

Sudakov log resummation for indirect detection of heavy WIMP dark matter

Martin Vollmann

Technische Universität München (TUM)



Based on
Beneke, Broggio, Hasner, MV
arXiv:[1805.07367](https://arxiv.org/abs/1805.07367)

Outline

- Motivation
- The wino model as a concrete example
- Soft-collinear effective field theory (SCET) approach
- Factorization formula
- Results

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Motivation I

Precise determination
of σv



Learn about / “measure” J

Screenshot of previous talk

$$\begin{aligned}\frac{d\Phi}{dE} &= \frac{1}{4\pi} \frac{dN}{dE} \int_{\Delta\Omega} d\Omega \int d\ell \\ &\quad \int d^3v_1 \frac{f(\vec{r}(\ell, \Omega), \vec{v}_1)}{m_x} \int d^3v_2 \frac{f(\vec{r}(\ell, \Omega), \vec{v}_2)}{m_x} \\ &\quad \times \frac{\sigma_A |\vec{v}_1 - \vec{v}_2|}{2} \\ &= \frac{\langle \sigma_A v \rangle}{8\pi m_x^2} \frac{dN}{dE} \times J \\ J &\equiv \int_{\Delta\Omega} d\Omega \int d\ell [f(\vec{r}(\ell, \Omega))]^2 \\ f(\vec{r}) &= \int d^3v f(\vec{r}, \vec{v})\end{aligned}$$

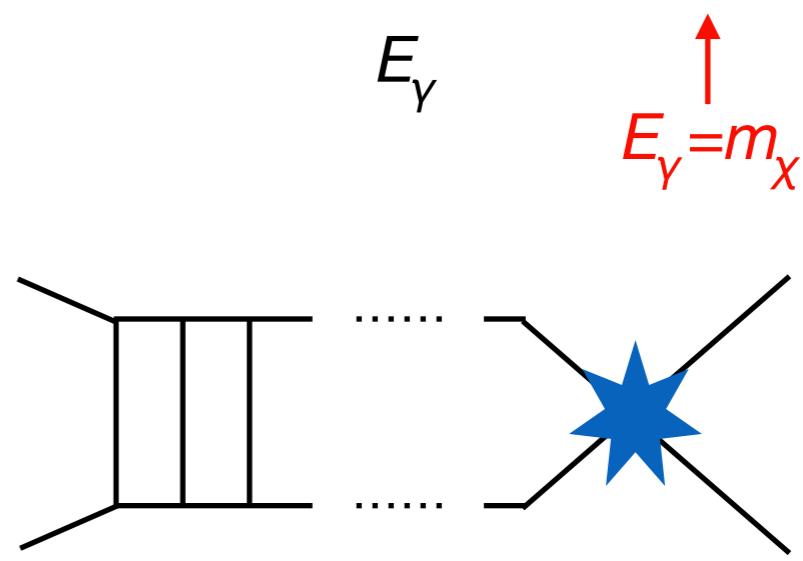
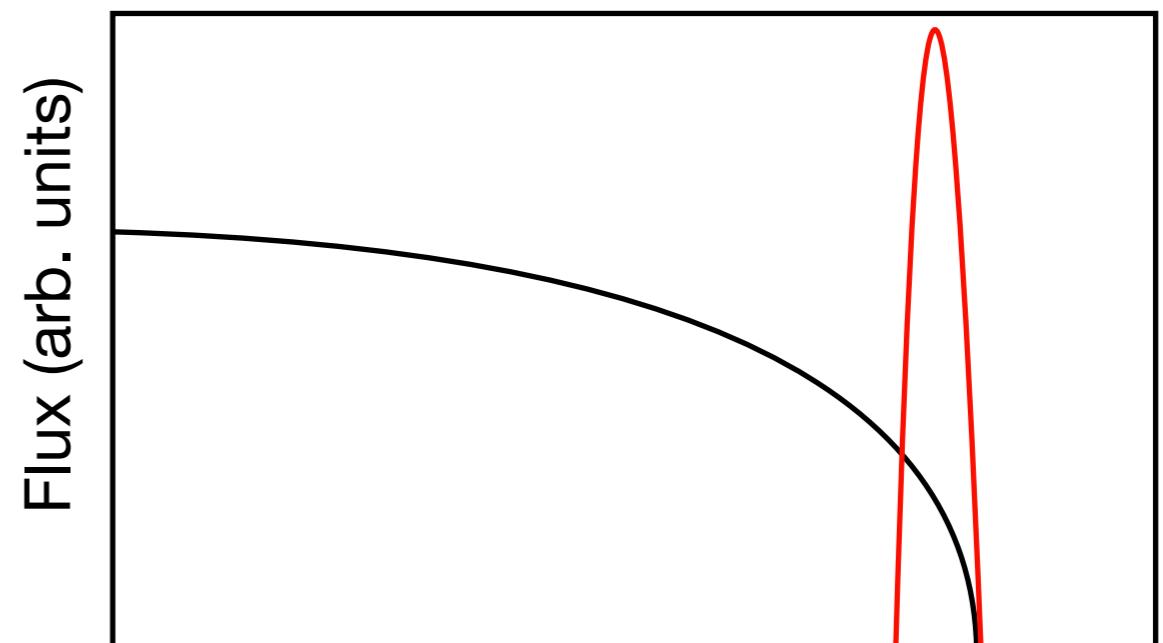
$f =$ dark matter velocity distribution

Motivation. Spectral γ -ray lines

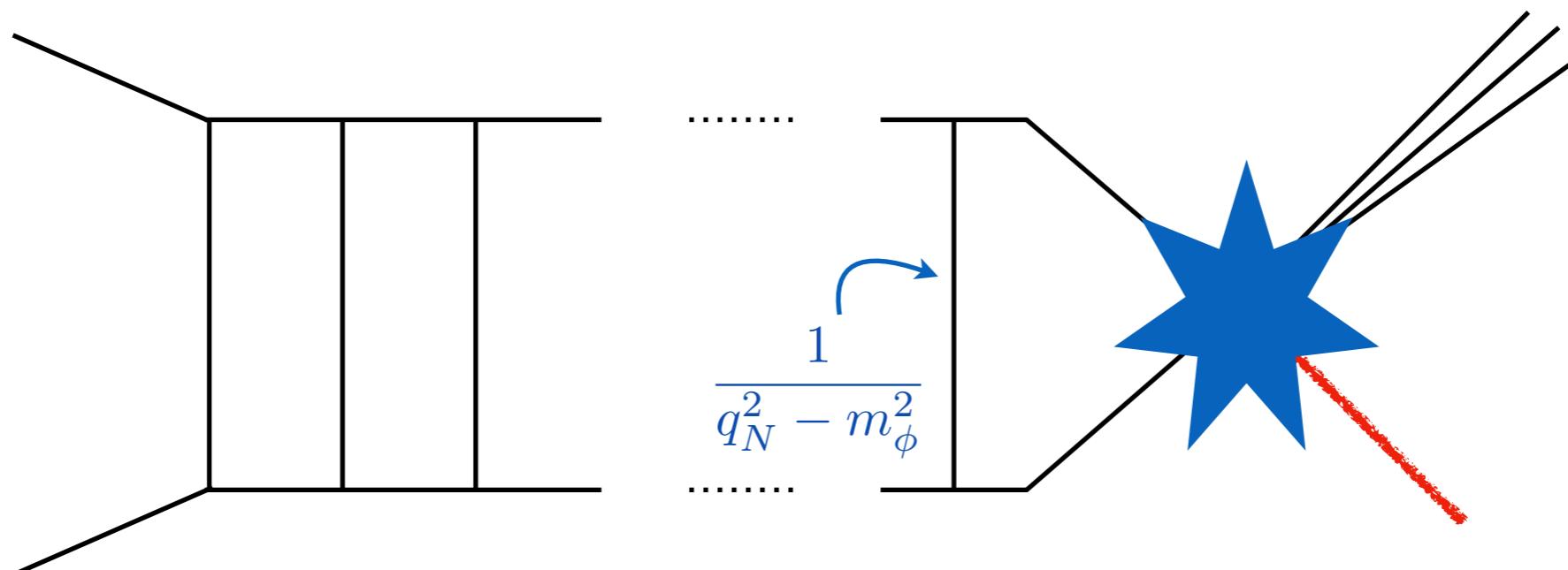
- Spectral-line feature in gamma rays → very promising prediction for indirect DM detection
- In some models the observationally limiting loop-suppression of $(\sigma v)_{\text{LINE}}$ can be lifted through non-perturbative effects



Sommerfeld enhancement



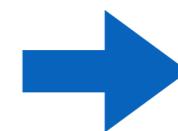
Motivation. Sommerfeld effect for exclusive annihilation



Two facts, one problem

• $m_\phi \ll m_\chi$ is generically required

• final state is *exclusive*



expect large
Sudakov logarithms!!

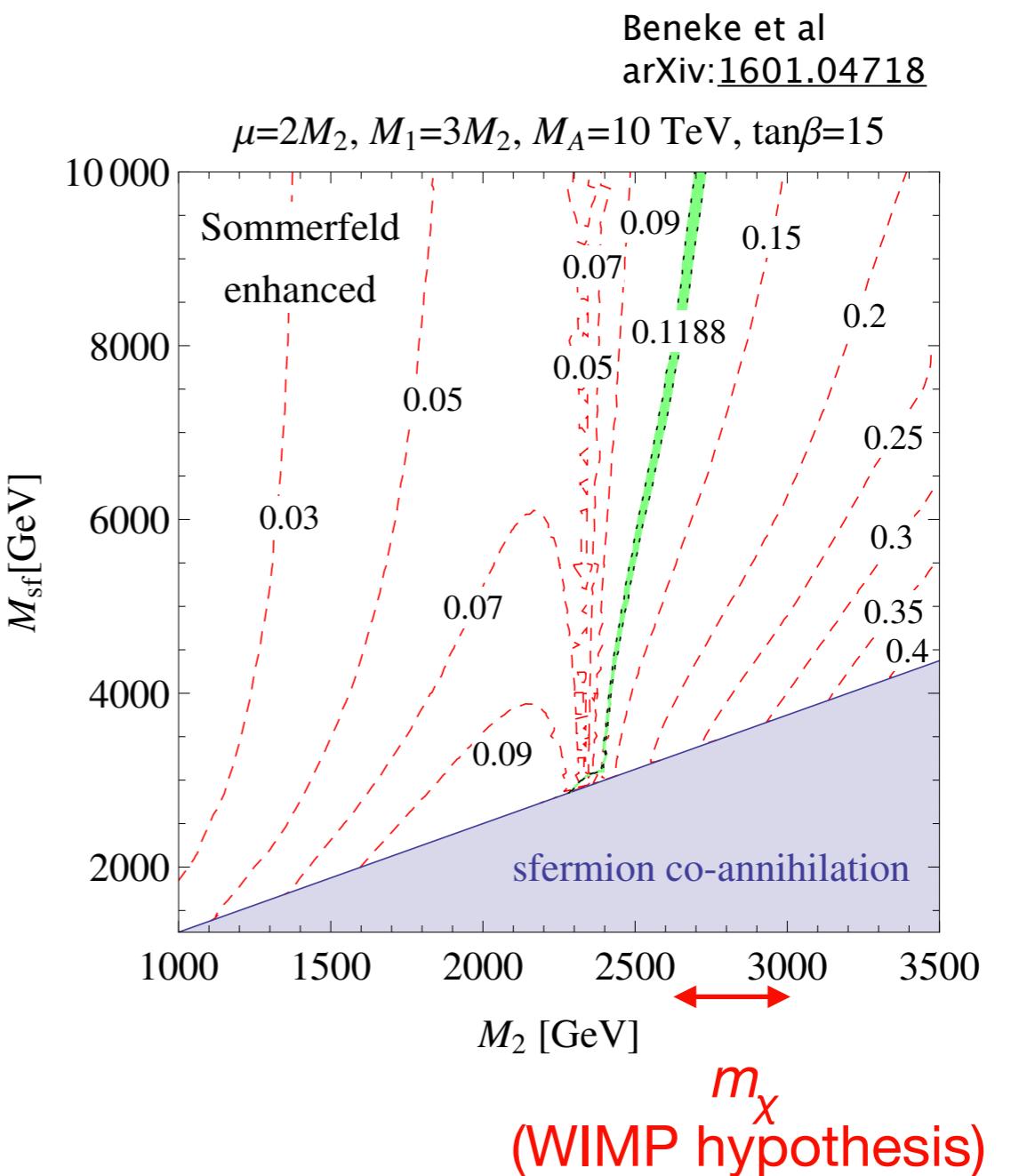
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The wino-like/MDM triplet model

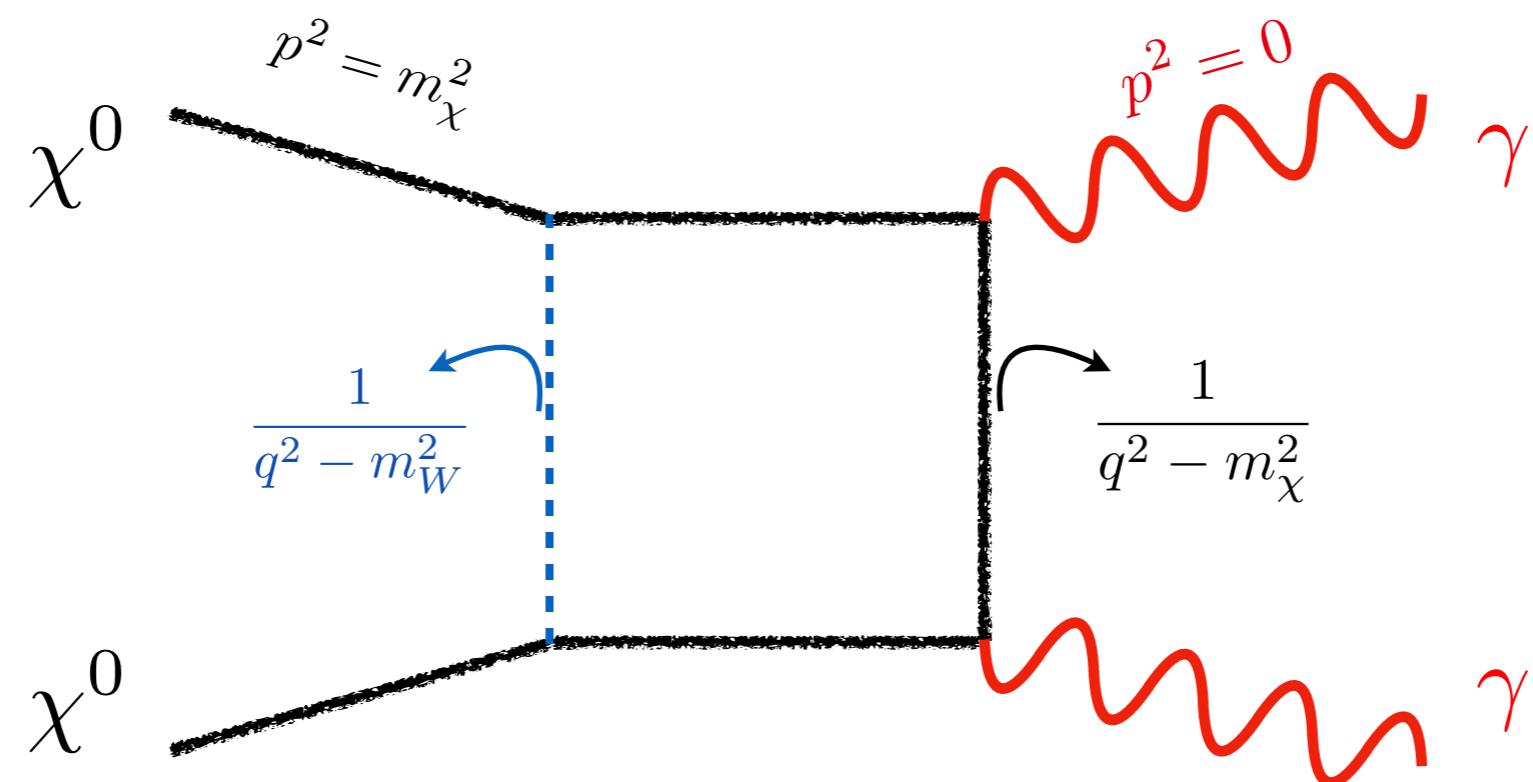
$$\delta\mathcal{L}_{\text{Wino}} = \frac{1}{2}\bar{\chi}(i\gamma^\mu D_\mu - m_\chi)\chi$$

- After EWSB get two almost-degenerate mass eigenstates χ^0 (the DM) and χ^+
- Limiting case of the SUSY neutralino ($M_{\text{sf}} > 20M_2$ in the plot)
- *Direct Detection*: below neutrino floor
- *LHC*: much too heavy to be observed



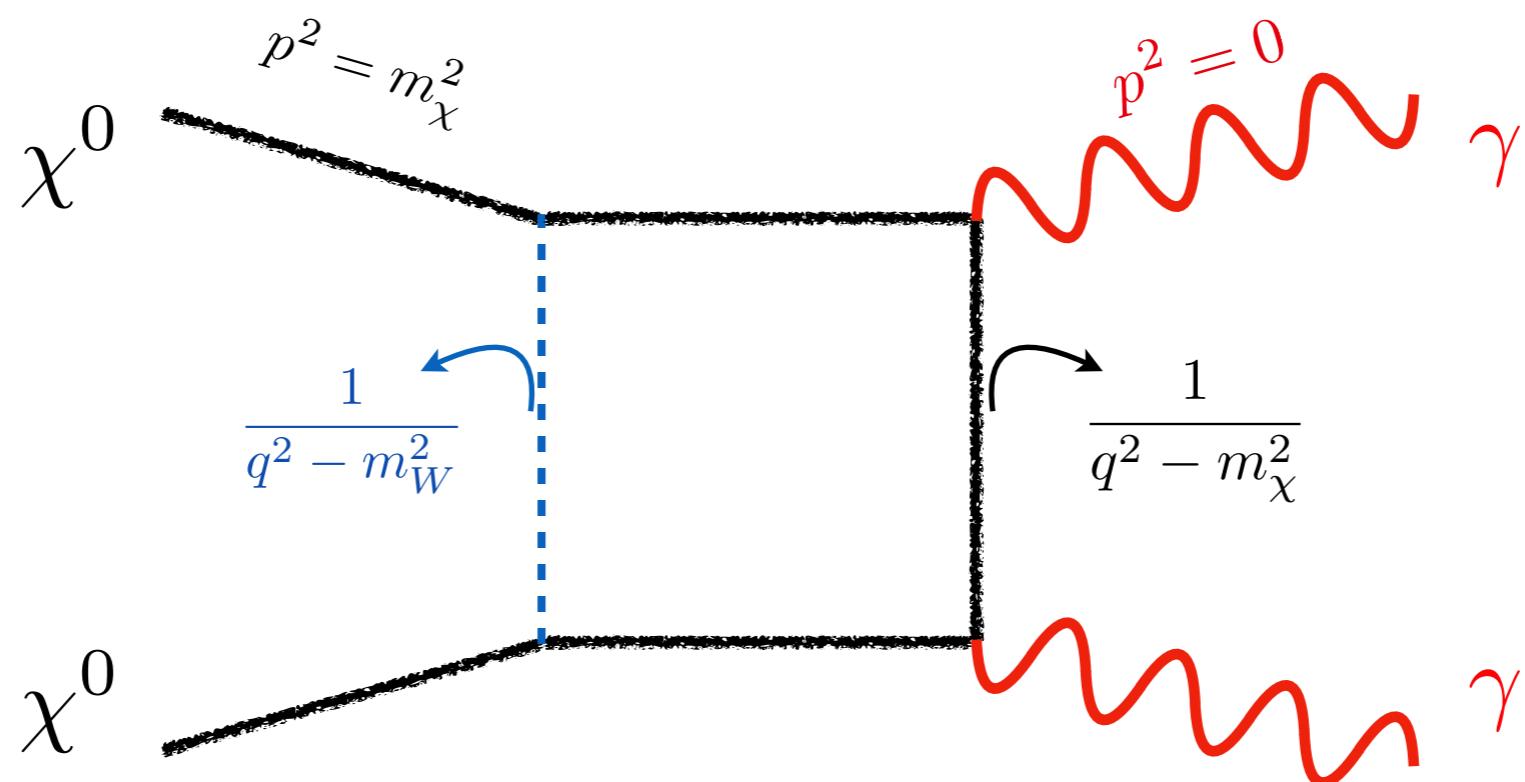
Fixed-order NLO calculation for

$$\sigma V_{\gamma\gamma}$$



Fixed-order NLO calculation for

$$\sigma V_{\gamma\gamma}$$

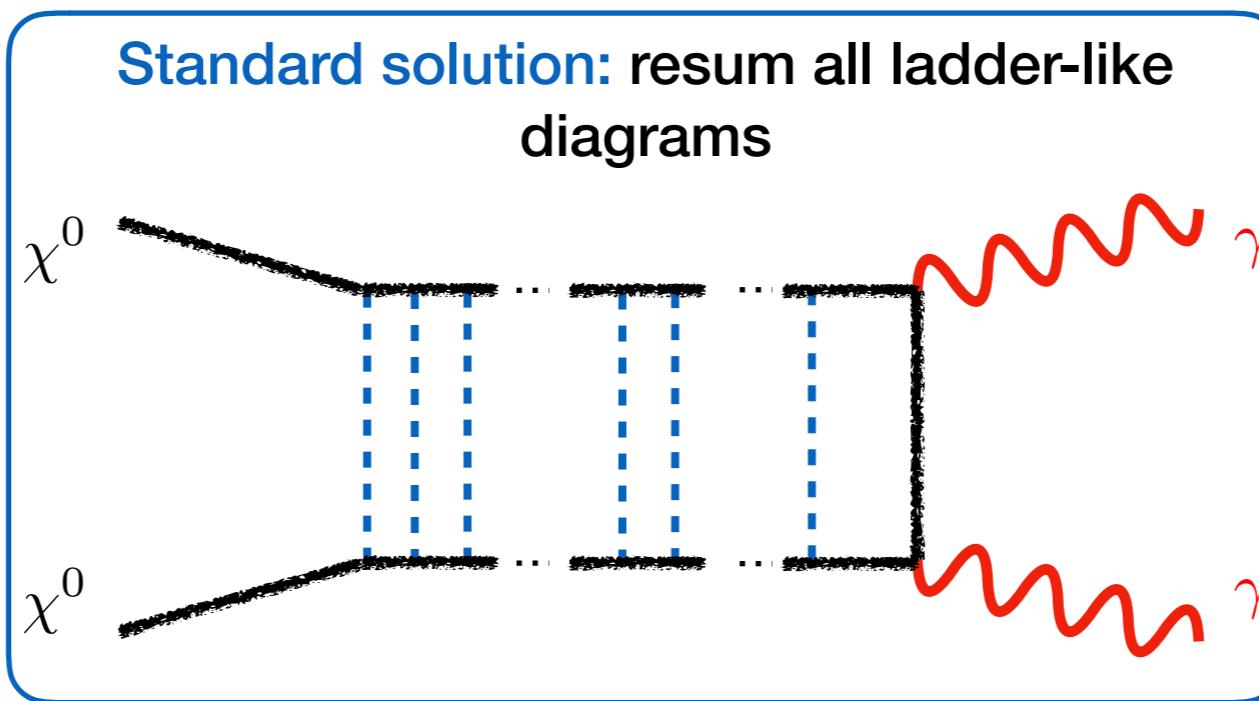


Sommerfeld

$$\mathcal{M}_{\text{So}} \sim \frac{g^4 m_\chi^2}{m_W^2} \gg g^2$$

Fixed-order NLO calculation for

$$\sigma V_{\gamma\gamma}$$

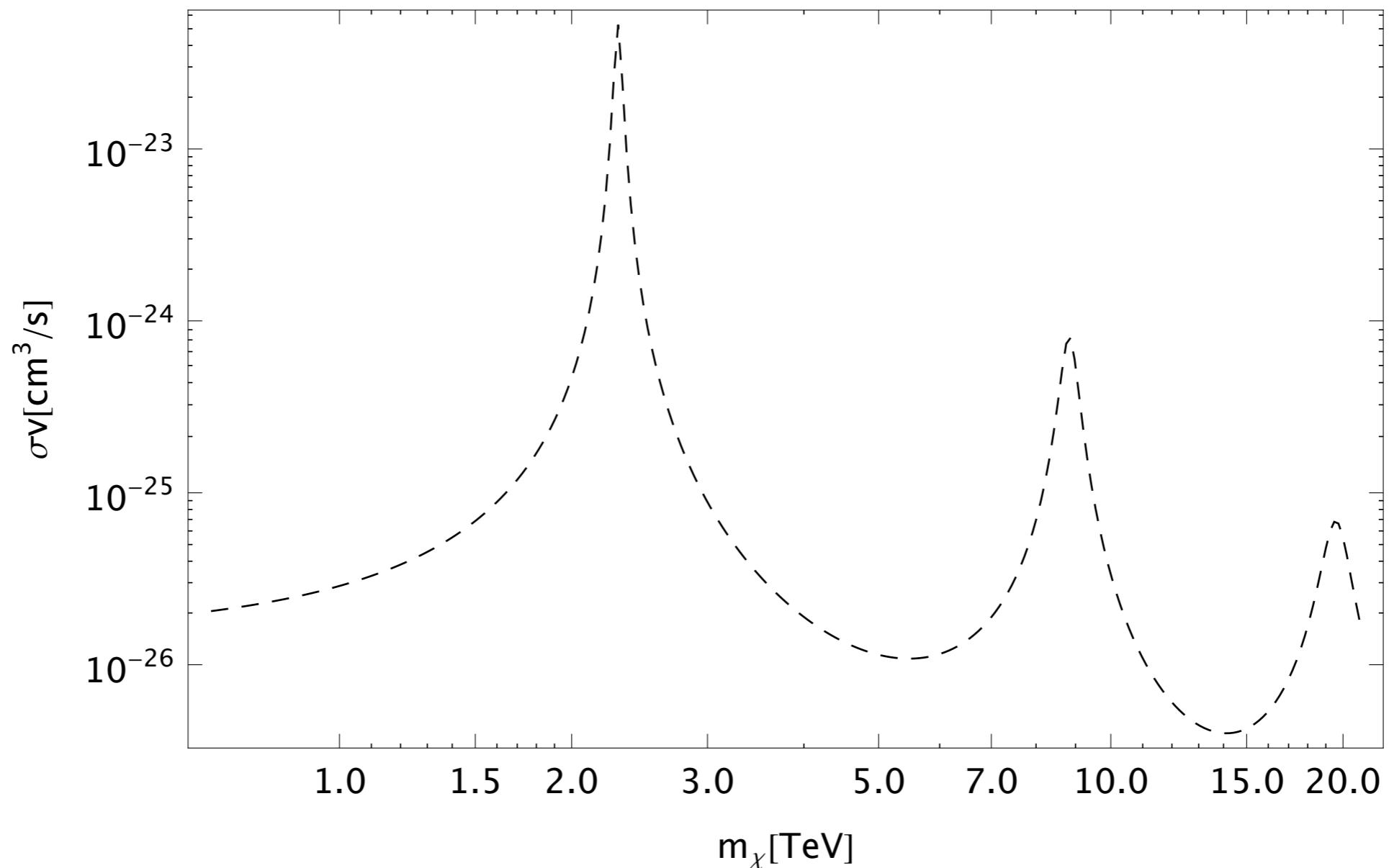


NREFT: Solve Schrödinger eq. with



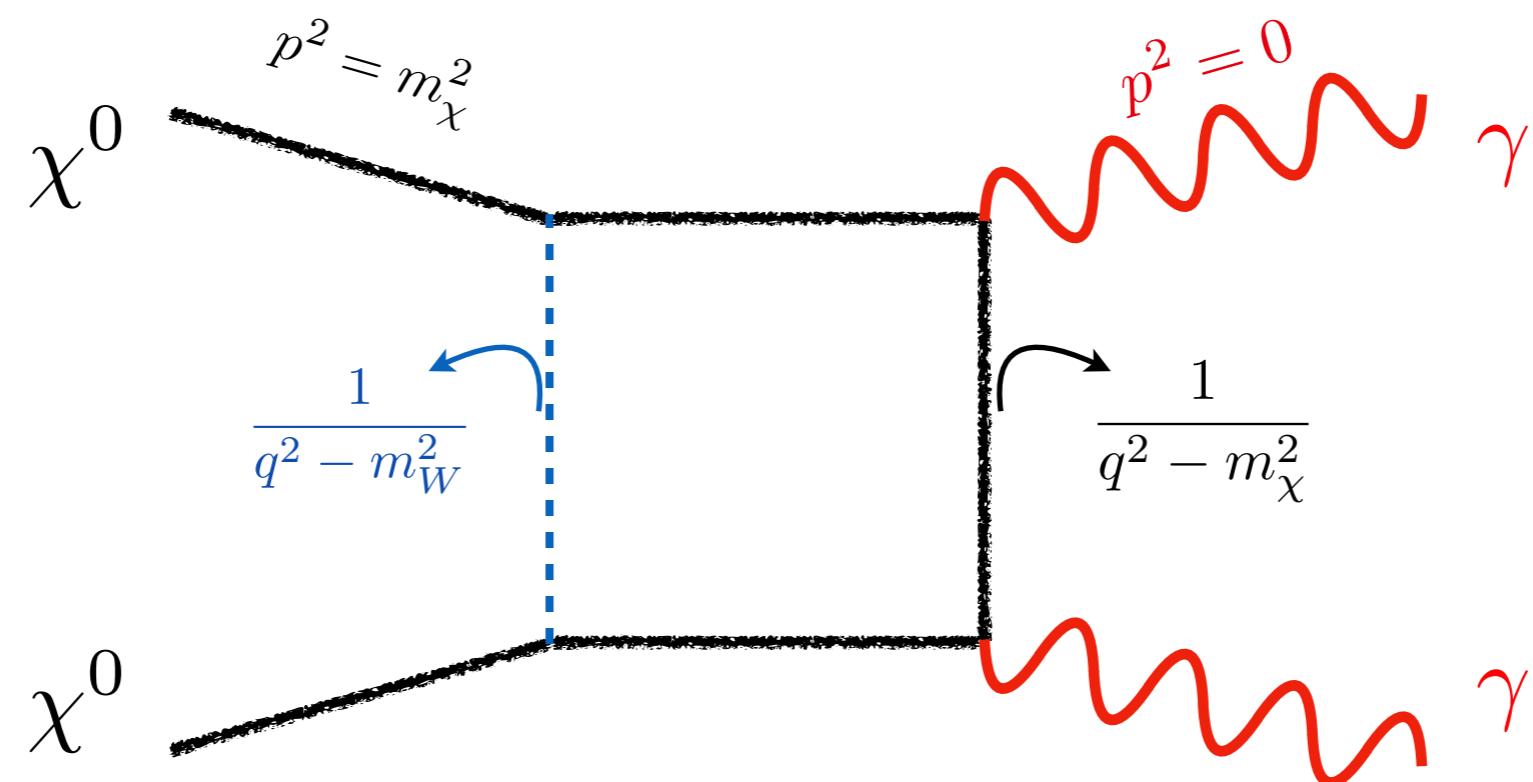
$$V(r) = \begin{pmatrix} 0 & -\sqrt{2}\alpha_2 \frac{e^{-m_W r}}{r} \\ -\sqrt{2}\alpha_2 \frac{e^{-m_W r}}{r} & -\frac{\alpha}{r} - \alpha_2 c_W^2 \frac{e^{-m_Z r}}{r} \end{pmatrix}$$

Sommerfeld enhancement



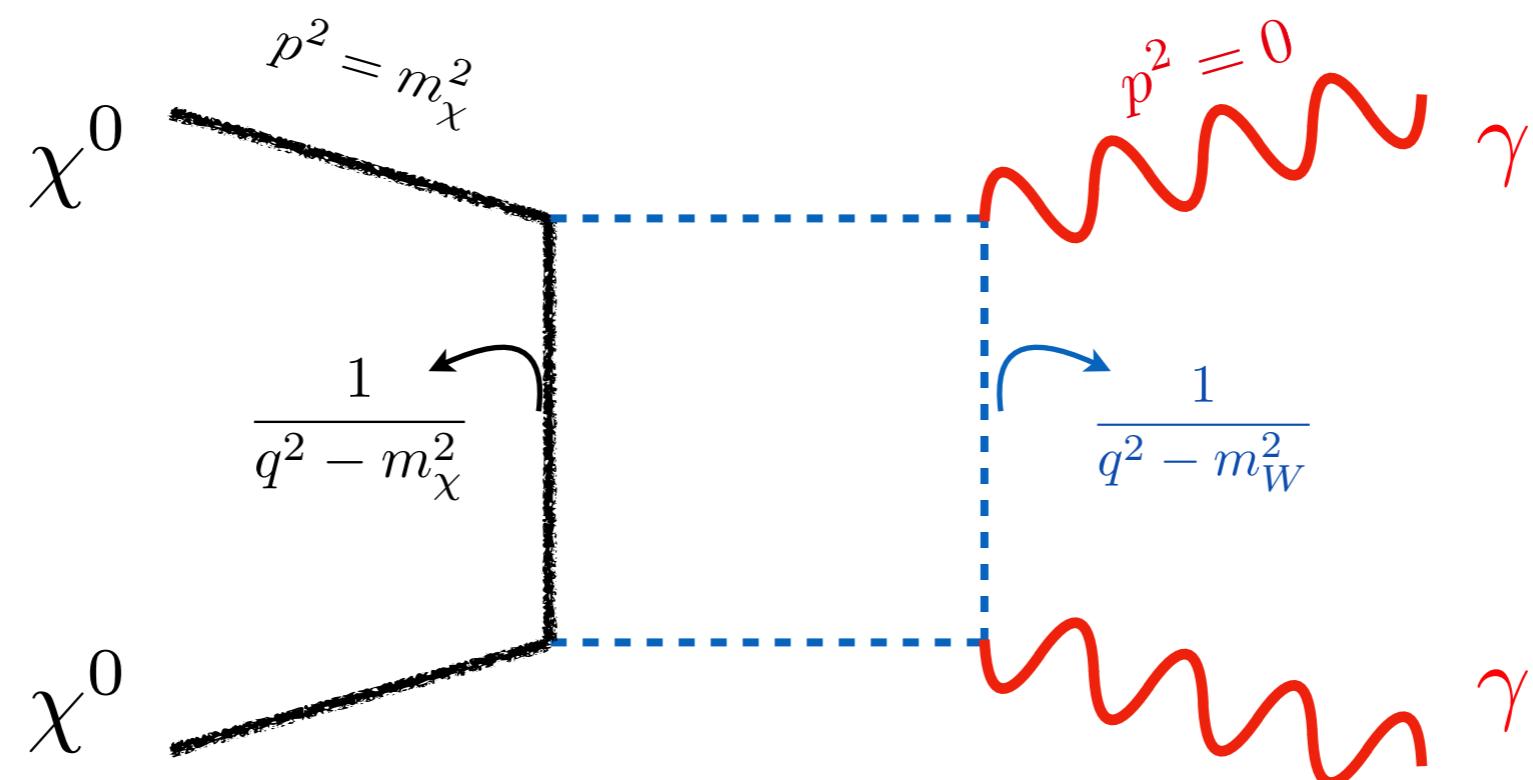
Fixed-order NLO calculation for

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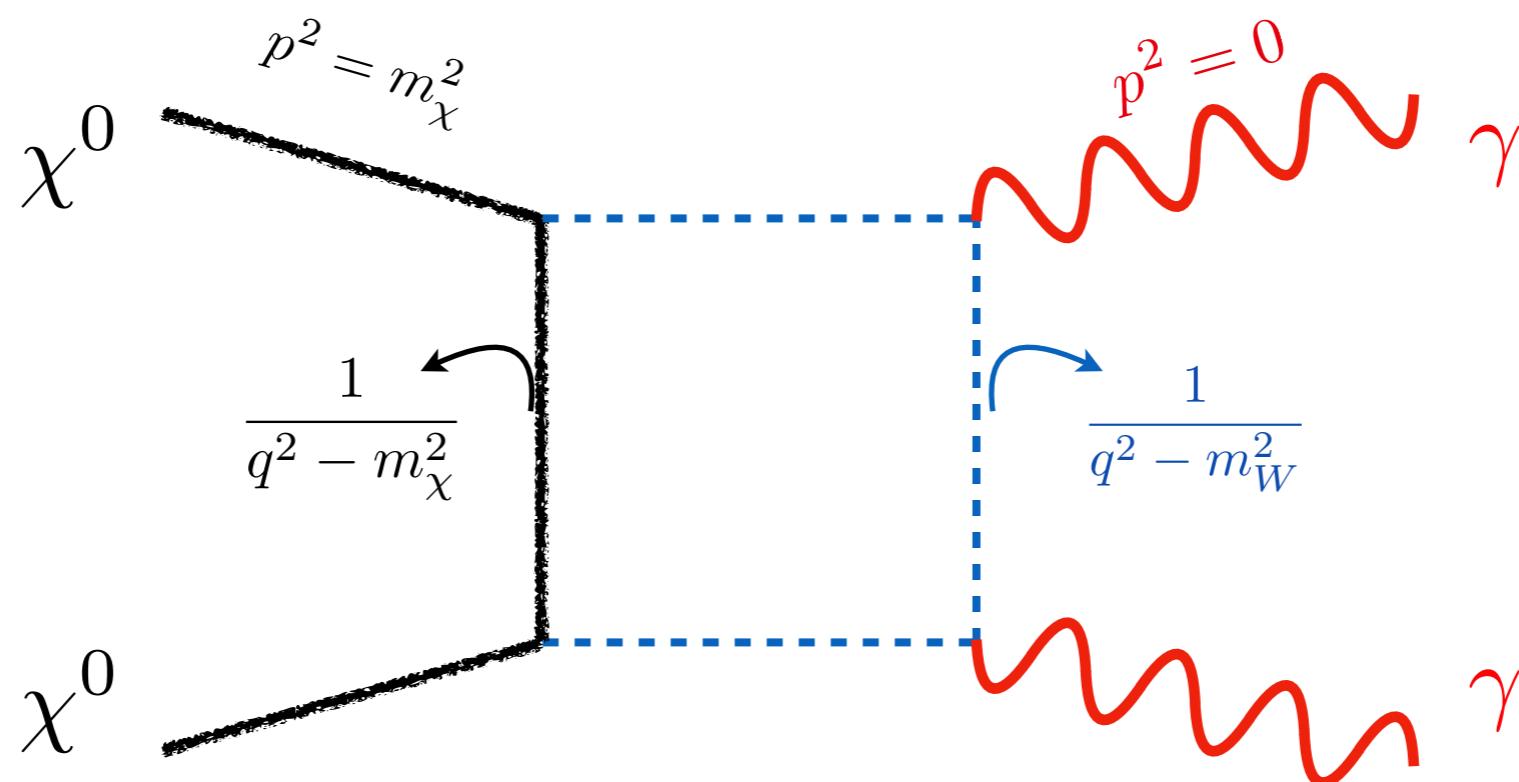
Fixed-order NLO calculation for

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Fixed-order NLO calculation for

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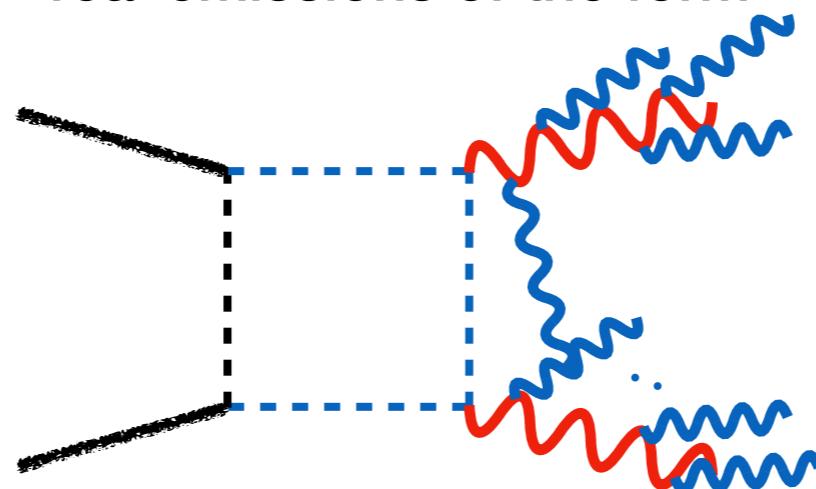


$$\text{Sudakov } \mathcal{M}_{\text{Su}} \sim g^4 \log^2 \frac{m_\chi^2}{m_W^2} \gg g^2$$

Fixed-order NLO calculation for

$$\sigma v_{\gamma\gamma}$$

Standard solution: resum soft virtual and real emissions of the form



Renormalization group eqs.
in a (soft collinear) effective theory

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Soft-collinear effective theory (SCET)

$$I_{\text{full}} =$$

A Feynman diagram consisting of a central square loop. Two horizontal lines enter from the left, and two horizontal lines exit to the right. Two diagonal lines enter from the bottom-left and bottom-right, and two diagonal lines exit to the top-left and top-right. The lines are represented by black strokes.

$$p_0 = m_\chi v \quad k^2 = m_W^2$$
$$p_0 = m_\chi v \quad k'^2 = m_W^2$$

SCET. Momentum regions

Expand the loop integral

$$I_{\text{full}} = \text{Diagram} = \int \frac{d^D q}{(2\pi)^D} \frac{1}{q^2 - m_\chi^2} \frac{1}{(q + p_0)^2 (q + p_0 - k)^2 (q - p_0)^2} \Big|_{k^2 \ll m_\chi^2}$$

as the sum

$$\text{Diagram}_h + \text{Diagram}_{s'} + \text{Diagram}_{hc} + \text{Diagram}_{\bar{h}\bar{c}} \Big|_{k^2 = 0}$$

where the variable momentum is expanded (at leading order in the m_W/m_χ expansion)

$$q_h \sim m_\chi (1, 1, 1) \quad q_s \sim m_W (1, 1, 1)$$

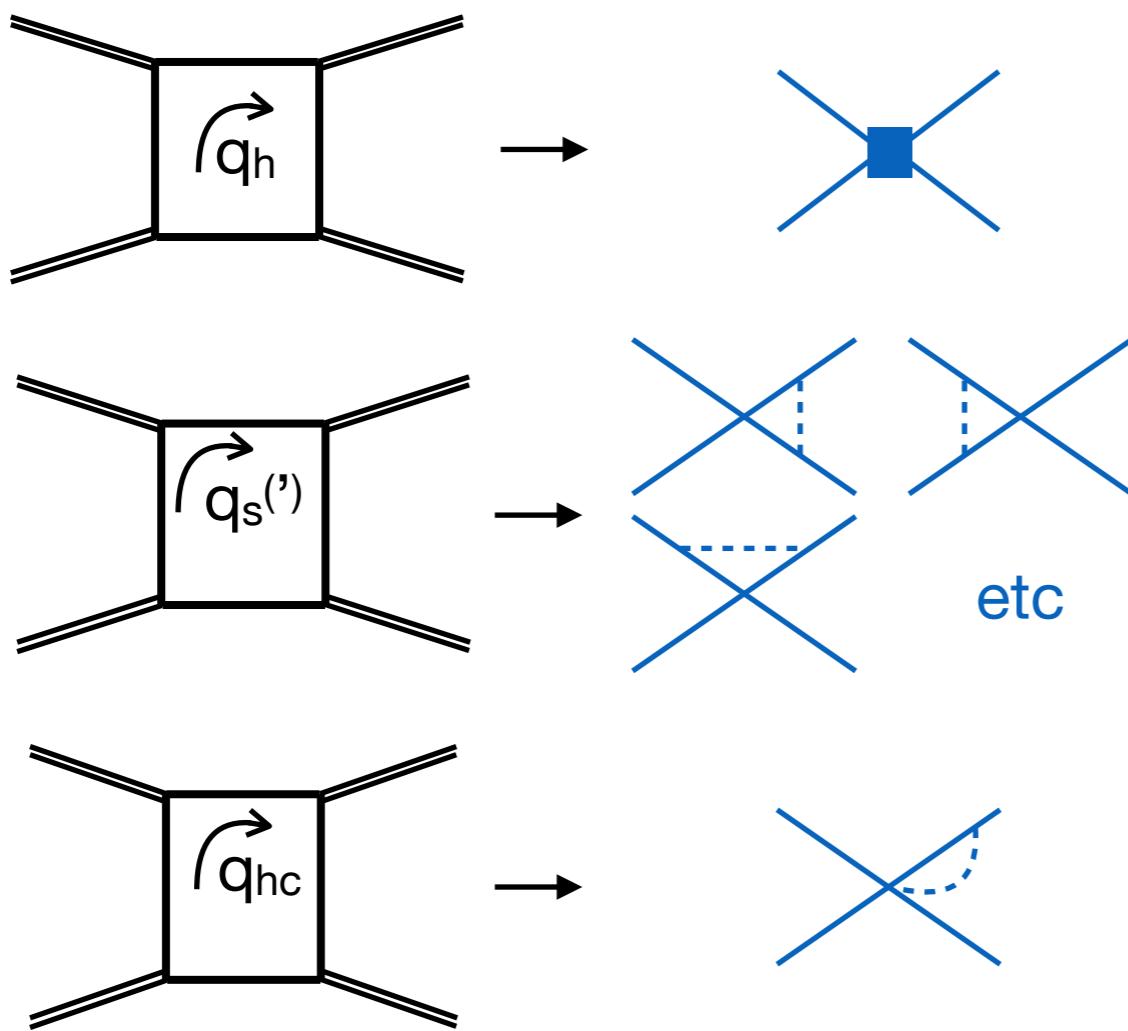
$$q_{hc} \sim (m_W, m_\chi, \sqrt{m_\chi m_W}) \quad q_{\bar{h}\bar{c}} \sim (m_\chi, m_W, \sqrt{m_\chi m_W})$$

Light-cone
coordinates

$$q = q_c n + q_{\bar{c}} \bar{n} + q_\perp \rightarrow (q_c, q_{\bar{c}}, q_\perp)$$

SCET. Roadmap to factorization

Interpret each expansion as a Feynman diagram of the SCET



Factorization

Wilson
coefficients

only depend on m_x

Soft
functions

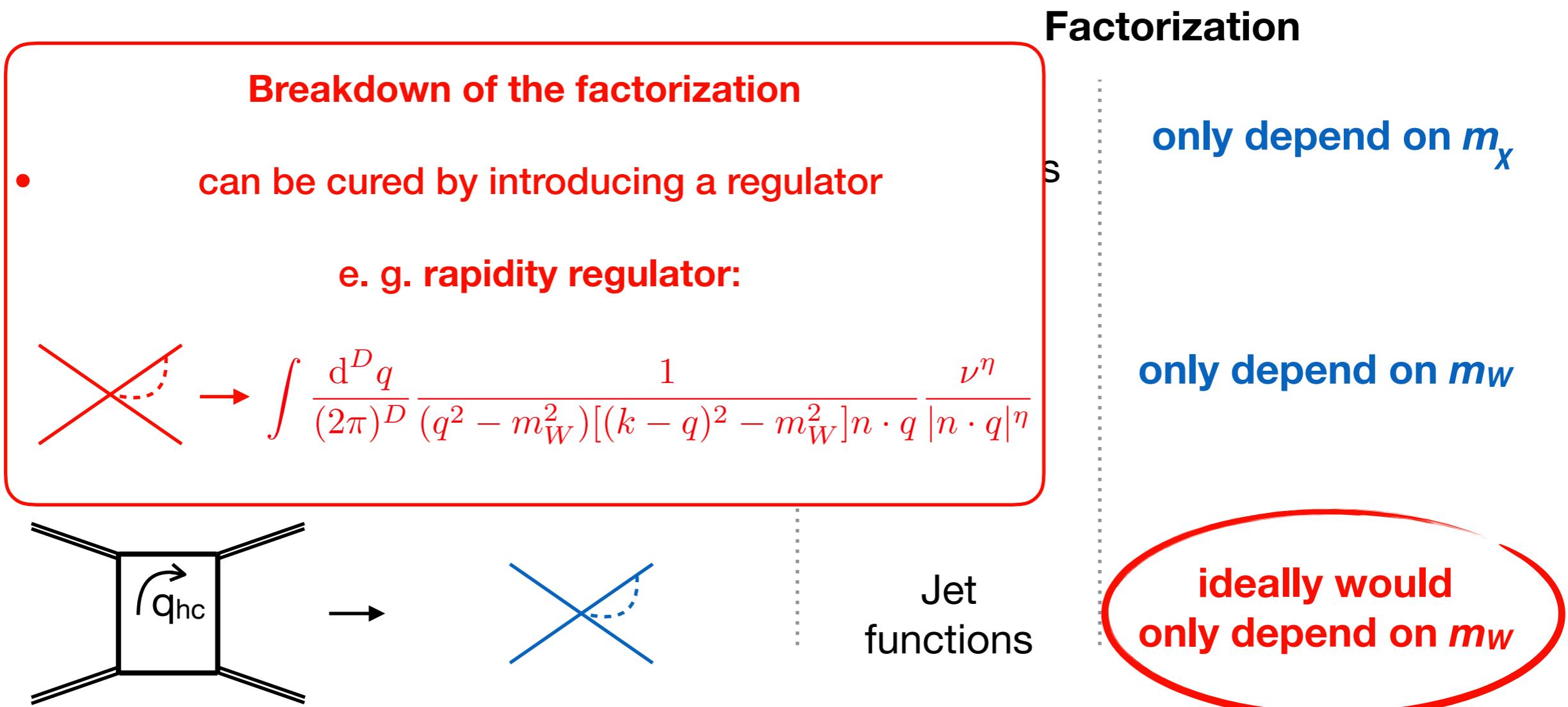
only depend on m_w

Jet
functions

**ideally would
only depend on m_w**

SCET. Roadmap to factorization

Interpret each expansion as a Feynman diagram of the SCET



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Factorization theorem. Exclusive Wino $\chi\chi \rightarrow \gamma + X$ annihilation

$$\frac{d(\sigma v_{\text{rel}})}{dE_\gamma} = \sum_{I,J} S_{IJ} \Gamma_{IJ}(E_\gamma)$$

Sommerfeld matrix
 $I, J = (\chi^0\chi^0) \text{ or } (\chi^+\chi^-)$

Factorization theorem. Exclusive Wino $\chi\chi \rightarrow \gamma + X$ annihilation

$$\frac{d(\sigma v_{\text{rel}})}{dE_\gamma} = \sum_{I,J} S_{IJ} \Gamma_{IJ}(E_\gamma)$$

$$\begin{aligned} \Gamma_{IJ}(E_\gamma) &= \frac{1}{4} \frac{2}{\pi m_\chi} \sum_{i,j=1,2} \sum_{V,W,X,Y} C_j^*(\mu_W) C_i(\mu_W) D_{J,XY}^{j*}(\mu_W, \nu_s) D_{I,VW}^i(\mu_W, \nu_s) \\ &\quad \times V(\mu_W, \nu_s, \nu_j) Z_\gamma^{YW} \int d\omega J^{XV}(4m_\chi(m_\chi - E_\gamma - \omega), \mu_W, \nu_j) S_\gamma(\omega) \end{aligned}$$

Factorization theorem. Exclusive Wino $\chi\chi \rightarrow \gamma + X$ annihilation

$$\frac{d(\sigma v_{\text{rel}})}{dE_\gamma} = \sum_{I,J} S_{IJ} \Gamma_{IJ}(E_\gamma)$$

Wilson coefficients
(after RGE resummation)

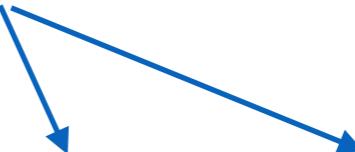
$$\begin{aligned} \Gamma_{IJ}(E_\gamma) &= \frac{1}{4} \frac{2}{\pi m_\chi} \sum_{i,j=1,2} \sum_{V,W,X,Y} C_j^*(\mu_W) C_i(\mu_W) D_{J,XY}^{j*}(\mu_W, \nu_s) D_{I,VW}^i(\mu_W, \nu_s) \\ &\times V(\mu_W, \nu_s, \nu_j) Z_\gamma^{YW} \int d\omega J^{XV}(4m_\chi(m_\chi - E_\gamma - \omega), \mu_W, \nu_j) S_\gamma(\omega) \end{aligned}$$

Factorization theorem. Exclusive Wino $\chi\chi \rightarrow \gamma + X$ annihilation

$$\frac{d(\sigma v_{\text{rel}})}{dE_\gamma} = \sum_{I,J} S_{IJ} \Gamma_{IJ}(E_\gamma)$$

Soft functions
(also require rapidity regulator)

$$\begin{aligned} \Gamma_{IJ}(E_\gamma) &= \frac{1}{4} \frac{2}{\pi m_\chi} \sum_{i,j=1,2} \sum_{V,W,X,Y} C_j^*(\mu_W) C_i(\mu_W) D_{J,XY}^{j*}(\mu_W, \nu_s) D_{I,VW}^i(\mu_W, \nu_s) \\ &\times V(\mu_W, \nu_s, \nu_j) Z_\gamma^{YW} \int d\omega J^{XV}(4m_\chi(m_\chi - E_\gamma - \omega), \mu_W, \nu_j) S_\gamma(\omega) \end{aligned}$$



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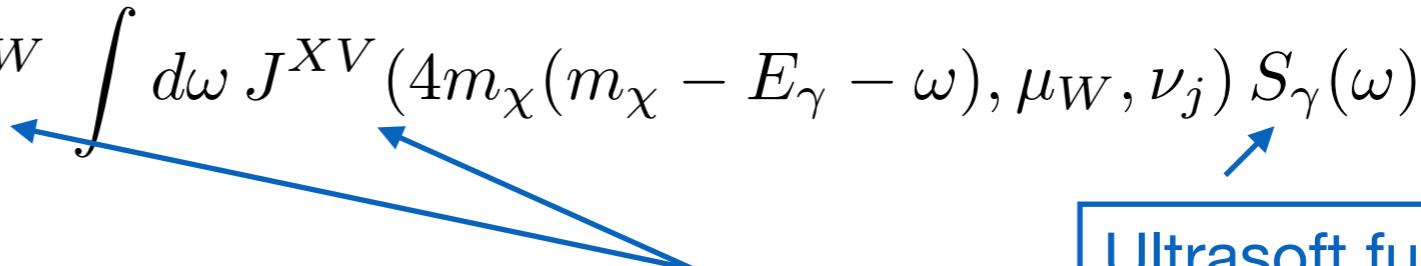
$$\times V(\mu_W, \nu_s, \nu_j) Z_\gamma^{YW} \int d\omega J^{XV}(4m_\chi(m_\chi - E_\gamma - \omega), \mu_W, \nu_j) S_\gamma(\omega)$$

Rapidity regulator RGE matrix
(resums those logs that break factorization)

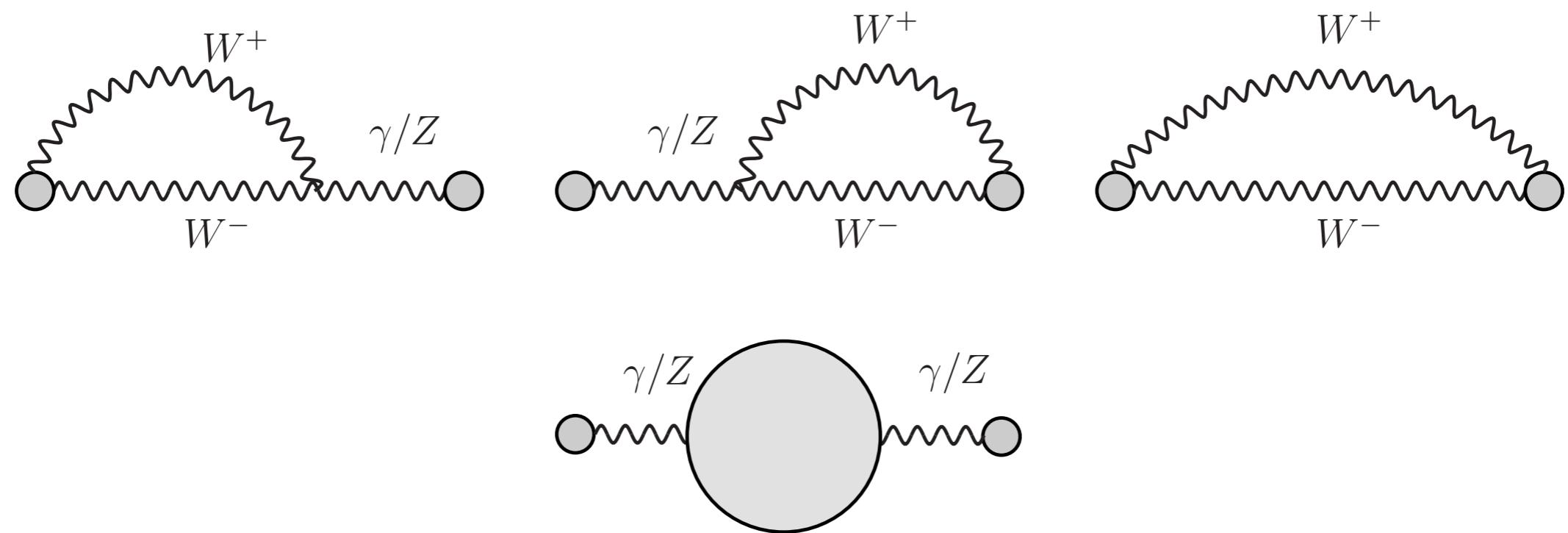
Factorization theorem. Exclusive Wino $\chi\chi \rightarrow \gamma + X$ annihilation

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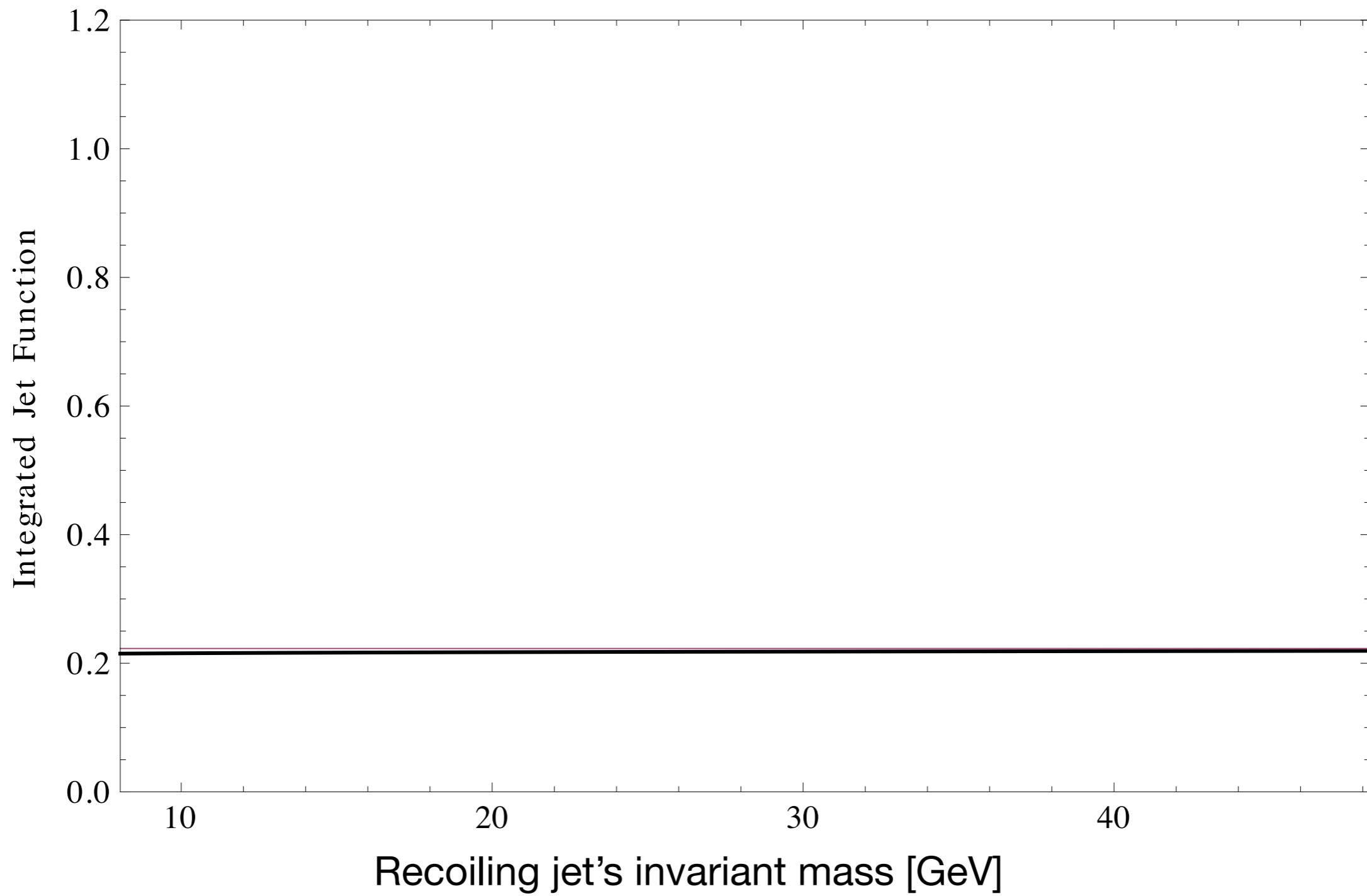
$$\begin{aligned} \Gamma_{IJ}(E_\gamma) &= \frac{1}{4} \frac{2}{\pi m_\chi} \sum_{i,j=1,2} \sum_{V,W,X,Y} C_j^*(\mu_W) C_i(\mu_W) D_{J,XY}^{j*}(\mu_W, \nu_s) D_{I,VW}^i(\mu_W, \nu_s) \\ &\times V(\mu_W, \nu_s, \nu_j) Z_\gamma^{YW} \int d\omega J^{XV}(4m_\chi(m_\chi - E_\gamma - \omega), \mu_W, \nu_j) S_\gamma(\omega) \end{aligned}$$


Jet functions Ultrasoft function

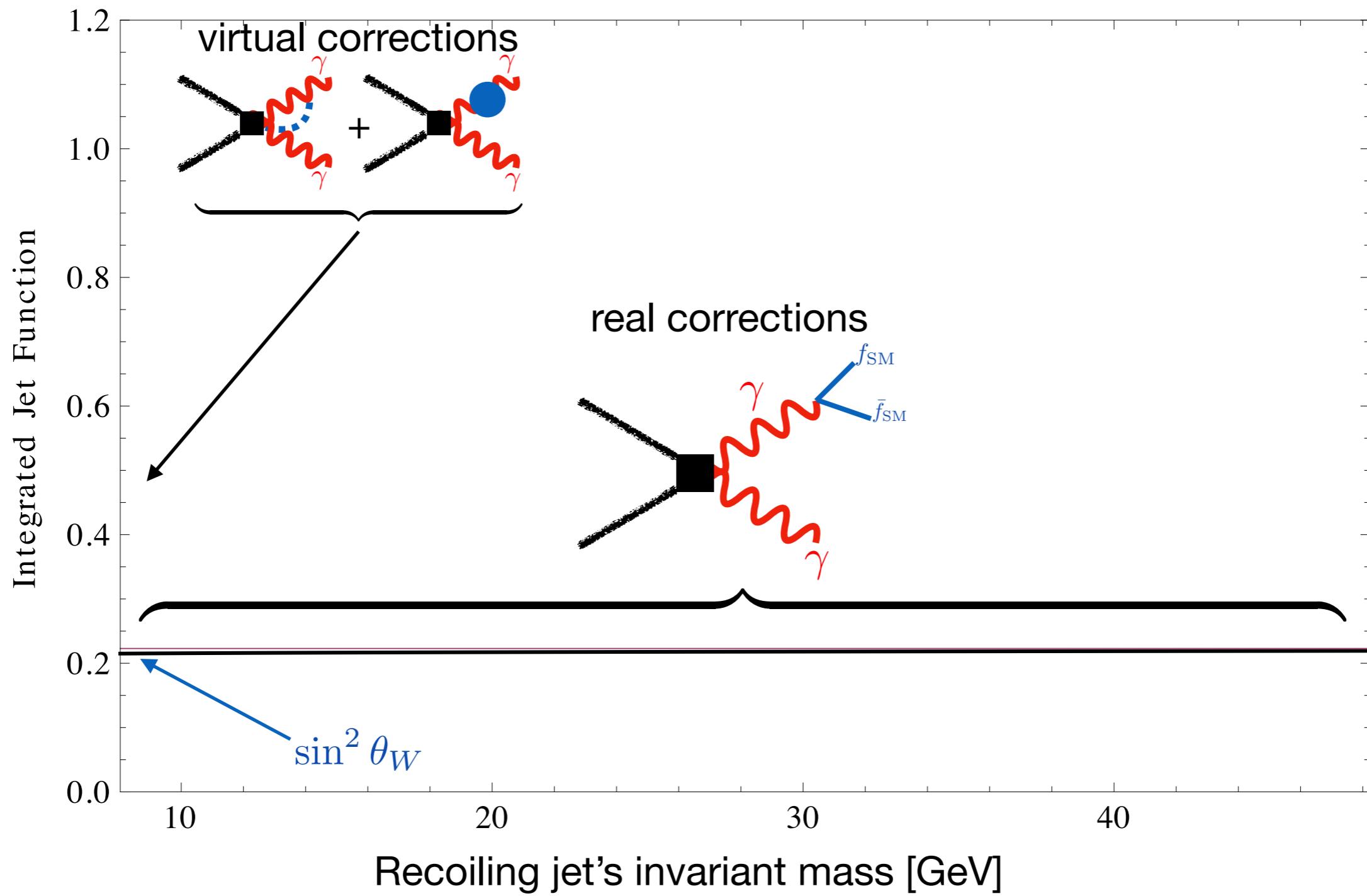
Jet function



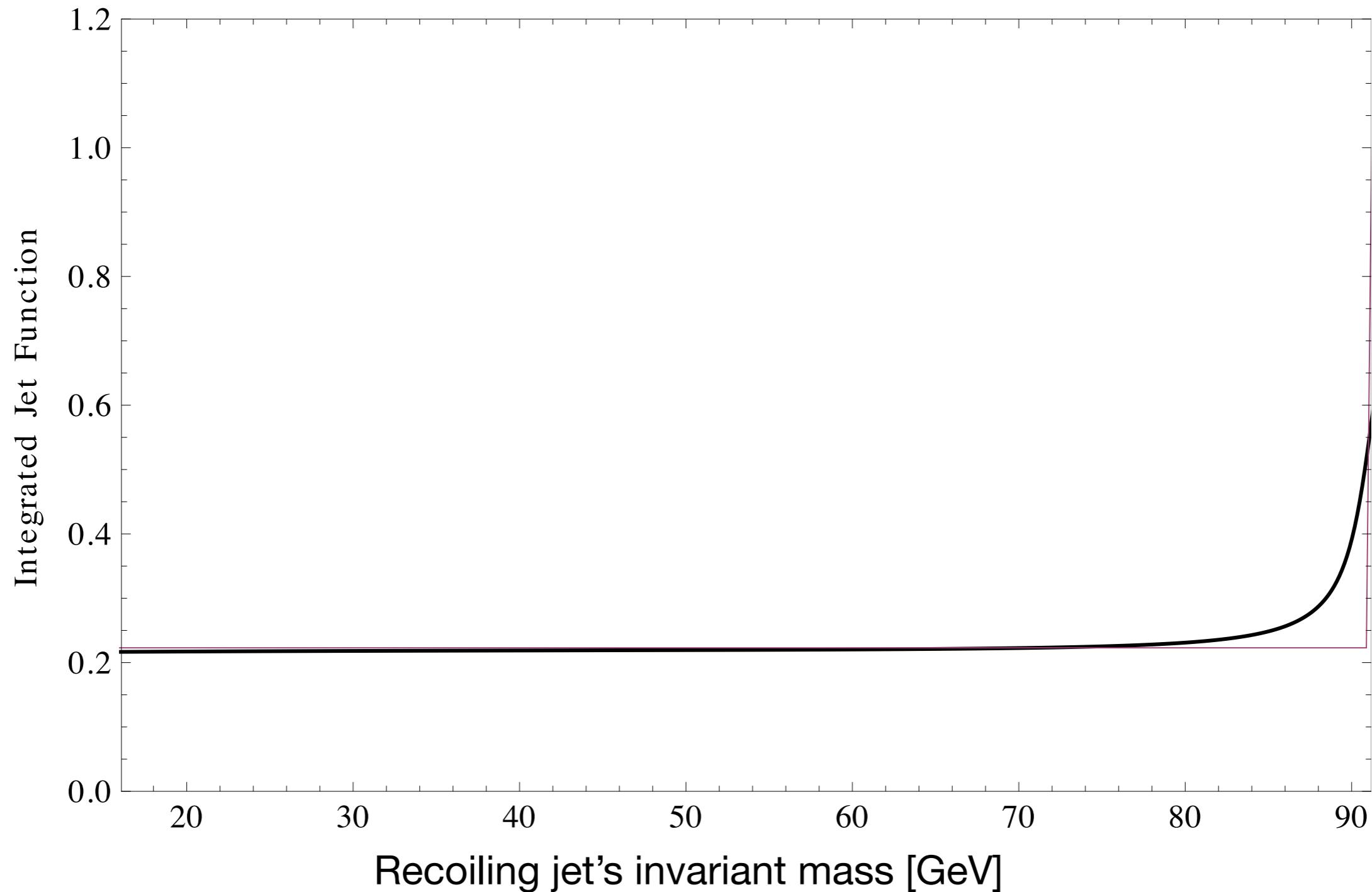
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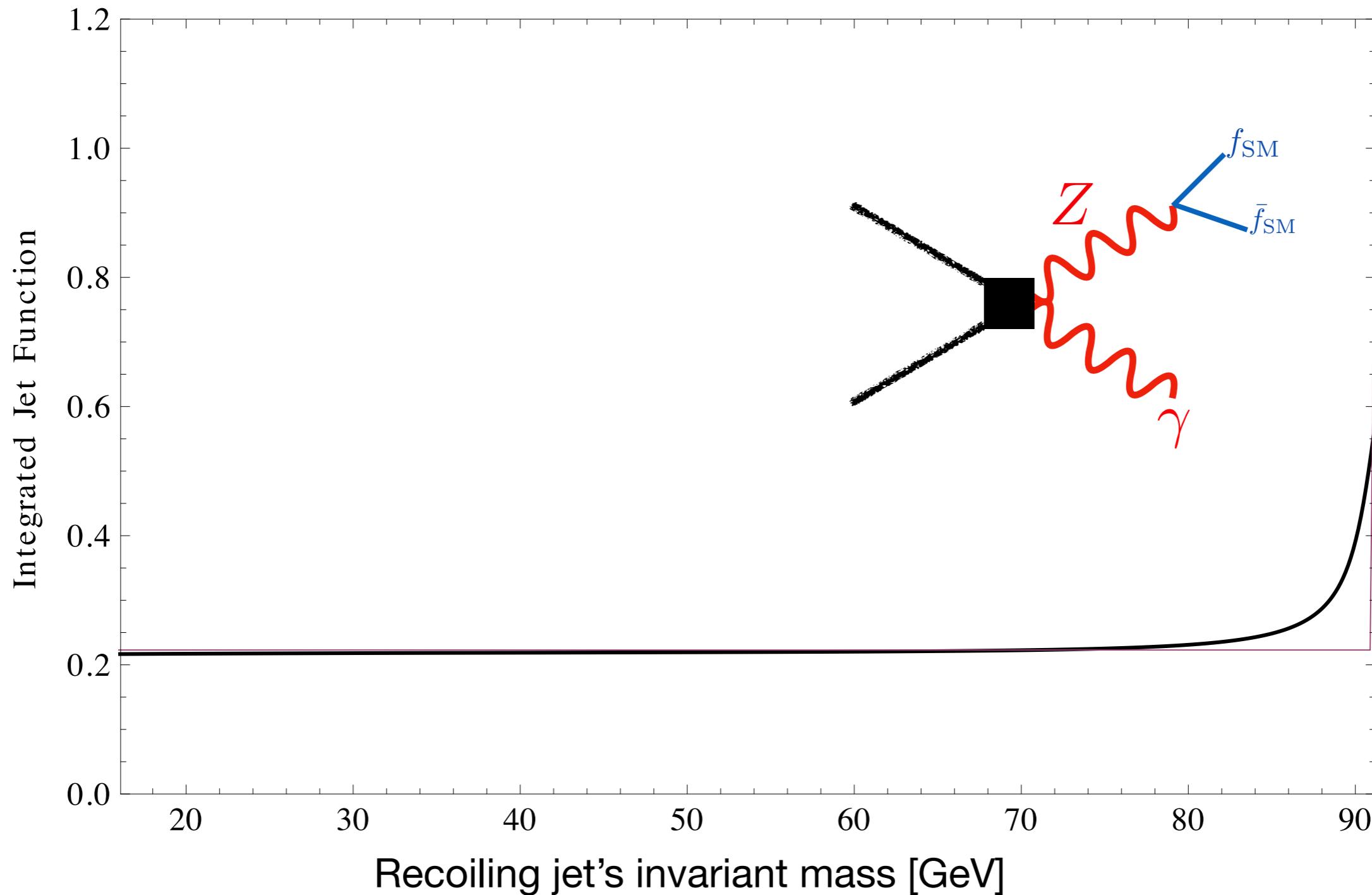
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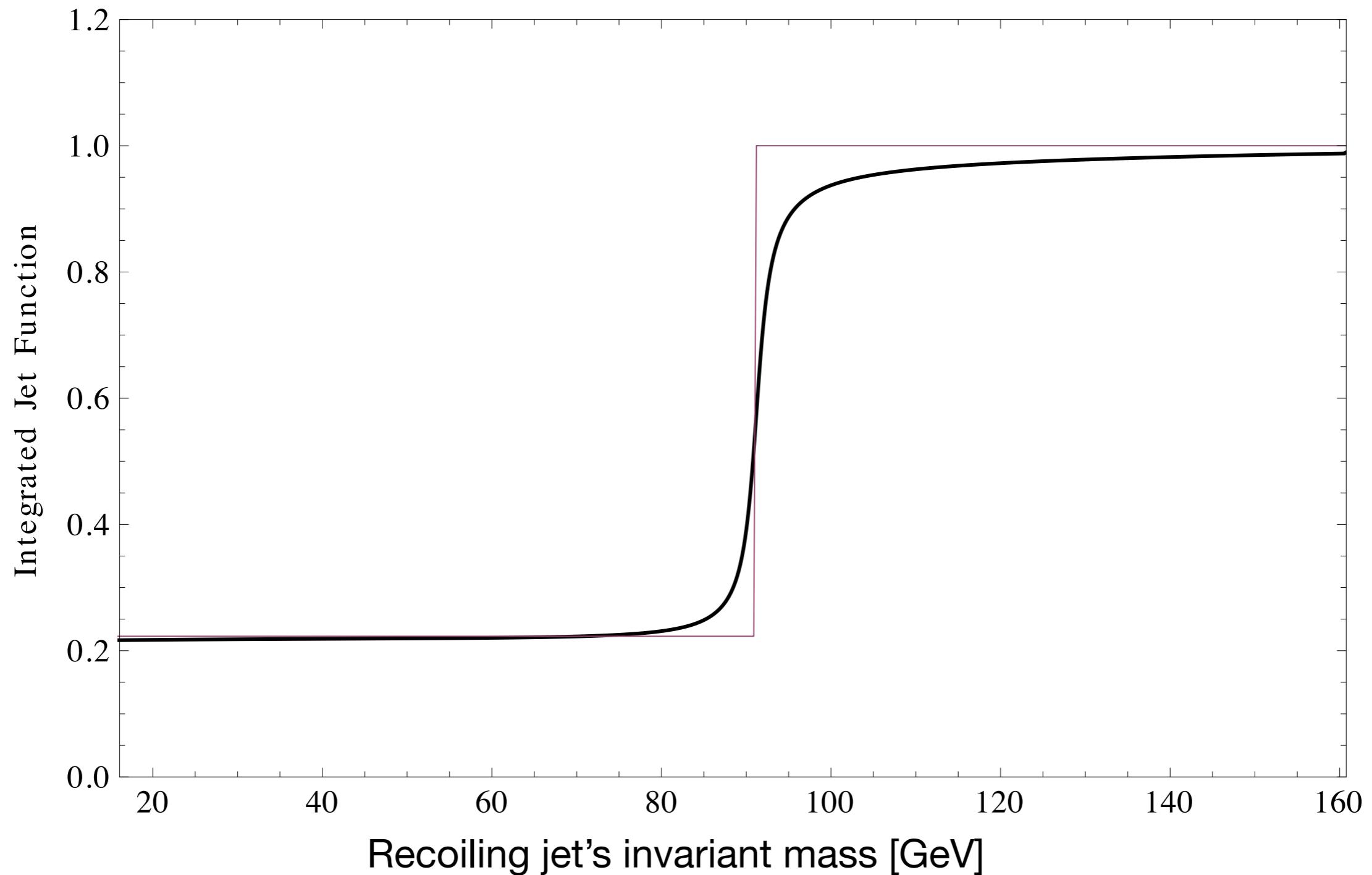
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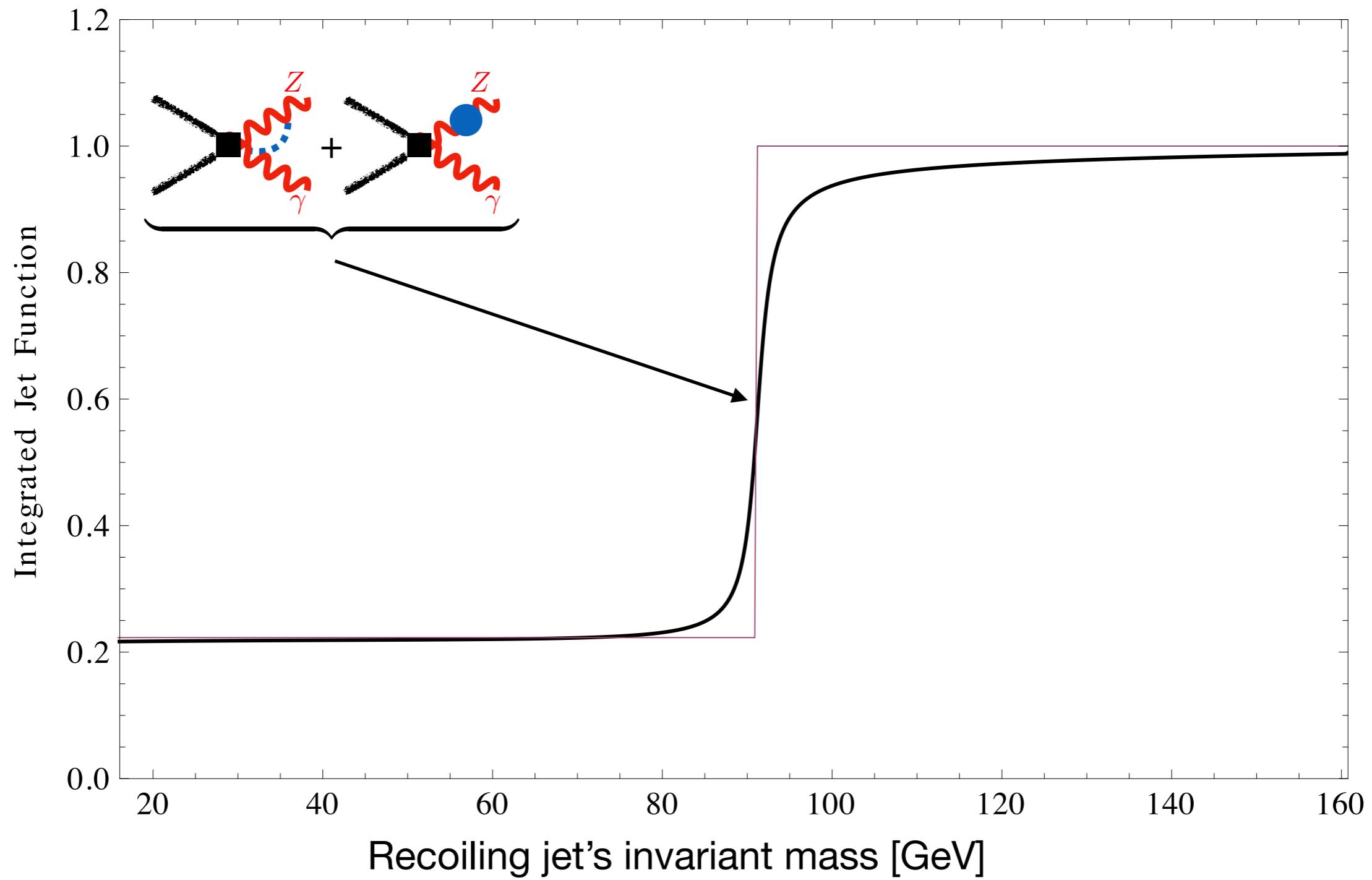
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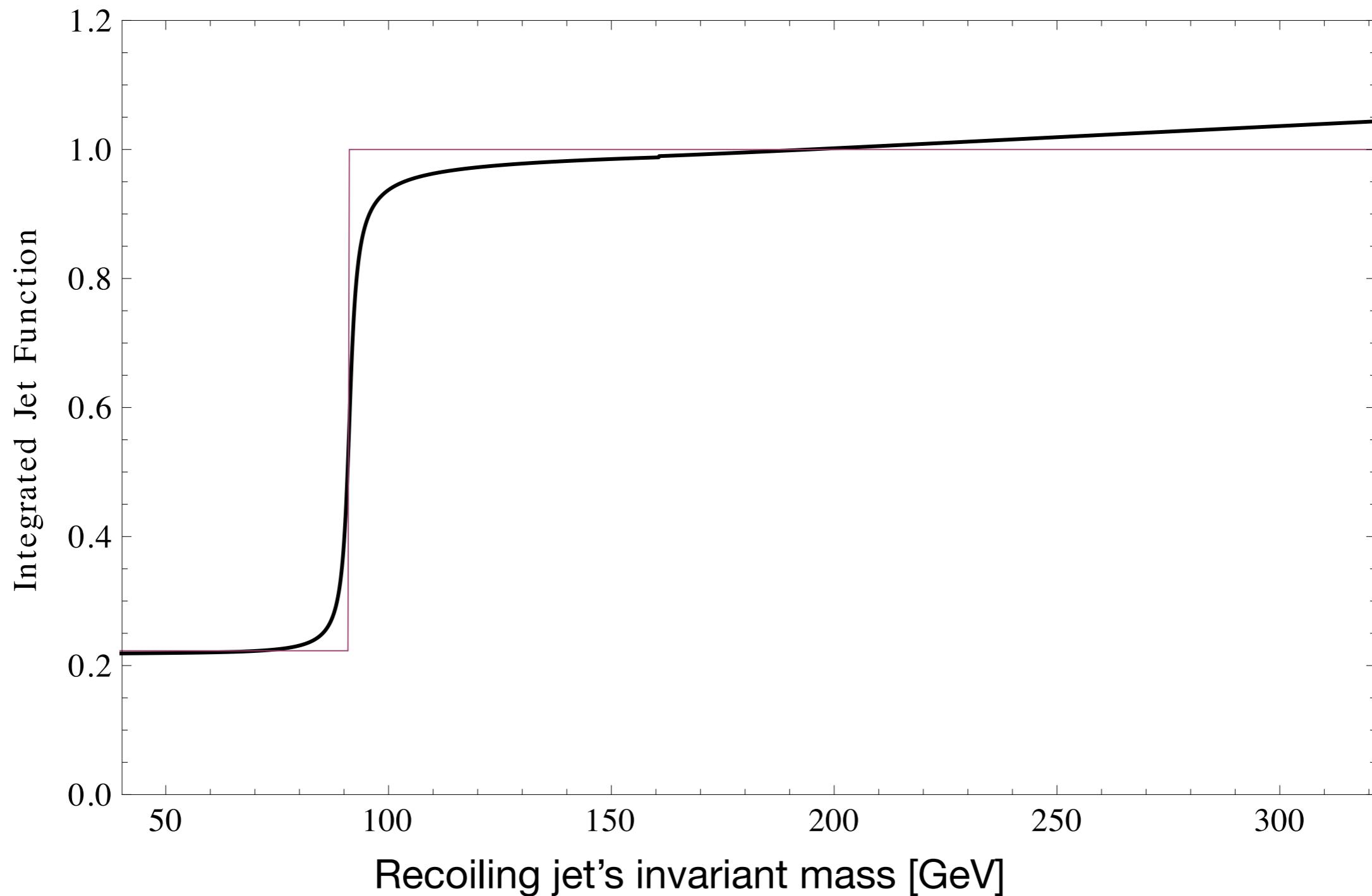
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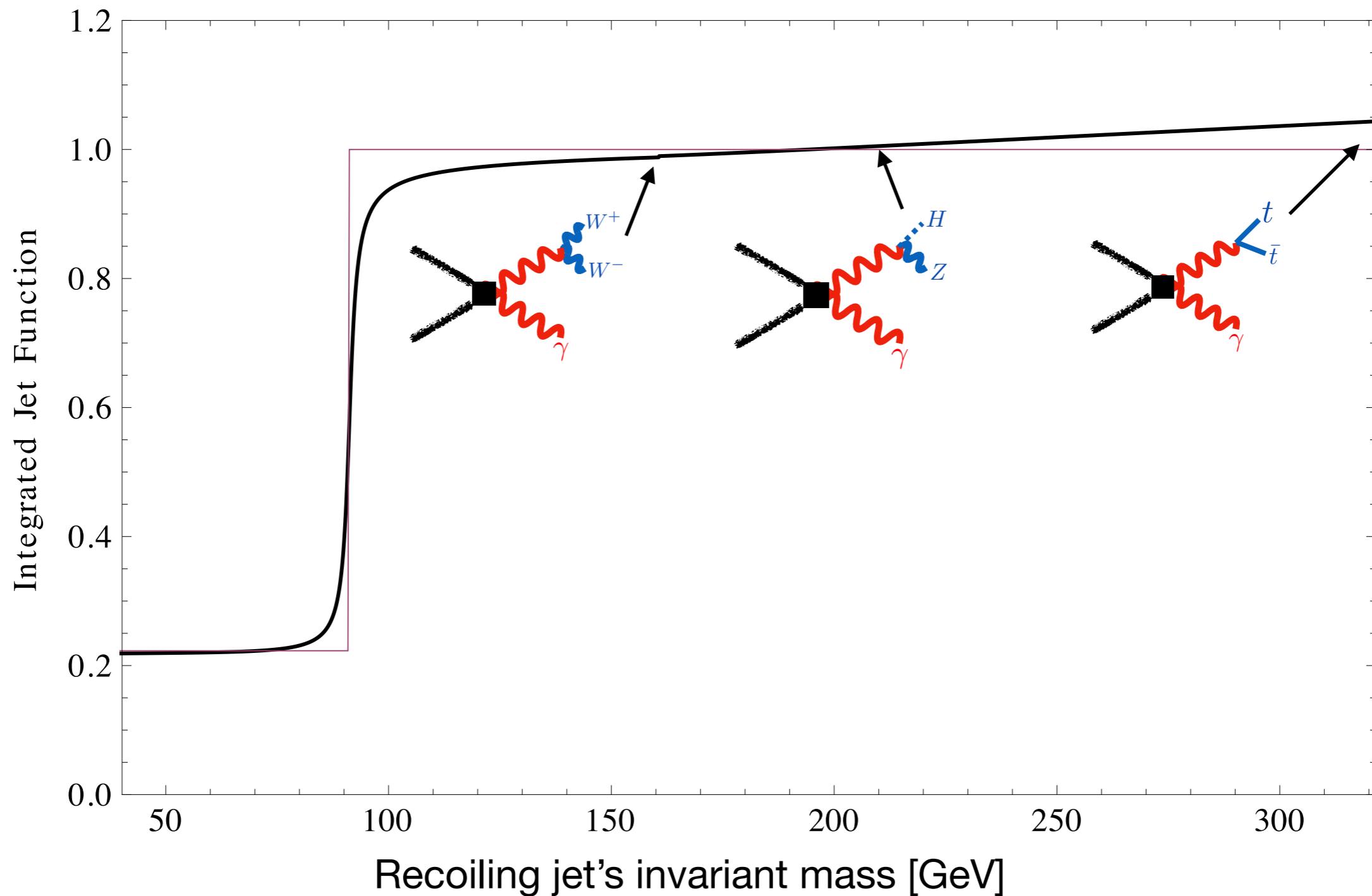
Jet function



Jet function



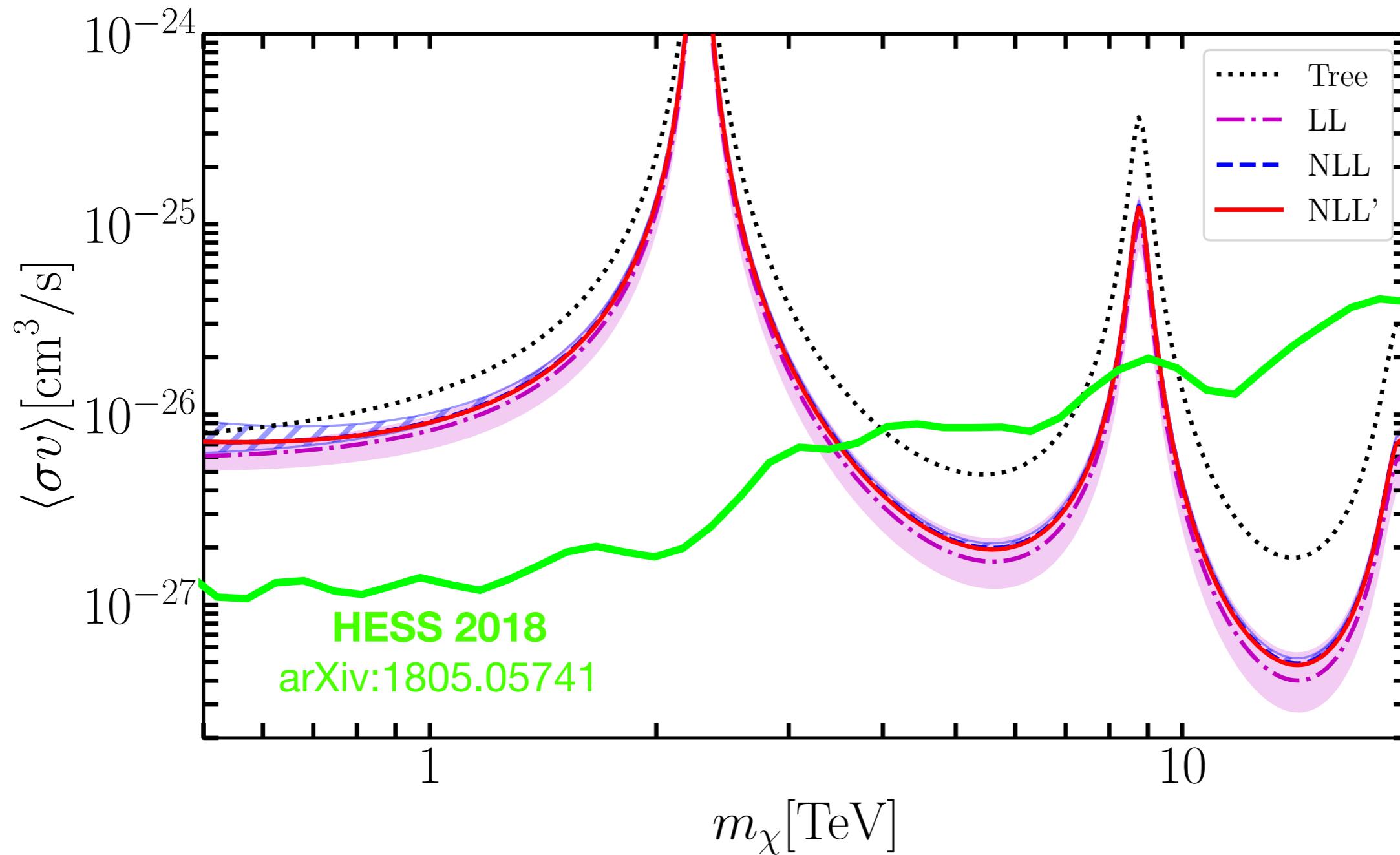
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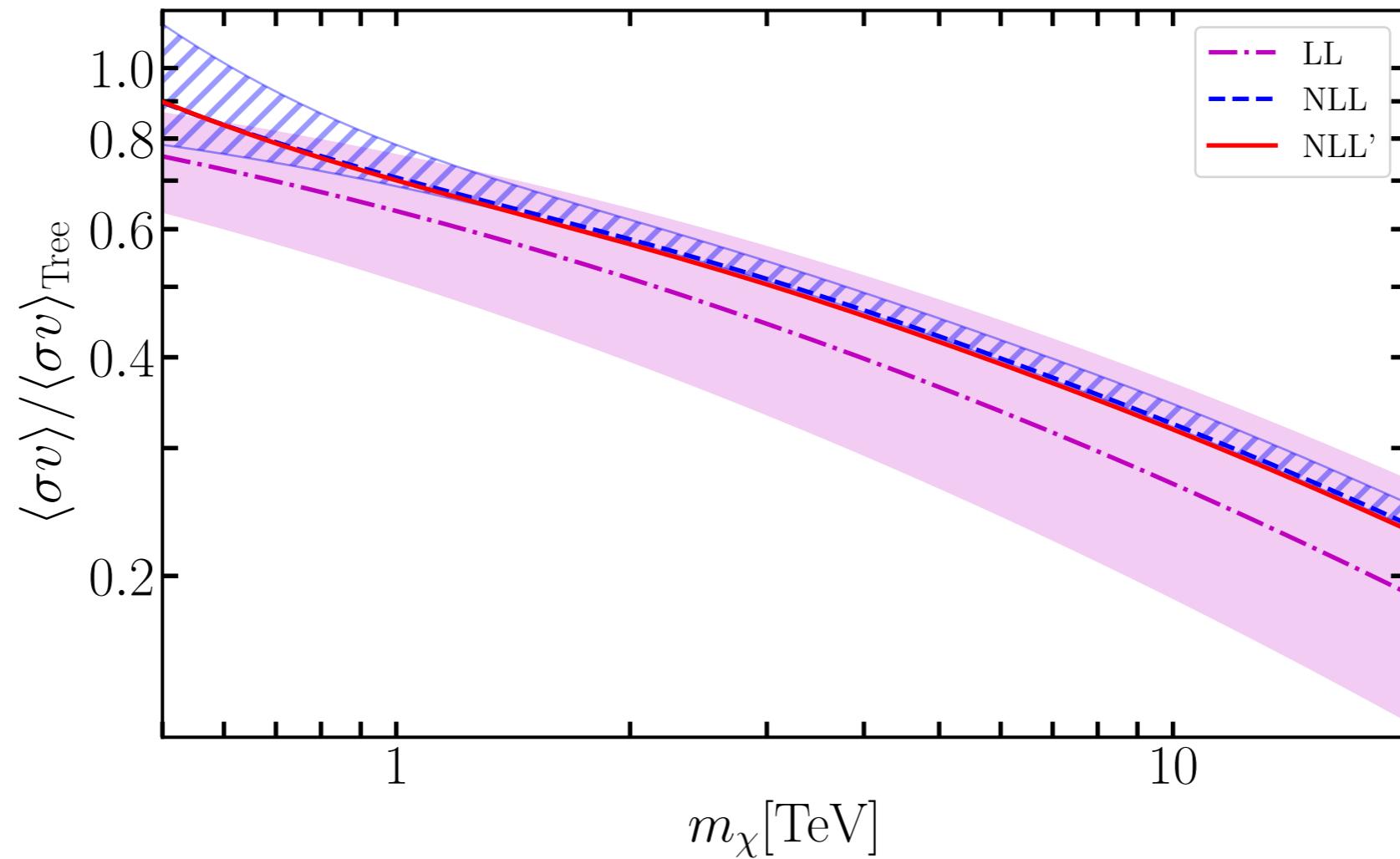
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Predicted cross section



Effect of the Sudakov resummation



What I did not mention

- Operator basis
- Subtleties as to the EW renormalization
 - Prescription to take into account the light fermion masses, etc.
- Poorer energy resolutions of gamma-ray telescopes
- Prospects with the Cerenkov Telescope Array (CTA)
- Further annihilation channels

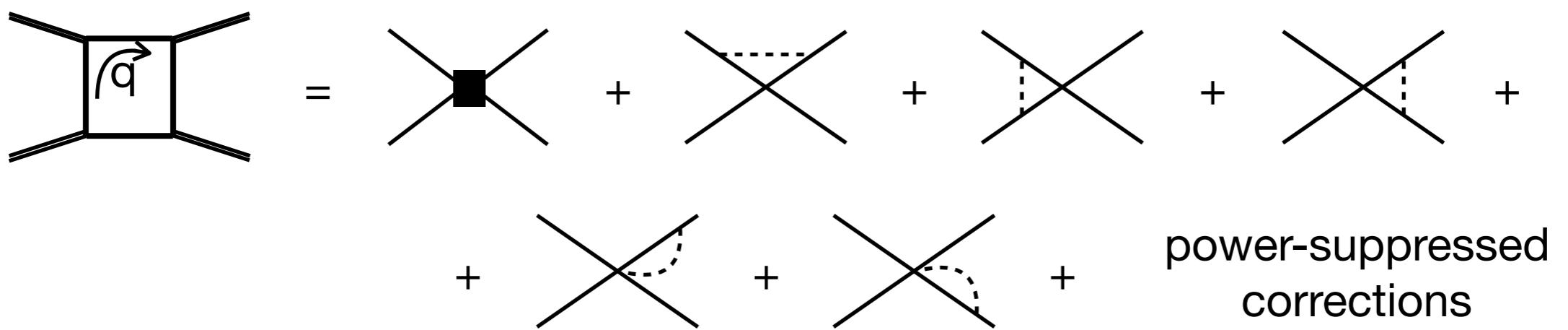
More papers coming soon!

Conclusions

- Radiative corrections relevant for indirect **TeV-scale** DM detection are typically very large
- Used the SCET approach to account for large Sudakov logarithms
 - Not that easy! e.g. need to introduce rapidity regulator
- Obtained factorization formula that is valid at all orders in perturbation theory
- Theoretical uncertainties are reduced to an unprecedented level
- Follow-up papers to come!

Backup slides

Soft-collinear effective theory



- LL Resummation $\alpha^n \log^{(n+1)}$
- NLL Resummation $\alpha^n \log^{(n+1)} + \alpha^n \log^n$
- NLL' \rightarrow NLL + NLO
- NNLL also possible provided the anomalous dimension of the relevant operators is known