Universidad Autónoma de Madrid

Instituto de Física Teórica UAM-CSIC

Matching BSM physics to the SMEFT and the Weak Hamiltonian with 1-loop accuracy

Pedro Ruiz-Femenía

Universidad Autónoma Madrid - IFT

Relevance of effective field theories: bottom-up

- Model independent approach. Physics above a given scale mapped into Wilson coefficients of higher dimension EFT ops.

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \sum_i \frac{1}{\Lambda^{d_i-4}} C_i\, \mathcal{O}_i$$

- LHC + EWPD data can be used to do global fits of Wilson coefficients. These bounds are model independent, any UV model must satisfy them, no need to re-do analysis for each model

- EFTs at NLO ?
  - ▷ With the increasing precision in some experimental observables, one-loop corrections can become important. Moreover, some EFT contributions are only generated at one-loop.
  - ▷ Ongoing efforts to extend the EFT analyses to NLO
    See for instance CERN Yellow Report 4
  - ▷ Radiative corrections to the EFT also important in Flavour Physics
    e.g. Feruglio, Paradisi, Pattori, arXiv:1606.00524; Pruna, Signer, JHEP 1410 (2014) 014

Mapping a UV model to the EFT: top-down

$$\mathcal{L}(\phi, \Phi)$$

M_Φ

E

$$\mathcal{L}_{\text{SMEFT}}(\phi, \mu = M_\Phi)$$

$\downarrow$ $RGEs$

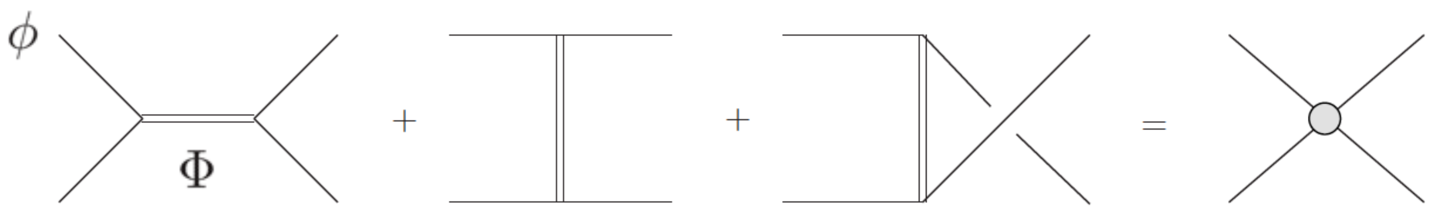
$$\mathcal{L}_{\text{SMEFT}}(\phi, \mu = E_{\text{exp}}) \longrightarrow \text{EFT analyses, global fits}$$

- This set-up translates the experimental constraints on the EFT parameter space (i.e. Wilson coeffs.) into those of the UV model. In specific models correlations among Wilson coeffs. expected
- Ideally, a simple and systematic framework to match any UV theory to the EFT would be needed

Diagrammatic vs functional matching

Two approaches to construct the low-energy EFT from the UV theory

- Matching the diagrammatic computation of Green functions with light particles in the external legs in the full theory and in the EFT



- Using functional methods to integrate out the heavy particle effects and extract the local contributions to the EFT

$$e^{i\Gamma_{\rm UV}} = \mathcal{N} \int \mathcal{D}\phi \mathcal{D}\Phi \, \exp\left[i \int dx \, \mathcal{L}(\phi, \Phi)\right]$$
$$e^{i\Gamma_{\rm EFT}} = \mathcal{N} \int \mathcal{D}\phi \, \exp\left[i \int dx \, \mathcal{L}_{\rm EFT}(\phi)\right]$$

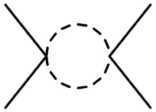
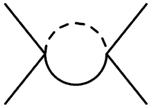
$\longrightarrow \Gamma_{\rm L,UV}[\hat{\phi}] = \Gamma_{\rm EFT}[\hat{\phi}]$
same 1LPI diagrams

They are equivalent, but functional methods are more systematic when one aims to determine many Green functions (no Feynman rules, symmetry factors...)

The revival of the functional approach

- Functional techniques to obtain the 1-loop effective action developed long ago
  Fraser '85; Aitchison, Fraser '85; Chan '86; Galliard '86; Cheyette '88

- The 1-loop heavy particle effects to the low-energy EFT were extracted from the full theory effective action in some cases:
  Ball '89; Bilenki, Santamaria '94; Dittmaier, Grosse-Knetter '95

- Functional methods have recently experienced a renaissance
  - ▷ The contribution to the EFT from pure heavy loops can be casted as a *universal* master formula Henning, Lu, Murayama '14; Drozd, Ellis, Quevillon, You '15
    
  - ▷ Variants of the functional approach that also account for heavy-light loops proposed; they require subtractions to remove terms already present in 1-loop EFT contributions Henning, Lu, Murayama '16; Ellis, Quevillon, You, Zhang '16
    

This talk: alternative functional method to obtain the complete one-loop EFT directly from the full theory effective action (i.e. without infrared subtractions) → "expansion by regions" Fuentes-Martín, Portolés, PRF, 1607.02142

Outline of the method

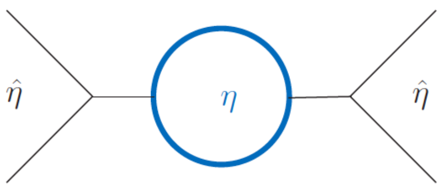
Preliminaries

Consider a general UV theory with heavy η_H and light η_L degrees of freedom

$$\eta = \begin{pmatrix} \eta_H \\ \eta_L \end{pmatrix}$$

(for charged degrees of freedom, the field and its complex conjugate enter as separate components in η)

- Split each field component: $\eta \to \hat{\eta} + \eta$



$\hat{\eta}$: Background field, classical solution of EOM

η : Quantum fluctuation (only in loops)

- At the one-loop level we only need to consider the Lagrangian up to $\mathcal{O}(\eta^2)$ in the generating functional

$$\mathcal{L} = \mathcal{L}^{\text{tree}}(\hat{\eta}) + \cancel{\left(\eta^\dagger \frac{\delta \mathcal{L}}{\delta \eta^*} + \frac{\delta \mathcal{L}}{\delta \eta} \eta \right)_{\eta = \hat{\eta}}} + \frac{1}{2} \eta^\dagger \underbrace{\left. \frac{\delta^2 \mathcal{L}}{\delta \eta^* \, \delta \eta} \right|_{\eta = \hat{\eta}}}_{\equiv \mathcal{O}} \eta + \mathcal{O}\left(\eta^3\right)$$

(EOM)

$\equiv \mathcal{O}$
fluctuation operator

One-loop effective action

$$e^{i\Gamma^{\rm 1loop}_{\rm UV}} = \mathcal{N}\int \mathcal{D}\eta \, \exp\left[i \int dx \, \frac{1}{2}\eta^\dagger \mathcal{O}\, \eta \right]$$

Generic form of the fluctuation operator:

$$\mathcal{O} = \begin{pmatrix} \Delta_H & X^\dagger_{LH} \\ X_{LH} & \Delta_L \end{pmatrix}$$

Δ_H, Δ_L : heavy and light loops

X_{LH} : heavy-light loops

- We want to compute the 1-loop heavy particle effects in Green functions of the light fields as an expansion in $1/m_H$

- We can factor out the loops involving heavy fields by writing $\mathcal{O}$ in block-diagonal form

$$\eta \to P\eta \qquad P = \begin{pmatrix} I & 0 \\ -\Delta_L^{-1} X_{LH} & I \end{pmatrix} \implies P^\dagger \mathcal{O} P = \begin{pmatrix} \widetilde{\Delta}_H & 0 \\ 0 & \Delta_L \end{pmatrix}$$

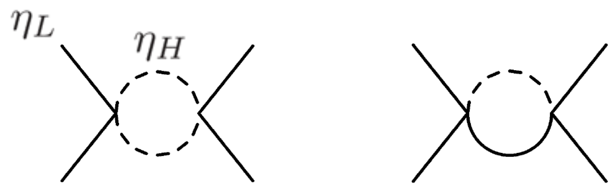
where $\widetilde{\Delta}_H = \Delta_H - X^\dagger_{LH}\Delta_L^{-1}X_{LH}$

One-loop effective action

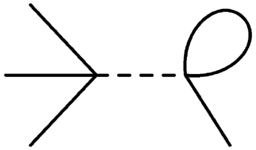
$$e^{i\Gamma_{\rm UV}^{\rm 1loop}} = \underbrace{\int \mathcal{D}\eta_H \, \exp\left[i\int dx\, \frac{1}{2}\eta_H^\dagger \widetilde{\Delta}_H\, \eta_H\right]} \mathcal{N}\int \mathcal{D}\eta_L \, \exp\left[i\int dx\, \frac{1}{2}\eta_L^\dagger \Delta_L\, \eta_L\right]$$

$$= \left(\det \widetilde{\Delta}_H\right)^{-c} \quad (c = 1/2, -1 \text{ for bosons/fermions})$$

$\rightarrow$ all 1-loop heavy-particle effects contained in $\det \widetilde{\Delta}_H$



The integral over Δ_L accounts for loops of light particles $\longrightarrow$ (heavy fields only as tree lines)



Part of the $\Gamma_{\rm UV}^{\rm 1loop}$ coming from loops involving heavy fields:

$$e^{i\Gamma_H} = \left(\det \widetilde{\Delta}_H\right)^{-\frac{1}{2}} \quad \text{(assume } \eta_H \text{ is bosonic in what follows)}$$

One-loop effective action

$$e^{i\Gamma_H} = \left(\det \widetilde{\Delta}_H\right)^{-\frac{1}{2}}$$

One-loop effective action

$$\Gamma_H = \frac{i}{2} \ln \det \widetilde{\Delta}_H$$

One-loop effective action

$$\Gamma_H = \frac{i}{2}\mathrm{Tr}\ln\widetilde{\Delta}_H$$

Evaluating the functional determinant

$$\Gamma_H = \tfrac{i}{2} \mathrm{Tr} \ln \widetilde{\Delta}_H$$

Rewrite the functional trace using momentum eigenstates

$$\begin{aligned}
\Gamma_H &= \frac{i}{2} \,\mathrm{tr} \int \frac{d^d p}{(2\pi)^d} \langle p | \ln \widetilde{\Delta}_H | p \rangle \\
&= \frac{i}{2} \,\mathrm{tr} \int d^d x \int \frac{d^d p}{(2\pi)^d} \, e^{-ipx} \ln \left(\widetilde{\Delta}_H \left(x, \partial_x \right) \right) e^{ipx} \\
&= \frac{i}{2} \,\mathrm{tr} \int d^d x \int \frac{d^d p}{(2\pi)^d} \, \ln \left(\widetilde{\Delta}_H \left(x, \partial_x + ip \right) \right) \mathbb{1}
\end{aligned}$$

For scalars: $\widetilde{\Delta}_H(x, \partial_x) = -\hat{D}^2 - m_H^2 - U(x, \partial_x)$

Evaluating the functional determinant

$$\Gamma_H = \frac{i}{2}\,\mathrm{tr}\int d^dx \int \frac{d^dp}{(2\pi)^d}\;\ln\left[-\left(\hat{D}-ip\right)^2 - m_H^2 - U\left(x, \partial_x + ip\right)\right]\mathbb{1}$$

Evaluating the functional determinant

$$\Gamma_H = -\frac{i}{2}\int d^dx \sum_{n=1}^{\infty}\frac{1}{n}\int\frac{d^dp}{(2\pi)^d}\,\mathrm{tr}\left\{\left(\frac{2ip\hat{D}+\hat{D}^2+U\left(x,\partial_x+ip\right)}{p^2-m_H^2}\right)^n \mathbb{1}\right\}$$

Evaluating the functional determinant

$$\Gamma_H = -\frac{i}{2}\int d^dx \sum_{n=1}^{\infty}\frac{1}{n}\int \frac{d^dp}{(2\pi)^d}\,\mathrm{tr}\left\{\left(\frac{2ip\hat{D}+\hat{D}^2+U(x,\partial_x+ip)}{p^2-m_H^2}\right)^n \mathbb{1}\right\}$$

This expression generates **all one-loop amplitudes with at least one heavy-particle propagator** in the loop (n : # of heavy propagators)

$$U(x,\partial_x) = -\hat{D}^2 - m_H^2 - \widetilde{\Delta}_H(x,\partial_x)$$

$$U_H = -\hat{D}^2 - m_H^2 - \Delta_H \qquad \text{(heavy loops)}$$

$$U_{LH} = X_{LH}^{\dagger}\Delta_L^{-1}X_{LH} \qquad \text{(heavy-light loops)}$$

The operator Δ_L^{-1} contains the light-particle propagator ($\Delta_L = \widetilde{\Delta}_L + X_L$)

$$\Delta_L^{-1} = \sum_{m=0}^{\infty}(-1)^m\left(\widetilde{\Delta}_L^{-1}X_L\right)^m\widetilde{\Delta}_L^{-1}$$

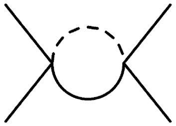
$\widetilde{\Delta}_L^{-1}$: Light-field propagator

X_L: Interaction term

Evaluating the functional determinant: **method of regions**

$$\Gamma_H = -\frac{i}{2}\int d^dx \sum_{n=1}^{\infty}\frac{1}{n}\int\frac{d^dp}{(2\pi)^d}\,\mathrm{tr}\left\{\left(\frac{2ip\hat{D}+\hat{D}^2+U(x,\partial_x+ip)}{p^2-m_H^2}\right)^n \mathbb{1}\right\}$$

• Loops with heavy particles receive contributions from the **hard** ($p\sim m_H$) and **soft** ($p\sim m_L, p_i$) loop momentum regions



• In dim. reg. the two contributions can be computed separately

→ **expansion by regions:** contribution from each region obtained by Taylor expanding the integrand with respect the parameters that are small there, and then integrating over the full d-dimensional space. Beneke, Smirnov '98

$$\Gamma_H \quad = \quad \Gamma_H^{\rm hard} \quad + \quad \Gamma_H^{\rm soft}$$

$p\sim m_H \gg m_L, p_i$

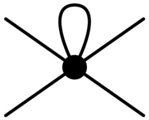
→ polynomial in $\frac{\partial}{m_H}, \frac{m_L}{m_H}$

(pure short-distance!)

$p\sim m_L \ll m_H$

→ heavy propagator expanded in p/m_H

(same as EFT loop diagrams from tree vertices)



Matching the full and EFT effective actions

Full-theory effective action

$$\Gamma_{\rm L,UV}[\hat\eta_L] = \int d^dx\, \mathcal{L}^{\rm tree}_{\rm UV} + \Gamma_H + \tfrac{i}{2}\ln\det\Delta_L \qquad \text{with } \hat\eta_H = \hat\eta_H[\hat\eta_L]$$

EFT effective action

$$\Gamma_{\rm EFT}[\hat\eta_L] = \int d^dx \left(\mathcal{L}^{\rm tree}_{\rm EFT} + \mathcal{L}^{\rm 1loop}_{\rm EFT}\right) + \frac{i}{2}\ln\det\mathcal{O}^{\rm tree}_{\rm EFT}$$

Tree-level matching

$$\mathcal{L}^{\rm tree}_{\rm EFT}(\hat\eta_L) = \mathcal{L}^{\rm tree}_{\rm UV}(\hat\eta)\Big|_{\text{local expansion of } \hat\eta_H = \hat\eta_H[\hat\eta_L]}$$

example: two real scalars η_L, η_H with a $\eta_L^3\eta_H$ interaction

$$\eta_H = \frac{g}{\partial^2 - m_H^2}\,\eta_L^3 = -\frac{g}{m_H^2}\,\eta_L^3 - \frac{g}{m_H^4}\,\partial^2\eta_L^3 + \ldots$$

Matching the full and EFT effective actions

Full-theory effective action

$$\Gamma_{\mathrm{L,UV}}[\hat{\eta}_L] = \int d^dx\, \mathcal{L}_{\mathrm{UV}}^{\mathrm{tree}} + \Gamma_H + \tfrac{i}{2}\ln\det\Delta_L \qquad \text{with } \hat{\eta}_H = \hat{\eta}_H[\hat{\eta}_L]$$

EFT effective action

$$\Gamma_{\mathrm{EFT}}[\hat{\eta}_L] = \int d^dx \left(\mathcal{L}_{\mathrm{EFT}}^{\mathrm{tree}} + \mathcal{L}_{\mathrm{EFT}}^{\mathrm{1loop}}\right) + \frac{i}{2}\ln\det\mathcal{O}_{\mathrm{EFT}}^{\mathrm{tree}}$$

One-loop matching

$$\int d^dx\, \mathcal{L}_{\mathrm{EFT}}^{\mathrm{1loop}} = \Gamma_H^{\mathrm{hard}} + \left(\Gamma_H^{\mathrm{soft}} + \frac{i}{2}\ln\det\Delta_L - \frac{i}{2}\ln\det\mathcal{O}_{\mathrm{EFT}}^{\mathrm{tree}}\right)$$

Matching the full and EFT effective actions

Full-theory effective action

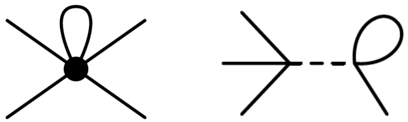
$$\Gamma_{\mathrm{L,UV}}[\hat{\eta}_L] = \int d^dx\, \mathcal{L}_{\mathrm{UV}}^{\mathrm{tree}} + \Gamma_H + \tfrac{i}{2} \ln\det\Delta_L \qquad \text{with } \hat{\eta}_H = \hat{\eta}_H[\hat{\eta}_L]$$

EFT effective action

$$\Gamma_{\mathrm{EFT}}[\hat{\eta}_L] = \int d^dx \left(\mathcal{L}_{\mathrm{EFT}}^{\mathrm{tree}} + \mathcal{L}_{\mathrm{EFT}}^{\mathrm{1loop}}\right) + \frac{i}{2} \ln\det\mathcal{O}_{\mathrm{EFT}}^{\mathrm{tree}}$$

One-loop matching

$$\int d^dx\, \mathcal{L}_{\mathrm{EFT}}^{\mathrm{1loop}} = \Gamma_H^{\mathrm{hard}} + \left(\Gamma_H^{\mathrm{soft}} + \frac{i}{2}\ln\det\Delta_L - \frac{i}{2}\ln\det\mathcal{O}_{\mathrm{EFT}}^{\mathrm{tree}}\right)$$



$\rightarrow$ same 1loop amplitude as one would obtain from $\mathcal{L}_{\mathrm{EFT}}^{\mathrm{tree}}$!!!

Matching the full and EFT effective actions

Full-theory effective action

$$\Gamma_{\mathrm{L,UV}}[\hat\eta_L] = \int d^dx\, \mathcal{L}^{\mathrm{tree}}_{\mathrm{UV}} + \Gamma_H + \tfrac{i}{2}\ln\det\Delta_L \qquad \text{with } \hat\eta_H = \hat\eta_H[\hat\eta_L]$$

EFT effective action

$$\Gamma_{\mathrm{EFT}}[\hat\eta_L] = \int d^dx \left(\mathcal{L}^{\mathrm{tree}}_{\mathrm{EFT}} + \mathcal{L}^{\mathrm{1loop}}_{\mathrm{EFT}}\right) + \frac{i}{2}\ln\det\mathcal{O}^{\mathrm{tree}}_{\mathrm{EFT}}$$

One-loop matching

$$\int d^dx\, \mathcal{L}^{\mathrm{1loop}}_{\mathrm{EFT}} = \Gamma^{\mathrm{hard}}_H + \left(\Gamma^{\mathrm{soft}}_H + \frac{i}{2}\ln\det\Delta_L - \frac{i}{2}\ln\det\mathcal{O}^{\mathrm{tree}}_{\mathrm{EFT}}\right)$$

for a formal proof see Zhang, 1610.00710

$$\Longrightarrow \boxed{\int d^4x\, \mathcal{L}^{\mathrm{1loop}}_{\mathrm{EFT}} = \Gamma^{\mathrm{hard}}_H}$$

one-loop Wilson coeffs. determined by hard part of full-theory eff. action

Evaluating the **hard** part of the functional determinant

$$\Gamma_H = -\frac{i}{2}\int d^dx \sum_{n=1}^{\infty}\frac{1}{n}\int\frac{d^dp}{(2\pi)^d}\,\mathrm{tr}\left\{\left(\frac{2ip\hat{D}+\hat{D}^2+U\,(x,\partial_x+ip)}{p^2-m_H^2}\right)^n \mathbb{1}\right\}$$

- To compute $\Gamma_H^{\rm hard}$ we introduce the counting

$$p,\, m_H \sim \zeta$$

and expand the integrand in Γ_H up to a given order ζ^{-k} with $k>0$

- Only a finite number of terms contributes because U at most $\mathcal{O}(\zeta)$

  for example: to obtain the dim. 6 operators , i.e. $\mathcal{O}\big(1/m_H^2\big)$,
  it is enough to compute U up $\mathcal{O}(\zeta^{-4})$ (recall that $d^4p \sim \zeta^4$)

A simple example

1st example: scalar toy model

$$\mathcal{L}(\varphi,\phi) = \frac{1}{2}\left(\partial_\mu\phi\,\partial^\mu\phi - M^2\,\phi^2\right) + \frac{1}{2}\left(\partial_\mu\varphi\,\partial^\mu\varphi - m^2\,\varphi^2\right) - \frac{\kappa}{4!}\,\varphi^4 - \frac{\lambda}{3!}\,\varphi^3\,\phi$$

Interesting for two reasons:

- Very simple. Excellent testing ground
- Only heavy-light loops contribute

Tree level: $\mathcal{L}_{\text{EFT}}^{\text{tree}} = \mathcal{L}(\hat{\phi},\hat{\varphi})$

EOM: $\hat{\phi} = -\frac{\lambda}{6M^2}\,\hat{\varphi}^3 + \mathcal{O}(M^{-4})$

$$\Longrightarrow \mathcal{L}_{\text{EFT}}^{\text{tree}} = \frac{1}{2}\left(\partial_\mu\hat{\varphi}\,\partial^\mu\hat{\varphi} - m^2\,\hat{\varphi}^2\right) - \frac{\kappa}{4!}\,\hat{\varphi}^4 + \frac{\lambda^2}{72M^2}\,\hat{\varphi}^6 + \mathcal{O}(M^{-4})$$

One loop: $\mathcal{L}_{\text{EFT}}^{\text{1loop}}$?

Scalar toy model: functional integration

$$\mathcal{L}(\varphi,\phi) = \frac{1}{2}\left(\partial_\mu\phi\,\partial^\mu\phi - M^2\,\phi^2\right) + \frac{1}{2}\left(\partial_\mu\varphi\,\partial^\mu\varphi - m^2\,\varphi^2\right) - \frac{\kappa}{4!}\,\varphi^4 - \frac{\lambda}{3!}\,\varphi^3\,\phi$$

1. Collect the fields in a multiplet: $\eta = \begin{pmatrix}\phi\\ \varphi\end{pmatrix}$

2. Compute the fluctuation operator:

$$\mathcal{O} = \left.\frac{\delta^2\mathcal{L}}{\delta\eta^*\,\delta\eta}\right|_{\eta=\hat{\eta}} \Longrightarrow \begin{cases} \Delta_H = -\partial^2 - M^2 \\ \Delta_L = -\partial^2 - m^2 - \frac{\kappa}{2}\,\hat{\varphi}^2 - \lambda\,\hat{\varphi}\,\hat{\phi} \\ X_{LH} = -\frac{\lambda}{2}\,\hat{\varphi}^2 \end{cases}$$

3. Calculate $U = U_H + U_{LH}$

$$U_H = -\partial^2 - M^2 - \Delta_H = 0$$

$$U_{LH} = X_{LH}^\dagger \Delta_L^{-1} X_{LH}$$

Scalar toy model: functional integration

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3. Calculate $U = U_H + U_{LH}$ and compute $U(x, \partial_x + ip)$ up to ζ^{-4}

$$U_H = -\partial^2 - M^2 - \Delta_H = 0$$
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3. Calculate $U = U_H + U_{LH}$ and compute $U(x,\partial_x + ip)$ up to ζ^{-4}

$$U(x,\partial_x+ip) = \frac{\lambda^2}{4}\,\hat\varphi^2\left[\frac{1}{p^2}\left(1+\frac{m^2}{p^2}\right) + \frac{1}{p^4}\left(2i\,p_\mu\partial^\mu + \partial^2 + \frac{\kappa}{2}\hat\varphi^2\right) - 4\,\frac{p_\mu p_\nu}{p^6}\,\partial^\mu\partial^\nu\right]\hat\varphi^2 + \mathcal{O}(\zeta^{-5})$$

Scalar toy model: functional integration

3. Calculate $U = U_H + U_{LH}$ and compute $U(x, \partial_x + ip)$ up to ζ^{-4}

$$U(x, \partial_x + ip) = \frac{\lambda^2}{4}\,\hat{\varphi}^2\left[\frac{1}{p^2}\left(1+\frac{m^2}{p^2}\right)+\frac{1}{p^4}\left(2i\,p_\mu\partial^\mu+\partial^2+\frac{\kappa}{2}\hat{\varphi}^2\right)-4\,\frac{p_\mu p_\nu}{p^6}\,\partial^\mu\partial^\nu\right]\hat{\varphi}^2 + \mathcal{O}(\zeta^{-5})$$

4. Insert $U(x, \partial_x + ip)$ in the general formula

$$\mathcal{L}_{\text{EFT}}^{\text{1loop}} = -\frac{i}{2}\sum_{n=1}^{\infty}\frac{1}{n}\int\frac{d^dp}{(2\pi)^d}\,\text{tr}\left\{\left(\frac{2ip\,\partial+\partial^2+U\,(x,\partial_x+ip)}{p^2-m_H^2}\right)^n\mathbb{1}\right\}$$

Scalar toy model: functional integration

3. Calculate $U = U_H + U_{LH}$ and compute $U(x, \partial_x + ip)$ up to ζ^{-4}

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4. Insert $U(x, \partial_x + ip)$ in the general formula **(only $n = 1$ contributes)**

$$\mathcal{L}_{\text{EFT}}^{\text{1loop}} = -\frac{i}{2}\sum_{n=1}^{\infty}\frac{1}{n}\int\frac{d^d p}{(2\pi)^d}\,\text{tr}\left\{\left(\frac{2ip\,\partial + \partial^2 + U\,(x,\partial_x + ip)}{p^2 - m_H^2}\right)^n \mathbb{1}\right\}$$

Scalar toy model: functional integration

3. Calculate $U = U_H + U_{LH}$ and compute $U(x, \partial_x + ip)$ up to ζ^{-4}

$$U(x,\partial_x + ip) = \frac{\lambda^2}{4}\,\hat{\varphi}^2\left[\frac{1}{p^2}\left(1+\frac{m^2}{p^2}\right) + \frac{1}{p^4}\left(2i\,p_\mu\partial^\mu + \partial^2 + \frac{\kappa}{2}\hat{\varphi}^2\right) - 4\,\frac{p_\mu p_\nu}{p^6}\,\partial^\mu\partial^\nu\right]\hat{\varphi}^2 + \mathcal{O}(\zeta^{-5})$$

4. Insert $U(x, \partial_x + ip)$ in the general formula **(only $n = 1$ contributes)**

$$\mathcal{L}_{\text{EFT}}^{1\text{loop}} = -\frac{i}{2}\int \frac{d^d p}{(2\pi)^d}\,\frac{U(x,\partial_x + ip)}{p^2 - M^2} + \mathcal{O}(M^{-4})$$

Scalar toy model: functional integration

3. Calculate $U = U_H + U_{LH}$ and compute $U(x, \partial_x + ip)$ up to ζ^{-4}

$$U(x,\partial_x+ip) = \frac{\lambda^2}{4}\,\hat{\varphi}^2\left[\frac{1}{p^2}\left(1+\frac{m^2}{p^2}\right)+\frac{1}{p^4}\left(2i\,p_\mu\partial^\mu+\partial^2+\frac{\kappa}{2}\hat{\varphi}^2\right)-4\,\frac{p_\mu p_\nu}{p^6}\,\partial^\mu\partial^\nu\right]\hat{\varphi}^2 + \mathcal{O}(\zeta^{-5})$$

4. Insert $U(x, \partial_x + ip)$ in the general formula **(only $n = 1$ contributes)**

$$\mathcal{L}_{\text{EFT}}^{1\text{loop}} = -\frac{i}{2}\int\frac{d^dp}{(2\pi)^d}\,\frac{U(x,\partial_x+ip)}{p^2-M^2} + \mathcal{O}(M^{-4})$$

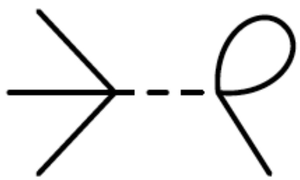
The integral is straightforward. In $\overline{\text{MS}}$ with $\mu = M$

$$\mathcal{L}_{\text{EFT}}^{1\text{loop}} = \frac{\lambda^2}{16(16\pi^2)}\left[2\left(1+\frac{m^2}{M^2}\right)\hat{\varphi}^4 - \frac{1}{M^2}\hat{\varphi}^2\partial^2\hat{\varphi}^2 + \frac{\kappa}{M^2}\hat{\varphi}^6\right]$$

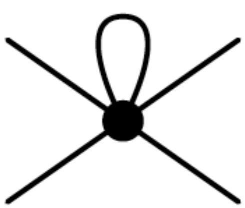
Scalar toy model: diagrammatic matching

4-point Green function $\qquad \mathcal{L}_{\rm EFT} = \mathcal{L}_{\rm EFT}^{\rm tree} + \frac{\alpha}{4!}\,\hat{\varphi}^4 + \frac{\beta}{4!M^2}\,\hat{\varphi}^2\partial^2\hat{\varphi}^2 + \frac{\gamma}{6!M^2}\,\hat{\varphi}^6$

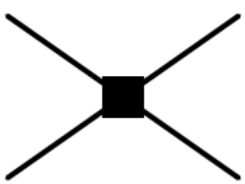
$$= \frac{i}{16\pi^2}\,\lambda^2\left[3 + \frac{s+t+u}{2M^2} + 3\,\frac{m^2}{M^2}\ln\left(\frac{m^2}{M^2}\right)\right] + \mathcal{O}(M^{-4})$$



$$= \frac{i}{16\pi^2}\,\lambda^2\left[-2\,\frac{m^2}{M^2} + 2\,\frac{m^2}{M^2}\ln\left(\frac{m^2}{M^2}\right)\right] + \mathcal{O}(M^{-4})$$



$$= \frac{i}{16\pi^2}\,\lambda^2\left[-5\,\frac{m^2}{M^2} + 5\,\frac{m^2}{M^2}\ln\left(\frac{m^2}{M^2}\right)\right] + \mathcal{O}(M^{-4})$$



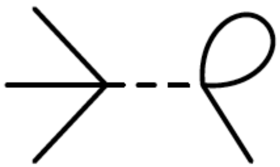
$$= i\,\alpha - i\,\frac{\beta}{3M^2}\,(s+t+u)$$

Scalar toy model: diagrammatic matching

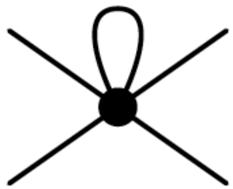
4-point Green function $\mathcal{L}_{\rm EFT} = \mathcal{L}_{\rm EFT}^{\rm tree} + \frac{\alpha}{4!}\hat\varphi^4 + \frac{\beta}{4!M^2}\hat\varphi^2\partial^2\hat\varphi^2 + \frac{\gamma}{6!M^2}\hat\varphi^6$

$$= \frac{i}{16\pi^2}\lambda^2\left[3 + 3\,\frac{m^2}{M^2} + \frac{s+t+u}{2M^2}\right]\Bigg|_{\rm hard}$$

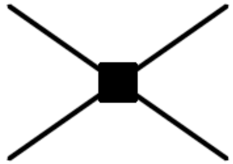
$$+ \frac{i}{16\pi^2}\lambda^2\left[-3\,\frac{m^2}{M^2} + 3\,\frac{m^2}{M^2}\ln\left(\frac{m^2}{M^2}\right)\right]\Bigg|_{\rm soft} + \mathcal{O}(M^{-4})$$



$$= \frac{i}{16\pi^2}\lambda^2\left[-2\,\frac{m^2}{M^2} + 2\,\frac{m^2}{M^2}\ln\left(\frac{m^2}{M^2}\right)\right]\Bigg|_{\rm soft} + \mathcal{O}(M^{-4})$$



$$= \frac{i}{16\pi^2}\lambda^2\left[-5\,\frac{m^2}{M^2} + 5\,\frac{m^2}{M^2}\ln\left(\frac{m^2}{M^2}\right)\right] + \mathcal{O}(M^{-4})$$



$$= i\,\alpha - i\,\frac{\beta}{3M^2}\,(s+t+u)$$

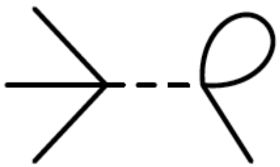
Scalar toy model: diagrammatic matching

4-point Green function

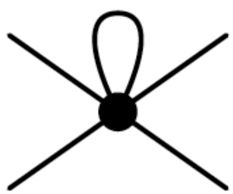
$$\mathcal{L}_{\rm EFT} = \mathcal{L}_{\rm EFT}^{\rm tree} + \frac{\alpha}{4!}\,\hat{\varphi}^4 + \frac{\beta}{4!M^2}\,\hat{\varphi}^2\partial^2\hat{\varphi}^2 + \frac{\gamma}{6!M^2}\,\hat{\varphi}^6$$

$$= \frac{i}{16\pi^2}\,\lambda^2\left[3 + 3\,\frac{m^2}{M^2} + \frac{s+t+u}{2M^2}\right]\Bigg|_{\rm hard}$$

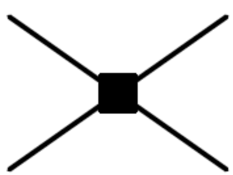
$$+ \frac{i}{16\pi^2}\,\lambda^2\left[-3\,\frac{m^2}{M^2} + 3\,\frac{m^2}{M^2}\ln\left(\frac{m^2}{M^2}\right)\right]\Bigg|_{\rm soft} + \mathcal{O}(M^{-4})$$



$$= \frac{i}{16\pi^2}\,\lambda^2\left[-2\,\frac{m^2}{M^2} + 2\,\frac{m^2}{M^2}\ln\left(\frac{m^2}{M^2}\right)\right]\Bigg|_{\rm soft} + \mathcal{O}(M^{-4})$$



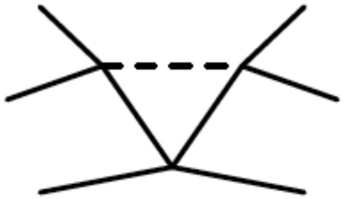
$$= \frac{i}{16\pi^2}\,\lambda^2\left[-5\,\frac{m^2}{M^2} + 5\,\frac{m^2}{M^2}\ln\left(\frac{m^2}{M^2}\right)\right] + \mathcal{O}(M^{-4})$$



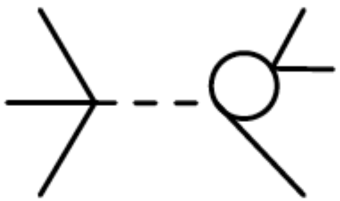
$$= i\,\alpha - i\,\frac{\beta}{3M^2}\,(s+t+u) \quad \Longrightarrow \quad \alpha = \tfrac{3}{16\pi^2}\lambda^2\left(1+\tfrac{m^2}{M^2}\right), \quad \beta = -\tfrac{3}{16\pi^2}\,\tfrac{\lambda^2}{2}$$

Scalar toy model: diagrammatic matching

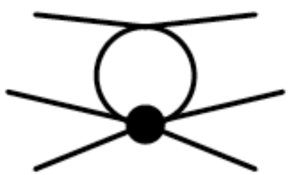
6-point Green function $\quad \mathcal{L}_{\rm EFT} = \mathcal{L}_{\rm EFT}^{\rm tree} + \frac{\alpha}{4!}\,\hat{\varphi}^4 + \frac{\beta}{4!M^2}\,\hat{\varphi}^2\partial^2\hat{\varphi}^2 + \frac{\gamma}{6!M^2}\,\hat{\varphi}^6$



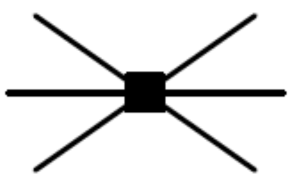
$$= \frac{i}{16\pi^2}\,45\,\frac{\kappa\,\lambda^2}{M^2}\bigg|_{\rm hard} + \frac{i}{16\pi^2}\,45\,\frac{\kappa\,\lambda^2}{M^2}\,\ln\left(\frac{m^2}{M^2}\right)\bigg|_{\rm soft} + \mathcal{O}(M^{-4})$$



$$= \frac{i}{16\pi^2}\,30\,\frac{\kappa\,\lambda^2}{M^2}\,\ln\left(\frac{m^2}{M^2}\right)\bigg|_{\rm soft} + \mathcal{O}(M^{-4})$$



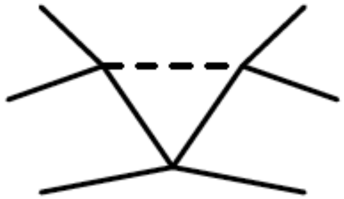
$$= \frac{i}{16\pi^2}\,75\,\frac{\kappa\,\lambda^2}{M^2}\,\ln\left(\frac{m^2}{M^2}\right) + \mathcal{O}(M^{-4})$$



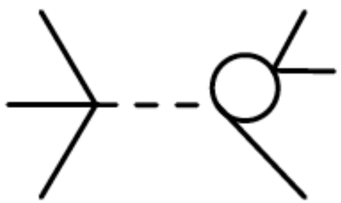
$$= i\,\frac{\gamma}{M^2}$$

Scalar toy model: diagrammatic matching

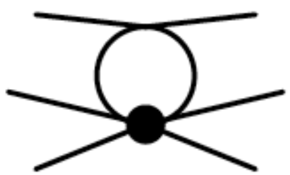
6-point Green function $\mathcal{L}_{\rm EFT} = \mathcal{L}_{\rm EFT}^{\rm tree} + \frac{\alpha}{4!}\hat{\varphi}^4 + \frac{\beta}{4!M^2}\hat{\varphi}^2\partial^2\hat{\varphi}^2 + \frac{\gamma}{6!M^2}\hat{\varphi}^6$



$$= \frac{i}{16\pi^2}\,45\,\frac{\kappa\,\lambda^2}{M^2}\Big|_{\rm hard} + \frac{i}{16\pi^2}\,45\,\frac{\kappa\,\lambda^2}{M^2}\ln\left(\frac{m^2}{M^2}\right)\Big|_{\rm soft} + \mathcal{O}(M^{-4})$$



$$= \frac{i}{16\pi^2}\,30\,\frac{\kappa\,\lambda^2}{M^2}\ln\left(\frac{m^2}{M^2}\right)\Big|_{\rm soft} + \mathcal{O}(M^{-4})$$



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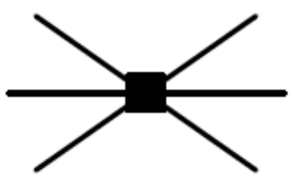


$$= i\frac{\gamma}{M^2} \quad \Longrightarrow \quad \gamma = \frac{45}{16\pi^2}\,\kappa\,\lambda^2$$

no need to compute EFT diagrams!!

Two applications

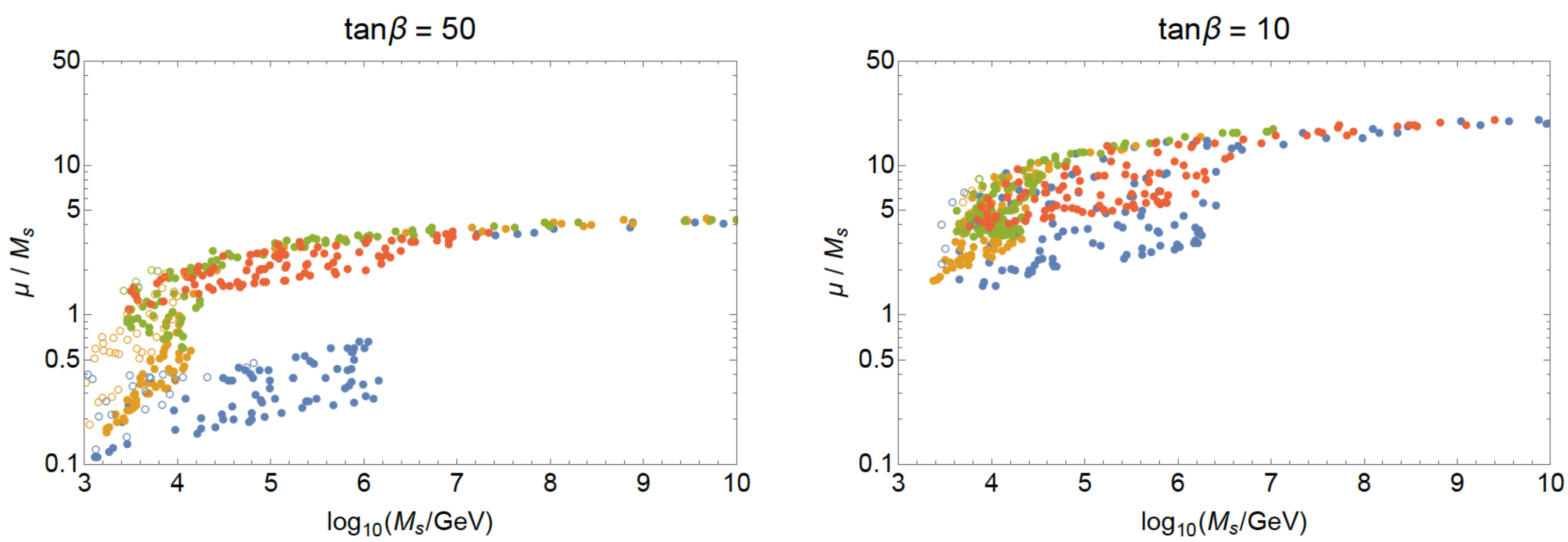
Integrate out BSM states in the MSSM

Wells, Zhang, 1711.04774

- computes the one-loop contributions to the SM renormalizable operators when heavy BSM states in the MSSM are integrated out using the functional approach

→ SUSY threshold corrections to SM parameters

- study what regions of parameter space can realize gauge coupling and Yukawa unification, while allowing consistent matching onto SMEFT parameters (those in Higgs potential in particular)



Integrate out BSM states in the MSSM

Wells, Zhang, 1711.04774

$U(x, \partial_x)$

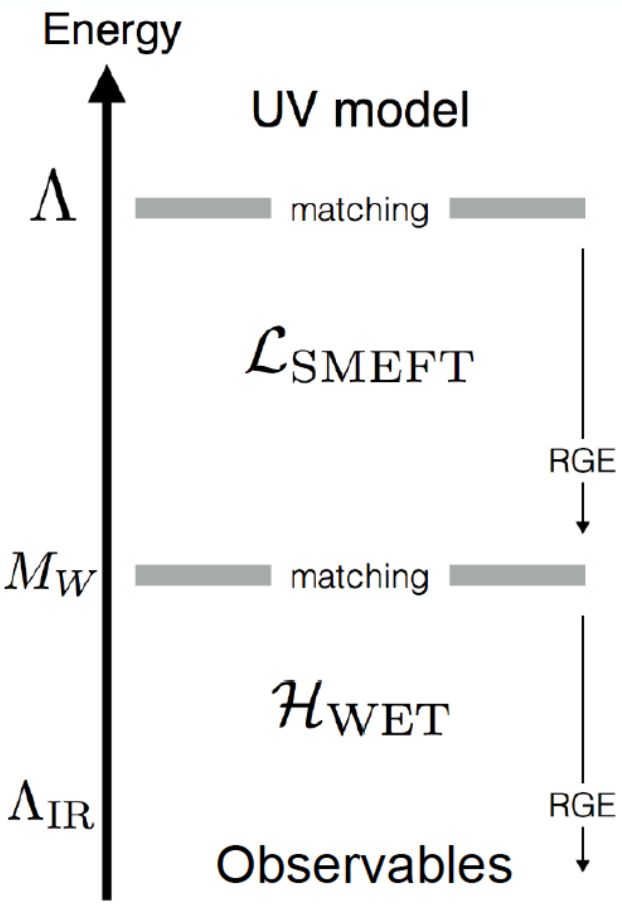
	Φ	$\tilde{q}$	$\tilde{u}$	$\tilde{d}$	$\tilde{l}$	$\tilde{e}$	$\tilde{\chi}$	$\tilde{g}$	$\tilde{W}$	$\tilde{B}$	ϕ	q	u	d	l	e	G	W	B
Φ	φ^2										φ^2	u, d	q	q	e	l		$D\Phi$	$D\Phi$
$\tilde{q}$		φ^2	φ	φ			u, d	q	q	q									
$\tilde{u}$		φ	φ^2	$\Phi\phi$			q	u		u									
$\tilde{d}$		φ	$\Phi\phi$	φ^2			q	d		d									
$\tilde{l}$					φ^2	φ	e		l	l									
$\tilde{e}$					φ	φ^2	l			e									
$\tilde{\chi}$		u, d	q	q	e	l			φ	φ									
$\tilde{g}$		q	u	d															
$\tilde{W}$		q			l		φ												
$\tilde{B}$		q	u	d	l	e	φ												
ϕ	φ^2										φ^2	u, d	q	q	e	l		$D\phi$	$D\phi$
q	u, d										u, d		φ	φ			q	q	q
u	q										q	φ					u		u
d	q										q	φ					d		d
l	e										e					φ		l	l
e	l										l				φ				e
G												q	u	d			$G_{\mu\nu}$		
W	$D\Phi$										$D\phi$	q			l			$W_{\mu\nu}, \phi^2, \Phi^2$	
B	$D\Phi$										$D\phi$	q	u	d	l	e			ϕ^2, Φ^2

Matching SMEFT and the Weak (low-energy) EFT

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda}\mathcal{L}_5 + \frac{1}{\Lambda^2}\mathcal{L}_6 + \cdots$$

Weinberg operator
generates ν Majorana masses

dim.-6 operators
parametrize deviations from the SM
Buchmüller, Wyler '86; Grzadkowsi et al. '10



- We would like to connect BSM constraints on the SMEFT $\mathcal{L}_6$ with low-energy observations

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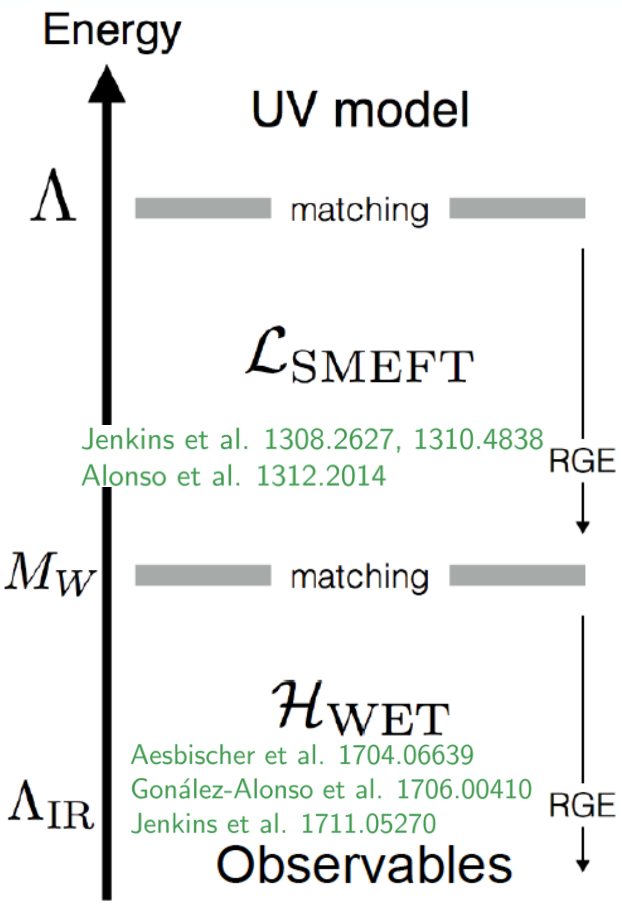

- We would like to connect BSM constraints on the SMEFT $\mathcal{L}_6$ with low-energy observations
- RGE take care of the (possibly large) logs. . .

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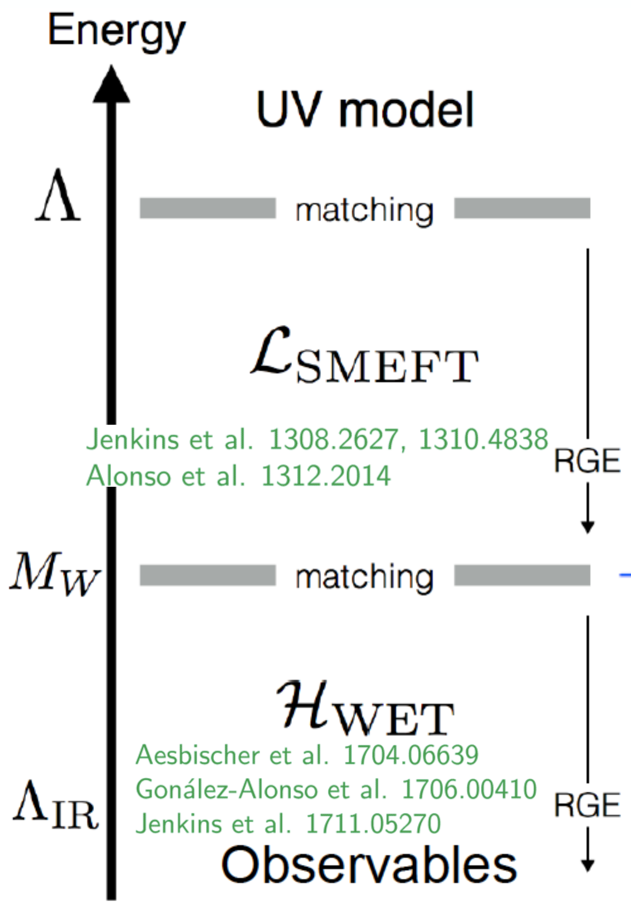

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$\longrightarrow$ known at tree-level Aesbischer et al. 1512.02830 Jenkins et al. 1709.04486

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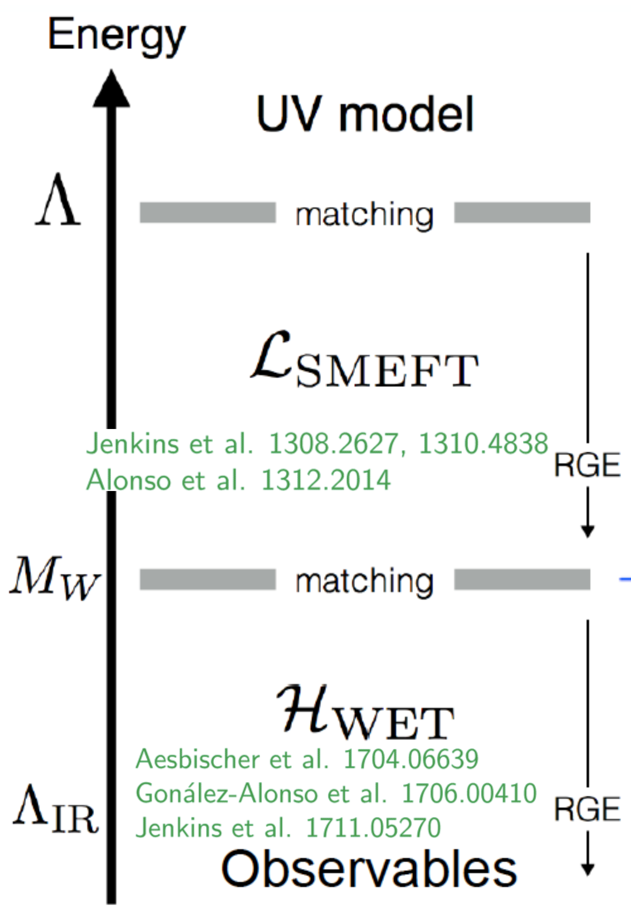

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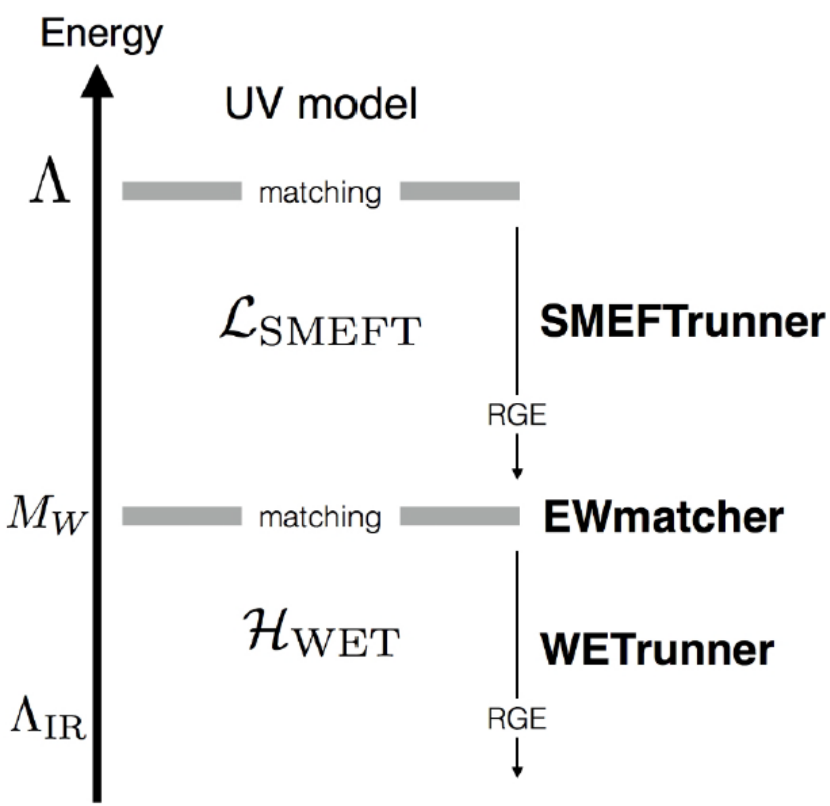
$\longrightarrow$ known at tree-level Aesbischer et al. 1512.02830 Jenkins et al. 1709.04486

- including one-loop pieces that are not log-enhanced could be important
$\longrightarrow$ **one-loop matching**
(w.i.p., non-trivial tree-level matching checked!)

Public code: DsixTools

A Mathematica package for the matching and RGE evolution from the new physics scale to the scale of low energy observables

Manual: arXiv:1704.04504
Website: https://dsixtools.github.io/



DsixTools

Celis, Fuentes-Martín, Vicente, Virto '17

Modular structure

Each module can be used independently

Summary

- We have provided a functional method to construct the EFT that results from integrating out the heavy part of the spectrum

- The method succesfully adresses some issues regarding the treatment of the terms that mix heavy and light quantum fluctuations, and simplifies the technical modus operandi:

  - ▷ only the hard component of the functional determinant is needed for the one-loop matching coeffs.

  - ▷ does not require subtractions of one-loop EFT contributions (as opposed to other methods proposed recently)

Outlook

- ▷ large amount of algebra involved in the computation of the functional trace, **automation** required for realistic models

- ▷ 1-loop matching between SMEFT and Weak Hamiltonian (work in progress)

Summary

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Thank you!

Evaluating the functional determinant

$$\Gamma_H = \frac{i}{2}\,\mathrm{tr}\int d^dx \int \frac{d^dp}{(2\pi)^d}\, \ln\left[-(\hat{D}-ip)^2 - m_H^2 - U\left(x,\partial_x + ip\right)\right]\mathbb{1}$$

This expression is **not** manifestly covariant (**open covariant derivatives**):

- If $U = U(x)$ (**momentum independent**) we can rewrite it in a manifestly covariant form

  Galliard 86', Cheyette 87'. Also used in Henning et al. JHEP 1601 (2016) 023

$$e^{i\hat{D}_\mu \partial_{p_\mu}} \ln\left[-\left(\hat{D}+ip\right)^2 - U(x)\right] e^{-i\hat{D}_\mu \partial_{p_\mu}} = \ln\left[-\left(\widetilde{G}_{\mu\nu}\,\partial_{p_\nu} + ip\right)^2 - \widetilde{U}\right]$$

$\widetilde{G}_{\mu\nu}$: Field-strength tensor

$\widetilde{U}$: Covariant derivatives only in commutators

- However U is **momentum dependent** when heavy-light loops are present and this "trick" is not useful

Long story short

In summary, the procedure goes as follows:

1. Collect all fields in a field multiplet, $\eta = (\eta_H \; \eta_L)^\intercal$. Charged fields and their conjugates should appear as separate components
2. Compute the fluctuation operator

$$\mathcal{O} = \left.\frac{\delta^2 \mathcal{L}}{\delta\eta^* \, \delta\eta}\right|_{\eta=\hat{\eta}} = \begin{pmatrix} \Delta_H & X^\dagger_{LH} \\ X_{LH} & \Delta_L \end{pmatrix}$$

3. Calculate $U = U_H + U_{LH}$ from $\mathcal{O}$

$$U_H = -\hat{D}^2 - m_H^2 - \Delta_H \qquad \text{(heavy loops)}$$

$$U_{LH} = X^\dagger_{LH} \Delta_L^{-1} X_{LH} \qquad \text{(heavy-light loops)}$$

   and expand $U(x, \partial_x + ip)$ to a given order in $\zeta \sim p, m_H$, e.g. $\mathcal{O}\left(\zeta^{-4}\right)$ for $d = 6$ operators

Long story short

In summary, the procedure goes as follows:

4. Insert $U(x, \partial_x + ip)$ in the general formula

$$\Gamma_H = -\frac{i}{2}\int d^dx \sum_{n=1}^{\infty}\frac{1}{n}\int\frac{d^dp}{(2\pi)^d}\,\text{tr}\left\{\left(\frac{2ip\hat{D}+\hat{D}^2+U\left(x,\partial_x+ip\right)}{p^2-m_H^2}\right)^n \mathbb{1}\right\}$$

and expand the integrand a given order in $\zeta \sim p, m_H$, e.g. $\mathcal{O}\left(\zeta^{-6}\right)$ for $d = 6$ operators. The computation of the integrals is straightforward

Disclaimer: Terms with open covariant derivatives should be kept and will combine into terms with field-strength tensors

Comparison with other approaches

Henning, Lu and Murayama approach

Henning, Lu, Murayama, arXiv:1604.01019

The authors implicitly perform the following shift

$$P = \begin{pmatrix} I & -\Delta_H^{-1} X_{LH}^\dagger \\ 0 & I \end{pmatrix} \Longrightarrow P^\dagger \mathcal{O} P = \begin{pmatrix} \Delta_H & 0 \\ 0 & \widetilde{\Delta}_L \end{pmatrix}$$

with $\widetilde{\Delta}_L = \Delta_L - \boldsymbol{X_{LH}\Delta_H^{-1}X_{LH}^\dagger}$. To obtain the heavy-light contributions from $\widetilde{\Delta}_L$ they have to subtract the contributions from the EFT

$$\frac{1}{-D^2 - M^2} = \left(\frac{1}{-D^2 - M^2}\right)_{\rm truncated} + \left(\frac{1}{-D^2 - M^2}\right)_{\rm rest}$$

Similar to Bilenki, Santamaria, Nucl. Phys. B420 (1994) 47

- ✗ Further diagonalizations are needed in the case of mixed statistics
- ✗ The truncation has to be defined for each order in the EFT. Intermediate steps are more involved
  (cancellations on non-analytic terms in the light masses must take place to get infrared-finite matching coeffs.)

Ellis, Quevillon, You and Zhang method

Ellis, Quevillon, You, Zhang, Phys. Lett. B 762 (2016) 166

The authors perform the functional integration of the full $\mathcal{O}$ (with no prior block-diagonalization) and subtract the contributions from the loops in the EFT in a similar fashion as Henning, Lu and Murayama

- ✓ Compact expression in terms of $U(x)$... but its validity is limited
- ✗ Further diagonalizations are needed in the case of mixed statistics (among heavy and/or light fields)
- ✗ The truncation has to be defined for each order in the EFT. Intermediate steps are more involved
- ✗ The method cannot be applied to cases where the heavy-light interactions contain derivatives (e.g. gauge interactions)

A recent paper [Zhang, arXiv:1610.00710] adopts our method and proposes a diagrammatic method for bookkeeping the algebra

2nd example: SM + heavy scalar triplet

Extension of the SM with and extra scalar sector Φ^a, $a = 1, 2, 3$ Gelmini, Roncadelli '81

$$\mathcal{L} = \mathcal{L}_{\mathsf{SM}} + \frac{1}{2} D_\mu \Phi^a D^\mu \Phi^a - \frac{1}{2} M^2 \Phi^a \Phi^a - \frac{\lambda_\Phi}{4} \left(\Phi^a \Phi^a\right)^2 + \kappa \left(\phi^\dagger \tau^a \phi\right) \Phi^a - \eta \left(\phi^\dagger \phi\right) \Phi^a \Phi^a$$

Tree level: $\mathcal{L}_{\mathsf{EFT}}^{\rm tree} = \mathcal{L}(\hat{\varphi}_{\rm SM}, \hat{\Phi})$

- We choose $\kappa \sim M$

EOM: $\hat{\Phi}^a = \frac{\kappa}{M^2} \left(\hat{\phi}^\dagger \tau^a \hat{\phi}\right) - \frac{\kappa}{M^4} \left[\hat{D}^2 + 2\eta \left(\hat{\phi}^\dagger \hat{\phi}\right)\right] \left(\hat{\phi}^\dagger \tau^a \hat{\phi}\right) + \mathcal{O}\left(\frac{\kappa}{M^6}\right)$

One loop: $\mathcal{L}_{\mathsf{EFT}}^{\rm 1loop}$

- We need to fix the gauge of the quantum gauge fields

BGF: $\mathcal{L}_{\mathsf{GF}} = -\frac{1}{2\xi_W} \left(\hat{D}_\mu W^{\mu\,a}\right)^2$

SM + heavy scalar triplet

$$\mathcal{L} = \mathcal{L}_{\mathsf{SM}} + \frac{1}{2} D_\mu \Phi^a D^\mu \Phi^a - \frac{1}{2} M^2 \Phi^a \Phi^a - \frac{\lambda_\Phi}{4} (\Phi^a \Phi^a)^2 + \kappa \left(\phi^\dagger \tau^a \phi\right) \Phi^a - \eta \left(\phi^\dagger \phi\right) \Phi^a \Phi^a$$

1. Collect the fields in a multiplet: $\eta = \begin{pmatrix} \Phi^a \\ \phi \\ \phi^* \\ W_\mu^b \end{pmatrix}$

2. Compute the fluctuation operator:

$$\mathcal{O} = \left.\frac{\delta^2 \mathcal{L}}{\delta \eta^* \, \delta \eta}\right|_{\eta = \hat{\eta}} \Longrightarrow \begin{cases} \Delta_H = \Delta_{\Phi\Phi}^{ab} \\ \Delta_L = \begin{pmatrix} \Delta_{\phi^*\phi} & X_{\phi\phi}^\dagger & \left(X_{W\phi}^{\nu\, d}\right)^\dagger \\ X_{\phi\phi} & \Delta_{\phi^*\phi}^\intercal & \left(X_{W\phi}^{\nu\, d}\right)^\intercal \\ X_{W\phi}^{\mu\, c} & \left(X_{W\phi}^{\mu\, c}\right)^* & \Delta_W^{\mu\nu\, cd} \end{pmatrix} \\ X_{LH}^\dagger = \left(\left(X_{\phi^*\Phi}^a\right)^\dagger \; \left(X_{\phi^*\Phi}^a\right)^\intercal \; \left(X_{W\Phi}^{\nu\, da}\right)^\intercal \right) \end{cases}$$

SM + heavy scalar triplet

3. Calculate $U = U_H + U_{LH}$ and compute $U(x, \partial_x + ip)$ up to ζ^{-4}

$$U_H = -\hat{D}^2 - M^2 - \Delta_H$$
$$U_{LH} = X^\dagger_{LH} \Delta_L^{-1} X_{LH}$$

- The computation is very lengthy but simple, mostly algebraic
- Using the Landau gauge (i.e. $\xi_W = 0$) simplifies the expressions, since the gauge propagator becomes transverse

4. Insert $U(x, \partial_x + ip)$ in the general formula

$$\Gamma_H = -\frac{i}{2} \int d^d x \sum_{n=1}^{\infty} \frac{1}{n} \int \frac{d^d p}{(2\pi)^d} \,\mathrm{tr} \left\{ \left(\frac{2ip\hat{D} + \hat{D}^2 + U\,(x, \partial_x + ip)}{p^2 - m_H^2} \right)^n \mathbb{1} \right\}$$

after a bit of algebra... for example, dim. 6 Higgs ops. proportional to g^2:

$$\mathcal{L}_{\rm EFT}^{\rm 1loop}\Big|_W = \frac{1}{16\pi^2} \frac{g^2 \kappa^2}{M^4} \left[-\frac{25}{16} \left(\hat{\phi}^\dagger \hat{\phi}\right) \partial^2 \left(\hat{\phi}^\dagger \hat{\phi}\right) + \frac{5}{4} \left[\left(\hat{\phi}^\dagger \hat{\phi}\right) \left(\hat{\phi}^\dagger \hat{D}^2 \hat{\phi}\right) + h.c. \right] - \frac{5}{4} \left| \hat{\phi}^\dagger \hat{D}_\mu \hat{\phi} \right|^2 \right]$$