Effects of quantum statistics on relic density of Dark Radiation

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Outline

- Motivation
- Freeze-out of Dark Radiation particles
  - Boltzmann equations
  - approximate methods
- Weinberg's Higgs portal model
  - numerical results
- Conclusions

Motivation

Directly, locally measured (low z) value of the Hubble constant

$$73.24 \pm 1.74 \ \frac{\mathrm{km}}{\mathrm{s\,Mpc}}$$

Riess et al. 2016

is more than 3σ bigger than the Hubble constant inferred from the CMB measurements and the standard ΛCDM model (high z)

$$67.8 \pm 0.9 \ \frac{\mathrm{km}}{\mathrm{s\,Mpc}}$$

Planck Collaboration 2016

Such discrepancy may be explained by additional contribution to energy density from dark radiation (DR)

$$\rho_{\mathrm{R}} = \rho_\gamma + \rho_\nu + \rho_{\mathrm{DR}} = \left[1 + \left(N_{\mathrm{eff}}^{\mathrm{SM}} + \Delta N_{\mathrm{eff}}\right) \frac{7}{8} \left(\frac{4}{11}\right)^{4/3}\right] \rho_\gamma$$

$N_{\mathrm{eff}} = N_{\mathrm{eff}}^{\mathrm{SM}} + \Delta N_{\mathrm{eff}}$ - effective number of neutrinos

$N_{\mathrm{eff}}^{\mathrm{SM}} \approx 3.045$

Salas Pastor 2016

Motivation

The present bounds on $N_{\rm eff}$ are not very tight

and depend on the set of experimental results used in the analysis

2.99 ± 0.20	3.15 ± 0.23	$3.29^{+0.11}_{-0.17}$
PlanckTT + lowP + BAO	Planck TT, TE, EE+lowP	...+H_0+SZ+lensing
Planck Collaboration 2016		*Feng et al. 2017*

Future experiments will determine $N_{\rm eff}$ more precisely

Precision better than 0.03 expected from the CMB-S4 experiment

CMB-S4 Collaboration 2016

In order to test models of DR we will need theoretical predictions with comparable accuracy

One of the effects to be taken into account comes from the quantum statistics of all particles involved in DR freeze-out process

Boltzmann equation in FRW metric:

$$E(\partial_t - pH\partial_p)f(p,t) = C_{\rm E}(p,t) + C_{\rm I}(p,t)$$

Massless DR particle χ annihilating into SM particles: $\chi\bar{\chi} \to N\bar{N}$
Distribution functions:

$$f_\chi = \left(e^{E/T+z} \pm 1\right)^{-1} \qquad f_N = \left(e^{E_N/T} \pm 1\right)^{-1}$$

z – chemical pseudopotential describes decoupling from equilibrium

Bernstein Brown Feinberg 1992

Dolgov Kainulainen 1993

Evolution of z given by integro-differential equation:

$$\frac{{\rm d}z}{{\rm d}x} = -\frac{x}{J_2(z,x)}\left[\frac{1}{3}\frac{g'_{*s}(x)}{g_{*s}(x)}J_3(z,x) + \frac{\sqrt{45}}{256\pi^{11/2}}\frac{M_{\rm Pl}}{m}\frac{\sinh z}{g\sqrt{g_*(x)}}\left(1 - \frac{x}{3}\frac{g'_{*s}(x)}{g_{*s}(x)}\right)\tilde{S}_I(z,x)\right]$$

$(x \equiv m_N/T)$

Boltzmann equation

$$J_n(z,x) = \int_0^\infty \mathrm{d}y\, y^n \frac{e^{xy}}{(e^{xy+z} \pm 1)^2}$$

$$\tilde{S}_I(z,x) = 2\pi \int \mathscr{D}\Phi \sum_{\text{spins}} |M_{\chi\bar{\chi}\to N\bar{N}}|^2$$

If $M_{\chi\bar{\chi}\to N\bar{N}}$ depends only on s then $\mathscr{D}\Phi$ may be reduced to a 2-dim. integral (which in general case is 5-dim. and much more complicated)

$$\mathscr{D}\Phi = \frac{1}{x^4}\int_{2x}^\infty \mathrm{d}p \int_0^{\sqrt{p^2-4x^2}} \mathrm{d}q\, \frac{1}{(e^{p+2z} - 1)(1 - e^{-p})} \cdot$$

$$\cdot \ln\left[\frac{\cosh\left(\frac{1}{2}(p+q)+z\right) \pm 1}{\cosh\left(\frac{1}{2}(p-q)+z\right) \pm 1}\right] \ln\left[\frac{\cosh\left(\frac{1}{2}(p+qV)\right) \pm 1}{\cosh\left(\frac{1}{2}(p-qV)\right) \pm 1}\right]$$

where: $V = \sqrt{1 - \frac{4x^2}{p^2-q^2}}$ $\quad s = \frac{p^2-q^2}{x^2} m_N^2$

From $z(x)$ we obtain $\quad n_\chi(x) = g\frac{m^3}{2\pi^2}\int_0^\infty \frac{y^2\mathrm{d}y}{e^{xy+z} \pm 1}$

Boltzmann equation – approximations

$$Y = e^{-z} Y_{\rm eq} \qquad Y_{\rm eq} = g\,\frac{45}{2\pi^4}\frac{\zeta^\pm}{g_{*s}} \qquad \zeta^\pm \equiv \zeta(3)\begin{cases}3/4 & \textbf{FD}\\ 1 & \textbf{BE}\end{cases}$$

$$Y'(x) = -\sqrt{\frac{\pi}{45}}\,\frac{g_{*s}(x)}{\sqrt{g_*(x)}}\,\frac{M_{\rm Pl} m}{x^2}\left(Y^2(x) - Y^2_{\rm eq}(x)\right)\langle\sigma v\rangle\,\frac{1}{\zeta^\pm}$$

- pure MB: $\pm 1 \to 0$, $\pm 1 \to 0$ everywhere ($\Rightarrow \zeta^\pm \to 1$)
- fractional fBE/fFD: $\zeta^\pm$ in $Y_{\rm eq}$, $\langle\sigma v\rangle = \langle\sigma v\rangle_{\rm MB}$
- partial pBE/pFD: $\zeta^\pm$ in $Y_{\rm eq}$, $\langle\sigma v\rangle = \langle\sigma v\rangle_{\rm p}$

$$\langle\sigma v\rangle_{\rm MB} = \frac{1}{512\pi}\frac{x^5}{m^5}\int_{4m^2}^{\infty} {\rm d}s\,\sqrt{1-\frac{4m^2}{s}}\sum_{\rm spins}|M(s)|^2\sqrt{s}\,{\rm K}_1\left(\frac{x\sqrt{s}}{m}\right)$$

$\langle\sigma v\rangle_{\rm p}$ similar to $\langle\sigma v\rangle_{\rm BM}$ with the substitution

$$\sqrt{s}\,{\rm K}_1\left(\frac{x\sqrt{s}}{m}\right) \to \int_{\sqrt{s}}^{\infty}\frac{{\rm d}E_+}{(\zeta^\pm)^2}\,\frac{e^{-\frac{x}{2m}E_+}}{\sinh\left(\frac{x}{2m}E_+\right)}\,\ln\left[\frac{{\rm fh}\left(\frac{x}{4m}\left(E_+ + \sqrt{E_+^2 - s}\right)\right)}{{\rm fh}\left(\frac{x}{4m}\left(E_+ - \sqrt{E_+^2 - s}\right)\right)}\right]$$

fh = sinh/cosh for BE/FD statistics of DR particles

statistics of SM particles N is ignored

Weinberg's Higgs portal model

Standard Model
+ scalar ϕ and Dirac fermion ψ charged under global $U(1)_{\mathrm{dark}}$:
$Q_{\mathrm{dark}}(\psi) = 1$, $Q_{\mathrm{dark}}(\phi) = 2$ *Weinberg 2013*

$$\mathcal{L}_{\mathrm{scalar}} = (D_\mu H)^\dagger (D^\mu H) + \mu_H^2 H^\dagger H - \lambda_H (H^\dagger H)^2$$
$$+\partial_\mu \phi^* \partial^\mu \phi + \mu_\phi^2 (\phi^* \phi)^2 - \lambda_\phi (\phi^* \phi)^2 - \kappa (H^\dagger H)(\phi^* \phi)$$

Spontaneous breaking of $U(1)_{\mathrm{dark}}$ gives:

- **massless Goldstone boson σ – DR particle**
- **massive scalar which mixes with the neutral component of the doublet H giving two mass eigenstates: h and ρ**
- **two massive Majorana fermions - lighter plays the role of DM**

Imposing two constraints, $v_H = 246$ GeV and $m_h = 125$ GeV, on $\mathcal{L}_{\mathrm{scalar}}$ we are left with 3 independent parameters: κ, λ_ϕ, m_ρ

It is often assumed that Goldstone bosons σ
"go out of equilibrium while kT is still above the mass of muons but below the mass of all other particles"
and that this leads to $\Delta N_{\text{eff}} = \frac{4}{7}\left(\frac{43}{57}\right)^{4/3} \approx 0.39$

Problems:

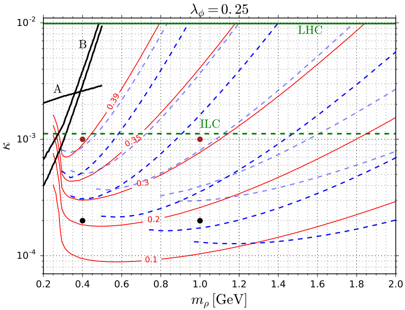
- **105 MeV $\approx m_\mu \ll kT \ll m_\pi \approx$ 135 MeV**
- **how the results change if pions are taken into account?**

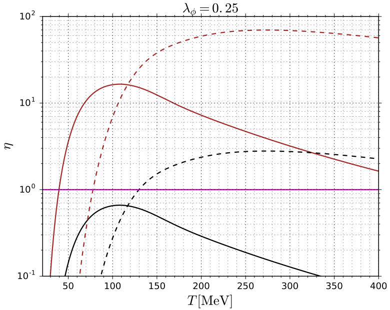
In addition, it is interesting to check:

- **what is the result of decoupling at different temperatures**
- **how different approximations (used in the literature) change the results with respect to those obtained with the full inclusion of quantum statistics of involved particles**

Instantaneous freeze-out approximation: $Y_\infty = Y^{\rm eq}(x_f)$
where the freeze-out temperature, x_f, is defined by the condition $\eta(x_f) = 1$
$\eta(x) = \left.\frac{\Gamma}{H}\right|_x = \left.\frac{n\langle\sigma v\rangle}{H}\right|_x$
Processes $\sigma\sigma \leftrightarrow \mu^+\mu^-$, $\pi\pi$ are dominated by ρ exchange which in the narrow resonance approximation may be described by

$$\sum_{\rm spins} |M|^2 \to 2\pi\kappa^2 m_\rho^3\,\delta(s-m_\rho^2)\frac{m_{\mu^\pm}^2(m_\rho^2-4m_{\mu^\pm}^2)+\frac{1}{27}(m_\rho^2+\frac{11}{2}m_\pi^2)^2}{\Gamma_\rho\left[(m_\rho^2-m_h^2)^2+\Gamma_h^2 m_h^2\right]}$$





Effects of statistics

Statistics influences the phase-space integrals:

$$\mathscr{D}\Phi_{\chi N} \approx \mathscr{D}\Phi_{\rm MB} \cdot \tanh^{\pm 1 \pm 1}\left(\frac{m_\rho x}{4 m_N}\right)$$
$$\mathscr{D}\Phi_{\rm p} \approx \mathscr{D}\Phi_{\rm MB} \cdot (\zeta^{\pm})^{-1} \tanh^{\pm 1}\left(\frac{m_\rho x}{4 m_N}\right)$$
$$\mathscr{D}\Phi_{\rm f} \approx \mathscr{D}\Phi_{\rm MB} \cdot \zeta^{\pm}$$

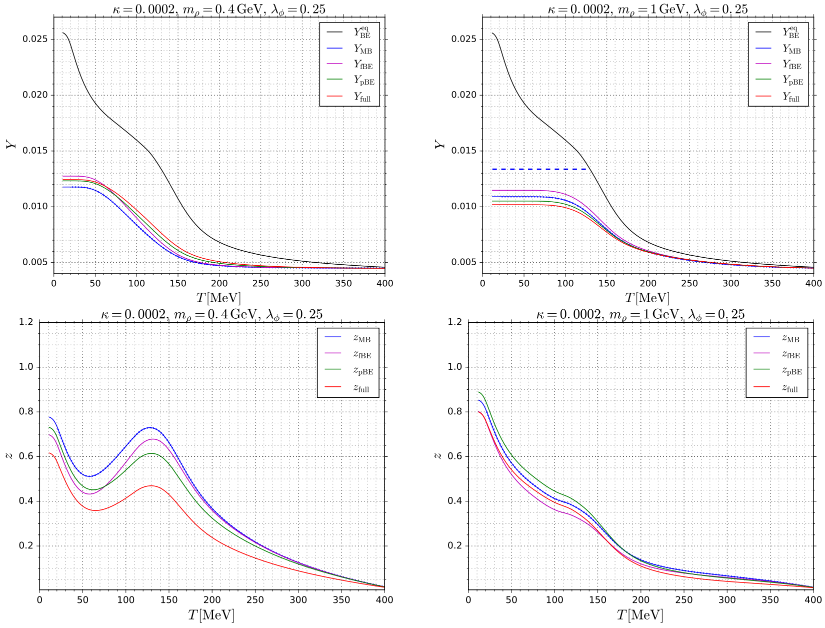
In Weinberg's Higgs portal model

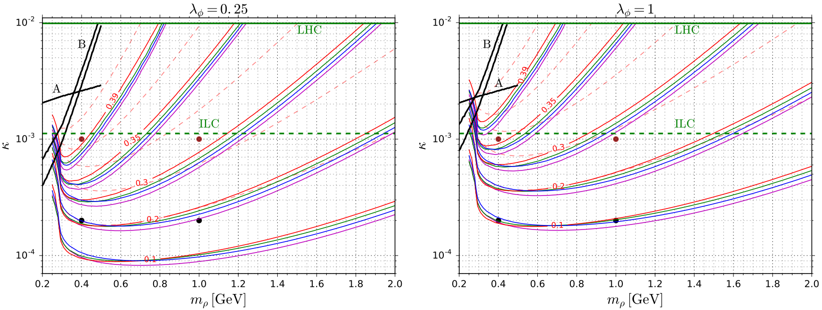
- $\mathscr{D}\Phi_{\sigma\mu} \approx \mathscr{D}\Phi_{\rm MB}$
- $\mathscr{D}\Phi_{\sigma\pi} \approx \mathscr{D}\Phi_{\rm MB} \cdot \coth\left(\frac{m_\rho x}{4 m_\pi}\right) \coth\left(\frac{m_\rho x}{4 m_\pi}\right)$
  - substantial effect for small $m_\rho x \ll 4 m_\pi$

Statistics (of DR particles) enters also via

$$J_n(z,x) \equiv \int_0^\infty {\rm d}y\, y^n \frac{e^{xy}}{(e^{xy+z} \pm 1)^2} \qquad n_\chi(x) = g \frac{m^3}{2\pi^2} \int_0^\infty \frac{y^2 {\rm d}y}{e^{xy+z} \pm 1}$$

Numerical examples



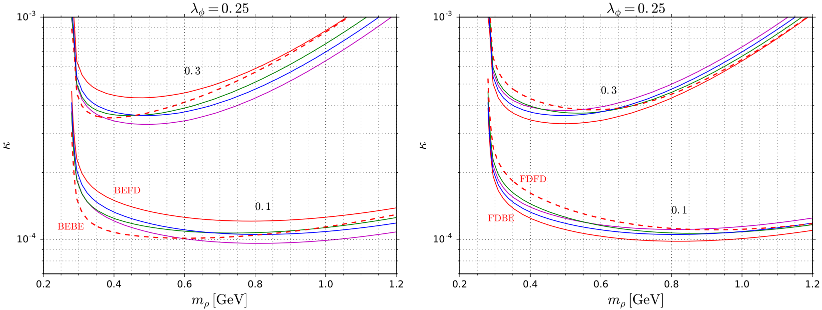


- **Approximate methods with only limited inclusion of statistics effects:**
  - **usually overestimate $\Delta N_{\rm eff}$**
  - **underestimate $\Delta N_{\rm eff}$ for small κ and m_ρ**
- **Inclusion of pions in the analysis is very important**
- **ILC sensitive to big part of the parametr space with $\Delta N_{\rm eff} \approx 0.39$**

Conclusions

- Dark Radiation may explain the difference between values of H_0 obtained from direct observations and inferred from CMB
- We compared a few methods of calculating $\Delta N_{\rm eff}$
    - statistics of all involved particles (DR and their annihilation products) is important
    - limited inclusion of statistics typically overestimates (underestimates) $\Delta N_{\rm eff}$ for bosonic (fermionic) DR
    - instantaneous freeze-out approximation overestimates $\Delta N_{\rm eff}$ (and sometimes fails at all)
- Weinberg's Higgs portal model
    - limited inclusion of statistics may change $\Delta N_{\rm eff}$ by ~ 0.05
    - effects from pions very important
    - parts of parameter space giving $\Delta N_{\rm eff} \approx 0.4$ within reach of ILC
- Boltzmann equation with full statistics should be used to compare models of DR with data from future experiments - like CMB-S4

Backup


$\lambda_\phi = 0.25$
10^{-3}
10^{-4}
κ
0.3
0.1
BEFD
BEBE
0.2
0.4
0.6
0.8
1.0
1.2
m_ρ [GeV]
$\lambda_\phi = 0.25$
10^{-3}
10^{-4}
κ
0.3
0.1
FDFD
FDBE
0.2
0.4
0.6
0.8
1.0
1.2
m_ρ [GeV]