Electroweak Symmetry Breaking by a Neutral Sector : Dynamical Relaxation of the Little Hierarchy Problem

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- The naturalness problem of EW scale and Higgs boson mass has been the most important issue for last four decades.

- The MSSM has been the most promising BSM candidate.

- No evidence of BSM has been observed yet at LHC.

→ Theoretical puzzles raised in the SM still remain UNsolved.

- A barometer of the solution to the naturalness problem is the stop mass .

  The stop mass bound has been already > 1 TeV.

  (The gluino mass bound has exceeded > 2 TeV.)

→ They start threatening the traditional status of SUSY as a solution to the naturalness problem of the EW phase transition.

- ATLAS and CMS have discovered the SM(-like) Higgs with 125 GeV mass, which is too heavy as a SUSY Higgs.

- According to the recent analyses, 10-20 TeV stop mass is necessary for the 125 GeV Higgs mass (without a large stop mixing).

$$\Delta m_{h_u}^2|_{1-\mathrm{loop}} \approx \frac{3|y_t|^2}{8\pi^2}\widetilde{m}_t^2 \log\left(\frac{\widetilde{m}_t^2}{\Lambda^2}\right)\left[1+\frac{1}{2}\frac{A_t^2}{\widetilde{m}_t^2}\right],$$

$$\Delta m_H^2|_{1-\mathrm{loop}} \approx \frac{3m_t^4}{4\pi^2 v_h^2}\left[\log\left(\frac{\widetilde{m}_t^2}{m_t^2}\right)+\frac{A_t^2}{\widetilde{m}_t^2}\left(1-\frac{1}{12}\frac{A_t^2}{\widetilde{m}_t^2}\right)\right],$$

$$\frac{1}{2}m_Z^2 = \frac{m_{h_d}^2 - m_{h_u}^2\tan^2\beta}{\tan^2\beta - 1} - |\mu|^2.$$

- ATLAS and CMS have discovered the SM(-like) Higgs with 125 GeV mass, which is too heavy as a SUSY Higgs.
- According to the recent analyses, 10-20 TeV stop mass is necessary for the 125 GeV Higgs mass (without a large stop mixing).

A fine-tuning of 10^{-3} – 10^{-4}

seems to be unavoidable !! ??

- Recently some new ideas (without SUSY) have been suggested to relax the gauge hierarchy problem.

- For UV completion, however, embedding them in SUSY also have been discussed.

- Recently some new ideas (without SUSY) have been suggested to relax the gauge hierarchy problem.

- For UV completion, however, embedding them in SUSY also have been discussed.

We will attempt to address the (little) hierarchy problem in the SUSY framework.

Little Hierarchy Problem

$$|\mu|^2 + \frac{1}{2}M_Z^2 = \frac{m_{h_d}^2 - m_{h_u}^2 \tan^2\beta}{\tan^2\beta - 1}$$

Why is M_Z^2 [$=(g_2^2+g_Y^2)(v_u^2+v_d^2)/2$] so small compared to the soft masses ?

[$v_u^2 + v_d^2 \equiv \langle |H|^2 \rangle = (174\ \text{GeV})^2$]

Problems in SUSY models

Gravity Mediated SUSY Breaking mech.

μ and $B\mu$ terms are O.K.

But Flavor and CP problems would arise.

Gauge Mediated SUSY Breaking mech.

Flavor and CP problems are absent.

But μ and $B\mu$ problems would be serious.

Model

$$W = (\lambda_1 X + \lambda_2 \phi + \mu)\, h_u h_d + M\, XY + (\kappa/2)\, Y \phi^2$$

Model

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$$W_{\rm UV} \supset \Psi \left(y_1 X_1 + y_2 X_2\right) Z + y_3 \Psi^c Z \phi$$
$$+ \left(y_4 X_1 + y_5 X_2\right) h_u h_d + \frac{(\Psi^c)^2}{M_P} \left(y_6 X_1 + y_7 X_2\right) Y + \frac{\kappa}{2} Y \phi^2,$$

Superfields	$X_{1,2}$	Y	Z	ϕ	Ψ	Ψ^c
$\mathrm{U}(1)_{\rm PQ}$	-1	$4/3$	$5/6$	$-2/3$	$1/6$	$-1/6$

Model

$$W = (\lambda_1 X + \lambda_2 \phi + \mu)\, h_u h_d + M\, XY + (\kappa/2)\, Y\phi^2$$

$$V \supset |H|^2|\lambda_1 X + \lambda_2\phi + \mu|^2 + |\lambda_1 h_u h_d + MY|^2 + \left|\frac{\kappa}{2}\phi^2 + MX\right|^2$$
$$+|\lambda_2 h_u h_d + \kappa Y\phi|^2 + m_X^2|X|^2 + m_Y^2|Y|^2 + m_\phi^2|\phi|^2$$
$$+\left\{(\lambda_1 a_1 X + \lambda_2 a_2 \phi)h_u h_d + MbXY + \frac{\kappa}{2}aY\phi^2 + \text{h.c.}\right\},$$

where $|H|^2 \equiv |h_u|^2 + |h_d|^2$

Model

W = (λ_1X + λ_2φ + μ) huhd + M XY + (κ/2) Yφ^2

$$V \supset |H|^2|\lambda_1 X + \lambda_2\phi + \mu|^2 + |\lambda_1 h_u h_d + MY|^2 + \underline{\underline{\left|\frac{\kappa}{2}\phi^2 + MX\right|^2}}$$
$$+|\lambda_2 h_u h_d + \kappa Y\phi|^2 + m_X^2|X|^2 + m_Y^2|Y|^2 + m_\phi^2|\phi|^2$$
$$+\left\{(\lambda_1 a_1 X + \lambda_2 a_2\phi)h_u h_d + MbXY + \frac{\kappa}{2}aY\phi^2 + \text{h.c.}\right\},$$

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Model

$$W = (\lambda_1 X + \lambda_2 \phi + \mu)\, h_u h_d + M\, XY + (\kappa/2)\, Y\phi^2$$

$$V \supset |H|^2 |\lambda_1 X + \lambda_2 \phi + \mu|^2 + |\lambda_1 h_u h_d + MY|^2 + \left|\frac{\kappa}{2}\phi^2 + MX\right|^2$$

$$+|\lambda_2 h$$

$$\boxed{(\kappa/2)\phi^2 + MX = 0}$$

***FLAT direction** (= modulus-like)*

in SUSY limit, with $h_u = h_d = Y = 0$

where $|H|^2 = |$

Model

$$W = (\lambda_1 X + \lambda_2 \phi + \mu)\, h_u h_d + M\, XY + (\kappa/2)\, Y\phi^2$$

$$V \supset |H|^2 |\lambda_1 X + \lambda_2 \phi + \mu|^2 + |\lambda_1 h_u h_d + MY|^2 + \left|\frac{\kappa}{2}\phi^2 + MX\right|^2$$

$+|\lambda \ldots$

Suppose

m_X^2, m_Y^2, m_ϕ^2, a, b, etc.

$\sim O(m^2)$ << MSSM soft para.

where $|H|^2 \equiv |h_u|^2 + |h_d|^2$

Effective mu and Bmu

$$\mu_{\text{eff}} = \lambda_1 \langle X \rangle + \lambda_2 \langle \phi \rangle + \mu,$$

$$B\mu_{\text{eff}} = \left(\lambda_1 M^* + \lambda_2 \kappa^* \langle \phi^* \rangle\right) \langle Y^* \rangle + \lambda_1 a_1 \langle X \rangle + \lambda_2 a_2 \langle \phi \rangle + B\mu,$$

Extreme Conditions

extreme conditions for X, Y, and ϕ

$$\begin{cases} \mathcal{M}_X^2 X + M^* b^* Y^* = -\frac{\kappa}{2} M^* \phi^2 - (\lambda_2 \phi + \mu) \lambda_1^* |H|^2 \\ \qquad\qquad\qquad\qquad -\lambda_1^* a_1^* h_u^* h_d^*, \\ \mathcal{M}_Y^2 Y^* + M b X = -\frac{\kappa}{2} a \phi^2 - (\lambda_1^* M + \lambda_2^* \kappa \phi)\, h_u^* h_d^*, \\ \left(|\kappa Y|^2 + |\lambda_2 H|^2 + m_\phi^2 \right) \phi + \left(\frac{\kappa}{2} \phi^2 + M X \right) \kappa^* \phi^* \\ \qquad\qquad + (\lambda_1 X + \mu) \lambda_2^* |H|^2 + \lambda_2^* a_2^* h_u^* h_d^* \\ \qquad\qquad\quad + (\lambda_2 h_u h_d + a^* \phi^*) \kappa^* Y^* = 0. \end{cases}$$

Solutions of Extrm. Condi.

$$X \approx \frac{-\kappa\phi^2}{2\mathcal{M}_X^2} M^* \left[1 - \frac{(a-b)b^*}{\mathcal{M}_Y^2} + \frac{2(\lambda_2\phi + \mu)\lambda_1^*|H|^2}{\kappa\phi^2 M^*} \right],$$

$$Y^* \approx \frac{-\kappa\phi^2}{2\mathcal{M}_Y^2}(a-b) - \frac{(\lambda_1^* M + \lambda_2^*\kappa\phi)h_u^* h_d^*}{\mathcal{M}_Y^2}.$$

where $\mathcal{M}_X^2 \equiv |\lambda_1 H|^2 + m_X^2 + |M|^2 \qquad (\approx |M|^2)$

$$\mathcal{M}_Y^2 \equiv |\kappa\phi|^2 + m_Y^2 + |M|^2$$

Extrm. Condi. for Φ

The extremum condition for $T_\zeta\ (\equiv \kappa\phi/M)$

$$\boxed{\begin{aligned}&\frac{1}{2}|T_\zeta|^2\left(|\lambda_1 H|^2+m_X^2\right)-T_\zeta^*\left(\lambda_2+\frac{\mu}{\phi}\right)\lambda_1^*|H|^2-\frac{1}{2}T_\zeta\lambda_2^*\lambda_1|H|^2\\&+\frac{\mu}{\phi}\lambda_2^*|H|^2+\left(|\lambda_2 H|^2+m_\phi^2\right)\approx\frac{|T_\zeta|^2(|T_\zeta|^2+2)}{4(|T_\zeta|^2+1)^2}\,|a-b|^2,\end{aligned}}$$

Extrm. Condi. for Φ

The extremum condition for $\underline{T_\zeta \ (\equiv \kappa\phi/M)}$

$$\frac{1}{2}|T_\zeta|^2\left(|\lambda_1 H|^2 + \underline{m_X^2}\right) - T_\zeta^*\left(\lambda_2 + \frac{\mu}{\phi}\right)\lambda_1^*|H|^2 - \frac{1}{2}T_\zeta\lambda_2^*\lambda_1|H|^2$$

$$\text{or} \quad |H|^2 \approx \frac{-m_\phi^2 - \frac{1}{2}\left(m_X^2 - |a-b|^2\right)|T_\zeta|^2}{\left(\lambda_2 - \frac{1}{2}T_\zeta\lambda_1 + \frac{\mu}{\phi}\right)\left(\lambda_2^* - T_\zeta^*\lambda_1^*\right)},$$

Extrm. Condi. for Φ

$$|H|^2 \sim -m_{\phi,X}^2 / |\lambda_{2,1}|^2$$

$$\text{or} \quad |H|^2 \approx \frac{-m_\phi^2 - \frac{1}{2}\left(m_X^2 - |a-b|^2\right)|T_\zeta|^2}{\left(\lambda_2 - \frac{1}{2}T_\zeta\lambda_1 + \frac{\mu}{\phi}\right)\left(\lambda_2^* - T_\zeta^*\lambda_1^*\right)},$$

Dynamical Relaxation

$$\boxed{\mu_{\text{eff}} = \lambda_1 \langle X \rangle + \lambda_2 \langle \phi \rangle + \mu,} \quad \boxed{\approx \frac{MT_\zeta}{\kappa}\left(\lambda_2 - \frac{1}{2}\lambda_1 T_\zeta\right) + \mu,}$$

μ_{eff}

$$\boxed{|\mu|^2 + \frac{1}{2}M_Z^2 = \frac{m_{h_d}^2 - m_{h_u}^2 \tan^2\beta}{\tan^2\beta - 1}} \; (\approx -m_{h_u}^2)$$

satisfying the conditions for EW symmetry breaking,

$B\mu_{\text{eff}}$

$$2B\mu \;<\; (m_{h_u}^2 + |\mu|^2) + (m_{h_d}^2 + |\mu|^2)$$

$$(B\mu)^2 \;>\; (m_{h_u}^2 + |\mu|^2)(m_{h_d}^2 + |\mu|^2)$$

Dynamical Relaxation

$$\mu_{\rm eff} = \lambda_1\langle X\rangle + \lambda_2\langle \phi\rangle + \mu, \qquad \approx \frac{MT_\zeta}{\kappa}\left(\lambda_2 - \frac{1}{2}\lambda_1 T_\zeta\right) + \mu,$$

$\mu_{\rm eff}$

$$|\mu|^2 + \frac{1}{2}M_Z^2 = \frac{m_{h_d}^2 - m_{h_u}^2\tan^2\beta}{\tan^2\beta - 1} \quad (\approx -m_{h_u}^2$$

sati ... g.

For $|\lambda_2 M/\kappa|^2 >> -m_{hu}^2$

$|T_\zeta(1-T_\zeta\lambda_1/2\lambda_2)| << 1$

$B\mu_{\rm eff}$

$(B$

Little Hierarchy Problem

$$|\mu|^2 + \frac{1}{2}M_Z^2 = \frac{m_{h_d}^2 - m_{h_u}^2 \tan^2\beta}{\tan^2\beta - 1}$$

Why is M_Z^2 $[=(g_2^2+g_Y^2)(v_u^2+v_d^2)/2]$ so small compared to the soft masses ?

$[\, v_u^2 + v_d^2 \equiv \langle |H|^2 \rangle = (174\ \mathrm{GeV})^2 \,]$

Little Hierarchy Problem

It is because m_ϕ, m_χ are so small compared to the MSSM soft masses.

Why is M_Z^2 $[=(g_2^2+g_Y^2)(v_u^2+v_d^2)/2]$ so small compared to the soft masses ?

$[\, v_u^2 + v_d^2 \equiv \langle |H|^2 \rangle = (174\ \mathrm{GeV})^2 \,]$

For small enough $m_{\phi,X}^2$

Introduce Gauge Med. SUSY Breaking
as well as Gravity Med. SUSY breaking

Gauge Med. $\longrightarrow$ Heavy MSSM soft masses
avoiding Exp. Bounds and SUSY flavor and CP problems

Gravity Med. $\longrightarrow$ Small MSSM singlet masses and $B\mu$ term

For small enough $m_{\phi,X}{}^2$

Introduce Gauge Med. SUSY Breaking
as well as Gravity Med. SUSY breaking

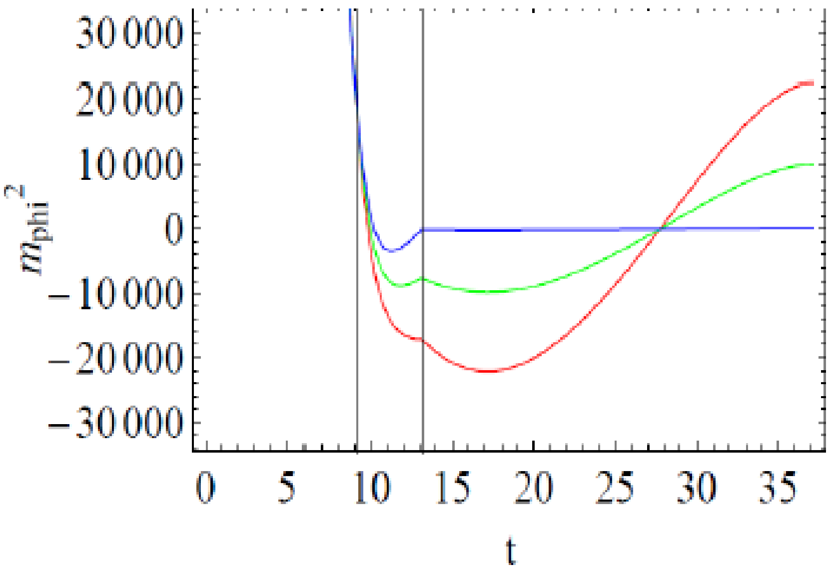
- ***Messenger Scale** of the **Gauge Med.***
  *needs to be **LOW** enough.*
- *λ_2 needs to be **SMALL** enough.*

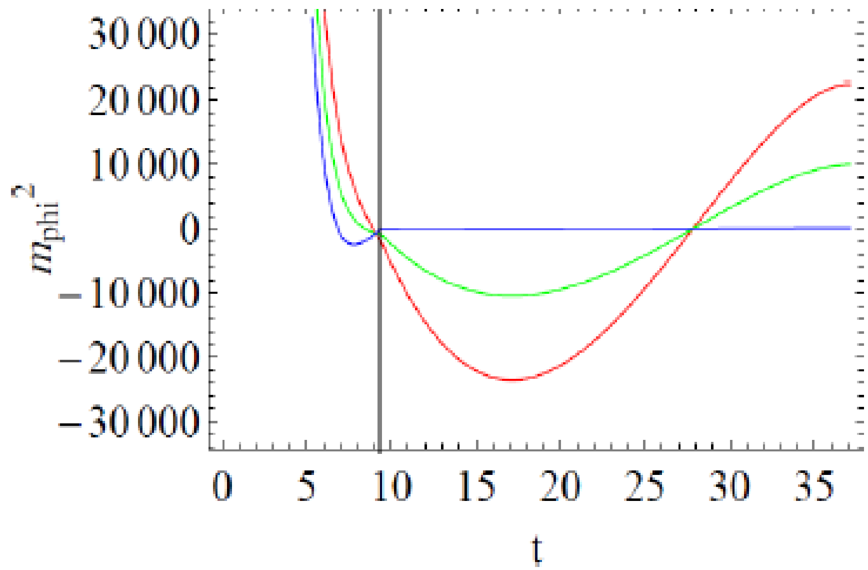
lems

Gravi

term

Focus Point
(λ_1=0.7 ≫ λ_2=0.02)





RG evolutions of $m_\phi{}^2$ under various trial $m_0{}^2$s.

the messenger scale = 500 TeV (L) and 12 TeV (R).

In both cases, the stop mass scales = 10 TeV.

Focus Point

Case I $\lambda_2^2 = 5 \cdot 10^{-4}$ $\lambda_1^2 = 0.5$	$\tan\beta = 10$ $\tilde{m}_t^2 = (10\,\mathrm{TeV})^2$ $\Lambda_M = 15\,\mathrm{TeV}$	Case II $\lambda_2^2 = 8 \cdot 10^{-3}$ $\lambda_1^2 = 0.5$	$\tan\beta = 40$ $\tilde{m}_t^2 = (20\,\mathrm{TeV})^2$ $\Lambda_M = 25\,\mathrm{TeV}$
$\boldsymbol{\Delta}_{\mathbf{m_0^2}}$	19.1	$\boldsymbol{\Delta}_{\mathbf{m_0^2}}$	79.6
$\boldsymbol{\Delta}_{\mathrm{M}_{\mathbf{1/2}}}$	83.2	$\boldsymbol{\Delta}_{\mathrm{M}_{\mathbf{1/2}}}$	28.6
$\boldsymbol{\Delta}_{\lambda_{\mathbf{2}}^{\mathbf{2}}}$	59.7	$\boldsymbol{\Delta}_{\lambda_{\mathbf{2}}^{\mathbf{2}}}$	56.5
$\boldsymbol{\Delta}_{\mathrm{GM}}$	37.1	$\boldsymbol{\Delta}_{\mathrm{GM}}$	153.5
$\boldsymbol{\Delta}_{\Lambda_{\mathrm{M}}}$	6.0	$\boldsymbol{\Delta}_{\Lambda_{\mathrm{M}}}$	21.3

Focus Point

Case I	$\tan\beta = 10$	Case II	$\tan\beta = 40$
$\lambda_2^2 = 5 \cdot 10^{-4}$	$\tilde{m}_t^2 = (10\,\mathrm{TeV})^2$	$\lambda_2^2 = 8 \cdot 10^{-3}$	$\tilde{m}_t^2 = (20\,\mathrm{TeV})^2$
$\lambda_1^2 = 0.5$	$\Lambda_M = 15\,\mathrm{TeV}$	$\lambda_1^2 = 0.5$	$\Lambda_M = 25\,\mathrm{TeV}$

$$F_{gauge}/(16\pi^2\Lambda_M) \equiv GM \approx 5.1\,\mathrm{TeV}\ (\mathrm{I}) \text{ and } 10.5\,\mathrm{TeV}\ (\mathrm{II}),$$

$$F_{gauge} \ll F_{gravity},$$

$$m_{3/2} \approx F_{gravity}/(\sqrt{3}\,M_{Pl}) \approx m_0 = 30.4\,\mathrm{GeV}\ (\mathrm{I}) \text{ and } 124.7\,\mathrm{GeV}\ (\mathrm{II}),$$

We set $M_{1/2} = 125\,m_0$ (I) and $54\,m_0$ (II) at the GUT scale.

Focus Point

Case I	$\tan\beta = 10$	Case II	$\tan\beta = 40$
$\lambda_2^2 = 5 \cdot 10^{-4}$	$\tilde{m}_t^2 = (10\,\mathrm{TeV})^2$	$\lambda_2^2 = 8 \cdot 10^{-3}$	$\tilde{m}_t^2 = (20\,\mathrm{TeV})^2$
$\lambda_1^2 = 0.5$	$\Lambda_M = 15\,\mathrm{TeV}$	$\lambda_1^2 = 0.5$	$\Lambda_M = 25\,\mathrm{TeV}$

$(M_G\ M_W\ M_B) \approx (12, 5, 3)$ TeV (I) and $(22, 9, 5)$ TeV (II)

$\mu_{eff} \approx 2.5$ TeV (I) and 2.3 TeV (II)

The SUSY particles' masses of the 1st and 2nd generations are much heavier.

Mass Matrix (fermion)

$$
\begin{pmatrix}
\kappa Y & \kappa\phi & 0 & \lambda_2 h_u & \lambda_2 h_d \\
\kappa\phi & 0 & M & 0 & 0 \\
0 & M & 0 & \lambda_1 h_u & \lambda_1 h_d \\
\lambda_2 h_u & 0 & \lambda_1 h_u & 0 & \mu_{\mathrm{eff}} \\
\lambda_2 h_d & 0 & \lambda_1 h_d & \mu_{\mathrm{eff}} & 0
\end{pmatrix}
$$

in the basis of $\{\phi, Y, X, h_d, h_u\}$

Mass Matrix (fermion)

$$\left(\begin{array}{ccc|cc} \kappa Y & \kappa\phi & 0 & \lambda_2 h_u & \lambda_2 h_d \\ \kappa\phi & 0 & M & 0 & 0 \\ 0 & M & 0 & \lambda_1 h_u & \lambda_1 h_d \\ \hline \lambda_2 h_u & 0 & \lambda_1 h_u & 0 & \mu_{\text{eff}} \\ \lambda_2 h_d & 0 & \lambda_1 h_d & \mu_{\text{eff}} & 0 \end{array}\right)$$

in the basis of $\{\phi, Y, X, h_d, h_u\}$

Mass Matrix (fermion)

$$\left(\begin{array}{ccc|cc}
\kappa Y & \kappa\phi & 0 & \lambda_2 h_u & \lambda_2 h_d \\
\kappa\phi & 0 & M & 0 & 0 \\
0 & M & 0 & \lambda_1 h_u & \lambda_1 h_d \\
\hline
\lambda_2 h_u & 0 & \lambda_1 h_u & 0 & \mu \\
\lambda_2 h_d & 0 & \lambda_1 h & &
\end{array}\right)$$

$$\frac{\kappa\langle Y\rangle}{1+T_\zeta^2} \approx \frac{1}{2}\, T_\zeta^2\ (b^* - a^*)$$

<u>The smallest mass E. value</u>

≈ sub GeV or lighter,

The lightest E. state plays the role of DM.

Mass Matrix (scalar)

$$\begin{pmatrix} m_H^2 & \lambda_2 H \mu_{\text{eff}}^* & \lambda_1 H \mu_{\text{eff}}^* \\ \lambda_2^* H^* \mu_{\text{eff}} & m_\phi^2 + |\lambda_2 H|^2 + |\kappa\phi|^2 & \lambda_2^* \lambda_1 |H|^2 + \kappa^* \phi^* M \\ \lambda_1^* H^* \mu_{\text{eff}} & \lambda_1^* \lambda_2 |H|^2 + \kappa\phi M^* & m_X^2 + |\lambda_1 H|^2 + |M|^2 \end{pmatrix}$$

in the basis of $\{H, \phi, X\}$

Mass Matrix (scalar)

$$\mathcal{O}_3^T \cdot \text{diag.}(m_1^2, m_2^2, M_3^2) \cdot \mathcal{O}_3$$

yields a symmetric matrix $\mathcal{M}_{ij}^2$ $(= \mathcal{M}_{ji}^2)$ with the following elements:

$$\begin{aligned}
\mathcal{M}_{11}^2 &\approx M_3^2\ \varepsilon_2^2 + m_2^2\ \varepsilon_1^2 + m_1^2, \\
\mathcal{M}_{12}^2 &\approx M_3^2\ \varepsilon_2\ \sin\theta + m_2^2\ \varepsilon_1\ \cos\theta - m_1^2\ \epsilon_1, \\
\mathcal{M}_{13}^2 &\approx M_3^2\ \varepsilon_2\ \cos\theta - m_2^2\ \varepsilon_1\ \sin\theta - m_1^2\ \epsilon_2, \\
\mathcal{M}_{23}^2 &\approx \Delta M_{32}^2\,(1 - \bar{\epsilon}^2)\ \sin\theta\ \cos\theta, \\
\mathcal{M}_{22}^2 &\approx \Delta M_{32}^2\ \sin^2\theta + m_2^2 - \{\Delta M_{32}^2\ \sin^2\theta + \Delta m_{21}^2\}\ \epsilon_1^2, \\
\mathcal{M}_{33}^2 &\approx \Delta M_{32}^2\ \cos^2\theta + m_2^2 - \{\Delta M_{32}^2\ \cos^2\theta + \Delta m_{21}^2\}\ \epsilon_2^2 \\
&\qquad -\Delta M_{32}^2\ \sin 2\theta\ \epsilon_1 \epsilon_2,
\end{aligned}$$

Mass Matrix (scalar)

$$\mathcal{O}_3^T \cdot \mathrm{diag.}(m_1^2, m_2^2, M_3^2) \cdot \mathcal{O}_3$$

yields a sym

$$\mathcal{O}_3 = \begin{pmatrix} c_1 c_2 & -s_1 & -c_1 s_2 \\ c_2 c_3 s_1 - s_2 s_3 & c_1 c_3 & -c_3 s_1 s_2 - c_2 s_3 \\ c_3 s_2 + c_2 s_1 s_3 & c_1 s_3 & c_2 c_3 - s_1 s_2 s_3 \end{pmatrix}$$

$$\mathcal{M}^2_{11} \approx$$

$$\mathcal{M}^2_{12} \approx$$

$$\mathcal{M}^2_{13} \approx M_3^2\ \varepsilon_2 \cos\theta - m_2^2\ \varepsilon_1 \sin\theta - m_1^2\ \epsilon_2,$$

$$\mathcal{M}^2_{23} \approx \Delta M^2_{32}\,(1 - \bar{\epsilon}^2)\ \sin\theta\ \cos\theta,$$

$$\mathcal{M}^2_{22} \approx \Delta M^2_{32}\ \sin^2\theta + m_2^2 - \{\Delta M^2_{32}\ \sin^2\theta + \Delta m^2_{21}\}\ \epsilon_1^2,$$

$$\mathcal{M}^2_{33} \approx \Delta M^2_{32}\ \cos^2\theta + m_2^2 - \{\Delta M^2_{32}\ \cos^2\theta + \Delta m^2_{21}\}\ \epsilon_2^2$$
$$-\Delta M^2_{32}\ \sin 2\theta\ \epsilon_1 \epsilon_2,$$

Mass Matrix (scalar)

$$\mathcal{O}_3^T \cdot \mathrm{diag.}(m_1^2, m_2^2, M_3^2) \cdot \mathcal{O}_3$$

yields a symmetric matrix $\mathcal{M}^2_{ij}$ $(= \mathcal{M}^2_{ji})$ with the following elements:

$$\mathcal{M}^2_{11} \approx M_3^2\ \varepsilon_2^2 + m_2^2\ \varepsilon_1^2 + m_1^2,$$

$$\mathcal{M}^2_{12} \approx M_3^2\ \varepsilon_2\ \sin\theta + m_2^2\ \varepsilon_1\ \cos\theta - m_1^2\ \epsilon_1,$$

$\mathcal{M}^2_{13}$

$\mathcal{M}^2_{23}$

$\mathcal{M}^2_{22}$

$\mathcal{M}^2_{33}$

$$\varepsilon_{2,1} \equiv \epsilon_{2,1}\ \cos\theta \pm \epsilon_{1,2}\ \sin\theta,$$

$$\bar{\epsilon}^2 \equiv \frac{1}{2}\left(\epsilon_1^2 + \epsilon_2^2\right) + \tan\theta\ \epsilon_1\epsilon_2 + \frac{2(m_2^2 - m_1^2)\ \epsilon_1\epsilon_2}{(M_3^2 - m_2^2)\ \sin 2\theta},$$

$$\Delta M_{32}^2 \equiv M_3^2 - m_2^2\ , \quad \text{and} \quad \Delta m_{21}^2 \equiv m_2^2 - m_1^2.$$

Mass Matrix (scalar)

(1,1) :

$$m_H^2 \approx M_Z^2 \cos^2 2\beta + \left| \frac{\kappa\phi^2 |\lambda_1|^2}{|M|^2} \right|^2 |H|^2 + \Delta m_H^2 \,,$$

$$\left(\boxed{\left| \frac{\kappa\phi^2 |\lambda_1|^2}{|M|^2} \right|^2 |H|^2 \approx M_3^2 \varepsilon_2^2 + M_2^2 \varepsilon_1^2 .} \right)$$

(1,2), (1,3) :

$$\boxed{\begin{aligned} \lambda_2 H \mu_{\text{eff}} &\approx M_3^2 \ \varepsilon_2 \ \sin\theta + M_2^2 \ \varepsilon_1 \ \cos\theta, \\ \lambda_1 H \mu_{\text{eff}} &\approx M_3^2 \ \varepsilon_2 \ \cos\theta - M_2^2 \ \varepsilon_1 \ \sin\theta, \end{aligned}}$$

$$\boxed{\frac{\varepsilon_1}{\varepsilon_2} \equiv \frac{\epsilon_1 \cos\theta - \epsilon_2 \sin\theta}{\epsilon_2 \cos\theta + \epsilon_1 \sin\theta} = \frac{M_3^2}{M_2^2} \ \frac{\frac{\lambda_2}{\lambda_1} - \tan\theta}{1 + \frac{\lambda_2}{\lambda_1} \tan\theta},}$$

Mass Matrix (scalar)

(2,3) : $\kappa\phi M = M_\Sigma^2 \sin\theta \cos\theta, \quad \lambda_1\lambda_2|H|^2 = \delta M^2 \sin\theta \cos\theta,$

$$M_\Sigma^2 + \delta M^2 \approx \Delta M_{32}^2 (1 - \bar{\epsilon}^2),$$

Mass Matrix (scalar)

<u>(2,3)</u> : $\kappa\phi M = M_\Sigma^2 \sin\theta \, \cos\theta, \quad \lambda_1\lambda_2|H|^2 = \delta M^2 \sin\theta \, \cos\theta,$

$$\boxed{M_\Sigma^2 + \delta M^2 \approx \Delta M_{32}^2(1 - \bar{\epsilon}^2),}$$

$$\boxed{\frac{\kappa\phi}{M} \equiv \tan\zeta, \quad \frac{\lambda_2}{\lambda_1} \equiv \tan\xi,}$$

T_ζ

Mass Matrix (scalar)

$$m_\phi^2 \approx m_2^2 + \Delta M_{32}^2 \left(\bar{\epsilon}^2 - \epsilon_1^2\right) \sin^2\theta - \Delta m_{21}^2 \epsilon_1^2$$
$$+ M_\Sigma^2 \sin\theta \, \cos\theta \left(\tan\theta - \tan\zeta\right)$$
$$+ \delta M^2 \sin\theta \, \cos\theta \left(\tan\theta - \tan\xi\right),$$
$$m_X^2 \approx m_2^2 + \Delta M_{32}^2 \left(\bar{\epsilon}^2 - \epsilon_2^2\right) \cos^2\theta - \Delta m_{21}^2 \epsilon_2^2$$
$$- \Delta M_{32}^2 \, \sin 2\theta \, \epsilon_1 \epsilon_2 + M_\Sigma^2 \sin\theta \, \cos\theta \left(\cot\theta - \cot\zeta\right)$$
$$+ \delta M^2 \sin\theta \, \cos\theta \left(\cot\theta - \cot\xi\right)$$

Around $\theta = \frac{\pi}{2} + \zeta$, i.e. when $\theta = \frac{\pi}{2} + \delta\theta$ $(|\delta\theta|, |\zeta| \ll 1)$,

(2,2), $$m_\phi^2 \approx M_3^2 - M_2^2 \left[\frac{\epsilon_1^2 + \epsilon_2^2}{2} + (\epsilon_1\epsilon_2 + \zeta - \delta\theta)\,\delta\theta\right],$$

(3,3) : $$m_X^2 \approx M_2^2 \left[1 - 2\epsilon_1\epsilon_2\delta\theta - \epsilon_2^2 - \frac{\delta\theta}{\zeta} - \mathcal{O}(\delta\theta^2)\right],$$

Mass Matrix (scalar)

$$m_\phi^2 \approx m_2^2 + \Delta M_{32}^2\left(\vec{\epsilon}^2 - \epsilon_1^2\right)\sin^2\theta - \Delta m_{21}^2\epsilon_1^2$$
$$+ M_\Sigma^2 \sin\theta\,\cos\theta\,(\tan\theta - \tan\zeta)$$
$$+ \delta M^2 \sin\theta\,\cos\theta\,(\tan\theta - \tan\xi)\,.$$

$$\boxed{\delta\theta \approx \frac{\zeta\,(1-\epsilon_2^2)}{1+2\epsilon_1\epsilon_2\zeta}\,, \qquad M_3^2 \approx M_2^2\left[\frac{\epsilon_1^2+\epsilon_2^2}{2} + \epsilon_1\epsilon_2\zeta\right]}$$

$$+ \delta M^2 \sin\theta\,\cos\theta\,(\cot\theta - \cot\xi)$$

Around $\theta = \frac{\pi}{2} + \zeta$, i.e. when $\theta = \frac{\pi}{2} + \delta\theta$ ($|\delta\theta|, |\zeta| \ll 1$),

(2,2), $\quad m_\phi^2 \approx M_3^2 - M_2^2\left[\frac{\epsilon_1^2+\epsilon_2^2}{2} + (\epsilon_1\epsilon_2 + \zeta - \delta\theta)\,\delta\theta\right]$,

(3,3) : $\quad m_X^2 \approx M_2^2\left[1 - 2\epsilon_1\epsilon_2\delta\theta - \epsilon_2^2 - \frac{\delta\theta}{\zeta} - \mathcal{O}(\delta\theta^2)\right]$,

Mass Matrix (scalar)

We can fulfill the constraints e.g. with

- $\lambda_1 \approx$ **0.7**, $|\lambda_2 / \lambda_1| = 0.03$, $\tan\zeta < 10^{-1}$,
- $M_3 \sim 500$ GeV, $M_2 \sim$ **5 TeV**,
- $\boldsymbol{\varepsilon \sim 10^{-1} - 10^{-2}}$, $|\tan\theta| > 10^{+1}$.

The mixing btw H and the singlets can be suppressed enough.

The mixing btw ϕ and X is almost the maximal.

Conclusion

- The MSSM μ term is dynamically adjusted by singlets such that the min. cond. of the Higgs is fulfilled.
  A ***FLAT DIRECTION*** compensates m_{hu}^2, while the ***SM Higgs*** does m_ϕ^2.

- A relatively small soft mass of a singlet (m_ϕ^2 or m_χ^2) is responsible for the small <H> (or small M_Z). Possible by Small Gravity Medi. Effects!

- The MSSM SUSY ptl.s are heavy enough to avoid Exp. Bounds and FCNC. Possible by Large Gauge Medi. Effects!

- A sub-GeV fermionic DM is predicted, while the Higgsino is quite heavy.

- The Mixings btw the Higgs and singlets can be suppressed enough by introducing several singlets.

Thank You !!