Studying flow and nonflow in small systems with AMPT and TMC

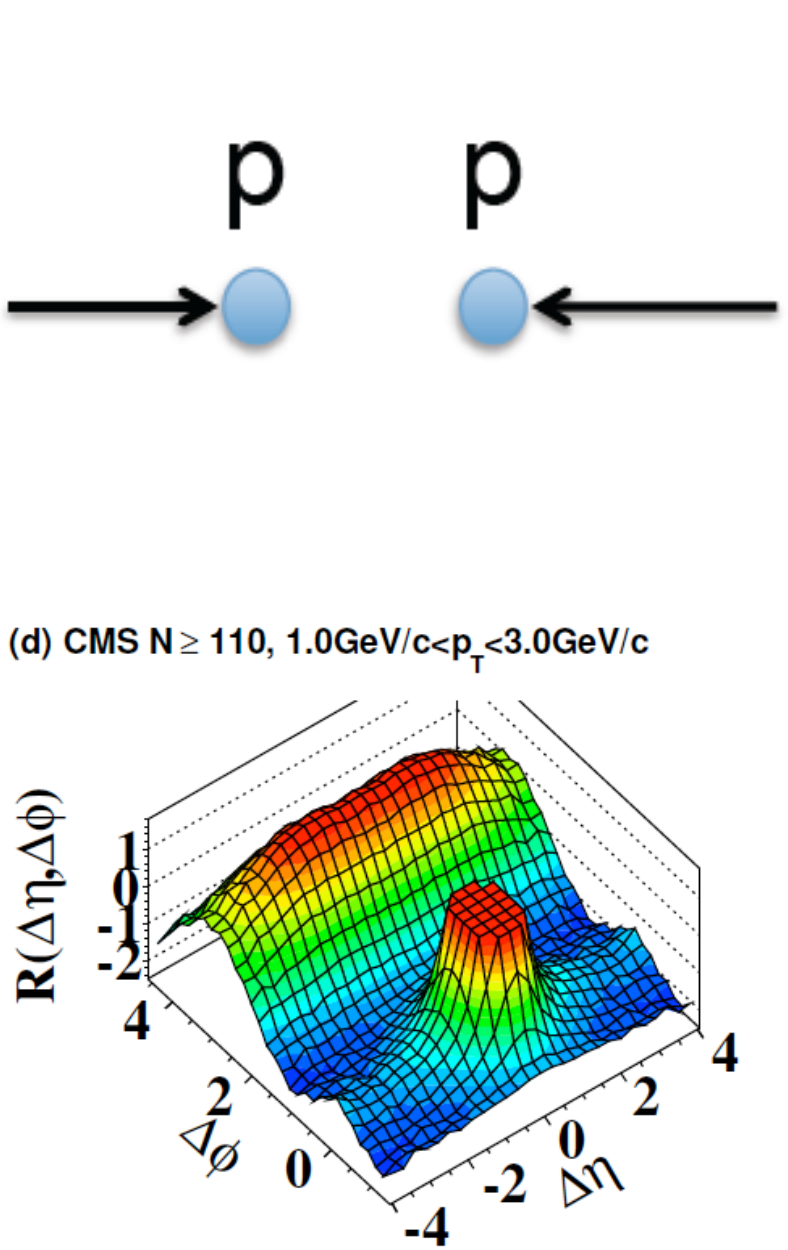
Guo-Liang Ma
（马国亮）

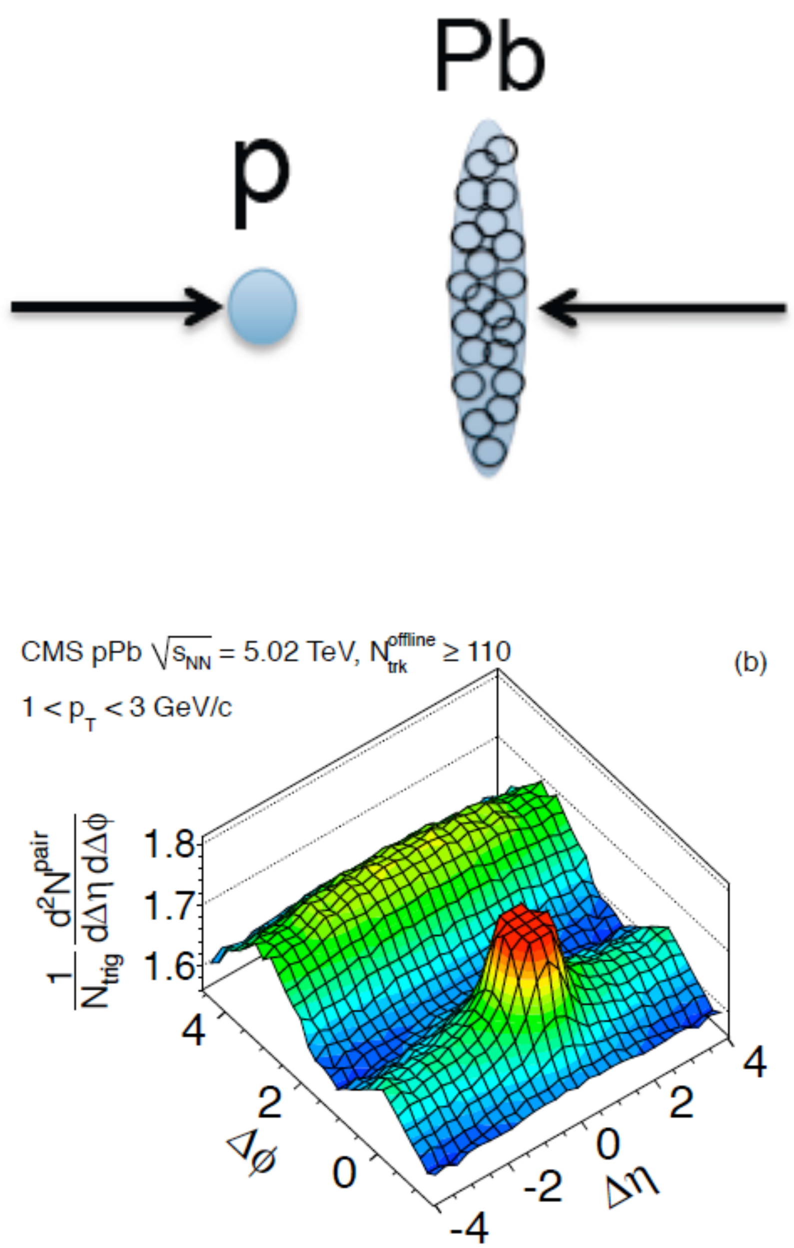
中国科学院上海应用物理研究所
SINAP
Shanghai Institute of Applied Physics, Chinese Academy of Sciences

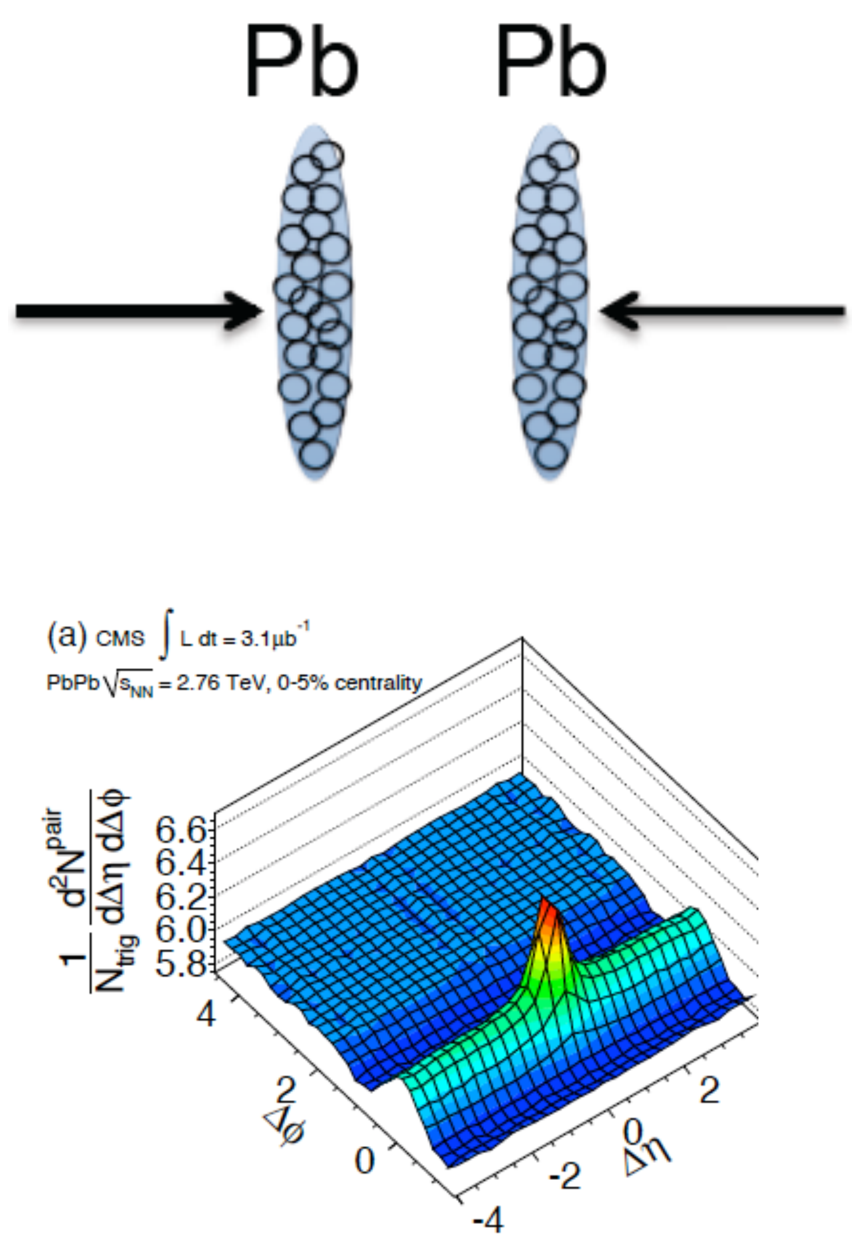
Outline

- **Flow measurements in small systems**
- **Numerical results based on AMPT**
- **Analytical results based on TMC⊕v2**
- **Summary**

Long-range correlation in p+p, p+Pb, and Pb+Pb

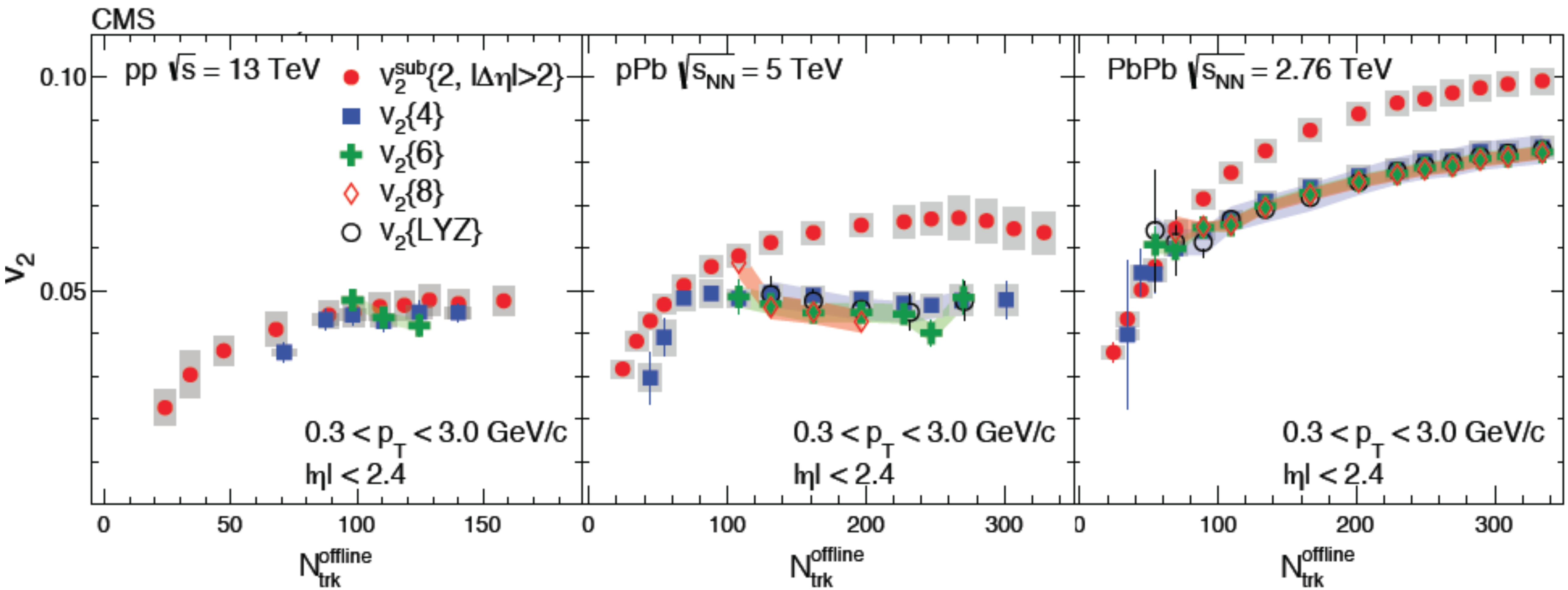






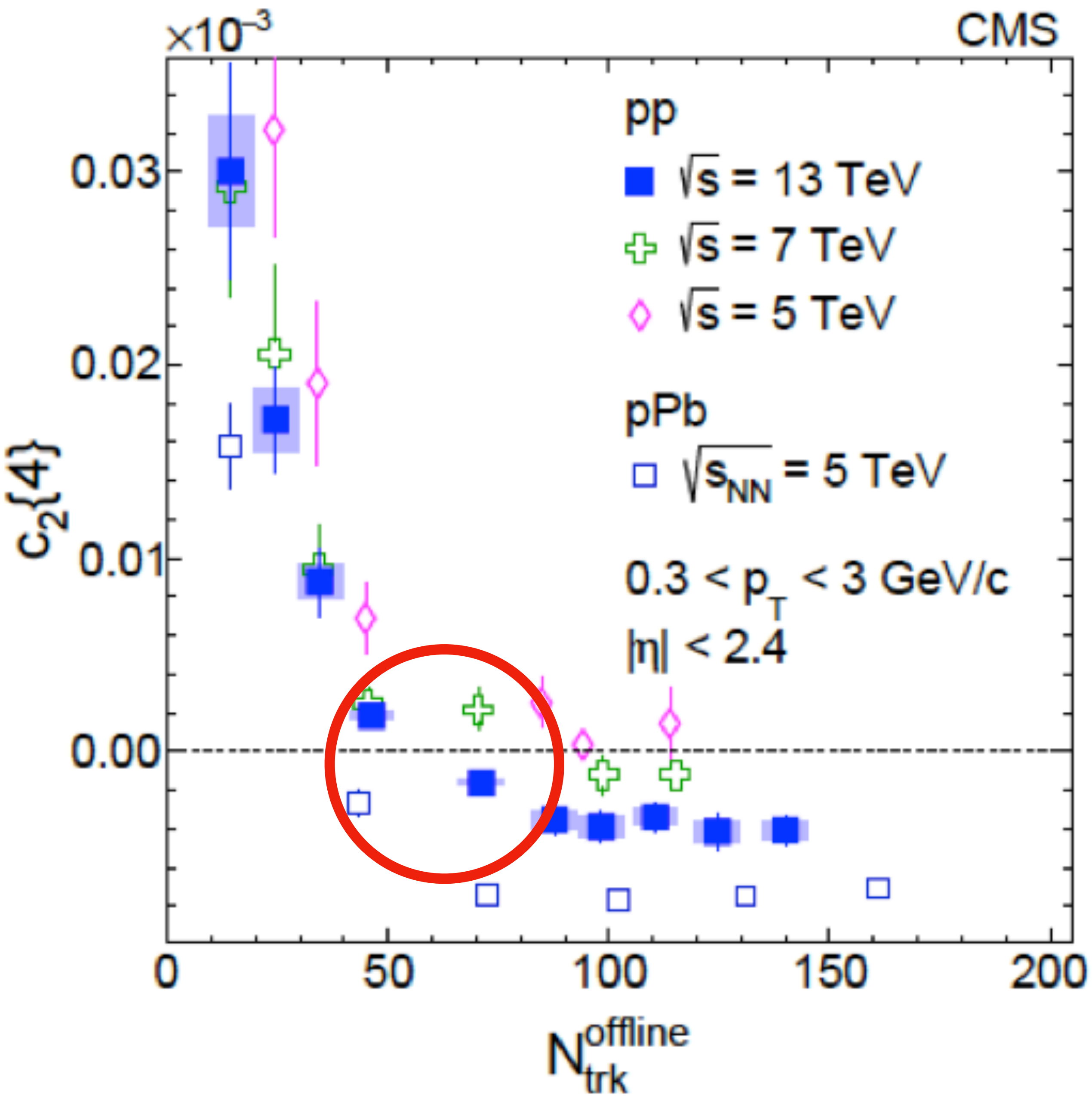
- Similar long-range correlations in p+p, p+Pb and Pb+Pb

Cumulant v2 in p+p, p+Pb, and Pb+Pb



- Multi-particle cumulant v2 are less than 2-particle v2=> **Multi-particle cumulants suppress non-flow**
- Similar v2 for 4,6,8-particle cumulants=> **multi-particle correlation**

Sign change of c2{4} in small systems



$$\langle\langle 2\rangle\rangle \equiv \langle\langle e^{in(\phi_1-\phi_2)}\rangle\rangle,$$

$$\langle\langle 4\rangle\rangle \equiv \langle\langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)}\rangle\rangle,$$

$$c_n\{4\} = \langle\langle 4\rangle\rangle - 2\times\langle\langle 2\rangle\rangle^2,$$

$$v_n\{4\} = \sqrt[4]{-c_n\{4\}},$$

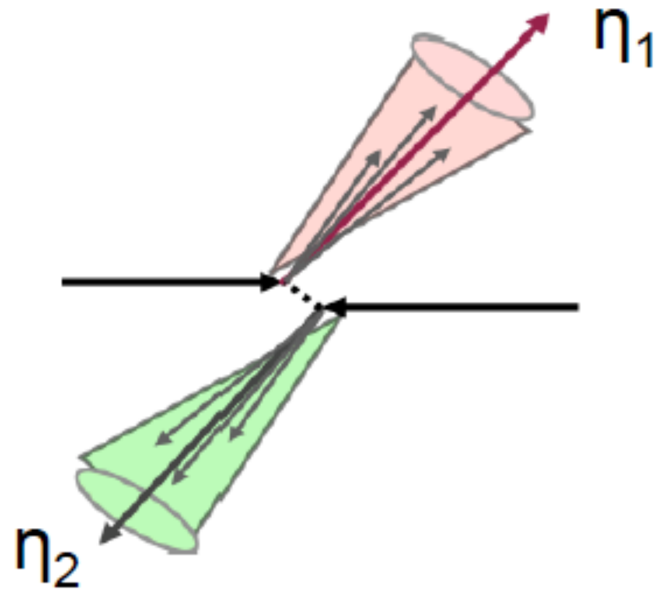
- c2{4} changes sign at a Ntrack!

=> the onset of collectivity in small system?

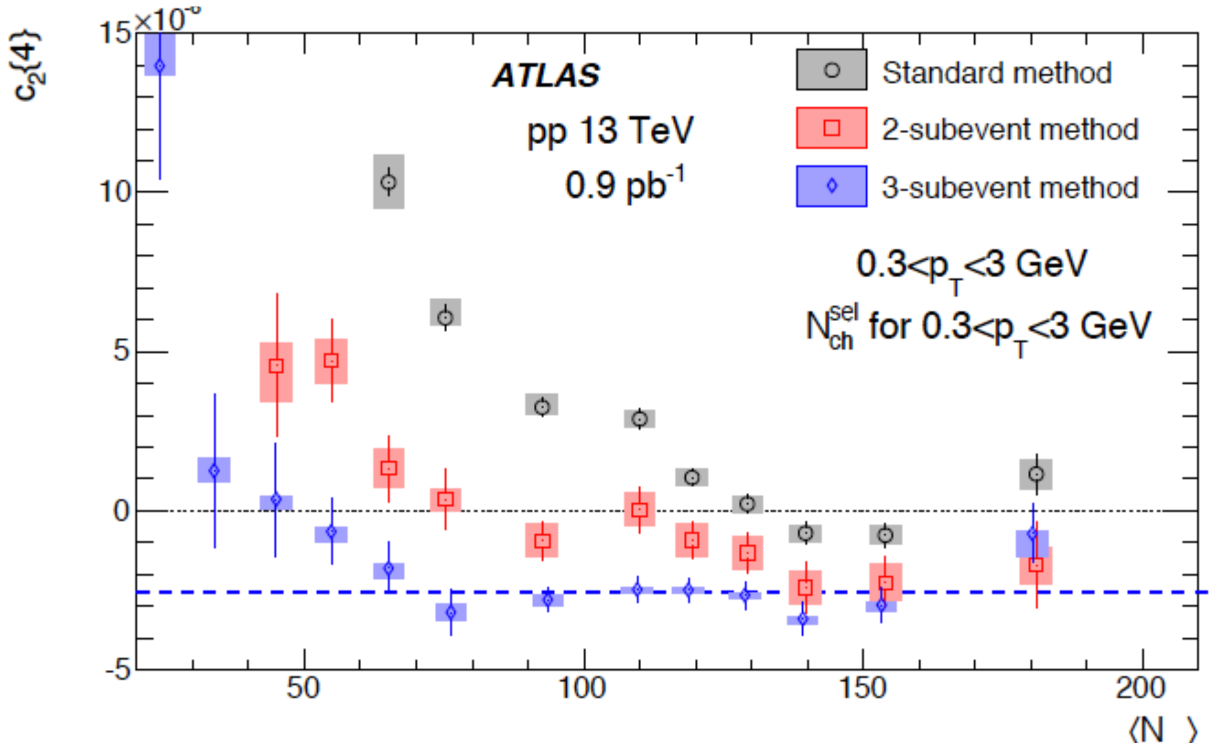
Subevent cumulant with less non-flow

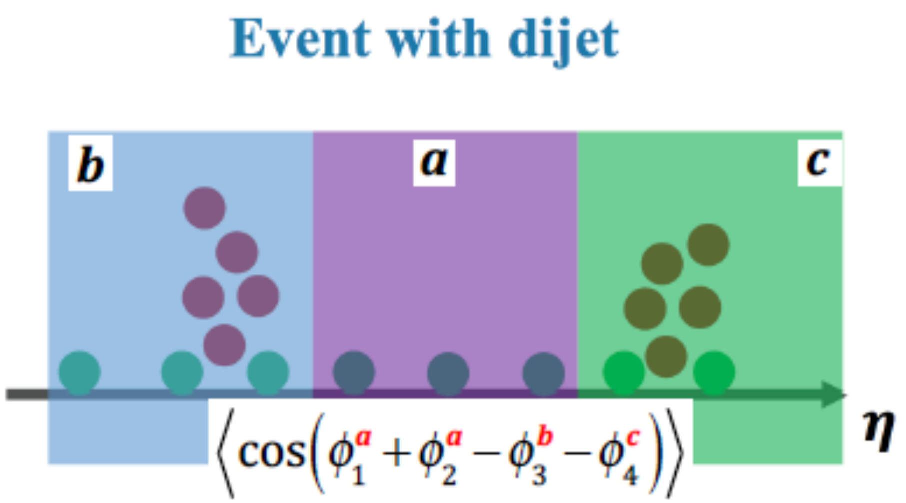
flow`=flow⊕non-flow

Jets and dijets contribute to non-flow, which are confined in one or two η regions



reduced by requiring simultaneous correlation between multiple η regions





3 sub-event

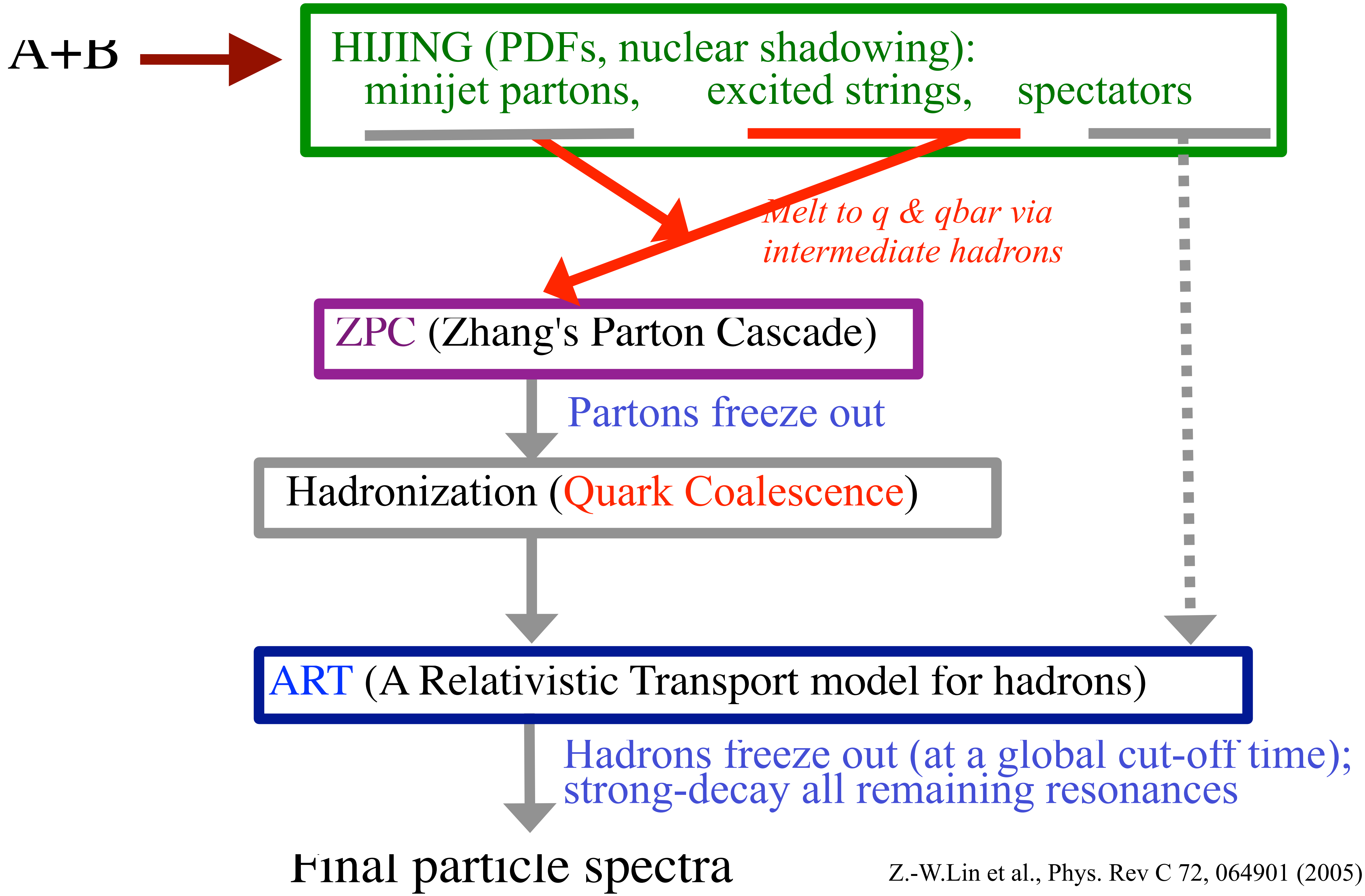
removes inter-jet correlations

- c2{4} changes sign at a smaller Ntrack with subevent method.

=> Non-flow effects need to be carefully suppressed (to see the onset).

- **Flow measurements in small systems**
- **Numerical results based on AMPT**
- **Analytical results based on TMC⊕v2**
- **Discussion && Summary**

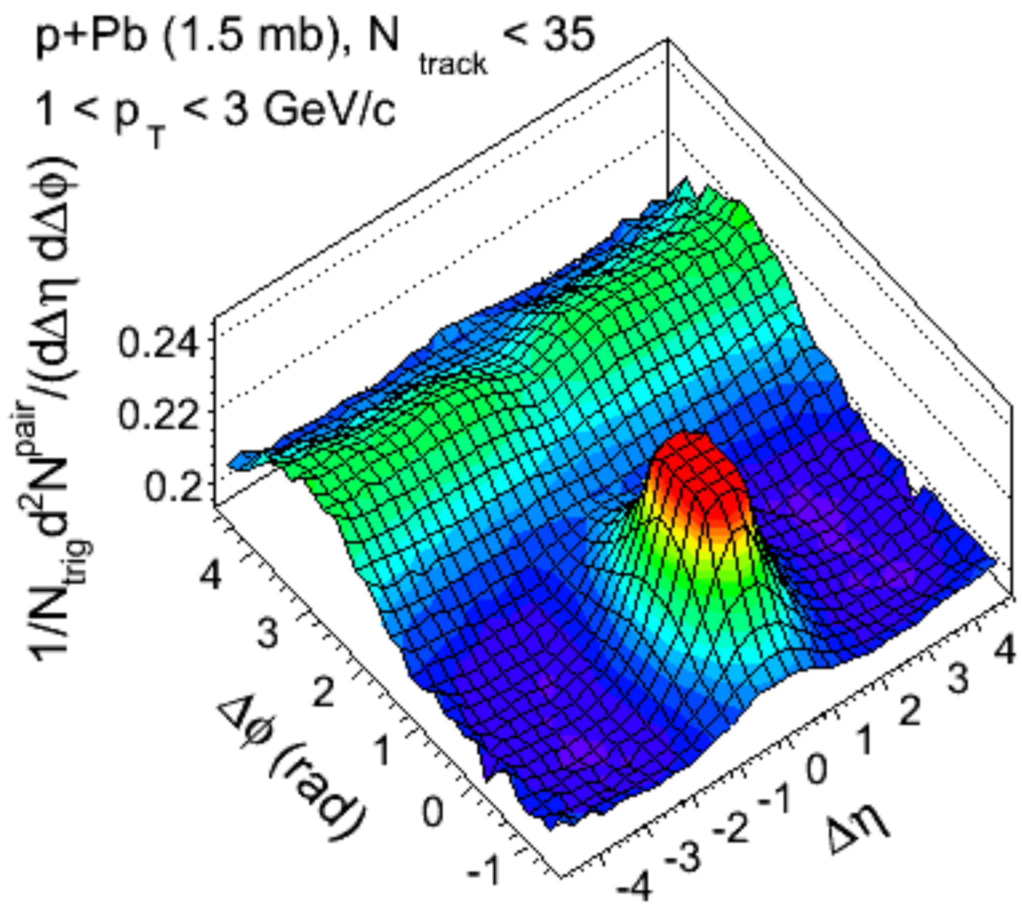
Structure of string-melting AMPT model



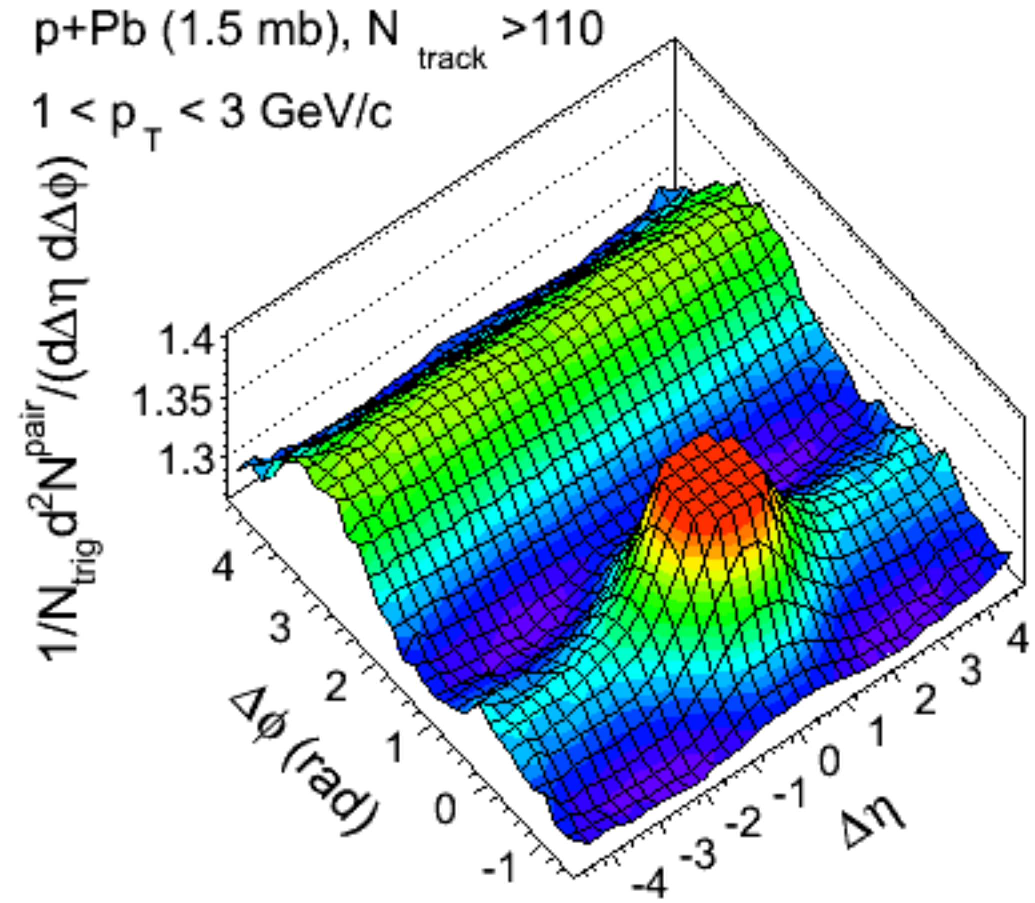
Z.-W.Lin et al., Phys. Rev C 72, 064901 (2005)

AMPT results on long-range correlation in p+Pb

G.-L. Ma and A. Bzdak, PLB 739, 209 (2014)



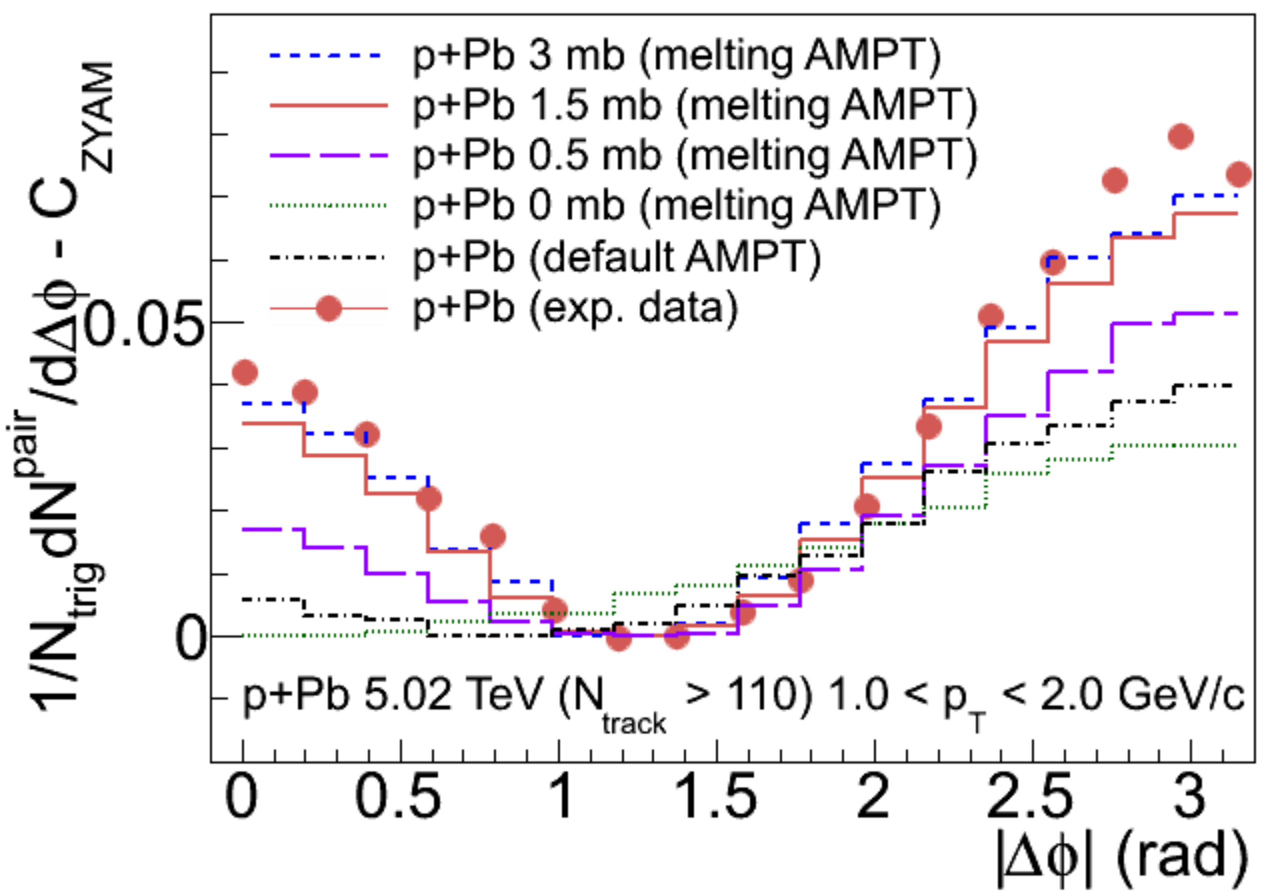
- No long-range correlation in low-multiplicity p+Pb.



- Clear long-range correlation in high-multiplicity p+Pb.

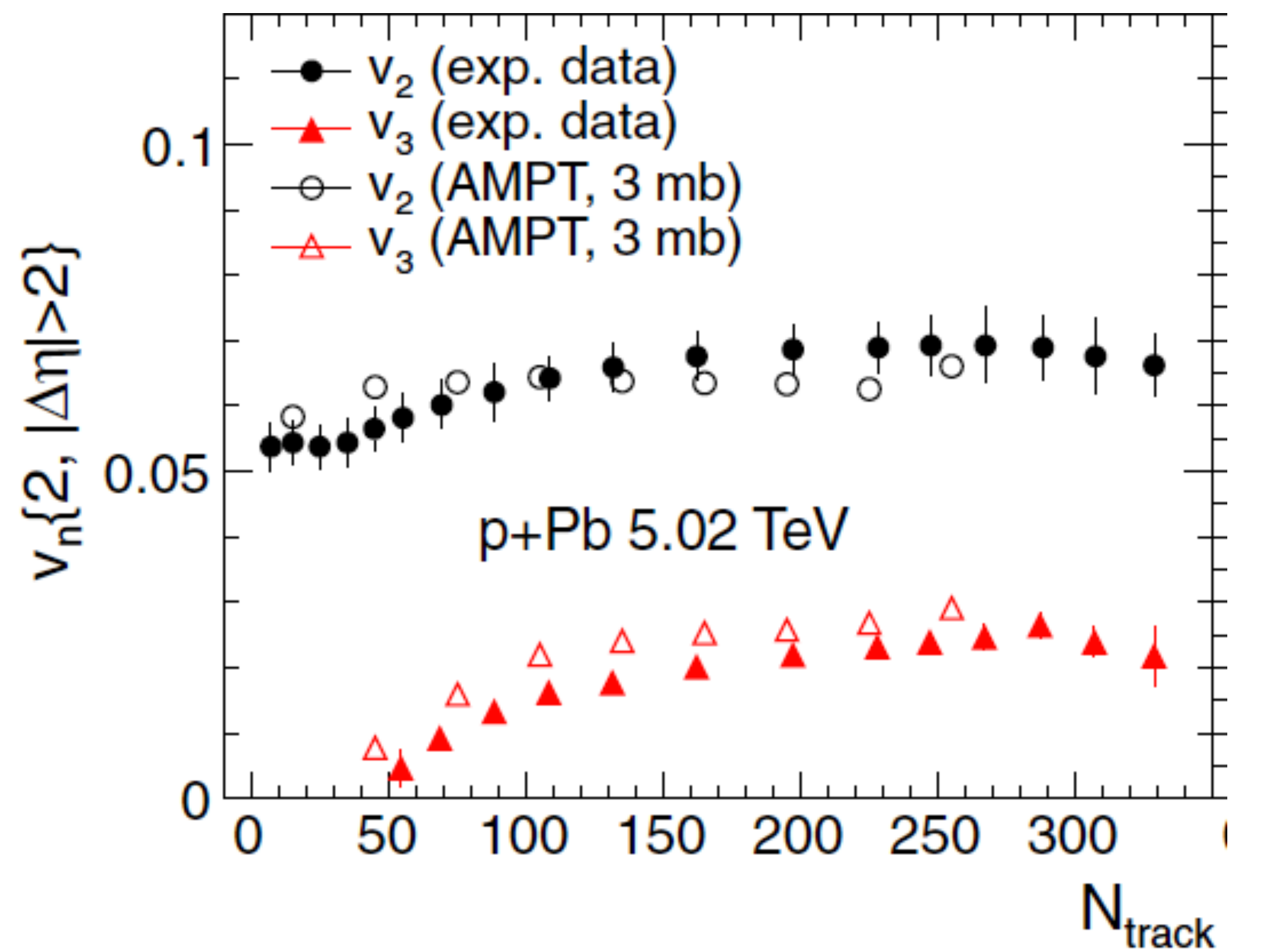
AMPT results on long-range correlation and v_n

G.-L. Ma and A. Bzdak, Phys.Lett. B 739 (2014) 209.



- The two-particle correlation in p+Pb can be well described by σ=1.5-3 mb.
- The signal strength increases with σ and vanishes for σ = 0 mb.=>**Long-range correlation is produced by parton cascade.**

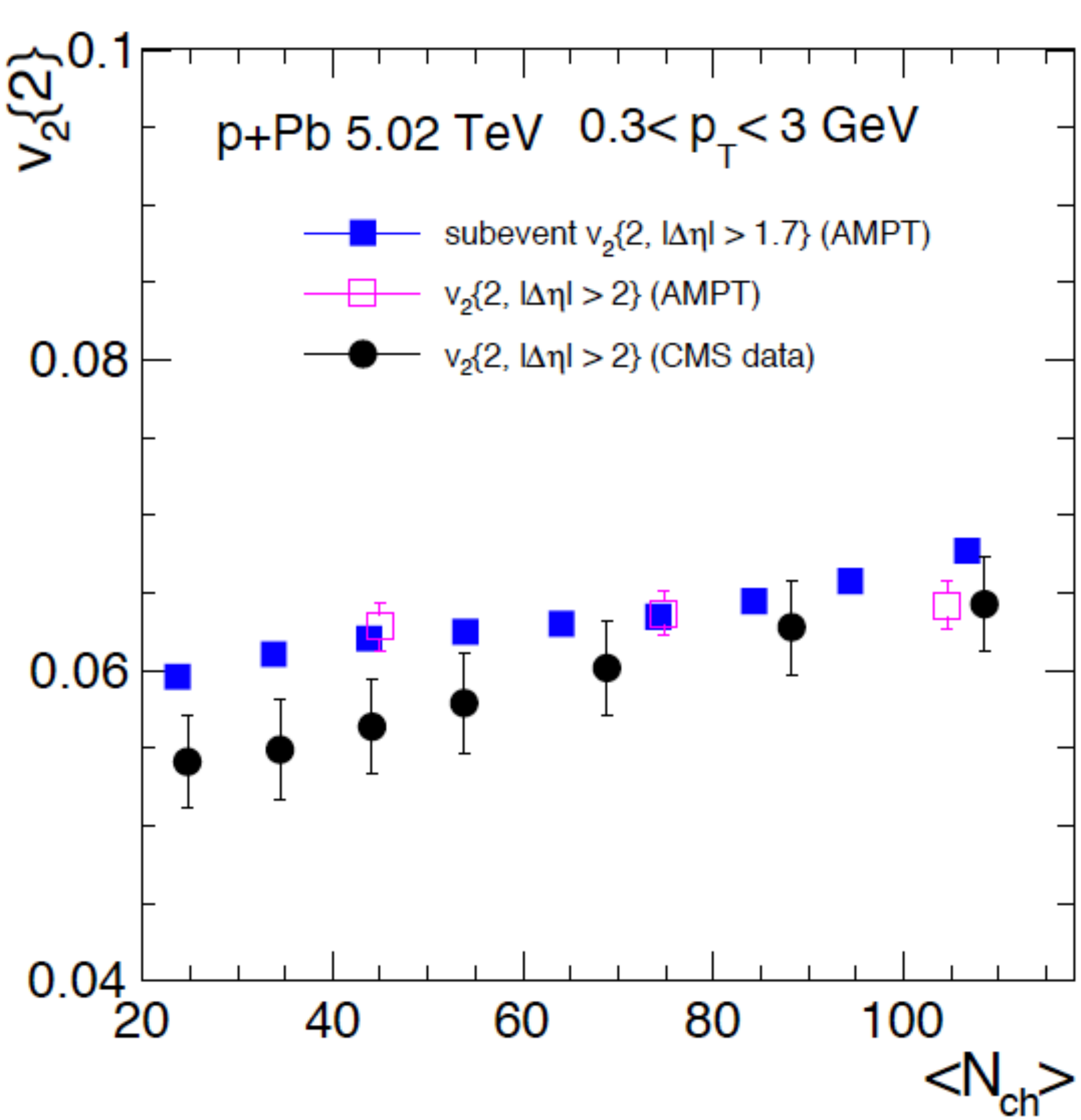
A. Bzdak and G.-L. Ma, Phys.Rev.Lett. 113, (2014) 252301.

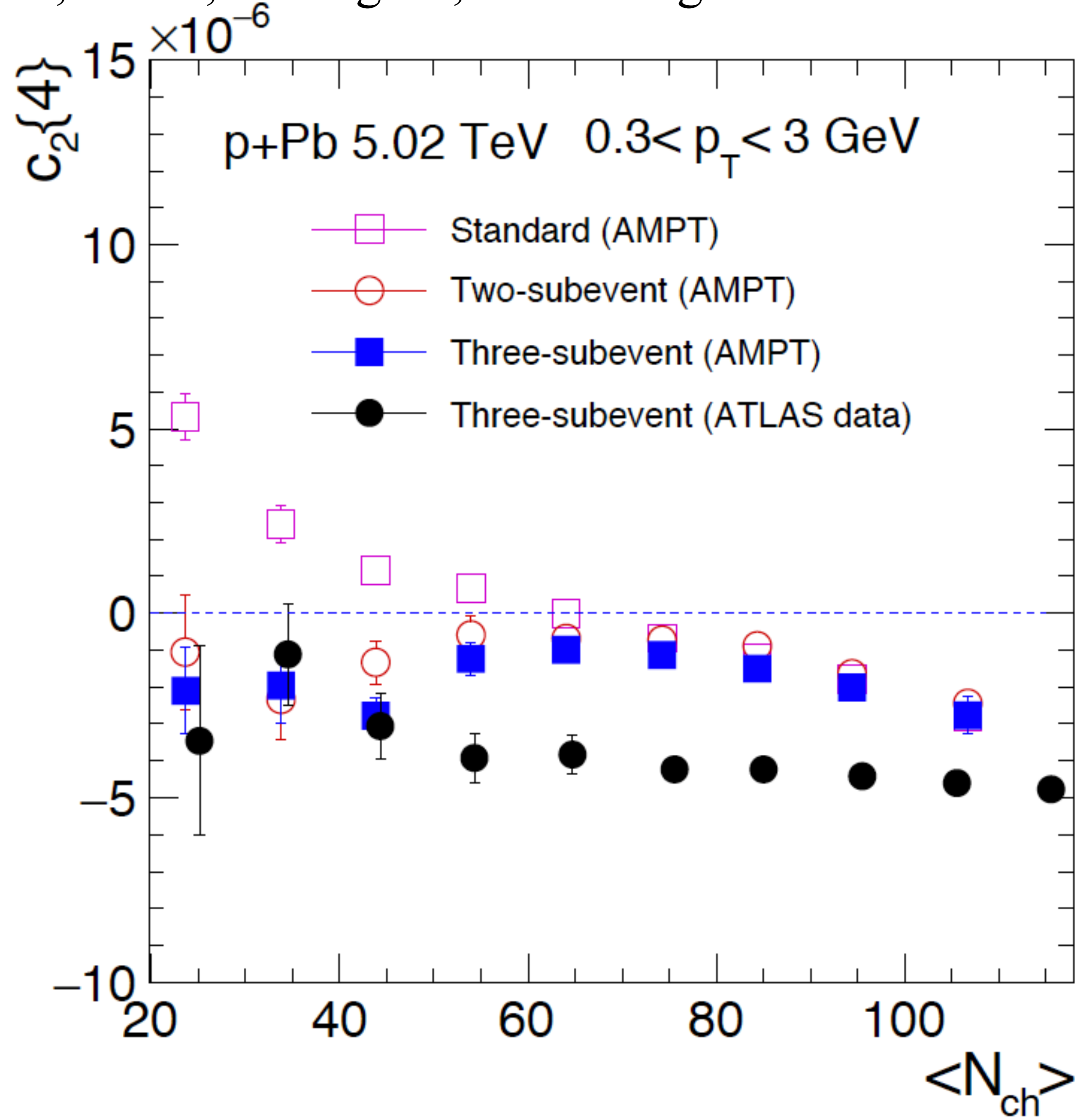


- For p+Pb, AMPT (σ=3 mb) reproduces the integrated two-particle v2 and v3.

AMPT results on subevent cumulants

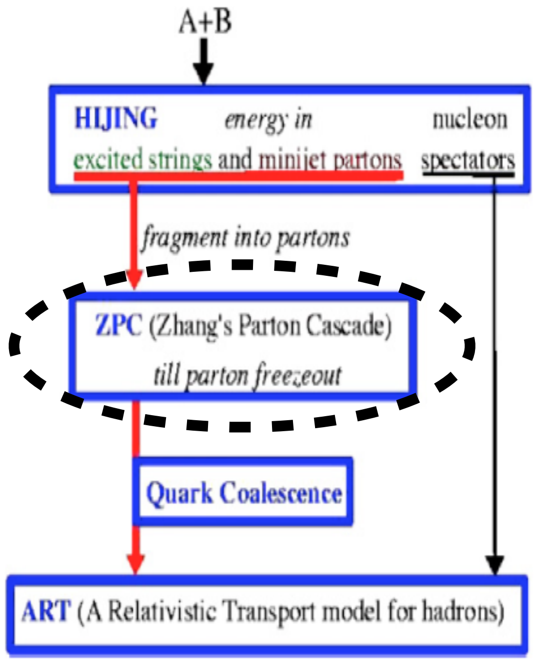
M. Nie,P. Huo, Jiarong Jia, Guo-Liang Ma arXiv:1802.00374

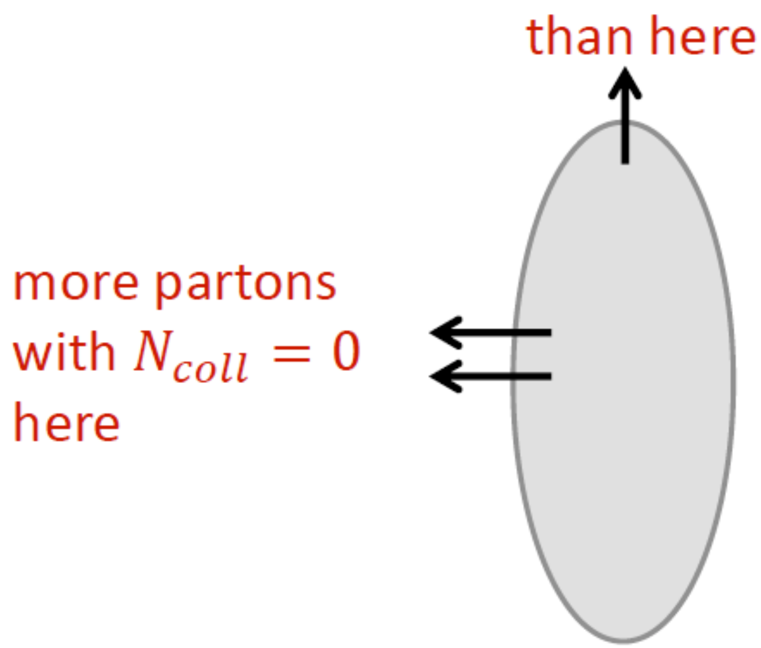




- For p+Pb, AMPT (3 mb) reproduces 2-particle v2{2}.
- AMPT (3 mb) underestimate 4-particle c2{4}.=>**smaller collectivity OR missing of non-gaussian flow fluctuations in AMPT**

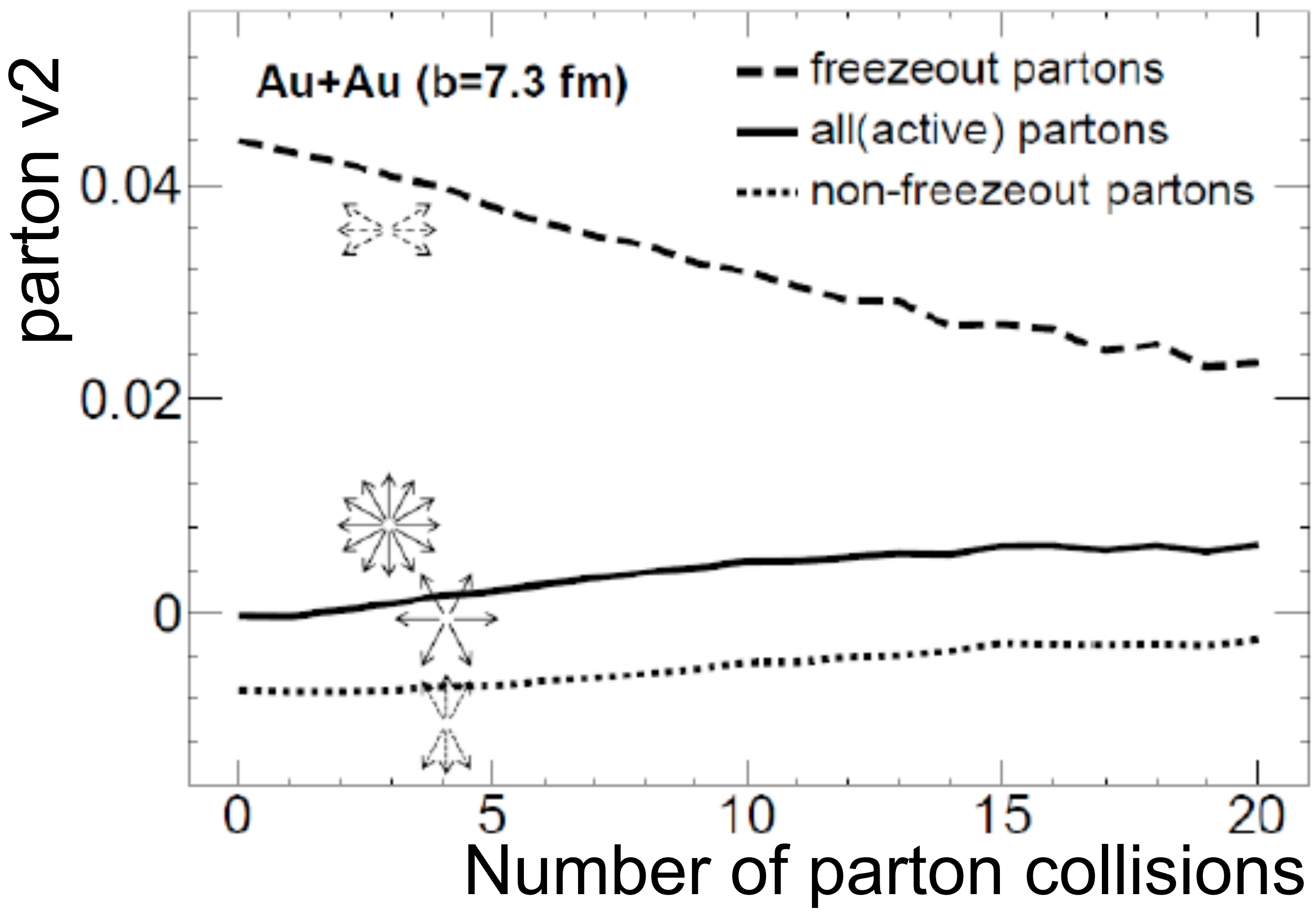
Escape mechanism





L. He, T. Edmonds, Zi-Wei Lin, F. Liu, D. Molnar, Fuqiang Wang, Phys.Lett. B753 (2016) 506.

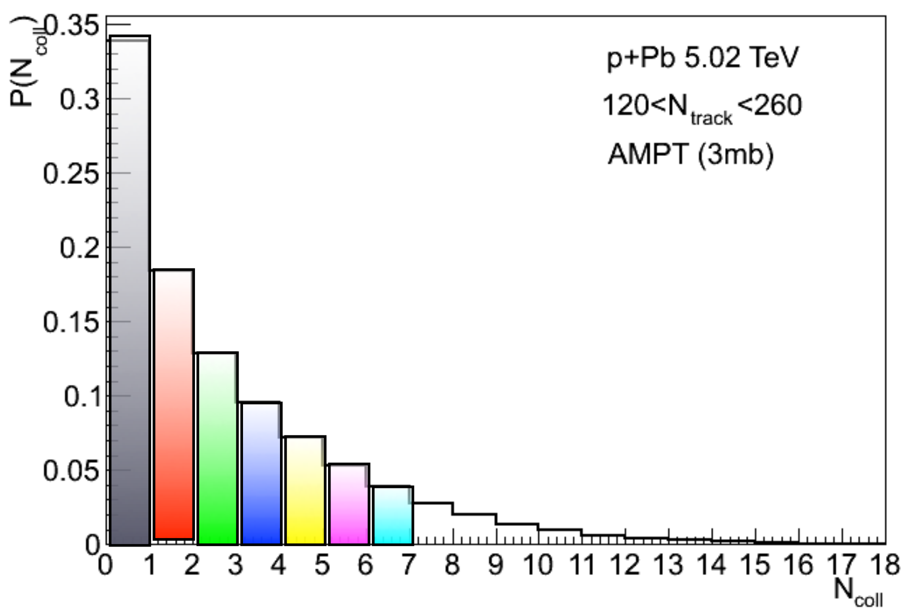
larger probability for partons to escape along the short axis



Features:

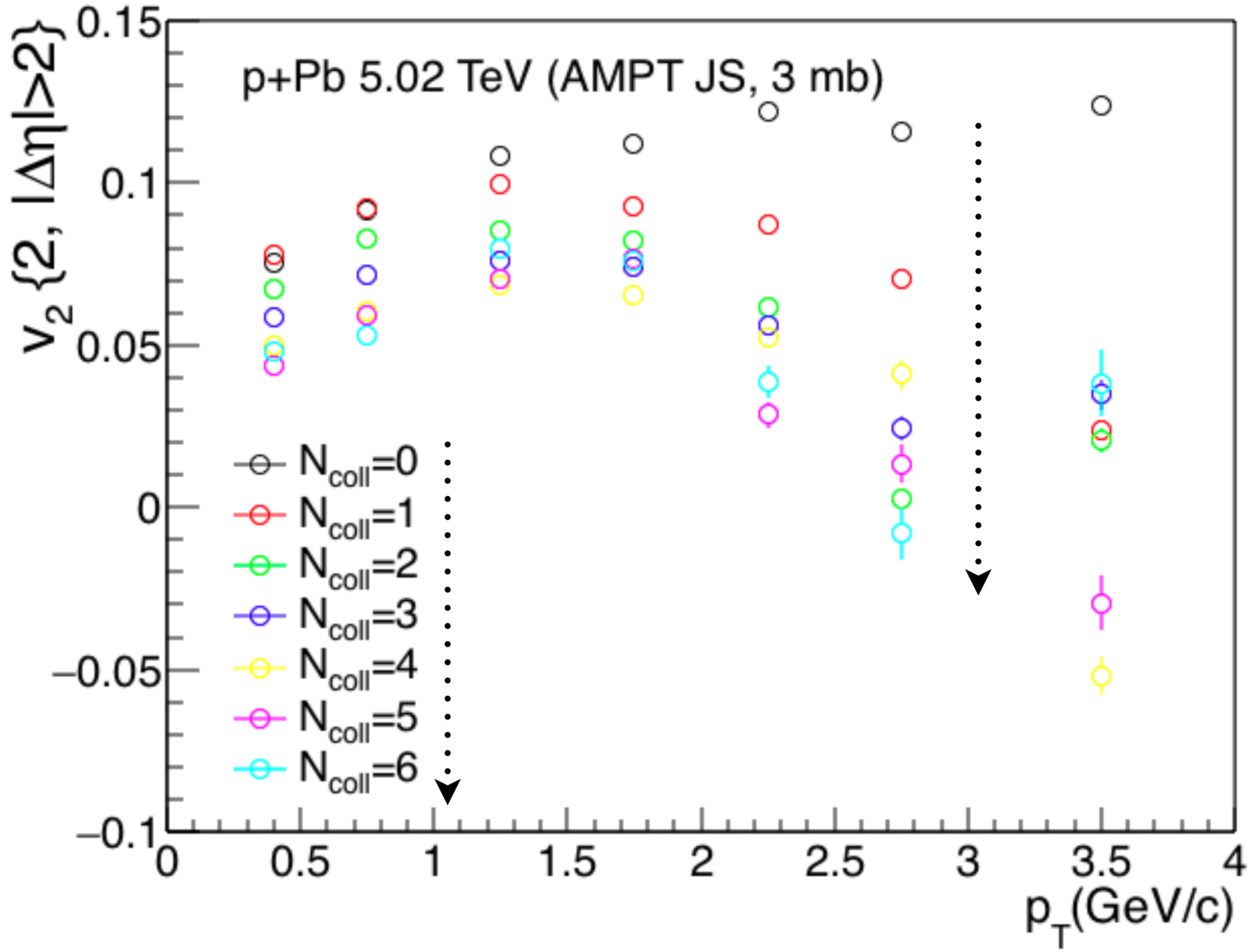
- Ncoll=0 partons freeze out with large positive v2
- Remaining partons go from negative v2 to small v2 after collisions.
- The escape contribution to total v2 is large (~70%) in mid-central Au+Au.
- Larger for d+Au (~90%).

Final partons’ v2 with different Ncoll



partons = Ncoll=0 + Ncoll=1 + Ncoll=2 + Ncoll=3 + Ncoll=4 + Ncoll=5 + Ncoll=6 + ...

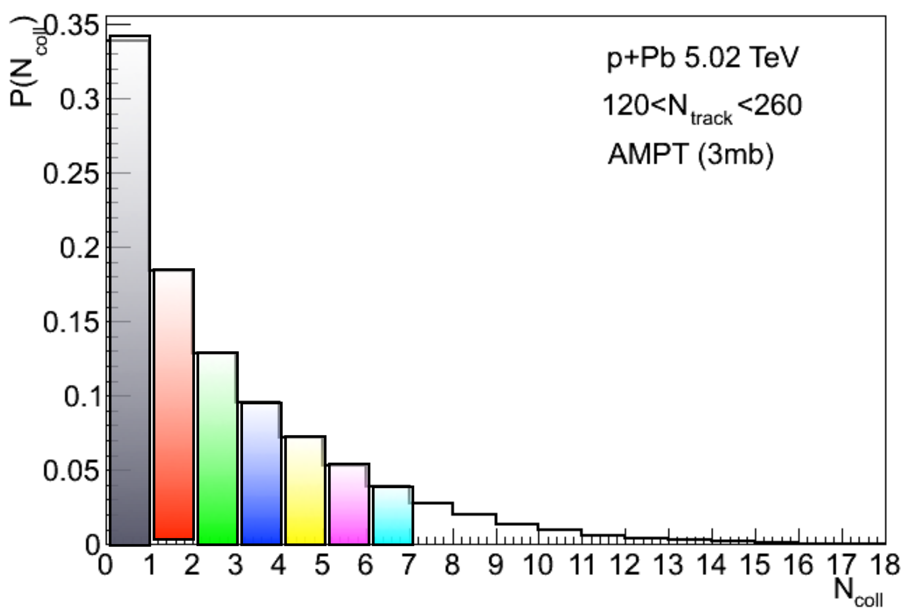
Final freezeout partons



- Final partons’ v2 decreases with Ncoll.

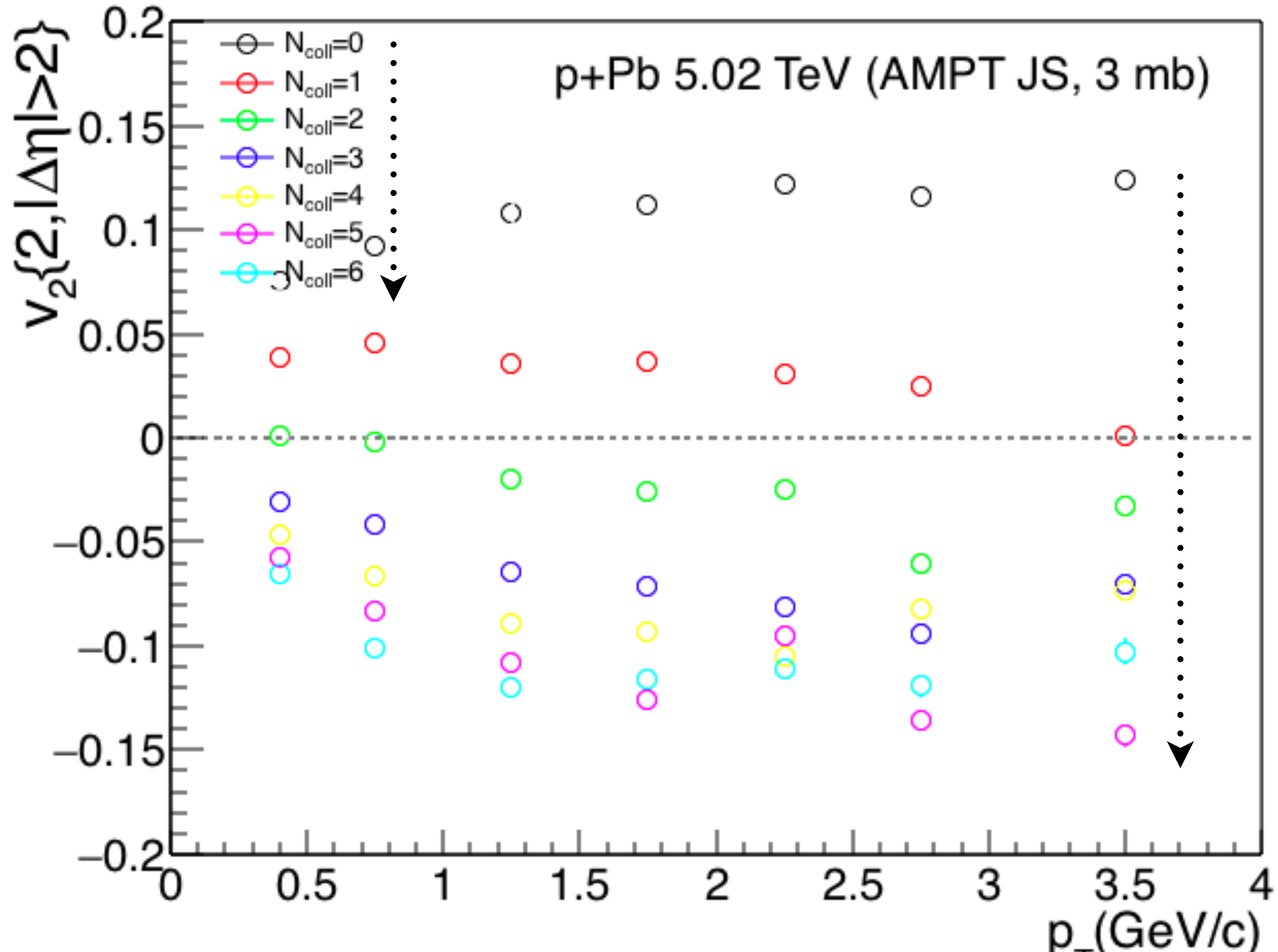
Guo-Liang Ma, Adam Bzdak, Nucl. Phys. A 956 (2016) 745–748

Initial partons' v2 with different Ncoll



partons = Ncoll=0 + Ncoll=1 + Ncoll=2 + Ncoll=3 + Ncoll=4 + Ncoll=5 + Ncoll=6 + ...

Initial partons

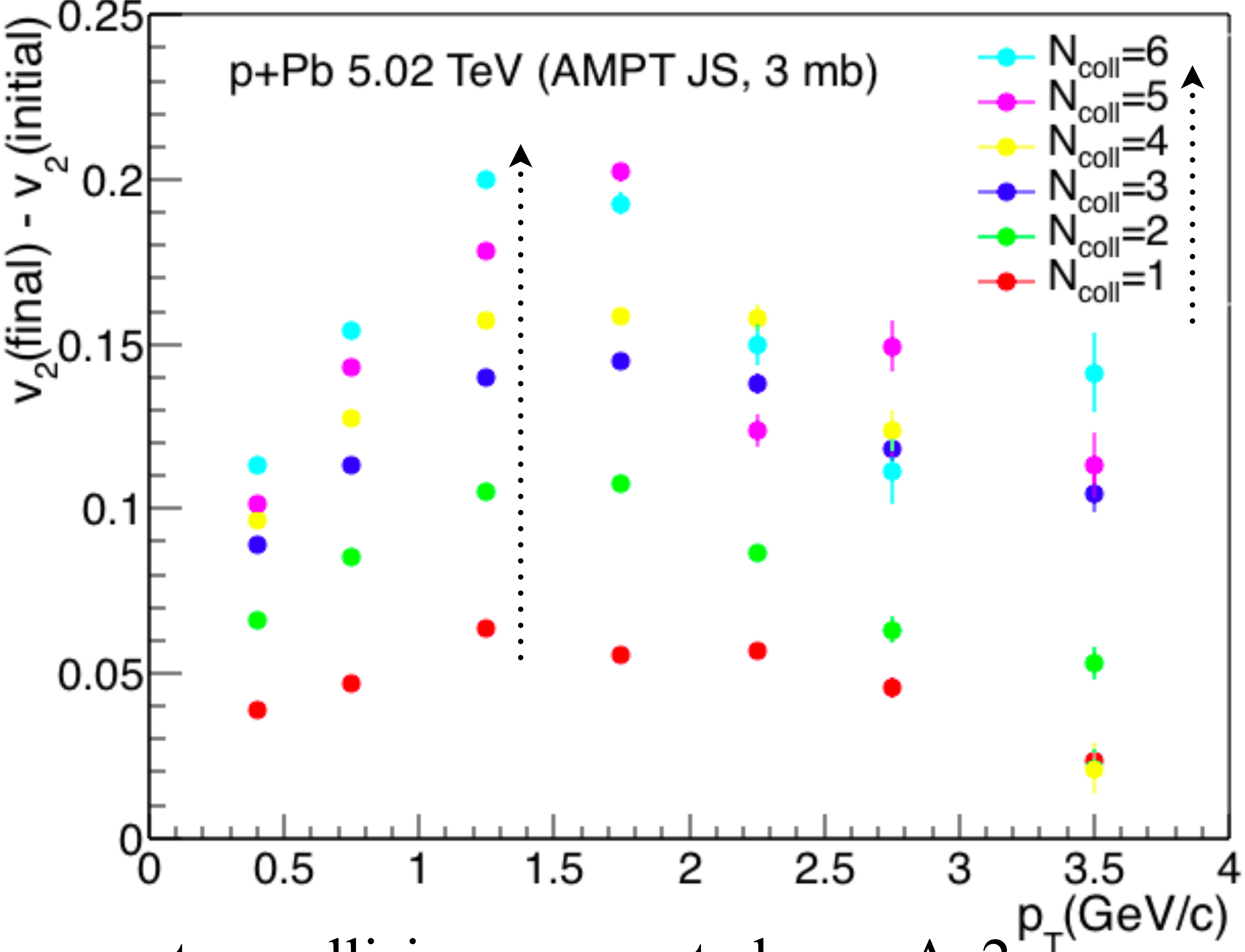


- In the initial state, v2 (small Ncoll)>0 and v2 (large Ncoll)<0 since the average v2 must be zero.

Guo-Liang Ma, Adam Bzdak, Nucl. Phys. A 956 (2016) 745–748

Partons' Δv2 with diff Ncoll

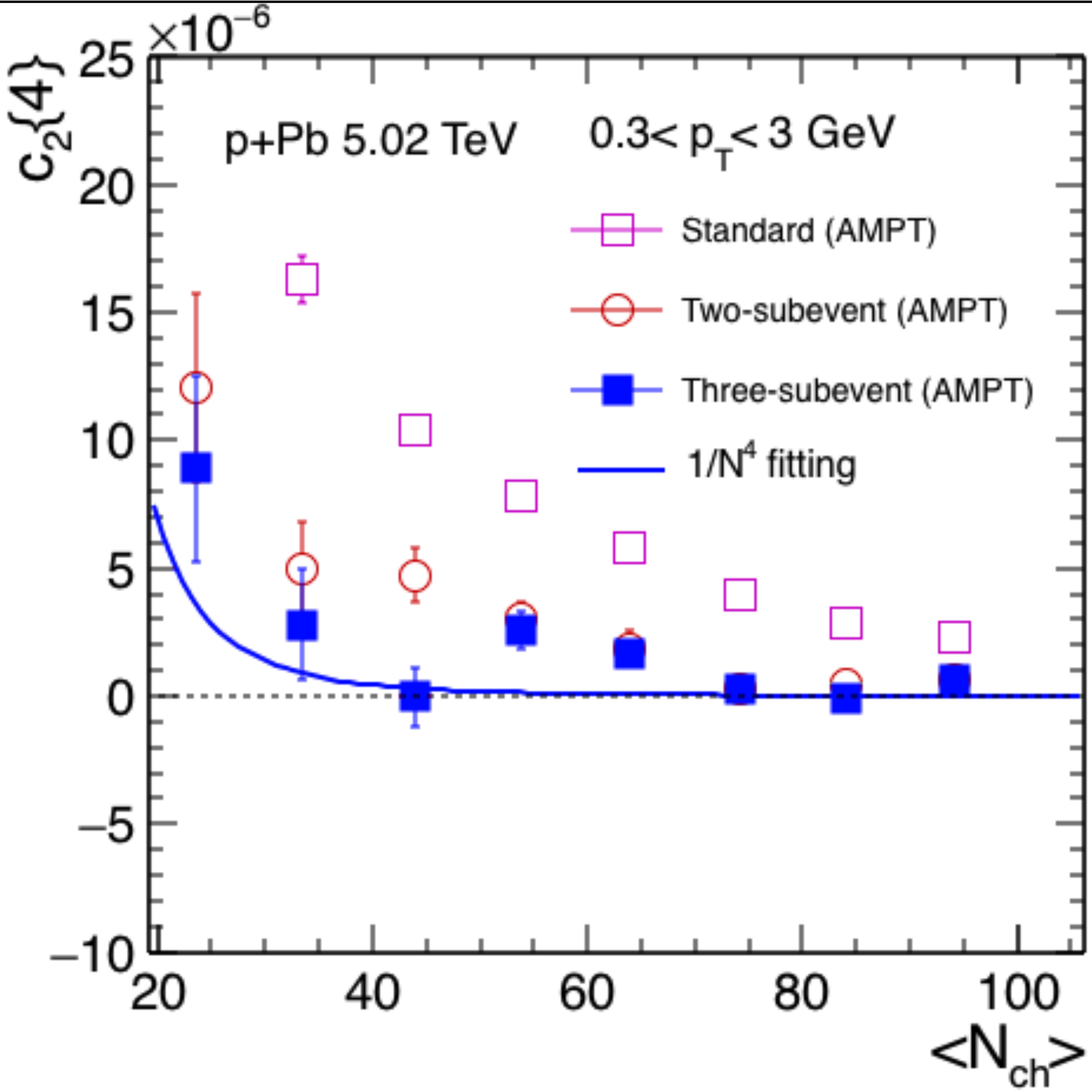
Guo-Liang Ma, Adam Bzdak, Nucl. Phys. A 956 (2016) 745.



- More parton collisions generate larger Δv2.
  =>Escape mechanism needs collisions.

$$\text{flow}` = \text{flow}(\text{escape} \oplus \text{hydro} \oplus \text{CGC}) \oplus \text{non-flow}(?)$$

Non-flow in AMPT model

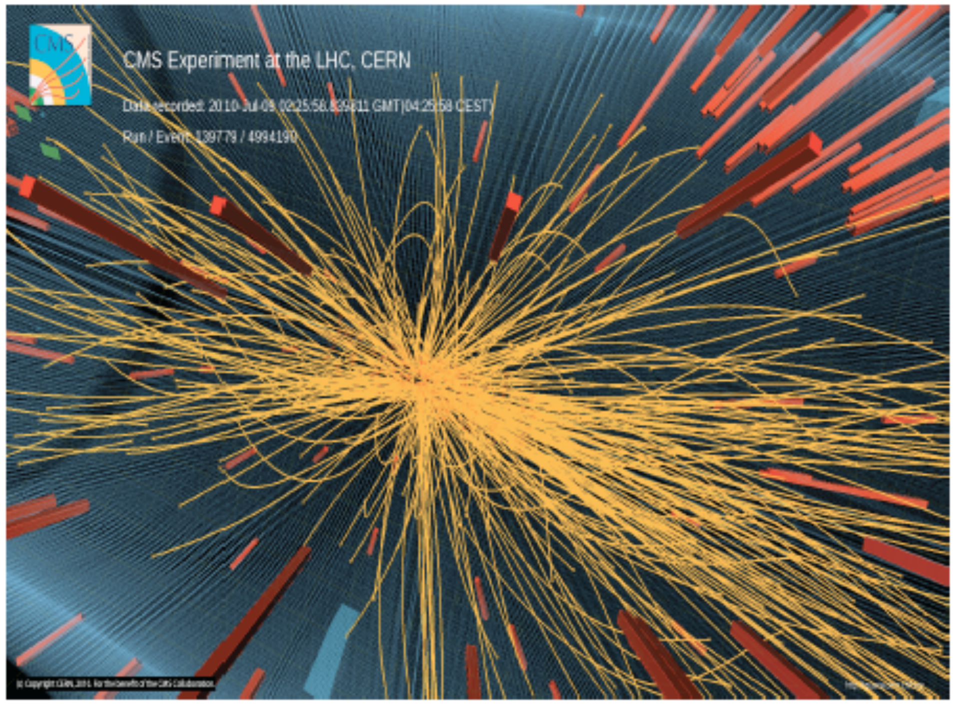


- By turning off parton cascade and hadron rescatterings, we can study non-flow due to jets , resonance decays, TMC, etc..
- Subevent cumulants suppress both jets and resonance decays.
- **The three-subevent c2{4} obeys ~1/N^4.<=TMC effect**

- **Flow measurements in small systems**
- **Numerical results based on AMPT**
- **Analytical results based on TMC⊕v2**
- **Discussion && Summary**

NOT FROM AMPT!

Particle production under TMC



- All produced N particle must obey the transverse momentum conservation(TMC).
- But ones only can experimentally measure part of them, i.e. k particles (k<N), due to the limits of acceptance and resolution.

N-particle momentum probability distribution:

$$f_N(\vec{p}_1, ..., \vec{p}_N) = \frac{1}{A}\delta^2(\vec{p}_1 + \ldots + \vec{p}_N) f(\vec{p}_1) \cdots f(\vec{p}_N),$$

$$A = \int_F \delta^2(\vec{p}_1 + \ldots + \vec{p}_N) f(\vec{p}_1) \cdots f(\vec{p}_N) d^2\vec{p}_1 \cdots d^2\vec{p}_N,$$

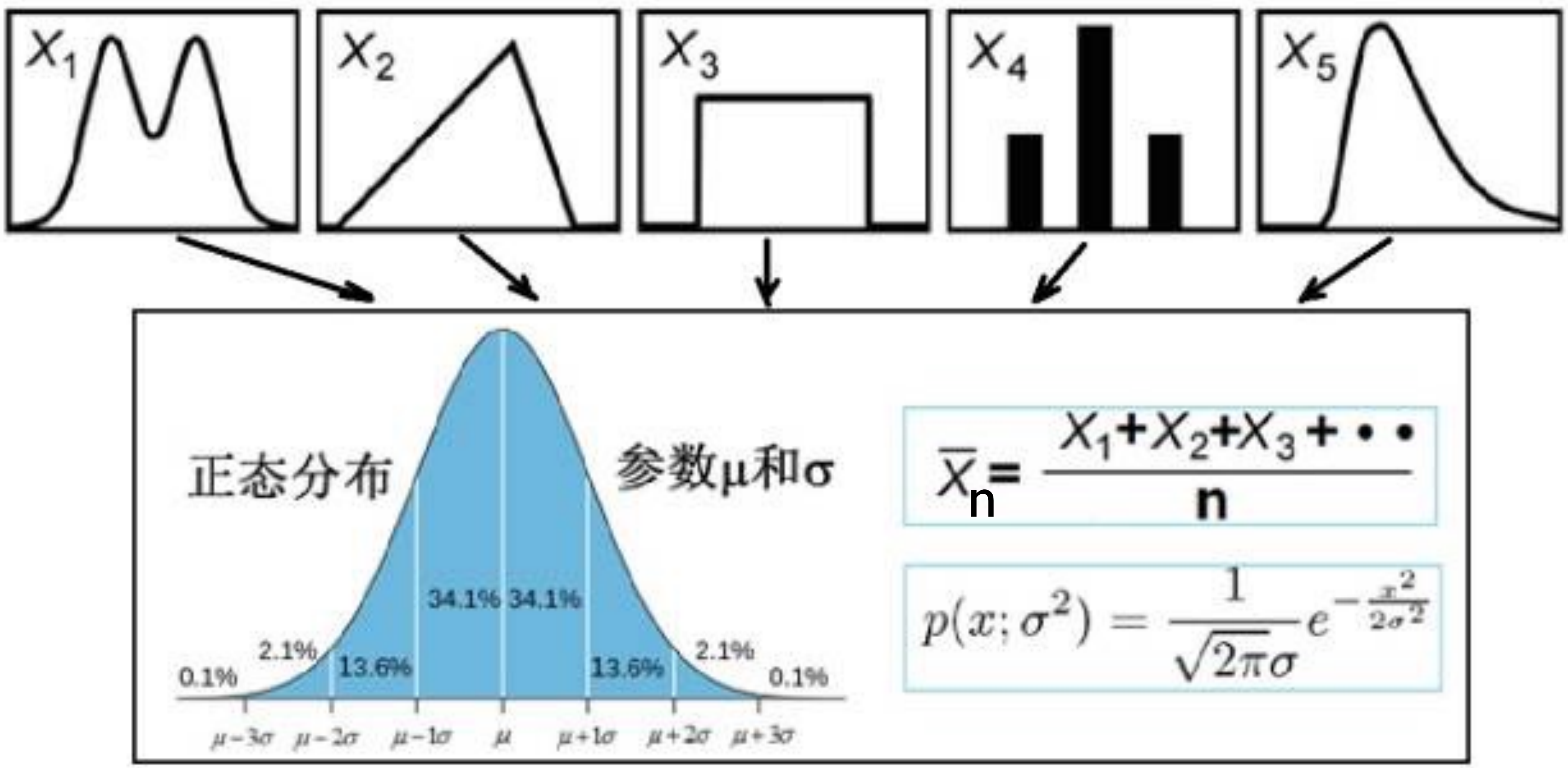
?

k-particle momentum probability distribution:

$$f_k(\vec{p}_1, ..., \vec{p}_k) = \frac{1}{A} f(\vec{p}_1) \cdots f(\vec{p}_k) \int_F \delta^2(\vec{p}_1 + \ldots + \vec{p}_N) f(\vec{p}_{k+1}) \cdots f(\vec{p}_N) d^2\vec{p}_{k+1} \cdots d^2\vec{p}_N$$

Central limit theorem

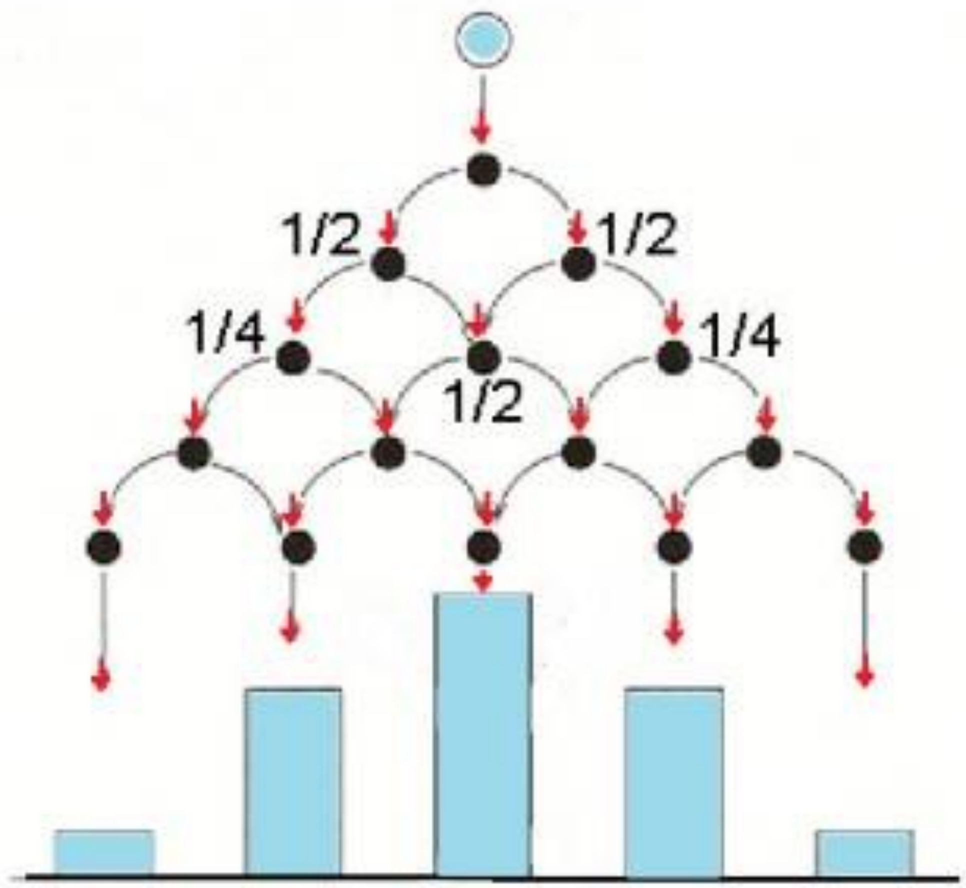
Why always normal distribution?

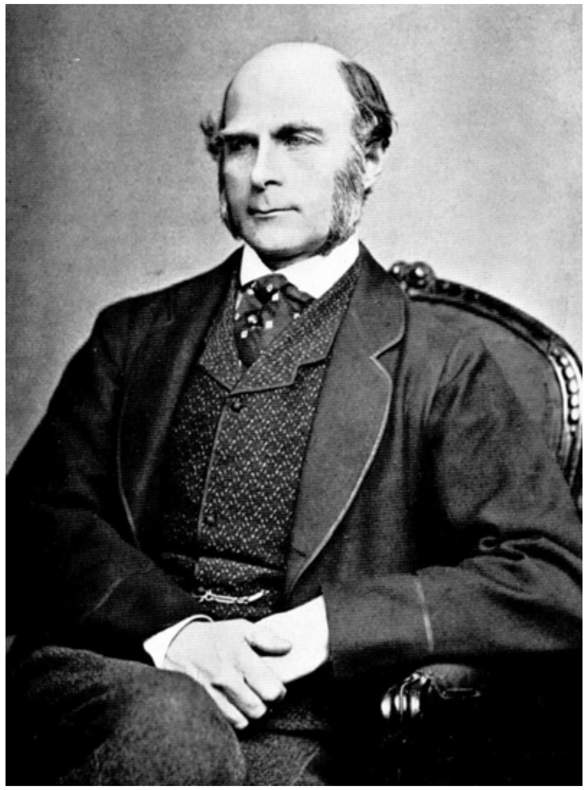


Abraham de Moivre
法国数学家**棣莫佛**
(1667-1754)

- For large enough n, the distribution of $\overline{X}_n$ is close to the normal distribution with mean μ and variance σ^2/n.

Central limit theorem

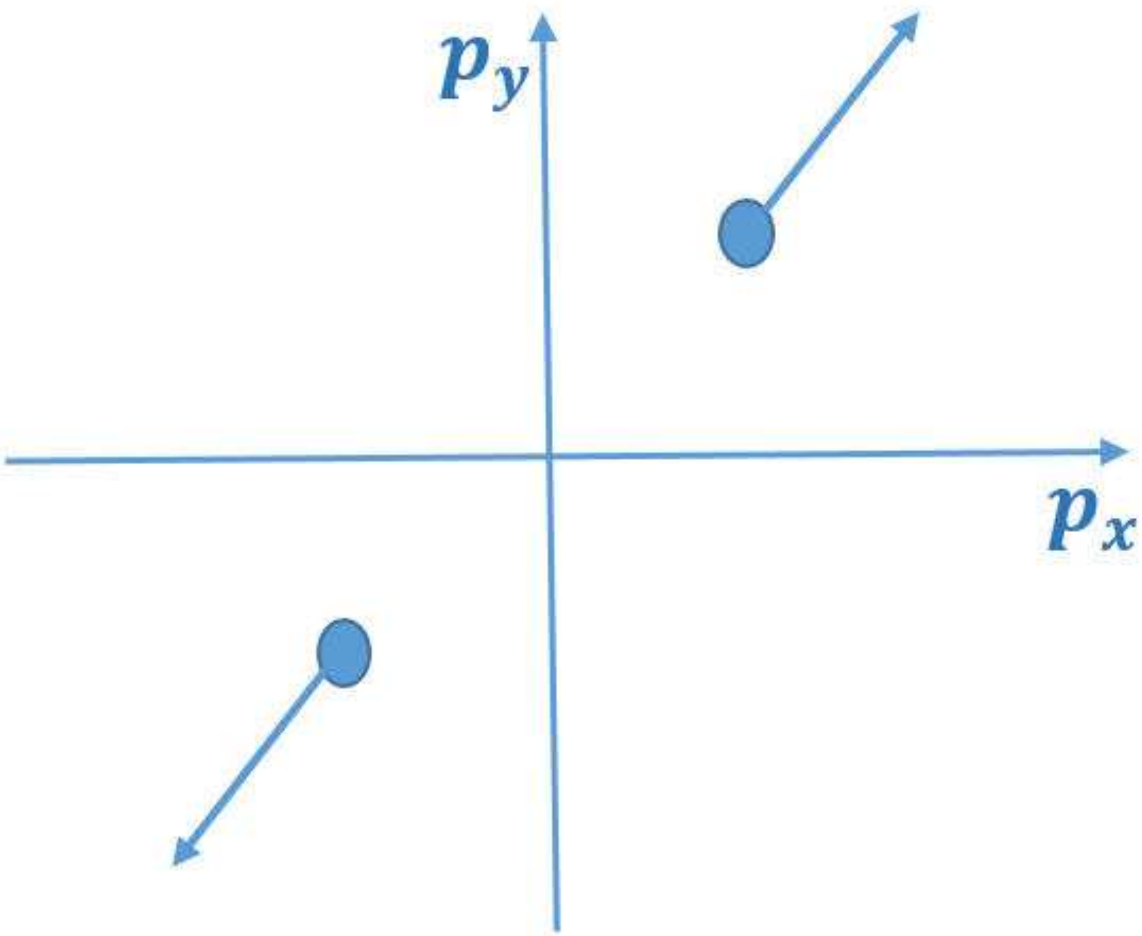


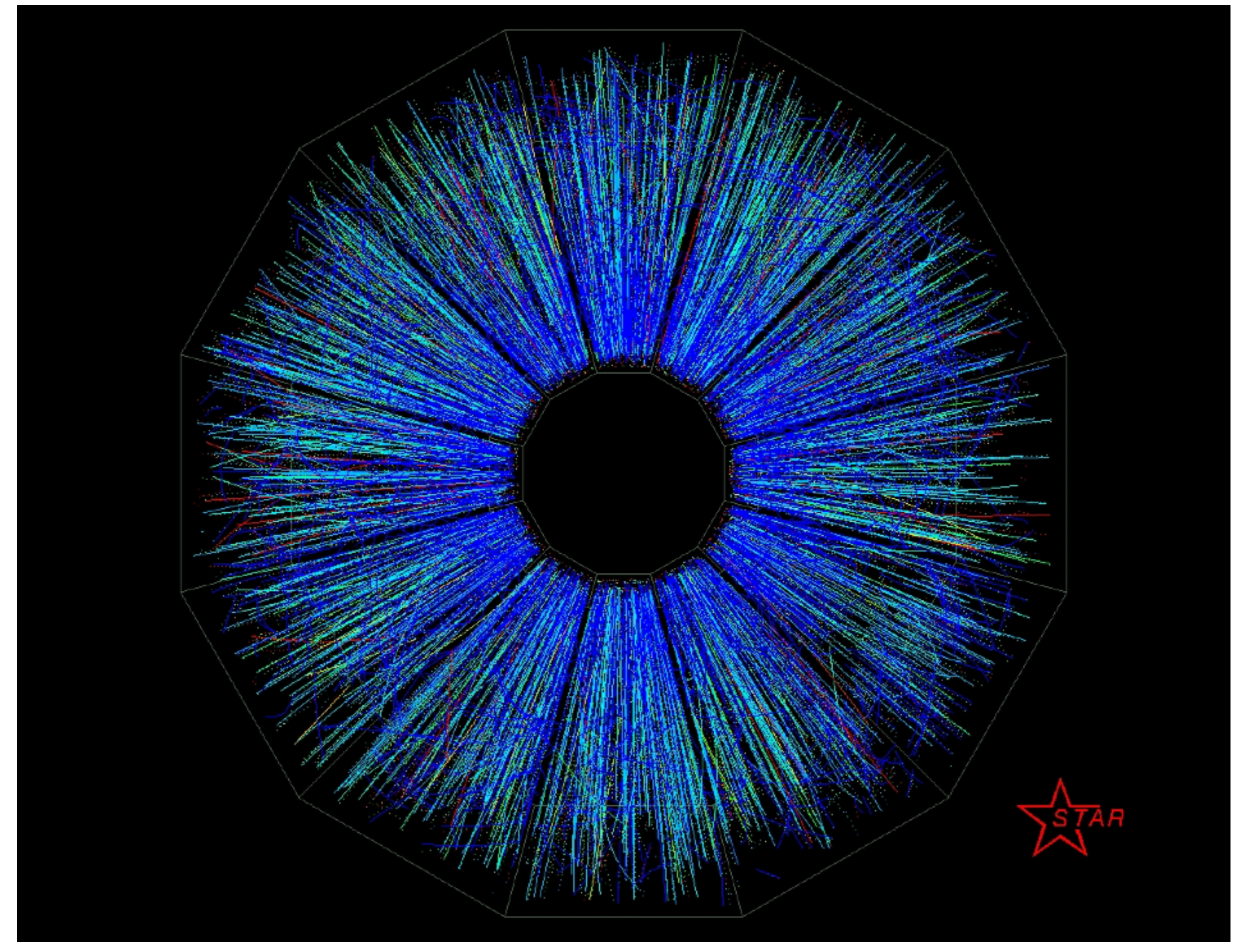


Sir Francis Galton
(1822-1911)

- The bean machine physically demonstrates the central limit theorem.
  =>Here a normal distribution is a limit response to many binomial distributions.

Multi-particle correlation due to TMC



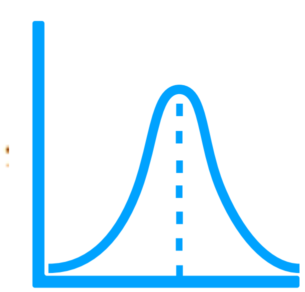


$$f_k(\vec{p}_1, ..., \vec{p}_k) = \frac{1}{A} f(\vec{p}_1) \cdots f(\vec{p}_k) \int_F \delta^2(\vec{p}_1 + \ldots + \vec{p}_N) f(\vec{p}_{k+1}) \cdots f(\vec{p}_N) d^2\vec{p}_{k+1} \cdots d^2\vec{p}_N$$

$<p>=0$

$\sigma^2=<p^2>$

$$f_k(\vec{p}_1, ..., \vec{p}_k) = f(\vec{p}_1) \cdots f(\vec{p}_k) \frac{N}{N-k} \exp\left(-\frac{(\vec{p}_1 + ... + \vec{p}_k)^2}{(N-k)\langle p^2 \rangle_F}\right)$$



c2{2} from TMC

$$c_2\{2\} = \left\langle e^{i2(\phi_1-\phi_2)} \right\rangle = \frac{\int_0^{2\pi} f_2(\vec{p}_1,\vec{p}_2) e^{i2(\phi_1-\phi_2)} d\phi_1 d\phi_2}{\int_0^{2\pi} f_2(\vec{p}_1,\vec{p}_2) d\phi_1 d\phi_2}$$

$$f_2(\vec{p}_1,\vec{p}_2) = f(\vec{p}_1) f(\vec{p}_2) \frac{N}{N-2} \exp\left(-\frac{p_1^2+p_2^2+2p_1p_2\cos(\phi_1-\phi_2)}{(N-2)\left\langle p^2\right\rangle_F}\right)$$

$$c_2\{2\}|_{p_1,p_2} = \frac{I_2(x)}{I_0(x)}, \quad x = \frac{2p_1p_2}{(N-2)\left\langle p^2\right\rangle_F}$$

($I_k(x)$ is the modied Bessel function of the the 1st kind.)

$$c_2\{2\}|_{p_1,p_2} \approx \frac{p_1^2 p_2^2}{2(N-2)^2 \left\langle p^2\right\rangle_F^2}, \quad \textit{if} \;\; p_1p_2 < \frac{1}{2}(N-2)\langle p^2\rangle_F .$$

c2{k} from TMC

$$c_2\{2\}|_{p_1,p_2} \approx \frac{p_1^2 p_2^2}{2(N-2)^2 \left\langle p^2 \right\rangle_F^2};$$

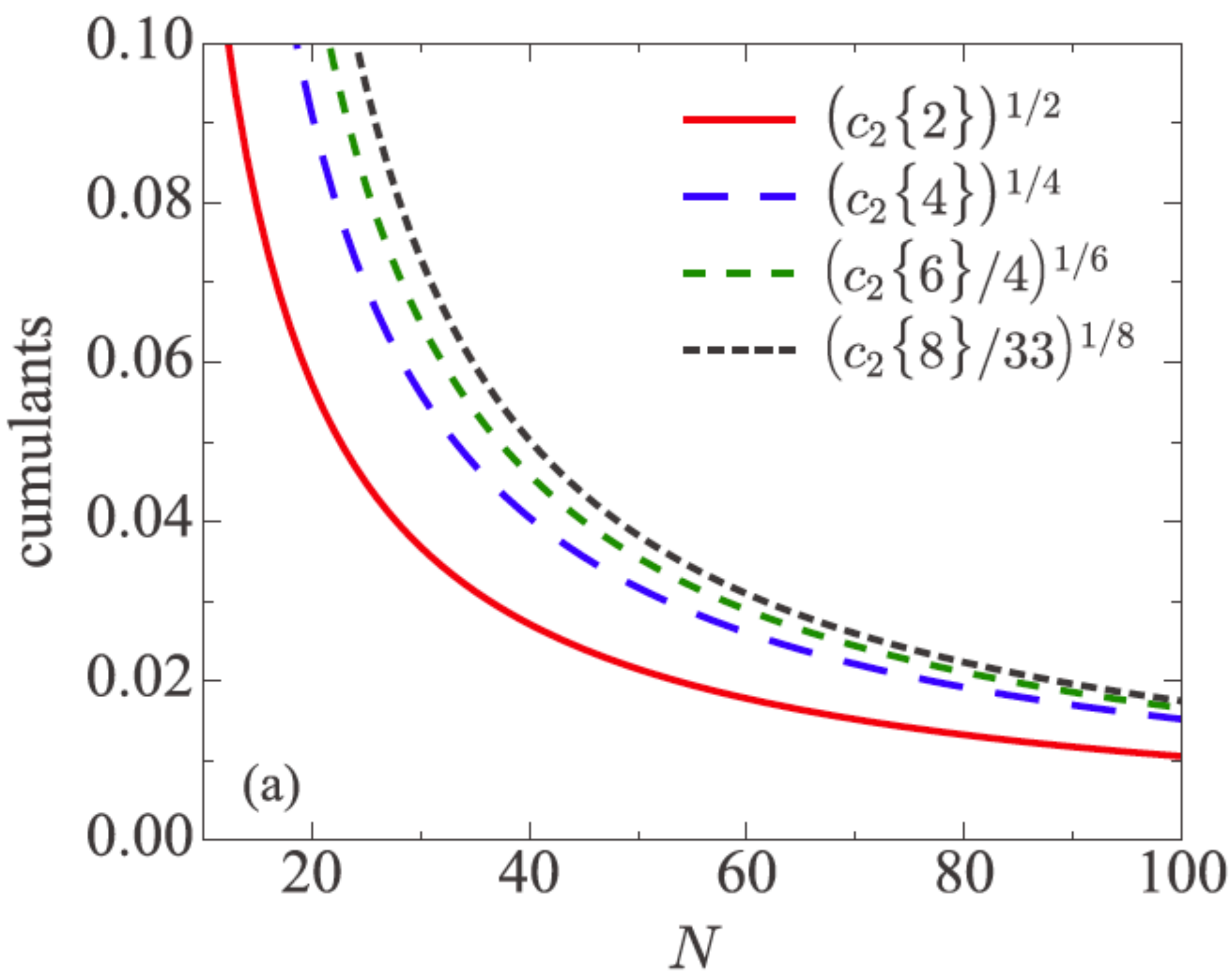
$$c_2\{4\}|_{p_1,p_2,p_3,p_4} \approx \frac{(p_1 p_2 p_3 p_4)^2}{(N-4)^4 \left\langle p^2 \right\rangle_F^4}.$$

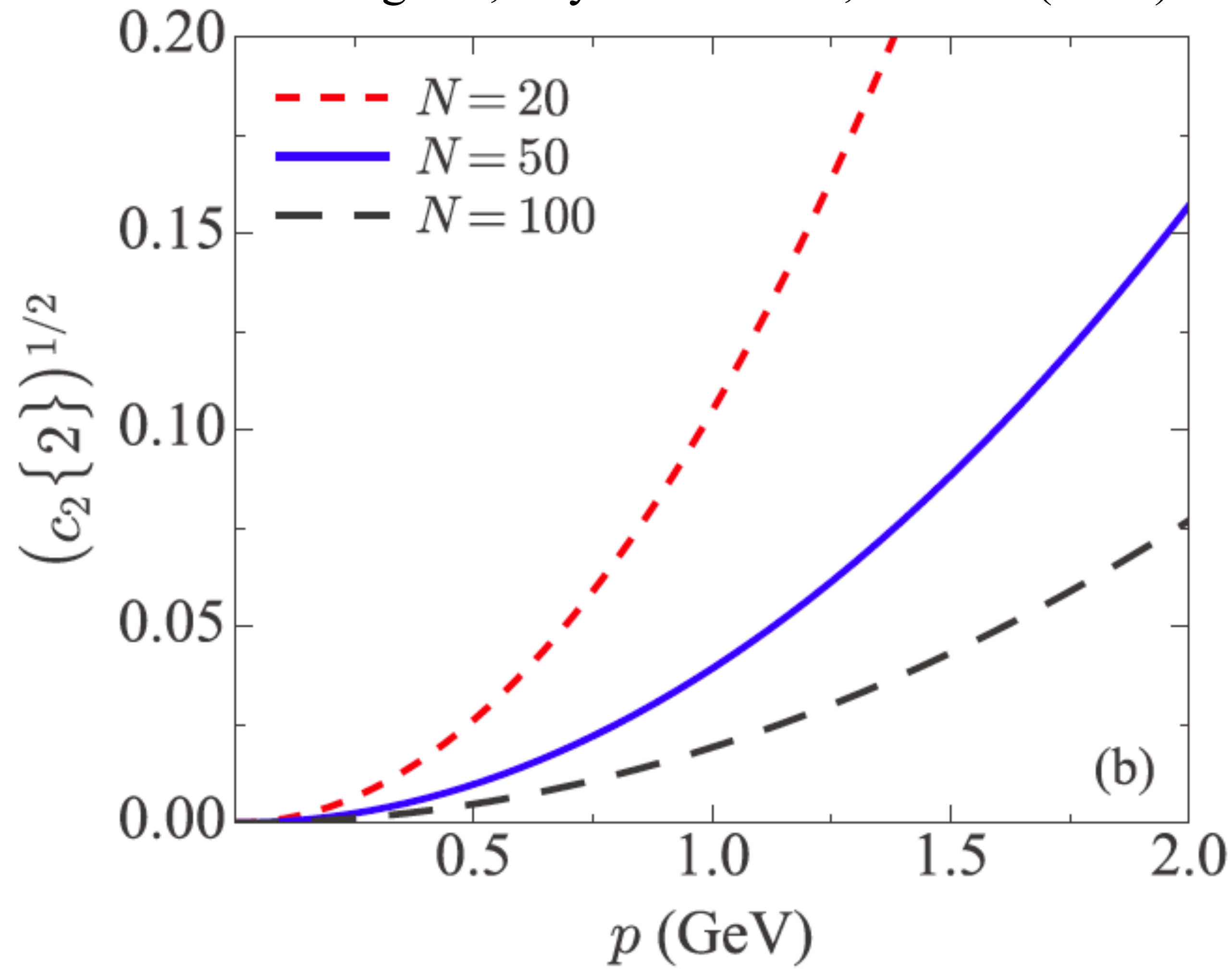
$$\frac{1}{4} c_2\{6\}|_{p_1,\ldots,p_6} \approx \frac{3}{2} \frac{(p_1 p_2 p_3 p_4 p_5 p_6)^2}{(N-6)^6 \left\langle p^2 \right\rangle_F^6}$$

$$\frac{1}{33} c_2\{8\}|_{p_1,\ldots,p_8} \approx \frac{24}{11} \frac{(p_1 p_2 p_3 p_4 p_5 p_6 p_7 p_8)^2}{(N-8)^8 \left\langle p^2 \right\rangle_F^8}$$

Properties of c2{k} from TMC

Adam Bzdak and Guo-Liang Ma, Phys. Rev. C 97, 014903 (2018)



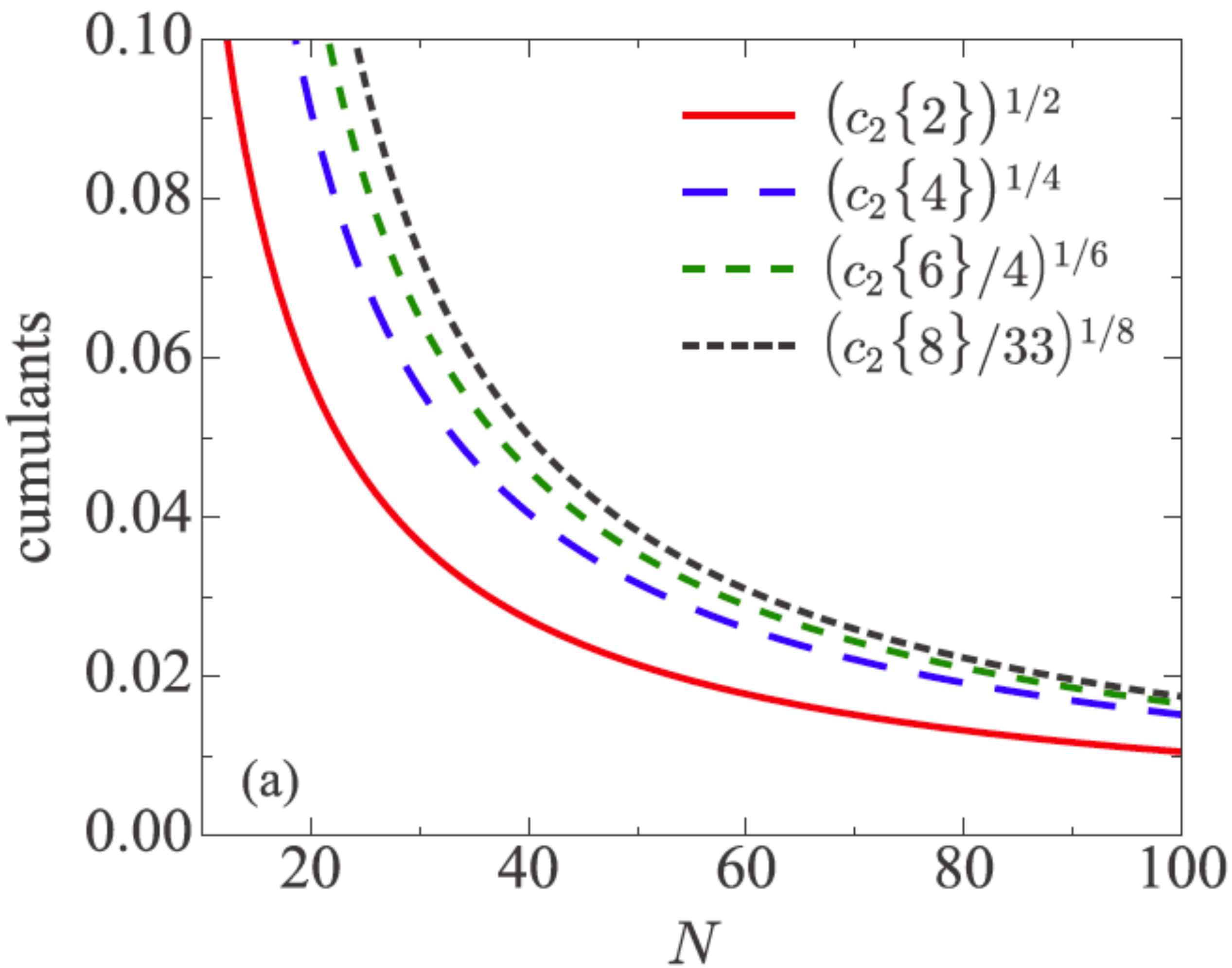


- Cumulants from TMC: **$c_2\{k\} \propto 1/N^k > 0$**
- The magnitudes increase with the order of cumulant.

- TMC influence on c2{2}: more significant for particles with higher momenta (parabolic dependence) and smaller N.

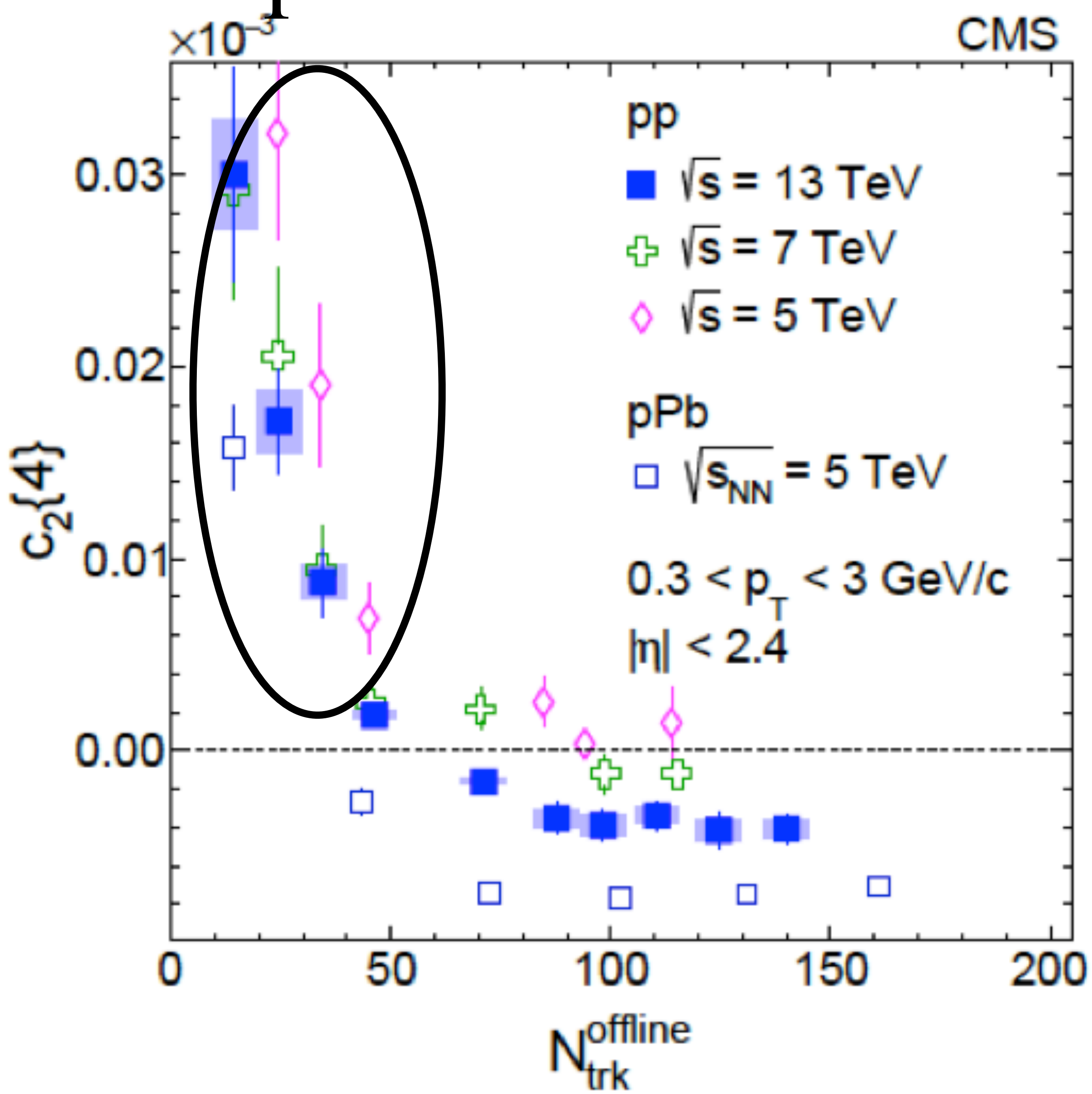
c2{4} from TMC vs the data

c2{k} due to TMC:



- Always c2{k} > 0

Exp. data:

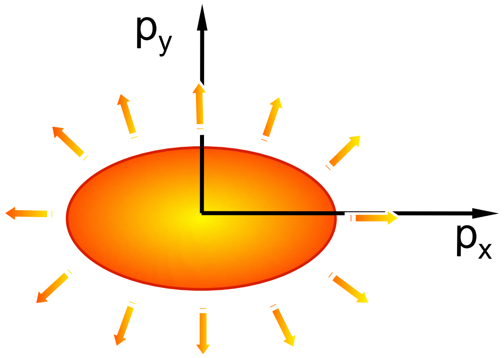


- c2{4} changes from positive to negative with Ntrack.

- **=> Data = TMC⊕ negative part?**

c2{2} from TMC⊕**flow(v2)**

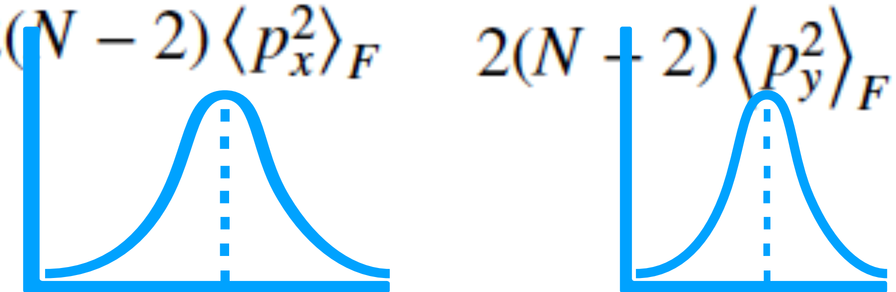
Momentum space



$$f(p,\phi)=\frac{g(p)}{2\pi}\left[1+2v_2(p)\cos(2\phi-2\Psi_2)\right],$$

CLT(<p>=0, $\boldsymbol{\sigma}_x^2$=<p_x^2>, $\boldsymbol{\sigma}_y^2$=<p_y^2>)

$$f_2(p_1,\phi_1,p_2,\phi_2)=f(p_1,\phi_1)f(p_2,\phi_2)\frac{N}{N-2}\exp\left(-\frac{(p_{1,x}+p_{2,x})^2}{2(N-2)\langle p_x^2\rangle_F}-\frac{(p_{1,y}+p_{2,y})^2}{2(N-2)\langle p_y^2\rangle_F}\right).$$



$$\left\langle p_x^2\right\rangle_F = \frac{1}{2}\left\langle p^2\right\rangle_F(1+\bar{\bar{v}}_{2,F})$$
$$\left\langle p_y^2\right\rangle_F = \frac{1}{2}\left\langle p^2\right\rangle_F(1-\bar{\bar{v}}_{2,F})$$

$c_2\{2\}=$

$$\langle e^{2i(\phi_1-\phi_2)}\rangle|_{p_1,p_2}=\frac{\int_0^{2\pi}f_2(p_1,\phi_1;p_2,\phi_2)e^{2i(\phi_1-\phi_2)}d\phi_1 d\phi_2}{\int_0^{2\pi}f_2(p_1,\phi_1;p_2,\phi_2)d\phi_1 d\phi_2}$$

$$c_2\{2\}\approx(v_2(p))^2-\frac{p^2v_2(p)[2v_2(p)-\bar{\bar{v}}_{2,F}]}{(N-2)\langle p^2\rangle_F}+\frac{p^4}{2(N-2)^2\langle p^2\rangle_F^2}.$$

Flow TMC

c2{4} from TMC⊕flow(v2)

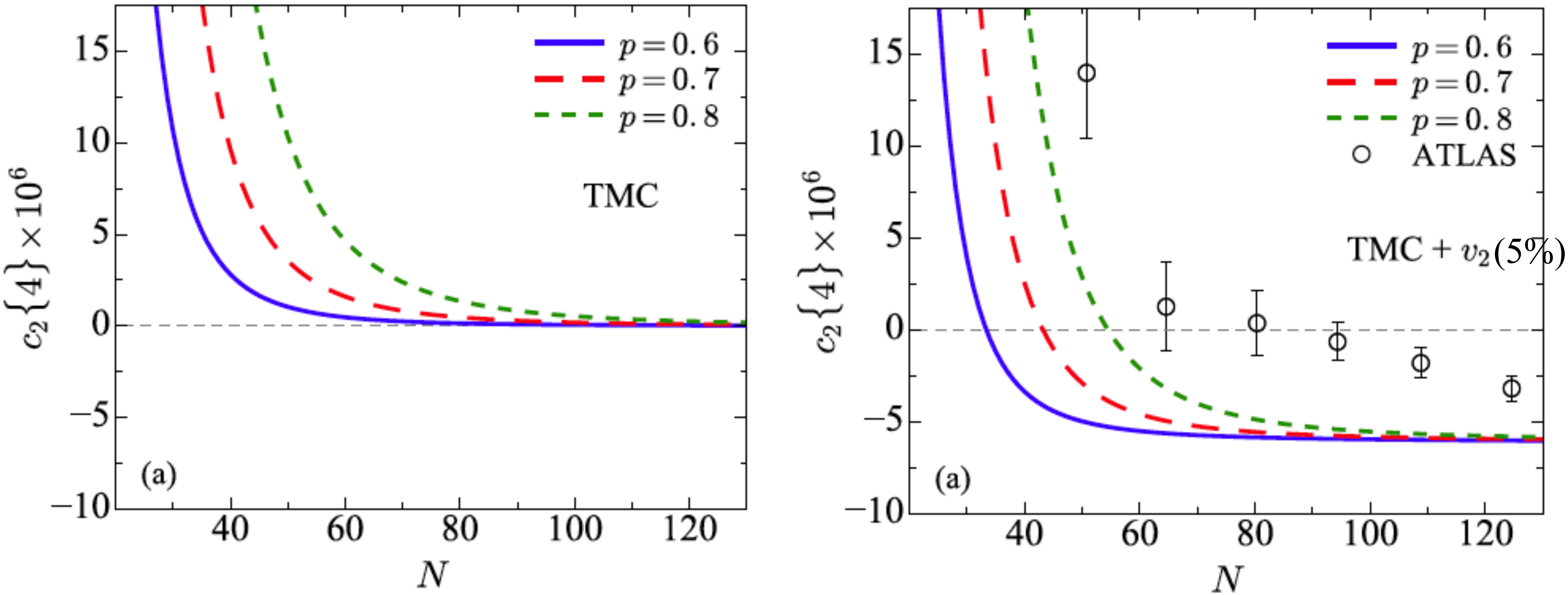
$$f_4(p_1,\phi_1,...,p_4,\phi_4) = f(p_1,\phi_1)\cdots f(p_4,\phi_4)\frac{N}{N-4}\times \exp\left(-\frac{(p_{1,x}+...+p_{4,x})^2}{2(N-4)\langle p_x^2\rangle_F}-\frac{(p_{1,y}+...+p_{4,y})^2}{2(N-4)\left\langle p_y^2\right\rangle_F}\right)$$

$$\langle e^{2i(\phi_1+\phi_2-\phi_3-\phi_4)}\rangle|_{p_1,p_2,p_3,p_4} = \frac{\int_0^{2\pi} f_4(p_1,\phi_1,...,p_4,\phi_4)e^{2i(\phi_1+\phi_2-\phi_3-\phi_4)}d\phi_1\cdots d\phi_4}{\int_0^{2\pi} f_4(p_1,\phi_1,...,p_4,\phi_4)d\phi_1\cdots d\phi_4}$$

$$c_2\{4\} \approx (v_2(p))^4-\frac{2p^2(v_2(p))^3[2v_2(p)-\bar{\bar{v}}_{2,F}]}{(N-4)\langle p^2\rangle_F}+\frac{2p^4(v_2(p))^2}{(N-4)^2\langle p^2\rangle_F^2}-\frac{2p^6v_2(p)[8v_2(p)-3\bar{\bar{v}}_{2,F}]}{(N-4)^3\langle p^2\rangle_F^3}+\frac{p^8[442(v_2(p))^2-360v_2(p)\bar{\bar{v}}_{2,F}+27(\bar{\bar{v}}_{2,F})^2]}{6(N-4)^4\langle p^2\rangle_F^4}+\frac{3p^8}{2(N-4)^4\langle p^2\rangle_F^4}-2(c_2\{2\})^2,$$

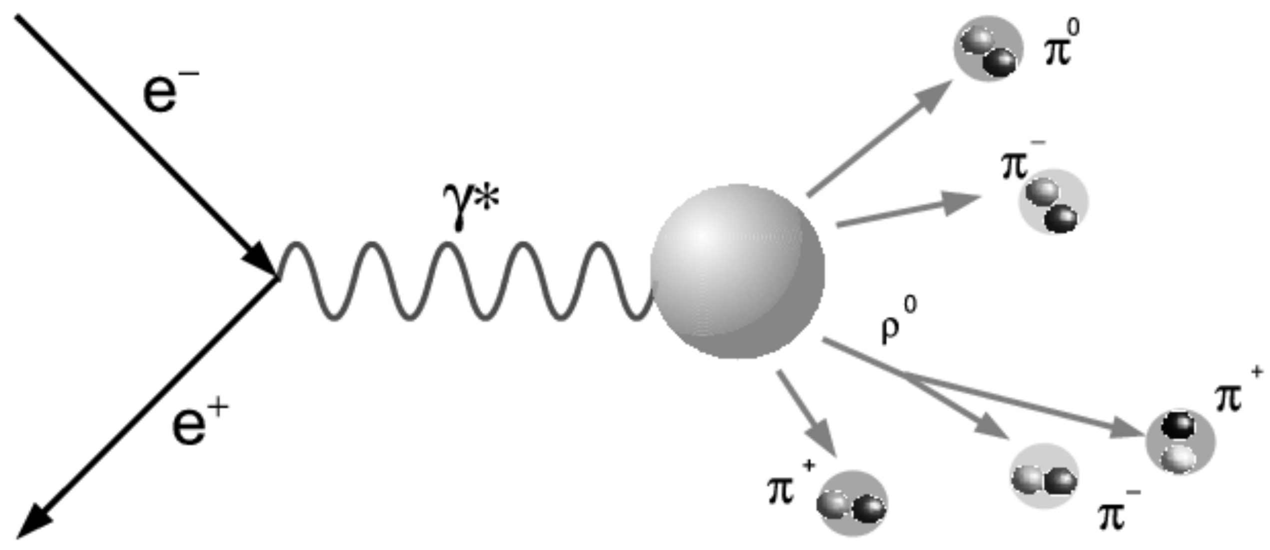
c2{4} from TMC⊕flow(v2)

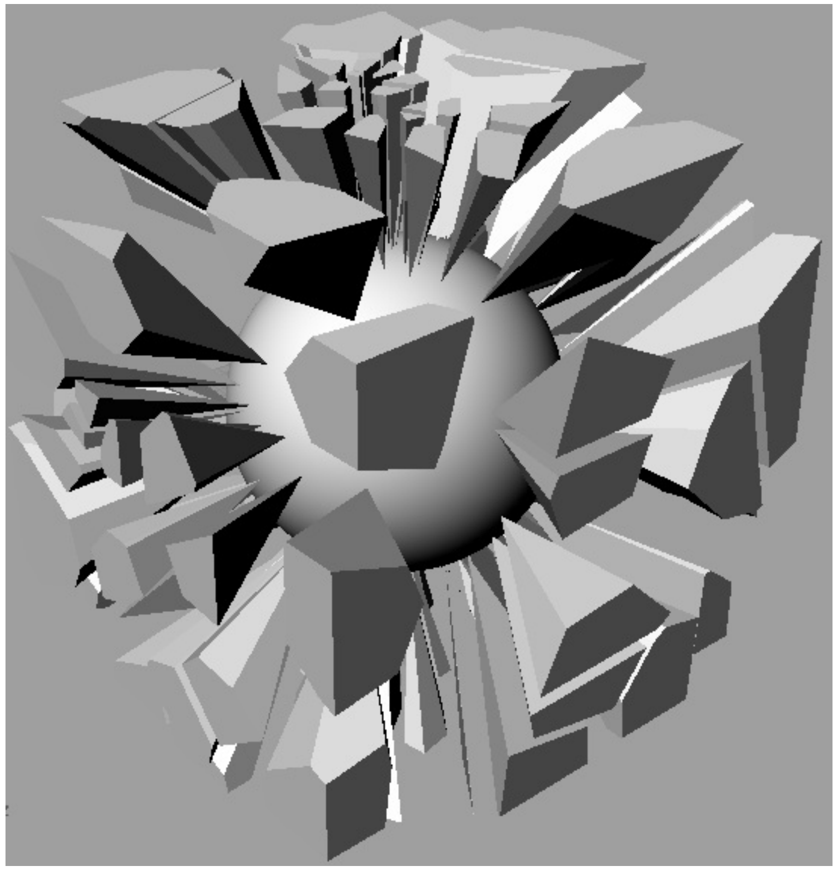
Adam Bzdak and Guo-Liang Ma, Phys. Lett. B 781, 117 (2018)



- The azimuthal cumulant, c2{4} , originating from TMC ⊕ flow(v2), qualitatively agrees with the ATLAS subevent p+p data.
- The sign change of c2{4} depends on v2(N,p), $v2_F$ and $p^2/\langle p^2\rangle_F$, [P(v2) in progress], ….
- **v2(N,p)=> the onset of collectivity in small system.**

Possible other applications





e^{+}+e^{-} collisions

small explosions

•Our results generally holds for any system with a small number of particles/fragments as long as the system respects the TMC.

=>TMC system has positive azimuthal cumulant flow , i.e. $c_2\{k\} \propto 1/N^k > 0$.

Summary

FLOW`=FLOW(*hydro*⊕*escape*⊕*CGC*⊕…)⊕**NON-FLOW**(*TMC*⊕*jet*⊕*resonance*⊕…)

AMPT

- The elastic scattering of partons, with σ =1.5-3mb, naturally explains the long-range correlations and flow in small systems.
- The signals arise from escape⊕hydro due to parton collisions in AMPT.

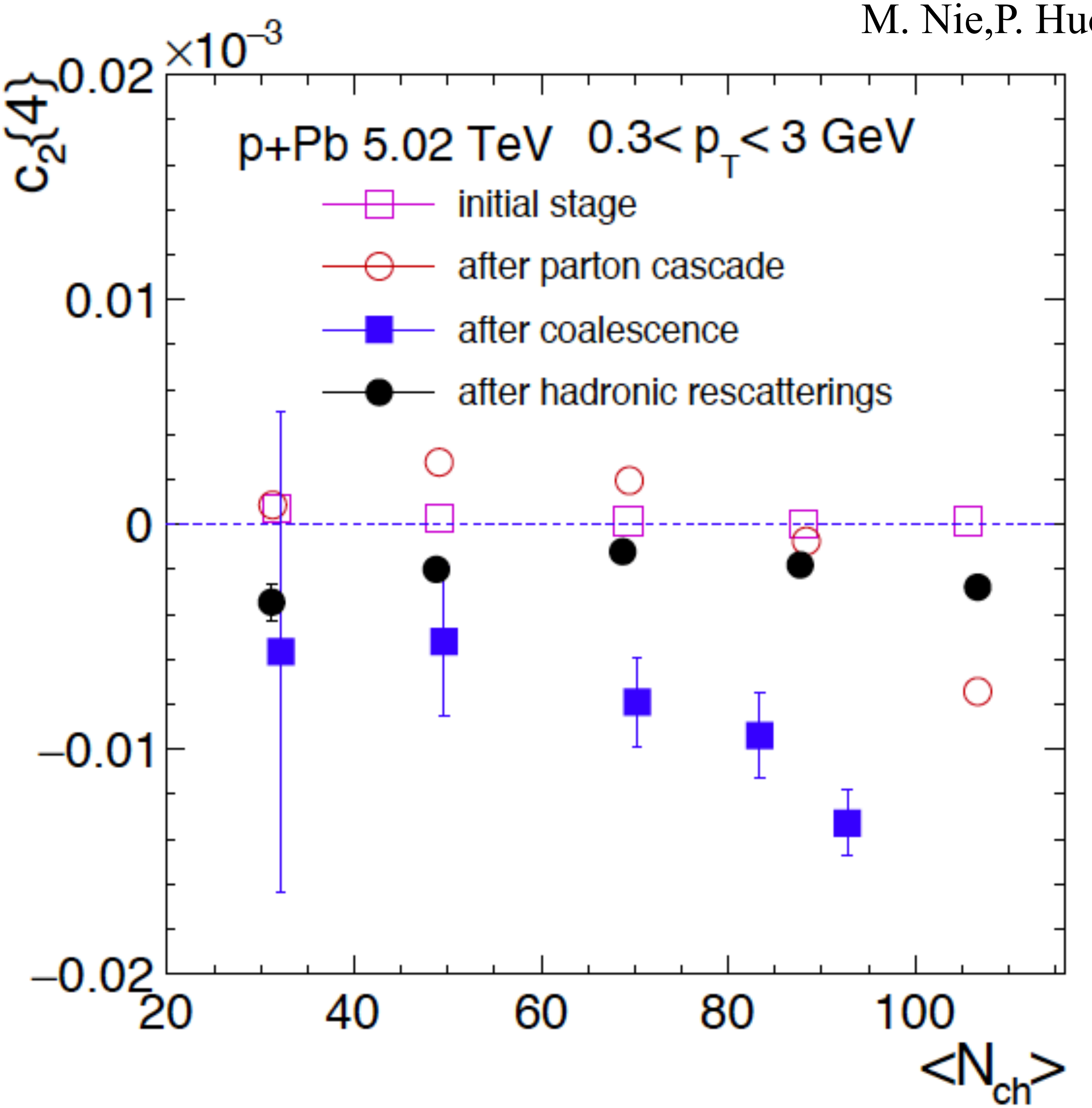
TMC⊕ v2

- TMC brings positive azimuthal cumulant flow , $c_2\{k\} \propto 1/N^k > 0$.
- TMC⊕flow(v2) reproduces the sign change of $c_2\{4\}$, qualitatively agrees with exp. data.=> the onset of collectivity in small system?

Thanks!

c2{4} stage evolution in the AMPT model

M. Nie,P. Huo, Jiarong Jia, Guo-Liang Ma arXiv:1802.00374

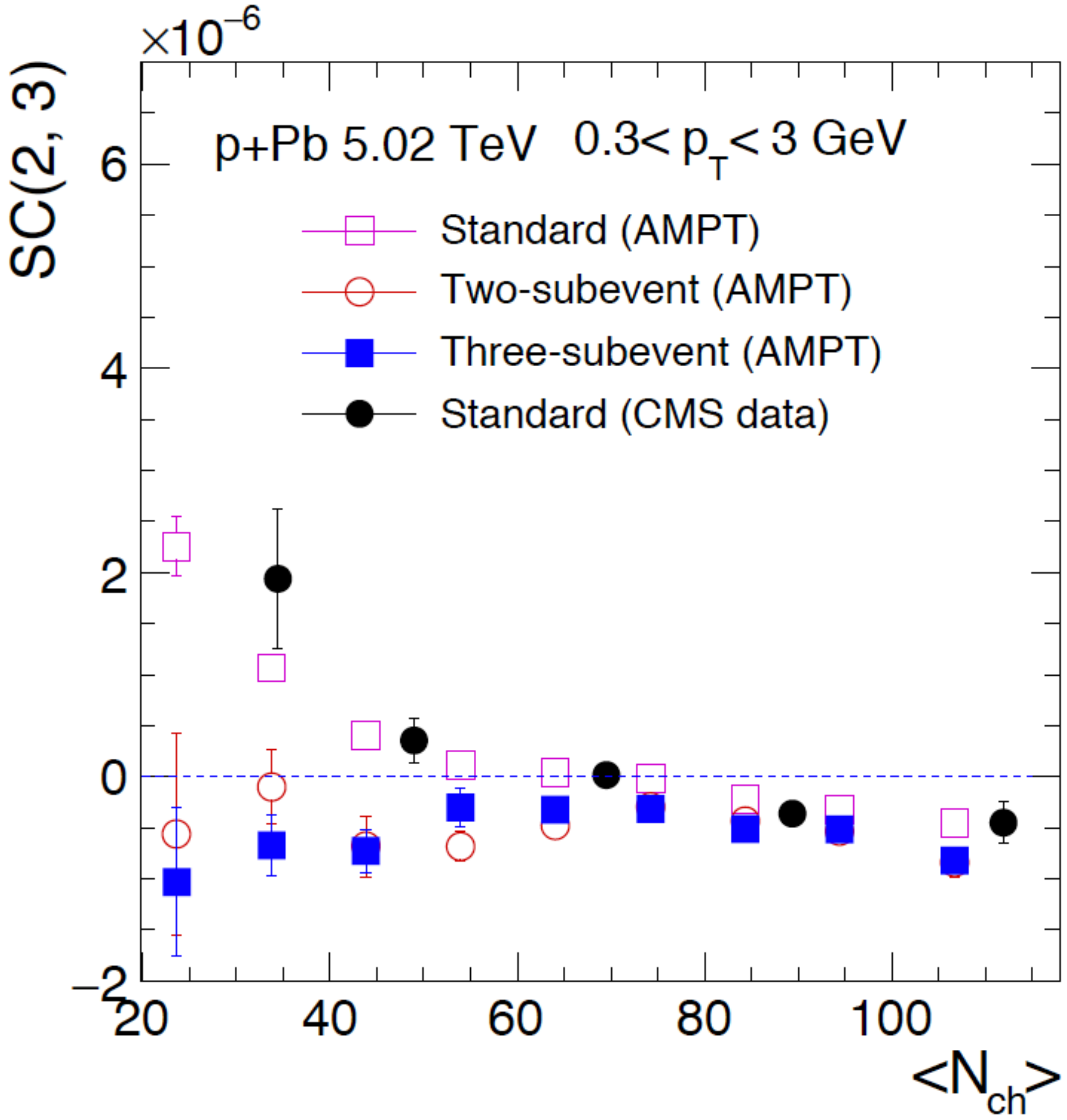


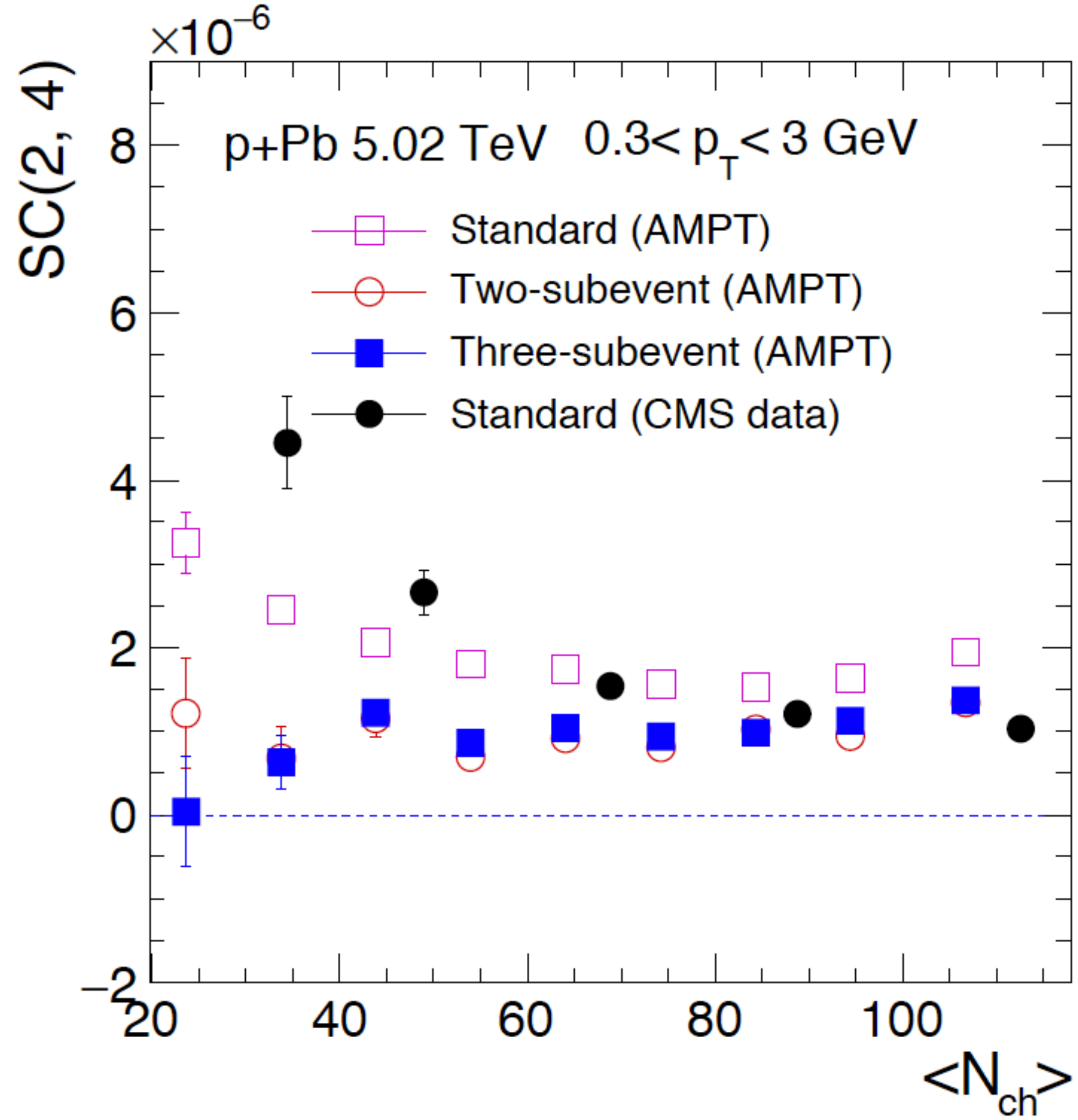
AMPT results about c2{4} for different stages with 3-subevent method:

- c2{4} of initial partons is consistent with the expectation from TMC only.
- c2{4} of final partons changes sign due to the formation of flow.
- c2{4} of hadrons changes due to hadronization and rescatterings.=>**The real case may not be so simple.**

Symmetric Cumulants in AMPT model

M. Nie,P. Huo, Jiarong Jia, Guo-Liang Ma arXiv:1802.00374





- The standard SC(2,3) and SC(2,4) are consistent with the CMS data.
- The subevent SC(2,3) is negative, and subevent SC(2,4) is smaller than standard one.
  =>The standard SC cumulants may be contaminated by non-flow effects.

k-particle cumulant elliptic flow c2{k}

$$c_2\{2\} = \left\langle e^{i2(\phi_1-\phi_2)} \right\rangle$$

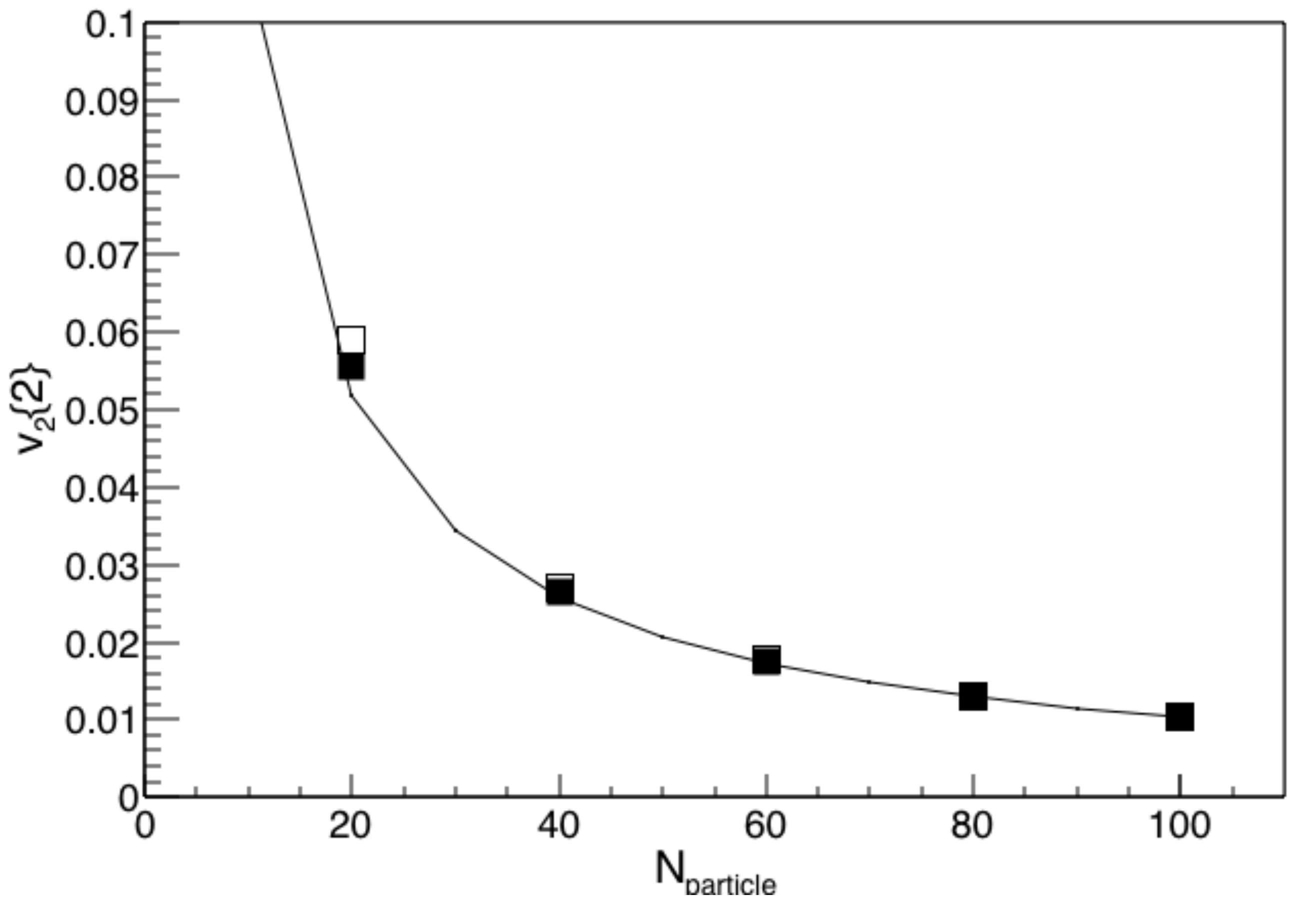
$$c_2\{4\} = \left\langle e^{i2(\phi_1+\phi_2-\phi_3-\phi_4)} \right\rangle - 2\left\langle e^{i2(\phi_1-\phi_2)} \right\rangle^2$$

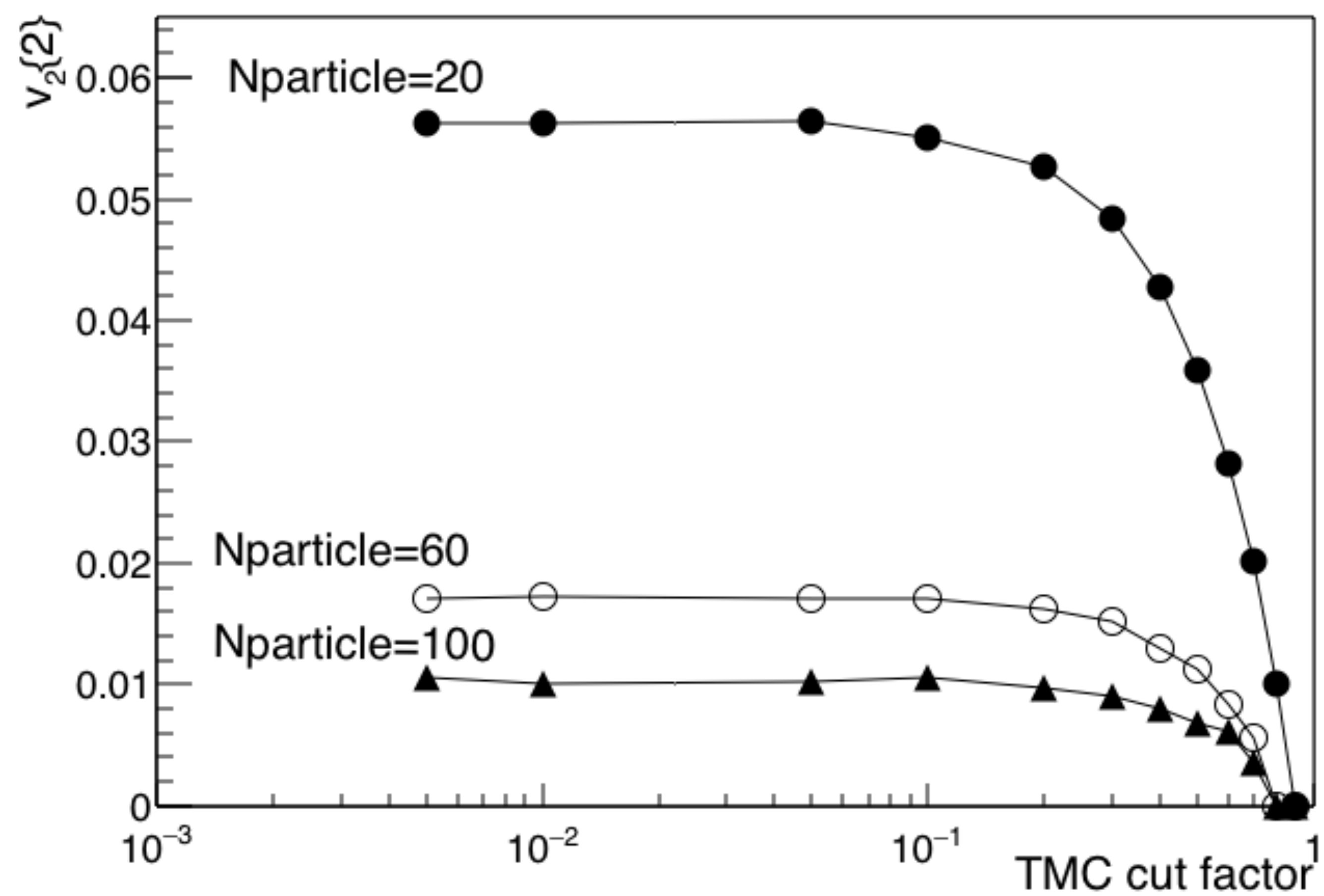
$$c_2\{6\} = \left\langle e^{i2(\phi_1+\phi_2+\phi_3-\phi_4-\phi_5-\phi_6)} \right\rangle - 9\left\langle e^{i2(\phi_1-\phi_2)} \right\rangle \left\langle e^{i2(\phi_1+\phi_2-\phi_3-\phi_4)} \right\rangle + 12\left\langle e^{i2(\phi_1-\phi_2)} \right\rangle^3$$

$$c_2\{8\} = \left\langle e^{i2(\phi_1+\phi_2+\phi_3+\phi_4-\phi_5-\phi_6-\phi_7-\phi_8)} \right\rangle - 16\left\langle e^{i2(\phi_1-\phi_2)} \right\rangle \left\langle e^{i2(\phi_1+\phi_2+\phi_3-\phi_4-\phi_5-\phi_6)} \right\rangle - 18\left\langle e^{i2(\phi_1+\phi_2-\phi_3-\phi_4)} \right\rangle^2 + 144\left\langle e^{i2(\phi_1-\phi_2)} \right\rangle^2 \left\langle e^{i2(\phi_1+\phi_2-\phi_3-\phi_4)} \right\rangle - 144\left\langle e^{i2(\phi_1-\phi_2)} \right\rangle^4,$$

$$(v_2\{2\})^2 = c_2\{2\}, \quad (v_2\{4\})^4 = -c_2\{4\}, \quad (v_2\{6\})^6 = \frac{c_2\{6\}}{4}, \quad (v_2\{8\})^8 = -\frac{c_2\{8\}}{33}.$$

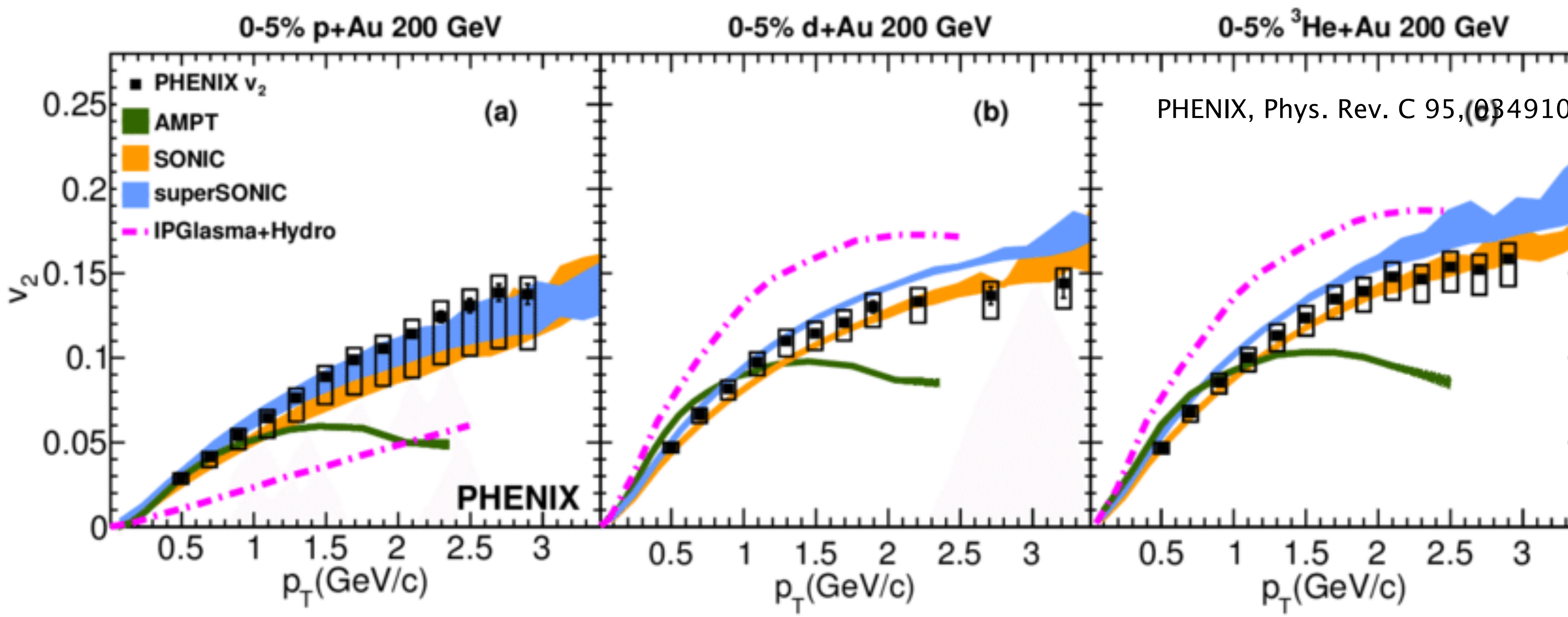
Discussion I: numerical proof



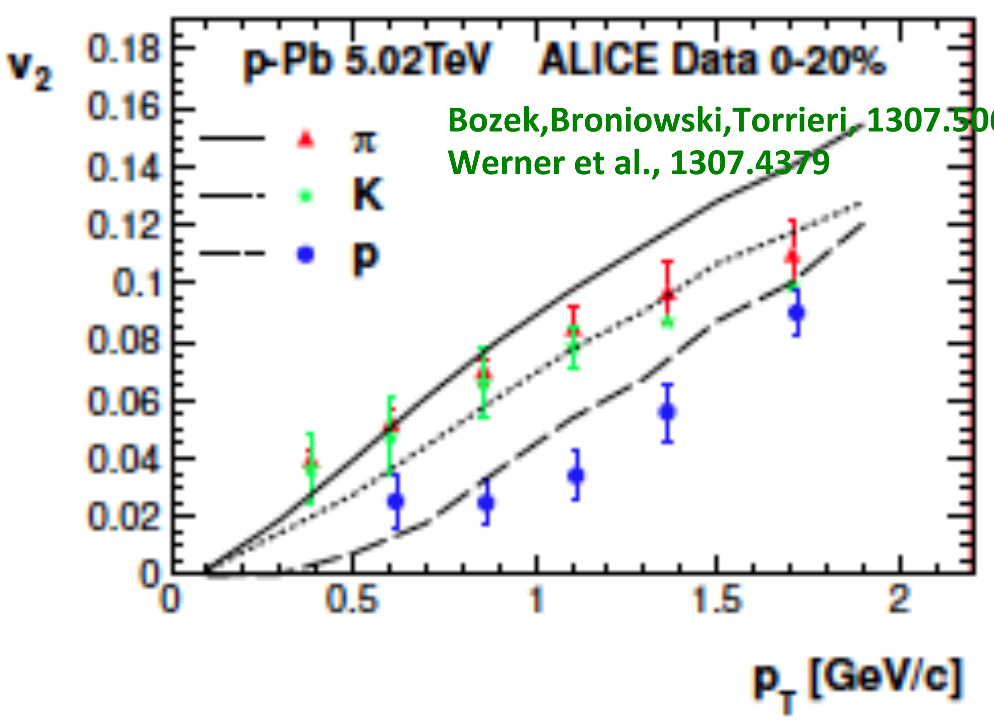


- We numerically demonstrate that our analytic result is correct, by sampling N particles which obey the TMC within a TMC cut.

Hydrodynamical results in small systems



PHENIX, Phys. Rev. C 95, 034910 (2017)

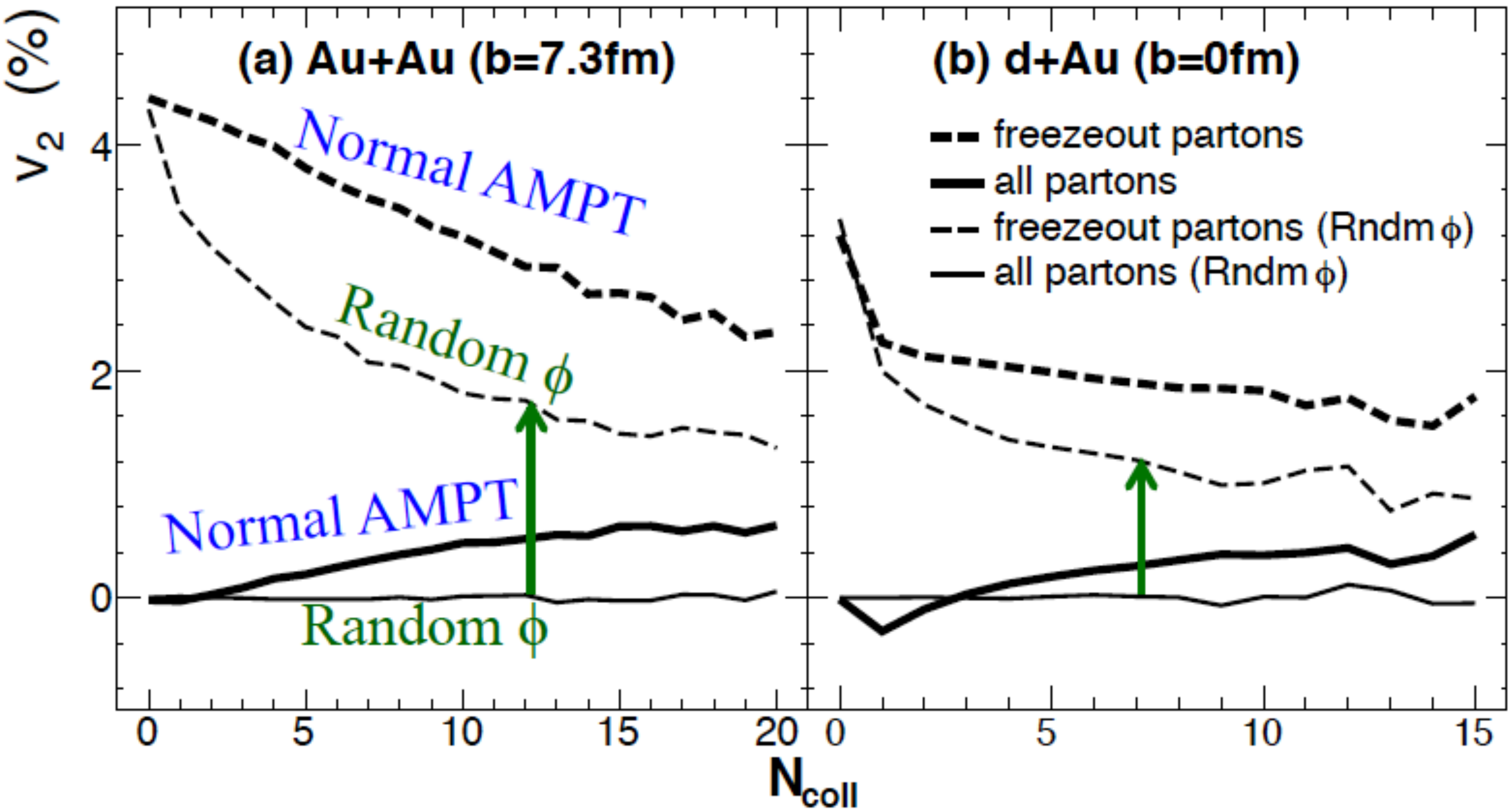


Bozek,Broniowski,Torrieri, 1307.5060
Werner et al., 1307.4379

- Hydrodynamics can well describe elliptic flow v2 in p+A at RHIC and LHC energies.

Escape mechanism in AMPT model

v_2 from **Random Test:** purely from the escape mechanism



	Normal $\langle v_2 \rangle$	$\langle v_2 \rangle$ for random-ϕ	**Ratio** ~ contribution from pure escape	<Ncoll>
Au+Au	3.9%	2.7%	**69%**	4.6 (*modest*)
d+Au	2.7%	2.5%	**93%**	1.2 (*low*)