Spontaneous Leptogenesis in Continuum-Clockwork Axion Models

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[IBS-CTPU]

[arXiv:1808.XXXXX]

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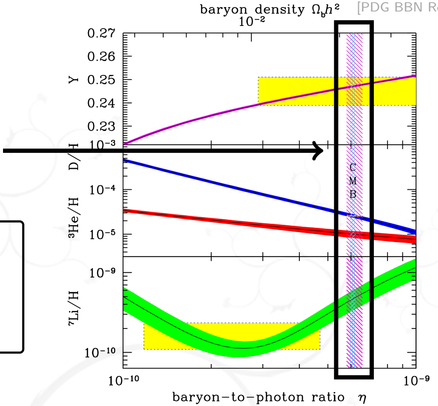
Outstanding Issues

matter-antimatter asymmetry:

need to explain baryon-to-photon ratio:

$$\eta_B^0 \equiv \frac{n_b^0 - n_{\bar{b}}^0}{n_\gamma^0} \approx \frac{n_b^0}{n_\gamma^0} \approx 6 \times 10^{-10}$$

Typically need ***Sakharov conditions***:
- B and/or L violation
- C and CP violation
- departure from thermal equilibrium



[PDG BBN Review]

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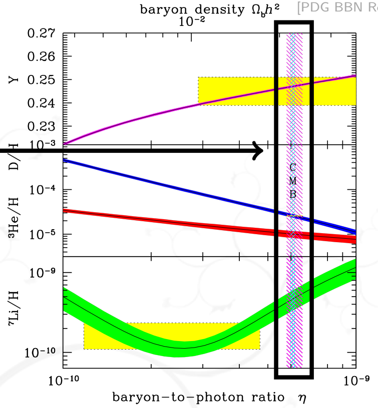

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alternatives exist if $\not{CPT}$ spontaneously,
e.g., with homogeneous scalar field ϕ:

$$\mathcal{L}_{\text{eff}} \supset \underbrace{\frac{1}{f}\partial_\mu\phi}_{\text{NG boson}} \overbrace{J_\ell^\mu}^{\text{lepton current}} \approx \frac{\dot{\phi}}{f}(n_\ell - n_{\bar{\ell}}) \equiv \mu_{\text{eff}} n_L$$

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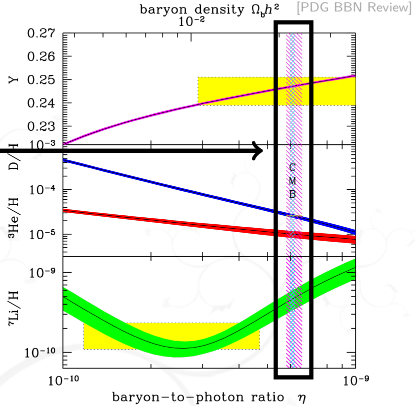

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lepton current

NG boson

effective **chemical potential**

$$\mu_{\text{eff}} \equiv \dot\phi / f$$

for lepton number density n_L

⇒ opportunity for asymmetry generation **even at equilibrium**

The $\mu_{\text{eff}} \neq 0$ **shifts equilibrium** n_L^{eq} value away from zero:

$$n_L^{\text{eq}} \propto \int \frac{d^3p}{(2\pi)^3} \left[\frac{1}{e^{(p-\mu_{\text{eff}})/T}+1} - \frac{1}{e^{(p+\mu_{\text{eff}})/T}+1} \right] \approx \frac{1}{6}\mu_{\text{eff}} T^2$$

when L-violation occurs sufficiently fast $\Gamma_L \gg H$ but in general

$$\dot{n}_L + 3Hn_L \;=\; -\,\Gamma_L \left(n_L^{\text{eq}} - n_L\right)$$

$\Rightarrow$ need to specify a **source of L-violation**:

Assume Weinberg operators, with heavy $M_* \sim \Lambda_{\text{GUT}} \gg T$ right-handed neutrinos:

$$\mathcal{L}_{\text{eff}} \supset -\frac{(LH)^2}{2M_*} \;,$$

then the rate is fixed by experiment:

$$\Gamma_L = 4 n_\ell^{\text{eq}} \langle \sigma_L v \rangle \sim \mathcal{O}(10^8\,\text{GeV}) \left(\frac{T}{10^{13}\,\text{GeV}}\right)^3 \;.$$

Spontaneous Leptogenesis via Axions [Kusenko '18]

- An **axion-like field** is a natural candidate for $\phi(x)$:

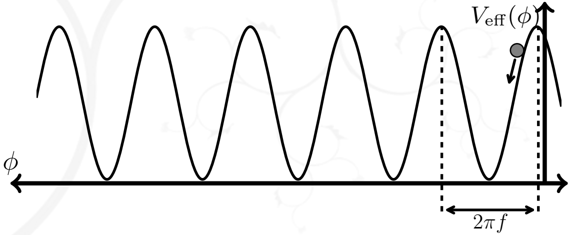
$$\mathcal{L}_{\rm eff} \supset \frac{1}{f}\partial_\mu \phi J^\mu_\ell \sim -\frac{\phi}{f}\partial_\mu J^\mu_\ell$$

$\Rightarrow$ recast using $U(1)_L$ anomalies:

$$-\frac{\phi}{f}\partial_\mu J^\mu_\ell = \frac{\phi}{f}\Big(\underbrace{\frac{N_f g_2^2}{8\pi^2} W_{\mu\nu}\widetilde{W}^{\mu\nu}}_{\text{weak}} - \underbrace{\frac{N_f g_1^2}{8\pi^2} B_{\mu\nu}\widetilde{B}^{\mu\nu}}_{\text{hypercharge}} \Big)$$

which can readily arise [*e.g.*, string axion models]

- With typical $V(\phi) \sim \Lambda^4 \cos(\phi/f)$ potential, successful leptogenesis requires **heavy** $m_\phi \gtrsim 10^8$ GeV $\Rightarrow$ axion decays and is **not suitable DM candidate**



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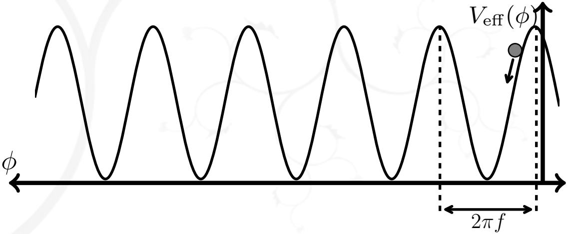
consider physics that deforms effective potential such that $V_{\rm eff}(\phi)$ **retains curvature** at edges but mass m_ϕ is **suppressed**



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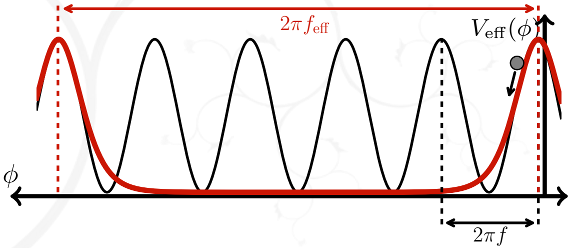

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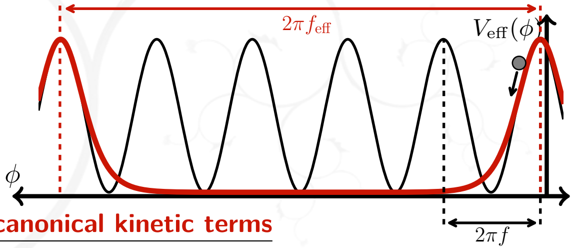


how?: this can appear via **non-canonical kinetic terms**

Non-Canonical Kinetic Term

- As a toy model, consider a periodic scalar field $\theta(x)$ with non-trivial wavefunction renormalization $Z(\theta)$:

$$\begin{aligned}\mathcal{L} &= -\frac{Z(\theta)}{2} f^2 (\partial_\mu \theta)^2 - \Lambda^4 (1-\cos\theta) + \frac{\theta}{8\pi^2}\left[F\widetilde{F}\right]_{\rm SM} \\ &\equiv -\frac{1}{2}(\partial_\mu \phi)^2 - V_{\rm eff}(\phi)\ , \end{aligned} \tag{1}$$

so that $\phi/f \equiv \int \sqrt{Z(\theta)}d\theta$ is canonically normalized.

- The effective potential $V_{\rm eff}(\phi)$ has the desired properties if $Z(\theta) \gg 1$ towards the minimum ($\theta \to 0$), and $Z(\theta) \to 1$ at the edges

for example:

$$Z(\theta) = \frac{1}{1+\epsilon-\cos\theta} \quad \text{for} \quad \epsilon \ll 1$$

$\Rightarrow$ suppresses mass $m_{\rm eff}^2 = \dfrac{\Lambda^4}{f^2}\epsilon$ and enhances lifetime $\tau \sim \dfrac{1}{\sqrt{\epsilon^5}}$

physical origin for this scenario?: $\longrightarrow$ **continuum-clockwork models**!

***discrete* clockwork mechanism:** [Choi *et al.* '14] [Kaplan, Rattazzi '15] [Choi, Im '15]

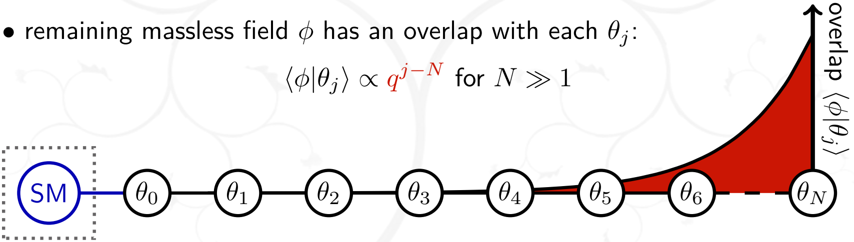
- $N+1$ scalars θ_j with "nearest-neighbor" interactions ($q \in \mathbb{Z}$):

$$\mathcal{L} \supset -\sum_{j=0}^{N-1} \mu^2 f^2 \cos\left(\theta_{j+1} - q\theta_j\right)$$

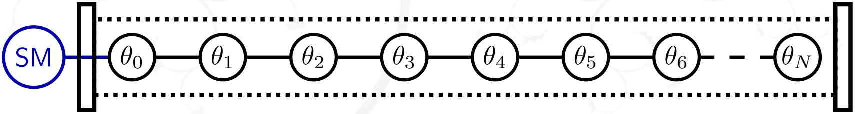
$\Rightarrow U(1)^{N+1}$ broken down to $U(1)_{\rm CW} : \theta_j \to \theta_j + \alpha q^j$

- remaining massless field ϕ has an overlap with each θ_j:

$$\langle \phi | \theta_j \rangle \propto q^{j-N} \text{ for } N \gg 1$$

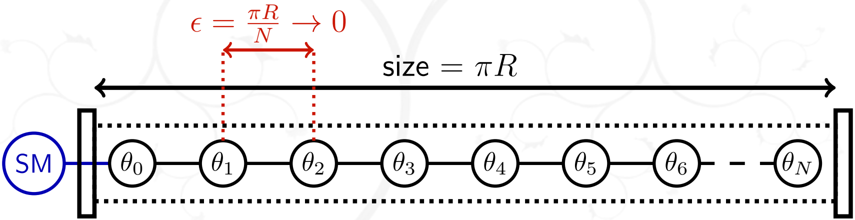


any couplings $q^{-N}\phi\mathcal{O}_{\rm SM}$ **exponentially suppressed**


SM
θ_0
θ_1
θ_2
θ_3
θ_4
θ_5
θ_6
θ_N

consider $N \to \infty$ **continuum limit**:

identify $y \equiv j\epsilon$ with **extra spatial coordinate**



Continuum-Clockwork Axion

[Giudice *et al.* '16] [Craig *et al.* '17] [Choi *et al.* '17]

In continuum limit, find action for resulting bulk field $\theta(x,y)$:

$$\mathcal{S} \;=\; -\frac{f_5^3}{2}\int d^5x\left[(\partial_\mu\theta)^2 + (\partial_y\theta - m\sin\theta)^2\right]$$

- massless 4D mode $\phi(x)$ realized as:

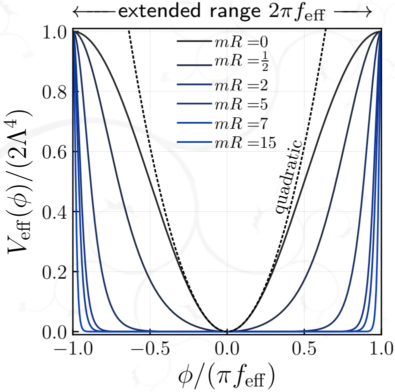
$$\tan\left[\frac{\theta(x,y)}{2}\right] = e^{my}\,\underbrace{u\left[\phi(x)\right]}_{ie^{-\pi m R}\mathrm{sn}\left[\frac{\phi(x)}{2if}\middle|e^{-2\pi m R}\right]}$$

- any small deviation in boundary masses generates a potential:

$$V_{\text{eff}}(\phi) = \Lambda^4\left\{1-\cos\left[\theta(x,0)\right]\right\}$$
$$= \frac{2\Lambda^4\left[u(\phi)\right]^2}{1+\left[u(\phi)\right]^2}$$

$$\Rightarrow \quad m_{\text{eff}}^2 \equiv \left.\frac{\partial^2 V_{\text{eff}}}{\partial\phi^2}\right|_{\phi=0} = e^{-2\pi m R}\frac{\Lambda^4}{f^2}$$

so curvature as $\phi\to 0$ is **suppressed**



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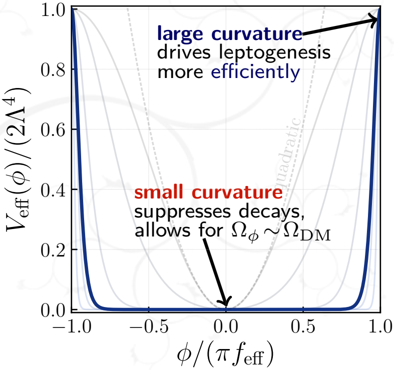
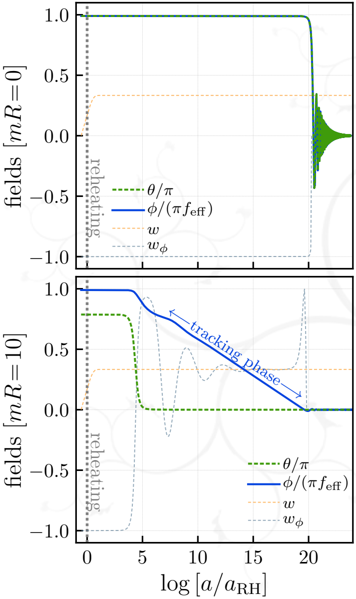


Some Interesting Early Dynamics



$m_{\rm eff} = 1$eV $\quad f_{\rm eff} = 10^{13}$GeV

taking $mR \neq 0$ shows **substantial** differences:

- *prior* to undergoing coherent oscillations, axion field enters **"tracking phase"** (similar to that found in models of quintessence):

$$\phi(t) \approx -2f \log\left[\frac{m_{\rm eff}t}{\sqrt{2}} + \text{constant}\right]$$

for many e-foldings that drives $w_\phi \to w = \frac{1}{3}$

- tracking **prevents $\mu_{\rm eff} = \dot{\theta}$ oscillations** that would wash out asymmetry

- deformation of $mR > 0$ potential triggers leptogenesis at **higher temperatures**

- n_L never reaches equilibrium value $n_L^{\rm eq}$ $\Rightarrow$ asymmetry generated through **"freeze-in"** process, in contrast to traditional models

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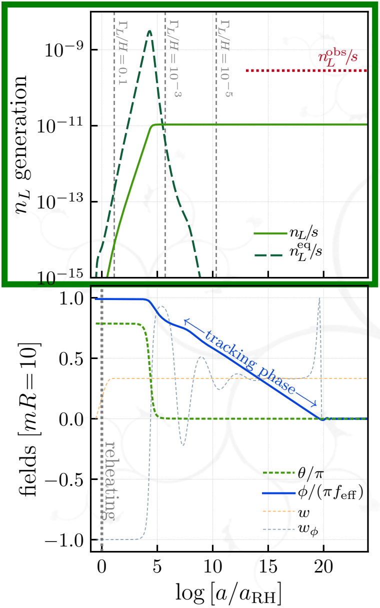

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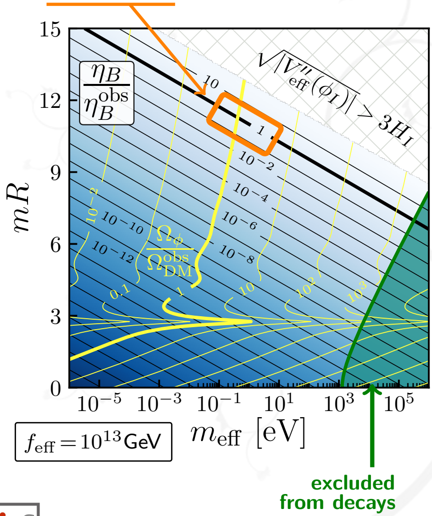
Viable Regions from Numerical Simulations

the **baryon asymmetry** η_B and **axion abundance** Ω_ϕ can be achieved simultaneously in observed amounts:



- decays suppressed not only by small effective mass $m_{eff} = \Lambda^2 e^{-\pi m R}/f$, but also by suppressed couplings

$$\Gamma_\phi \sim \frac{m_{eff}^3}{f^2} e^{-\pi m R}$$

- abundance at large mR only weakly depends on clockwork factor $e^{\pi m R}$:

$$\frac{\Omega_\phi}{\Omega_{DM}^{obs}} \approx \left[\frac{f_{eff}/\log\left(4e^{\pi m R}\right)}{10^{12}\text{GeV}}\right]^2 \sqrt{\frac{m_{eff}}{3.7\text{meV}}}$$

and **insensitive** to initial field

constraints from **isocurvature**?
at first glance, presents a *major concern* for this type of scenario

Isocurvature Perturbations

The axion is subject to de-Sitter quantum fluctuations during inflation

$$\delta\phi = \frac{H_I}{2\pi}$$

In our model, ultimately manifested in isocurvature mode *two ways*:

① **axion-photon** isocurvature

$$S_{\phi\gamma} \equiv \frac{\delta_\phi}{1+w_\phi} - \frac{3}{4}\delta_\gamma$$

② **baryon-photon** isocurvature

$$S_{B\gamma} \equiv \frac{\delta n_B}{n_B} - \frac{3}{4}\delta_\gamma$$

both show **significant departures** from standard $(mR = 0)$ case

Isocurvature Perturbations

Baryon Component

- When ϕ is slowly rolling most of baryon number density n_B is produced:

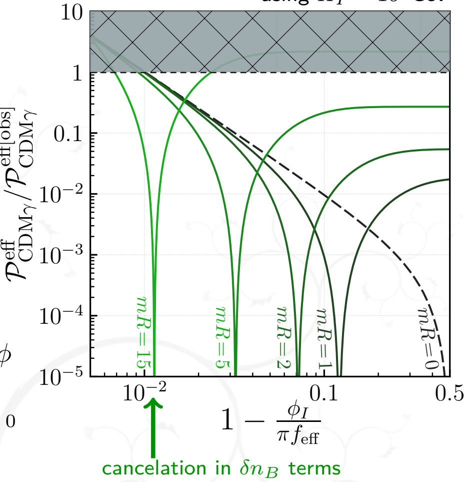
$$n_B \;\propto\; \dot{\theta} T^2 \;\propto\; \frac{[V'_{\text{eff}}(\phi)]^2}{\sqrt{\left[1 - \frac{V_{\text{eff}}}{2\Lambda^4}\right] \frac{V_{\text{eff}}}{2\Lambda^4}}}$$

$\Rightarrow$ perturbation at the end of slow roll:

$$\frac{\delta n_B}{n_B} \approx \left\{ 2\frac{V''_{\text{eff}}(\phi)}{V'_{\text{eff}}(\phi)} - \frac{1}{2}\frac{V'_{\text{eff}}(\phi)}{V_{\text{eff}}(\phi)} \left[\frac{1 - \frac{V_{\text{eff}}}{\Lambda^4}}{1 - \frac{V_{\text{eff}}}{2\Lambda^4}} \right] \right\} \delta\phi$$

inflection points/cancelations in terms can drive $\rightarrow 0$

using $H_I = 10^8$ GeV



- Numerical simulations show **additional suppression** by up to $\mathcal{O}(10)$ factor

Isocurvature Perturbations
Axion Component

• The tracking behavior in axion field implies a non-trivial evolution in $S_{\phi\gamma}$

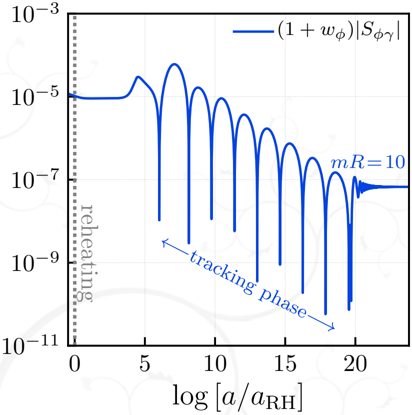
$$\frac{1}{2}\frac{d\left[(1+w_\phi)S_{\phi\gamma}\right]}{d\log a} = \Gamma$$

$$-2\left[(1+w_\phi)S_{\phi\gamma}\right] - \Gamma = \frac{d\Gamma}{d\log a}$$

where it is coupled to the intrinstic entropy perturbation:

$$\Gamma \equiv \frac{\overbrace{\delta P_\phi}^{\text{pressure perturbation}}/\rho_\phi - c_\phi^2\delta_\phi}{1-\underbrace{c_\phi^2}_{\text{adiabatic sound speed}}}$$

$\Rightarrow$ can be solved analytically to show amplitude of $S_{\phi\gamma} \propto 1/\sqrt{a}$ **falls** while axion follows **tracking trjaectory**



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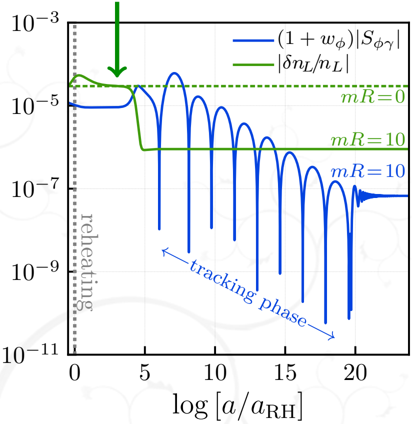
$\delta n_B/n_B$ can also be dynamically suppressed



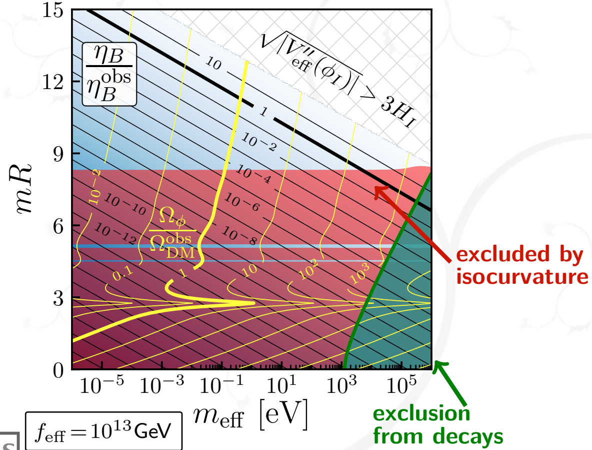
axion tracking dynamics lead generically to a ***suppression*** of the axion $S_{\phi\gamma}$ and baryon $S_{B\gamma}$ isocurvature modes

Revisiting the Viable Regions

Including Isocurvature Constraints

We cannot actually discriminate between $S_{B\gamma}$ and $S_{\phi\gamma}$ contributions in CMB observations — together these bound *effective* CDM isocurvature:

$$\mathcal{P}^{\text{eff}}_{\text{CDM}\gamma}(k_*) \equiv \left[\frac{\Omega_B}{\Omega_{\text{CDM}}} S_{B\gamma} + \frac{\Omega_\phi}{\Omega_{\text{CDM}}} S_{\phi\gamma}\right]^2 \lesssim 8.8 \cdot 10^{-11}$$



$f_{\text{eff}} = 10^{13}$ GeV

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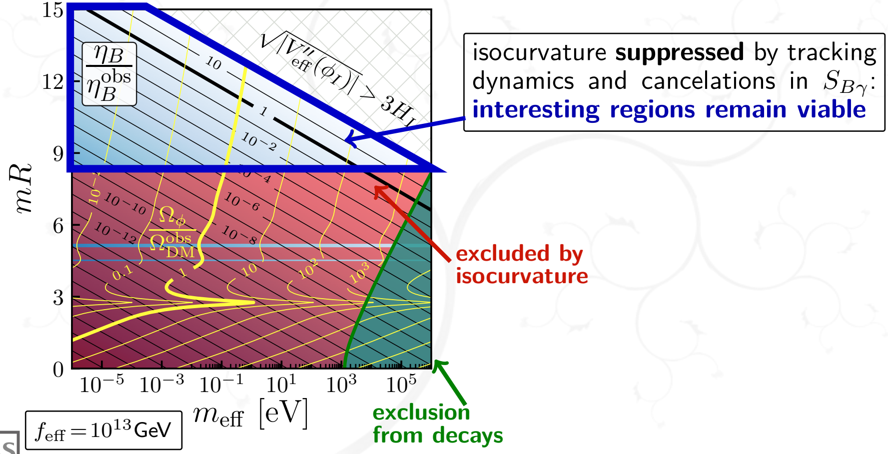


TAKE-AWAY MESSAGE:

Taking advantage of deformations in the effective potential of **continuum-clockwork axion**, we can produce appropriate **matter-antimatter asymmetry** via spontaneous leptogenesis, with axion serving as **dark matter** candidate with proper abundance

○ axion exhibits **"tracking behavior"** — driven towards radiation-like equation of state prior to coherent oscillations

○ **suppression of isocurvature** and more efficient asymmetry production due to these dynamics

○ we have provided an example, but ***in general*** *this can arise from models with non-canonical kinetic terms*

THANK YOU FOR YOUR ATTENTION!

[BEGIN BACK-UP SLIDES]

The *Discrete*-Clockwork [Choi, Kim, Yun '14] [Choi, Im '15] [Kaplan, Rattazzi '15]

A Brief Review for Context

In the *discrete* clockwork mechanism $N+1$ axions θ_j are coupled by "nearest-neighbor" interactions (with $q \in \mathbb{Z}$):

$$\mathcal{L} = -\frac{1}{2}\sum_{j=0}^{N} f^2 \left(\partial_\mu \theta_j\right)^2 + \sum_{j=0}^{N-1} \mu^2 f^2 \cos\left(\theta_{j+1} - q\theta_j\right)$$

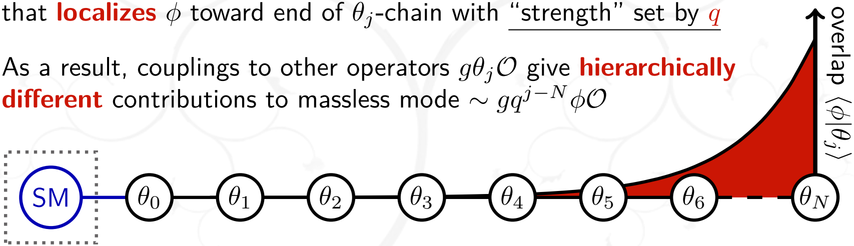
$\Rightarrow$ symmetry $U(1)^{N+1}$ broken down to a single $U(1)_{\mathrm{CW}} : \theta_j \to \theta_j + \alpha q^j$

The massless field ϕ associated with $U(1)_{\mathrm{CW}}$ has an overlap with each θ_j:

$$\langle \phi | \theta_j \rangle \propto q^{j-N} \text{ for } N \gg 1$$

that **localizes** ϕ toward end of θ_j-chain with "strength" set by q

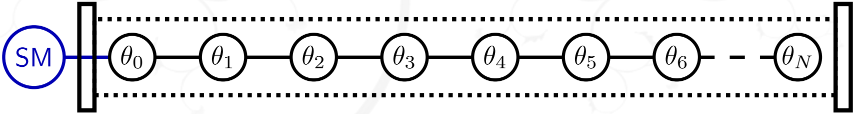
As a result, couplings to other operators $g\theta_j\mathcal{O}$ give **hierarchically different** contributions to massless mode $\sim g q^{j-N} \phi \mathcal{O}$



any couplings $q^{-N}\phi\mathcal{O}_{\mathrm{SM}}$ **exponentially suppressed**

The *Discrete*-Clockwork [Choi, Kim, Yun '14] [Choi, Im '15] [Kaplan, Rattazzi '15]

A Brief Review for Context

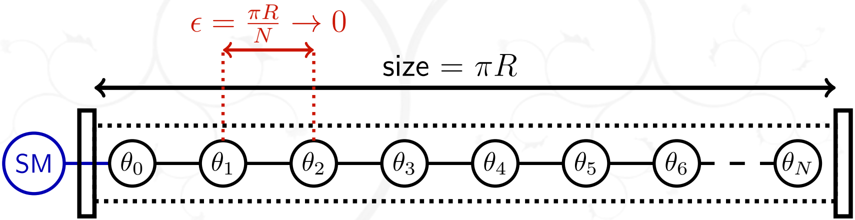


The *Discrete*-Clockwork [Choi, Kim, Yun '14] [Choi, Im '15] [Kaplan, Rattazzi '15]

A Brief Review for Context

consider $N \to \infty$ **continuum limit**:

identify $y \equiv j\epsilon$ with **extra spatial coordinate**



The *Continuum*-Clockwork Axion [Giudice *et al.* '16] [Craig *et al.* '17] [Choi *et al.* '17]

Taking the number of clockwork fields $N \to \infty$ we can construct a **continuum limit**:

$$\mathcal{S} = -\frac{f_5^3}{2} \int d^5x \left[(\partial_\mu \theta)^2 + (\partial_y \theta - m \sin\theta)^2 \right]$$

with y interpreted as coordinate of an $S^1/\mathbb{Z}_2$ flat **extra dimension** of size πR, and $\theta = \theta(x, y)$ is now a bulk angular field with periodicity $\theta \to \theta + 2\pi$

- The bulk/boundary masses above furnish a **massless 4D axion** $\phi(x)$:

$$\tan\left[\frac{\theta(x,y)}{2}\right] = e^{my} u\left[\phi(x)\right]$$

where

$$u\left[\phi(x)\right] = i e^{-\pi m R} \mathrm{sn}\left[\frac{\phi(x)}{2if} \middle| e^{-2\pi m R}\right]$$

sn: Jacobi elliptic function

$$f \equiv \sqrt{\frac{f_5^3}{2m}(1 - e^{-2\pi m R})}$$

- A mode expansion for $\delta\theta \equiv \theta - \langle\theta\rangle$

$$\delta\theta(x,y) = f_0(y)\delta\phi(x) + \sum_{n=1}^{\infty} f_n(y)\delta\phi_n(x)$$

shows **localization** of the massless mode $\delta\phi$ is **retained**:

$$f_0(y) \propto \mathrm{sech}\left[m\left(y - y_0\right)\right]$$

$$y_0 \equiv -\log\left[\langle u(\phi)\rangle\right]$$

localization dependent on 4D axion VEV $\langle\phi\rangle$

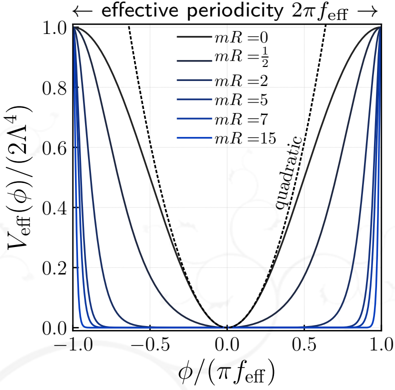
Effective 4D-Axion Potential [Choi, Im, Shin '17]

- While this axion ϕ is massless, a small deviation in boundary mass **generates** an effective potential:

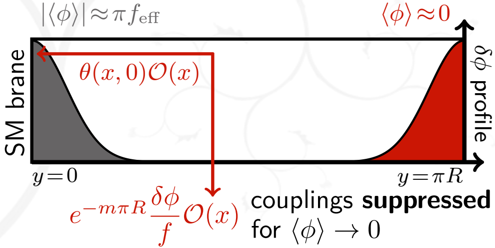
$$V_{\text{eff}}(\phi) = \Lambda^4 \left\{1 - \cos\left[\theta(x,0)\right]\right\}$$
$$= \frac{2\Lambda^4 \left[u(\phi)\right]^2}{1 + \left[u(\phi)\right]^2}$$

"clockwork factor" analogous to q^{2N}

$$\Rightarrow \quad m_{\text{eff}}^2 \equiv \left.\frac{\partial^2 V_{\text{eff}}}{\partial \phi^2}\right|_{\phi=0} = e^{-2\pi m R}\,\frac{\Lambda^4}{f^2}$$



shape of potential $V_{\text{eff}}(\phi)$ is **contorted** as mR deviates from zero, since position of localization set by $\langle\phi\rangle$



couplings **suppressed** for $\langle\phi\rangle \to 0$

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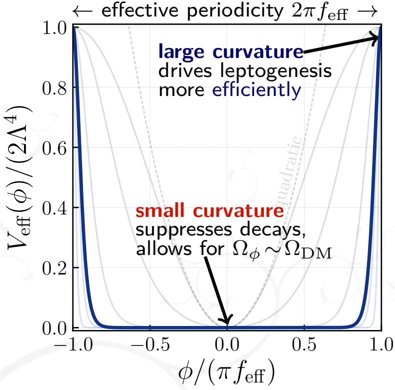

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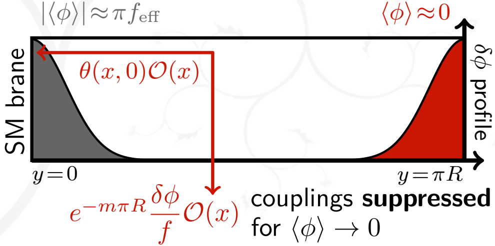

Summary of the Model

The effective four-dimensional model then appears as

$$\mathcal{S}_{\text{eff}} = \int d^4x \sqrt{-g}\Big[-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V_{\text{eff}}(\phi) - \mu_{\text{eff}} n_L + \cdots\Big]$$

The leptons couple to $\theta \equiv \theta(x,0)$:

$$\mu_{\text{eff}} \equiv \dot{\theta} = \frac{\frac{\text{sgn}\{\phi\}}{2\Lambda^4} V'_{\text{eff}}(\phi)}{\sqrt{\left(1-\frac{V_{\text{eff}}}{2\Lambda^4}\right)\frac{V_{\text{eff}}}{2\Lambda^4}}}\dot{\phi}$$

$\Rightarrow n_L^{\text{eq}} \propto \mu_{\text{eff}}$ non-linear function of both $\{\phi, \dot{\phi}\}$ which **significantly** alters dynamics from the standard scenario:

$$\frac{d(n_L/s)}{d\log T} = \underbrace{\frac{T}{T_L}}_{T_L \approx 1.73\cdot 10^{13}\ \text{GeV}}\left(\frac{n_L}{s} - \frac{n_L^{\text{eq}}}{s}\right)$$

To model during/after reheating:

$$\overbrace{\dot{\rho}_\varphi + 3H\rho_\varphi = -\Gamma_\varphi \rho_\varphi}^{\text{inflaton}}$$

$$\underbrace{\dot{\rho}_R + 4H\rho_R}_{\text{radiation}} = +\Gamma_\varphi\rho_\varphi + \Gamma_\phi\rho_\phi$$

and the axion evolution goes as

$$\ddot{\phi} + (3H + \Gamma_\phi)\dot{\phi} + \frac{\partial V_{\text{eff}}}{\partial \phi} = \underbrace{\frac{\partial \mu_{\text{eff}}}{\partial \dot{\phi}}\Gamma_L(n_L - n_L^{\text{eq}})}_{\text{backreaction usually negligible}}$$