Collectivity in small systems

(selected results from ALICE)

Second international workshop on Collectivity in Small Collision Systems

Wuhan, China
June 2018

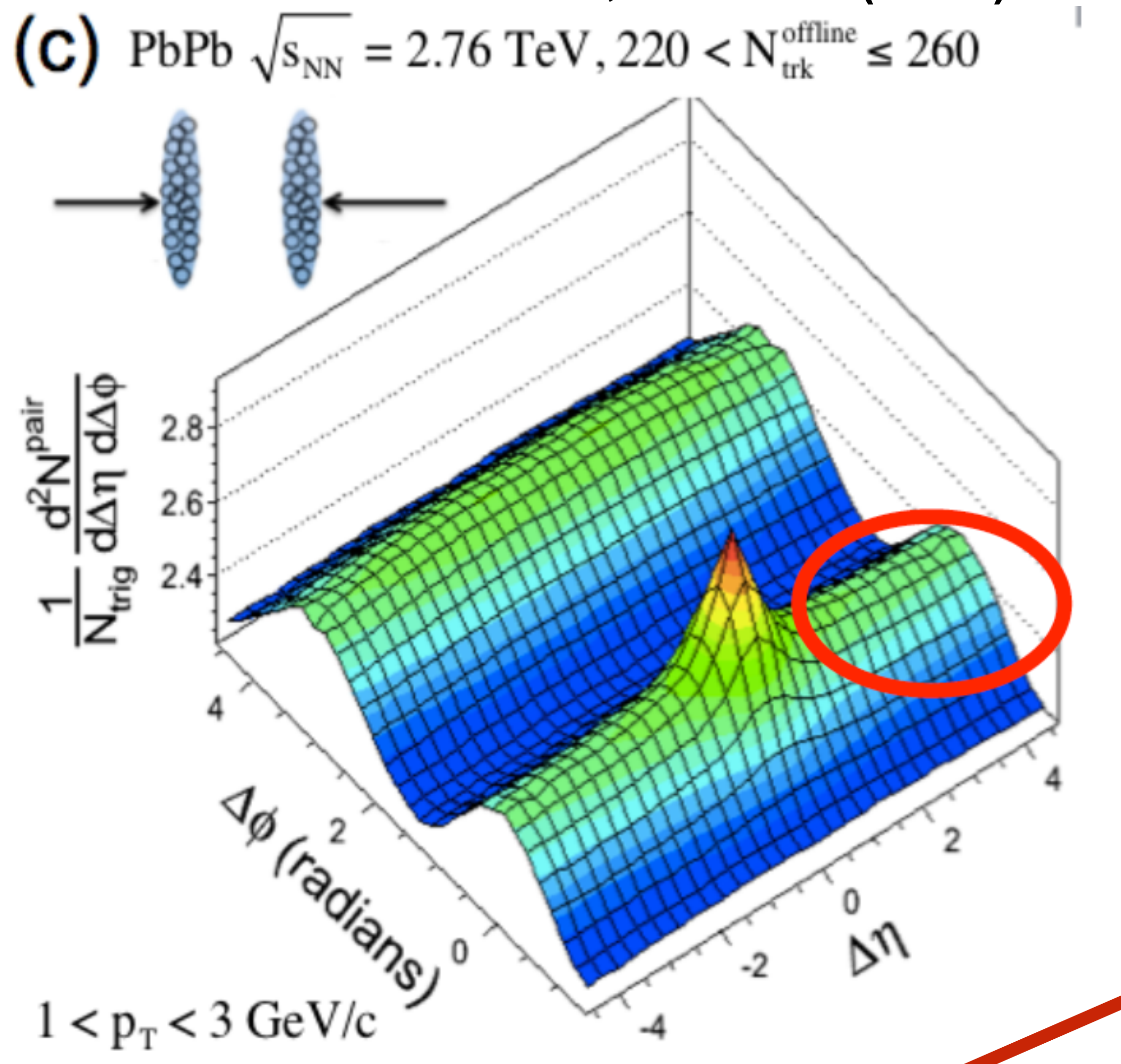
Katarina Gajdosova
Niels Bohr Institute, Copenhagen

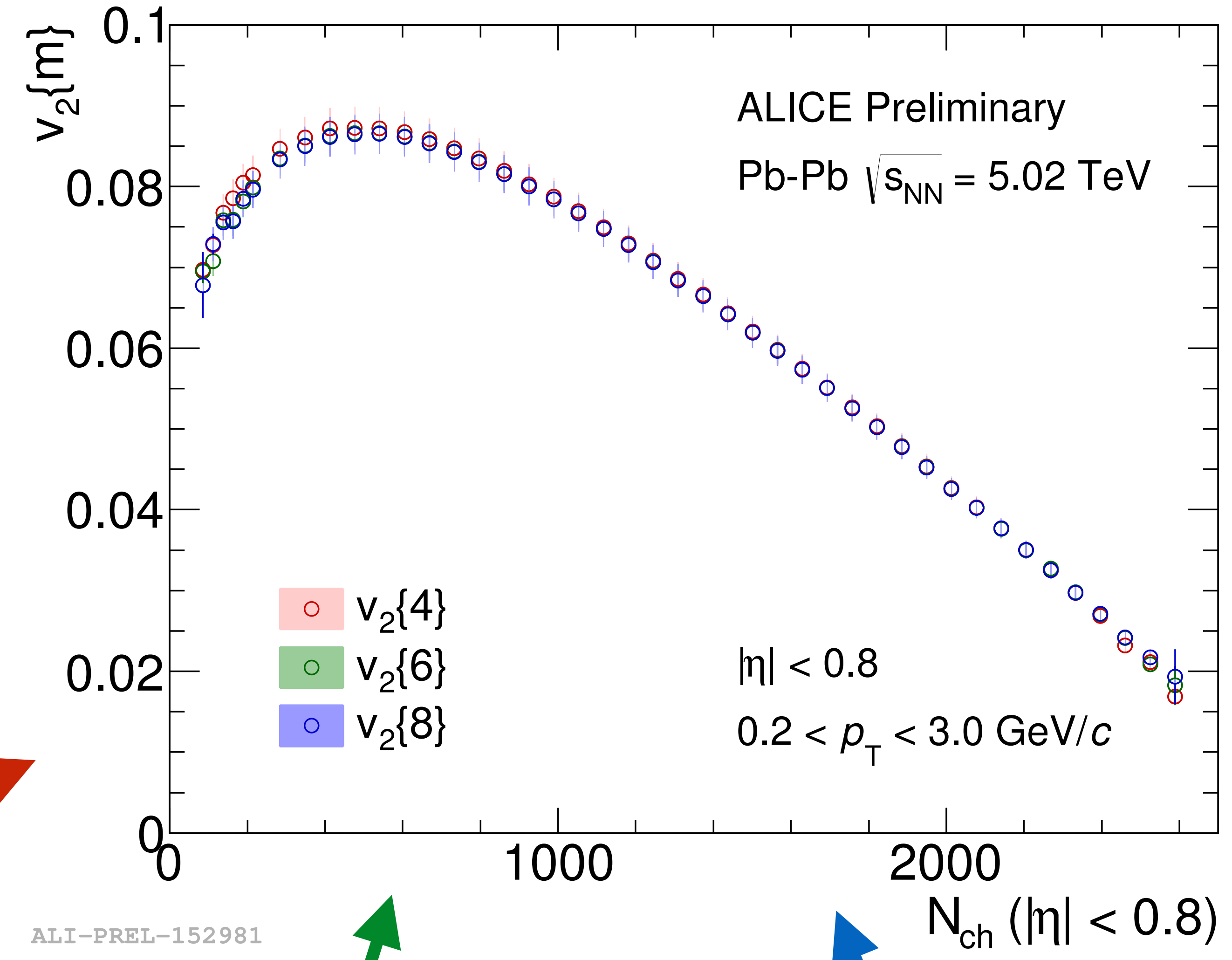
Collectivity in heavy-ion collisions

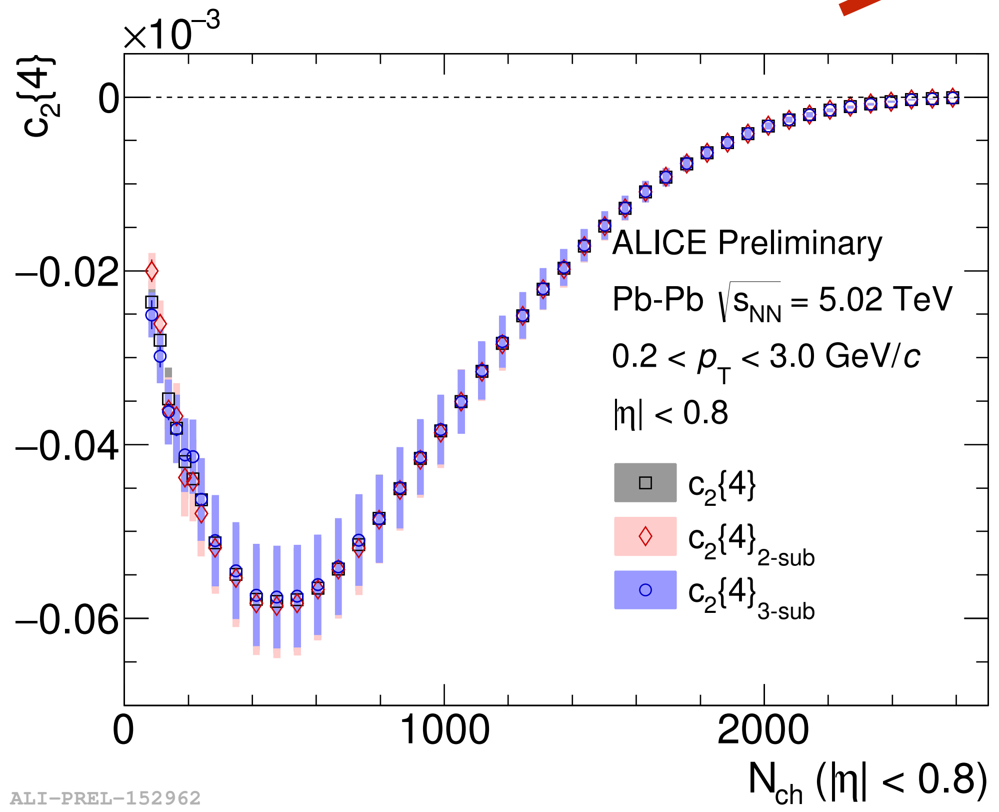
Charged hadrons

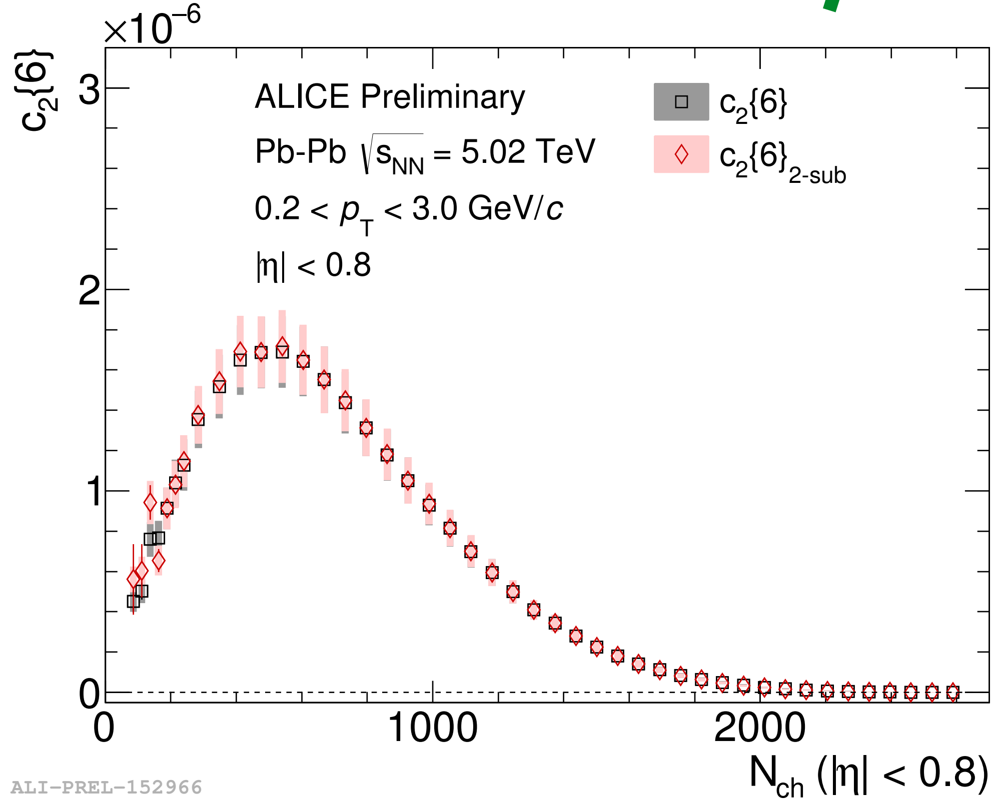
- **Long-range** correlations
- **Multi-particle** correlations

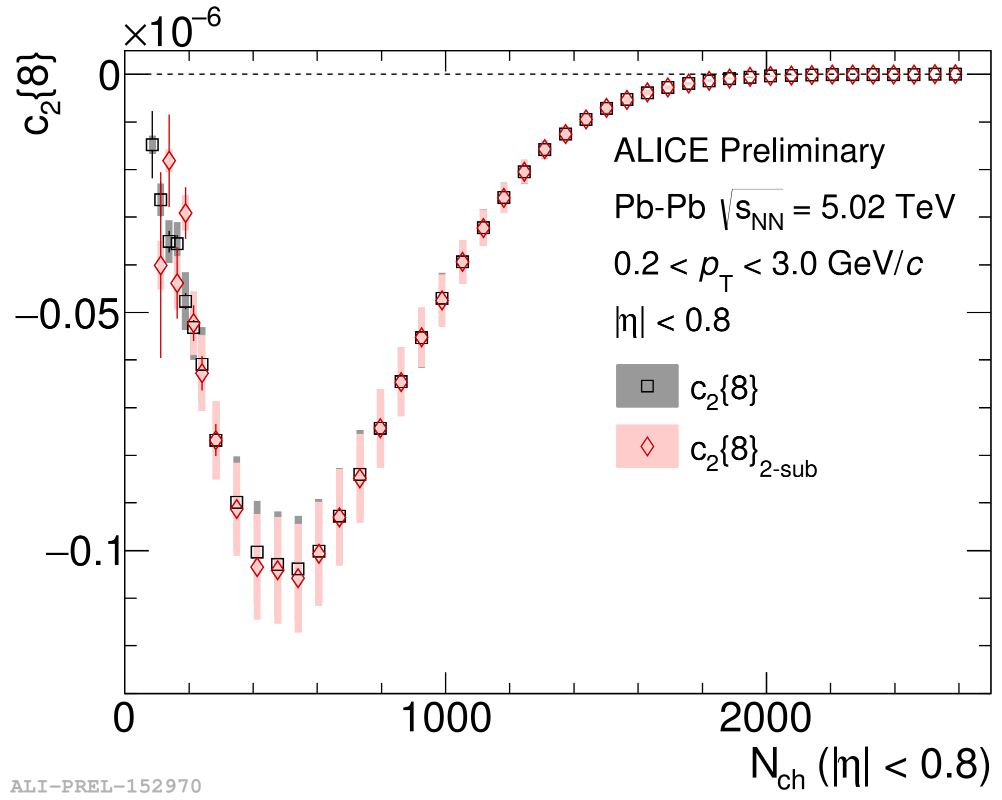
CMS, PLB 724 (2013) 213



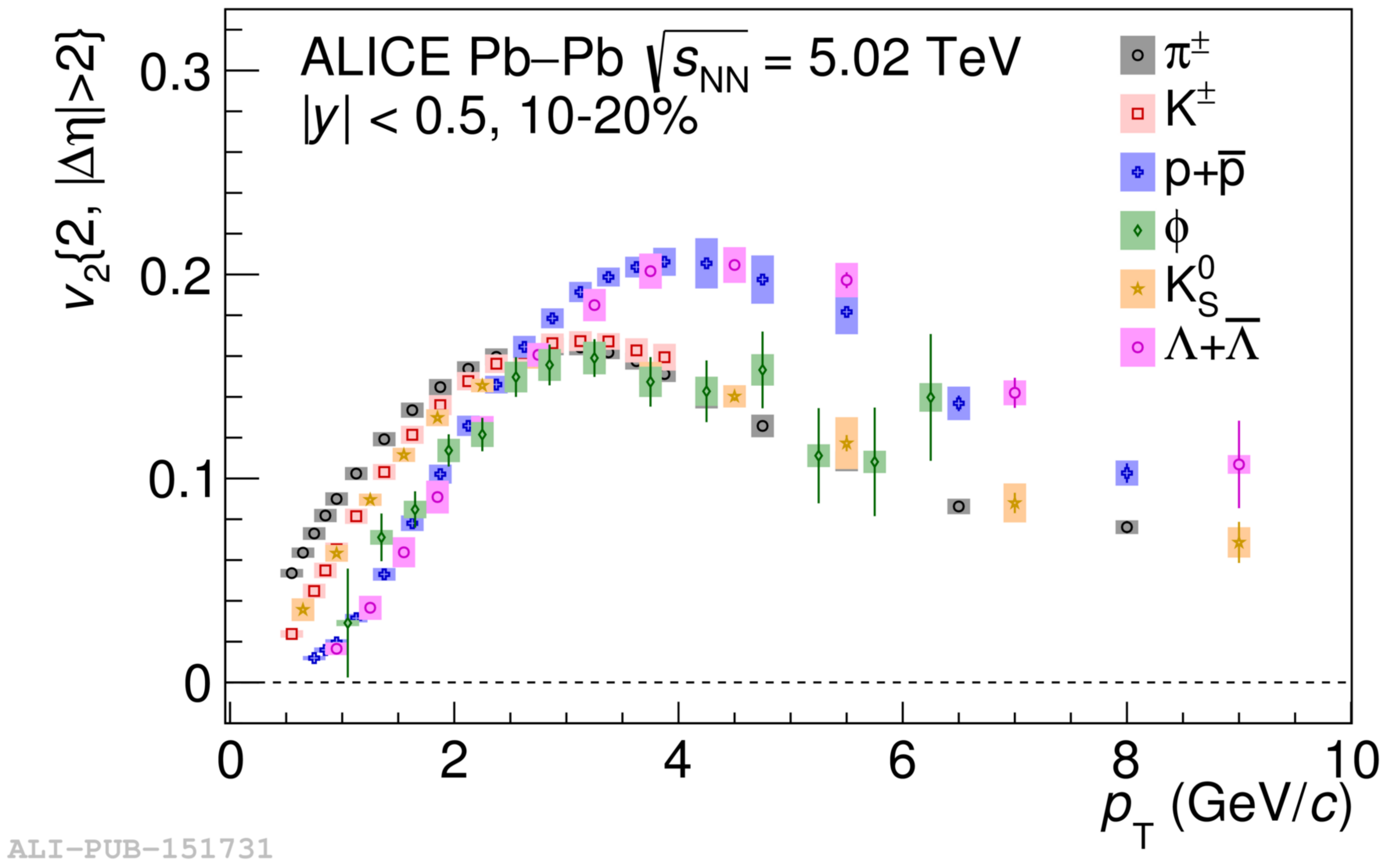








Identified hadrons



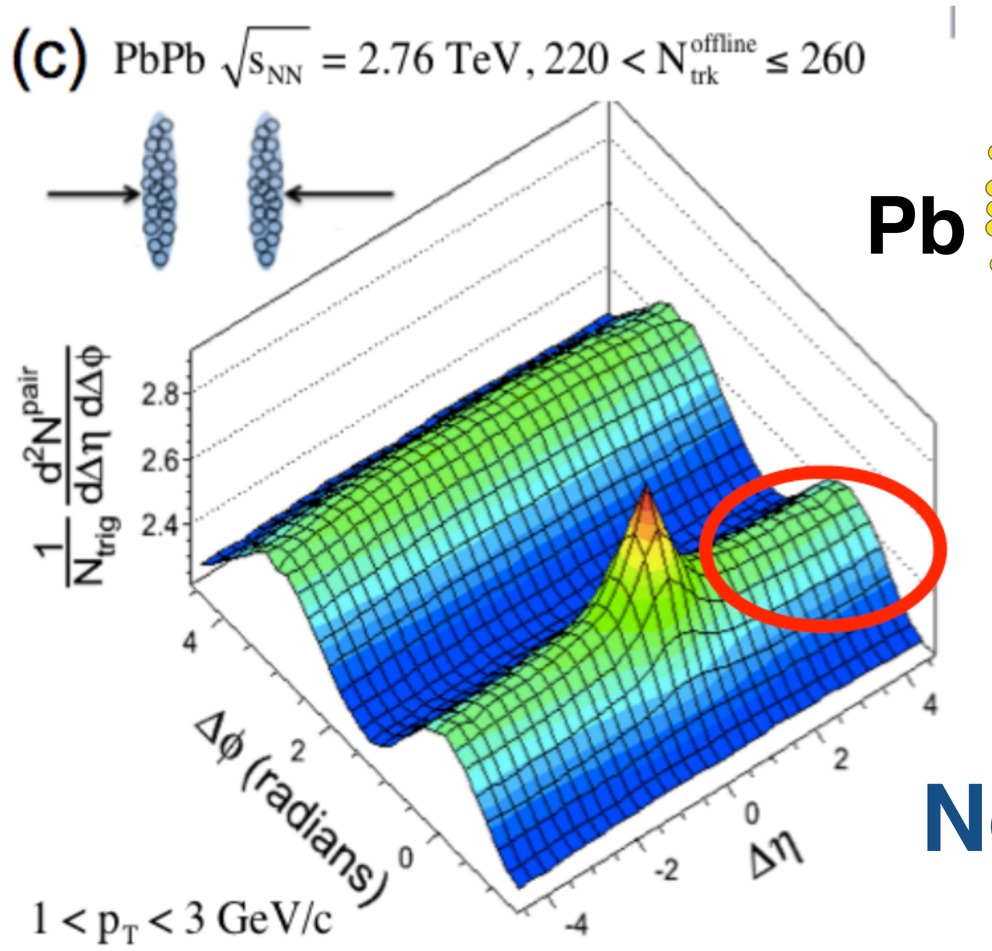
- **Mass ordering** at low p_T
- **Baryon/meson grouping** at intermediate p_T

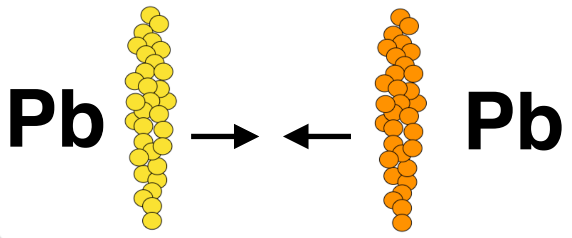
Collectivity in small systems? (charged particles)

- Traditional simplified picture:
  - Large collision systems -> QGP
  - Small collision systems -> no QGP -> baseline & CNM

Flow:

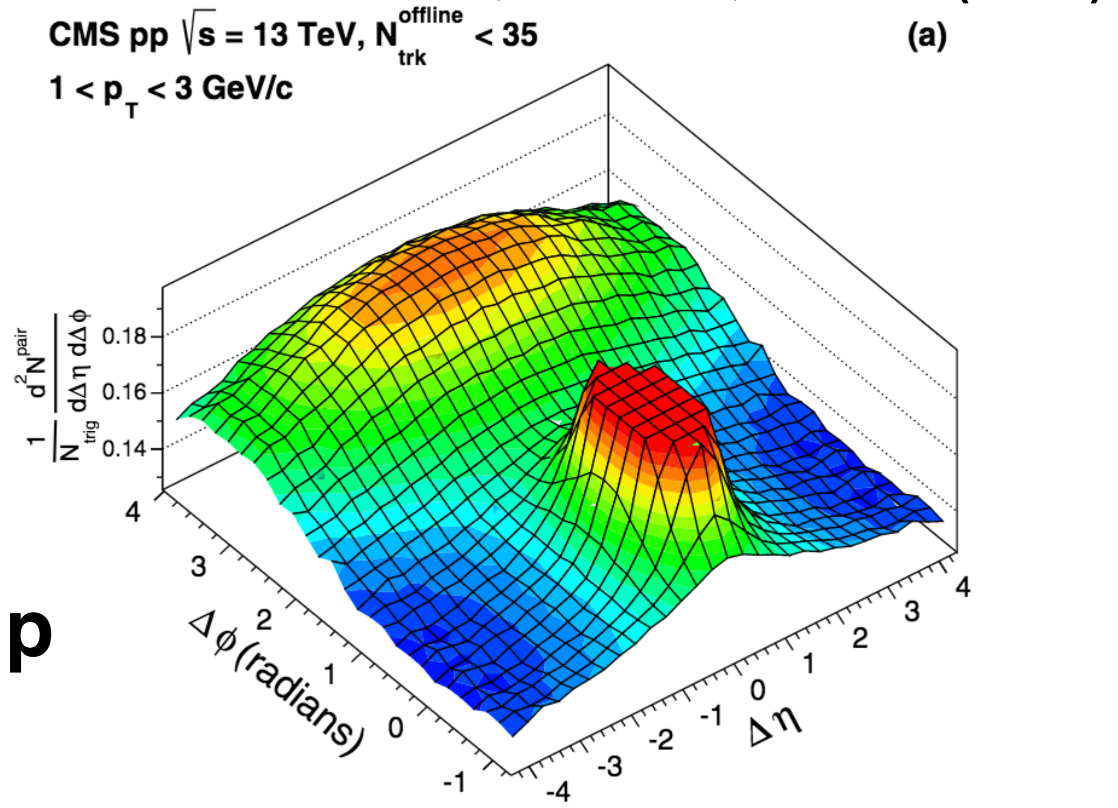
CMS, PLB 724 (2013) 213

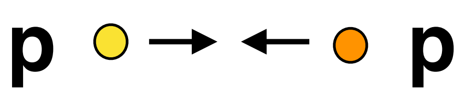




No flow:

CMS, PRL 116, 172302 (2016)





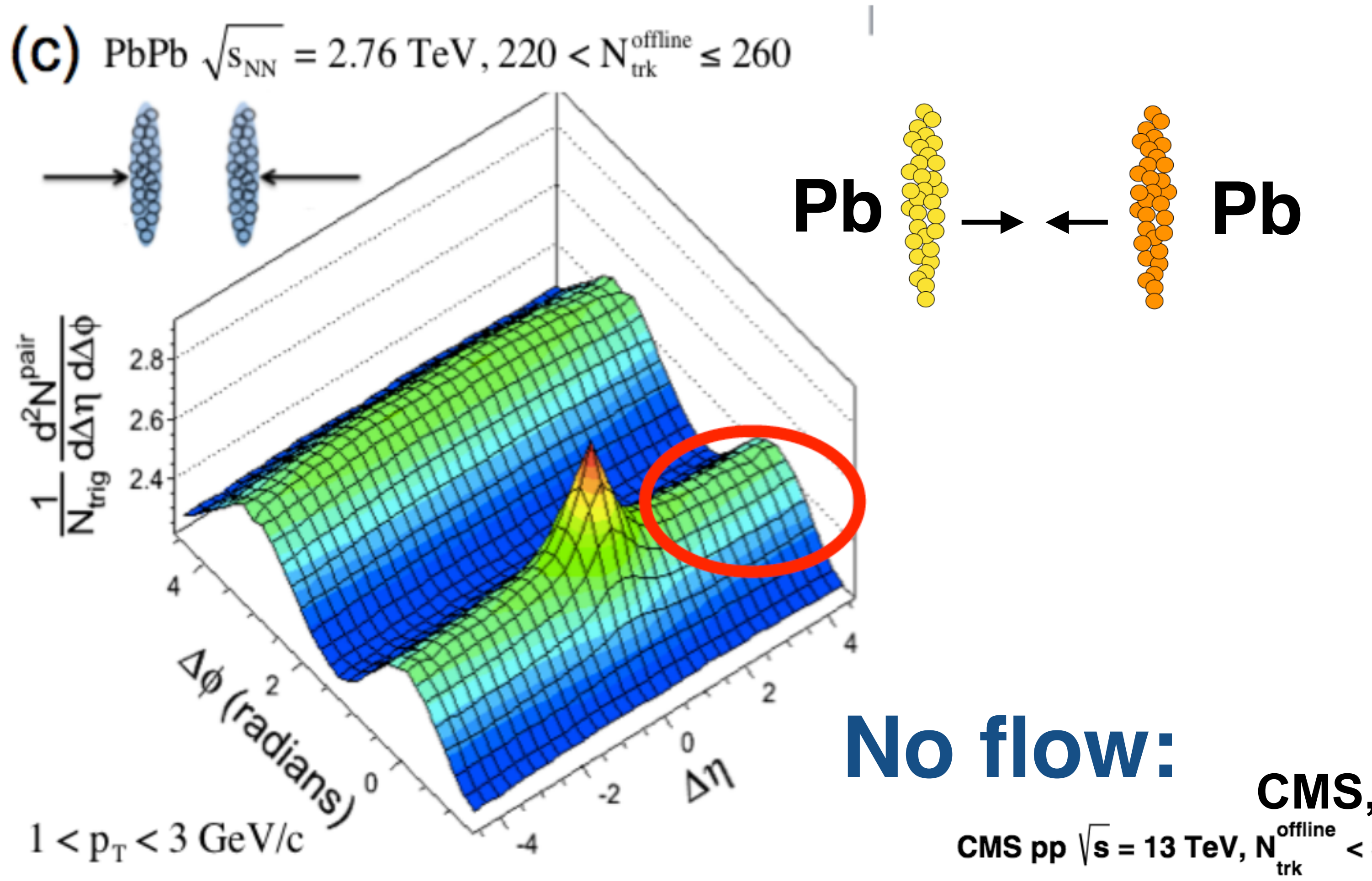
Collectivity in small systems? (charged particles)

- Traditional simplified picture:
  - Large collision systems -> QGP
  - Small collision systems -> no QGP -> baseline & CNM

- New picture:
  - Large collision systems -> QGP
  - Small collision systems at **high multiplicity** -> ?

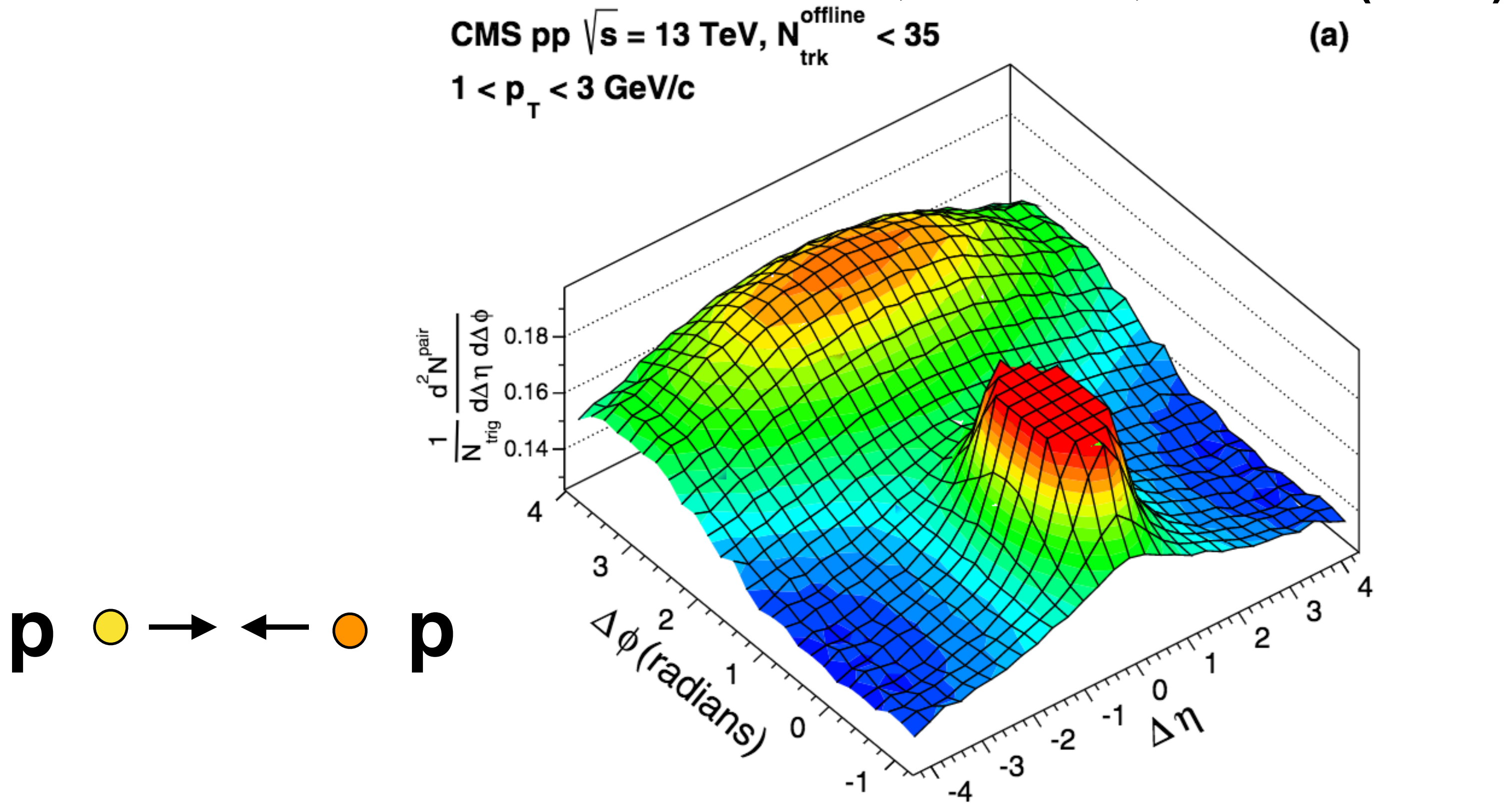
Flow:

CMS, PLB 724 (2013) 213

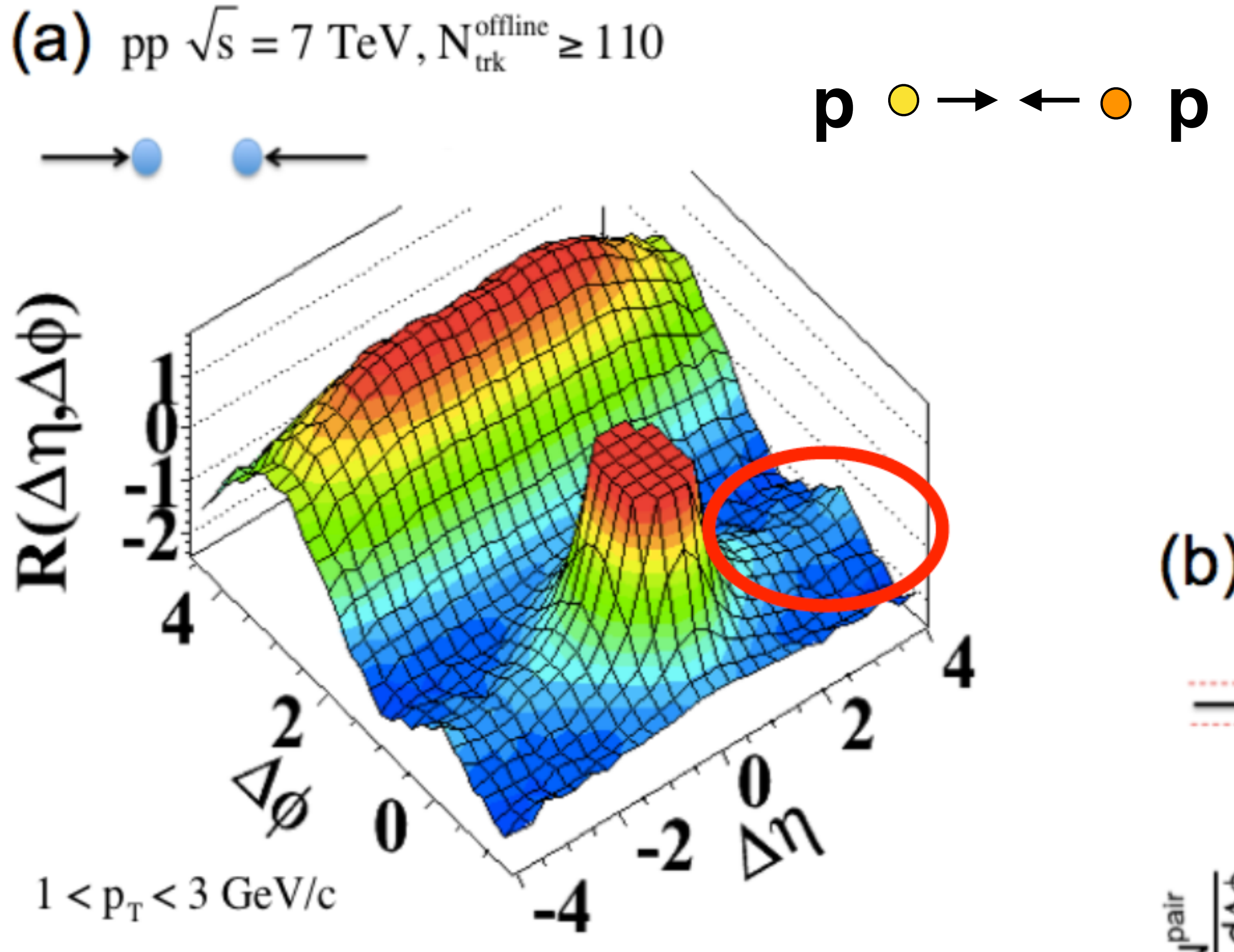


No flow:

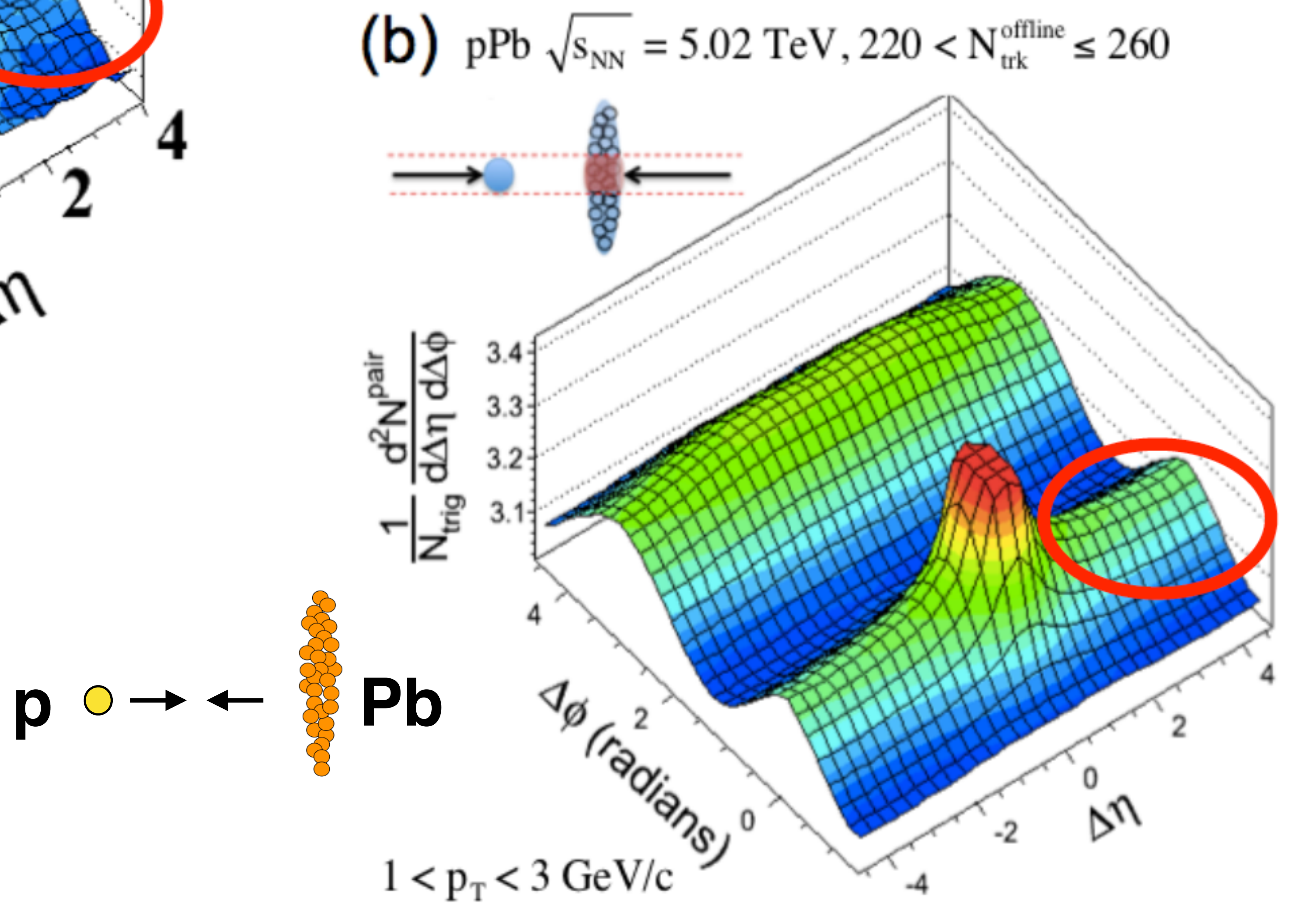
CMS, PRL 116, 172302 (2016)



CMS, JHEP 09 (2010) 091



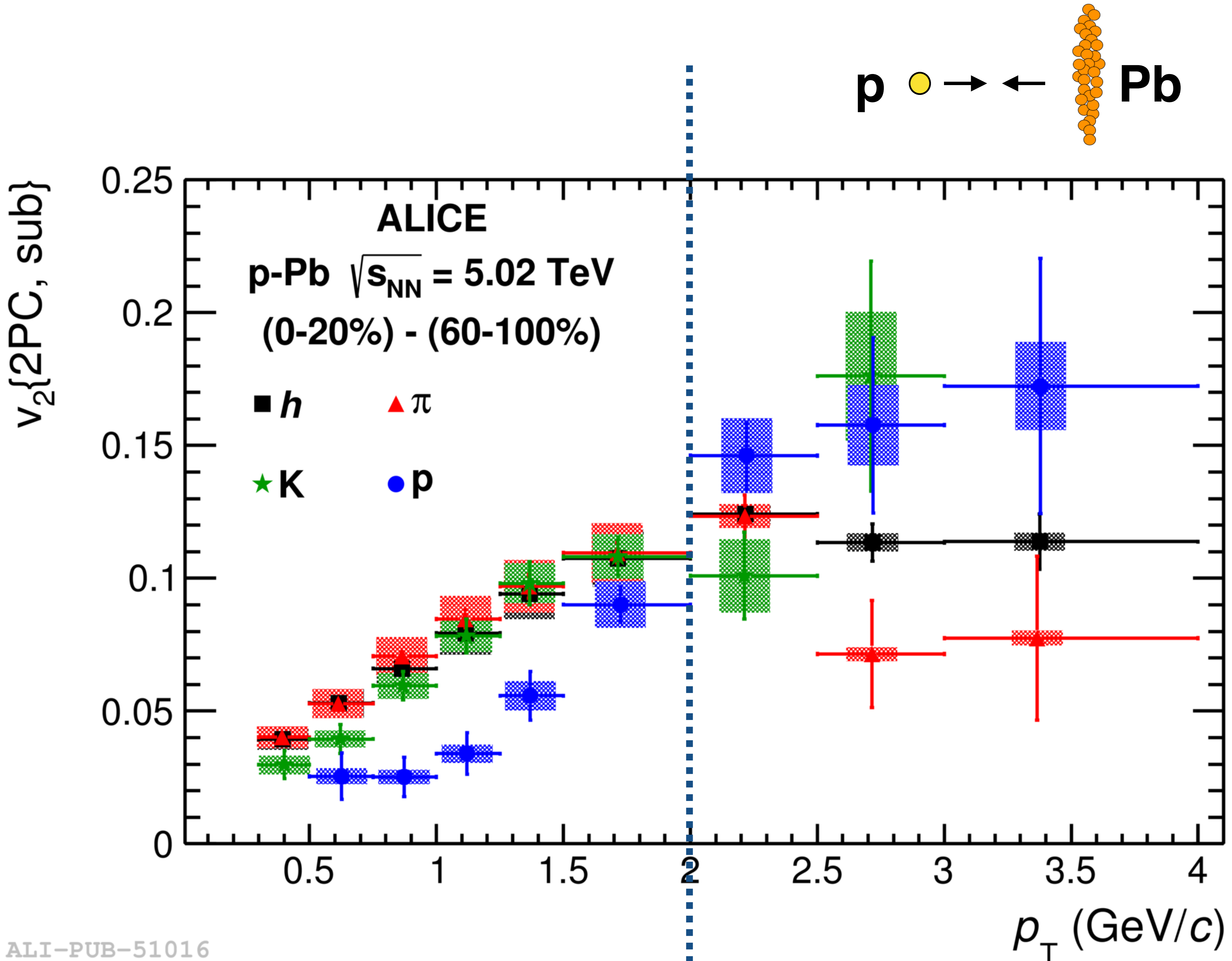
CMS, PLB 724 (2013) 213



Collectivity in small systems? (identified particles)

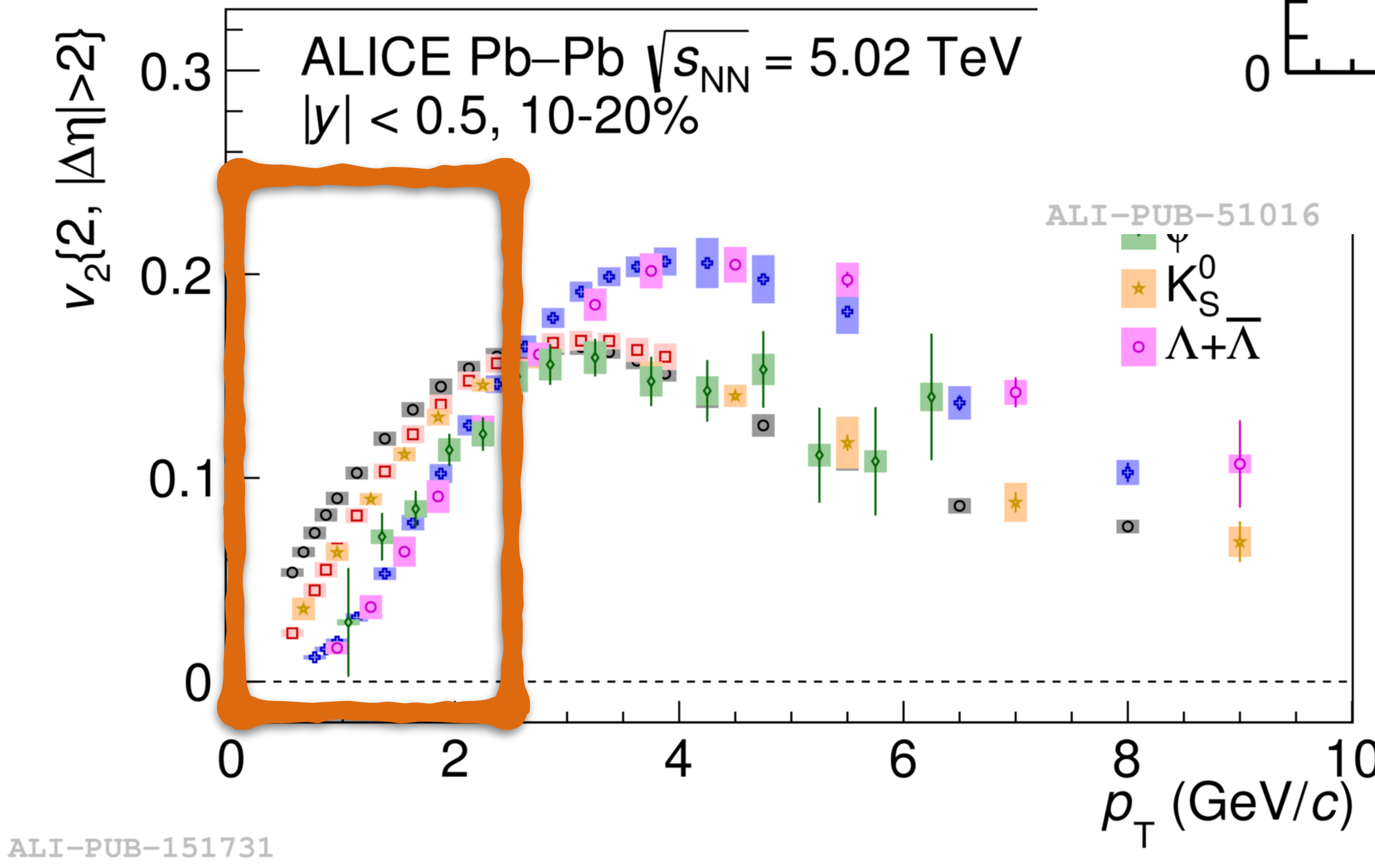
Mass ordering

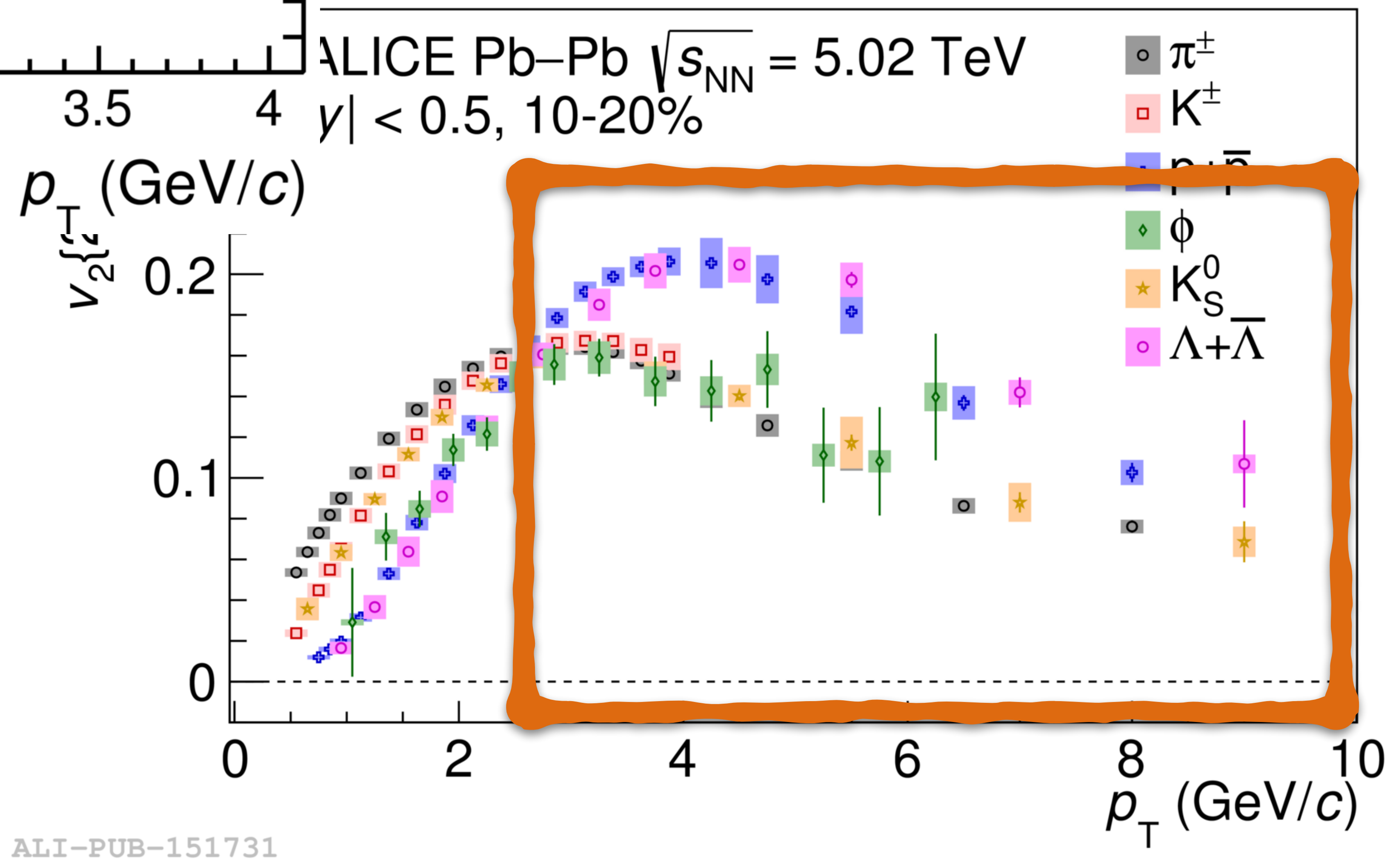
- Pb-Pb collisions
  - hydrodynamic flow, hadron re-scattering
- p-Pb collisions
  - Mass ordering observed too
  - What is its origin?



Baryon/meson grouping

- Pb-Pb collisions
  - recombination/coalescence
- p-Pb collisions
  - Is there baryon/meson grouping or not?

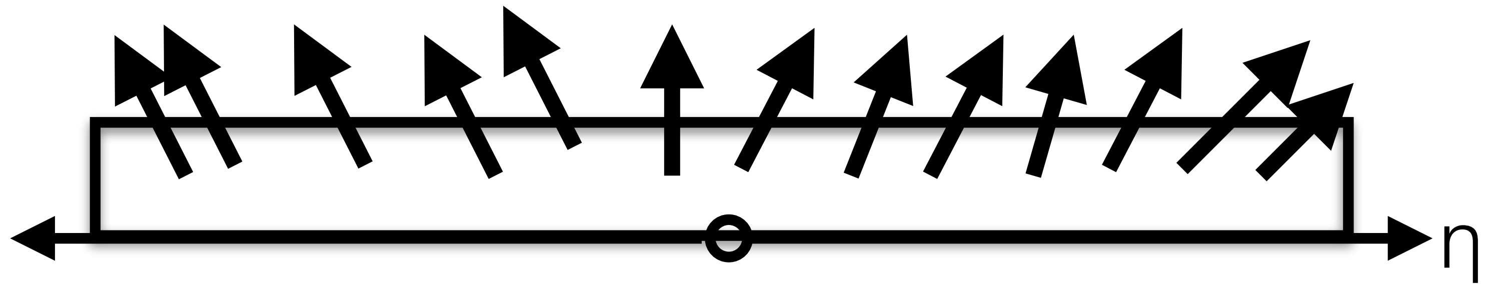




for more details see
R. Bertens talk **Friday@9:00**

Reference flow (charged particles)

m-particle correlation



$$\langle\langle 2\rangle\rangle_n = \langle\langle \cos n(\varphi_1 - \varphi_2)\rangle\rangle$$

$$\langle\langle 4\rangle\rangle_n = \langle\langle \cos n(\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4)\rangle\rangle$$

$$\langle\langle 6\rangle\rangle_n = \langle\langle \cos n(\varphi_1 + \varphi_2 + \varphi_3 - \varphi_4 - \varphi_5 - \varphi_6)\rangle\rangle$$

$$\langle\langle 8\rangle\rangle_n = \langle\langle \cos n(\varphi_1 + \varphi_2 + \varphi_3 + \varphi_4 - \varphi_5 - \varphi_6 - \varphi_7 - \varphi_8)\rangle\rangle$$

$$\langle\langle 4\rangle\rangle_{m,n} = \langle\langle \cos(m\varphi_1 + n\varphi_2 - m\varphi_3 - n\varphi_4)\rangle\rangle$$

m-particle cumulant

$$c_n\{2\} = \langle\langle 2\rangle\rangle_n$$

$$c_n\{4\} = \langle\langle 4\rangle\rangle_n - 2\cdot\langle\langle 2\rangle\rangle_n^2$$

$$c_n\{6\} = \langle\langle 6\rangle\rangle - 9\cdot\langle\langle 2\rangle\rangle\cdot\langle\langle 4\rangle\rangle + 12\cdot\langle\langle 2\rangle\rangle^3$$

$$c_n\{8\} = \langle\langle 8\rangle\rangle - 16\cdot\langle\langle 6\rangle\rangle\langle\langle 2\rangle\rangle - 18\cdot\langle\langle 4\rangle\rangle^2 + 144\cdot\langle\langle 4\rangle\rangle\langle\langle 2\rangle\rangle^2 - 144\cdot\langle\langle 2\rangle\rangle^4$$

Symmetric Cumulant

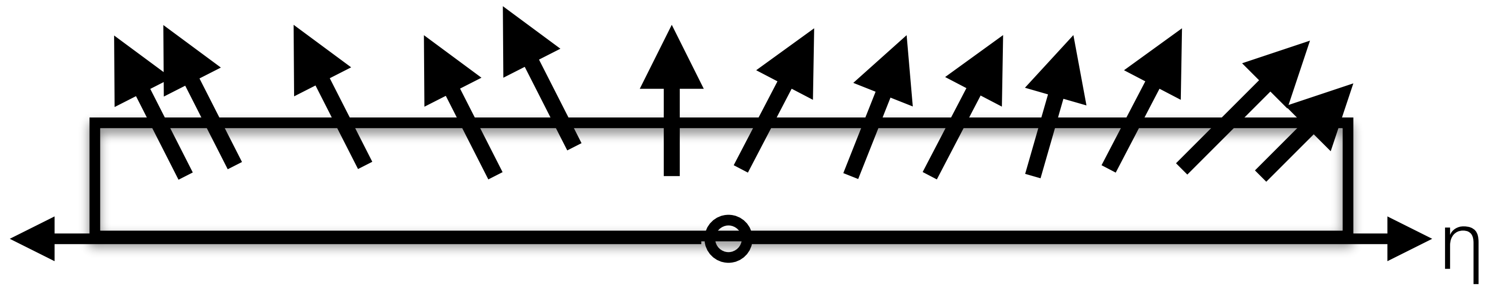
$$SC(m,n) = \langle\langle 4\rangle\rangle_{m,n} - \langle\langle 2\rangle\rangle_m\langle\langle 2\rangle\rangle_n$$

flow coefficients

$$v_n\{2\} = \sqrt{c_n\{2\}} \qquad v_n\{4\} = \sqrt[4]{-c_n\{4\}} \qquad v_n\{6\} = \sqrt[6]{\frac{1}{4}c_n\{6\}} \qquad v_n\{8\} = \sqrt[8]{-\frac{1}{33}c_n\{8\}}$$

m-particle correlation

m-particle correlation



$$\langle\langle 2\rangle\rangle_n = \langle\langle \cos n(\varphi_1 - \varphi_2)\rangle\rangle$$

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$$\langle\langle 6\rangle\rangle_n = \langle\langle \cos n(\varphi_1 + \varphi_2 + \varphi_3 - \varphi_4 - \varphi_5 - \varphi_6)\rangle\rangle$$

$$\langle\langle 8\rangle\rangle_n = \langle\langle \cos n(\varphi_1 + \varphi_2 + \varphi_3 + \varphi_4 - \varphi_5 - \varphi_6 - \varphi_7 - \varphi_8)\rangle\rangle$$

PHYSICAL REVIEW C **89**, 064904 (2014)

Generic framework for anisotropic flow analyses with multiparticle azimuthal correlations

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(Received 20 December 2013; revised manuscript received 6 May 2014; published 9 June 2014)

```
for(int i=0; i<nTracks; i++)
{
  for(int j=0; i<nTracks; j++)
  {
    for(int k=0; k<nTracks; k++)
    {
      ...
    }
  }
}
```

- Efficient method to avoid nested loops: **Q-cumulants**
  - Calculates any correlation with only one loop over tracks (using Q-vectors)

$$Q_n = \sum_{k=1}^{M} e^{in\varphi_k}$$

Generic Framework

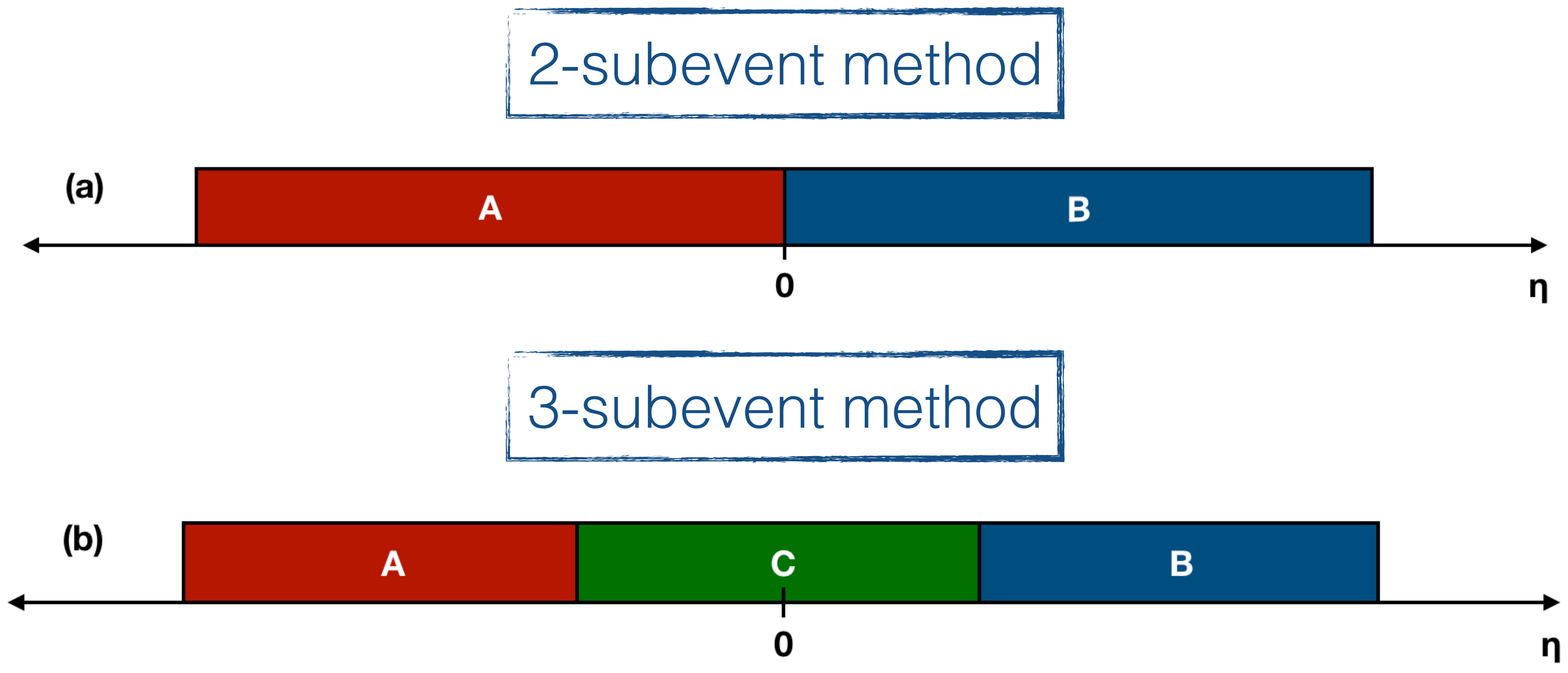
- Universal implementation able to calculate any type and order of correlation, including corrections (which was not possible to do with Q-cumulant method)

$$Q_{n,p} = \sum_{k=1}^{M} w_k^p e^{in\varphi_k}$$

Contamination with non-flow effects

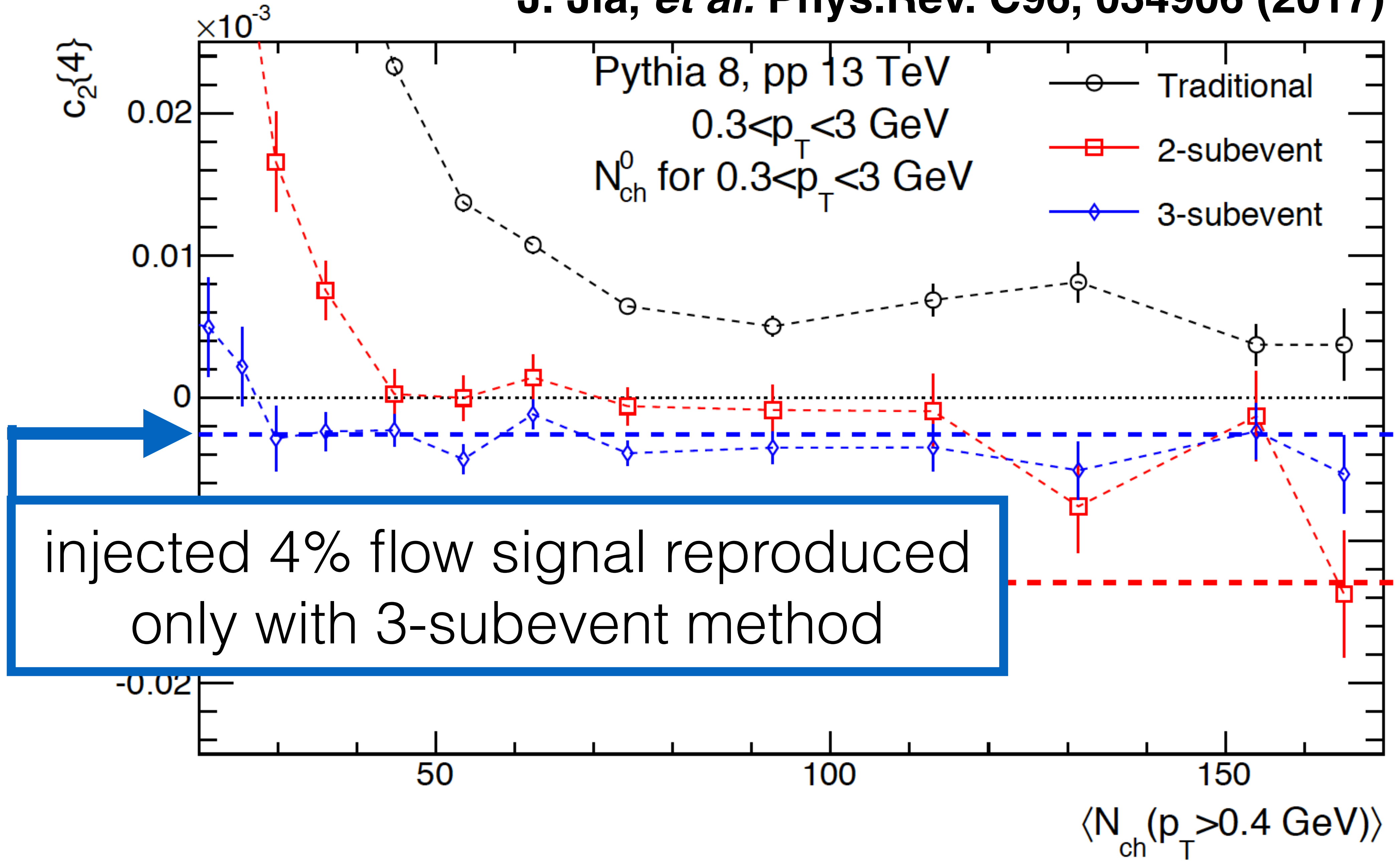
- **Non-flow**: correlations not associated with the common symmetry plane
- Multi-particle cumulants are able to suppress lower order non-flow
  - Same order non-flow remains
- Small systems are strongly dominated by non-flow effects

- **Subevent method**: enforces large space separation between particles that are being correlated
  - This should remove most of the short-range correlations (which includes non-flow)



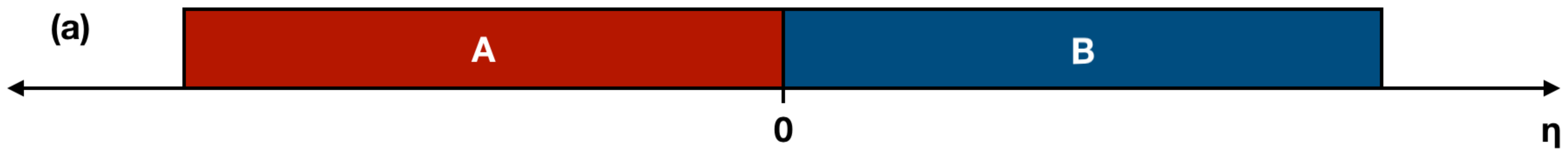
K. Gajdosova, M. Zhou, QM17 (2017)

J. Jia, *et al.* Phys.Rev. C96, 034906 (2017)



- PYTHIA with injected flow
  - Decrease of the $c_2\{4\}$ signal with subevent method
  - Shows further suppression of non-flow effects in multi-particle cumulants
  - Only 3-subevent method was able to reproduce the flow signal

Subevent method in 2-particle cumulants (with GF)



$$\langle\langle 2\rangle\rangle_n = \langle\langle \cos n(\varphi_1 - \varphi_2)\rangle\rangle$$

- Standard vs. subevent method: the same procedure to obtain flow coefficients
  - However, in subevent method we correlate only particles from certain regions = subevents
- It was successfully implemented in Generic Framework

Standard method:

$$\mathrm{Two}\,(n_1, n_2) = \frac{Q_{n_1,1}Q_{n_2,1} - Q_{n_1+n_2,2}}{Q_{0,1}^2 - Q_{0,2}}$$

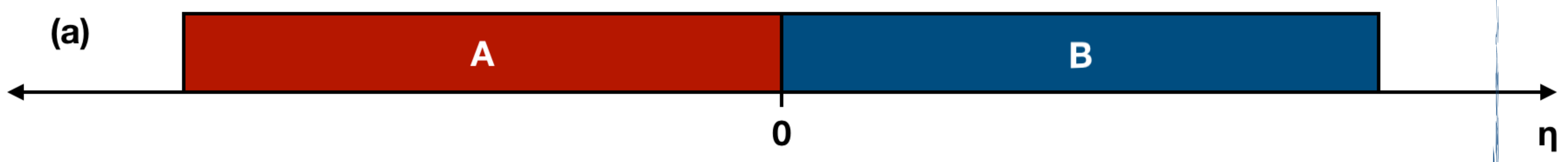
autocorrelation terms

Subevent method:

- Autocorrelation terms can be removed, because particles don't share the same subevent

$$\mathrm{TwoSubevent}\,(n_1, n_2) = \frac{Q_{n_1,1}Q_{n_2,1}}{Q_{0,1}^2}$$

Subevent method in multi-particle cumulants (with GF)



$$\langle\langle 2\rangle\rangle_n = \langle\langle \cos n(\varphi_1 - \varphi_2)\rangle\rangle$$
$$\langle\langle 4\rangle\rangle_n = \langle\langle \cos n(\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4)\rangle\rangle$$

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.
.

- Standard vs. subevent method: the same procedure to obtain flow coefficients
  - However, in subevent method we correlate only particles from certain regions = subevents

- It was successfully implemented in Generic Framework

Standard method:

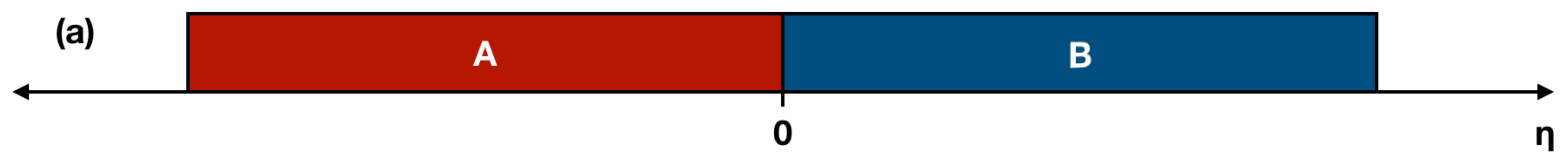
$$\mathrm{Four}(\mathrm{n}_1,\mathrm{n}_2,\mathrm{n}_3,\mathrm{n}_4) = \frac{\begin{aligned} &Q_{n_1,1}Q_{n_2,1}Q_{n_3,1}Q_{n_4,1} - Q_{n_1+n_2,2}Q_{n_3,1}Q_{n_4,1} - Q_{n_2,1}Q_{n_1+n_3,2}Q_{n_4,1} \\ &- Q_{n_1,1}Q_{n_2+n_3,2}Q_{n_4,1} + 2Q_{n_1+n_2+n_3,3}Q_{n_4,1} - Q_{n_2,1}Q_{n_3,1}Q_{n_1+n_4,2} \\ &+ Q_{n_2+n_3,2}Q_{n_1+n_4,2} - Q_{n_1,1}Q_{n_3,1}Q_{n_2+n_4,2} + Q_{n_1+n_3,2}Q_{n_2+n_4,2} \\ &+ 2Q_{n_3,1}Q_{n_1+n_2+n_4,3} - Q_{n_1,1}Q_{n_2,1}Q_{n_3+n_4,2} + Q_{n_1+n_2,2}Q_{n_3+n_4,2} \\ &+ 2Q_{n_2,1}Q_{n_1+n_3+n_4,3} + 2Q_{n_1,1}Q_{n_2+n_3+n_4,3} - 6Q_{n_1+n_2+n_3+n_4,4}, \end{aligned}}{Q_{0,1}^4 - 6Q_{0,1}^2 Q_{0,2} + 3Q_{0,2}^2 + 8Q_{0,1}Q_{0,3} - 6Q_{0,4}}$$

$\mathrm{Six}(\mathrm{n}_1,\mathrm{n}_2,\mathrm{n}_3,\mathrm{n}_4,\mathrm{n}_5,\mathrm{n}_6) =$ 203 terms !

$\mathrm{Eight}(\mathrm{n}_1,\mathrm{n}_2,\mathrm{n}_3,\mathrm{n}_4,\mathrm{n}_5,\mathrm{n}_6,\mathrm{n}_7,\mathrm{n}_8) =$ 4140 terms !

We cannot go term-by-term and remove autocorrelation terms by hand

Subevent method in multi-particle cumulants (with GF)



$$\langle\langle 2\rangle\rangle_n = \langle\langle \cos n(\varphi_1 - \varphi_2)\rangle\rangle$$

$$\langle\langle 4\rangle\rangle_n = \langle\langle \cos n(\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4)\rangle\rangle$$

.
.
.

- Standard vs. subevent method: the same procedure to obtain flow coefficients
  - However, in subevent method we correlate only particles from certain regions = subevents
- It was successfully implemented in Generic Framework

Subevent method:

$$\text{FourSubevent}(n_1, n_2, n_3, n_4) = \text{Two}^A(n_1, n_2) \cdot \text{Two}^B(n_1, n_2)$$

$$\text{SixSubevent}(n_1, n_2, n_3, n_4, n_5, n_6) = \text{Three}^A(n_1, n_2, n_3) \cdot \text{Three}^B(n_1, n_2, n_3)$$

$$\text{EightSubevent}(n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8) = \text{Four}^A(n_1, n_2, n_3, n_4) \cdot \text{Four}^B(n_1, n_2, n_3, n_4)$$

Differential flow (identified particles)

2-particle correlation: $\langle\langle 2\rangle\rangle'_n = \langle\langle \cos n(\varphi_1 - \varphi_2^{p_T})\rangle\rangle$

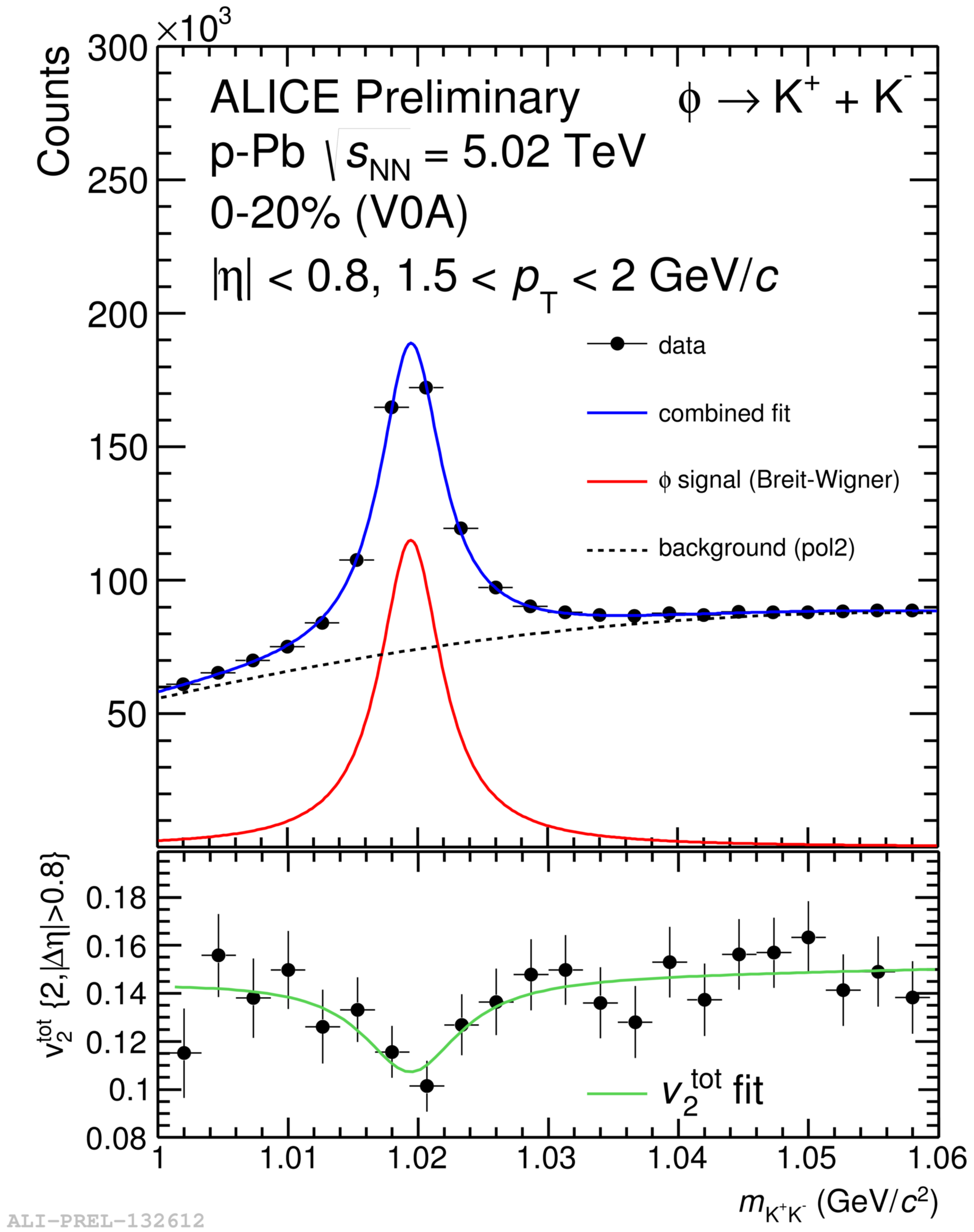
2-particle cumulant: $d_n\{2\}(p_T) = \langle\langle 2\rangle\rangle'_n$

flow coefficient:

$h^{\pm}, \pi^{\pm}, K^{\pm}, p(\overline{p})$ $\quad v_n\{2\}(p_T) = \dfrac{d_n\{2\}(p_T)}{\sqrt{c_n\{2\}}} = \dfrac{\langle v_n(p_T)\cdot v_n\rangle}{\sqrt{\langle v_n \cdot v_n\rangle}}$

$K^0_S, \Lambda(\overline{\Lambda}), \phi$ $\quad v_n^{tot}\{2\}(p_T, m_{inv}) = \dfrac{d_n\{2\}(p_T, m_{inv})}{\sqrt{c_n\{2\}}}$

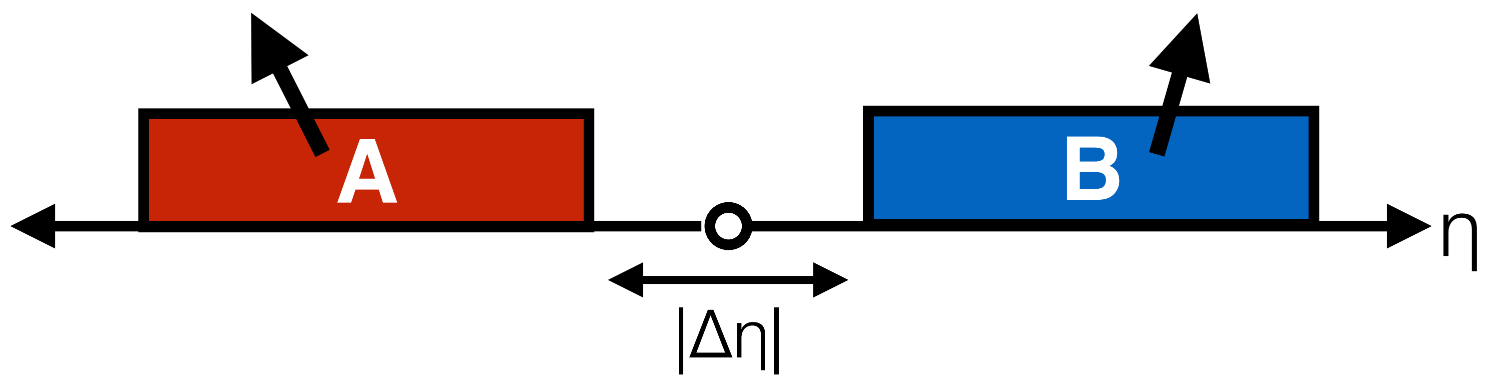
$$v_n^{tot}(m_{inv}) = \frac{N^{sig}(m_{inv})}{N_{tot}(m_{inv})}\cdot v_n^{sig} + \frac{N^{bg}(m_{inv})}{N^{tot}(m_{inv})}\cdot v_n^{bg}(m_{inv})$$



for more details see
R. Bertens talk **Friday@9:00**

Non-flow subtraction method

1. **Pseudorapidity separation**



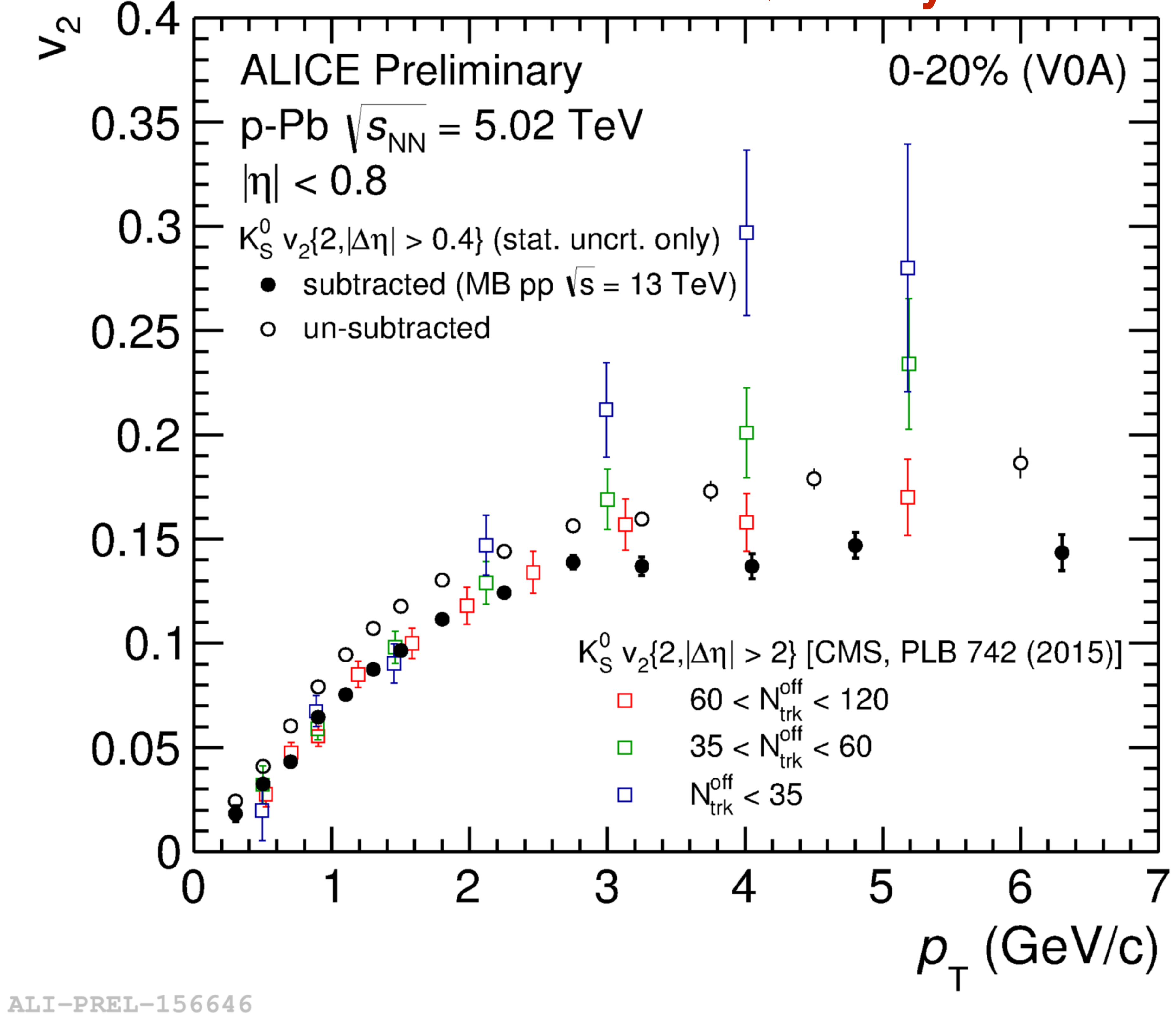
2. **Additional non-flow subtraction**
   - Non-flow estimation: MB pp collisions

$$v_2^{\mathrm{pPb,sub}}(p_T) = \frac{d_2^{\mathrm{pPb}}\{2\} - k \cdot d_2^{\mathrm{pp}}\{2\}}{\sqrt{c_2^{\mathrm{pPb}}\{2\} - k \cdot c_2^{\mathrm{pp}}\{2\}}}$$

   - Non-flow scaled by mean event multiplicities (Voloshin *et al.*, arXiv:0809.2949)

$$\delta_n \propto \frac{1}{M} \quad \longrightarrow \quad k = \frac{\langle M \rangle^{\mathrm{pp}}}{\langle M \rangle^{\mathrm{pPb}}}$$

shown @QM18 by V. Pacik

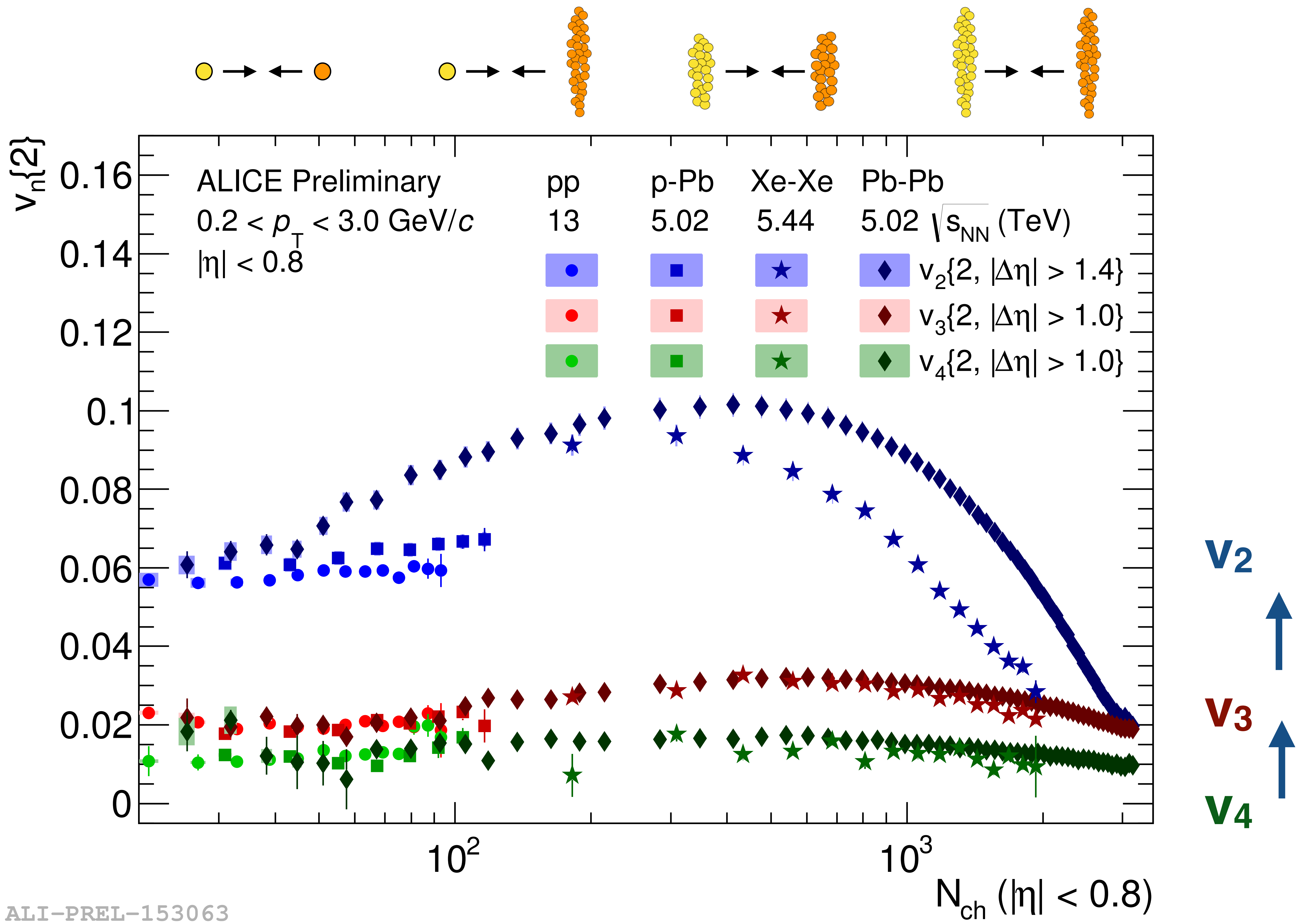


for more details see
R. Bertens talk **Friday@9:00**

Measurements of two-particle cumulants

$v_n\{2\}$ in small collision systems

charged hadrons



ALI-PREL-153063

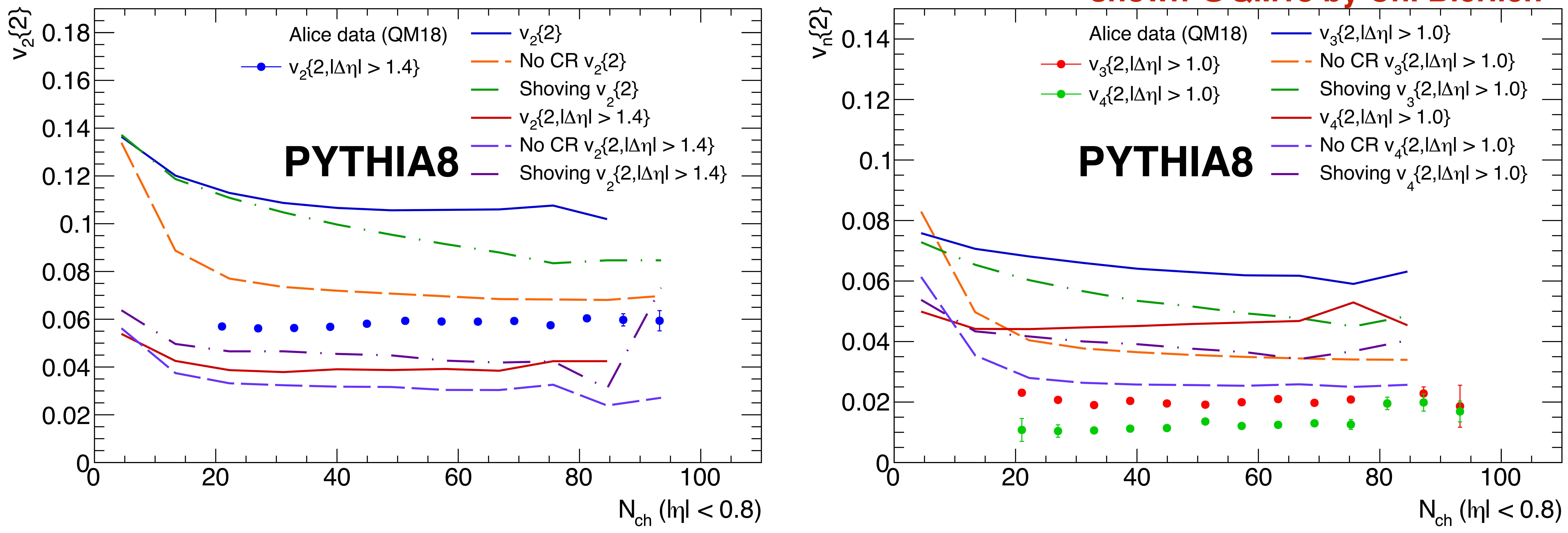
shown @QM18

- **Heavy-ion collisions**:
  - Clear multiplicity dependence of v_2 showing response to collision geometry
  - Ordering $v_2 > v_3 > v_4$
- **Small systems**:
  - Comparable values with Pb-Pb at low M
  - Weak multiplicity dependence
  - Ordering $v_2 > v_3 > v_4$

$v_n\{2\}$ in small collision systems

charged hadrons

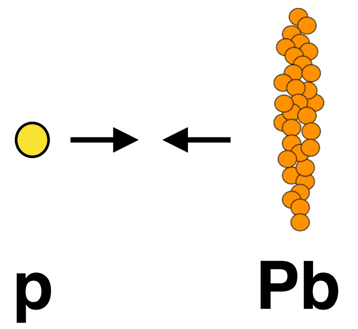
shown @QM18 by Ch. Bierlich



- Data are not reproduced by non-flow only model
  - Qualitative agreement with PYTHIA8 w/o color reconnection or with shoving mechanism
- Note: calculations from PYTHIA have different kinematic cuts ($|\eta| < 0.9$, $0.1 < p_T < 3.0$ GeV/c)

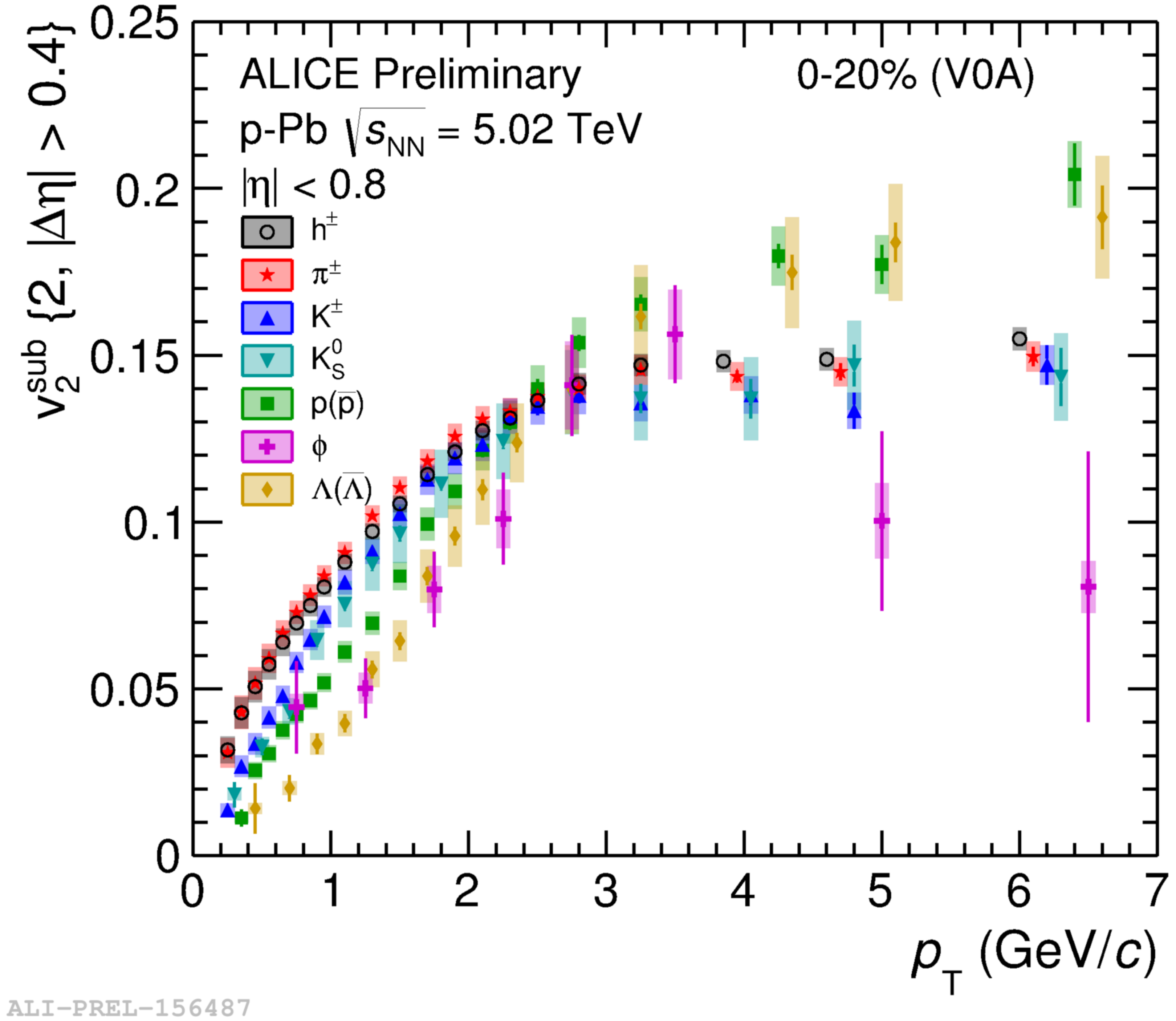
$v_2\{2\}(p_T)$ in small collision systems

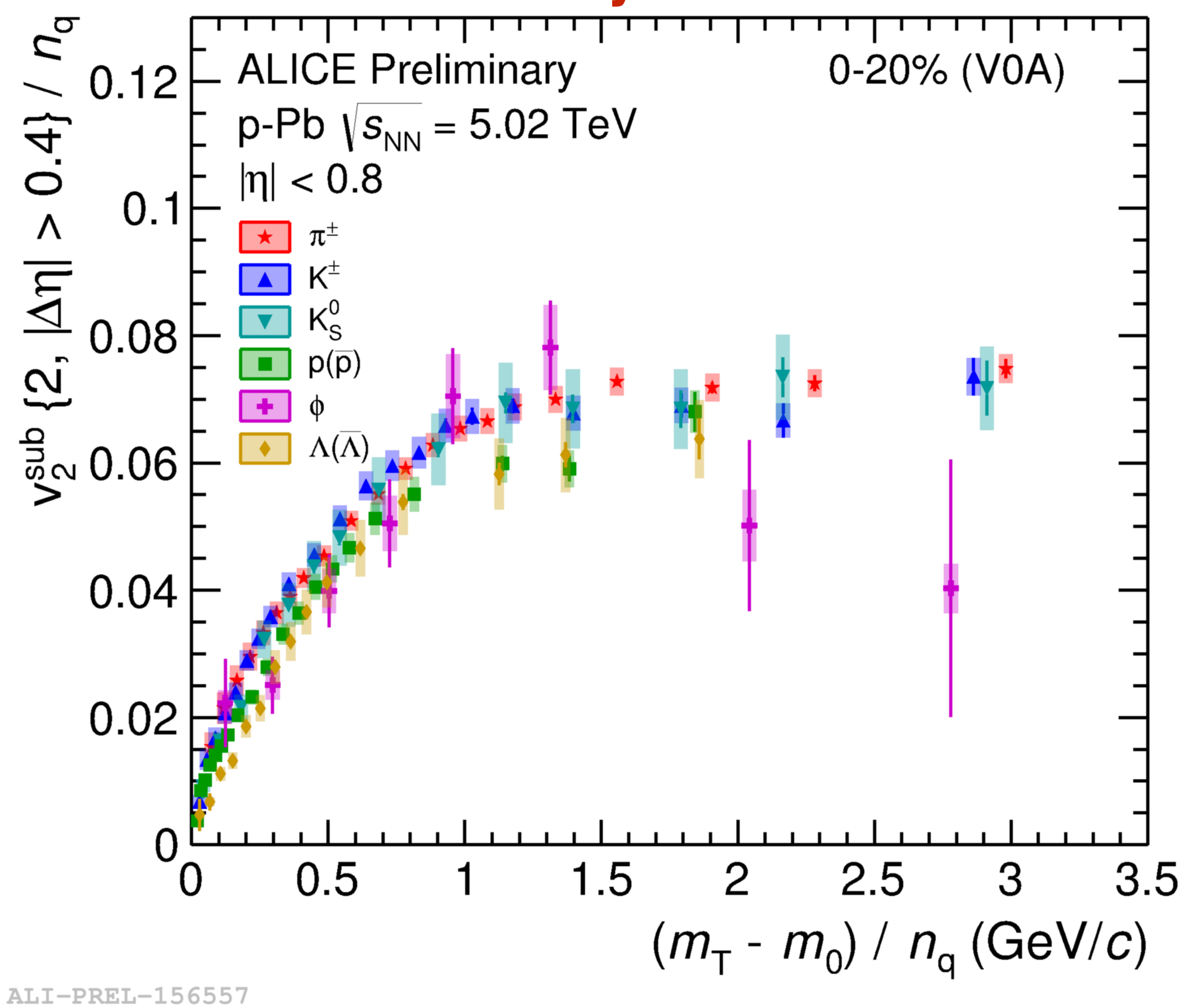
identified hadrons



- $v_2(p_T)^{subtracted}$ of identified hadrons in 5.02 TeV p-Pb collisions (using Run2 data)
  - First ALICE measurements of K^0_S, Λ and ϕ v_2 in small collision systems
- Similar observations as in Pb-Pb measurements
  - Clear **mass ordering** at low p_T
  - Indication of **baryon/meson grouping** at intermediate p_T
- Approximate **NCQ scaling** in p-Pb collisions
  - Similar to what is observed in Pb-Pb collisions

shown @QM18 by V. Pacik





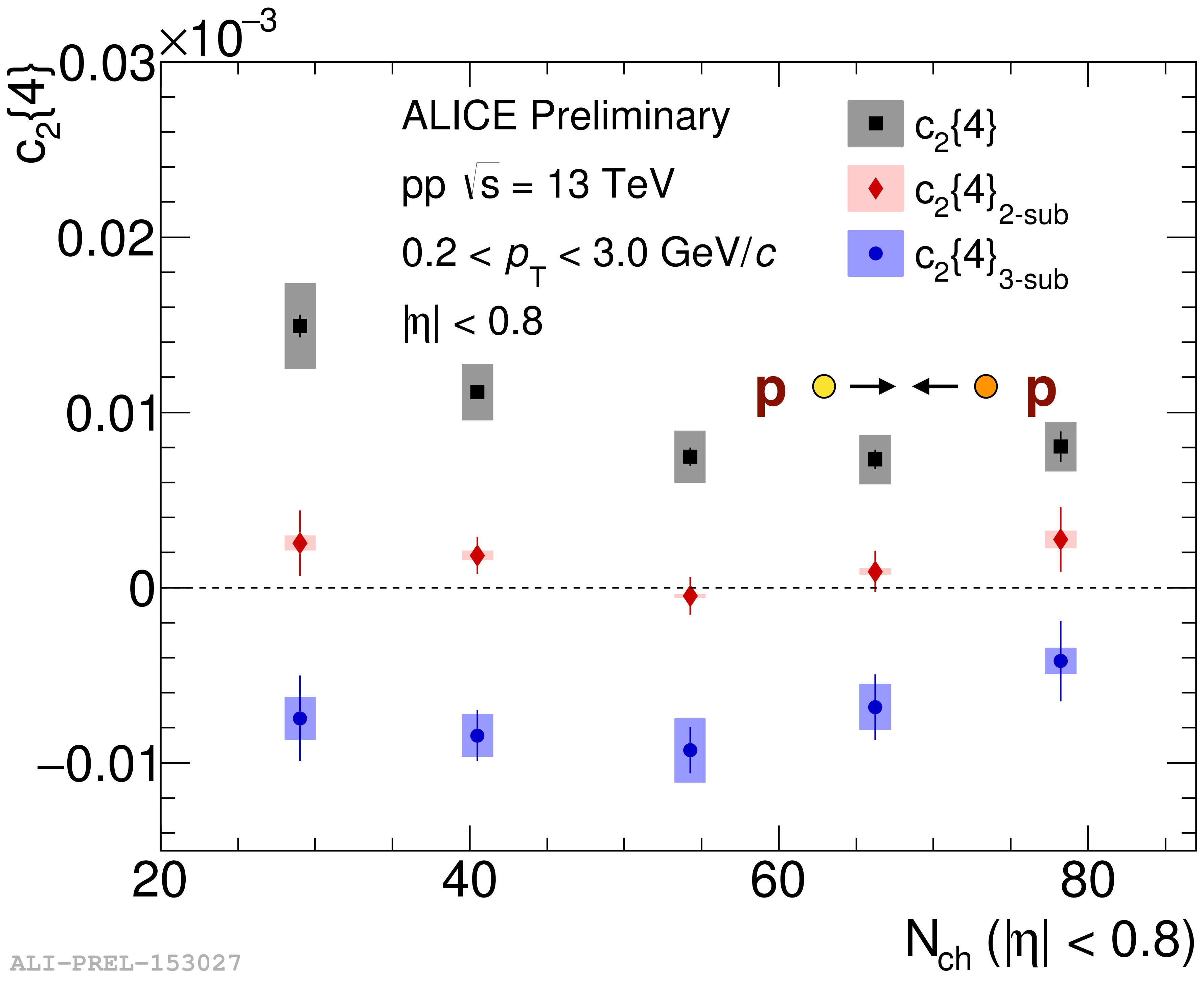
$$\mathrm{KE_T} = m_\mathrm{T} - m_0 = \sqrt{p_\mathrm{T}^2 - m_0^2} - m_0$$

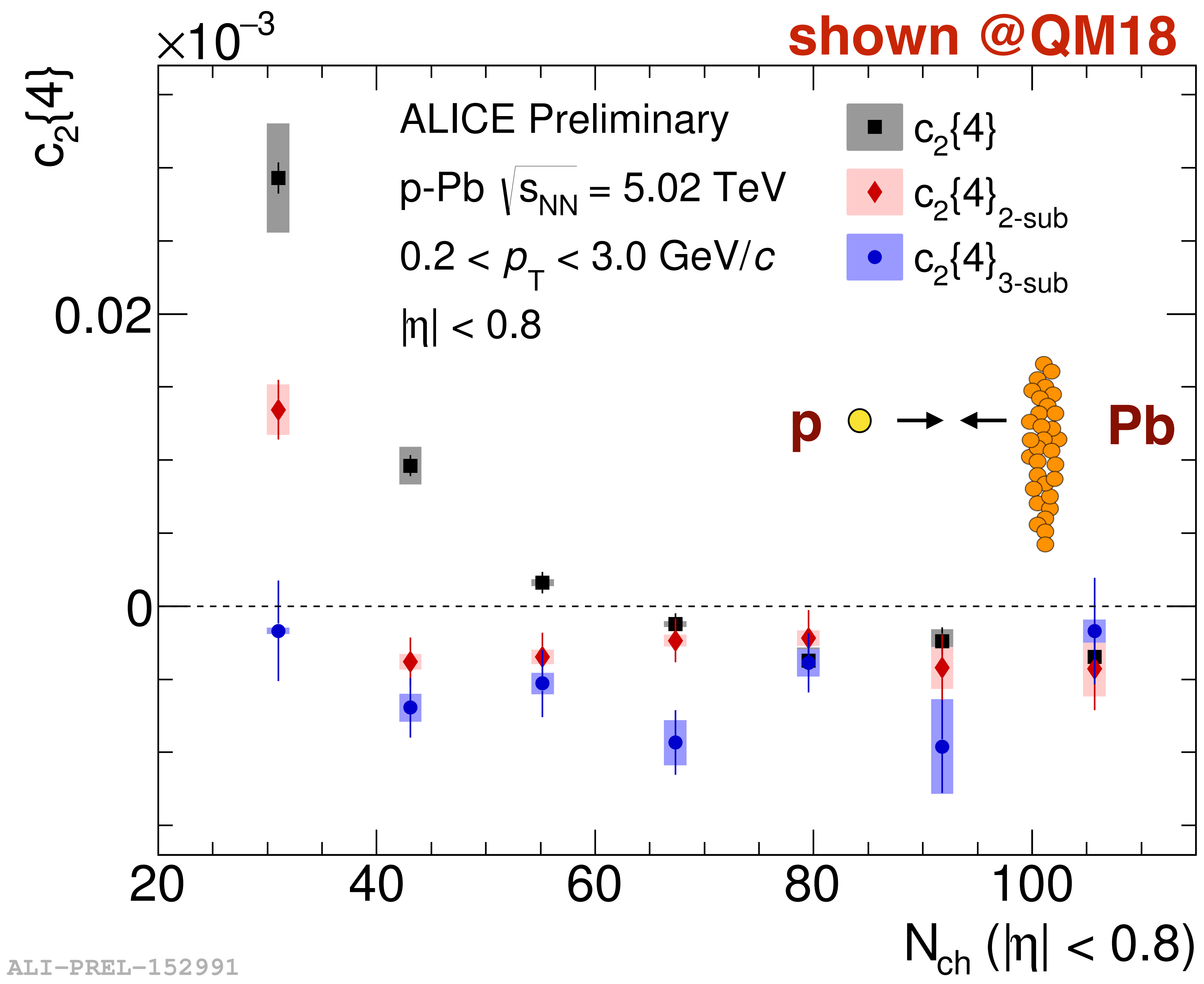
for more details see **R. Bertens** talk **Friday@9:00**

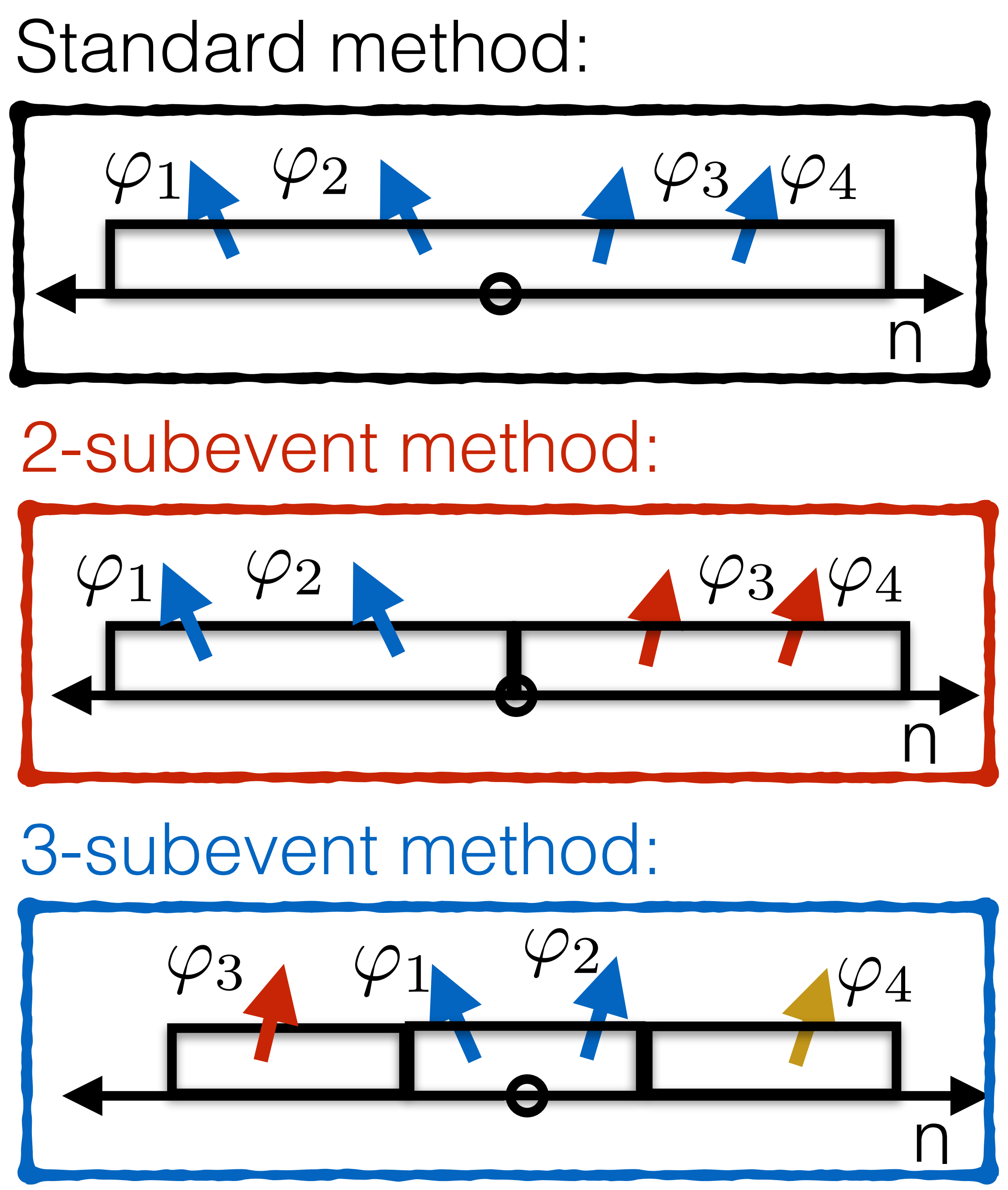
heavy-flavor μ results see **X. Zhang** talk **Friday@9:45**

Measurements of multi-particle cumulants

Measurements of $c_2\{4\}$ with subevent method

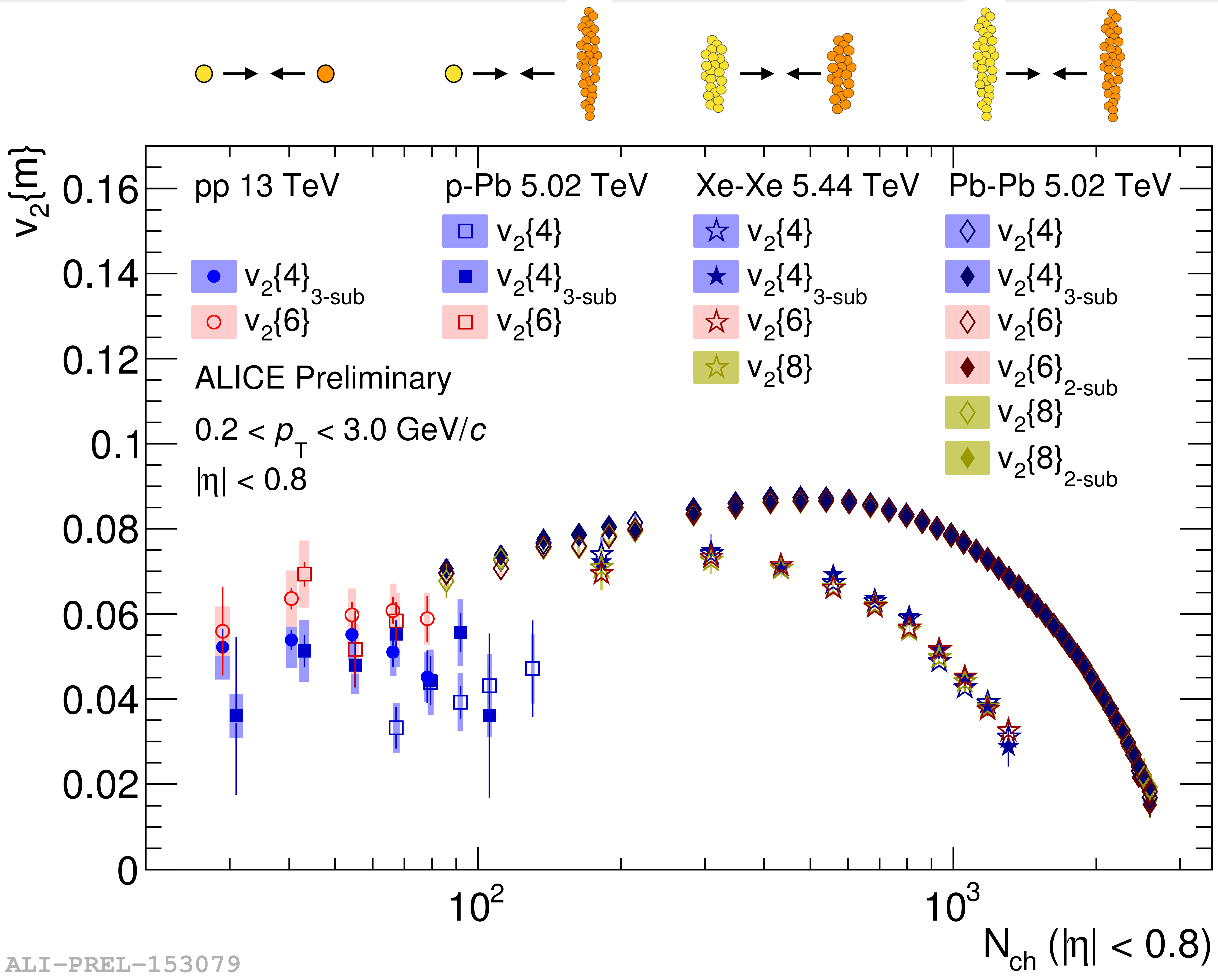






- Subevent method further suppresses non-flow in multi-particle cumulants in **pp collisions**
  - **Negative $c_2\{4\}_{3\text{-sub}}$** -> **real value** for **$v_2\{4\}_{3\text{-sub}}$**
- Non-flow can be largely suppressed also in **p-Pb collisions**
- No significant further decrease of $c_2\{4\}_{3\text{-sub}}$ with $|\Delta\eta| > 0.2$ between subevents

Flow coefficients from multi-particle cumulants



ALI-PREL-153079

shown @QM18

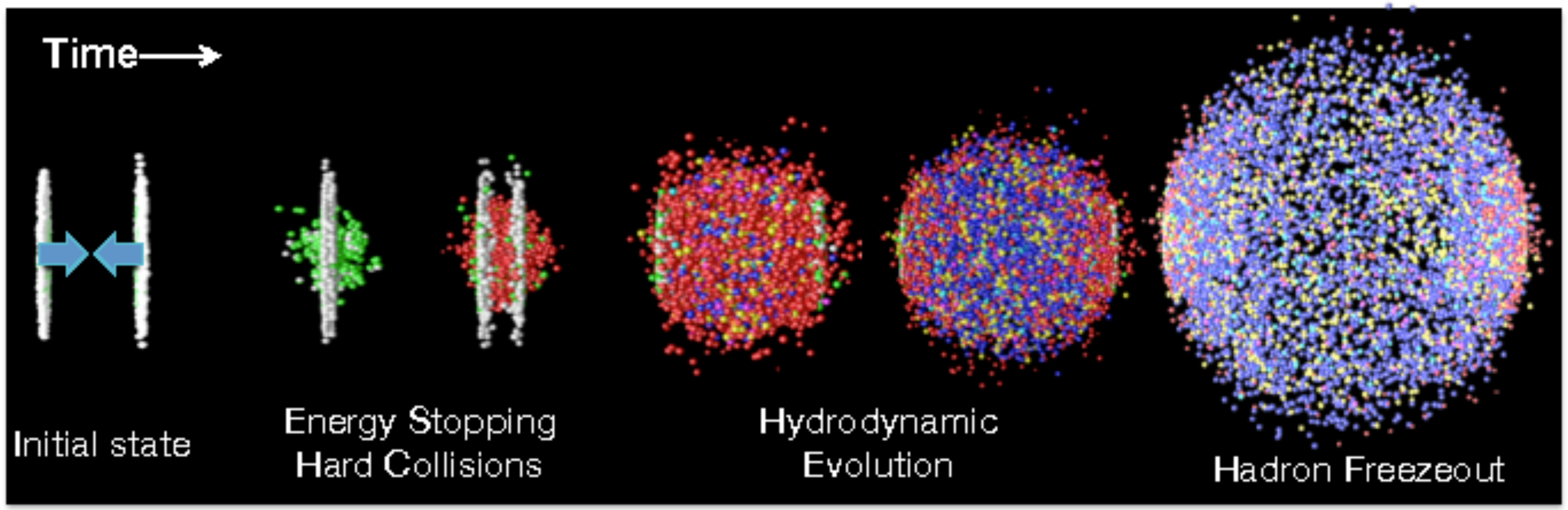
- **Heavy-ion collisions:**
  - Long-range: signal doesn't change anymore with subevent method
    - $v_2\{4\} \sim v_2\{4\}_{3\text{-sub}}$
    - $v_2\{6\} \sim v_2\{6\}_{2\text{-sub}}$
    - $v_2\{8\} \sim v_2\{8\}_{2\text{-sub}}$
  - Multi-particle: $v_2\{4\} \sim v_2\{6\} \sim v_2\{8\}$
- **p-Pb:** $v_2\{4\} < v_2\{4\}_{3\text{-sub}} \sim v_2\{6\}$
  - **Indication of collectivity**
- **pp:** real $v_2\{4\}_{3\text{-sub}}$ extracted for the first time in ALICE
  - $v_2\{4\}_{3\text{-sub}} \sim v_2\{6\}$ (Improved agreement can be done with subevent method in $v_2\{6\}$)
  - **Indication of collectivity**

Multi-particle cumulants show
evidence of long-range multi-particle correlations

Origin of collectivity in small collision systems

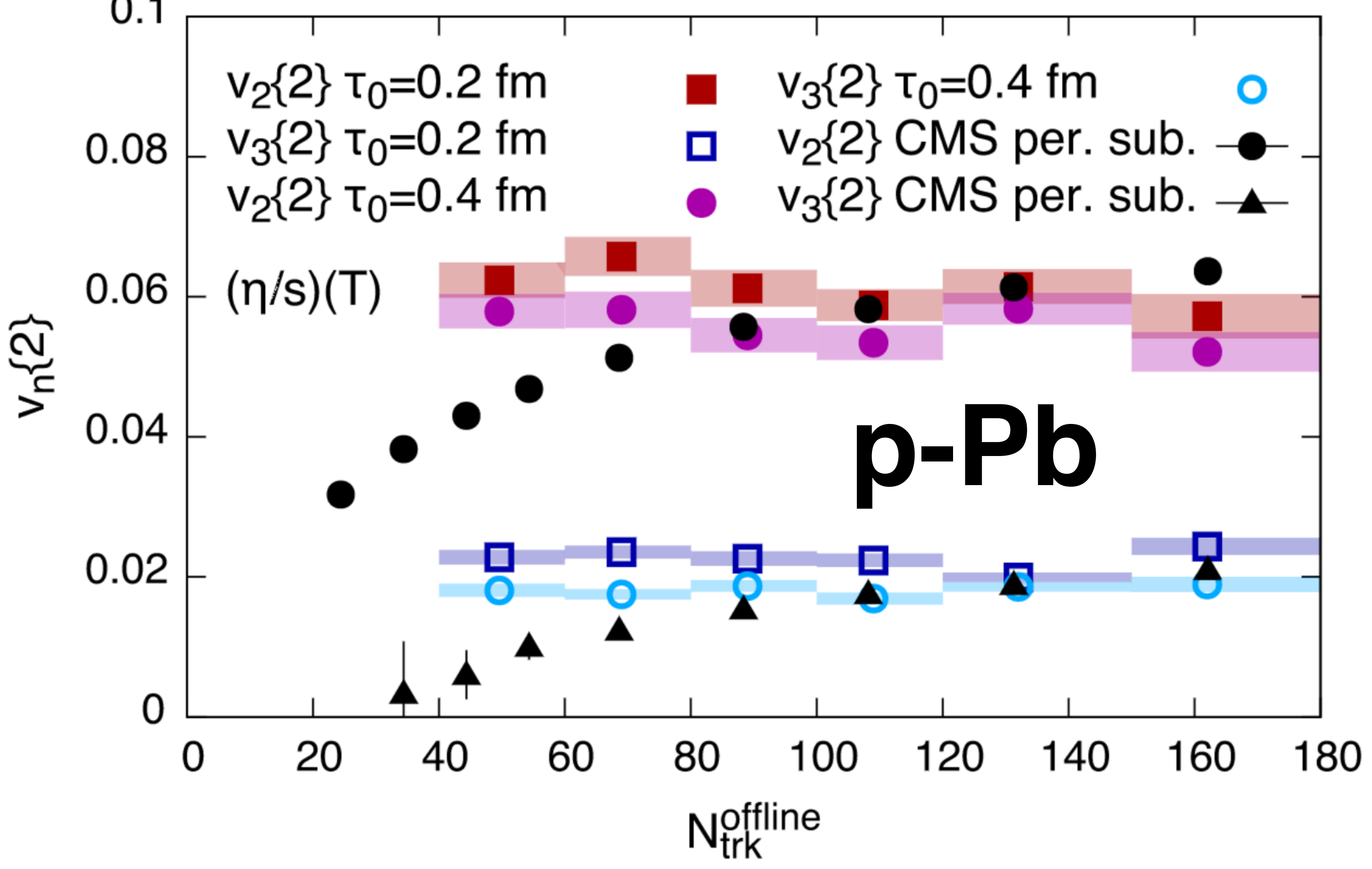
- What is the origin of the observed collectivity in small collision systems?

Initial stage effects **Final stage effects** **Both**

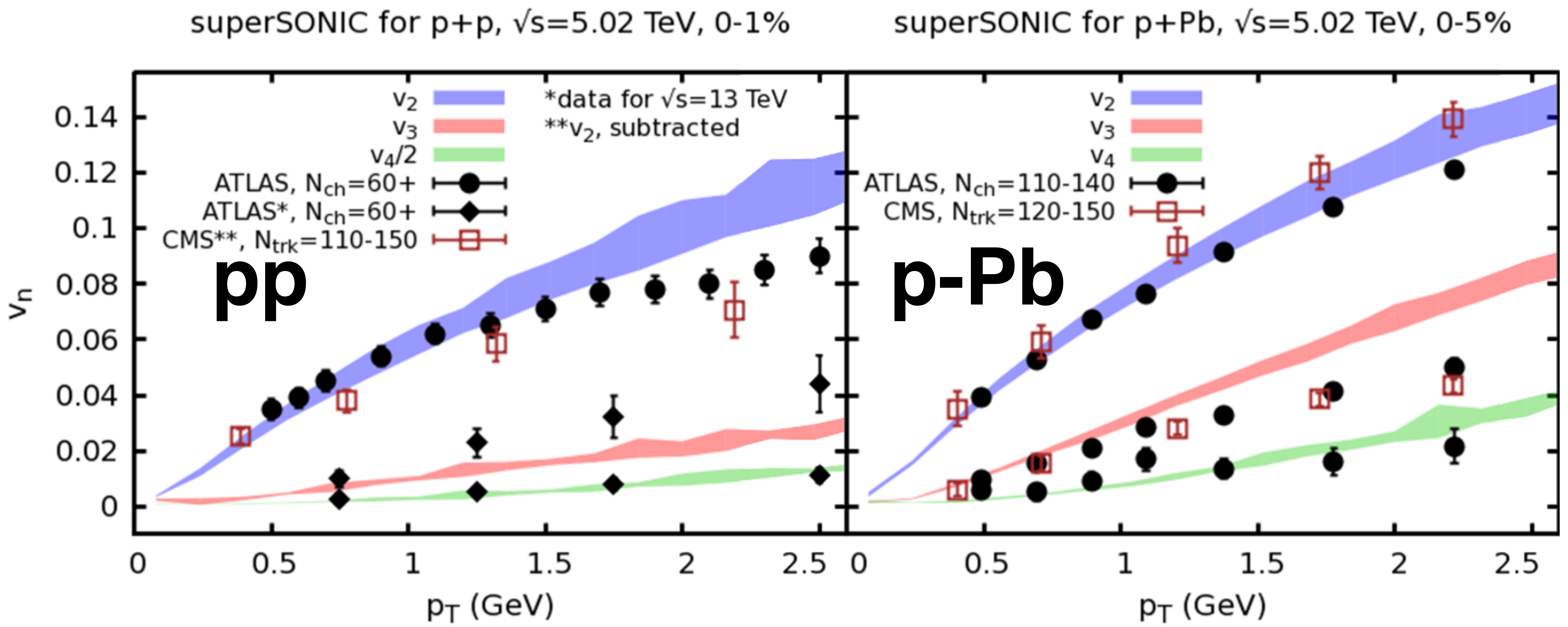


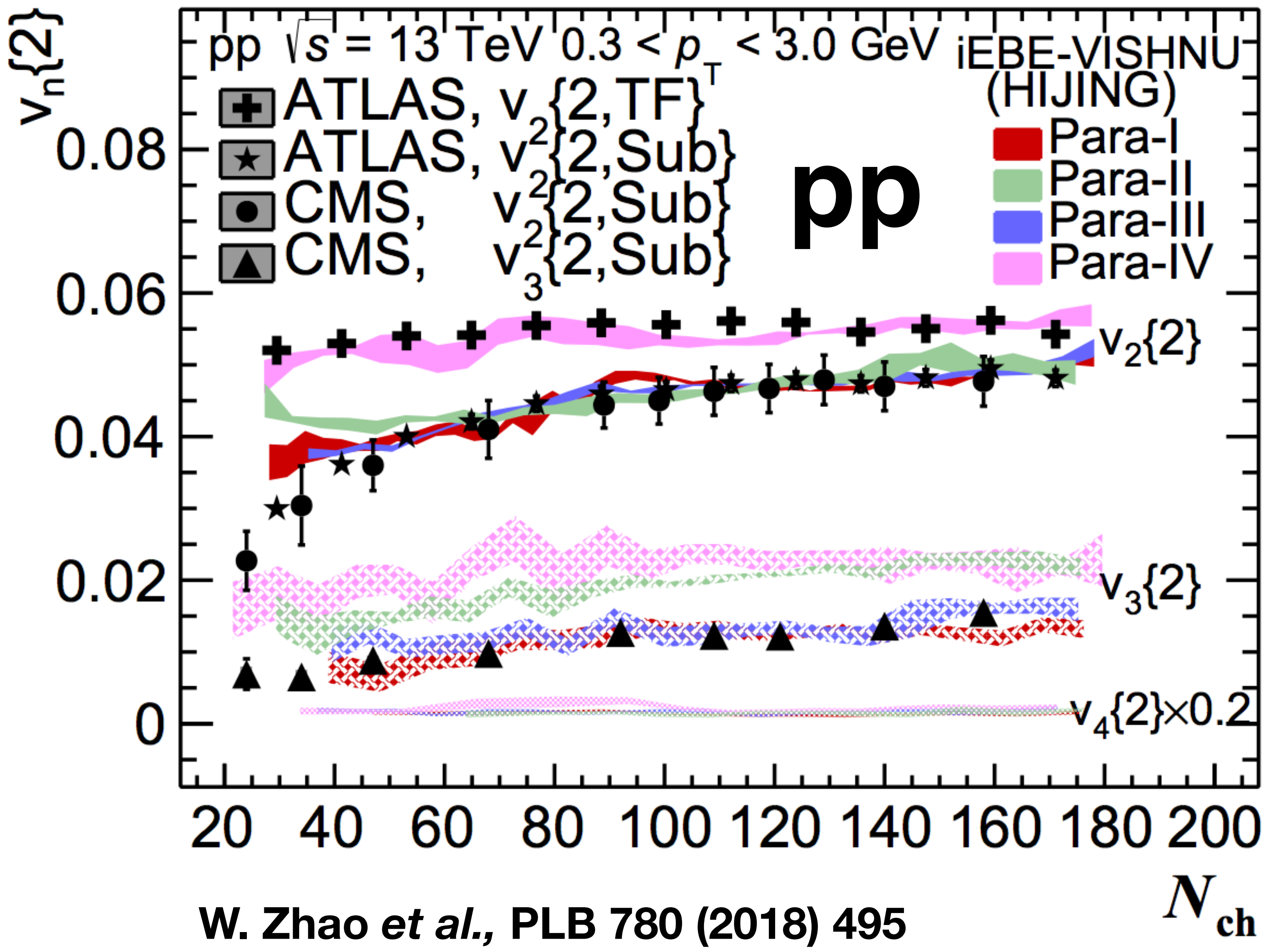
Two-particle correlations: charged hadrons

Mantysaari *et al.* PLB772, 681 (2017)



Weller *et al.* PLB 774 (2017) 351





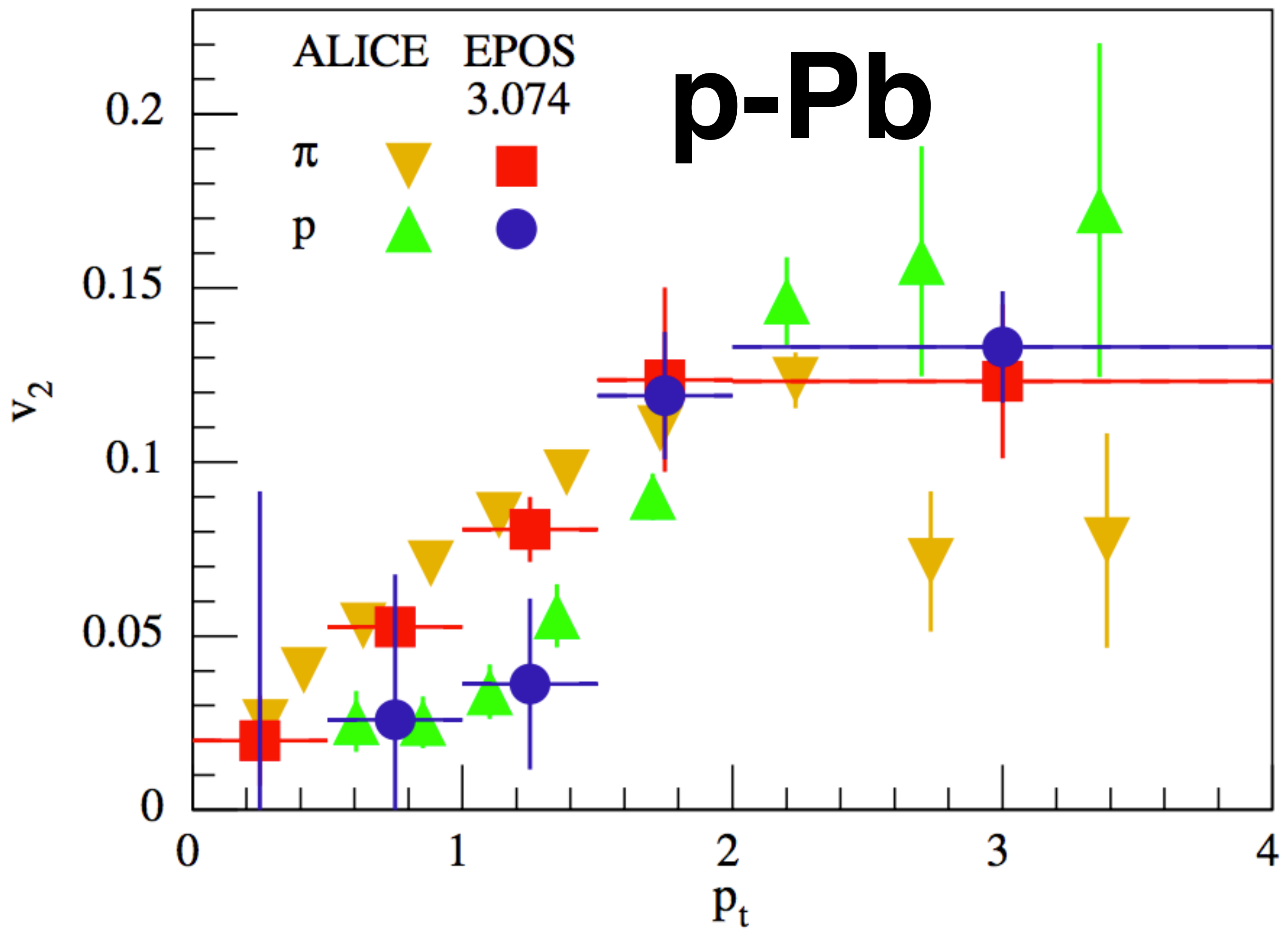
W. Zhao *et al.*, PLB 780 (2018) 495

Final stage effects

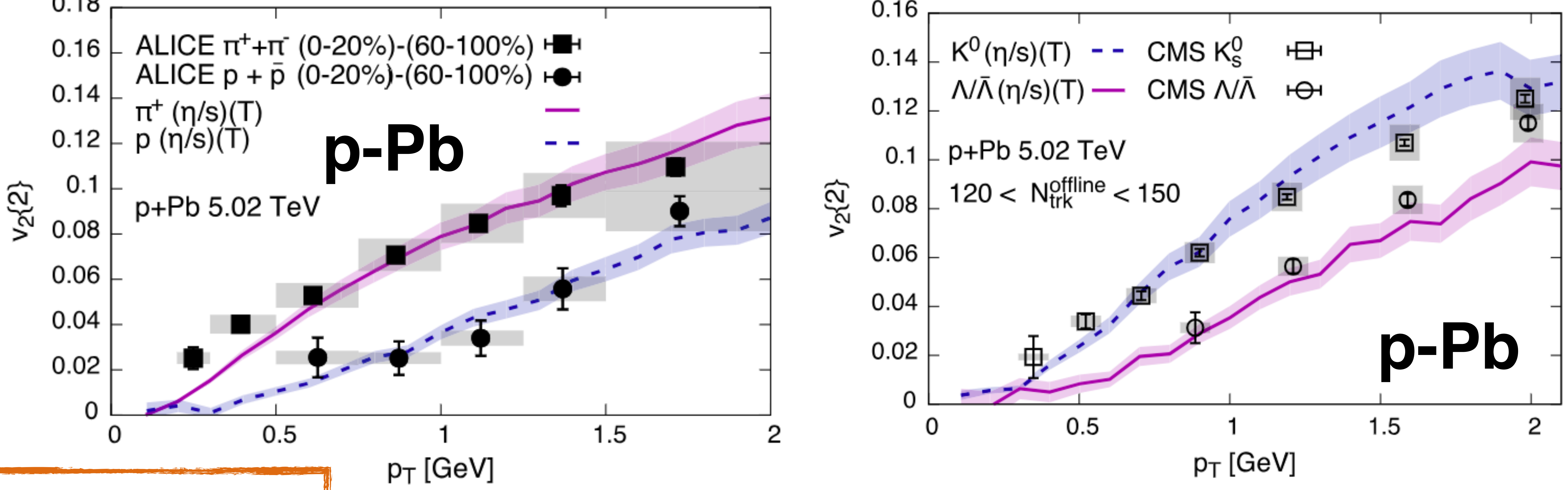
- Two-particle correlations are described fairly well:
  - Final stage models (hydrodynamics, parton escape, hadron interactions)
  - Rope and shoving mechanism
- The description is only successful after including sub-nucleon fluctuations
  - Constraining the initial conditions in small systems is important

Two-particle correlations: identified hadrons

K. Werner *et al.*, PRL 112, 232301 (2014)

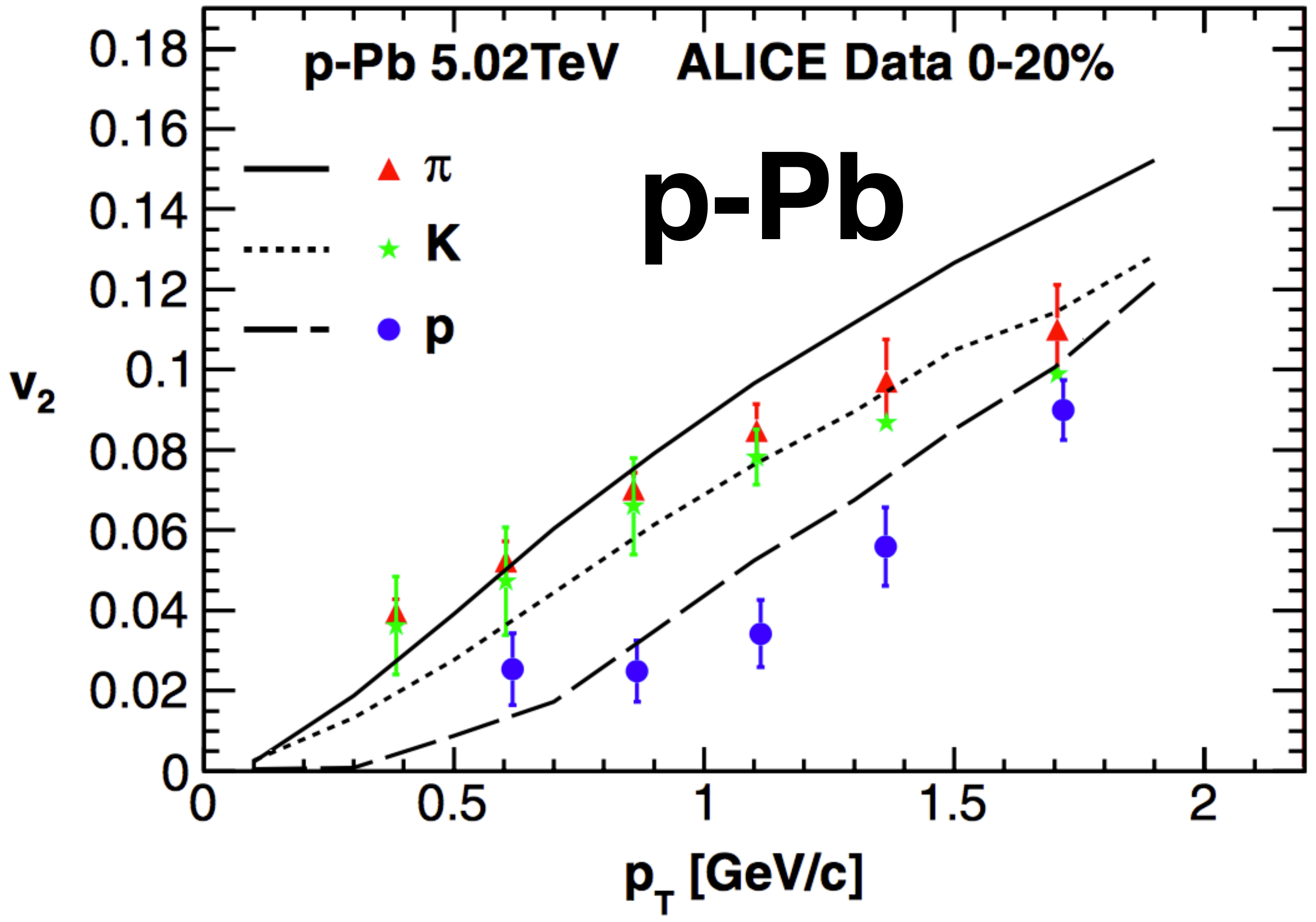


H. Mantysaari *et al.*, PLB 772 (2017) 681



Final stage effects

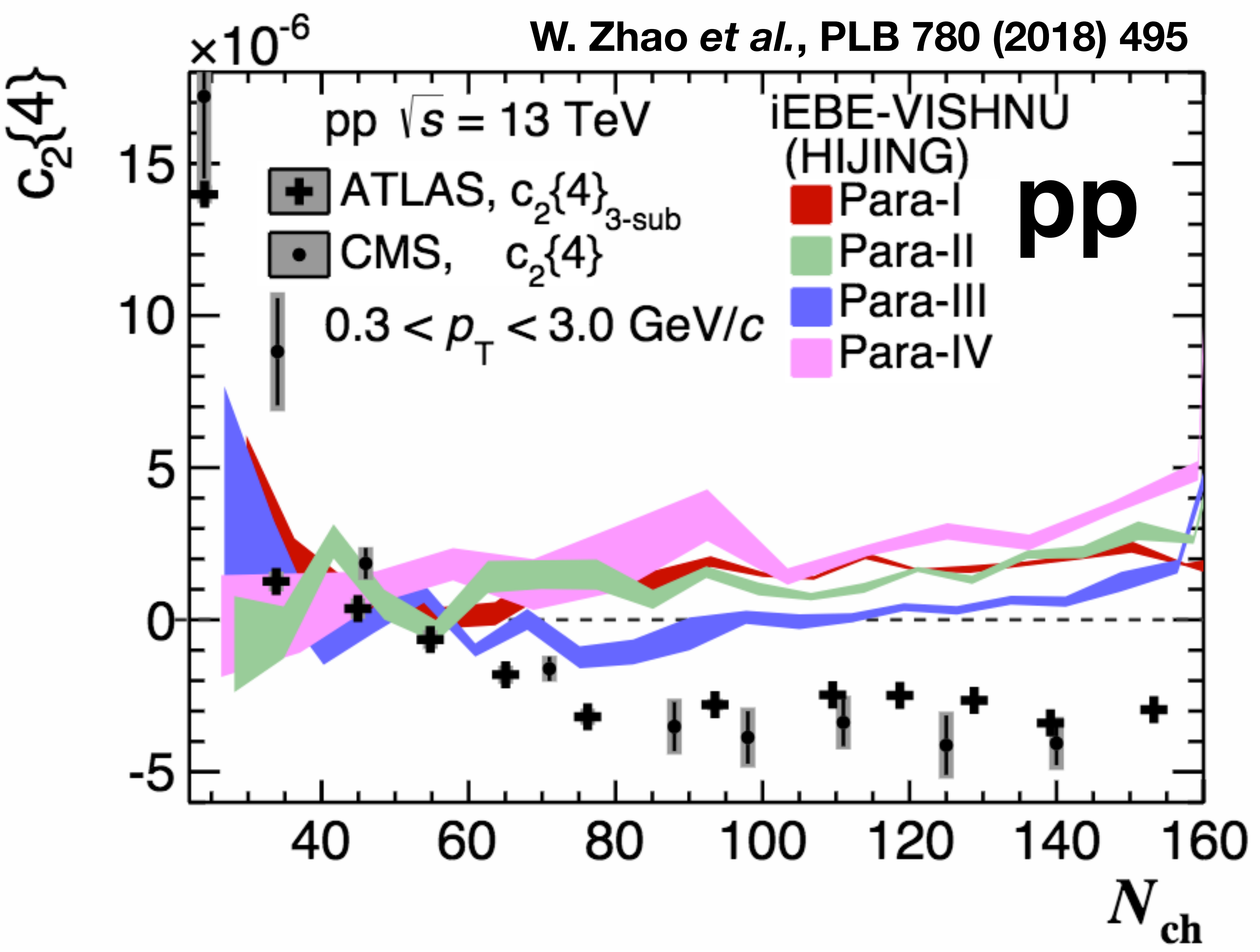
P. Bozek *et al.*, PRL 111, 172303 (2013)



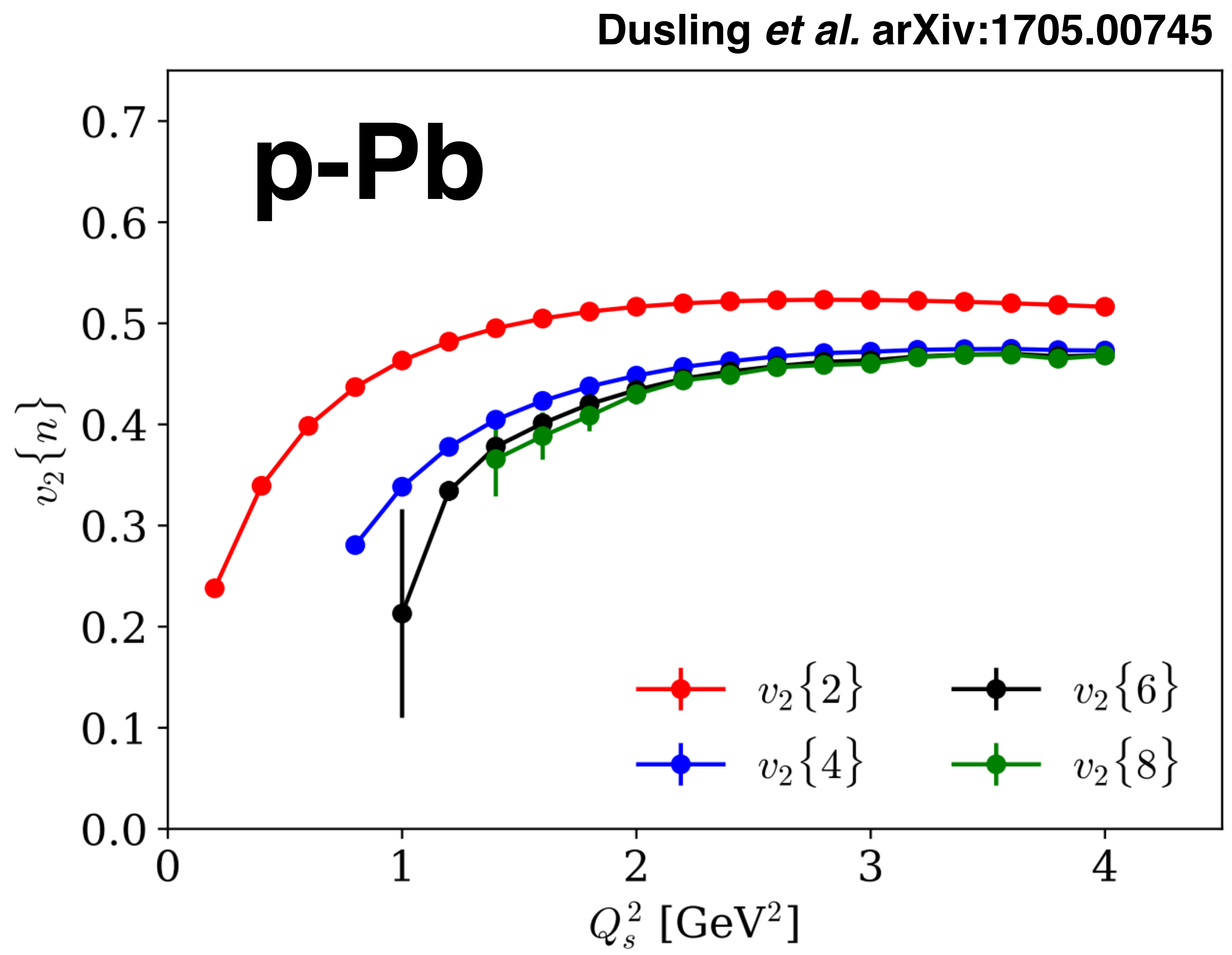
- Two-particle correlations of identified hadrons are also well described:
  - Hydrodynamic models with different initial conditions (Glauber, IP-Glasma, multiple parton scattering)
- Constraining initial conditions is important

Multi-particle correlations: charged hadrons

Final stage effects



Initial stage effects

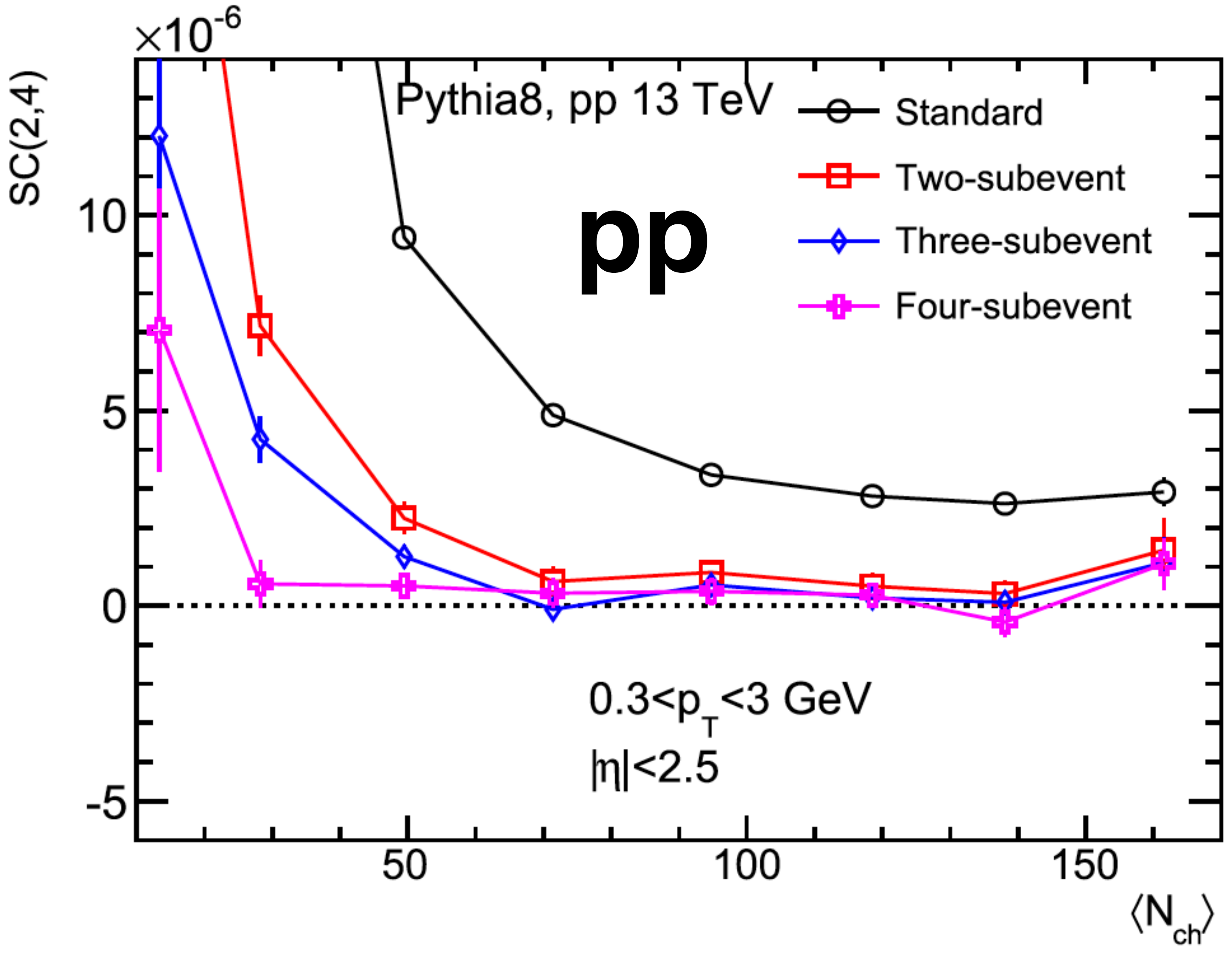


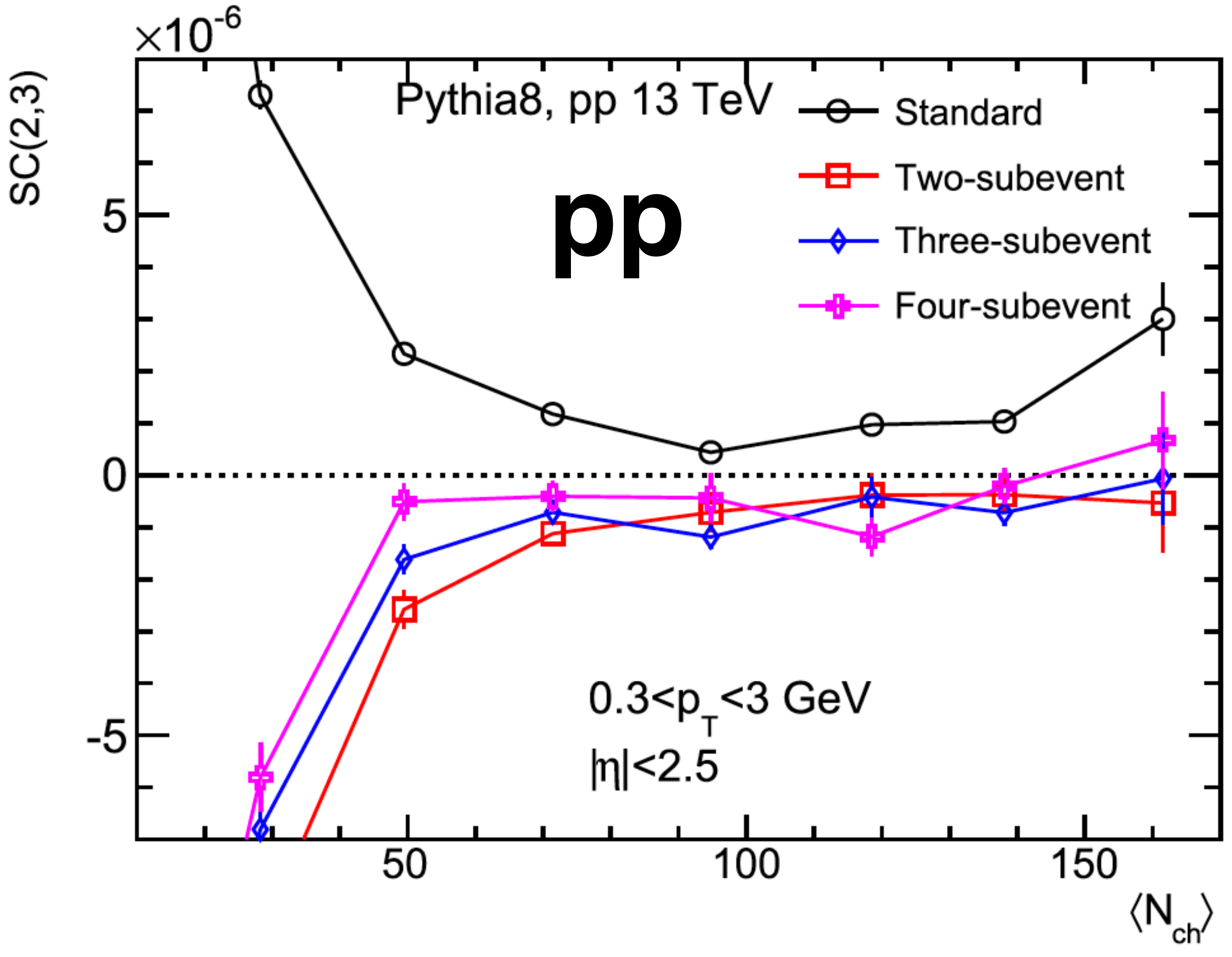
- There is no model that can describe multi-particle correlations measurements so far
- Final stage model
  - Hydrodynamic model produces positive $c_2\{4\}$
- Initial stage model
  - Qualitatively reproduces multi-particle cumulants, but overestimates the magnitude

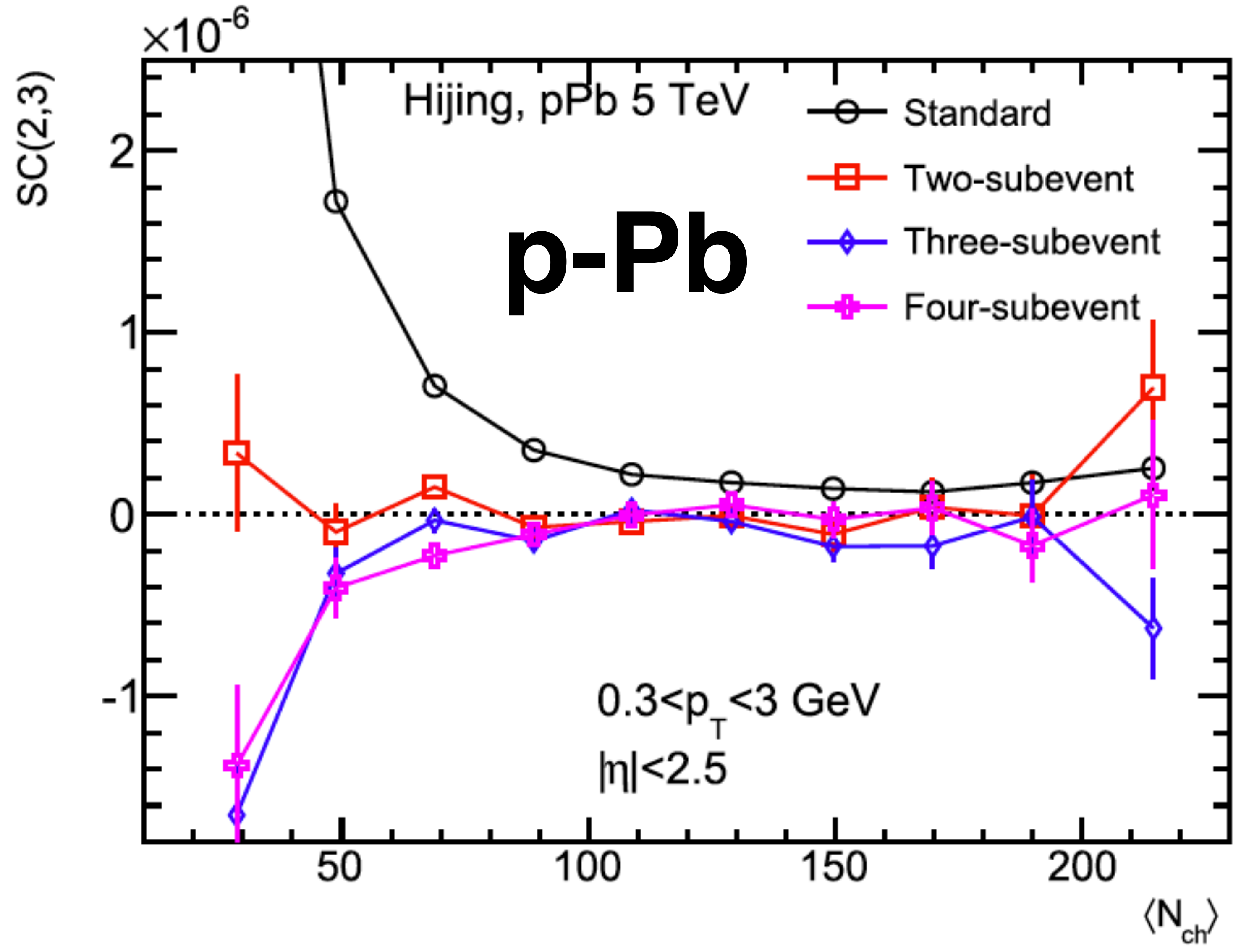
- Constraining initial stage effects in small systems is crucial to improve the understanding of the measurements
- Measurements of **Symmetric Cumulants** SC(m,n)
  - Quantify correlation between flow coefficients v_n and v_m
  - SC(3,2) is sensitive to initial stage effects, while SC(4,2) provides information about final stage effects

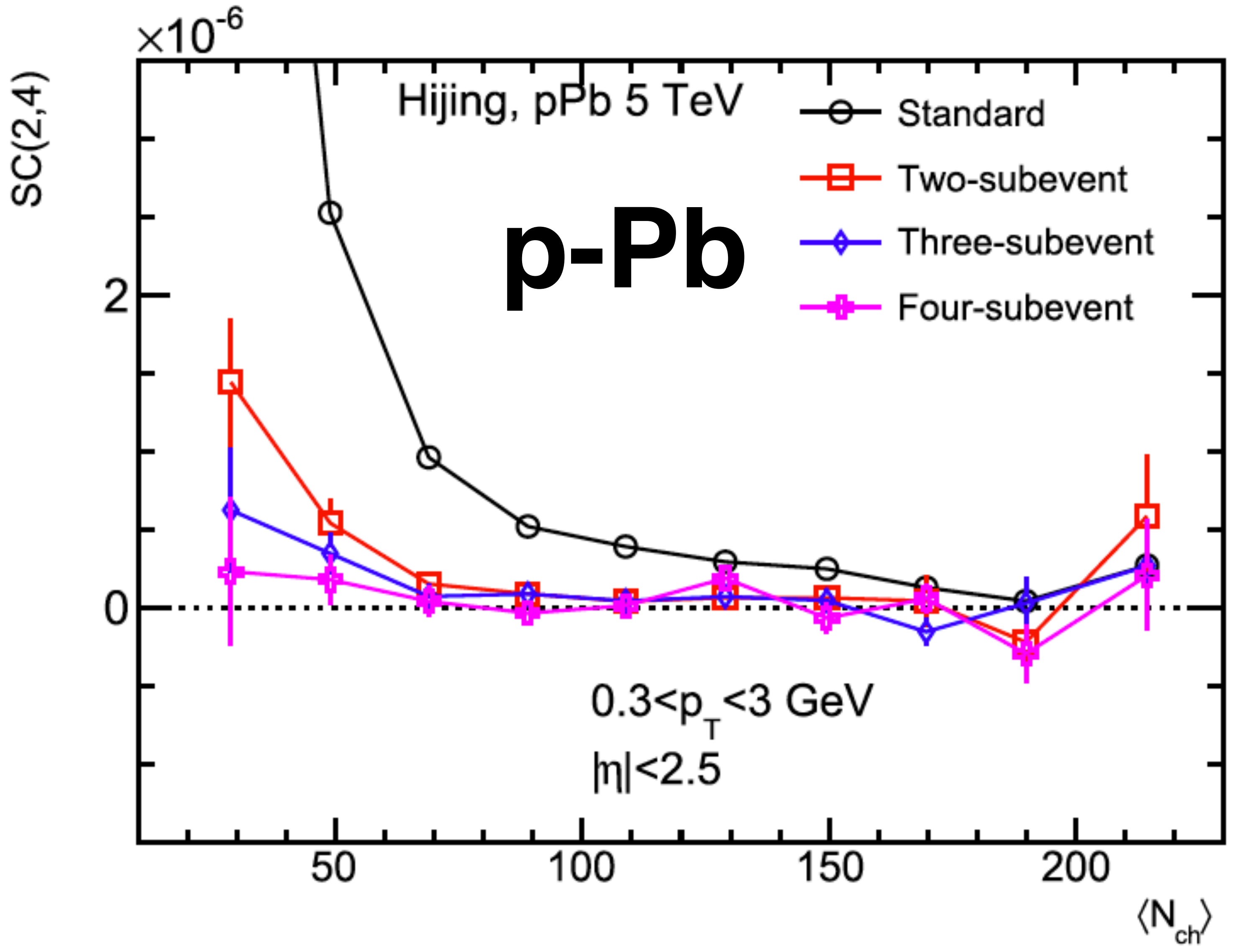
Non-flow in Symmetric Cumulants

Huo *et al.*, PLB 777 (2018) 201



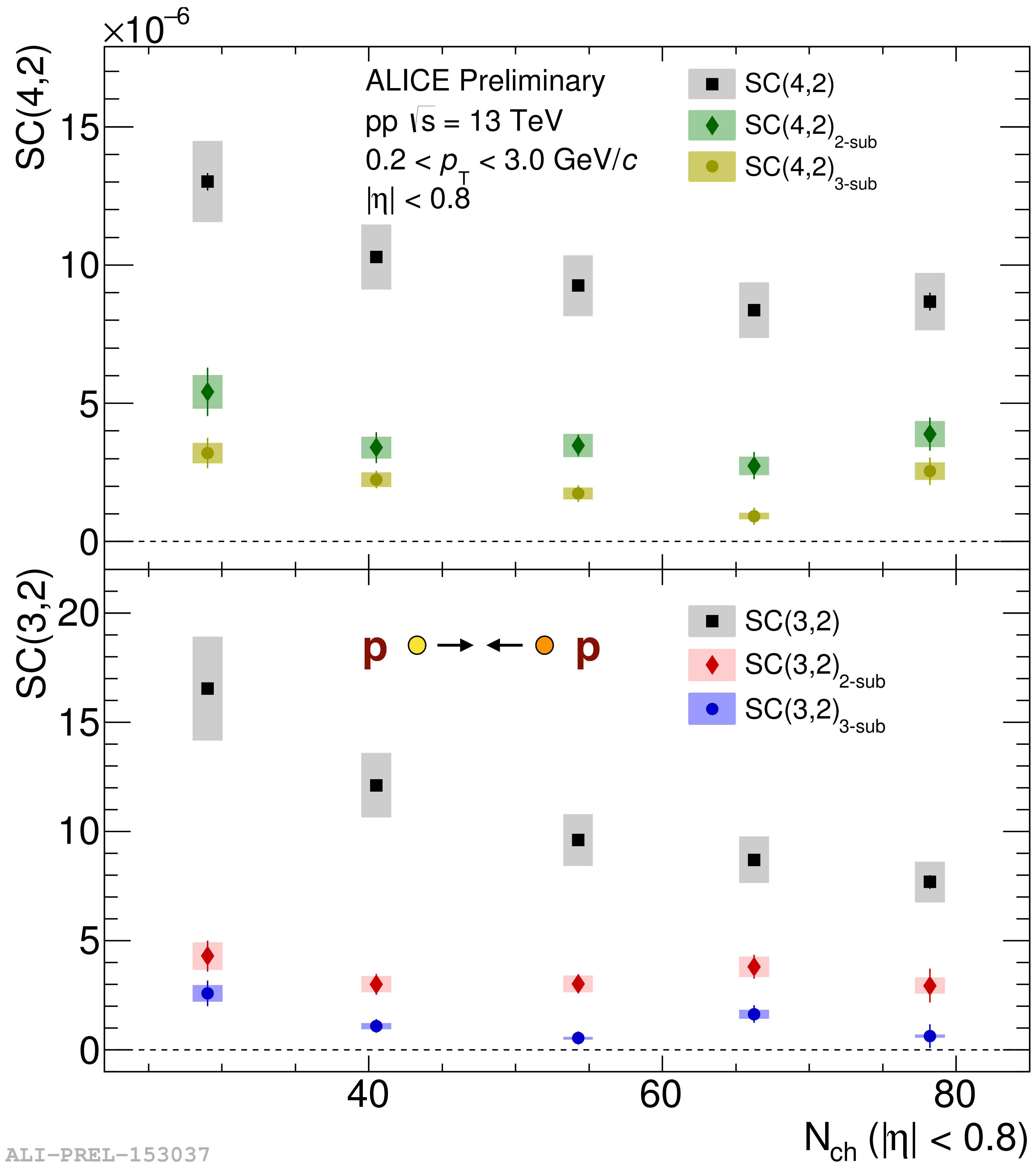


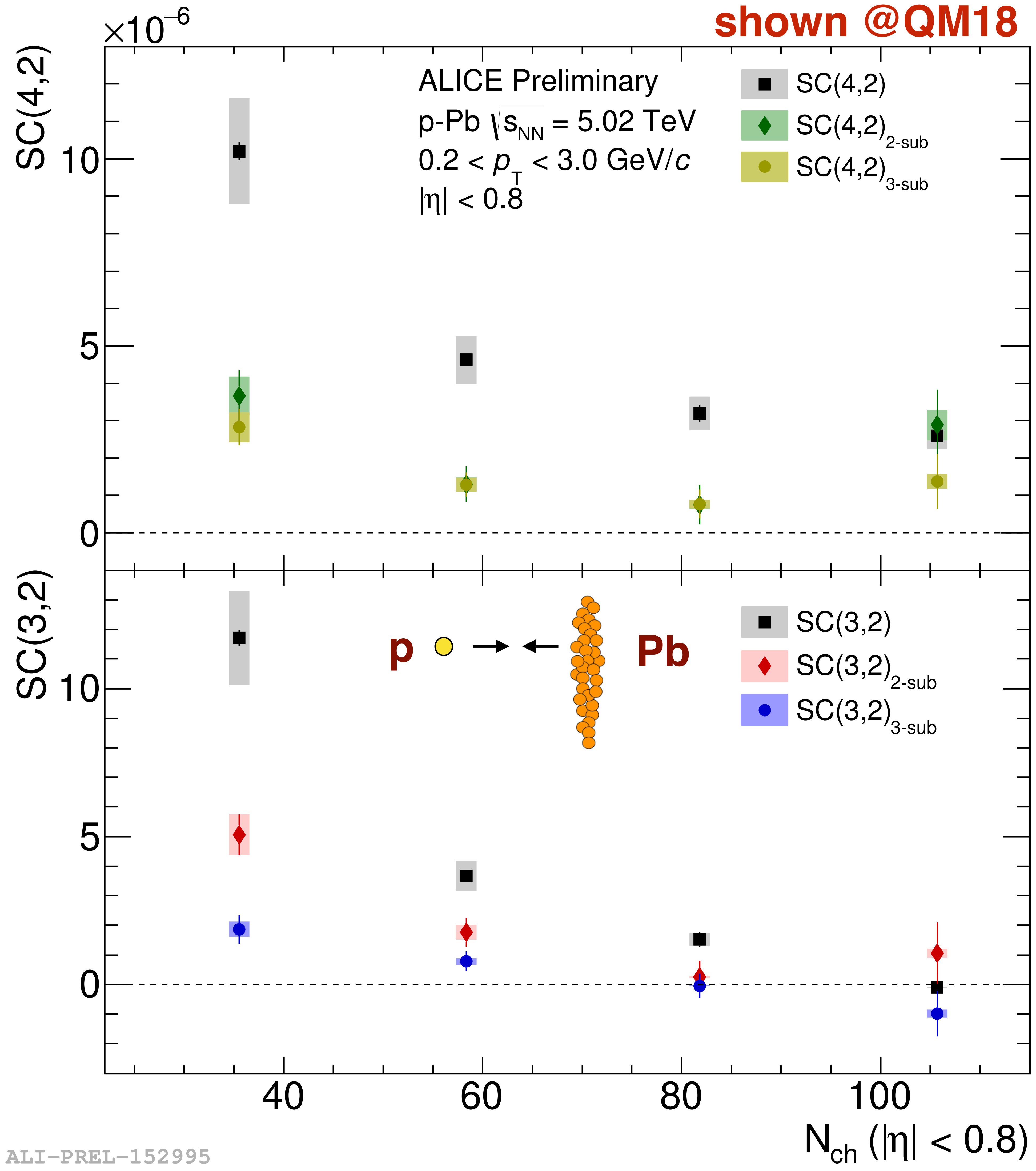




- Subevent method tested on Symmetric Cumulants with PYTHIA/HIJING:
  - Clear suppression of non-flow in SC(m,n) in both simulations
- Measurements are strongly affected by non-flow effects in small collision systems

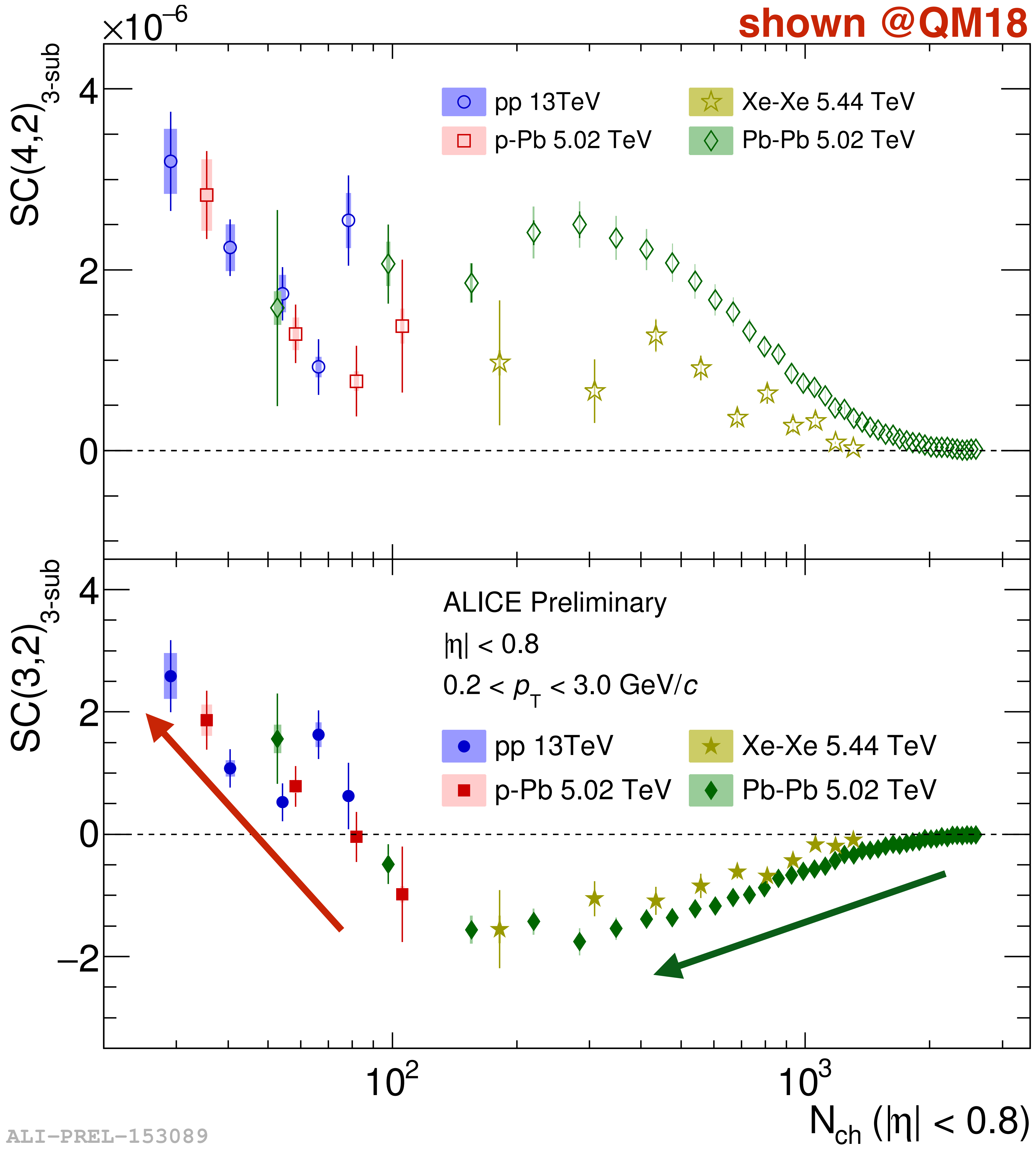
Measurements of SC(m,n) in small collision systems





- Clear suppression of non-flow effects

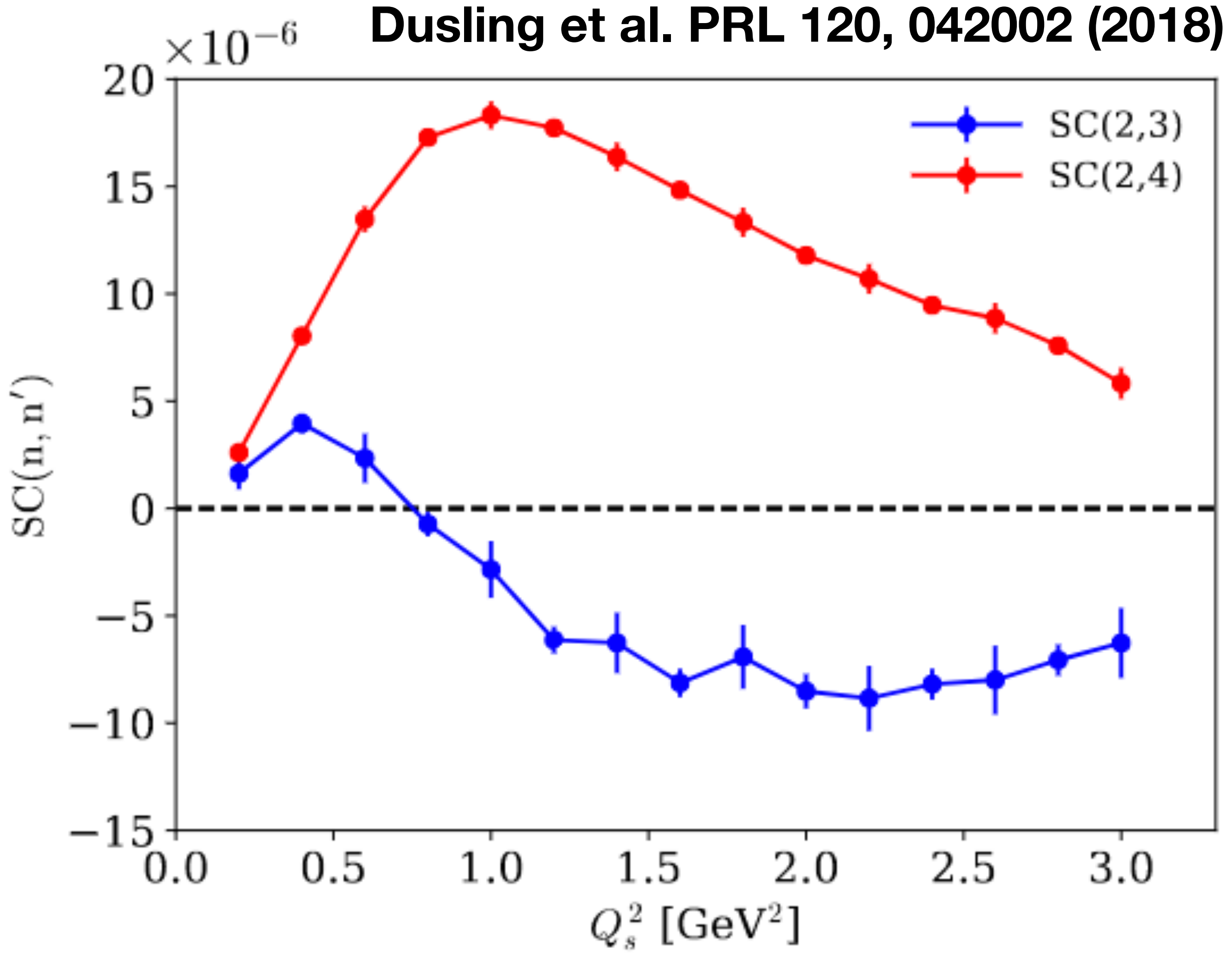
Full comparison: SC(m,n)$_{3\text{-sub}}$



- **SC(3,2):**
  - Anti-correlation between v_2 and v_3 observed at large multiplicities
  - Transition to positive values at low multiplicities (followed by all collision systems)
- **SC(4,2):**
  - Positive correlation between v_2 and v_4 in all collision systems

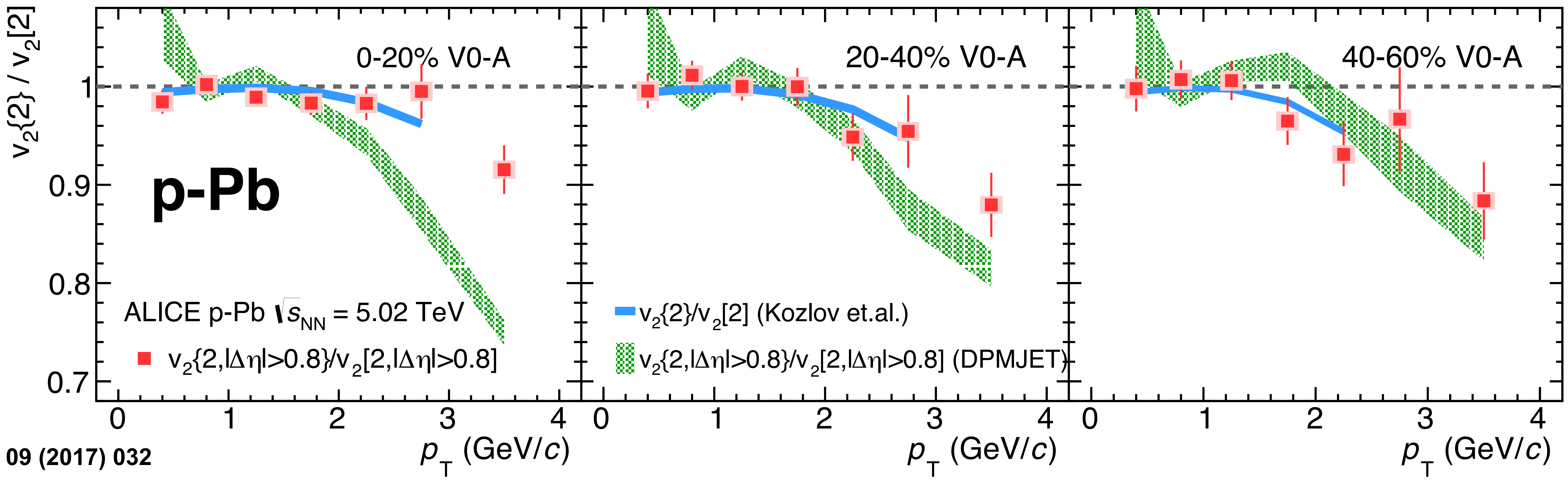
- Provide tight **constraints on initial conditions**, which are currently not well known in small systems
  - Qualitatively predicted by model with initial state correlations

Dusling et al. PRL 120, 042002 (2018)

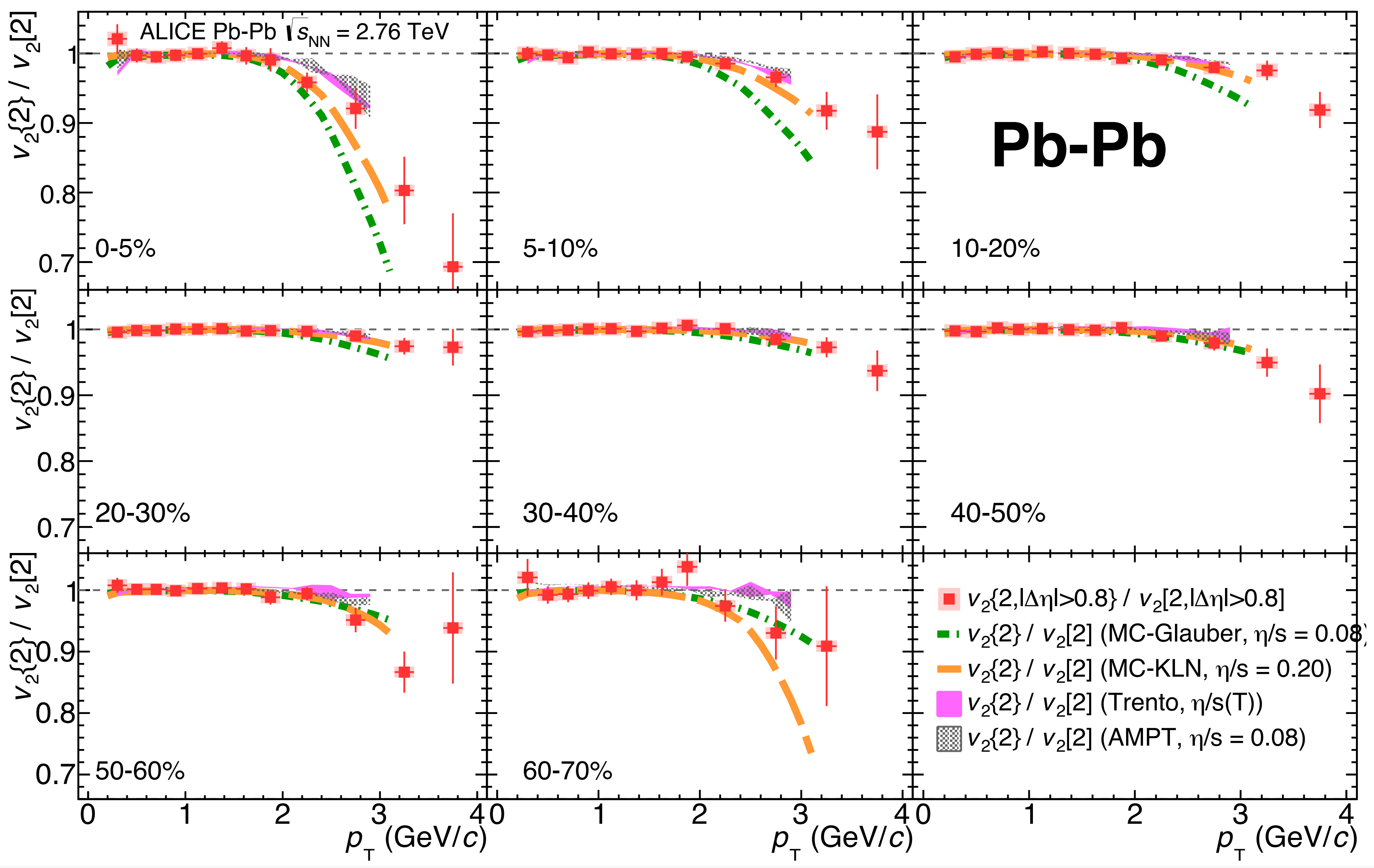


p_T - dependent fluctuations of V_n

charged hadrons



ALICE, JHEP 09 (2017) 032



$$\frac{v_n\{2\}}{v_n[2]}(p_T^a) = \frac{\langle v_n(p_T^a)\, v_n^{ref} \cos[n(\Psi_n(p_T^a) - \Psi_n)]\rangle}{\sqrt{\langle v_n(p_T^a)^2\rangle}\sqrt{\langle v_n^{ref\,2}\rangle}}$$

- Ratio < 1:
  - p_T dependent flow vector fluctuations

- Hints of deviations from unity at p_T > 2 GeV/c
- Measurements not described by DPMJET at high multiplicity class
- Hydrodynamics reproduces results in all multiplicity classes
- Hint of collectivity in p-Pb collisions

Summary

- **Charged particle** measurements of **$v_n\{m\}$** together with **SC(m,n)**
- **Charged particle** measurements of **$v_2\{2\}/v_2[2]$ (p_T)**
- **Identified particle** measurements of **$v_2\{2\}(p_T)$**
- All these results help us to explore the origin of the observed collectivity in small collision systems

initial state effects   final state effects   both

- Suppression of non-flow contamination is important -> subevent method provides the least biased measurements

- ***Understanding of the collectivity in small systems is yet to come***

Backup

Differential flow (charged particles)

- **Flow coefficient v_2 in a p_T interval *a***
  - Reference particles (RP) taken from a wide range of p_T, particles of interest (POI) from a certain small p_T region
  - If no p_T-dependent flow vector fluctuations ->

$$v_n\{2\}(p_T^a) = \sqrt{\langle v_n(p_T^a)^2 \rangle}$$

$$v_n\{2\}(p_T^a) = \frac{\langle\langle \cos\left[n(\varphi_1^a - \varphi_2^{ref})\right]\rangle\rangle}{\sqrt{\langle\langle \cos\left[n(\varphi_1^{ref} - \varphi_2^{ref})\right]\rangle\rangle}} = \frac{\langle v_n(p_T^a) v_n^{ref} \cos[n(\Psi_n(p_T^a) - \Psi_n)]\rangle}{\sqrt{\langle {v_n^{ref}}^2 \rangle}}$$

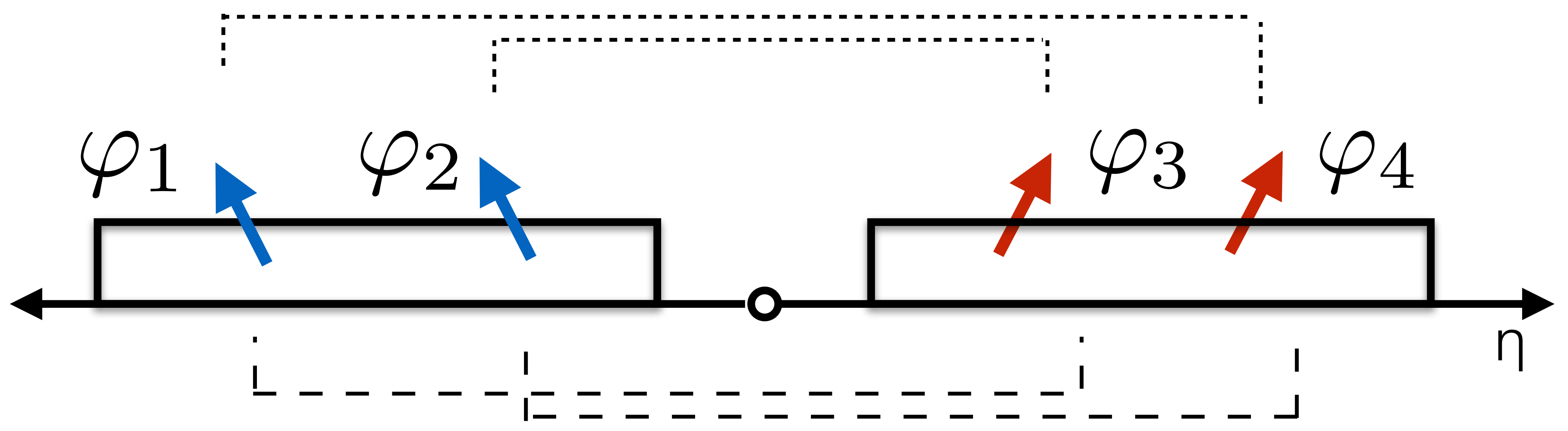
- New type of observable: **differential flow $v_n[2]$**
  - Both RP and POI taken from the certain small p_T region

$$v_n[2](p_T^a) = \sqrt{\langle\langle \cos\left[n(\varphi_1^a - \varphi_2^a)\right]\rangle\rangle} = \sqrt{\langle\langle \cos\left[n(\varphi_1^a - \Psi_n(p_T^a)) - n(\varphi_2^a - \Psi_n(p_T^a))\right]\rangle\rangle} = \sqrt{\langle v_n(p_T^a)^2 \rangle}$$

- Ratio of these observables
  - **Deviation from unity -> p_T dependent flow vector fluctuations**

$$\frac{v_n\{2\}}{v_n[2]}(p_T^a) = \frac{\langle v_n(p_T^a)\, v_n^{ref} \cos[n(\Psi_n(p_T^a) - \Psi_n)]\rangle}{\sqrt{\langle v_n(p_T^a)^2\rangle}\sqrt{\langle {v_n^{ref}}^2 \rangle}}$$

2-subevent method



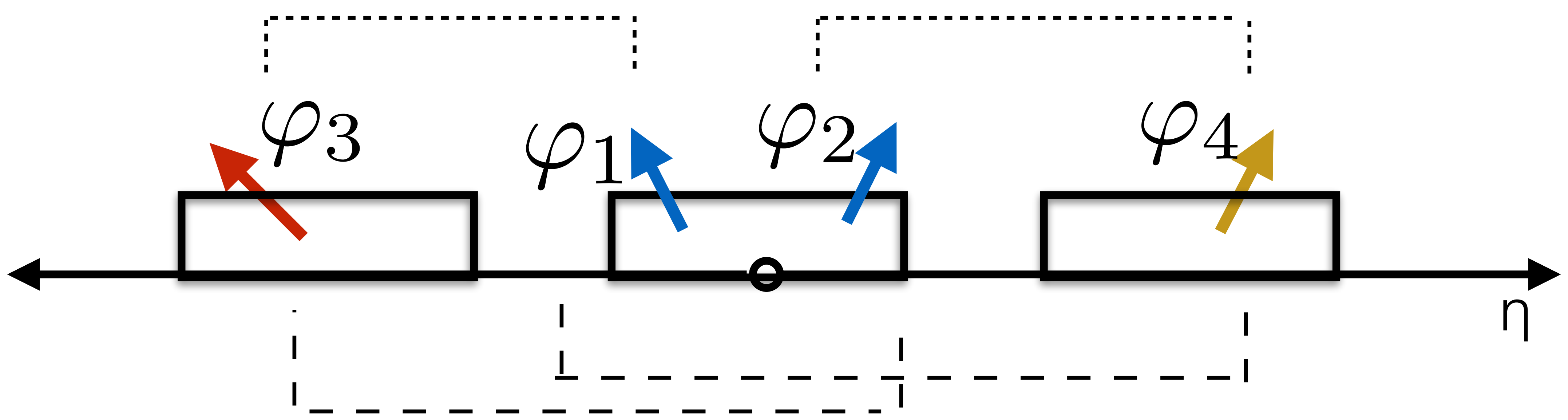
$$\langle\langle 4\rangle\rangle_{\text{2-sub}} = \langle\langle \cos n(\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4)\rangle\rangle$$

$$\langle\langle 2\rangle\rangle^2_{\text{2-sub}} = \langle\langle \cos n(\varphi_1 - \varphi_3)\rangle\rangle\langle\langle \cos n(\varphi_2 - \varphi_4)\rangle\rangle$$

$$\langle\langle 2\rangle\rangle^2_{\text{2-sub}} = \langle\langle \cos n(\varphi_1 - \varphi_4)\rangle\rangle\langle\langle \cos n(\varphi_2 - \varphi_3)\rangle\rangle$$

$$c_n\{4\}_{2-sub} = \langle\langle 4\rangle\rangle_{2-sub} - 2\cdot\langle\langle 2\rangle\rangle^2_{2-sub}$$

3-subevent method



$$\langle\langle 4\rangle\rangle_{\text{3sub}} = \langle\langle \cos n(\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4)\rangle\rangle$$

$$\langle\langle 2\rangle\rangle^2_{\text{3sub}} = \langle\langle \cos n(\varphi_1 - \varphi_3)\rangle\rangle\langle\langle \cos n(\varphi_2 - \varphi_4)\rangle\rangle$$

$$\langle\langle 2\rangle\rangle^2_{\text{3sub}} = \langle\langle \cos n(\varphi_1 - \varphi_4)\rangle\rangle\langle\langle \cos n(\varphi_2 - \varphi_3)\rangle\rangle$$

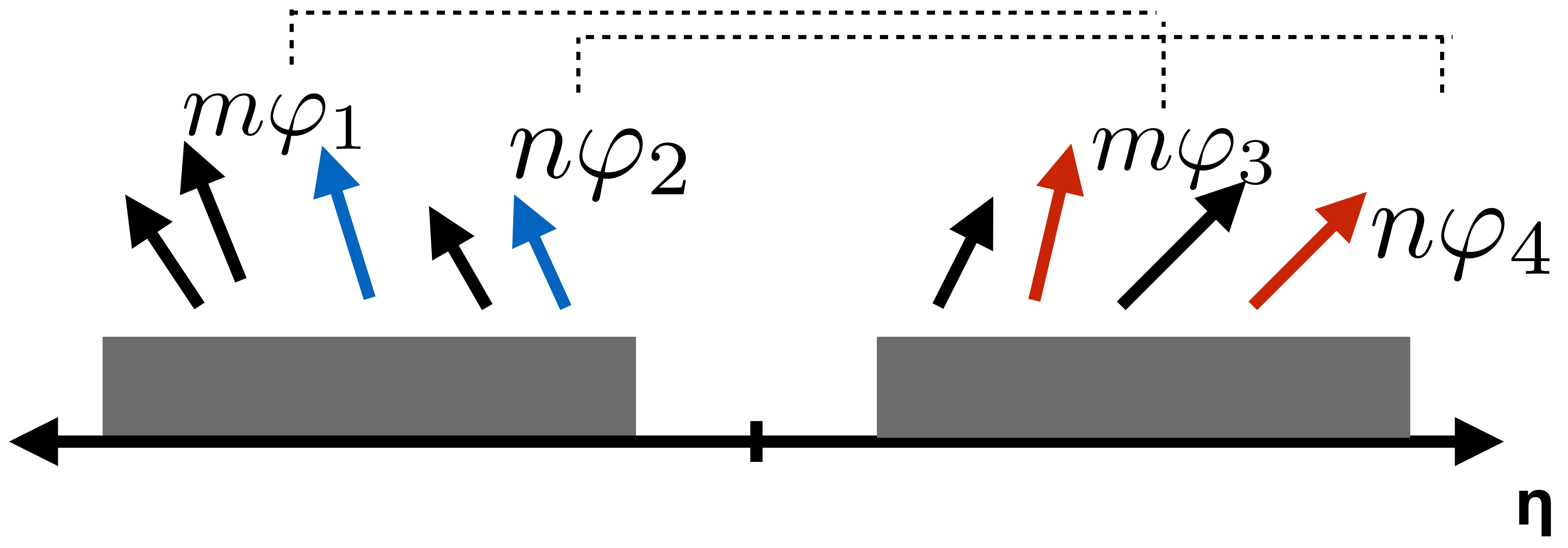
$$c_n\{4\}_{\text{3sub}} = \langle\langle 4\rangle\rangle_{\text{3sub}} - 2\cdot\langle\langle 2\rangle\rangle^2_{\text{3sub}}$$

Implemented with Generic Framework

- Similarly for SC(m,n)

Contamination with non-flow in SC(m,n)

- SC(m,n) measurements are based on 4-particle cumulant
- Clear contamination of standard $c_2\{4\}$ measurements -> **SC(m,n) is contaminated too**
- Method developed very recently by both ATLAS and ALICE (WPCF 2017, Phys.Lett. B777 (2018) 201-206)



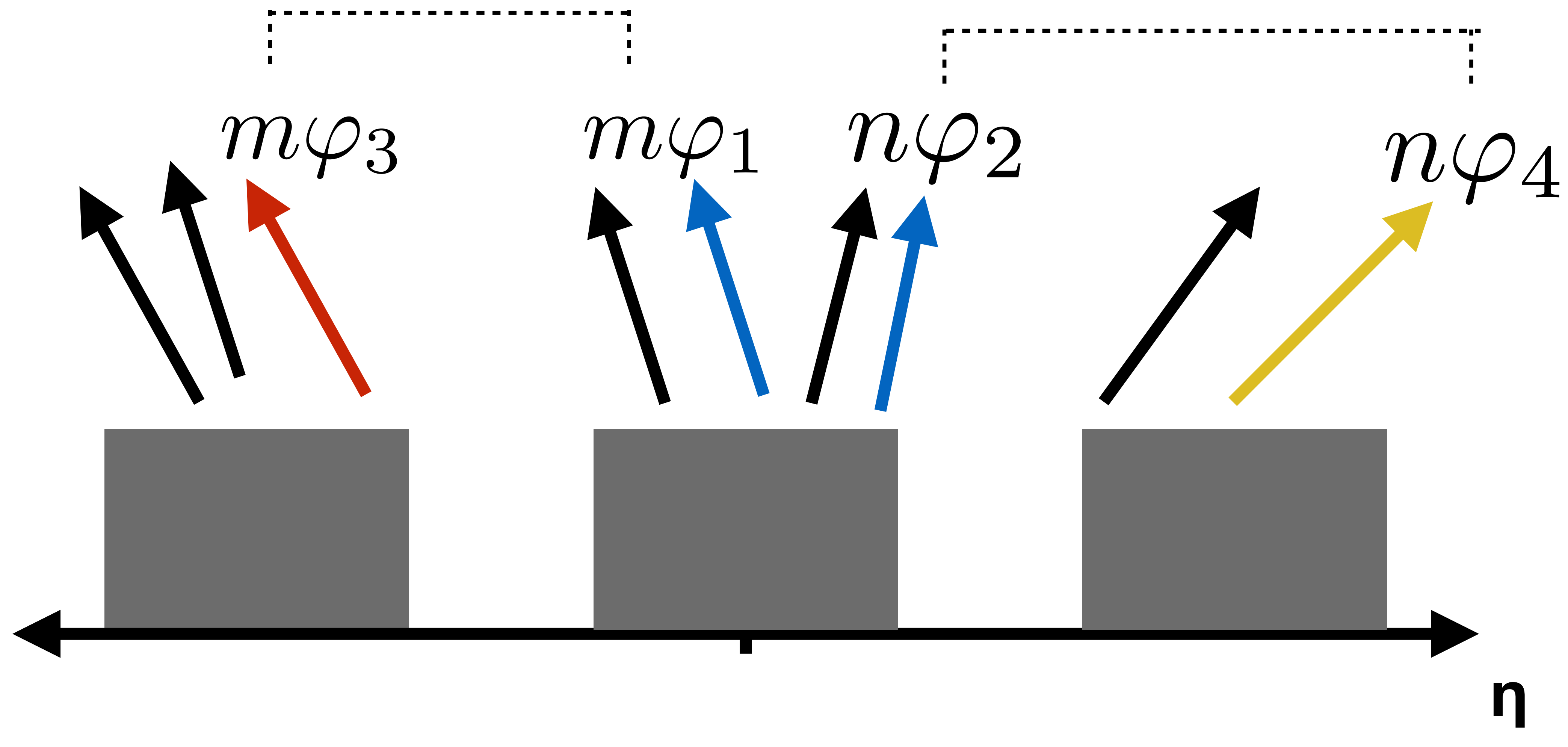
$$\langle\langle 4\rangle\rangle_{\text{2-sub}} = \langle\langle \cos\left(\mathrm{m}\varphi_1 + \mathrm{n}\varphi_2 - \mathrm{m}\varphi_3 - \mathrm{n}\varphi_4\right)\rangle\rangle$$

$$\langle\langle 2\rangle\rangle_{\text{2-sub}}\langle\langle 2\rangle\rangle_{\text{2-sub}} = \langle\langle \cos \mathrm{m}(\varphi_1 - \varphi_3)\rangle\rangle\langle\langle \cos \mathrm{n}(\varphi_2 - \varphi_4)\rangle\rangle$$

$$SC(m,n)_{2-sub} = \langle\langle 4\rangle\rangle_{2-sub} - \langle\langle 2\rangle\rangle_{2-sub}\langle\langle 2\rangle\rangle_{2-sub}$$

3-subevent method in the backup

3-subevent method in SC(m,n)

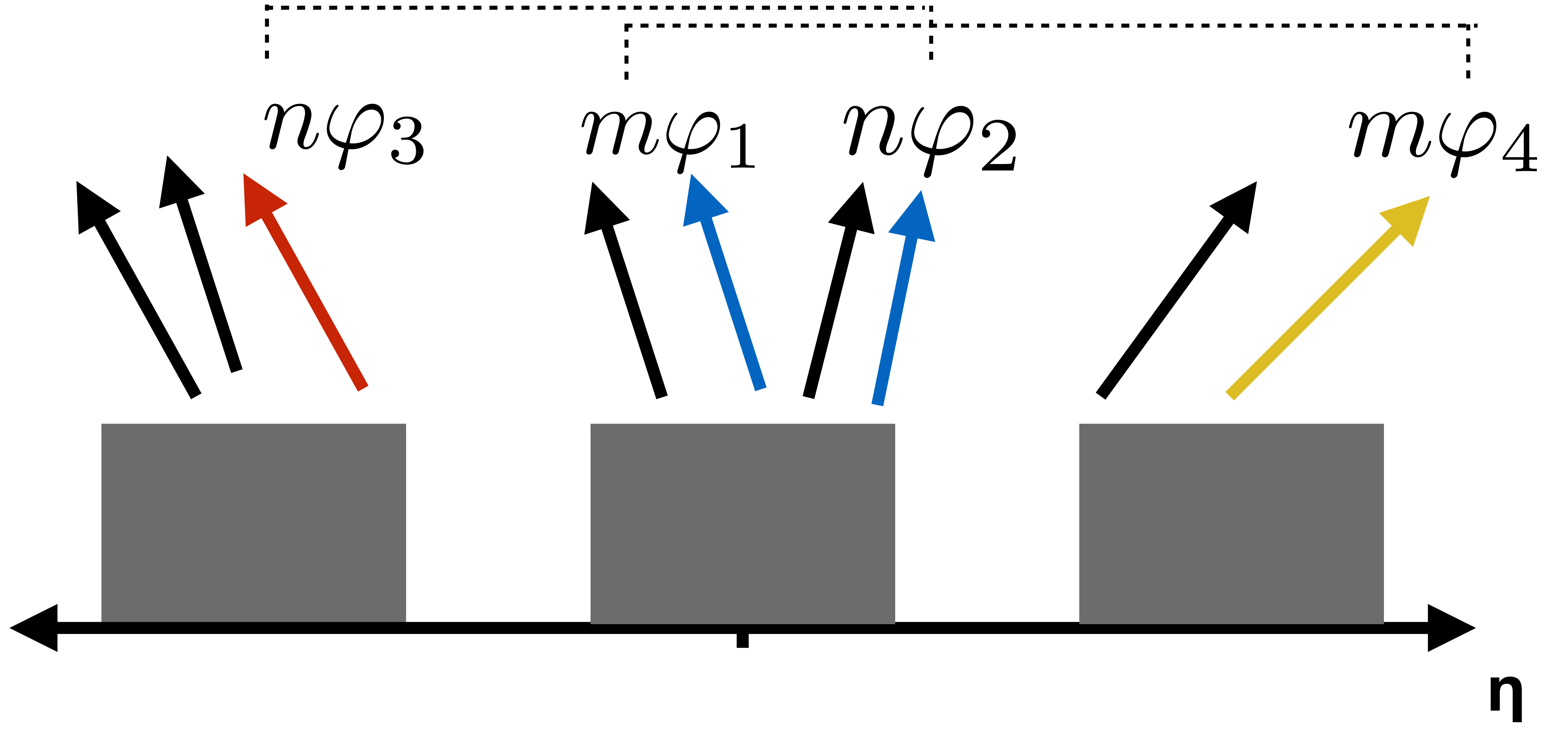


A.

$$\langle\langle 4\rangle\rangle_{m,n,-m,-n} = \langle\langle \cos(\mathrm{m}\varphi_1 + \mathrm{n}\varphi_2 - \mathrm{m}\varphi_3 - \mathrm{n}\varphi_4)\rangle\rangle$$

$$\langle\langle 2\rangle\rangle_{m,-m}\langle\langle 2\rangle\rangle_{n,-n} = \langle\langle \cos\mathrm{m}(\varphi_1 - \varphi_3)\rangle\rangle\langle\langle \cos\mathrm{n}(\varphi_2 - \varphi_4)\rangle\rangle$$

$$SC(m,n)_A = \langle\langle 4\rangle\rangle_{m,n,-m,-n} - \langle\langle 2\rangle\rangle_{m,-m}\langle\langle 2\rangle\rangle_{n,-n}$$



B.

$$\langle\langle 4\rangle\rangle_{m,n,-n,-m} = \langle\langle \cos(\mathrm{m}\varphi_1 + \mathrm{n}\varphi_2 - \mathrm{n}\varphi_3 - \mathrm{m}\varphi_4)\rangle\rangle$$

$$\langle\langle 2\rangle\rangle_{n,-n}\langle\langle 2\rangle\rangle_{m,-m} = \langle\langle \cos\mathrm{n}(\varphi_2 - \varphi_3)\rangle\rangle\langle\langle \cos\mathrm{m}(\varphi_1 - \varphi_4)\rangle\rangle$$

$$SC(m,n)_B = \langle\langle 4\rangle\rangle_{m,n,-n,-m} - \langle\langle 2\rangle\rangle_{n,-n}\langle\langle 2\rangle\rangle_{m,-m}$$

- $SC(m,n)_A$ and $SC(m,n)_B$ are then combined together