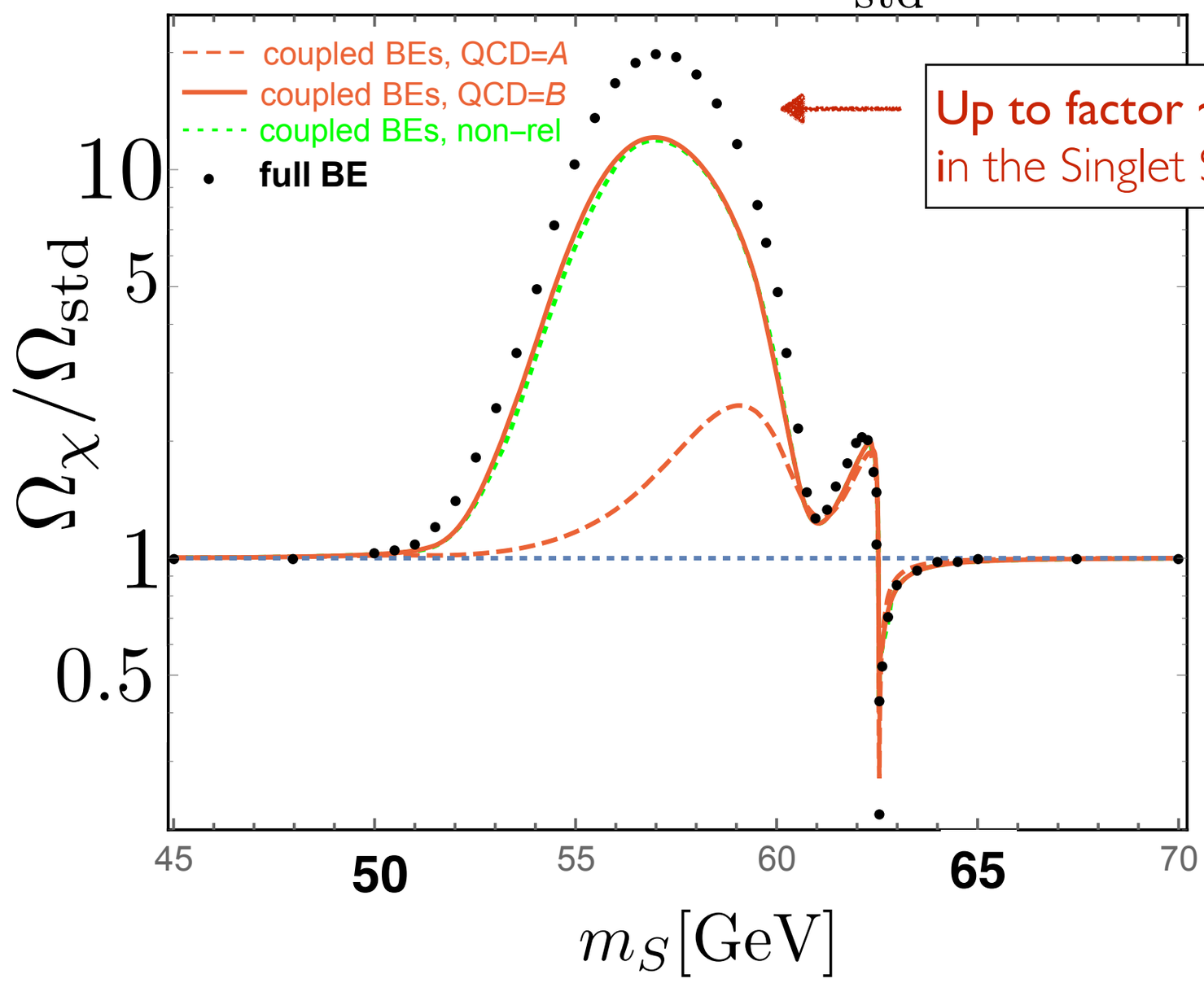
QCD phase-transition scenarios (dashed vs solid line)

QCD-A: All quarks are free and present in the plasma down to T_c =154 MeV

QCD-B: Only light quarks (u,d,s) available for scattering and only down to $4T_c$

Effect on the DM relic abundance

Relative Change in relic density: $\frac{\Omega_\chi}{\Omega_{\rm std}}$



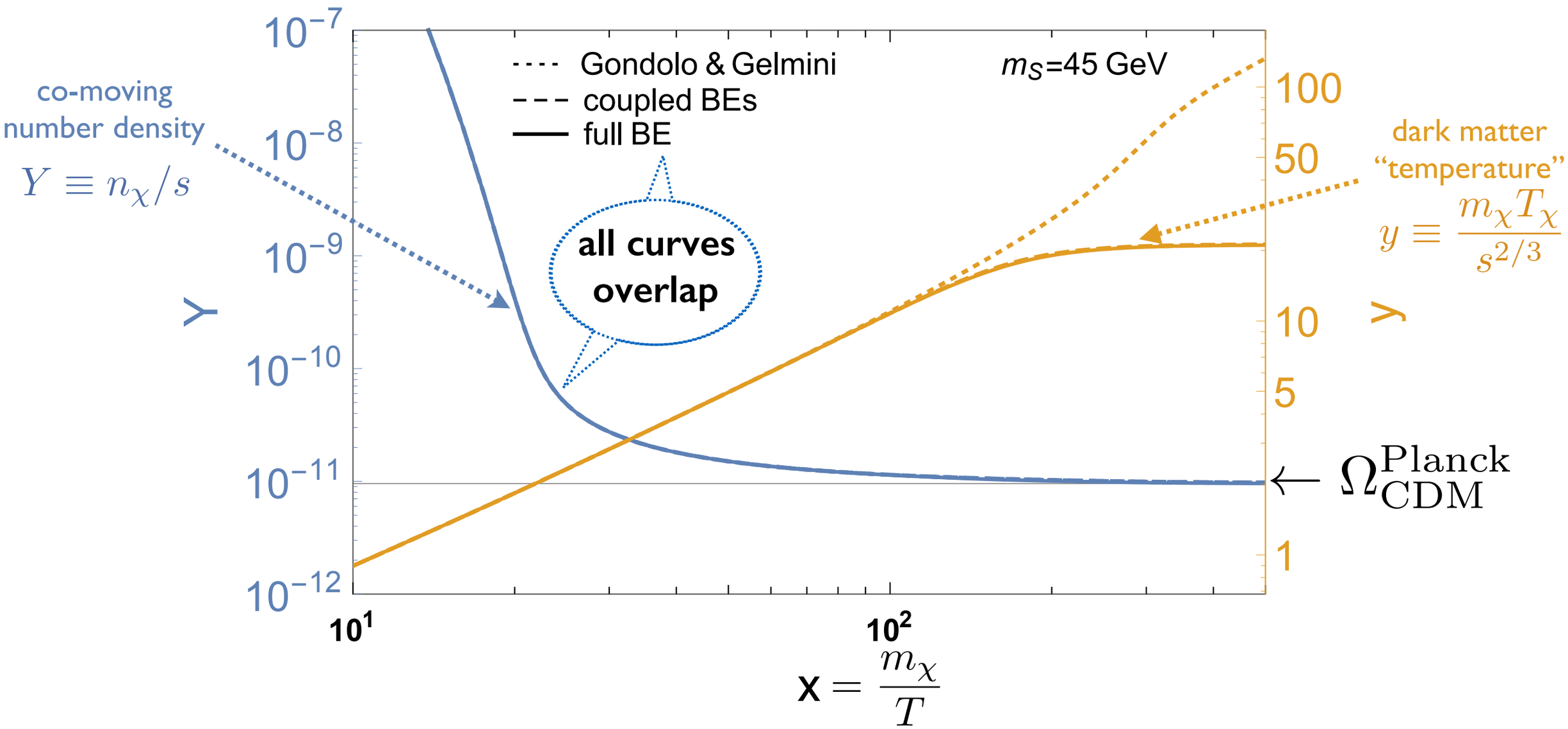
Up to factor ~10 effect!
in the Singlet Scalar model

→ **Significantly effects the relic abundance**

Coupled BEs (infinite self-scattering) / Full BE (zero self-scattering)

m_{DM} = 45 GeV — away from resonance

Late kinetic decoupling

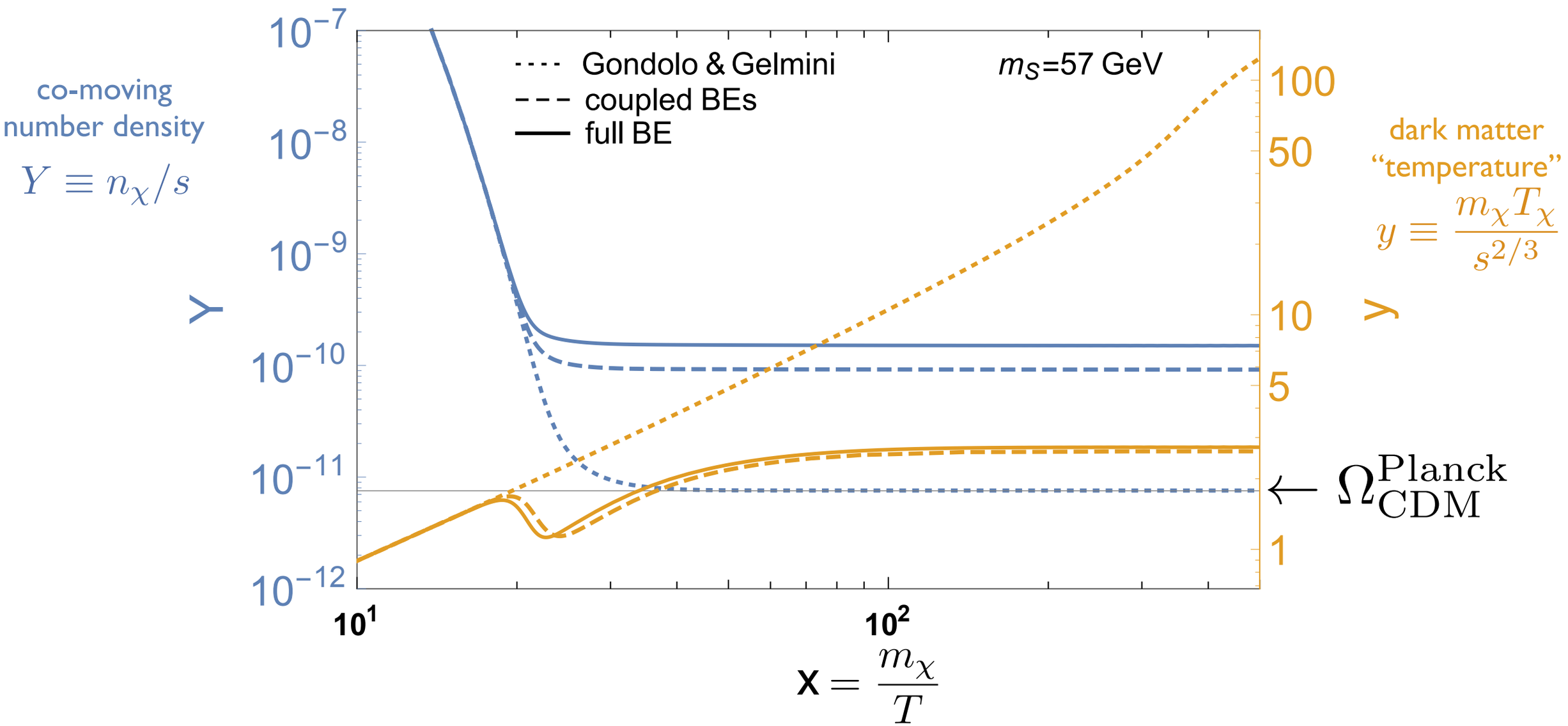


Standard formalism (= Gondolo & Gelmini) works excellent

DM chemical freeze-out ($x \approx 25$) before kinetic decoupling ($x \approx 150$)

m_{DM} = 57 GeV — close to the resonance

Early kinetic decoupling

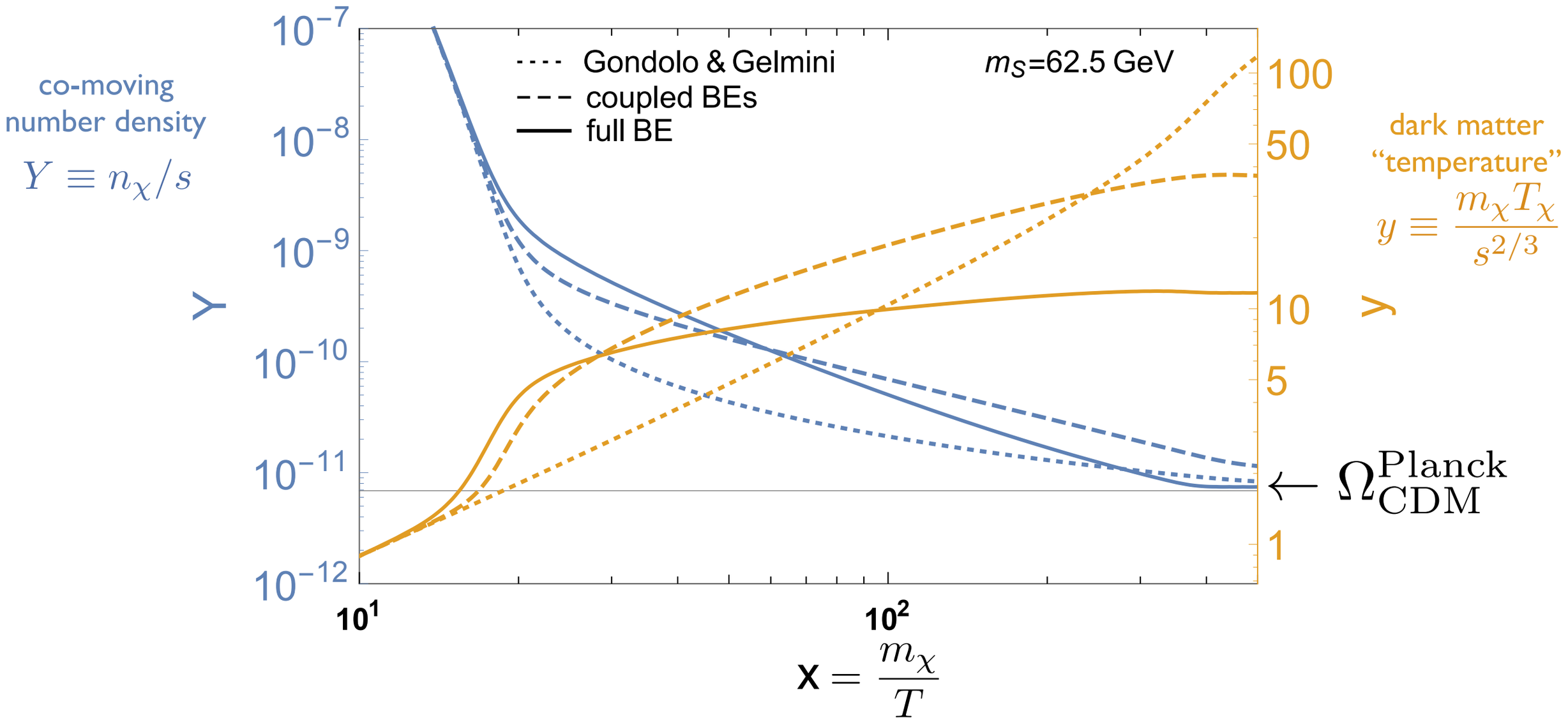


Resonant Annihilation most effective among high momenta particles

DM goes through a "cooling" phase and annihilation quickly loose efficiency

m_{DM} = 62.5 GeV — on resonance

Even earlier kinetic decoupling



Resonant annihilation very effective among low momenta particles

DM goes through a"heating" phase and a prolonged annihilation phase

Summary

Summary

- **Early kinetic decoupling** can impact DM abundances
  — a case not handled within standard relic density codes

- **Two methods** explored:

  1) **Coupled system of 0th and 2nd moments of Boltzmann Eq.** **Add to DarkSusy**
     — good accuracy under given assumptions

  2) **Numerical solver of the phase-space Boltzmann Eq.** **Make it public**
     — handles also non-trivial phase-space distributions

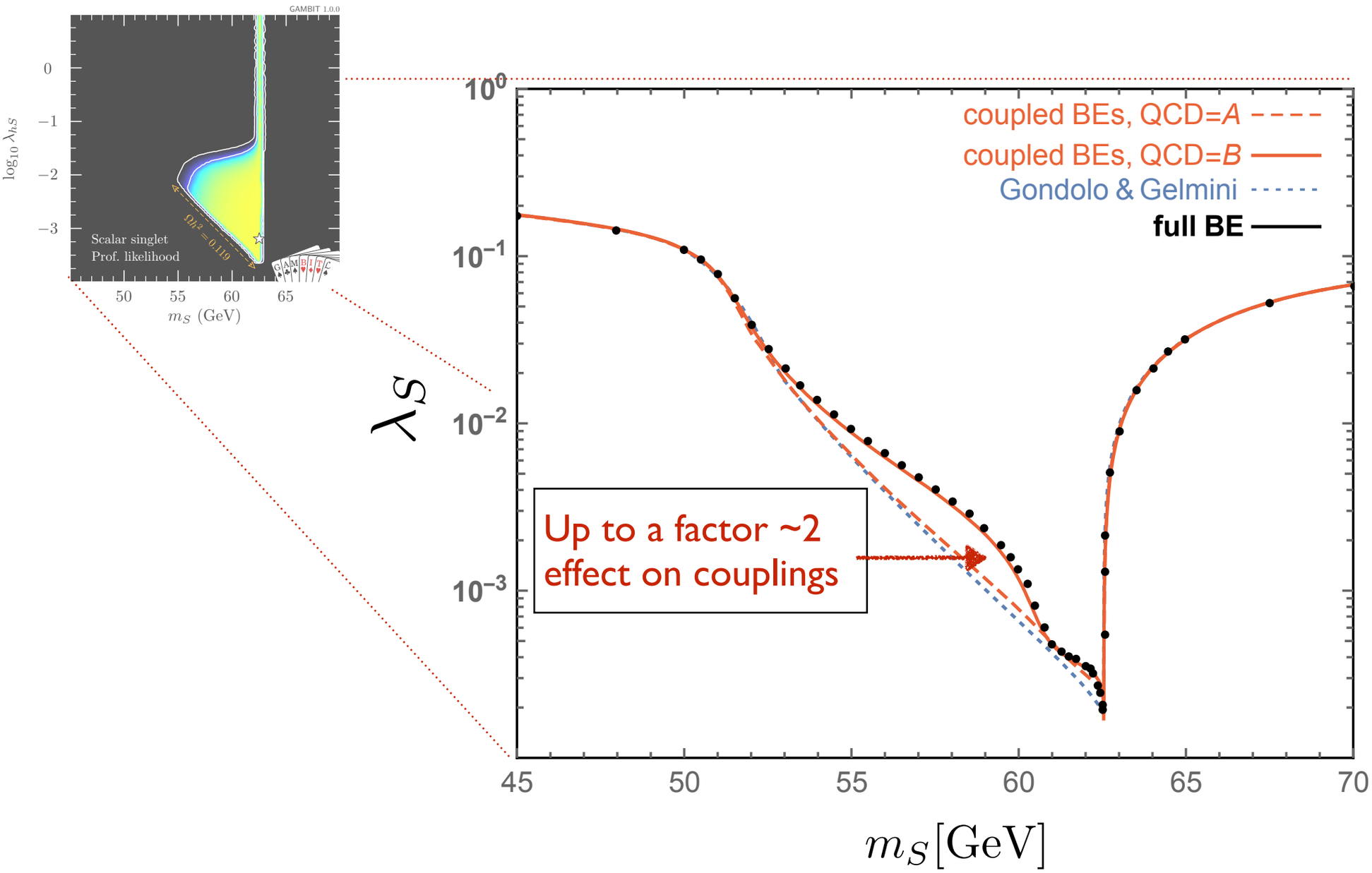
- Up to an **order of magnitude impact** on DM relic abundance in the Scalar singlet model

Kinetic decoupling during chemical freeze-out …

… an exception to lookout for

Backups

Impact on model parameters



Significant <u>modification</u> of the correct relic density contour in the Scalar Singlet DM model

→ larger coupling required → Increased signal in e.g. Direct Detection experiments