

New physics scales and local family symmetry in a quartification SUSY GUT

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Outline

- 1 Motivation
- 2 Q-GUT breaking steps
- 3 Estimation of scales
- 4 Conclusions and outlook

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Motivation

Large mass and mixing hierarchies found in Nature

- $m_t, m_h, m_z, m_w \sim 10^2 \text{ GeV}$ (EW scale)
- $m_t \gtrsim 10^{11} m_\nu$
- $m_e \gtrsim 10^5 m_\nu$
- $m_\tau \sim 10^{1.2} m_\mu \sim 10^{3.5} m_e$
- $m_t \sim 10^2 m_c \sim 10^{4.6} m_u$
- $m_t \sim 10^{1.6} m_b \sim 10^{3.2} m_s \sim 10^{4.8} m_d$
-

$$V_{\text{CKM}} \sim \begin{pmatrix} \cos \theta_c & \sin \theta_c & 0 \\ -\sin \theta_c & \cos \theta_c & 0 \\ 0 & 0 & 1 \end{pmatrix} + \text{Perturbations} .$$

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IDEA

- Framework based on unification principles with a family symmetry and:
 - > hierarchial scales/VEVs,
 - > large parameters **tree-level generated**,
 - > small parameters and splitting **radiatively generated** (RGEs, threshold corrections).

The Quartification model (Q-GUT)

Our proposal: Extend the SUSY trinification model (Georgi, Glashow and De Rujula 1984) with a local family $SU(3)_F$ symmetry

$$[SU(3)_C \times SU(3)_L \times SU(3)_R] \times \mathbb{Z}_3 \times SU(3)_F$$

- Use the minimal field content:

$$(1, \mathbf{3}, \bar{\mathbf{3}}, \mathbf{3}) = (L^i)^l_r = \begin{pmatrix} H_u^0 & H_d^- & e_L \\ H_u^+ & H_d^0 & \nu_L \\ e_R & \nu_R & \phi \end{pmatrix}^i, \quad (\mathbf{3}, \bar{\mathbf{3}}, 1, \mathbf{3}) = (Q_L^i)^x_l = \begin{pmatrix} u_L^x & d_L^x & D_L^x \end{pmatrix}^i,$$

$$(\bar{\mathbf{3}}, 1, \mathbf{3}, \mathbf{3}) = (Q_R^i)^r_x = \begin{pmatrix} u_{Rx}^c & d_{Rx}^c & D_{Rx}^c \end{pmatrix}^{\top i}.$$

- Higgs and leptons unified in L due to SUSY
- $W_1 = \lambda_{27} \varepsilon_{ijk} (Q_L^i)^x_l (Q_R^j)^r_x (L^k)^l_r$
 - > **One** family of quarks and **all** leptons massless at tree-level \rightarrow **radiatively generated**,
 - > Exact Yukawa unification for all three families.

So far no mass scale and due to SUSY protection there is no minimum in the scalar potential consistent with SSB.

- Solution is to introduce adjoint superfields inspired by the embedding of $[SU(3)]^4$ into E_8 .

$$(\mathbf{8}, 1, 1, 1) = \Delta_C^a, \quad (1, \mathbf{8}, 1, 1) = \Delta_L^a, \quad (1, 1, \mathbf{8}, 1) = \Delta_R^a, \quad (1, 1, 1, \mathbf{8}) = \Delta_F^a$$

-

$$W_2 = \sum_{A=L,R,C} (\lambda_{78} d_{abc} \Delta_A^a \Delta_A^b \Delta_A^c + \mu_{78} \delta_{ab} \Delta_A^a \Delta_A^b) + (\lambda_1 d_{abc} \Delta_F^a \Delta_F^b \Delta_F^c + \mu_1 \delta_{ab} \Delta_F^a \Delta_F^b)$$

- $\mu_{78} \sim \mu_1 \sim M_{GUT}$
- No μ -like problem as Higgs sits in fundamental sector.

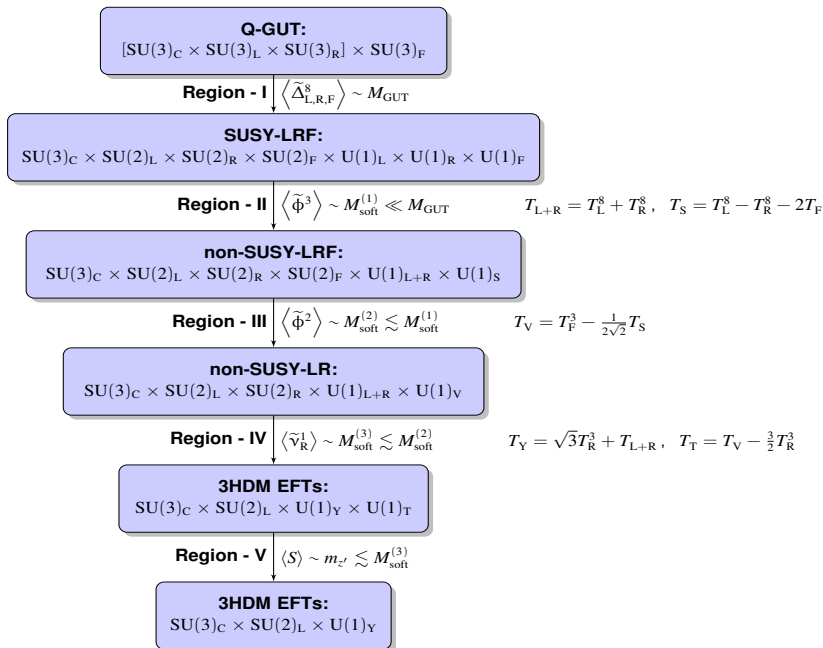
- Embedding of $[SU(3)]^4$ into $E_8 \Rightarrow$ **No gauge anomalies.**

$$[SU(3)]^3 \times SU(3)_F \subset E_6 \times SU(3)_F \subset E_8$$

- We motivate our field content in the Katsuki et. al. \mathbb{Z}_3 -orbifold for the breaking $E_8 \rightarrow E_6 \times SU(3)_F$ ([Prog.Theor.Phys. 82 \(1989\) 171](#))
 - > massless physical $(\mathbf{27}, \mathbf{3})$,
 - > massive adjoint $(\mathbf{78}, 1) \oplus (1, \mathbf{8})$,
 - > $(\overline{\mathbf{27}}, \overline{\mathbf{3}})$ removed by orbifolding.
 - > No dangerous $(\mathbf{27}, \mathbf{3}) \cdot (\overline{\mathbf{27}}, \overline{\mathbf{3}})$ terms and low energy limit is chiral.

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- $U(1)_T$ is a family symmetry at low-energy scales.
- After $\langle \tilde{\Delta}_{L,R,F}^8 \rangle$ VEVs, the components of adjoint superfields remain heavy and are integrated out.
 - Neutral components still relevant for neutrino sector (See-Saw mechanism)
- After Q-GUT breaking the SUSY-LRF theory reads

$$W = \varepsilon_{ijk} \{ y_{1-3} \Phi^i \mathbf{D}_L^j \mathbf{D}_R^k + y_{4-6} (\mathbf{H}^i)^L_R (\mathbf{q}_L^j)_L (\mathbf{q}_R^k)^R + y_{7-9} (\mathbf{E}_L^i)^L (\mathbf{q}_L^j)_L \mathbf{D}_R^k + y_{10-12} (\mathbf{E}_R^i)_R \mathbf{D}_L^j (\mathbf{q}_R^k)^R \} .$$

- Mass scale in the fundamental sector emerges solely from soft-SUSY breaking interactions.
- SUSY stabilizes the $M_{\text{GUT}} \gg M_{\text{soft}}^{(1)}$ hierarchy.

Why 3HDM EFTs?

The simplest scenario that automatically provides CKM mixing with Cabibbo form.

$$\langle L^1 \rangle \sim \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \omega & 0 \end{pmatrix} \quad \langle L^2 \rangle \sim \frac{1}{\sqrt{2}} \begin{pmatrix} v_2 & 0 & 0 \\ 0 & v_3 & 0 \\ 0 & 0 & f \end{pmatrix} \quad \langle L^3 \rangle \sim \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & p \end{pmatrix}.$$

$$M_{\text{EW}} \sim v_{1,2,3} \ll \omega \lesssim f \lesssim p \ll M_{\text{GUT}}, \quad (p, f, \omega) \sim M_{\text{soft}}^{(1,2,3)}$$

Classical approach: ($y_{1-12} \rightarrow \lambda_{27}$)

$$m_{c,t}^2 = \frac{1}{2} \lambda_{27}^2 (v_1^2 + v_2^2), \quad m_b^2 = 3m_s^2 = \frac{1}{2} \lambda_{27}^2 v_3^2, \quad m_{u,d}^2 = 0, \quad \tan \theta_C = \frac{v_1}{v_2}$$

$$m_B^2 = \frac{1}{2} \lambda_{27}^2 (2p^2 + f^2 + \omega^2), \quad m_S^2 = \frac{1}{2} \lambda_{27}^2 (p^2 + f^2), \quad m_D^2 = \frac{1}{2} \lambda_{27}^2 \omega^2.$$

Outline

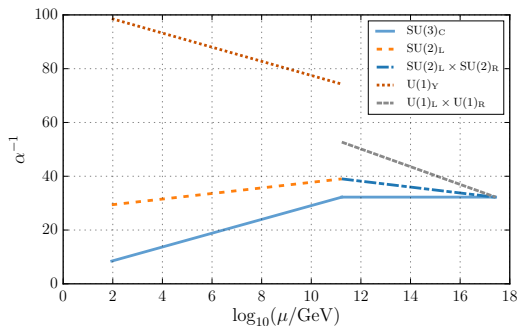
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Estimation of $\omega = \langle \tilde{\nu}_R^1 \rangle$, $f = \langle \tilde{\phi}^2 \rangle$, $p = \langle \tilde{\phi}^3 \rangle$ scales

Use 1-loop running and three-level matching to **estimate** scale hierarchies.

- Consider for simplicity $\omega \simeq f \simeq p = M_{\text{soft}}$
- Unification condition: $\alpha_{\tilde{g}_{L,R}}^{-1}(M_{\text{GUT}}) = \alpha_{\tilde{g}_{L,R,C}}^{-1}(M_{\text{GUT}}) = \alpha_U^{-1}$

$$\alpha_i^{-1}(\mu_2) = \alpha_i^{-1}(\mu_1) - \frac{b_i}{2\pi} \ln\left(\frac{\mu_2}{\mu_1}\right)$$



- Threshold conditions:

$$\alpha_{\tilde{g}_{L+R}}^{-1}(p) = \alpha_{\tilde{g}_L}^{-1}(p) + \alpha_{\tilde{g}_R}^{-1}(p)$$

$$\alpha_{\tilde{g}_Y}^{-1}(\omega) = \alpha_{\tilde{g}_R}^{-1}(\omega) + \frac{1}{3} \alpha_{\tilde{g}_{L+R}}^{-1}(\omega)$$

$$\alpha_{\tilde{g}_Y}^{-1}(m_z) = \cos^2 \theta_W \alpha_{EM}^{-1}$$

$$\alpha_{\tilde{g}_L}^{-1}(m_z) = \sin^2 \theta_W \alpha_{EM}^{-1}$$

- Solution:

$$M_{\text{soft}} \sim 8.8 \cdot 10^{10} \text{ GeV},$$

$$M_{\text{GUT}} \sim 4.9 \cdot 10^{17} \text{ GeV},$$

$$\alpha_U^{-1} \sim 31.5,$$

Effect of decoupling scales

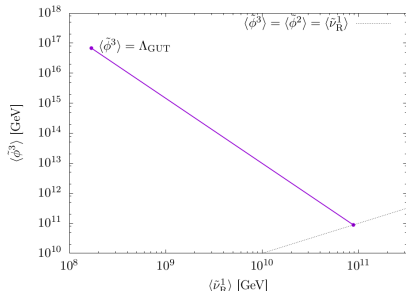
● Solve RGEs for ω

$$\omega = m_Z \exp \left\{ 20.69 - \frac{1}{19} \ln \left(\frac{p}{f} \right) \left[4b_{g_C}^{\text{II}} - 9b_{g_L}^{\text{II}} + 3b_{g_R}^{\text{II}} + b_{g_{L+R}}^{\text{II}} \right] - \frac{1}{19} \ln \left(\frac{p}{f} \right) \left[4b_{g_C}^{\text{III}} - 9b_{g_L}^{\text{III}} + 3b_{g_R}^{\text{III}} + b_{g_{L+R}}^{\text{III}} \right] \right\}.$$

● To minimize ω need to maximise $b_{g_C}^{\text{II,III}}$, $b_{g_R}^{\text{II,III}}$, $b_{g_{L+R}}^{\text{II,III}}$ and minimise $b_{g_L}^{\text{II,III}}$.

> Scalar content in regions II and III without $(\tilde{u}_L, \tilde{d}_L)^{1,2,3}$, $(\tilde{e}_L, \tilde{\nu}_L)^{1,2,3}$ and $H_{u,d}^3$

$$> b_{g_C}^{\text{II,III}} = -\frac{13}{3}, \quad b_{g_L}^{\text{II,III}} = -\frac{2}{3}, \quad b_{g_R}^{\text{II,III}} = \frac{4}{3}, \quad b_{g_{L+R}}^{\text{II,III}} = \frac{40}{3}.$$



- purple line: run to measured values
- ω decreases with increasing p
- **Optimal scenario when $p = f = \omega$**

How to lower the ω , f and p scales?

- Consider unification at E_6 level relaxing \mathbb{Z}_3 in the original $[\text{SU}(3)]^3 \times \mathbb{Z}_3$ unification.
- Consider correction to gauge-kinetic terms from dim-5 operators (Chakraborty, Raychaudhuri 0812.2783 [hep-ph])

$$\mathcal{L}_{d5} = -\frac{\eta}{M_{\text{Pl}}} \left[\frac{1}{4c} \text{Tr} (F_{\mu\nu} \Phi_R F^{\mu\nu}) \right]$$

- Φ_R sits in $(\mathbf{78} \otimes \mathbf{78})_{\text{sym}} = \mathbf{1} \oplus \mathbf{650} \oplus \mathbf{2430}$
- $\mathbf{650}$ contains two $[\text{SU}(3)]^3$ singlets which provide linearly independent contributions whose VEVs break $E_6 \rightarrow [\text{SU}(3)]^3$.
- In general we consider $\Phi_R = \kappa_1 \Phi_{\mathbf{1}} + \kappa_{650} \Phi_{\mathbf{650}} + \kappa_{650'} \Phi_{\mathbf{650}'} + \kappa_{2430} \Phi_{\mathbf{2430}}$ with $\kappa_1^2 + \kappa_{650}^2 + \kappa_{650'}^2 + \kappa_{2430}^2 = 1$
- Unification condition modified

$$\alpha_{\text{C}}^{-1} (1 + \epsilon \delta_{\text{C}})^{-1} = \alpha_{\text{L}}^{-1} (1 + \epsilon \delta_{\text{L}})^{-1} = \alpha_{\text{R}}^{-1} (1 + \epsilon \delta_{\text{R}})^{-1} \quad \epsilon \sim \frac{M_{E_6}}{M_{\text{Pl}}}$$

$$\delta_{\text{C}} = -\frac{1}{\sqrt{2}} \kappa_{650} - \frac{1}{\sqrt{26}} \kappa_{2430}$$

$$\delta_{\text{L,R}} = \frac{1}{2\sqrt{2}} \kappa_{650} \pm \frac{3}{2\sqrt{2}} \kappa_{650'} - \frac{1}{\sqrt{26}} \kappa_{2430}$$

A possible solution: (not unique)

$$\epsilon = 0.66 \quad M_{3333} = 10^{17.5} \text{ GeV} \quad p = 10^6 \text{ GeV}$$

$$f = 10^{5.5} \text{ GeV} \quad \omega = 10^5 \text{ GeV} \quad m_{z'} = 10^3 \text{ GeV}$$

$$\Phi_R = -0.61\Phi_{650} + 0.75\Phi_{650'} + 0.27\Phi_{2430}$$

$$\alpha_C^{-1} = 1.15\alpha_L^{-1} = 1.89\alpha_R^{-1}$$

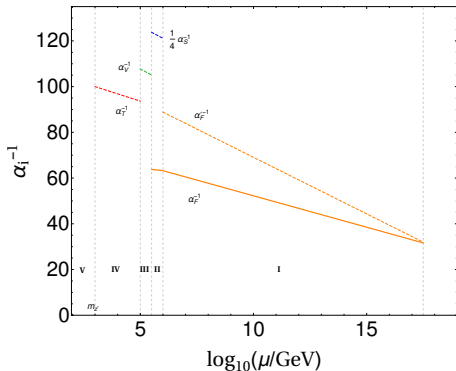
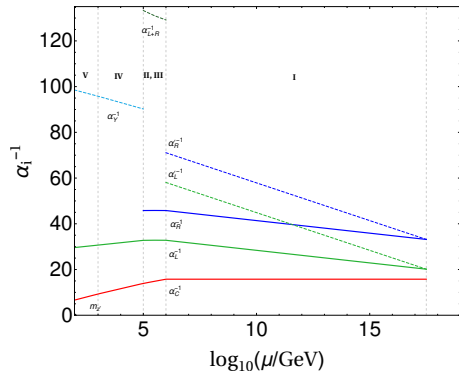
$$\alpha_{L+R}^{-1} = \alpha_L'^{-1} + \alpha_R'^{-1}$$

$$\alpha_S^{-1} = \alpha_L'^{-1} + \alpha_R'^{-1} + 4\alpha_F'^{-1}$$

$$\alpha_V^{-1} = \alpha_F^{-1} + \frac{1}{12}\alpha_S^{-1}$$

$$\alpha_T^{-1} = \frac{4}{9}\alpha_V^{-1} + \alpha_R^{-1}$$

$$\alpha_Y^{-1} = \frac{1}{3}\alpha_{L+R}^{-1} + \alpha_R^{-1}$$



This example considers:

- **Region I:** Keep all states in $\mathbf{27} = (L, Q_L, Q_R)$
- **Region II:** Keep all fermions in $\mathbf{27}$ and remove all squarks and $\tilde{\phi}^3$
- **Region III:** Keep all fermions in $\mathbf{27}$, all Higgs doublets and the $SU(2)_R$ doublet $(e_R^1 \ \nu_R^1)$
- **Regions IV and V contain:** (low-scale phenomenology)
 - > All chiral (SM-like) quarks and leptons,
 - > One generation of VLQs,
 - > Three generations of VLLs,
 - > Three Higgs doublets
 - > One $U(1)_T$ -charged complex singlet \rightarrow new Z' after $U(1)_T$ -breaking

Full classification of low-energy scale 3HDMs

3HDM	SU(3) _C	SU(2) _L	U(1) _Y	U(1) _T	extra	SU(3) _C	SU(2) _L	U(1) _Y	U(1) _T
q_{Li}	3	2	1/3	(3, -1, -1)	$D_{L,R}$	3	1	-2/3	-2
u_{Ri}	3	1	4/3	(0, 4, 4)	E_{Li}	1	2	-1	(-1, -5, -5)
d_{Ri}	3	1	-2/3	(-6, 2, -2)	E_{Ri}	1	2	-1	(-5, -1, -1)
ℓ_{Li}	1	2	-1	(3, -1, -1)	ν_{Rk}	1	1	0	(0, ..., 0 4, ..., 4 -4, ..., -4) <small>17 entries 6 entries 9 entries</small>
e_{Li}	1	1	-2	(-6, -2, -2)	T_i	1	3	0	(0, 0)
H_i	1	2	1	(x_1, x_2, x_3)	S	1	1	0	-4

$$(x_1, x_2, x_3) = \begin{cases} (5, 1, 5) & \text{Model I-A} \\ (1, 5, 1) & \text{Model I-B} \\ (5, 1, -3) & \text{Model II} \end{cases}$$

- U(1)_T must be broken by $\langle S \rangle$ providing a TeV scale Z' boson.
- **New exotic states offer rich phenomenology/cosmology:** EW-precision, flavour, collider, neutrino, dark-matter, baryogenesis, **primordial GW**.
 - > see my talk on Wed 25/07, Cosmology and GW section 1 (simplified to a THDSM inspired by Q-GUT).

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Conclusions and outlook

- Constructed a framework based on the quartification group
- Discussed how to achieve a consistent low-energy, spectrum/mixing via hierarchial VEVs and quantum effects.

Direction 1

- Phenomenology and cosmology studies on Models I-(A and B) and II (standalone)
 - Search for all viable regions (flavour, EW-precision),
 - New physics: exotic states (VLF, flavour non-universal Z' , scalars), DM, GW, EWBG.

Direction 2

- Linking Models I-(A and B) and II to the high scale Q-GUT
 - Matching and RG running analysis