Supersymmetric Dirac-Born-Infeld action from N=2 supergravity

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Talk Plan

- Introduction (motivation)
- Target : partial breaking (PB) model of N=2 supergravity = FGP model
  S. Ferrara, L. Girardello, M. Porrati, **Phys.Lett. B376 (1996) 275-281**
- Sugra Dirac-Born-Infeld (DBI) action from PB
- Summary

Introduction

- Susy DBI action has much interesting aspects, e.g., EFT of D-brane, duality structure,…

$$L = 1 - \sqrt{-\det\left(\eta_{ab} + F_{ab}\right)},$$

- Coupling with supergravity (sugra) is important for cosmological applications.
- There are a lot of works about sugra DBI :

e.g., S. Cecotti and S. Ferrara, (1987)
S. M. Kuzenko and S. A. McCarthy, (2003)
H. Abe, Y. Sakamura, Y. Yamada (2015)
…

Introduction

- N=1 susy DBI is related to partial breaking of N=2 susy (appears as effective action of Goldstone multiplet)
  J. Bagger and A.A. Galperin, (1997)
  M. Rocek and A.A. Tseytlin, (1999)
- Previous works focus on the remaining N=1 susy and extend susy DBI to supergravity
- Direct derivation from N=2 supergravity is still missing (our goal)→ the resultant action should respect the broken susy.

Let's start from the known N=2 sugra model which can cause PB = FGP model

S. Ferrara, L. Girardello, M. Porrati, (1996)

Set up

- One vector and one hyper multiplet

Gravity multiplet: $\{e^a_\mu, \psi^A_\mu, A^0_\mu\}$ $A = 1{,}2$

vector multiplet: $\{z, \lambda^A, A^1_\mu\}$

hyper multiplet: $\{b^u, \zeta_\alpha\}$ $u = 0{,}1{,}2{,}3, \alpha = 1{,}2$

- Vector sector

Kahler potential: $K = -\log i\left(\bar{X}^\Lambda F_\Lambda - \bar{F}_\Lambda X^\Lambda\right)$ $\Lambda, \Sigma = 0, 1,$

Holomorphic prepotential: F $F_\Lambda = \dfrac{\partial F}{\partial X^\Lambda}$

General expression: $F = -i(X^0)^2 f(X^1/X^0)$

$X^0 = \frac{1}{\sqrt{2}}, \quad X^1 = \frac{z}{\sqrt{2}},$ ⇒ $F_0 = -\frac{i}{\sqrt{2}}(2f - zf'), \quad F_1 = -\frac{i}{\sqrt{2}} f'$

S. Ferrara, L. Girardello, M. Porrati, (1996)

Set up

- There is a subtlety (EM dual)

$$\Omega(z) = \begin{pmatrix} X^\Lambda(z) \\ F_\Sigma(z) \end{pmatrix}, \quad \Lambda, \Sigma = 0, 1,$$

Symplectic trans:

$$\Omega \to \tilde{\Omega} = \mathcal{S}\Omega \qquad \mathcal{S} \in Sp(4, \mathbb{R})$$

- Here we choose

$$S = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\Rightarrow \quad X^\Lambda = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ if' \end{pmatrix}, \quad F_\Sigma = \frac{1}{\sqrt{2}} \begin{pmatrix} -i(2f - zf') \\ z \end{pmatrix},$$

Prepotential does not exist

S. Ferrara, L. Girardello, M. Porrati, (1996)

Set up

- Hyper sector (four real scalars: b^0-b^3, $SO(4,1)/SO(4)$)

Metric: $h_{uv} = \frac{1}{2(b^0)^2}\delta_{uv}$ ⇐ Invariant under constant translations of b^1, b^2, b^3

- Gauging (g_i ($i = 1,2,3$): coupling constant)

$$b^2 \rightarrow b^2 + \epsilon^2(g_2 + g_3),$$
$$b^3 \rightarrow b^3 + \epsilon^3 g_1,$$

Covariant derivative

$$D_\mu b^u = \partial_\mu b^u + A_\mu^\Sigma k_\Sigma^u. \qquad \Sigma = 0,1$$

Scalar potential

- Gauging produces scalar potential

$$V = \frac{e^K}{(b^0)^2} g^{z\bar{z}} D^x \bar{D}^x - \frac{e^K}{2(b^0)^2} \left(\mathcal{E}^x + i f' \mathcal{M}^x\right) \left(\mathcal{E}^x - i \bar{f}' \mathcal{M}^x\right)$$

$$D^x \equiv \frac{1}{\sqrt{2}} \left(\mathcal{E}^x K_z + \mathcal{M}^x (i f'' + i f' K_z)\right)$$

$$\mathcal{E}^x \equiv (0, g_2, g_1), \quad \mathcal{M}^x \equiv (0, g_3, 0)$$

- Minimum (Minkowski)

$$\langle f' \rangle = \frac{g_1}{g_3} + i \frac{g_2}{g_3}, \quad \langle f'' \rangle = 0.$$

Partial breaking

- Susy transformation

$$\langle\delta\psi_{A\mu}\rangle = \left\langle \frac{e^{K/2}g_1}{2\sqrt{2}b^0} \right\rangle \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} \gamma_\mu \begin{pmatrix} \epsilon^1 \\ \epsilon^2 \end{pmatrix},$$

$$\langle\delta\lambda^{zA}\rangle = \left\langle \frac{ie^{K/2}g_1}{\sqrt{2}b^0} g^{z\bar{z}} \right\rangle \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix},$$

$$\langle\delta\zeta_\alpha\rangle = \left\langle \frac{ie^{K/2}g_1}{\sqrt{2}b^0} \right\rangle \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix},$$

$$\langle\delta\psi_{-\mu}\rangle = \left\langle -\frac{e^{K/2}g_1}{\sqrt{2}b^0} \right\rangle \gamma_\mu \epsilon_-, \quad \langle\delta\psi_{+\mu}\rangle = 0, \qquad \phi_\pm = \frac{1}{\sqrt{2}}(\phi_1 \pm \phi_2), \; \phi = \{\psi, \lambda, \zeta, \epsilon\}$$

$$\langle\delta\lambda^{z-}\rangle = \left\langle -\frac{\sqrt{2}ie^{K/2}g_1}{b^0} g^{z\bar{z}} K_{\bar{z}} \right\rangle \epsilon_-, \quad \langle\delta\lambda^{z+}\rangle = 0,$$

$$\langle\delta\zeta_+\rangle = \left\langle \frac{\sqrt{2}ie^{K/2}g_1}{b^0} \right\rangle \epsilon_-, \quad \langle\delta\zeta_-\rangle = 0,$$

SUSY parameter

$$\chi_\bullet \equiv -\langle K_z \rangle \lambda^{z-} + 2\zeta_+ \qquad \delta\chi_\bullet = \left\langle \frac{3\sqrt{2}i}{b^0} e^{K/2} g_1 \right\rangle \epsilon_-$$

$$\eta_\bullet \equiv \langle K_z \rangle \lambda^{z-} + \zeta_+ \qquad \delta\eta_\bullet = 0$$

Goldstino

Mass

- Fermion mass

Goldstino is absorbed by superHiggs

$$\psi_\mu^- \to \psi_\mu^- - \frac{i}{6}\gamma_\mu \chi_\bullet$$

$$M_1 = \left| \left\langle \frac{\sqrt{2} g_1 e^{K/2}}{b^0} \right\rangle \right|$$

$$M_2 = \left| \left\langle \frac{e^{K/2}}{\sqrt{2} b^0} f''' g_3 g^{z\bar{z}} \right\rangle \right|$$

$$\mathcal{L}_{f,mass} = M_1 \left\{ \bar{\psi}_\mu^- \gamma^{\mu\nu} \psi_\nu^- - \frac{1}{3} \bar{\eta}_\bullet \eta_\bullet \right\} + M_2 \bar{\lambda}^{z+} \lambda^{z+} + \text{h.c.}$$

- Boson mass

Two vectors (${A_\mu}^0$, ${A_\mu}^1$) have the same mass with gravitino, M_1.
Complex scalar z has the same mass with λ^{z+}, M_2.

Spectrum

- Summary

$\mathcal{N}=1$ multiplet	component	mass
gravity	$e_\mu^a, \psi_{\mu+}$	0
massive spin 3/2	$\psi_{\mu-}, A_\mu^0, A_\mu^1, \eta_\bullet$	M_1
massive chiral	λ^{z+}, z	M_2
massless chiral	ζ_+, b^0, b^1	0

$$M_1 = \left| \left\langle \frac{\sqrt{2} g_1 e^{K/2}}{b^0} \right\rangle \right|$$

$$M_2 = \left| \left\langle \frac{e^{K/2}}{\sqrt{2} b^0} f''' g_3 g^{z\bar{z}} \right\rangle \right|$$

What we should do next

FGP model

$e^a_\mu, \psi_{\mu+}$	0
$\psi_{\mu-}, A^0_\mu, A^1_\mu, \eta_\bullet$	M_1
λ^{z+}, z	M_2
ζ_+, b^0, b^1	0

?

We may get
DBI with sugra corrections
(interactions should
respect broken susy)

Rigid limit
$M_{Pl} \to \infty$ [1]

[1]S. Ferrara, L. Girardello, M. Porrati, (1996)
L. Andrianopoli, P. Concha, R. D'Auria, E. Rodriguez, Mario Trigiante, (2015)
[2] I. Antoniadis, H. Partouche, T. R. Taylor, (1996)
[3] L. Andrianopoli, R. D'Auria, M. Trigiante, **(2015)**
I. Antoniadis, J. Derendinger, C. Markou, (2017)

APT model[2]:
Global PB model

$A^1_\mu, \eta_\bullet$	0
λ^{z+}, z	M_2

+ Hidden sector

Integrating out
z and λ^{z+} [3]

SUSY DBI

$A^1_\mu, \eta_\bullet$	0

$$\mathcal{L}^{(0)} = 1 - \sqrt{-\det\left(\eta_{ab} + F^1_{ab}\right)},$$

+ Volkov Akulov action

Linearized approximation

- Expand w.r.t $\mu \equiv M_{Pl}/\Lambda$

$$f = \frac{1}{2} + \frac{1}{\mu}z + \frac{1}{\mu^2}\phi(z) + \frac{1}{\mu^3}\psi(z) + \mathcal{O}(\frac{1}{\mu^4})$$

$$g_1 = \frac{\xi}{\mu^2}, \quad g_2 = \frac{e}{\mu^2}, \quad g_3 = \frac{2m}{\mu}$$

PB condition:

$$\langle f' \rangle = \frac{g_1}{g_3} + i\frac{g_2}{g_3}, \quad \langle f'' \rangle = 0.$$

$$\langle \phi' \rangle = \langle \phi'' \rangle = \langle \psi' \rangle = \langle \psi'' \rangle = 0,$$

$$e = 0, \quad \xi = 2m$$

- We have

$$\mathcal{L} = \mathcal{L}^{(0)} + \frac{1}{\mu}\mathcal{L}^{(1)}$$

Linearized approximation

- E.g., scalar potential

Rigid

$$V^{(0)} = \frac{2m^2}{(b^0)^2}\left[\frac{1}{2\mathrm{Im}\mathcal{F}''}(|\mathcal{F}''|^2+4)-2\right]$$

global prepotential: $\mathcal{F}(z) \equiv iz^2 - 2i\phi(z)$

Next order

$$V^{(1)} = \frac{m^2}{(b^0)^2}\Big[-4-2z-i\mathcal{F}'+\frac{-1}{\mathrm{Im}\mathcal{F}''}\Big\{\frac{1}{2}|\mathcal{F}''|^2+10+\frac{1}{4}\mathcal{F}'\mathcal{F}''-\frac{3}{2}\bar{\mathcal{F}}'\mathcal{F}''+2i\mathcal{F}''\bar{\psi}''$$
$$-4z+2i\mathcal{F}'-i\mathcal{F}''(z-\bar{z})+\mathrm{h.c.}\Big\}+\frac{1}{(\mathrm{Im}\mathcal{F}'')^2}\Big\{\psi''-i\mathcal{F}'-2i\mathcal{F}''-i\mathcal{F}''\bar{z}+\mathrm{h.c.}\Big\}\Big]$$

Corrections on DBI

- Linearized action is model dependent ($\psi, \ \mathcal{F}$)
- As the simplest model, we take

$$\psi = \text{const}, \quad \mathcal{F} = az^3 + bz^2 + cz$$

- Integrating out z (mass: $M_2^2 \sim m^2 \langle \mathcal{F}''' \rangle$)

EOM of z ⇒ $z = z^{(0)}(F^1_{\mu\nu}) + \frac{1}{\mu} z^{(1)}(F^1_{\mu\nu}, F^0_{\mu\nu})$

$$\mathcal{L}^{(0)} = \underbrace{1 - \sqrt{-\det\left(\eta_{\mu\nu} + F^1_{\mu\nu}\right)}}_{\equiv \mathcal{D}} - \frac{1}{4} F^0_{\mu\nu} F^{0\mu\nu} + \cdots ,$$

$$\mathcal{L}^{(1)} = g(\mathcal{D}) + h(\mathcal{D}) F^0_{\mu\nu} F^{1\mu\nu} + \cdots$$

Hidden sector couples (graviphoton couples to another vector)

Comments

- We could have corrections of graviphoton kinetic terms like $J(\mathcal{D})(F^0_{\mu\nu}F^{0\mu\nu})$, but they do not appear.
- Seems to be far from minimal coupling

$$1-\sqrt{-\det\left(\eta_{\mu\nu}+F^1_{\mu\nu}\right)}\longrightarrow\sqrt{-g}-\sqrt{-\det\left(g_{\mu\nu}+F^1_{\mu\nu}\right)}$$

- Two vectors and gravitino are still massless at this order.

Summary & discussion

- We discussed supergravity corrections on SUSY DBI at the linearized level.
- We focus on the direct derivation from N=2 supergravity
- We found some non trivial properties

  Coupling with graviphoton, difference with minimal coupling,…

- Nilpotency is not important(?)
- Off-shell formulation(?)