Bottom and strange Yukawa couplings at the NNLO in the MSSM

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Introduction

The MSSM Higgs sector

- 2 Higgs doublets, 5 physical Higgs bosons
- 2 input parameters at LO: M_A and $\mathrm{tg}\beta \equiv \frac{v_2}{v_1}$
- couplings of the Higgs bosons:

Φ		g_u^Φ	g_d^Φ	g_V^Φ
SM	H	1	1	1
MSSM	h	$\cos\alpha/\sin\beta$	$-\sin\alpha/\cos\beta$	$\sin(\beta-\alpha)$
	H	$\sin\alpha/\sin\beta$	$\cos\alpha/\cos\beta$	$\cos(\beta-\alpha)$
	A	$1/\mathrm{tg}\beta$	$\mathrm{tg}\beta$	0

- For large $\mathrm{tg}\beta$ the down-type Yukawa couplings are enhanced
- Higgs decay to $b\bar{b}$ and Higgs bremsstrahlung from b quarks are dominant

Introduction

Yukawa couplings in the large $\mathrm{tg}\beta$ case

- Down-type Yukawa couplings are enhanced
- LR mixing terms in the squark mass matrices can be enhanced
- Soft SUSY breaking: $-\lambda_u A_u \phi_2 \tilde{u}_R^\dagger \tilde{Q}_L \; -\lambda_d A_d \phi_1 \tilde{d}_R^\dagger \tilde{Q}_L$

$$\mathcal{M}^2_{\tilde{f}} = \begin{pmatrix} M^2_{LL} & M^2_{LR} \\ M^2_{RL} & M^2_{RR} \end{pmatrix} = \begin{pmatrix} M^2_{\tilde{f}_L} + m_f^2 & m_f(A_f - \mu r_f) \\ m_f(A_f - \mu r_f) & M^2_{\tilde{f}_R} + m_f^2 \end{pmatrix}$$

with $r_d = 1/r_u = \mathrm{tg}\beta$

Effective Lagrangian

$$
\begin{aligned}
\mathcal{L}_{eff} &= -\sum_{q=b,s} \lambda_q \bar{q}_R \left[(1+\Delta_{q,1})\phi_1^0 + \Delta_{q,2}\phi_2^{0*}\right] q_L + h.c. \\
&= -\sum_{q=b,s} m_q \bar{q}\left[1 + i\gamma_5 \frac{G^0}{v}\right] q - \frac{m_q/v}{1+\Delta_q}\bar{q}\left[g_q^h\left(1 - \frac{\Delta_q}{\mathrm{tg}\alpha\ \mathrm{tg}\beta}\right) h\right. \\
&\qquad \left. + g_q^H\left(1 + \Delta_q \frac{\mathrm{tg}\alpha}{\mathrm{tg}\beta}\right) H - g_q^A\left(1 - \frac{\Delta_q}{\mathrm{tg}^2\beta}\right) i\gamma_5 A\right] q
\end{aligned}
$$

1-loop contributions for the bottom:

$$
\begin{aligned}
\Delta_{b,1} &= -\frac{C_F}{2}\ \frac{\alpha_s(\mu_R)}{\pi}\ m_{\tilde{g}}\ A_b\ I(m^2_{\tilde{b}_1}, m^2_{\tilde{b}_2}, m^2_{\tilde{g}}) \\
\Delta_{b,2} &= \Delta^{QCD}_{b,2} + \Delta^{elw}_{b,2} \\
\Delta^{QCD}_{b,2} &= \frac{C_F}{2}\ \frac{\alpha_s(\mu_R)}{\pi}\ m_{\tilde{g}}\ \mu\ I(m^2_{\tilde{b}_1}, m^2_{\tilde{b}_2}, m^2_{\tilde{g}}) \\
\Delta^{elw}_{b,2} &= \frac{\lambda_t^2(\mu_R)}{(4\pi)^2}\ A_t\ \mu\ I(m^2_{\tilde{t}_1}, m^2_{\tilde{t}_2}, \mu^2)
\end{aligned}
$$

$$
\Delta_q = \frac{\Delta_{q,2}\ \mathrm{tg}\beta}{1+\Delta_{q,1}}
$$

$$
I(a,b,c) = \frac{ab\log\frac{a}{b} + bc\log\frac{b}{c} + ca\log\frac{c}{a}}{(a-b)(b-c)(a-c)}
$$

Carena, Garcia, Nierste and Wagner, Nucl.Phys. B577 (2000) 88
Guasch, Häfliger and Spira, Phys.Rev. D68 (2003) 115001

Resummation

The corrections $\Delta_{b,s}$ induce a modification of the relation between the mass m_b (m_s) and the Yukawa coupling λ_b (λ_s):

$$m_q = \frac{\lambda_q v_1}{\sqrt{2}} \left[1 + \Delta_{q,1} + \Delta_{q,2}\, \mathrm{tg}\beta\right] \qquad (q = b, s)$$

Resummed couplings:

$$\mathcal{L}_{eff} = - \sum_{q=b,s} \frac{m_q}{v}\, \bar{q} \left[\tilde{g}_q^h\, h + \tilde{g}_q^H\, H - \tilde{g}_q^A\, i\gamma_5\, A \right] q$$

$$\tilde{g}_q^h = \frac{g_q^h}{1+\Delta_q}\left[1 - \frac{\Delta_q}{\mathrm{tg}\alpha\,\mathrm{tg}\beta}\right]$$

$$\tilde{g}_q^H = \frac{g_q^H}{1+\Delta_q}\left[1 + \Delta_q \frac{\mathrm{tg}\alpha}{\mathrm{tg}\beta}\right] \qquad \Delta_q = \frac{\Delta_{q,2}\, \mathrm{tg}\beta}{1+\Delta_{q,1}}$$

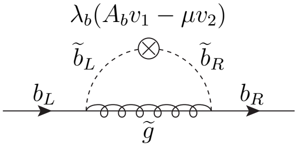
$$\tilde{g}_q^A = \frac{g_q^A}{1+\Delta_q}\left[1 - \frac{\Delta_q}{\mathrm{tg}\beta^2}\right]$$

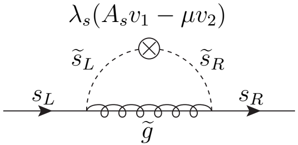
Carena, Garcia, Nierste and Wagner, Nucl.Phys. B577 (2000) 88
Guasch, Häfliger and Spira, Phys.Rev. D68 (2003) 115001

Low Energy theorem

- Calculate the self-energies at vanishing momentum
- Substitute:

$$
\begin{aligned}
v_1 &\rightarrow \sqrt{2}\phi_1^0 \\
v_2 &\rightarrow \sqrt{2}\phi_2^{0*}
\end{aligned}
$$

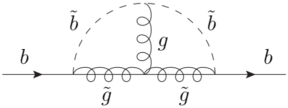


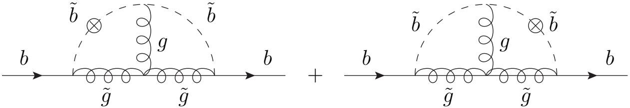


$$m_q = \frac{\lambda_q}{\sqrt{2}} v_1 + \Sigma_S(m_q^2) \qquad \Sigma_S(m_q^2) = \frac{\lambda_b}{\sqrt{2}} v_1 \; [\Delta_{q,1} + \Delta_{q,2} \mathrm{tg}\beta]$$

Ellis, Gaillard and Nanopoulos, Nucl.Phys. B106 (1976) 292
Shifman, Vainshtein, Voloshin and Zakharov, Sov.J.Nucl.Phys.30 (1979) 711

Bottom Yukawa coupling at NNLO





Extract leading terms for large $\mathrm{tg}\beta$:

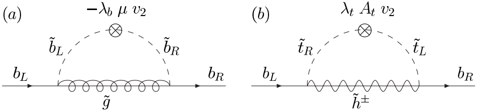
$$m_b = \frac{\lambda_b}{\sqrt{2}} v_1 + \Sigma_S(m_b)$$

$$\Sigma_S(m_b) = \frac{\lambda_b}{\sqrt{2}} v_1 \ \left(\Delta_{b,1} + \Delta_{b,2} \mathrm{tg}\beta\right)$$

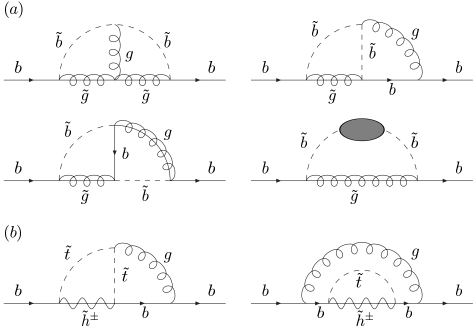
Mihaila and Reisser, JHEP 1008 (2010) 021
Noth and Spira, JHEP 1106 (2011) 084
Crivellin and Greub, Phys.Rev.D 87 (2013) 015013

Top-induced corrections

NLO



NNLO



Calculation: technical details

- dimensional regularization in $n = 4 - 2\varepsilon$ dimensions
- reduction to master integrals:

$$A_0(m) = \int \frac{d^n k}{(2\pi)^n} \frac{1}{k^2 - m^2}$$

$$T_{134}(m_1, m_3, m_4) = \int \frac{d^n k}{(2\pi)^n} \frac{d^n q}{(2\pi)^n} \frac{1}{(k^2 - m_1^2)[(k-q)^2 - m_3^2](q^2 - m_4^2)}$$

- dimensional regularization violates SUSY $\Rightarrow$ anomalous counter terms

$$\hat{g}_s = g_s \left[1 + \left(\frac{C_A}{6} - \frac{C_F}{8}\right) \frac{\alpha_s}{\pi}\right]$$

$$\lambda_{Hbb} = \lambda_{H\tilde{b}\tilde{b}} \left(1 + \frac{C_F}{4} \frac{\alpha_s}{\pi}\right) = \lambda_{\tilde{H}\tilde{b}b} \left(1 + \frac{3C_F}{8} \frac{\alpha_s}{\pi}\right)$$

Martin and Vaughn, Phys.Lett. B318 (1993) 331-337

Renormalization

- 1-loop corrections are finite
- 2-loop corrections are UV-divergent
- Counter terms:

$$
\begin{aligned}
m_{\tilde{q}_i}^{0\,2} &= m_{\tilde{q}_i}^2 + \delta m_{\tilde{q}_i}^2 \\
m_{\tilde{g}}^0 &= m_{\tilde{g}} + \delta m_{\tilde{g}} \\
\lambda_t^0 &= \lambda_t(\mu_R) + \delta\lambda_t \\
\alpha_s^0 &= \alpha_s(\mu_R) + \delta\alpha_s \\
A_t^0 &= A_t + \delta A_t
\end{aligned}
$$

- $m_{\tilde{b}_i}$, $m_{\tilde{t}_i}$, $m_{\tilde{g}}$: on-shell
- A_t: on-shell
- α_s, λ_t: $\overline{\mathrm{MS}}$ with 5 flavours

- Anomalous counter terms (restoring SUSY)

Noth and Spira, JHEP 1106 (2011) 084

Bottom Yukawa coupling at NNLO

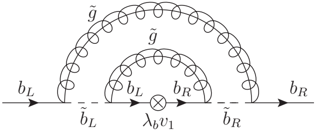
- After renormalization the SUSY–QCD corrections to the $\Delta_{b,1}$ terms are the same as for the $\Delta_{b,2}$:

$$\Delta_{b,2}^{QCD} = \mu\,\Delta^{NLO}\,[1+\delta_1]$$

$$\Delta_b = \frac{\mu\ \mathrm{tg}\beta\ \Delta^{NLO}\,[1+\delta_1] + \Delta_{b,2}^{elw}\ \mathrm{tg}\beta\,[1+\delta_2]}{1 - A_b^0\ \Delta^{NLO}\,[1+\delta_1]}$$

- Renormalization of A_b^0:

$$\begin{aligned} A_b^0 &= A_b(\mu_R^2) + \delta A_b \\ \delta A_b &= C_F \frac{\alpha_s}{\pi}\Gamma(1+\epsilon)\left(\frac{4\pi\mu^2}{\mu_R^2}\right)^{\epsilon}\frac{m_{\tilde{g}}}{\epsilon} \neq \mathcal{O}(A_b) \end{aligned}$$



Ghezzi, Glaus, Müller, Schmidt and Spira, arXiv:1711.02555

Strange Yukawa coupling at NNLO

- Same calculation as for the b, as both b and s are considered massless
- absence of sizeable SUSY–electroweak contributions, since $\lambda_c = 0$
- resummed terms:
  - $\mathcal{O}\left[(\alpha_s\,\mu\,\mathrm{tg}\beta)^n\right]$, $\mathcal{O}\left[\alpha_s^{n+1}\,(\mu\,\mathrm{tg}\beta)^n\right]$
  - $\mathcal{O}\left[(\alpha_s\,A_s)^n\right]$, $\mathcal{O}\left[\alpha_s^{n+1}\,A_s^n\right]$

$$\Delta_s = \frac{\Delta_{s,2}\mathrm{tg}\beta\,[1+\delta_s]}{1+\Delta_{s,1}\,[1+\delta_s]}$$

Ghezzi, Glaus, Müller, Schmidt and Spira, arXiv:1711.02555

Resummation

$$\Delta_q = \frac{\Delta_{q,2}\ \mathrm{tg}\beta}{1+\Delta_{q,1}}$$

- Resummation of $\mathrm{tg}\beta$-enhanced contributions:
  - $\mathcal{O}((\alpha_s \mu \mathrm{tg}\beta)^n)$@NLO
  - $\mathcal{O}(\alpha_s^{n+1}(\mu \mathrm{tg}\beta)^n)$@NNLO
- Resummation of top-induced $\mathrm{tg}\beta$-enhanced contributions:
  - $\mathcal{O}[(\lambda_t^2 A_t \mathrm{tg}\beta)^n]$@NLO
  - $\mathcal{O}[\alpha_s(\lambda_t^2 A_t \mathrm{tg}\beta)^n]$@NNLO
- Resummation of $A_{b,s}$-induced terms (can be important when $A_q \simeq \mu \mathrm{tg}\beta$):
  - $\mathcal{O}((\alpha_s A_{b,s})^n)$@NLO
  - $\mathcal{O}(\alpha_s^{n+1}(A_{b,s})^n)$@NNLO

Partial decay rates

Neutral Higgs bosons decay to $b\bar{b}$

$$\Gamma[\Phi \to b\bar{b}] = \frac{3 G_F M_\Phi}{4\sqrt{2}\pi} \overline{m}_b^2(M_\Phi) \left[1 + \delta_{\rm QCD} + \delta_t^\Phi\right] \tilde{g}_b^\Phi \left[\tilde{g}_b^\Phi + \delta_{SQCD}^{rem}\right] \qquad \Phi = h, H, A$$

Charged Higgs decay to $c\bar{s}$ (and production...)

$$\Gamma[H^+ \to c\bar{s}] = \frac{3 G_F M_{H^\pm}}{4\sqrt{2}\pi} |V_{cs}|^2 \left[\overline{m}_c^2(M_{H^\pm})(g_c^A)^2 + \overline{m}_s^2(M_{H^\pm})(\tilde{g}_s^A)^2\right] (1+\delta_{\rm QCD})$$

- $\delta_{\rm QCD}$: QCD corrections, N^{4}LO
- δ_t: top-induced correction, NNLO
- $\tilde{g}_{b,s}^\Phi$: resummed Yukawa couplings
- δ_{SQCD}^{rem}: remainder from the $\tilde{g}_b^\Phi$ resummation

Benchmark scenarios

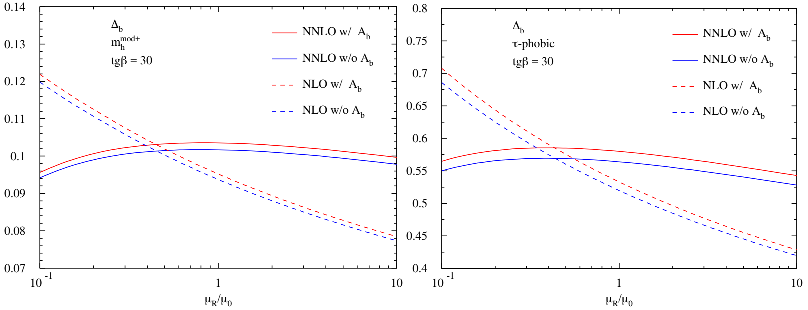
m_h^{mod+} scenario:

$$\mathrm{tg}\beta = 30, \qquad M_{\tilde{Q}} = 1\,\mathrm{TeV}, \qquad M_{\tilde{\ell}_3} = 1\,\mathrm{TeV}, \qquad M_{\tilde{g}} = 1.5\,\mathrm{TeV},$$
$$M_2 = 200\,\mathrm{GeV}, \qquad A_b = A_\tau = A_t = 1.607\,\mathrm{TeV}, \qquad \mu = 200\,\mathrm{GeV}.$$

τ-phobic scenario:

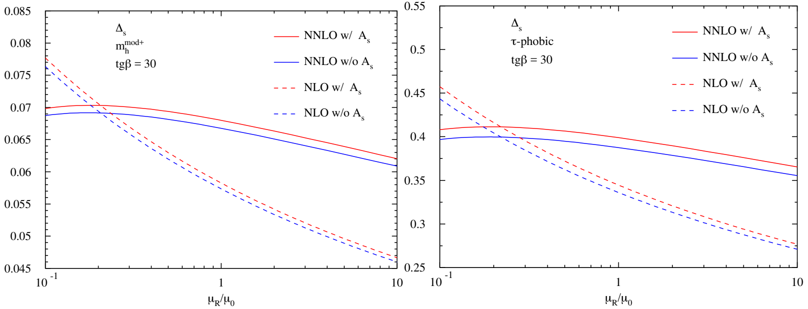
$$\mathrm{tg}\beta = 30, \qquad M_{\tilde{Q}} = 1.5\,\mathrm{TeV}, \qquad M_{\tilde{\ell}_3} = 500\,\mathrm{GeV}, \qquad M_{\tilde{g}} = 1.5\,\mathrm{TeV},$$
$$M_2 = 200\,\mathrm{GeV}, \qquad A_b = A_t = 4.417\,\mathrm{TeV}, \qquad A_\tau = 0, \qquad \mu = 2\,\mathrm{TeV}.$$

Scale dependence of Δ_b



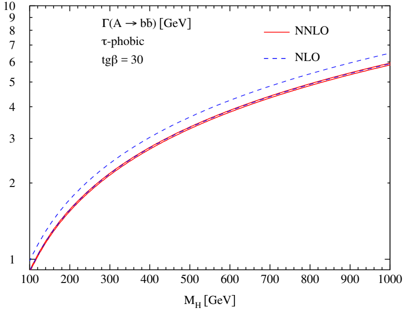
Ghezzi, Glaus, Müller, Schmidt and Spira, arXiv:1711.02555

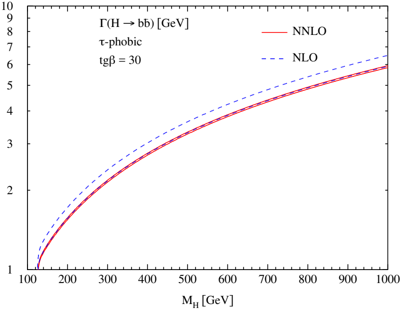
Scale dependence of Δ_s



Ghezzi, Glaus, Müller, Schmidt and Spira, arXiv:1711.02555

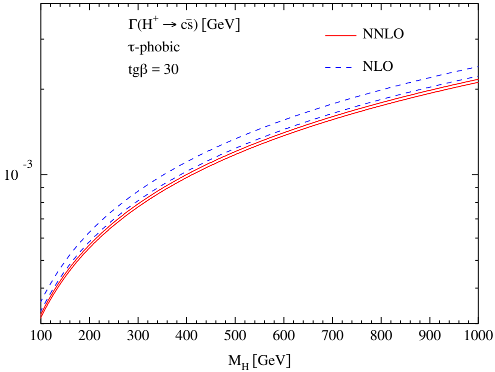
Partial decay widths of the neutral heavy Higgses





Ghezzi, Glaus, Müller, Schmidt and Spira, arXiv:1711.02555

Partial decay width of the charged Higgs



Ghezzi, Glaus, Müller, Schmidt and Spira, arXiv:1711.02555

Summary

- In large-$\mathrm{tg}\beta$ MSSM scenarios the down-type Yukawa couplings to the heavy Higgs bosons receive large corrections.
- These $\mathrm{tg}\beta$-enhanced corrections can be resummed in effective Yukawa couplings.
- They can be extracted from the calculation of self-energies with contributions from the off-diagonal terms of the squark mass matrices (LET).
- As the off-diagonal terms of the squark mass matrices are proportional to $A_q - \mu\mathrm{tg}\beta$, the resummation of $\mathrm{tg}\beta$-enhanced corrections and A_q-induced corrections go hand in hand.
- The calculation has been performed up to NNLO.
- The scale dependence was $\mathcal{O}(10\%)$ for the NLO predictions and is currently $\mathcal{O}(1\%)$ for the NNLO.