

Neutrino masses from Planck-scale lepton number breaking

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Introduction

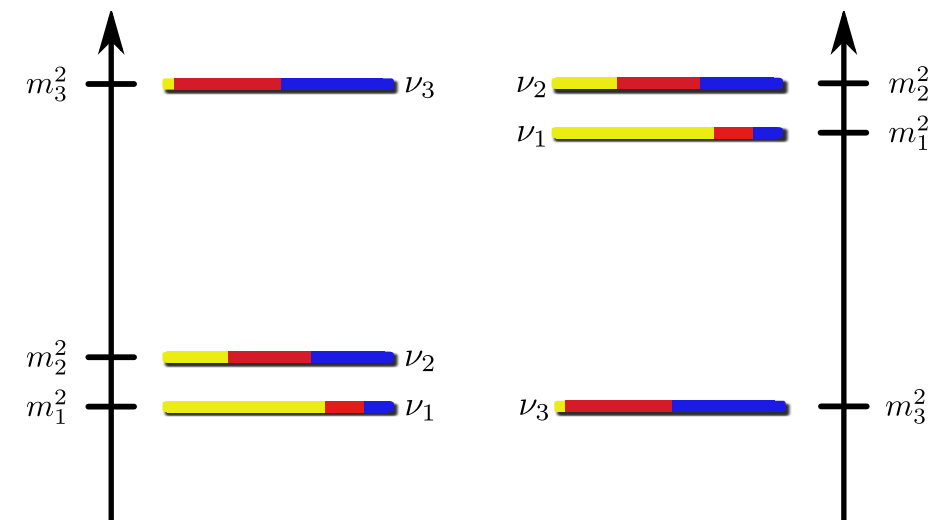
Neutrinos are massive. (massless in the Standard Model)

- Neutrino oscillation data

	NH	IH
$\sin^2 \theta_{12}$	$0.306^{+0.012}_{-0.012}$	$0.306^{+0.012}_{-0.012}$
$\sin^2 \theta_{23}$	$0.441^{+0.027}_{-0.021}$	$0.587^{+0.020}_{-0.024}$
$\sin^2 \theta_{13}$	$0.02166^{+0.00075}_{-0.00075}$	$0.02179^{+0.00076}_{-0.00076}$
Δm_{21}^2 [eV ²]	$7.50^{+0.19}_{-0.17} \times 10^{-5}$	$7.50^{+0.19}_{-0.17} \times 10^{-5}$
$\Delta m_{3\ell}^2$ [eV ²]	$2.524^{+0.039}_{-0.040} \times 10^{-3}$	$-2.514^{+0.038}_{-0.041} \times 10^{-3}$

Esteban et al. JHEP (2017)

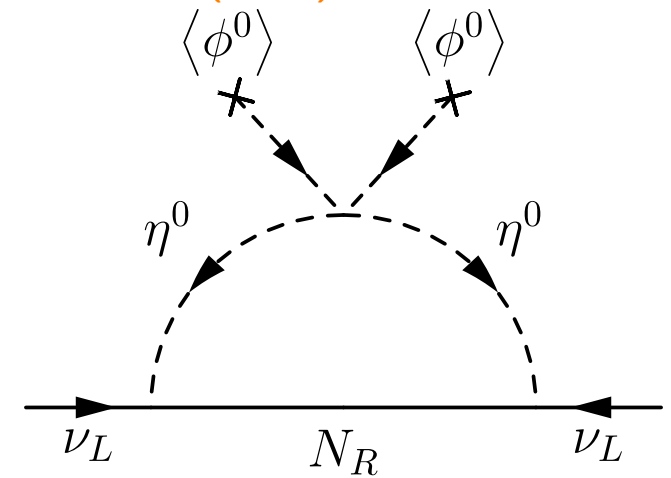
- Very small masses of neutrinos and large mixing angles.
- Mild hierarchy of two heaviest masses ~ 6 .
- \Rightarrow different mechanism of mass generation?



Introduction

Neutrino mass generation mechanisms

- Type-I, II, III seesaw mechanism [Minkowski, Yanagida, et al. \(1977\)](#)
- Inverse seesaw, Linear seesaw mechanisms, radiative generation of neutrino masses etc
[A. Zee \(1980\)](#), [K.S. Babu \(1988\)](#), [E. Ma, PRD \(2006\)](#) etc
- Ex. Type-I seesaw: $m_\nu \approx -m_D M^{-1} m_D^T$
diagonalized by U_{PMNS}
- We will show that radiative effects are important when right-handed neutrinos are very hierarchical.
- cf: split seesaw. [A. Kusenko, F. Takahashi, T. Yanagida, Phys.Lett. B \(2010\)](#)
 $M_1 \sim \text{keV}$, and $M_2, M_3 \sim 10^{12} \text{ GeV}$
- No intermediate scale



The Model (Type-I seesaw)

- Add three right-handed neutrinos.

$$\mathcal{L} = \frac{1}{2} \overline{N_i} \not{\partial} N_i - \frac{M_{ij}}{2} \overline{N_i^c} N_j - (Y_\nu)_{ij} \tilde{H} \overline{L_i} N_j + \text{H.c.}$$

- Assumption: rank-1 mass matrix at the Planck scale.

$$M = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & M_3 \end{pmatrix}, \quad M = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{pmatrix}$$

at Planck scale

at Electroweak scale

- M_1 and M_2 are generated by radiative effect.
 \Rightarrow Renormalization group equation (RGE) for M .

Renormalization Group Equation for M

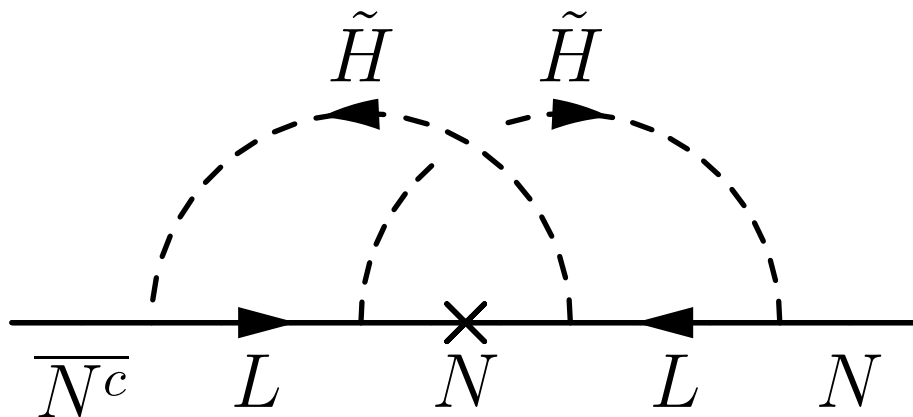
- At 1-loop, only one diagram contributes

$$\beta_M^{1\text{-loop}} = \frac{dM}{dt} = \frac{1}{(4\pi)^2} \left[(Y_\nu^\dagger Y_\nu)^T M + M (Y_\nu^\dagger Y_\nu) \right]$$

- At 2-loop, there are many contributions

$$\beta_M^{2\text{-loop}} = \frac{dM}{dt} = \frac{4}{(4\pi)^4} (Y_\nu^\dagger Y_\nu)^T M (Y_\nu^\dagger Y_\nu) + \dots$$

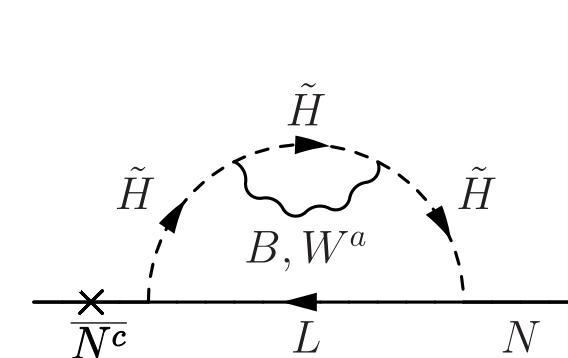
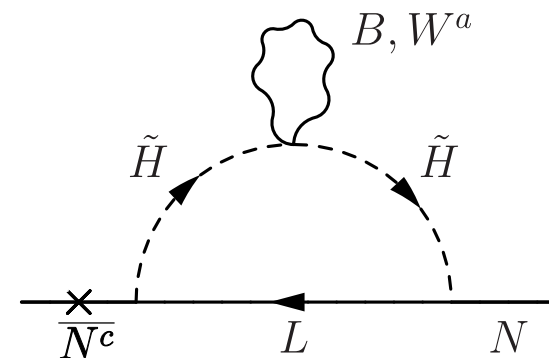
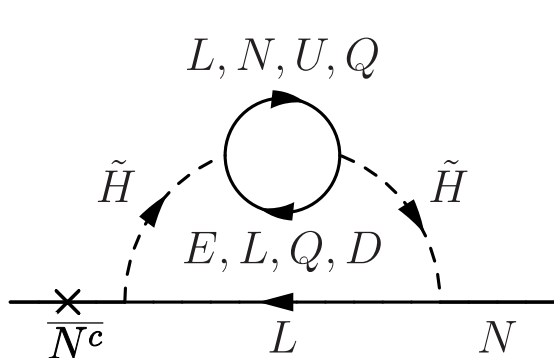
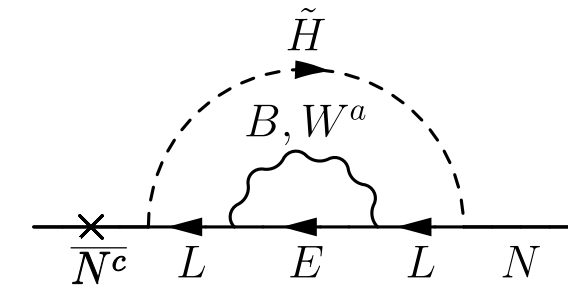
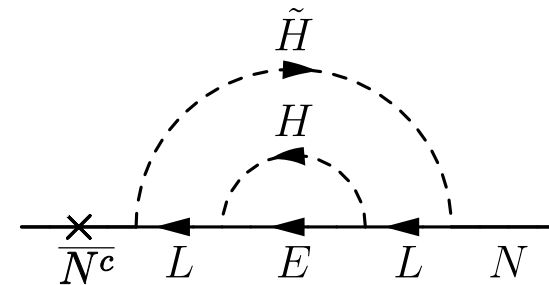
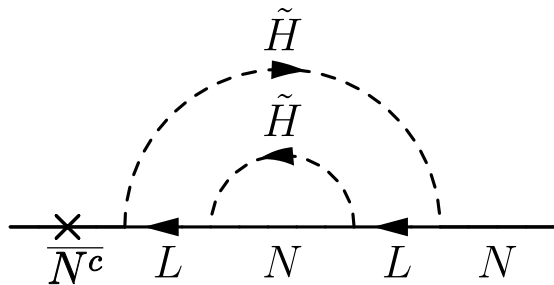
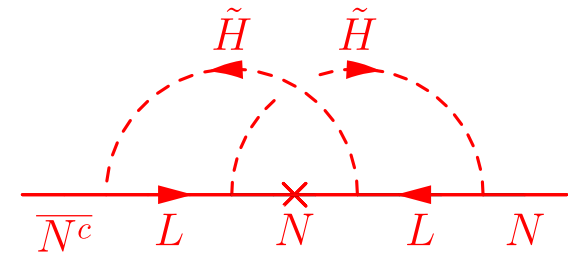
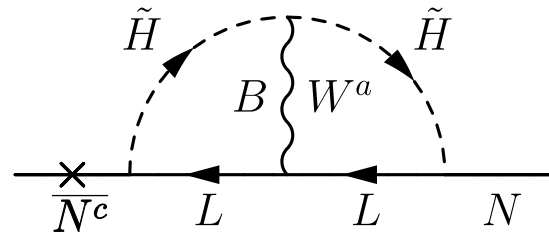
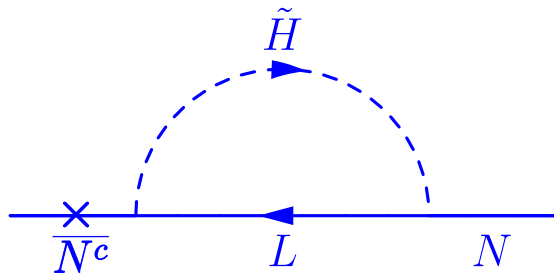
Rank increasing diagram



the other diagrams do not increase rank of M .

Renormalization Group Equation for M

All diagrams



Renormalization Group Equation for M

Full beta function

$$\begin{aligned}
 \frac{dM}{dt} = & \frac{1}{(4\pi)^2} \left[(Y_\nu^\dagger Y_\nu)^T M + M (Y_\nu^\dagger Y_\nu) \right] + \frac{4}{(4\pi)^4} (Y_\nu^\dagger Y_\nu)^T M (Y_\nu^\dagger Y_\nu) \\
 & + \frac{1}{(4\pi)^4} \left[\frac{17}{8} (g_Y^2 + g_2^2) (Y_\nu^\dagger Y_\nu) - \frac{1}{4} Y_\nu^\dagger Y_\nu Y_\nu^\dagger Y_\nu - \frac{1}{4} Y_\nu^\dagger Y_e Y_e^\dagger Y_\nu \right. \\
 & \left. - \frac{3}{2} \text{Tr} \left(Y_e^\dagger Y_e + Y_\nu^\dagger Y_\nu + 3Y_u^\dagger Y_u + 3Y_d^\dagger Y_d \right) (Y_\nu^\dagger Y_\nu) \right]^T M \\
 & + \frac{1}{(4\pi)^4} M \left[\frac{17}{8} (g_Y^2 + g_2^2) (Y_\nu^\dagger Y_\nu) - \frac{1}{4} Y_\nu^\dagger Y_\nu Y_\nu^\dagger Y_\nu - \frac{1}{4} Y_\nu^\dagger Y_e Y_e^\dagger Y_\nu \right. \\
 & \left. - \frac{3}{2} \text{Tr} \left(Y_e^\dagger Y_e + Y_\nu^\dagger Y_\nu + 3Y_u^\dagger Y_u + 3Y_d^\dagger Y_d \right) (Y_\nu^\dagger Y_\nu) \right]
 \end{aligned}$$

- We include only M and Y_ν .

Mass eigenvalues

- General parametrization at two-loop level:

$$\frac{dM}{dt} = P^T M + M P + Q^T M Q$$

where P, Q are matrices written by coupling constants.

Ex: $P = \frac{Y_\nu^\dagger Y_\nu}{(4\pi)^2}$ at 1-loop level, $Q = \frac{2Y_\nu^\dagger Y_\nu}{(4\pi)^2}$ at 2-loop level.

- \Rightarrow In terms of mass eigenvalues

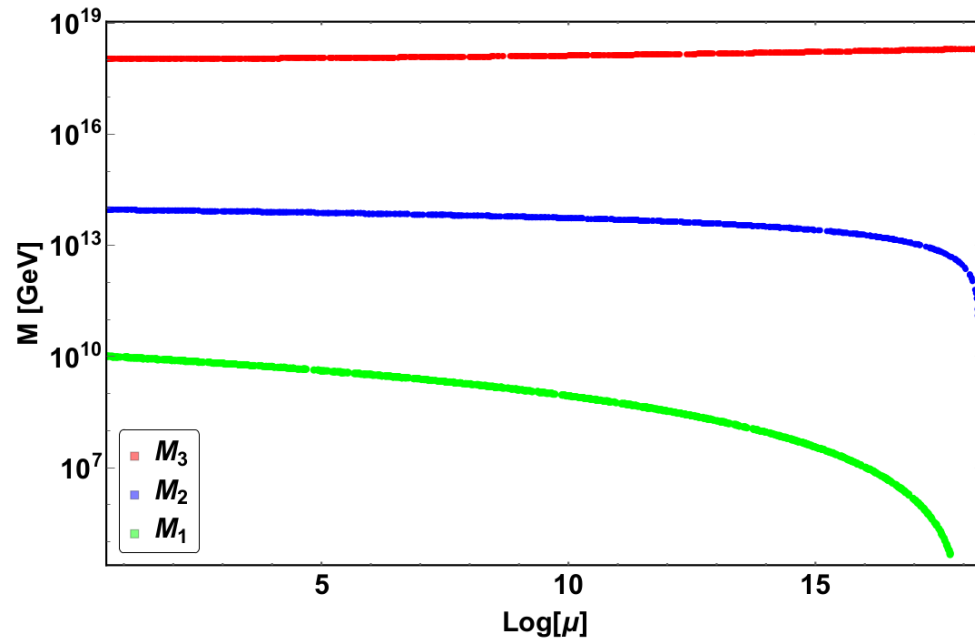
$$\frac{dM_i}{dt} = 2M_i \hat{P}_{ii} + \sum_k M_k \text{Re} \left[\hat{Q}_{ki}^2 \right] + \text{RGEs for } U$$

where $U^T M U = \text{diag}(M_1, M_2, M_3)$, $\hat{P} = U^\dagger P U$, $\hat{Q} = U^\dagger Q U$

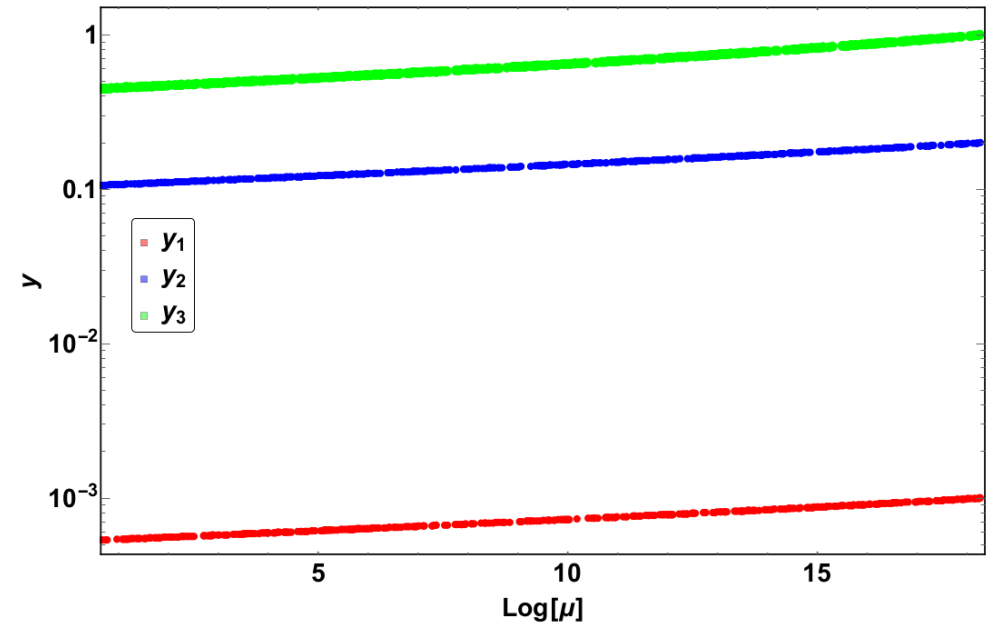
- Only $Q^T M Q$ term increases the rank of M .

Example Plots

RHN mass



Yukawa



- Parametrization: $Y_\nu = V_R \text{diag}(y_1, y_2, y_3) V_L^\dagger$
- Initial values at $\Lambda = M_P$: $\theta_{23} = 0.1$, $\theta_{13} = 0.2$, $\theta_{12} = 0.3$,
 $y_1 = 0.001$, $y_2 = 0.2$, $y_3 = 1$, $M_1 = M_2 = 0$, $M_3 = M_P$

Analytic results

- Formal RGE: $\frac{dM_i}{dt} = 2M_i\hat{P}_{ii} + \sum_k M_k \text{Re} \left[\hat{Q}_{ki}^2 \right]$
- Mass eigenvalues of right-handed neutrinos at scale μ obtained by iterative integration

$$M_1 \approx \frac{1}{2} M_3(M_P) \text{Re} \left(\hat{Q}_{32}^2 \right) \text{Re} \left(\hat{Q}_{21}^2 \right) \log^2 \left(\frac{\mu}{M_P} \right),$$

$$M_2 \approx M_3(M_P) \text{Re} \left(\hat{Q}_{32}^2 \right) \log \left(\frac{\mu}{M_P} \right),$$

$$M_3 = M_3(M_P)$$

only leading contribution is considered.

$\hat{Q} = U^\dagger Q U$ and U is unitary matrix diagonalizing M .

- M_1 is 4-loop order? \Rightarrow squared logarithm is dominant

Analytic results 2

For small neutrino masses: $m_\nu = -m_D M^{-1} m_D^T$ ($m_D = Y_\nu \langle H \rangle$)

- mass eigenvalues satisfy $m_1 m_2 m_3 = \frac{y_1^2 y_2^2 y_3^2 \langle H \rangle^6}{M_1 M_2 M_3}$

where parametrized as $Y_\nu = V_R Y_D V_L^\dagger$

- One can choose like: $m_3 \sim \frac{y_3^2 \langle H \rangle^2}{M_2}$, $m_2 \sim \frac{y_2^2 \langle H \rangle^2}{M_1}$, $m_1 \sim \frac{y_1^2 \langle H \rangle^2}{M_3}$

Heaviest neutrino mass: $m_3 \sim \mathcal{O}(0.1) \text{ eV} \left(\frac{y_3}{1}\right)^{-2} \left(\frac{M_3}{M_P}\right)^{-1}$

\Rightarrow correct mass scale is obtained with $y_3 = \mathcal{O}(1)$ because

$M_2 \sim M_P / (4\pi)^4 \sim 10^{14} \text{ GeV}$

Lightest neutrino mass: $m_1 \lesssim 10^{-5} \text{ eV}$ (almost massless).

m_2 and m_3 correspond to solar and atmospheric neutrino masses.

Rank increasement and flavor symmetry

- Number of rank increasement can be understood with global flavor symmetry

$$\mathcal{L} = - (Y_\nu)_{ij} H \overline{N}_i L_j - \frac{M_{ij}}{2} \overline{N}_i^c N_j + \text{H.c.}$$

- Neutrino masses are protected by $U(3)_L \times U(3)_N$ symmetry if $Y_\nu = M = 0$.

		rank Y_ν		
		1	2	3
rank M	1	$U(2)_L \times U(1)_N$	$U(1)_L$	No symmetry
	2	$U(2)_L$	$U(1)_L$	No symmetry
	3	$U(2)_L$	$U(1)_L$	No symmetry

Ex.1 $U(2)_L \times U(1)_N \Rightarrow$ two ν_L and one N are massless

Ex.2 $U(1)_L \Rightarrow$ one ν_L is massless

Number of Parameters

Parametrize $Y_\nu = V_R Y_D V_L^\dagger$

- For rank $Y_\nu = 3$ and rank $M = 3$ (usual case)
eigenvalues $y_{1,2,3}$, $M_{1,2,3}$ and 6 mixing angles and 6 CP phases
 \Rightarrow 18 parameters
- For rank $Y_\nu = 2$ and rank $M = 1$
eigenvalues $y_{2,3}$, M_3 and 5 mixing angles and 3 CP phases
 \Rightarrow 11 parameters (minimal number of parameters)
- CP phases are reduced: 6 \Rightarrow 3
Predictive Leptogenesis

Summary

- 1 If right-handed neutrino masses are highly hierarchical at the Planck scale, radiative corrections dominate right-handed neutrino masses at low scale.
- 2 Without any intermediate scale, small neutrino masses are naturally generated by two lighter RHNs.
- 3 Number of parameters are reduced and this framework leads predictive phenomenology.

Future Works

- 1 Application to leptogenesis, sterile neutrino dark matter.