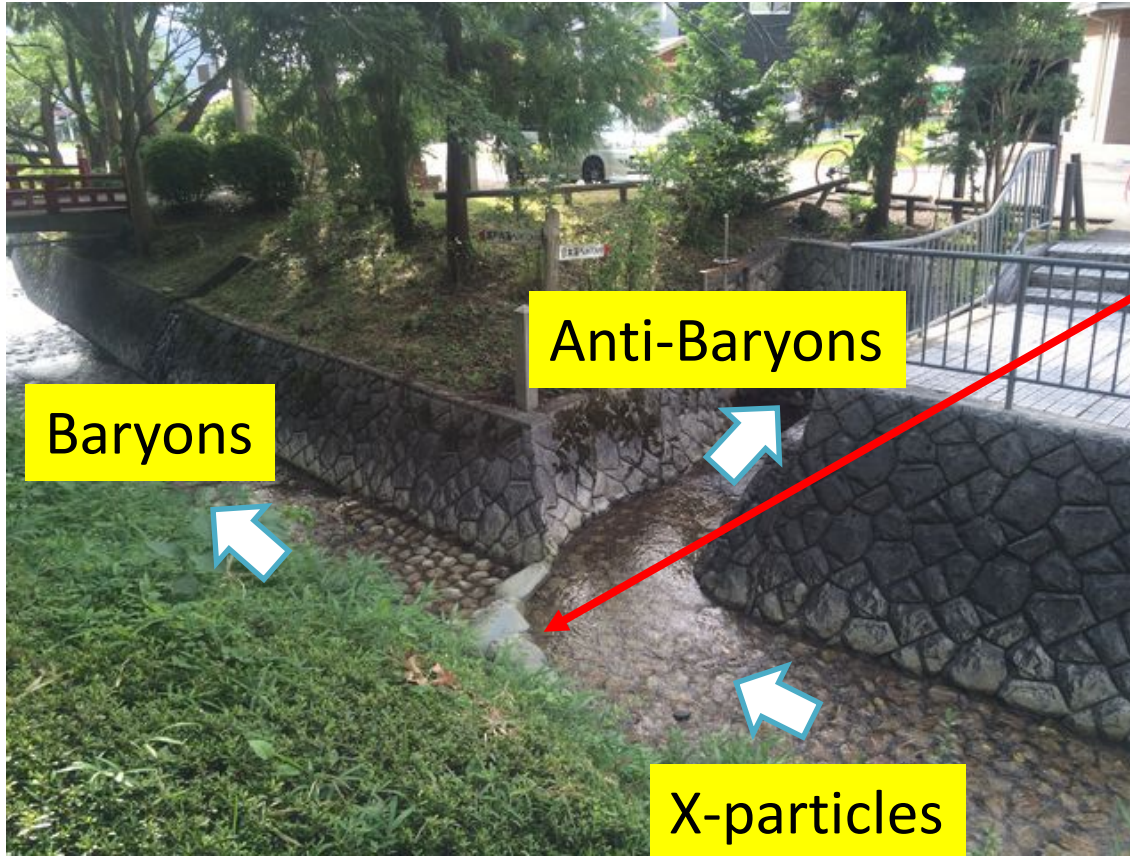


Baryogenesis with the non-adiabatic Berry phase

Tomohiro Matsuda / Saitama Institute of Technology
And
Seishi Enomoto / University of Florida

Origin of the asymmetry?



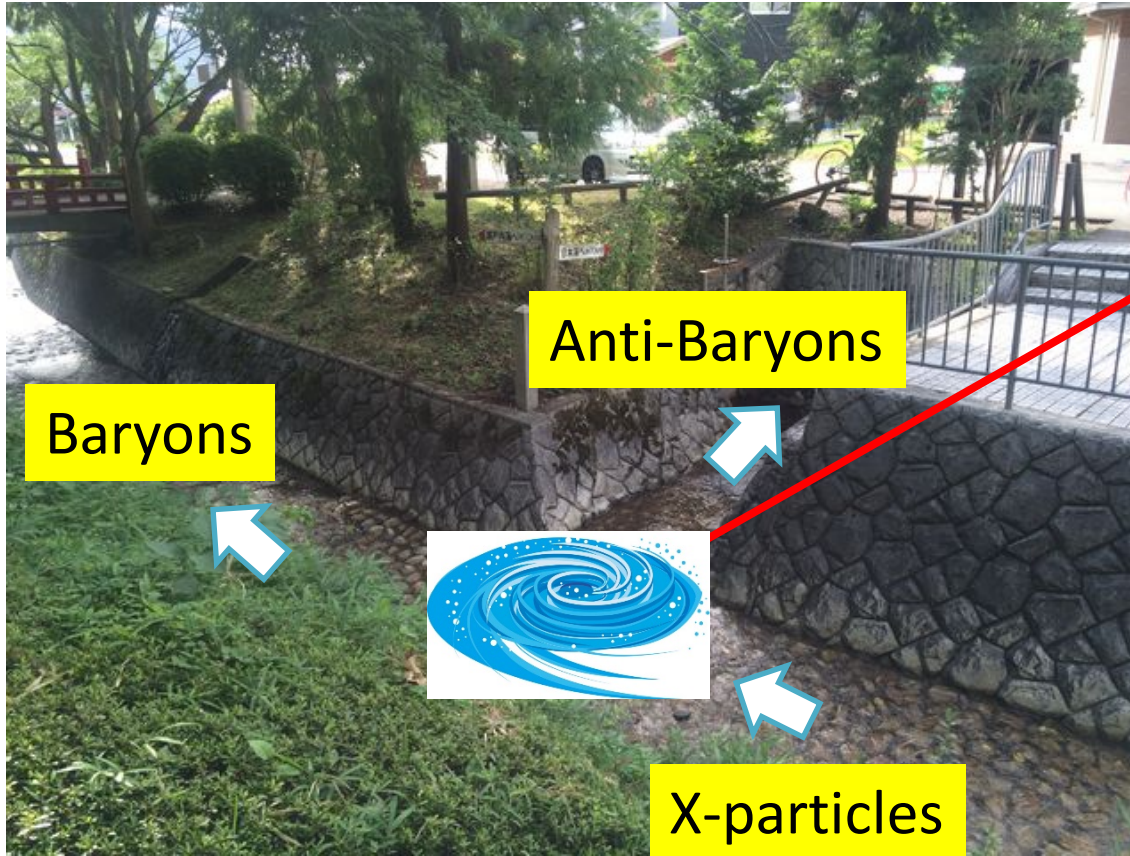
GUT Baryogenesis

$$g_B \neq g_{\bar{B}}$$

Different branching

If you find asymmetry, there should be something at the branching point.

Origin of the asymmetry?



Something is rotating
Spontaneous Baryogenesis

Our focus in this talk

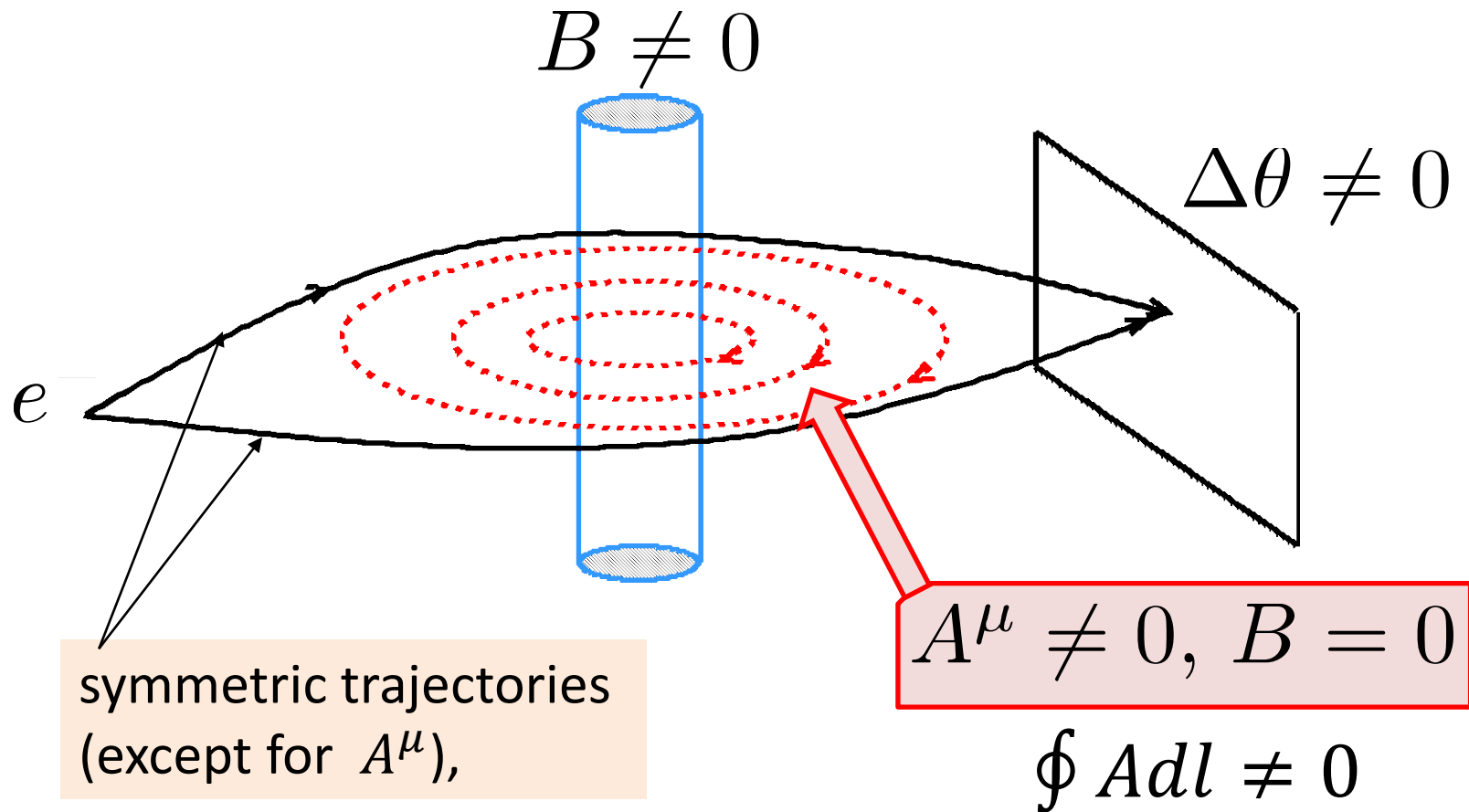
In this talk...

We reconsider the formalism of Spontaneous Baryogenesis
in terms of the Berry connection

Topics

1. The Berry Phase in a Kaon-like 2-state system
2. Chemical potential as the Berry connection
3. Merits of the Berry connection
4. Models and calculations
5. Conclusion

Aharonov–Bohm (AB) effect



Similar effect can be found in a Kaon-like 2-state model
* **Berry connection** appears instead of A^μ

Schrödinger equation for the state $\psi_0^t \equiv (K^0, \bar{K}^0)$

$$i \frac{d}{dt} \psi_0 = H \psi_0$$

$$H = \begin{pmatrix} M & \Delta e^{i\theta} \\ \Delta e^{-i\theta} & M \end{pmatrix}$$

This phase can be rotated away

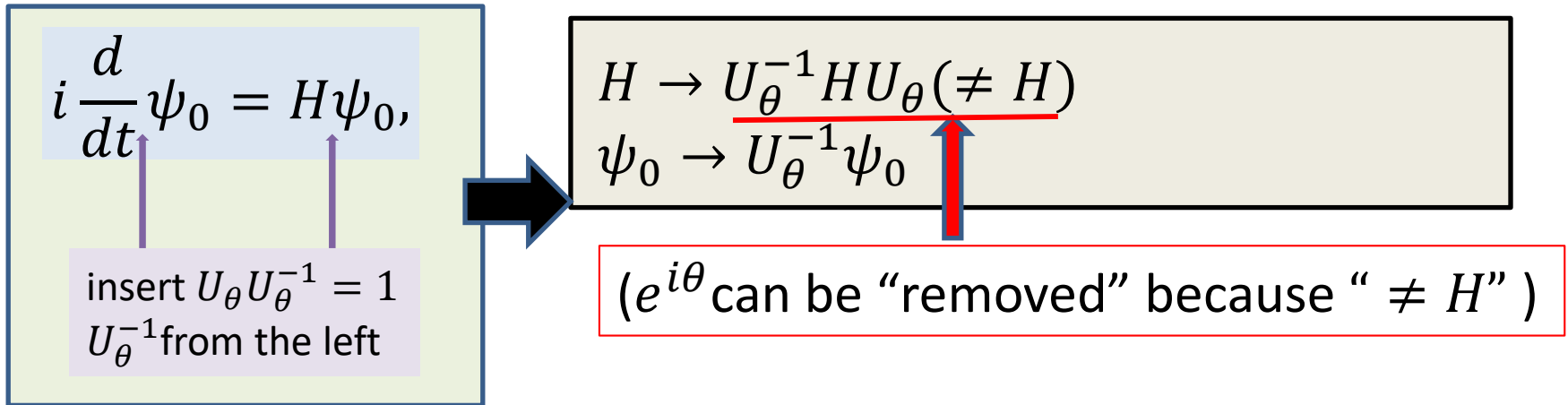
$$\psi^R \equiv U_\theta^{-1} \psi_0,$$

$$U_\theta \equiv \begin{pmatrix} e^{i\theta(t)/2} & 0 \\ 0 & e^{-i\theta(t)/2} \end{pmatrix}.$$

If θ depends on t , the **Berry connection** appears

Removing off-diagonal phase when $\theta(t)$

Consider $\psi^R = U_\theta^{-1}\psi_0$ to remove $e^{i\theta}$ in the off-diagonal elements



Global transformation (constant θ)

$$H \rightarrow U_\theta^{-1} H U_\theta,$$

Local transformation (time dependent θ)

$$H \rightarrow U_\theta^{-1} H U_\theta - i U_\theta^{-1} \partial_t U_\theta$$

Berry connection

(moved from the left-hand side)

For arbitrary U ,

Projective Hilbert space (equivalence class of state vectors)

$$\{U^{-1}\psi_0\}$$

An Equivalence class of Hamiltonians

$$\{H \rightarrow U^{-1}HU - iU^{-1} \partial_t U\}$$

These are defining different representations of the **identical** Schrödinger equation

We sometimes consider rotation of the fields to remove the phases in the theory

- Nothing happens If U is constant
- The Berry connection appears for $U(t)$

*Although this is not a common terminology

For later convenience we call this “The Berry transformation”

In our Kaon-like 2-state model,

The Berry Connection is $i\Gamma \equiv iU_\theta^{-1}\dot{U}_\theta$

$$i\Gamma \equiv \frac{1}{2} \begin{pmatrix} -\dot{\theta} & 0 \\ 0 & \dot{\theta} \end{pmatrix}.$$

In the **adiabatic** limit, the berry connection disappears

$$\Gamma \rightarrow 0$$

but its integral along the orbit may not vanish

$$\oint \Gamma dl \neq 0$$



Used in Topological science

We are considering “**non-adiabatic**” Berry connection

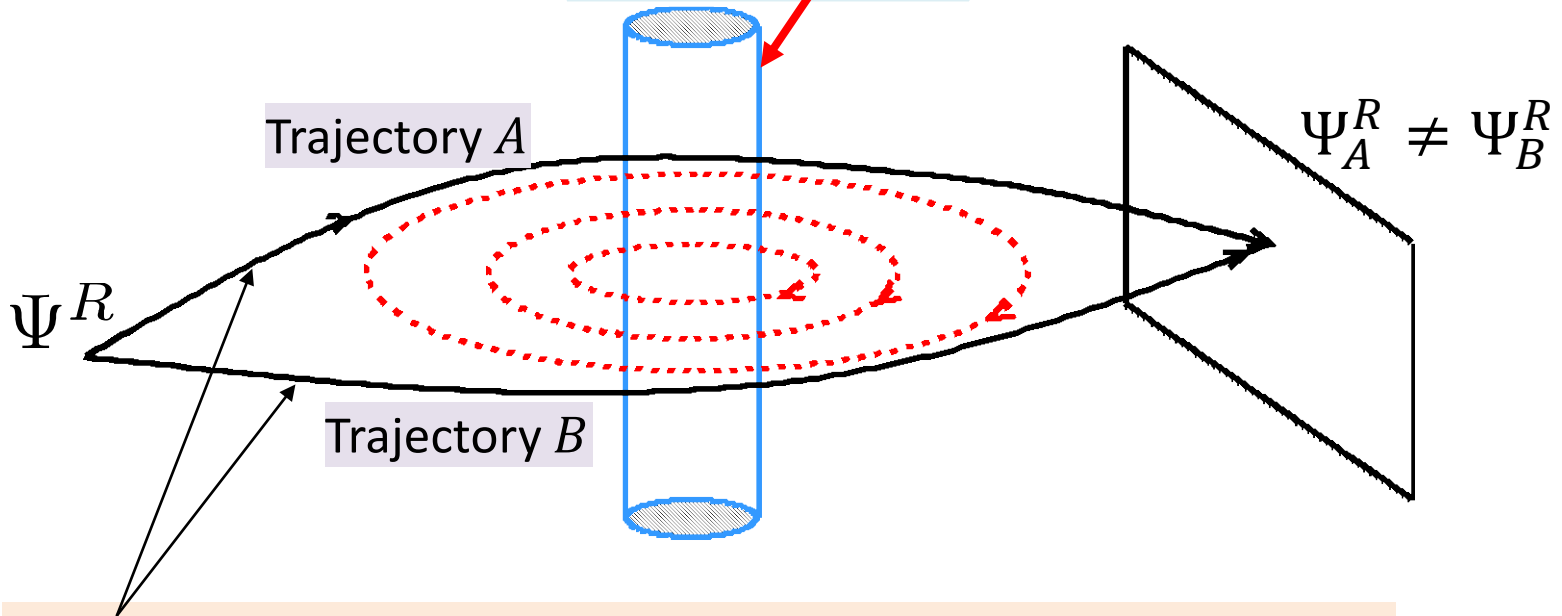
AB-like effect for the 2-state model

$$H = \begin{pmatrix} M & \Delta e^{i\theta} \\ \Delta e^{-i\theta} & M \end{pmatrix}$$

The phase can be removed by the redefinition $\Phi(x) \equiv A(x)e^{i\theta(x)}$

When Φ forms a string defect

$\Phi(x)$ -string $\oint \theta dl = 2n\pi$



The same Hamiltonian for Ψ^R , symmetric trajectory the Berry connection appears

The non-adiabatic Berry connection as the Chemical potential

The “chemical potential” in the Hamiltonian is given by

$$\begin{pmatrix} M & |\Delta| \\ |\Delta| & |\Delta| \end{pmatrix} \rightarrow \begin{pmatrix} M - \mu & |\Delta| \\ |\Delta| & M + \mu \end{pmatrix}$$

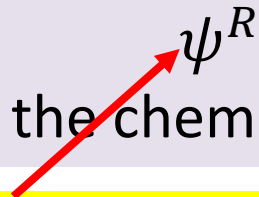
the Berry connection gives the same contribution

$$i \frac{d}{dt} \psi^R = (H^R - iU_\theta^{-1} \dot{U}_\theta) \psi^R,$$

$$H^R = \begin{pmatrix} M & |\Delta| \\ |\Delta| & M \end{pmatrix},$$

$$iU_\theta^{-1} \dot{U}_\theta = \frac{1}{2} \begin{pmatrix} \dot{\theta} & 0 \\ 0 & -\dot{\theta} \end{pmatrix},$$

The Berry connection of ψ^R mimics the chemical potential



If you are using another state vector the Berry connection may not mimic the “chemical potential”

“The Berry connection can mimic the chemical potential when the phase of the off-diagonal element is changing and is removed by the Berry transformation”

In the **Lagrangian** formalism, the process can be seen as...

The Berry phase and the chemical potential in the Lagrangian (for a simple **Single scalar boson**)

Starting from the Hamiltonian with the chemical potential

$$H = H_0 - \mu J^0 \quad H_0 = \int d^3x [\pi^* \pi + \phi^* \omega^2 \phi + V(\phi)]$$

$$J_0 = \int d^3x i \left(\phi^* \frac{\partial L}{\partial \dot{\phi}^*} - \frac{\partial L}{\partial \dot{\phi}} \phi \right) \quad \pi^* = \frac{\partial L}{\partial \dot{\phi}}$$

One can calculate the Lagrangian density

$$L = (\pi^* \dot{\phi} + \dot{\phi}^* \pi) - H$$

$$\Downarrow i\dot{\phi} = [\phi, H] = i(\pi + i\mu\phi)$$

$$= |\dot{\phi} - i\mu\phi|^2 - \dot{\phi}^* \omega^2 \phi - V(\phi)$$

$$\Downarrow \phi_{new} = \phi * e^{-i \int dt' \mu(t')}$$

$$= |\dot{\phi}_{new}|^2 - \dot{\phi}_{new}^* \omega^2 \phi_{new} - V(\phi_{new} * e^{+i \int dt' \mu(t')})$$

(Berry) Connection
 $\partial_0 \rightarrow \partial_0 - i\mu$

Chemical potential
 \Updownarrow
Time-dependent phase

The formalism looks very similar to “Spontaneous Baryogenesis”

Is the “formalism based on the **Berry phase**”
(θ is a parameter defined on the orbit)

equivalent to

the “formalism based on the **Nambu-Goldstone boson**”?
(θ is the NG boson and $\dot{\theta}$ comes from
the derivative coupling)

NO

Significant discrepancy appears
in the **Legendre** transformation
(Hamiltonian \Leftrightarrow Lagrangian)

Why Hamiltonian is important?

If the chemical potential appears in the Hamiltonian, bias appears for the number densities in the thermal equilibrium (Major quiz in many textbooks)

On the other hand...

If the chemical potential disappears in the Hamiltonian, the bias is not obvious.

The chemical potential comes from “derivative coupling”

The chemical potential defined using a field is

$$L_c = (\partial_\mu \phi) J_\phi^\mu$$

This term shifts the conjugate momentum

$$\pi = \frac{\partial L}{\partial \dot{\phi}}$$

Legendre Transformation

and the chemical potential disappears from the Hamiltonian.

The corresponding part is

$$H_c = \pi \dot{\phi} - L_c$$

A remedy is (Arbuzova, Dolgov and Novikov Phys. Rev. D **94** (2016) 123501)

$$(\dot{\phi})^2 \rightarrow (\pi - n_\phi)^2$$

which may complement the chemical potential, but this is not the chemical potential

The chemical potential is the Berry connection

The Berry phase is defined by the “parameter” defined along the classical orbit. Therefore the chemical potential is

$$\mu = \partial_t \langle \theta \rangle$$

The Berry connection defined for the “parameter” does not shift the conjugate momentum

$$\pi = \frac{\partial L}{\partial \dot{\phi}}$$

and the chemical potential **appears (successfully)** in the Hamiltonian.

*Of course the “Legendre transformation” and the “(trivial) Berry transformation” must **commute by definition**.

Without off-diagonal element, chemical potential gives nothing
(Just disappear after Berry Tr.)

When there is a Rotation of

- off-diagonal element
- interaction
- Majoran mass,

The Berry Transformation

- Defined for a “parameter”
- Gives the **Berry connection** $\Rightarrow \mu$
- Generates chemical potential in the Hamiltonian

When the same transformation is defined using a NG-“field”

- Gives a **Derivative coupling** $\Rightarrow \mu$
- Do **not** generate chemical potential in the Hamiltonian
- New Field is redefined by two fields (non-linear mixing)
- Definition of the number density?

What if the original Lagrangian already has the Derivative coupling?

- This term may disappear in the Hamiltonian
- Model Dependent argument

This case is not discussed in this talk

When there is a Rotation of

- off-diagonal element
- interaction
- Majoran mass,

The Berry Transformation

- Defined for a “parameter”
- Gives the **Berry connection** $\rightarrow \mu$
- Generates chemical potential in

Do not Mix!

These are different!

When the same transformation is

- Gives a **Derivative coupling** $\rightarrow \mu$
- Do **not** generate chemical potential in the Hamiltonian
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Merits of the Berry connection

1. Legendre transformation is consistent with the Berry transformation. (Commutative)
- ➔ 2. Non-trivial Multi-field extension
3. Simplifies the calculation / consistency relations

1. Multi-field Extension ($N_F = 3$) of a basic SB scenario

$$\mathcal{L} = i\bar{Q}_i(i\gamma^\mu\partial_\mu - m_Q^{ij})Q_j + \bar{L}_i(i\gamma^\mu\partial_\mu - m_L^{ij})L_j + (f^{ij}\bar{Q}_iL_j + h.c.),$$

*Multi-field extension of a basic scenario
Dolgov, Freese, Rangarajan, Srednicki,(1997)

m_Q, m_L can be diagonalized by

$$\hat{Q}_i = (U_Q^\dagger)^{ij} Q_j$$

$$\hat{L}_i = (U_L^\dagger)^{ij} L_j$$

Phases appear in

$$V^{ij} \equiv (U_Q^\dagger f U_L)^{ij}$$

Calculated from 3×3
Matrix diagonalization.
Complex function of the
original mass matrix
parameters

Then the interaction is written as

$$\mathcal{L}_{int} = ((U_Q^\dagger f U_L)^{ij} \hat{Q}_i \hat{L}_j + h.c.).$$

Phases in $V^{ij} \equiv (U_Q^\dagger f U_L)^{ij}$ is important for SB

A Clear Difference from the single-field model

Since V^{ij} is written using all the (complex) parameters of the original mass matrix,

$$\theta_i(a, b, c, \dots)$$

effective rotation($\dot{\theta}$)

$$\dot{\theta}_i(a, b, c, \dots) = \dot{a} \frac{\partial \theta_i}{\partial a} + \dot{b} \frac{\partial \theta_i}{\partial b} + \dots$$

can (theoretically) appear from a simple (not rotating) oscillation of a parameter

“Axionic oscillation” is not necessary for the asymmetry

A conventional oscillation after inflation is enough.

*confirmed by numerical calculation (S.E & T.M arXiv1707.05310)

Back to the 2-state model and consider

Simple models and calculations

Majorana Fermions and the Berry phase

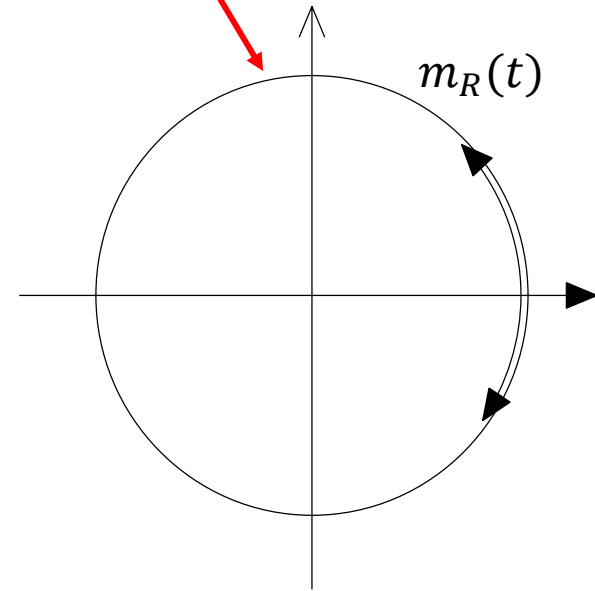
Models and Calculations 1 (the simplest example)

Majorana Fermion with a rotational oscillation

For $\Psi_R^t \equiv (\psi_R, \psi_L^c)$,

$$\mathcal{L}_m = \bar{\Psi}_R \begin{pmatrix} 0 & m_R \\ m_R^* & 0 \end{pmatrix} \Psi_R.$$

$$m_R(t) = M_R e^{i\theta(t)}$$



Expand

$$(\psi_R)_\alpha = \int \frac{d^3 k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}} \sum_{s=\pm} (e_{\mathbf{k}}^s)_\alpha \\ \times \left[u_{\mathbf{k}}^s(x^0) a_{\mathbf{k}}^s + s v_{\mathbf{k}}^s(x^0) a_{-\mathbf{k}}^{s\dagger} \right],$$

Equation of motion gives

$$(i\partial_t + s|\mathbf{k}|)u_{\mathbf{k}}^s = sm_R^* v_{\mathbf{k}}^s, \\ (i\partial_t + s|\mathbf{k}|)v_{\mathbf{k}}^{s*} = -sm_R^* u_{\mathbf{k}}^{s*}.$$

Unlike the Dirac fermion, mass term appears in the mixing term. Because of the time-dependent mixing term, standard process of decoupling fails in this case.

We need something ...

Using $\Psi^t \equiv (v_k^s, u_k^s)$, one can write them into

$$i \frac{d}{dt} \Psi = H \Psi$$

$$H = \begin{pmatrix} -s|k| & s M_R e^{-i\theta(t)} \\ s M_R e^{i\theta(t)} & s|k| \end{pmatrix}$$

$$\theta(t) = \theta_0(t) \cos m_\theta t$$

After the **Berry transformation**, we find

$$i \frac{d}{dt} \psi^R = \hat{H}^R \psi^R,$$

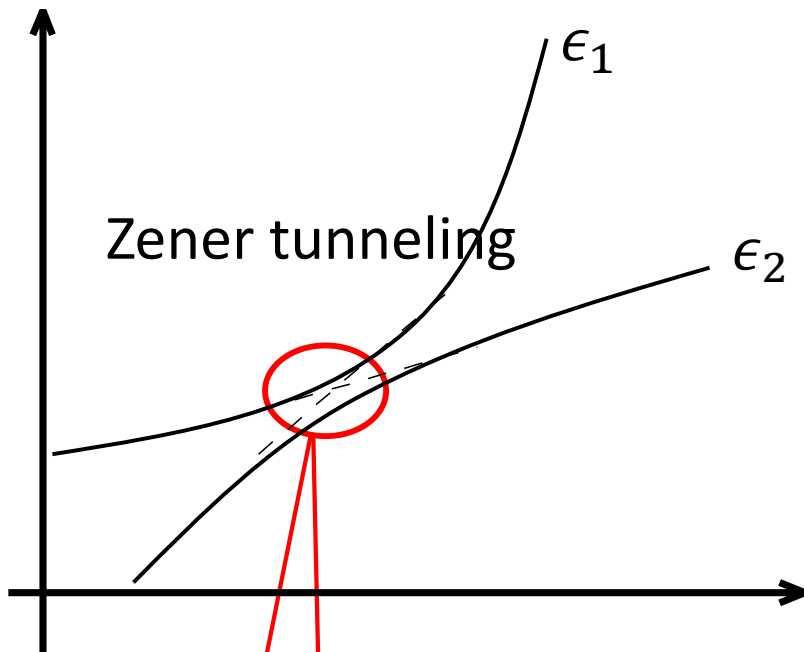
$$\hat{H}^R = \begin{pmatrix} -s|\mathbf{k}| + \frac{\theta_0 m_\theta}{2} \sin m_\theta t & s M_R \\ s M_R & s|\mathbf{k}| - \frac{\theta_0 m_\theta}{2} \sin m_\theta t \end{pmatrix}.$$

This model can be solved by Zener tunneling (next slide)

Or decoupling the equation

P. Adshead and E.I.Sfakianakis arXiv:1508.00891

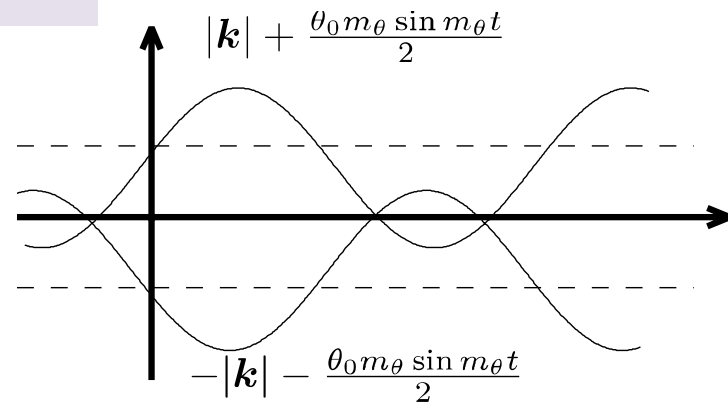
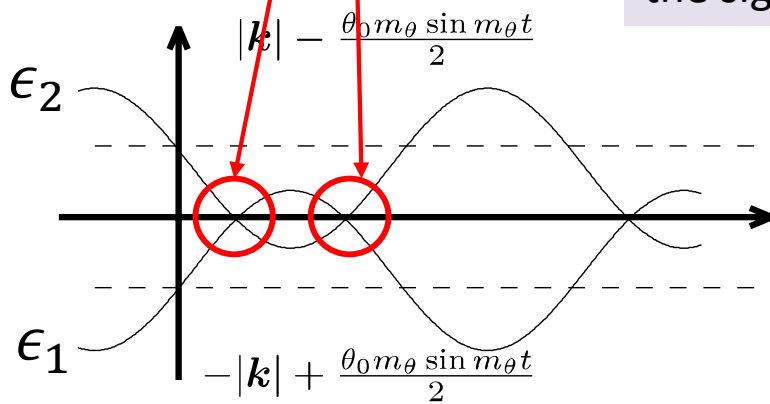
*Derivative coupling assumed=> not identical to our model



Zener tunneling gives the transition rate at the crossing point

Bogoliubov coefficients (number densities)

helicity flips the sign



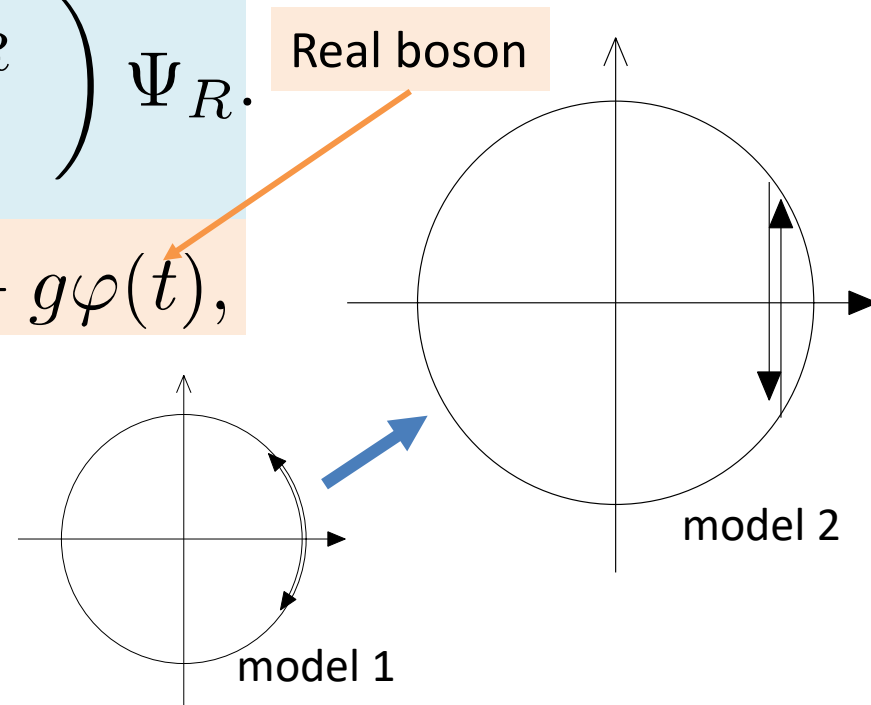
Models and Calculations 2

Majorana Fermion with a non-trivial Majorana Mass

For $\Psi_R^t \equiv (\psi_R, \psi_L^c)$,

$$\mathcal{L}_m = \bar{\Psi}_R \begin{pmatrix} 0 & m_R \\ m_R^* & 0 \end{pmatrix} \Psi_R.$$

$$m_R(t) = i\Lambda + g\varphi(t),$$



$$\dot{\theta} = \frac{d}{dt} \tan^{-1} \frac{g\varphi}{\Lambda} \neq 0 \rightarrow \text{Berry connection (chemical potential)}$$
$$\frac{d|m_R|}{dt} \neq 0 \rightarrow \text{Significant Particle production is possible}$$

Expand

$$(\psi_R)_\alpha = \int \frac{d^3 k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}} \sum_{s=\pm} (e_{\mathbf{k}}^s)_\alpha \times \left[u_{\mathbf{k}}^s(x^0) a_{\mathbf{k}}^s + s v_{\mathbf{k}}^s(x^0) a_{-\mathbf{k}}^{s\dagger} \right],$$

Equation of motion gives

$$(i\partial_t + s|\mathbf{k}|)u_{\mathbf{k}}^s = sm_R^* v_{\mathbf{k}}^s, \\ (i\partial_t + s|\mathbf{k}|)v_{\mathbf{k}}^{s*} = -sm_R^* u_{\mathbf{k}}^{s*}.$$

Again, $\dot{m}_R \neq 0$ prevents simple decoupling,

BUT if $\Lambda \neq 0$, one can rewrite equations using m_R^{-1}

*Berry is not enough to remove T-dep. in off-Diag.

$$(m_R^*)^{-1} (i\partial_t + s|\mathbf{k}|)u_{\mathbf{k}}^s = s v_{\mathbf{k}}^s,$$

to find decouple equations ($v_{\mathbf{k}}^s$ has m_R instead of m_R^*)

$$\left[\partial_t^2 - \frac{\dot{m}_R^*}{m_R^*} \partial_t + |\mathbf{k}|^2 + is|\mathbf{k}| \frac{\dot{m}_R^*}{m_R^*} \right] u_{\mathbf{k}}^s = -|m_R|^2 u_{\mathbf{k}}^s.$$

Substituting

$$u_{\mathbf{k}}^s = e^{\int \frac{\dot{m}_R^*}{2m_R^*} dt} U_{\mathbf{k}}^s,$$

we find

$$\ddot{U}_{\mathbf{k}}^s + \left[\partial_t \left(\frac{\dot{m}_R^*}{m_R^*} \right) - \frac{1}{4} \left(\frac{\dot{m}_R^*}{m_R^*} \right)^2 + |\mathbf{k}|^2 + |m_R|^2 + i s |\mathbf{k}| \frac{\dot{m}_R^*}{m_R^*} \right] U = 0.$$

Helicity($s = \pm$) appears here

Easy to solve this equation

*The Berry transformation suggests that one do not have to introduce both $\mu(t)$ and $m_R(t)$ at the same time.

Conclusions and discussions

Berry connection can mimic the Chemical potential

Berry connection \neq Derivative coupling **Do not Mix!**

Simple (not axionic) oscillation can generate asymmetry in Multi-field Extension

Berry transformation may Simplify the calculation
(because $\mu(t)$ can be removed)

Can be used to check the consistency of the calculation