QED Radiation in Vincia

Rob Verheyen

With Peter Skands & Ronald Kleiss

Work in progress

IMAPP   Nikhef   Radboud University

Introduction

Vincia is a parton shower plugin for Pythia based on antenna factorization

Currently Vincia only does QCD radiation

We want to include QED radiation too

Giele, Kosower, Skands:1102.2126

Gehrmann, Ritzmann, Skands:1108.6172

Current approaches to photon radiation

DGLAP

- Resums collinear photon logarithms
- Interleaving with QCD shower

YFS

- Resums soft photon logarithms
- Collinear logarithms can be included, but not resummed
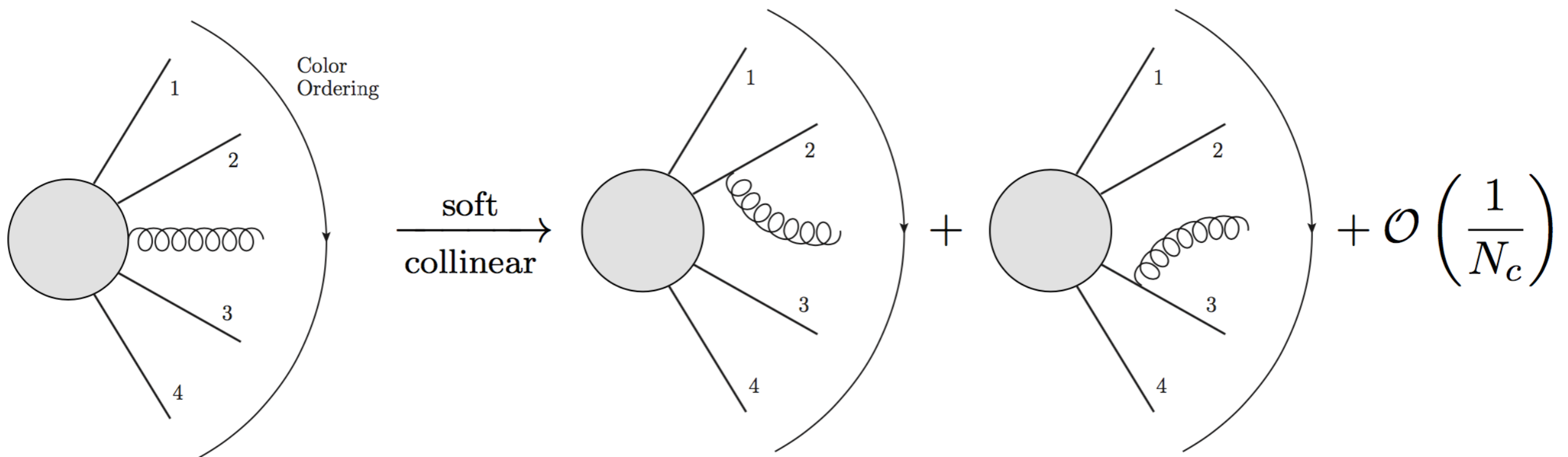- Afterburner to the shower

I'll discuss the three options for photon emission we're implementing

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Leading Color Gluon Emission

Factorization

$$|M(..,p_a,k,..)|^2 \xrightarrow{p_a \| k} g^2 C \frac{P(z)}{p_a \cdot k} |M(..,p_a+k,..)|^2$$

$$|M(..,p_a,k,p_b,..)|^2 \xrightarrow{k \to 0} g^2 C \left[\frac{2p_a \cdot p_b}{(p_a \cdot k)(k \cdot p_b)} - \frac{m_a^2}{(p_a \cdot k)^2} - \frac{m_b^2}{(p_b \cdot k)^2} \right] |M(..,p_a,p_b,..)|^2$$
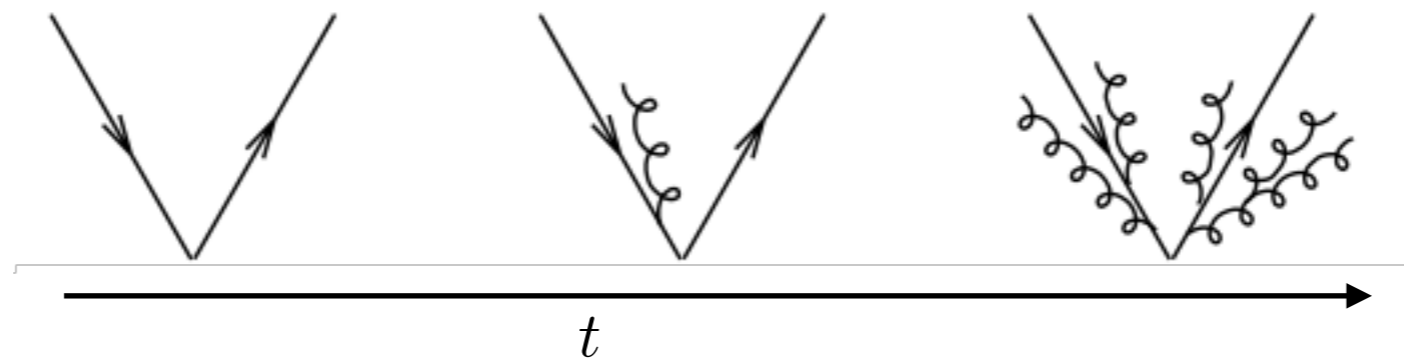
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Leading Color Gluon Emission

Factorization

$$|M(..,p_a,k,p_b,..)|^2 \approx g^2 C\, a_e^{QCD}(p_a,k,p_b)|M(..,p_a',p_b',..)|^2$$

$2 \to 3$ branching

Ordering scale

$$t = p_\perp^2 = 4\,\frac{p_a \cdot k\, p_b \cdot k}{m^2}$$

t

Illustration: G. Salam

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Photon Emission

$$|M(.., p_a, k, ..)|^2 \xrightarrow{p_a \| k} e^2 Q_a^2 \frac{P(z)}{p_a \cdot k} |M(.., p_a + k, ..)|^2$$

$$|M(\{p\}, k)|^2 \xrightarrow{k \to 0} -e^2 \sum_{[a,b]} Q_a Q_b \left[\frac{2 p_a \cdot p_b}{(p_a \cdot k)(k \cdot p_b)} - \frac{m_a^2}{(p_a \cdot k)^2} - \frac{m_b^2}{(p_b \cdot k)^2} \right] |M(\{p\})|^2$$

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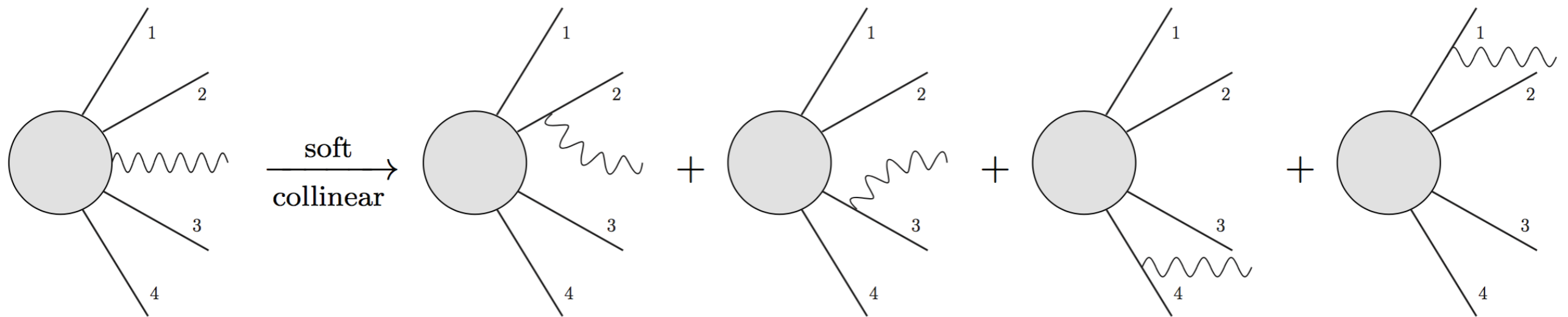
Photon Emission

Factorization

$$|M(\{p\}, k)|^2 \approx e^2 a_e^{QED}(\{p\}, k) \, |M(\{p'\})|^2$$

$$a_e^{QED}(\{p\}, k) = -\sum_{[a,b]} Q_a Q_b \, a_e^{QCD}(p_a, k, p_b)$$

$n \to n+1$ branching

Ordering scale
$$t = p_\perp^2 = 4 \frac{p_a \cdot k \, p_b \cdot k}{m^2}$$

Photon emissions are a multi-scale problem

Goal: recast this $n \to n+1$ branching into a (set of) $2 \to 3$ branchings

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Option 1:
Pairing

Incoherent Pairing

Pythia-like approach: Include only one antenna function for every fermion

$$a_e^{QED}(\{p\}, k) = Q_{f_1^+} Q_{f_1^-} a_e^{QCD}(p_{f_1^+}, k, p_{f_1^-}) + Q_{f_2^+} Q_{f_2^-} a_e^{QCD}(p_{f_2^+}, k, p_{f_2^-}) + ...$$
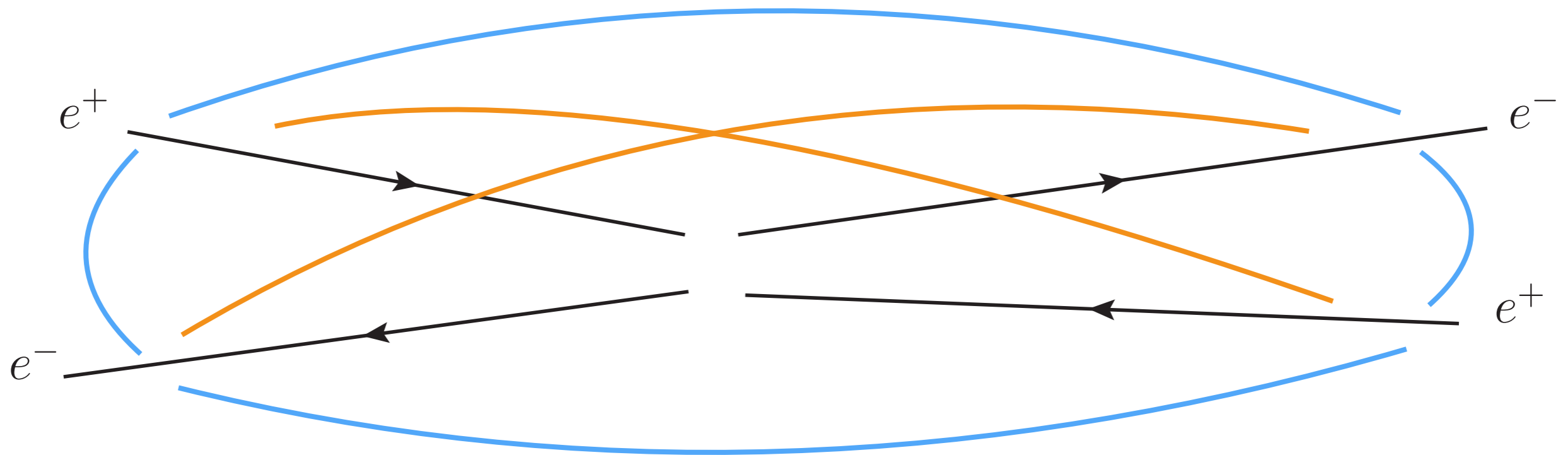
Competition between independent radiators

- Correct collinear behaviour
- Only includes some eikonal factors

Pair up the fermions to minimise $m^2_{f_1^+ f_1^-} + m^2_{f_2^+ f_2^-} + ...$

Incoherent Pairing

Account for destructive interference

Emission scales are kinematically restricted by the antenna mass

Pair up the fermions to minimise $m^2_{f_1^+ f_1^-} + m^2_{f_2^+ f_2^-} + \ldots$

Brute force $\mathcal{O}(n!)$ complexity…

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Turns out this is a well-known problem from graph theory!

$$
\begin{bmatrix} f_1^+ \\ f_2^+ \\ f_3^+ \end{bmatrix}
\begin{array}{ccc} f_1^- & f_2^- & f_3^- \end{array}
\begin{bmatrix}
m^2_{f_1^+ f_1^-} & m^2_{f_1^+ f_2^-} & m^2_{f_1^+ f_3^-} \\
m^2_{f_2^+ f_1^-} & m^2_{f_2^+ f_2^-} & m^2_{f_2^+ f_3^-} \\
m^2_{f_3^+ f_1^-} & m^2_{f_3^+ f_2^-} & m^2_{f_3^+ f_3^-}
\end{bmatrix}
$$

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Let's look at an example to see how it works

$$
\begin{array}{c}
\begin{bmatrix} f_1^- & f_2^- & f_3^- \end{bmatrix} \\
\begin{bmatrix} f_1^+ \\ f_2^+ \\ f_3^+ \end{bmatrix}
\begin{bmatrix}
35 & 50 & 30 \\
5 & 15 & 10 \\
35 & 50 & 20
\end{bmatrix}
\end{array}
$$

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The Hungarian Algorithm

Step 1: Subtract the lowest row element from all rows

$$
\begin{bmatrix} f_1^- & f_2^- & f_3^- \end{bmatrix}
$$

$$
\begin{bmatrix} f_1^+ \\ f_2^+ \\ f_3^+ \end{bmatrix}
\begin{bmatrix} 5 & 20 & 0 \\ 0 & 10 & 5 \\ 15 & 30 & 0 \end{bmatrix}
\begin{matrix} {\color{red}-30} \\ {\color{red}-5} \\ {\color{red}-20} \end{matrix}
$$

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The Hungarian Algorithm

Step 2: Subtract the lowest column element from all rows

$$\begin{array}{c} \\ \begin{bmatrix} f_1^+ \\ f_2^+ \\ f_3^+ \end{bmatrix} \end{array} \begin{array}{ccc} \begin{bmatrix} f_1^- & f_2^- & f_3^- \end{bmatrix} \\ \begin{bmatrix} 5 & 10 & 0 \\ 0 & 0 & 5 \\ 15 & 20 & 0 \end{bmatrix} \\ {\color{red}-10} \end{array}$$

The Hungarian Algorithm

Step 3: Find the minimal line covering

$$
\begin{array}{c}
\begin{bmatrix} f_1^- & f_2^- & f_3^- \end{bmatrix} \\
\begin{bmatrix} f_1^+ \\ f_2^+ \\ f_3^+ \end{bmatrix}
\begin{bmatrix} 5 & 10 & 0 \\ 0 & 0 & 5 \\ 15 & 20 & 0 \end{bmatrix}
\end{array}
$$

If the line covering is maximal (n=3), pairing with cost 0 can be found

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Step 4: Find the lowest uncovered element

$$
\begin{bmatrix} f_1^- & f_2^- & f_3^- \end{bmatrix}
$$

$$
\begin{bmatrix} f_1^+ \\ f_2^+ \\ f_3^+ \end{bmatrix}
\begin{bmatrix} 5 & 10 & 0 \\ 0 & 0 & 5 \\ 15 & 20 & 0 \end{bmatrix}
$$

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The Hungarian Algorithm

Step 4: Subtract that number from all uncovered element
Add it to all doubly covered elements

$$-5 \begin{bmatrix} f_1^- & f_2^- & f_3^- \end{bmatrix}$$

$$\begin{bmatrix} f_1^+ \\ f_2^+ \\ f_3^+ \end{bmatrix} \begin{bmatrix} 0 & 5 & 0 \\ 0 & 0 & 10 \\ 10 & 15 & 0 \end{bmatrix} +5$$

And go back to step 3

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The Hungarian Algorithm

$$\begin{bmatrix} f_1^- & f_2^- & f_3^- \end{bmatrix}$$

$$\begin{bmatrix} f_1^+ \\ f_2^+ \\ f_3^+ \end{bmatrix} \begin{bmatrix} \textcircled{0} & 5 & 0 \\ 0 & \textcircled{0} & 10 \\ 10 & 15 & \textcircled{0} \end{bmatrix}$$

Now we are able to find an optimal pairing!

$\mathcal{O}(n^3)$ complexity, so not computationally prohibitive
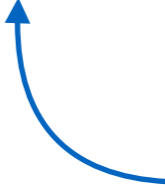
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Option 2: Coherent

Coherent Emission

Separate phase space into *sectors*

$$|M(\{p\}, k)|^2 \approx \sum_{[i,j]} \left(- \sum_{[a,b]} Q_a Q_b a_e^{QCD}(p_a, k, p_b) \right) \theta(p^2_{\perp\,ij}) |M(.., p'_i, p'_j, ..)|^2$$

1 if $p^2_{\perp\,ij}$ is the smallest
0 otherwise

Equivalent to ordering in

$$t = \min\left(p^2_{\perp\,ij}\right) = \min\left(4 \frac{p_i \cdot k \, p_j \cdot k}{m^2} \right)$$

But there's a problem…

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Sudakov Veto Algorithm

Want to sample from

$$f(t) \exp\left(-\int_t^u d\tau\, f(\tau)\right)$$

Find $g(t) \geq f(t)$

Set $u = t_{\text{start}}$

Sample t from $g(t) \exp\left(-\int_t^u d\tau\, g(\tau)\right)$
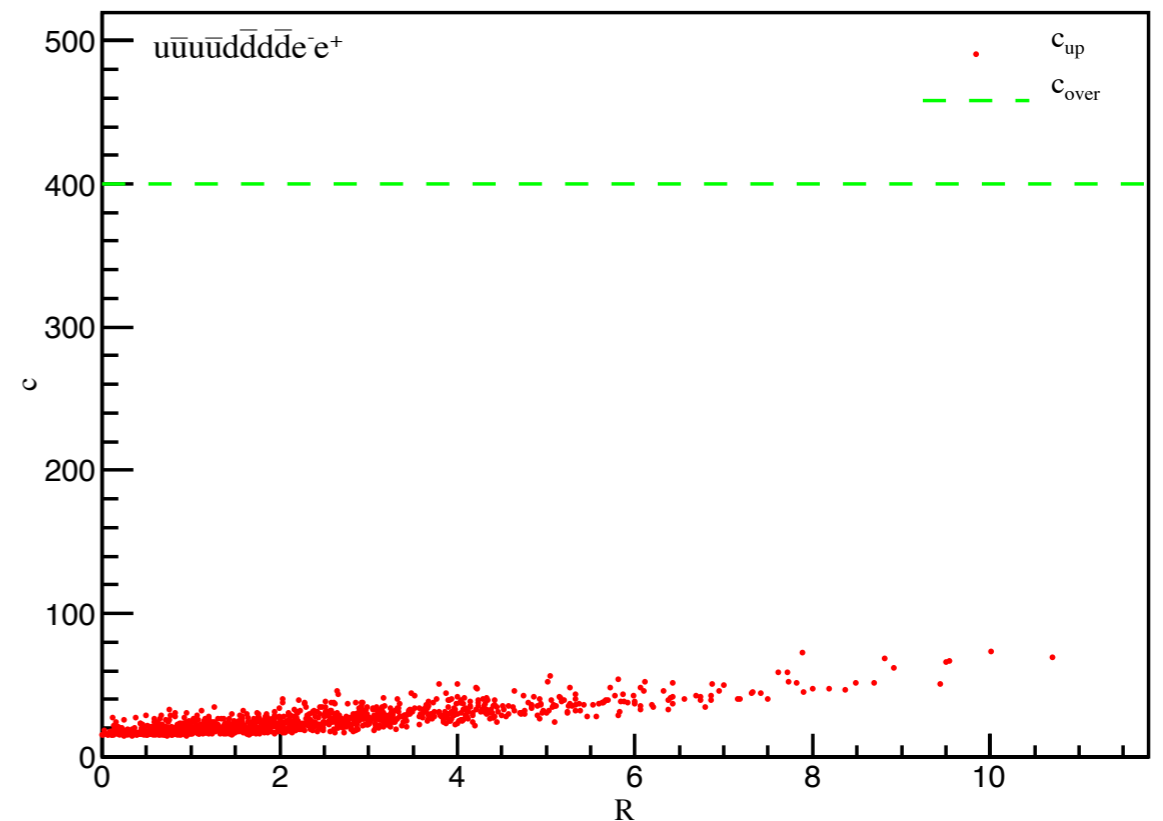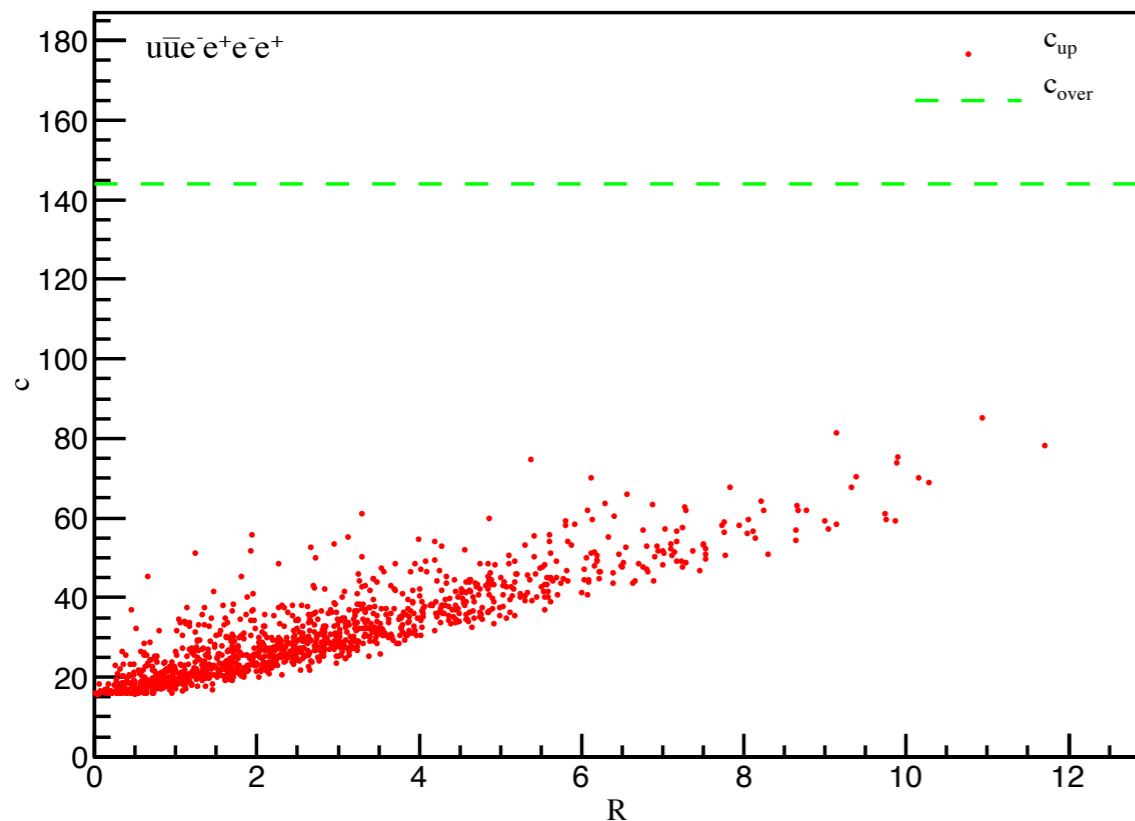
Set $u = t$

Accept with probability $\dfrac{f(t)}{g(t)}$

Done $\longrightarrow$ t

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Coherent Emission

We need an overestimate for the branching kernel

$$a_e^{QED} = -\sum_{[a,b]} Q_a Q_b \, a_e^{QCD}(p_a, k, p_b)$$

It's possible to find one, but…

The algorithm is slow!

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Option 3:
Coherent Weighted

Sudakov Veto Algorithm

Want to sample from

$$f(t) \exp\left(-\int_t^u d\tau\, f(\tau)\right)$$

Find $g(t) \not\geq f(t)$

Set $u = t_{\text{start}}$

Sample t from $g(t) \exp\left(-\int_t^u d\tau\, g(\tau)\right)$

Set $u = t$

Accept with probability $p(t)$

Apply weight

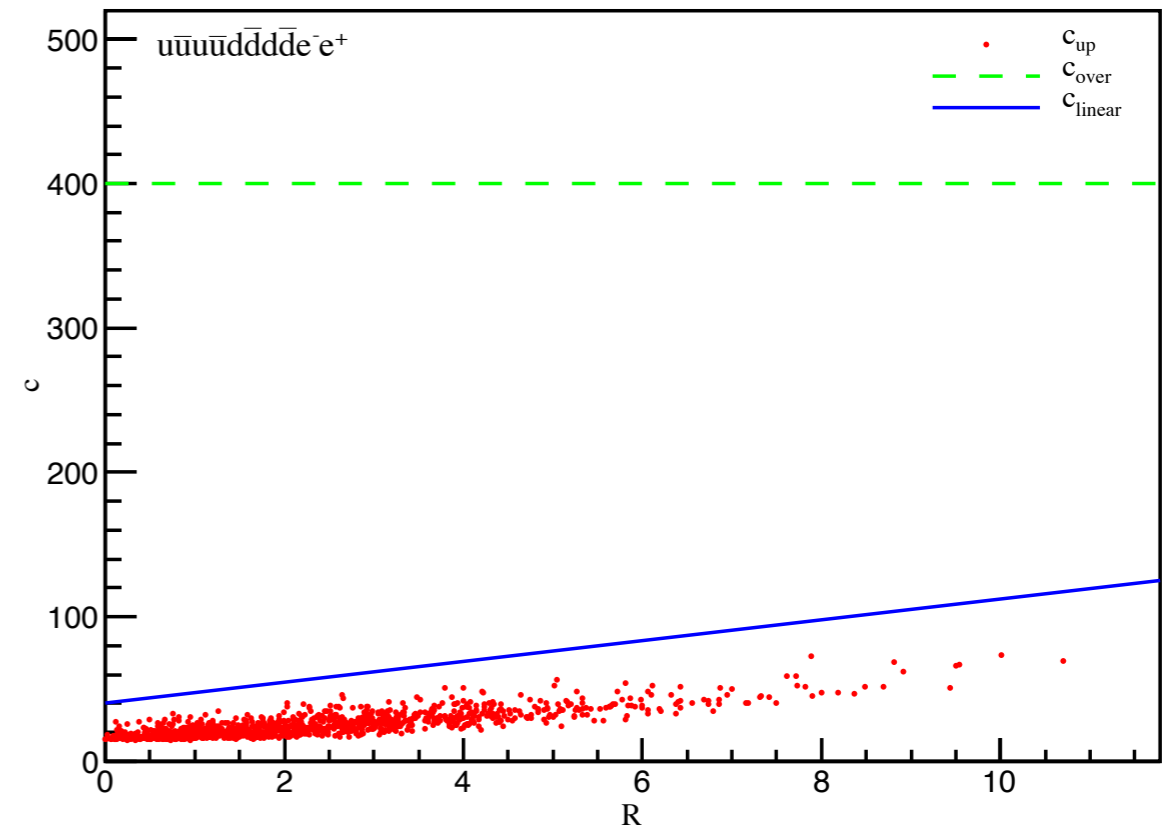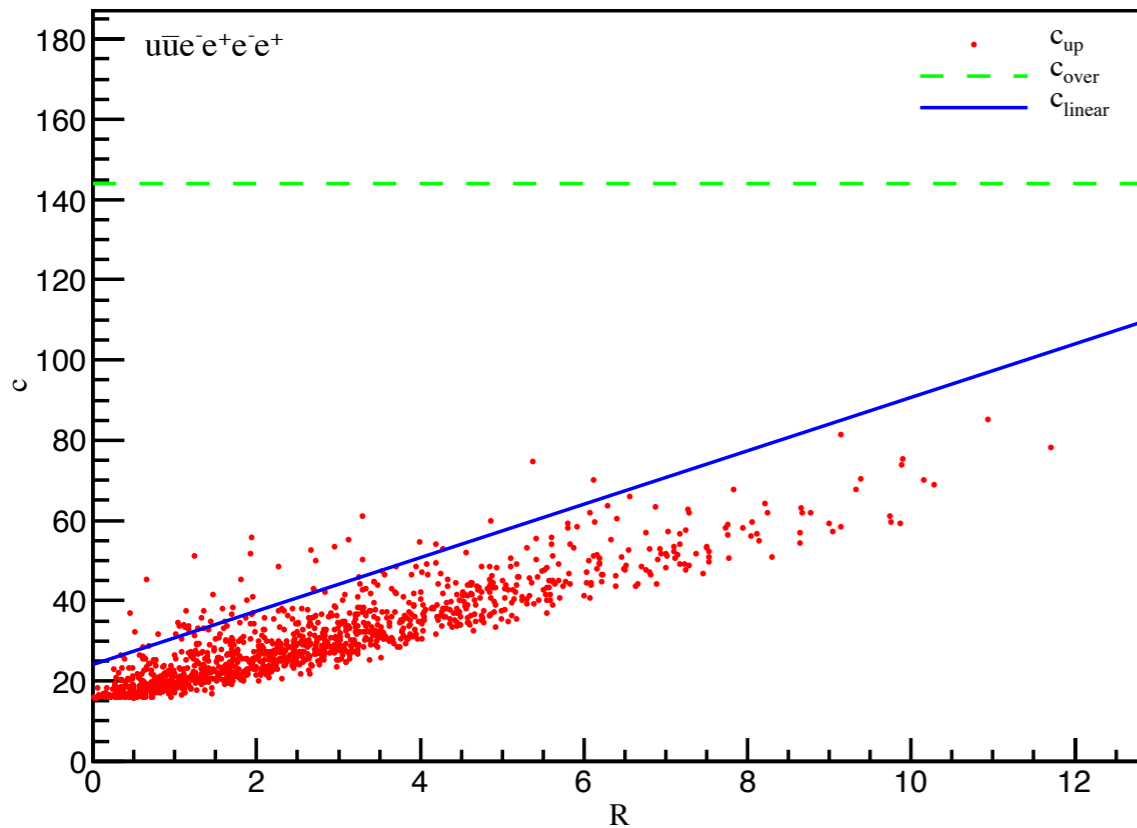$$\frac{1}{p(t)} \frac{f(t)}{g(t)}$$

Apply weight

$$\frac{1}{g(t)} \frac{g(t) - f(t)}{1 - p(t)}$$

Done $\longrightarrow t$

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Coherent Weighted Emission

Use an event-based incomplete overestimate

$$p(t) = \tanh\left(\frac{f(t)}{g(t)}\right)$$

$$R = -\sum_{[a,b]} Q_a Q_b \left(1 - \cos(\theta_{ab})\right)$$

Much faster, but events are weighted

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Summary

We're implementing three ways of doing photon emissions

1. Incoherent Pairing
   - Fast
   - Not coherent, but has most important eikonals

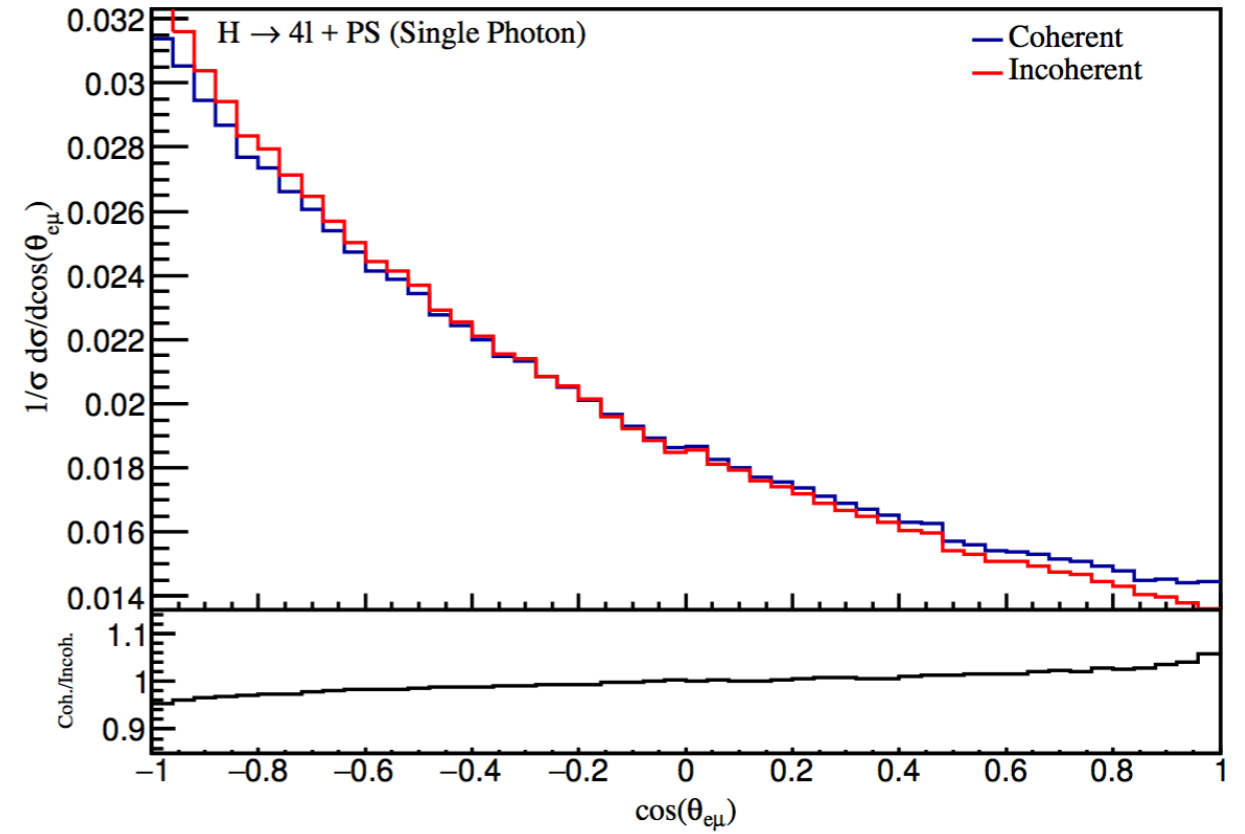2. Coherent Unweighted
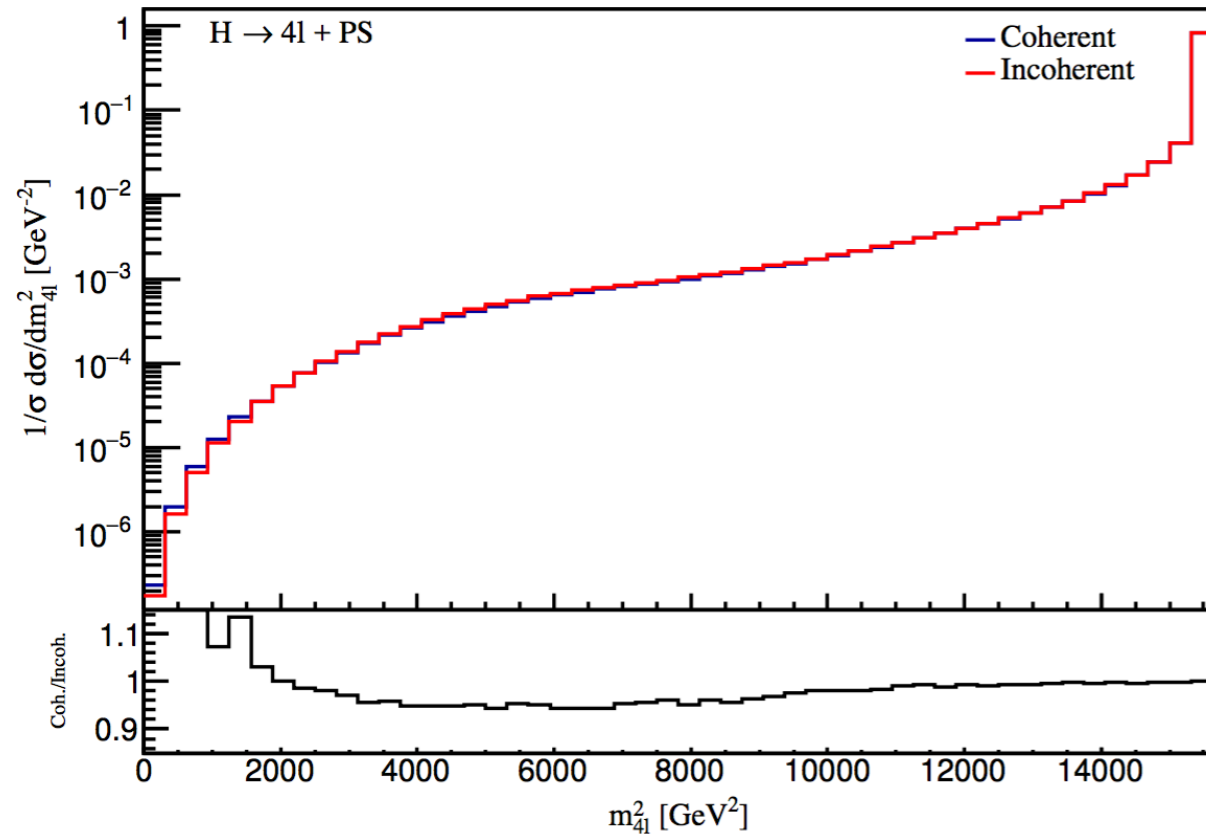   - Slow
   - Fully coherent

3. Coherent Weighted
   - Fast
   - Weighted events

Extra Slides

Comparison - Coherence

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Comparison - DGLAP equation

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