



Majorana vs Pseudo-Dirac Neutrinos at Electron-Positron Colliders

Joel Jones-Pérez

Pontificia Universidad Católica del Perú (PUCP)

Based on work in collaboration with
P. Hernández, O. Suárez-Navarro (180x.xxxxx)

Neutrino Masses

Type I Seesaw is probably most popular mechanism for neutrino masses

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + Y_\nu \left(\bar{\nu}_R L \cdot \tilde{H} \right) + \frac{1}{2} M_R (\bar{\nu}_R^c \nu_R) + \text{h.c.}$$

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$$m_\nu \sim m_D^T M_R^{-1} m_D$$



How can we test this?

Low Scale Seesaw

We can have GeV-scale heavy neutrinos with large couplings when mass matrix has a particular texture (broken LN symmetry?)

$$M_\nu = \left(\begin{array}{c|cc} 0 & m_D & 0 \\ \hline m_D^T & 0 & M \\ 0 & M & 0 \end{array} \right)$$

Light neutrinos: massless
Heavy neutrinos: one Dirac
LN Conserved

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$$M_\nu = \left(\begin{array}{c|cc} 0 & m_D & \varepsilon m'_D \\ \hline m_D^T & \mu' & M \\ \varepsilon m_D'^T & M & \mu \end{array} \right)$$

Light neutrinos: massive, Majorana
Heavy neutrinos: two Majorana
LN Violation: μ, μ', ε

Low Scale Seesaw

LVN Terms:

- μ, ϵ : Related to light neutrino masses \rightarrow very small
- $\mu' = M_5 - M_4$: Related to heavy neutrino mass splitting

Lopez-Pavon, Pascoli, Wong: 1209.5342 [hep-ph]

LVN should vanish if heavy neutrinos are mass-degenerate, or close (Pseudo-Dirac, Quasi-Dirac).

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Our objectives:

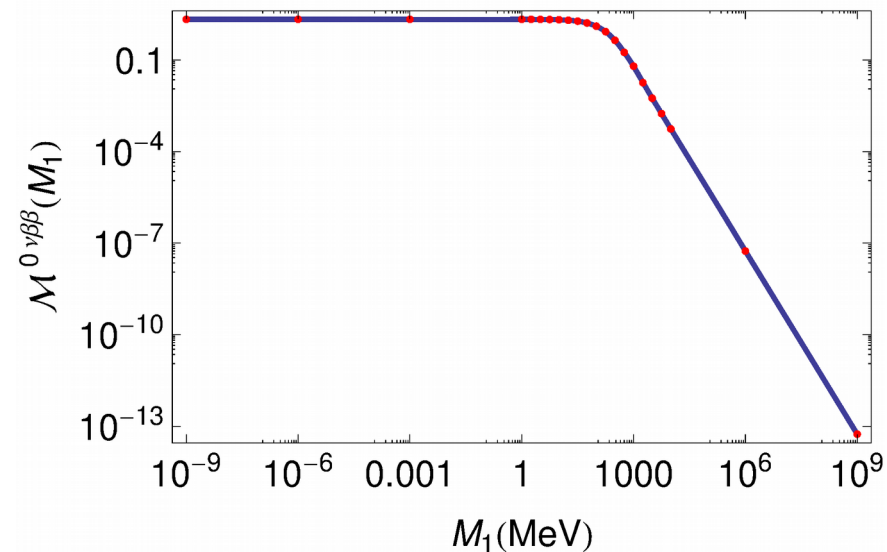
- Use LNV processes to set limits on heavy neutrino mass splittings.
- Probe the Pseudo-Dirac nature of heavy neutrinos at $e^+ e^-$ colliders.

Current Bounds

Neutrinoless Double Beta Decay

Clear signal of LNV. Can be affected by heavy neutrinos, provided they are light enough:

$$A_{\beta\beta} \propto \sum_{l_i=1}^3 m_{l_i} U_{el_i}^2 \mathcal{M}^{0\nu\beta\beta}(m_{l_i}) + \sum_{i=4}^5 M_i U_{eh_i}^2 \mathcal{M}^{0\nu\beta\beta}(M_i)$$



Neutrinoless Double Beta Decay

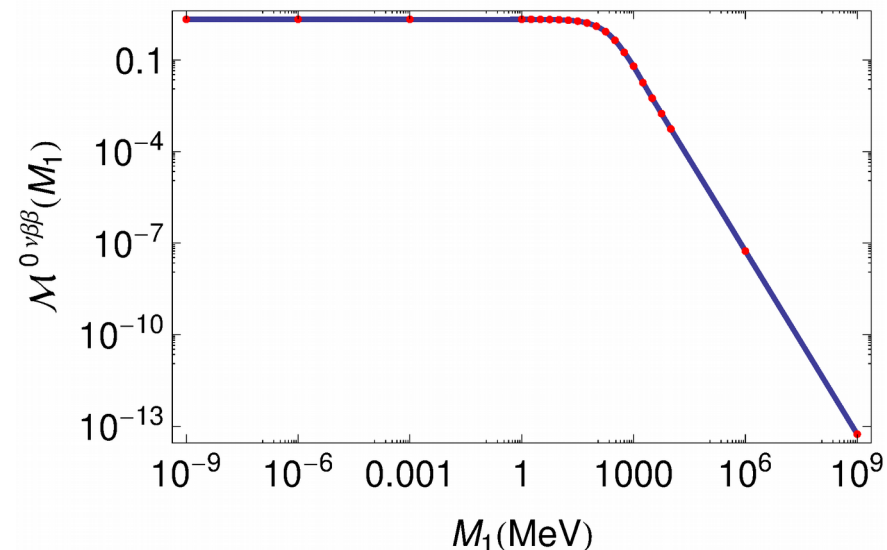
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Can be written:

$$A_{\beta\beta} \propto m_{\beta\beta} \Delta\mathcal{M}(0, M_5) + M_4 U_{e4}^2 \Delta\mathcal{M}(M_4, M_5)$$

$$\Delta\mathcal{M}(M_a, M_b) = \mathcal{M}^{0\nu\beta\beta}(M_a) - \mathcal{M}^{0\nu\beta\beta}(M_b)$$



Loop Corrections to Light Neutrino Masses

Heavy neutrinos can give sizeable corrections to light neutrino masses:

$$\delta m_{\text{loop}} = \frac{g^2}{64\pi^2 m_W^2} (m_D) M_h^{-1} \left(m_{\text{Higgs}}^2 \ln \left[\frac{M_h^2}{m_{\text{Higgs}}^2} \right] + 3m_Z^2 \ln \left[\frac{M_h^2}{m_Z^2} \right] \right) (m_D)^T$$

Grimus, Lavoura: hep-ph/0207229
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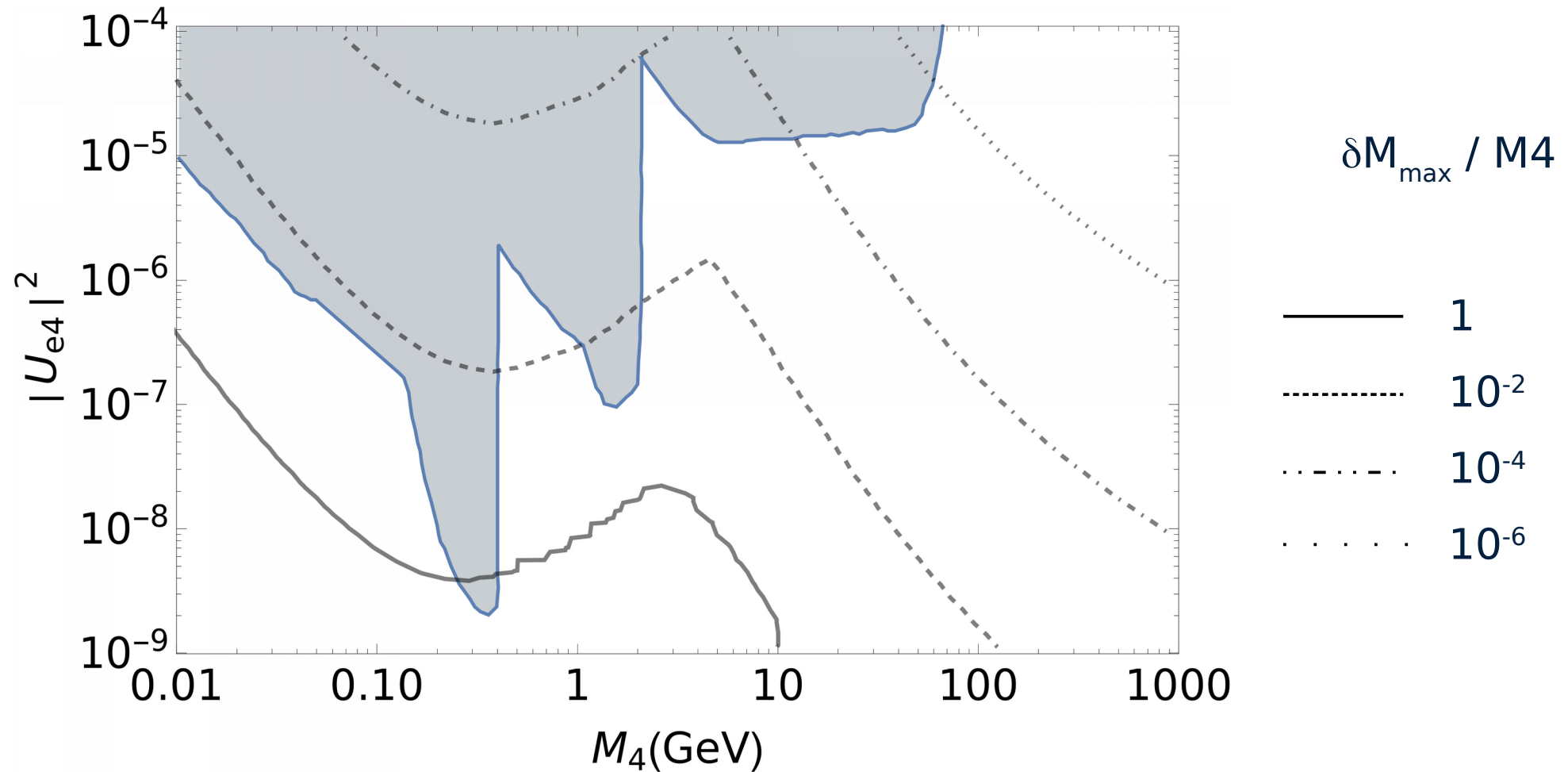
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The new contribution is large in the presence of new LNV contributions,
ie, μ' . Then:

$$\begin{aligned} M_5 \rightarrow M_4 &\Rightarrow \mu' \rightarrow 0 \\ &\Rightarrow \delta m_{\text{loop}} \rightarrow 0 \end{aligned}$$

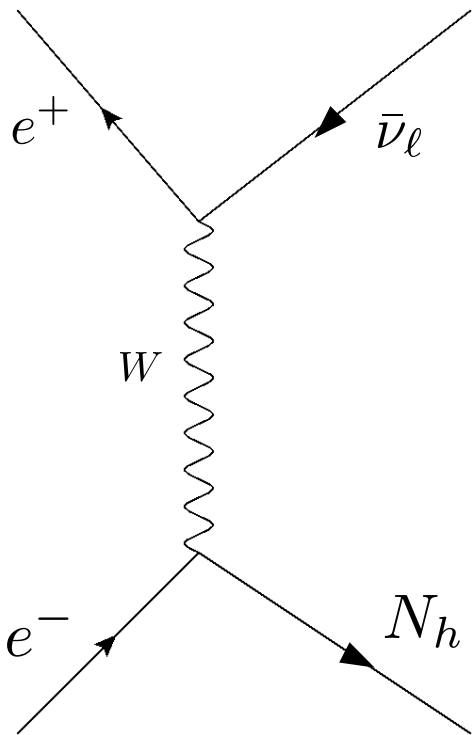
Combined Bounds



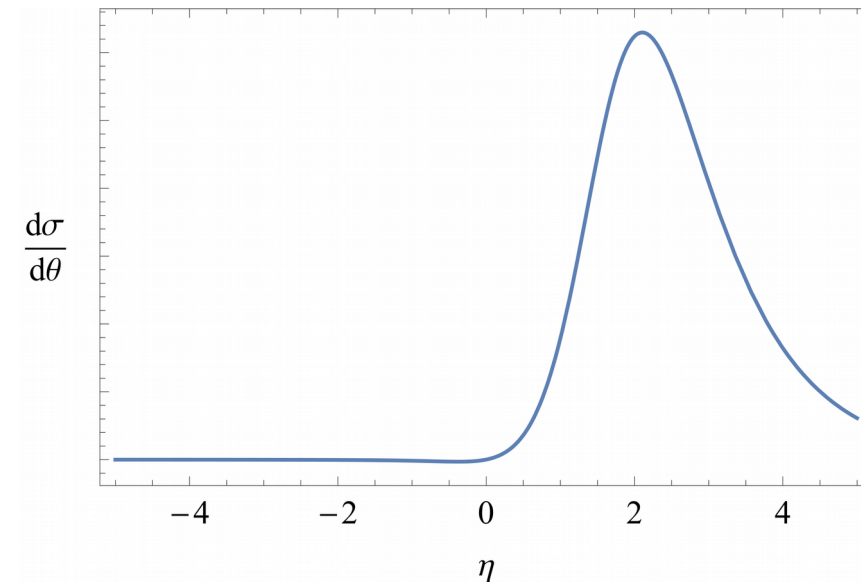
δM at Lepton Colliders

Direct Production

“Observation” of LNV at lepton colliders can imply that heavy neutrinos are Majorana.

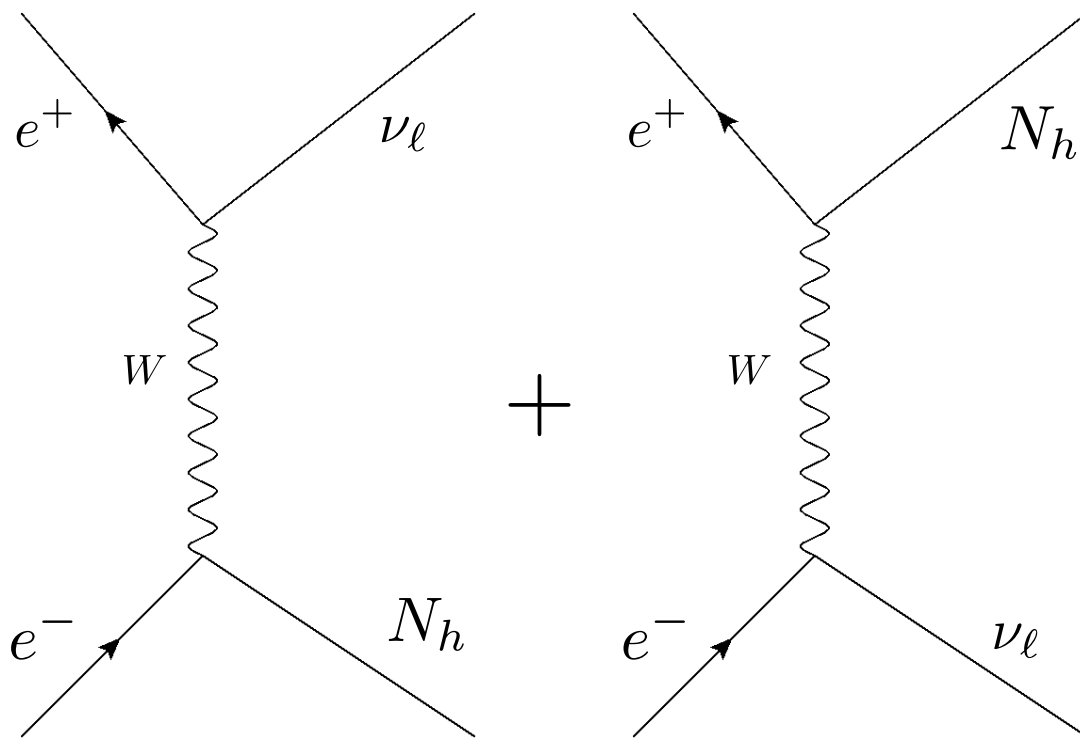


Dirac heavy neutrino

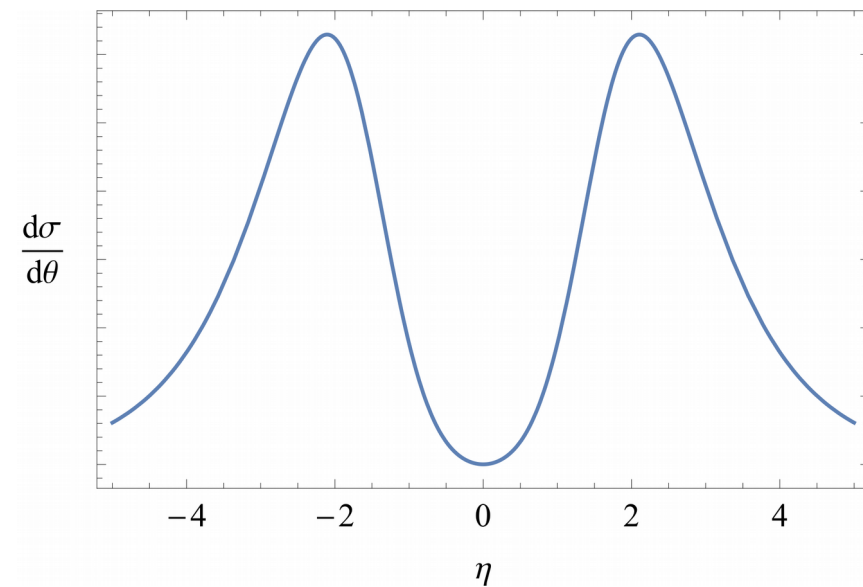


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Majorana heavy neutrino



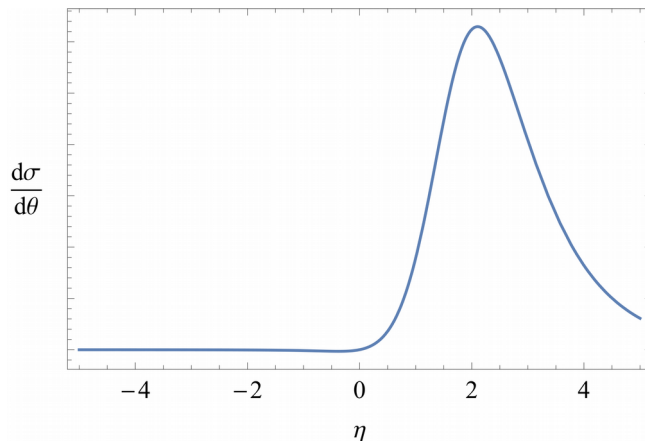
Direct Production

Heavy neutrinos are detected by their decay. We focus on:

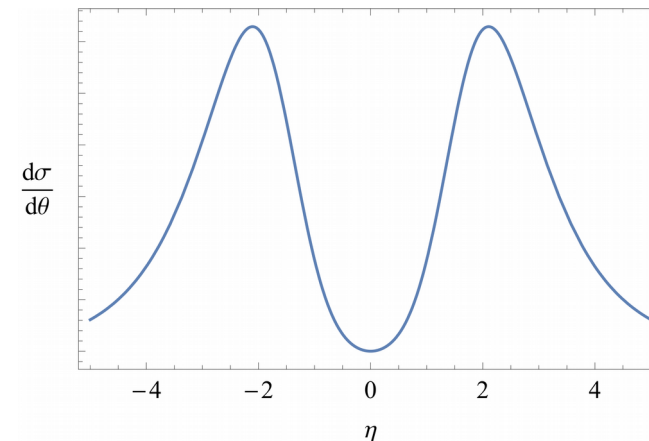
$$N_{4,5} \rightarrow \ell^- j j$$

The charged lepton follows the same η distribution.

Asymmetry in ℓ^-
 \Rightarrow (Pseudo)Dirac



Symmetry in ℓ^-
 \Rightarrow Majorana



Heavy Neutrinos as Virtual Particles

We want the asymmetry to appear in the Pseudo-Dirac Limit.

=> Treat heavy neutrinos as virtual particles!

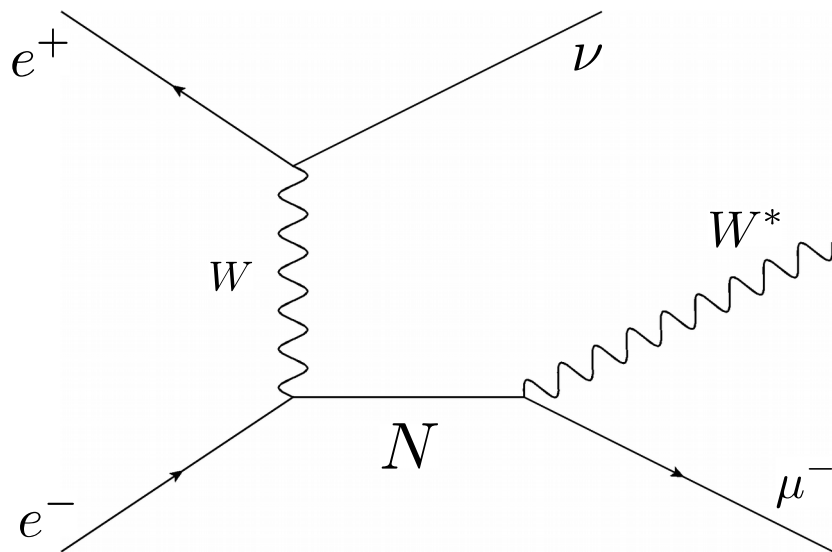


Diagram A (LNC)

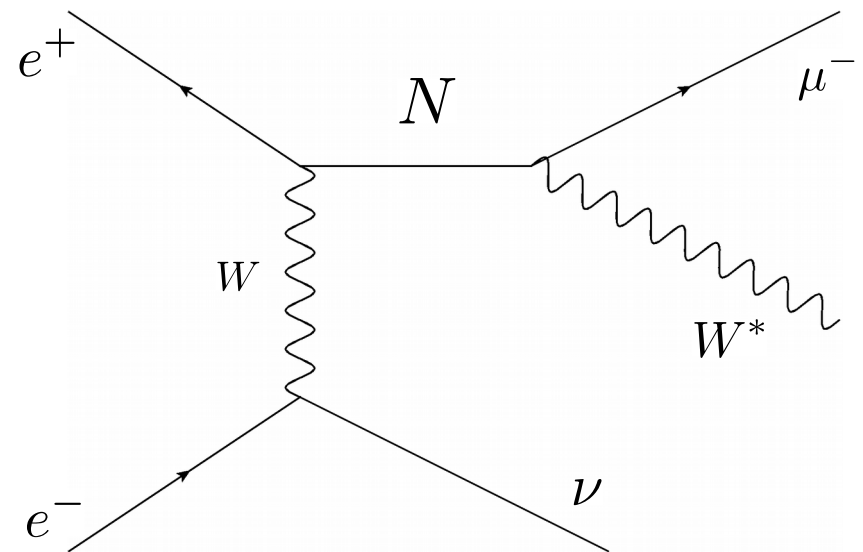


Diagram B (LNV)

Heavy Neutrinos as Virtual Particles

For Diagram A (LNC) we find:

$$|\mathcal{M}_A|^2 \propto \left(\frac{1}{M_4^2 |f(M_4)|^2} + \frac{2}{M_4 M_5} \Re \left[\frac{1}{f(M_4) f^*(M_5)} \right] + \frac{1}{M_5^2 |f(M_5)|^2} \right)$$
$$\frac{\not{q} + M_j}{q^2 - M_j^2 + iM_j \Gamma_j} \equiv \frac{\not{q} + M_j}{f(M_j)}$$

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$$\frac{q + M_j}{q^2 - M_j^2 + iM_j \Gamma_j} \equiv \frac{q + M_j}{f(M_j)}$$

On the limit when $M_4 = M_5$, $\Gamma_4 = \Gamma_5$ (*LNC Limit*) we evidently get:

$$|\mathcal{M}_A^{\text{LNC}}|^2 \propto \frac{4}{M_4^2 |f(M_4)|^2}$$

Heavy Neutrinos as Virtual Particles

For Diagram B (LNV) we find:

$$|\mathcal{M}_B|^2 \propto \left(\frac{1}{q^2 |f(M_4)|^2} - \frac{2}{q^2} \Re \left[\frac{1}{f(M_4) f^*(M_5)} \right] + \frac{1}{q^2 |f(M_5)|^2} \right)$$
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On the LNC Limit we find:

$$|\mathcal{M}_B^{\text{LNC}}|^2 = 0$$

LNV contribution vanishes, so we pass sanity check.

Heavy Neutrinos as Virtual Particles

On the limit when $M_5 \rightarrow M_4$ (*PseudoDirac Limit*) we can expand:

$$|\mathcal{M}_B^{\text{PD}}|^2 \propto 4 \frac{M_4^2}{q^2 |f(M_4)|^4} \left[\left(1 + \frac{\Gamma_4^2}{4M_4^2} \right) (\delta M)^2 + \frac{1}{4} (\delta \Gamma)^2 + \frac{\Gamma_4}{2M_4} \delta \Gamma \delta M \right]$$

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Comparing both contributions, and taking $q^2 \rightarrow M_4^2$ (on-shell):

$$\left(\frac{|\mathcal{M}_B^{PD}|^2}{|\mathcal{M}_A^{LNC}|^2} \right)_{q^2=M_4^2} \sim \left(1 + \frac{\Gamma_4^2}{4M_4^2} \right) \left(\frac{\delta M}{\Gamma_4} \right)^2 + \frac{1}{4} \left(\frac{\delta \Gamma}{\Gamma_4} \right)^2 + \frac{\Gamma_4}{2M_4} \frac{\delta \Gamma \delta M}{\Gamma_4^2}$$

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LNV effects shall be relevant when $\delta M / \Gamma$ is large.

Avoiding Backgrounds

The final state can have complicated backgrounds from $e^+e^- \rightarrow W^+ W^-$.
To avoid them, we require the heavy neutrino to lead to a displaced vertex.

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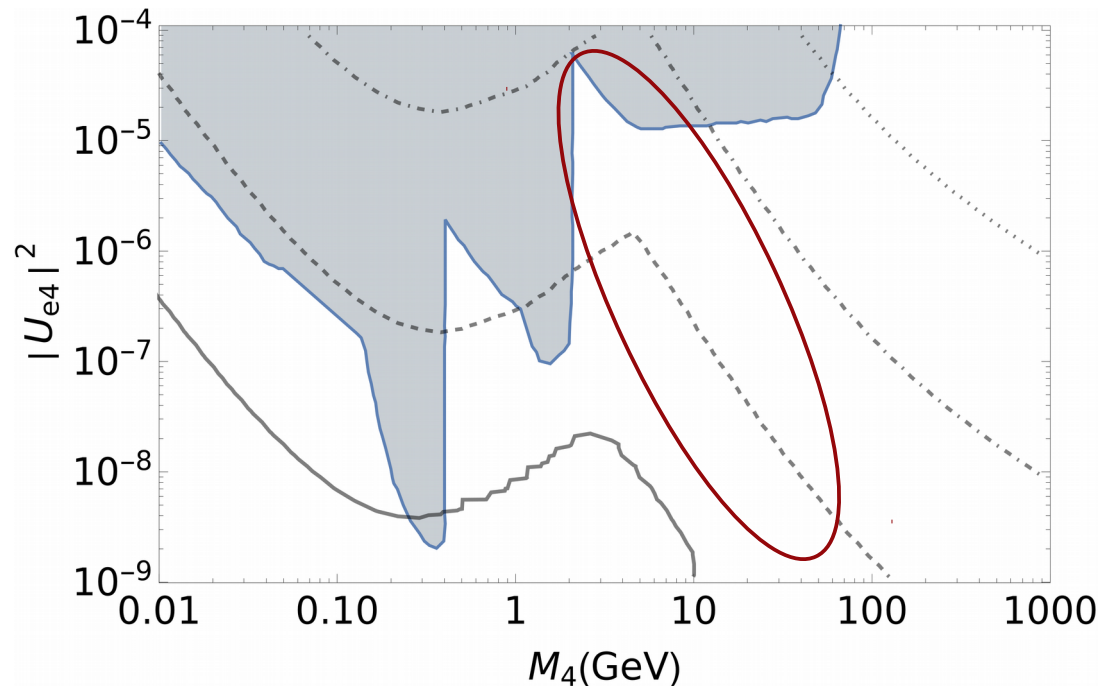
Parameter region where this is
feasible at the ILC is restricted:

$$10 \mu\text{m} < L_{xy} < 2.49 \text{ m}$$

$$L_z < 3 \text{ m}$$

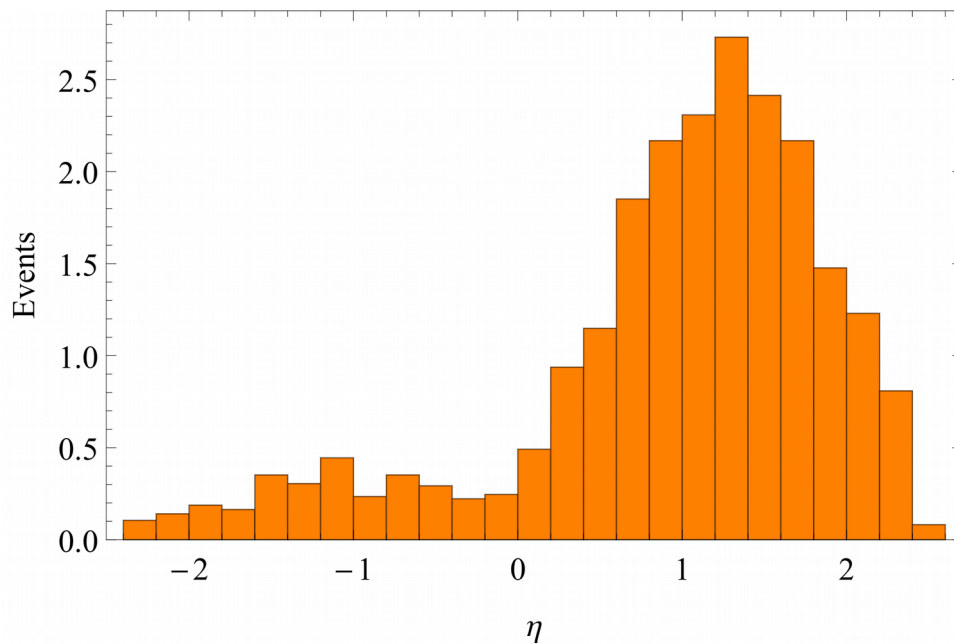
Charged lepton impact parameter:

$$d_\ell \equiv \frac{L_x p_y^\ell - L_y p_x^\ell}{p_T^\ell} > 6 \mu\text{m}$$

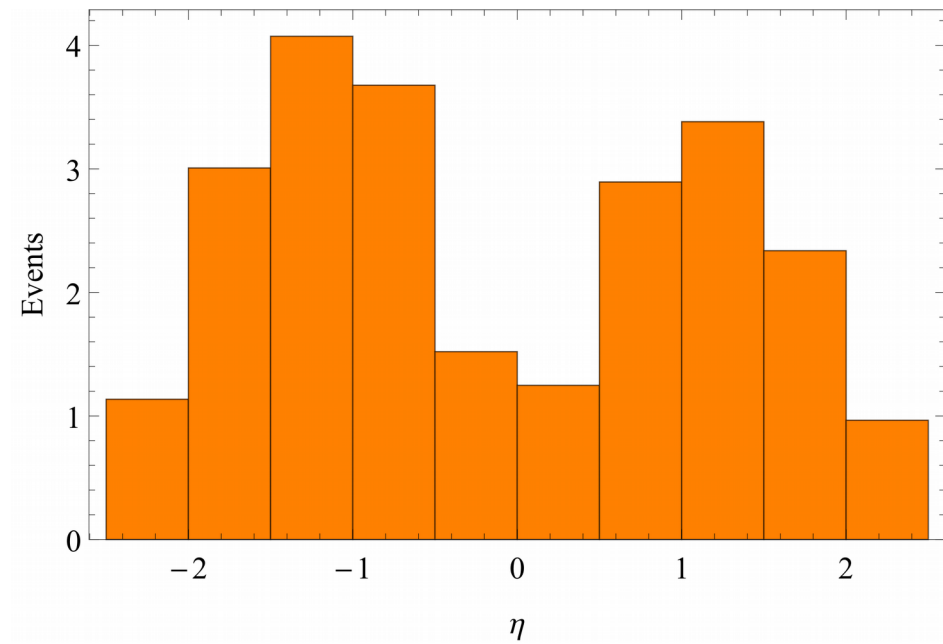


ℓ^- pseudorapidity distribution (benchmark 1)

Mass (GeV)	$ U_{\mu 4} ^2$	Γ_4 (meV)	$c\tau_4$ (mm)
10	1×10^{-5}	0.7	0.3



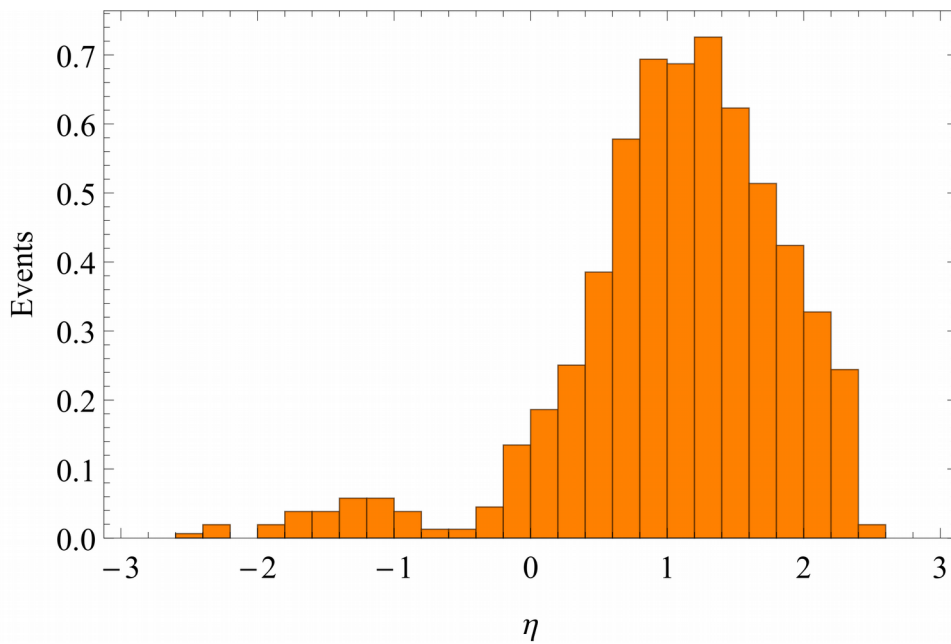
$\delta M \ll \Gamma_4$



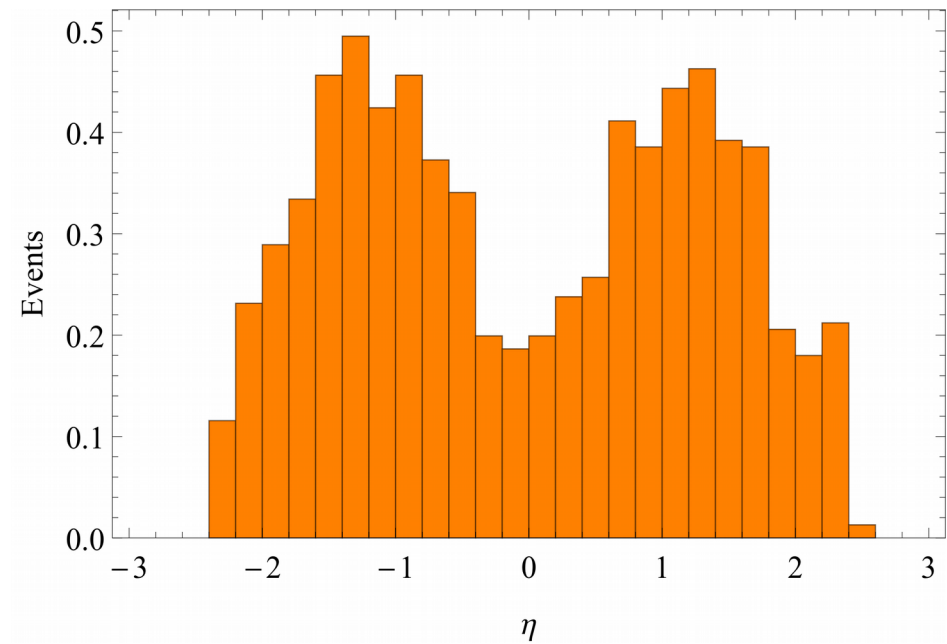
$\delta M \gg \Gamma_4$

ℓ^- pseudorapidity distribution (benchmark 2)

Mass (GeV)	$ U_{\mu 4} ^2$	Γ_4 (meV)	$c\tau_4$ (mm)
20	5×10^{-6}	20	0.01



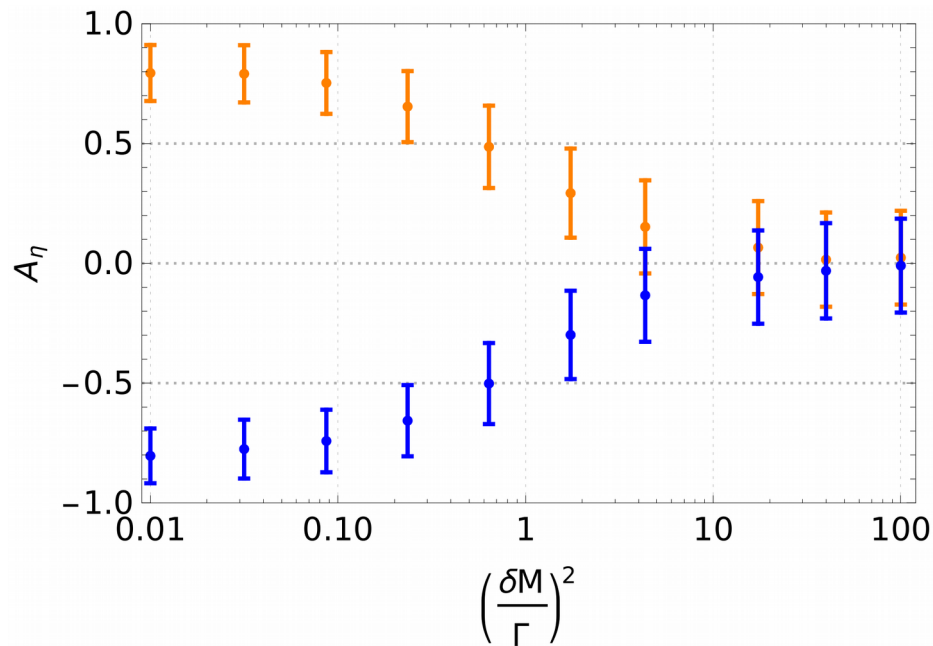
$\delta M \ll \Gamma_4$



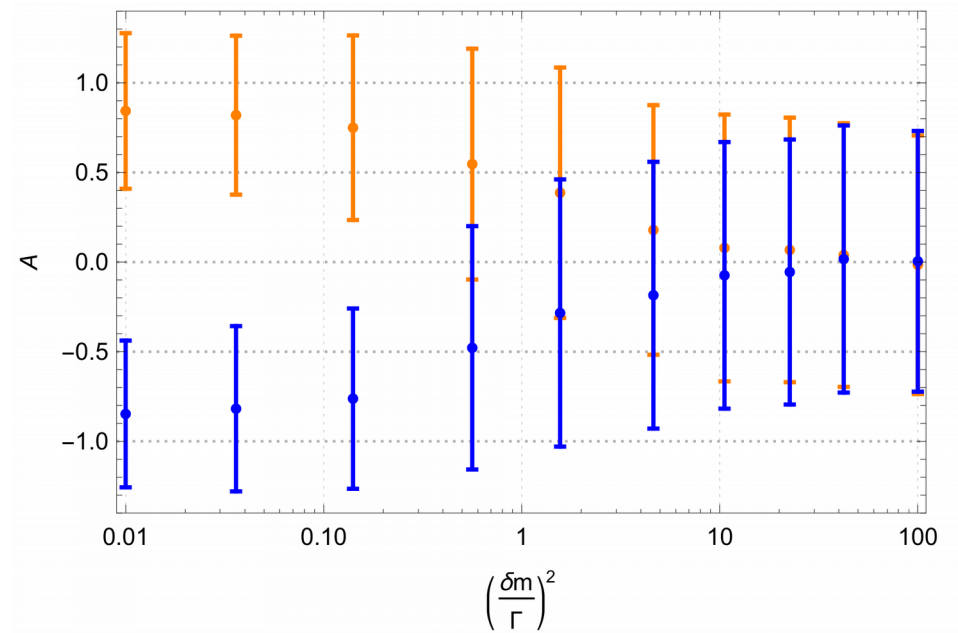
$\delta M \gg \Gamma_4$

Bounding the Mass Difference (2σ)

$$A_{\eta}^{\pm} = \frac{N^{\pm}(\eta > 0) - N^{\pm}(\eta < 0)}{N_{\text{tot}}^{\pm}}$$



Benchmark 1



Benchmark 2

Conclusions:

- We can probe the (Majorana) PseudoDirac nature of neutrinos by (not) observing an asymmetry in the η distribution of displaced leptons, at the ILC.
- The observation of the asymmetry depends on the ratio between δM and Γ .
- For both studied benchmarks, the δM “thresholds” are:

$$\delta M \sim 1 \text{ meV}$$

Benchmark 1

$$\delta M \sim 20 \text{ meV}$$

Benchmark 2



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Thanks!

Backup

Casas-Ibarra:

- By using the Casas-Ibarra parametrisation, one can write the Dirac masses as:

$$(m_D^{\text{new}})_{a4} \simeq \pm (Z_a^{\text{NH}})^* \sqrt{m_3 M_4} \cosh \gamma_{45} e^{\mp i \theta_{45}}$$

$$(m_D^{\text{new}})_{a5} \simeq -i (Z_a^{\text{NH}})^* \sqrt{m_3 M_5} \cosh \gamma_{45} e^{\mp i \theta_{45}}$$

- The mass matrix is:

$$M'_\nu = \begin{pmatrix} 0 & (m_D^{\text{new}})_{a4} & (m_D^{\text{new}})_{a5} \\ (m_D^{\text{new}})_{a4}^T & M_4 & 0 \\ (m_D^{\text{new}})_{a5}^T & 0 & M_5 \end{pmatrix}$$

Casas-Ibarra:

- With a change of basis:

$$V^T M'_\nu V = M_\nu$$

$$V = \begin{pmatrix} I & 0 & 0 \\ 0 & -i \cos \theta & i \sin \theta \\ 0 & \sin \theta & \cos \theta \end{pmatrix}$$

$$M_\nu = \left(\begin{array}{c|cc} 0 & m_D & \varepsilon m'_D \\ \hline m_D^T & \mu' & M \\ \varepsilon m_D'^T & M & \mu \end{array} \right)$$

$$\tan \theta = \sqrt{M_5/M_4}$$

$$\Rightarrow \mu' = M_5 - M_4$$

Casas-Ibarra:

- The elements of the mixing matrix:

$$U_{a4} \simeq \pm Z_a^{\text{NH}} \sqrt{\frac{m_3}{M_4}} \cosh \gamma_{45} e^{\mp i\theta_{45}}$$

$$U_{a5} \simeq i Z_a^{\text{NH}} \sqrt{\frac{m_3}{M_5}} \cosh \gamma_{45} e^{\mp i\theta_{45}}$$

$$Z_a^{\text{NH}} \equiv (U_{\text{PMNS}})_{a3} \pm i \sqrt{\frac{m_2}{m_3}} (U_{\text{PMNS}})_{a2}$$

Diagrams A and B:

- The matrix elements squared are:

$$|\mathcal{M}_A|^2 = \frac{1}{4} \left(\frac{g}{\sqrt{2}} \right)^6 \left[\sum_{j,k=4}^5 \Omega_{Aj} \Omega_{Ak}^* \right] G_A^{\lambda\delta} \epsilon_\lambda^*(p_4) \epsilon_\delta(p_4)$$

$$|\mathcal{M}_B|^2 = \frac{1}{4} \left(\frac{g}{\sqrt{2}} \right)^6 \left[\sum_{j,k=4}^5 \frac{M_j M_k}{q^2} \Omega_{Bj} \Omega_{Bk}^* \right] G_B^{\lambda\delta} \epsilon_\lambda^*(p_4) \epsilon_\delta(p_4)$$

$$\Omega_{Aj} \equiv \frac{U_{\mu j}^* U_{ej} U_{e\nu}^*}{f(M_j)}$$

$$\Omega_{Bj} \equiv \frac{U_{\mu j}^* U_{ej}^* U_{e\nu}}{f(M_j)}$$

Software

- SARAH 4.12.3: Model implementation
- SPheno 3.3.8: Calculation of masses, mixing and branching ratios
- WHIZARD 2.5.0: Event generation at ILC
- Pythia 6: Parton shower and hadronization (from WHIZARD)
- Delphes 3.4.1: Fast detector simulation
- DSiD Card: Delphes card for SiD detector.