Application of Global Bayesian Analysis to Large and Small systems

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Questions

1. How to determine the properties of a physics system with observations of the final states?
2. How accurate are our statements about its properties?

A schematic example: extract one parameter from one observation

A system with a state $x_{\rm true}$ leads to an observation y. $y = f_{phy}(x_{\rm true})$.

- Measurement of y comes with uncertainty $\rightarrow$ Exp uncertainty $y_{exp} \pm \sigma_{exp}$.
- Our model f_M of the true function f_{phy} is imperfect $\rightarrow$ Theory uncertainty σ_{th}.

A Bayesian point of view

- What is the probability of x to be the truth,

$$
\begin{aligned}
P(x = x_{true} | f_M, y_{exp}) &\propto \mathrm{Likelihood}(f_M(x) = y_{exp} | f_M, x) \times \mathrm{Prior}(x) \\
\log \mathrm{Likelihood} &= -\frac{(f_M(x) - y_{exp})^2}{2\sigma^2} + C, \sigma^2 = \sigma_{\rm exp}^2 + \sigma_{\rm th}^2 \quad (+\cdots)
\end{aligned}
$$

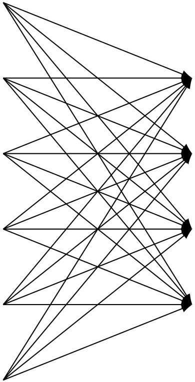
- Bayes' theorem $P(A|B) = P(B|A)P(A)/P(B)$.

A much more complex situation for heavy ion physics

Parameters	Observables		
Nucleon size			
Nucleon shape	Flows $v_2, v_3, \ldots$		
Pre-eq flow	mean-p_T, fluct-p_T	$\otimes$ Systems	$\otimes$ Beam energies
Shear viscosity	Particle yields	Large	200 GeV
		Small	2.76 TeV
Bulk Viscosity	Multiplicity		5.02 TeV
More ...			

A much more complex situation for heavy ion physics

Parameters | **Model very complex** | **Observables**



Parameters:
- Nucleon size
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- More ...

Observables:
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- mean-p_T, fluct-p_T
- Particle yields
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$\otimes$ Systems $\otimes$ Beam energies

Systems	Beam energies
Large	200 GeV
Small	2.76 TeV
	5.02 TeV

Given parameters and predict observables →

A much more complex situation for heavy ion physics

Parameters | **Model very complex** | **Observables**

Nucleon size

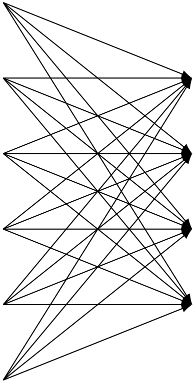
Nucleon shape

Pre-eq flow

Shear viscosity

Bulk Viscosity

More ...



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mean-p_T, fluct-p_T

Particle yields

Multiplicity

$\otimes$ Systems $\otimes$ Beam energies

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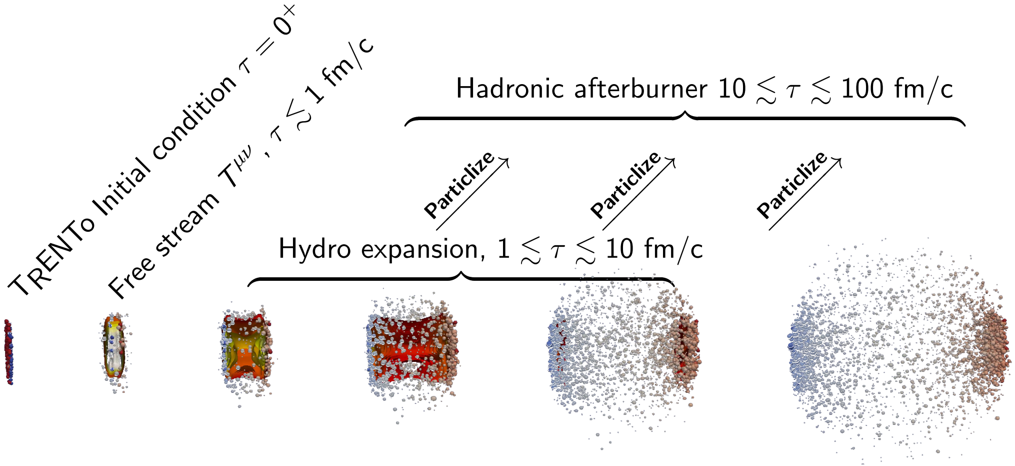
	5.02 TeV

Infer parameters by comparing to data

Work through a real example: extract $\eta/s, \zeta/s$ from AA collisions @LHC

Physics models: a multistage hybrid simulation.



*Thesis work by JE Bernhard `arXiv:1804.06469`.

Physics model: TRENTo initial condition

Prepare two nuclei

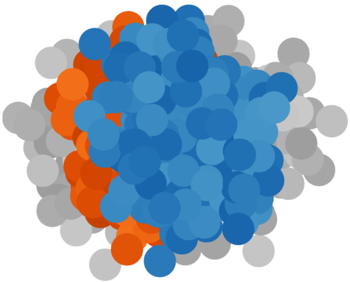
nucleon positions sampled from a Woods-Saxon function

↓

Determine binary collisions

$$P_{\mathrm{coll}}(b) = 1 - \exp(-\sigma_{\mathrm{eff}} T_{pp}(b))$$

σ_{eff} fixed by fitting $\sigma_{\mathrm{pp,\ inel}}$



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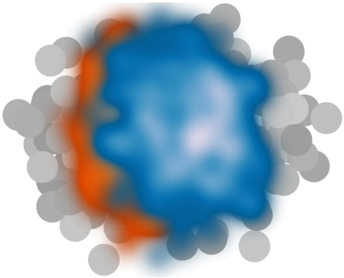
Calculate participant density

$$T_{A,B}(x) = \sum_{i \in N_{\mathrm{part}}} \gamma_i \rho(x - x_i).$$



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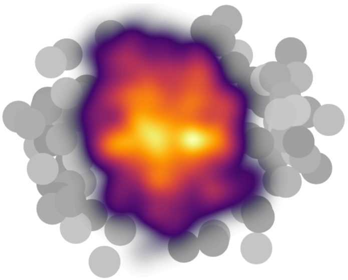
Entropy deposition

Generalized mean ansatz:

$$\frac{dS}{\tau dx_\perp^2 d\eta_s} \propto \left(\frac{T_A^p + T_B^p}{2} \right)^{1/p}.$$



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→

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Features

- Interpolates a family of entropy deposition mappings at mid-rapidity.
- Gaussian proton shape.
- Parameters: nucleon width (w), fluctuation (σ), entropy deposition (p).

Physics model: pre-equilibrium free streaming and hydro

Pre-equilibrium stage

- Before τ_{fs} free stream initial energy density and match it to hydrodynamic $T^{\mu\nu}$ at $\tau = \tau_{\mathrm{fs}}$.
- Shall use more realistic model in the future such as an effective kinetic theory.
- A tunable parameter τ_{fs}.

Hydrodynamics

- 2+1D relativistic viscous hydrodynamics (OSU hydro).
- EoS: interpolate between HRG and Lattice results (HotQCD).
- Parametrization of η/s and ζ/s

$$\frac{\eta}{s} = (\eta/s)_{\mathrm{min}} + (\eta/s)_{\mathrm{slope}}(T - T_c)\left(\frac{T}{T_c}\right)^{(\eta/s)_{\mathrm{curv}}}, \frac{\zeta}{s} = \frac{(\zeta/s)_{\mathrm{max}}}{\left(\frac{T-T_0}{(\zeta/s)_{\mathrm{width}}}\right)^2 + 1}.$$

Particularization and hadronic rescattering

Particularization: `frzout` by JE Bernhard

- Hadrons are sampled from fluid cell with Cooper-Frye prescription at $T = T_{\rm sw}$.
- Include the effect of hadron finite widths.
- Non-δf type shear and bulk corrections.
- Parameter: particularization temperature ($T_{\rm sw}$).

Hadronic rescattering

- An Ultra-relativistic Quantum Molecular Dynamics model (`UrQMD`).

A summary of parameters

Group	Parameters	Explanation
IC	p	Entropy deposition parameter
	Norm(s)	Overall entropy normalizations
	σ fluct	Gamma fluctuation std of N-N collision
	w [fm]	Gaussian nucleon width
	d min [fm]	Nucleon-nucleon minimum distance in nucleus
Pre-eq	τ_{fs} [fm/c]	Freestreaming time
QGP	η/s min	value of $\eta/s(T_c)$
	η/s slope [GeV^{-1}]	slope parameter of $\eta/s(T)$
	η/s curv	curvature $\eta/s(T)$
	ζ/s max	max value of $\zeta/s(T)$
	ζ/s width [GeV]	width parameter of $\zeta/s(T)$
	ζ/s T_0 [GeV]	peak temperature of $\zeta/s(T)$
Particularization	T switch [GeV]	switching temperature from hydro to UrQMD.

A sophisticated Bayesian parameter estimation

Input parameters
parameter space $\vec{p}$

Physics models
Initial stages, hydro, transport, etc

Experimental data
Yield, flow, $\langle p_T \rangle$, etc

Computing models
MB event-by-event simulations

Bayes' Theorem
Posterior = Likelihood × Prior

$$\text{Likelihood}(\vec{p}) \propto e^{-\frac{1}{2}(y_{\text{exp}} - y_{\vec{p}})^T \Sigma^{-1} (y_{\text{exp}} - y_{\vec{p}})}$$

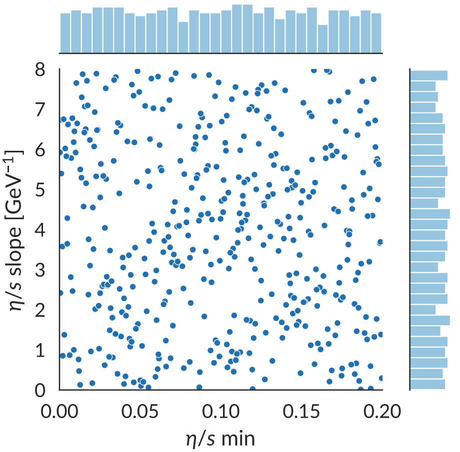
Gaussian process emulator
surrogate model

Markov chain Monte Carlo
Maginalize onto interested parameters

Posterior distribution
quantitative knowledge of each parameter w/ uncertainty

Calibration to LHC 2.76 TeV and 5.02 TeV Pb+Pb data (new!)

Sample design input parameters



- For a high dimensional ($n = 14$) space, grid interpolate is impossible $N \sim \mathcal{O}(10^n)$.
- A smarter sampling technique: Latin Hyper cube sampling.

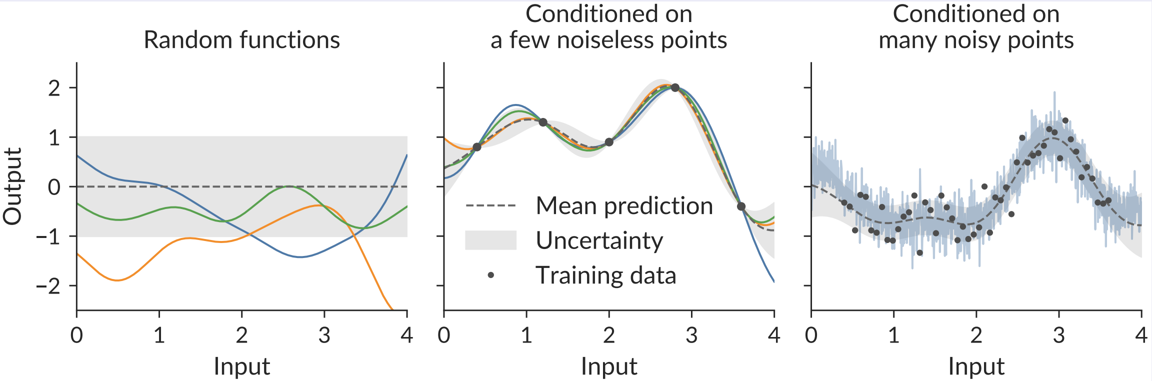
1. Optimized random samples.
2. Maximize the mini. dist. between pairs of points.
3. The marginalization on any parameter is uniform.
4. $N \sim \mathcal{O}(10 \times n)$ for smooth function over the space.

- Perform full model calculation on design points.

Calibration to LHC 2.76 TeV and 5.02 TeV Pb+Pb data (new!)

Surrogate model: Gaussian process emulator

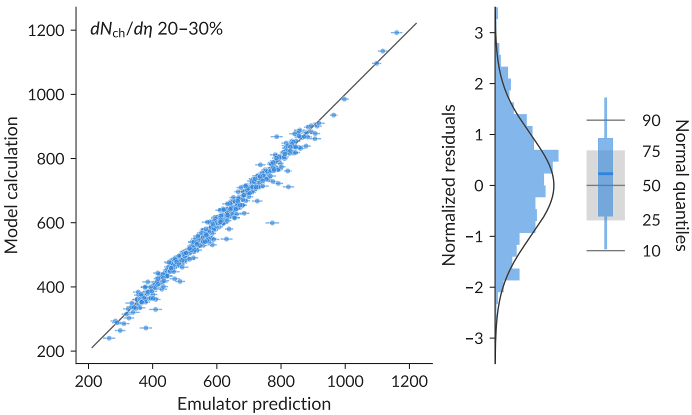
- After full model evaluation ($\vec{Y}_i$) on design points ($\vec{x}_i$): $\vec{x}_i \longrightarrow \vec{Y}_i$.
- How to get arbitrary $\vec{Y}(\vec{x})$ for "arbitrary" parameters $\vec{x}$?
- Non-parametric interpolation through Gaussian process emulator.
- A mean prediction + interpolation uncertainty.



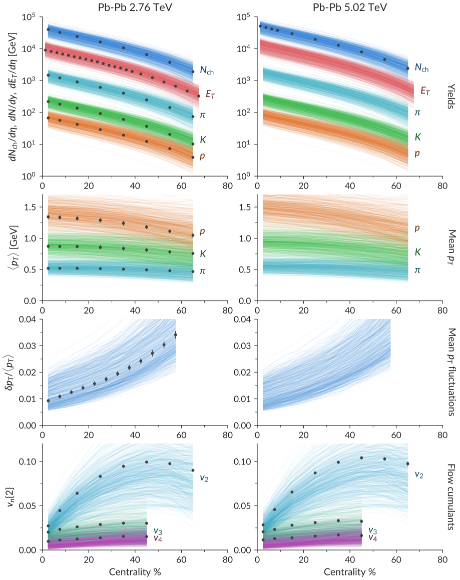
Calibration to LHC 2.76 TeV and 5.02 TeV Pb+Pb data (new!)

Validate the performance of Gaussian process emulator

- Predict observables with newly generated inputs and compare with actually calculation.
- Validate that the trained emulator predictions reproduce the dependence of $\vec{Y}$ on $\vec{x}$.
- Emulator do have uncertainty!



Calibration to LHC 2.76 TeV and 5.02 TeV Pb-Pb data (New)



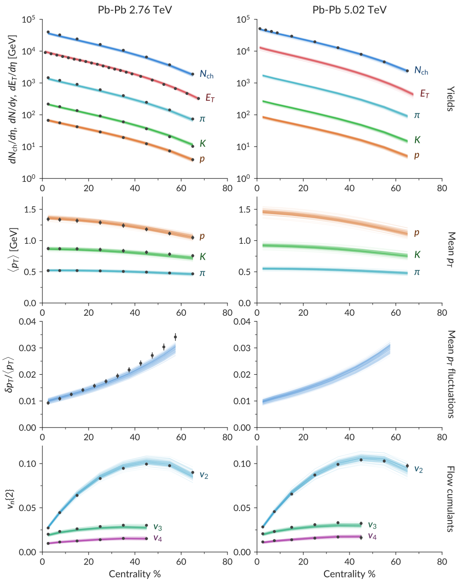
Calculation of design points (prior)

- 500 design points.
- Observables:
  multiplicity, transverse energy, p, π, K yield and mean-p_T, mean-p_T EbE fluctuation, v_2, v_3, v_4.
- Parameter design ranges are wide enough to spread over all data points.

The likelihood function

$$
\begin{aligned}
\Delta y &= (y_{\mathrm{exp}} - y_{\mathrm{p}}) \\
\log \mathrm{Likelihood} &= C - \frac{1}{2}\det\Sigma - \frac{1}{2}\left(\Sigma^{-1}\right)_{ij}\Delta y_i \Delta y_j. \\
\Sigma &= \sigma^2_{\mathrm{stat}} + \sigma^2_{\mathrm{sys}} + \sigma^2_{\mathrm{GP}} + \sigma^2_{\mathrm{model}}
\end{aligned}
$$

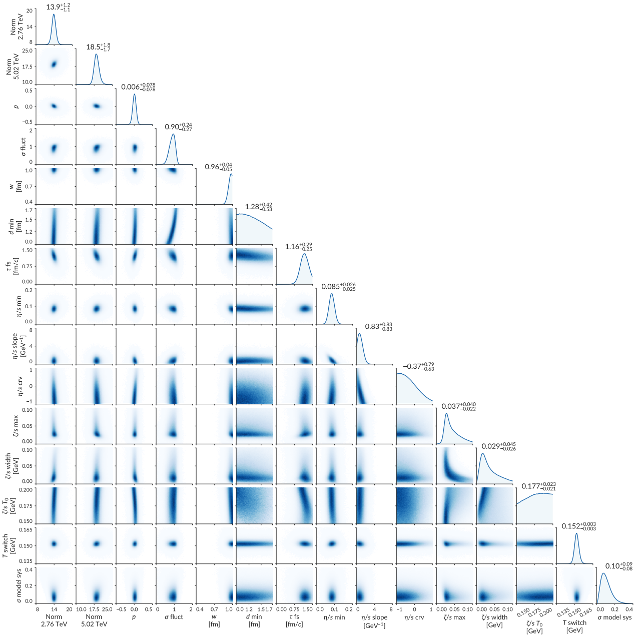
Calibration to LHC 2.76 TeV and 5.02 TeV Pb-Pb data (New)



After model-to-data comparison (posterior, from emulator)

- After comparing to experimental data, the emulator can predict observables from the calibrated distribution of parameters.
- Achieve a global agreement with data.

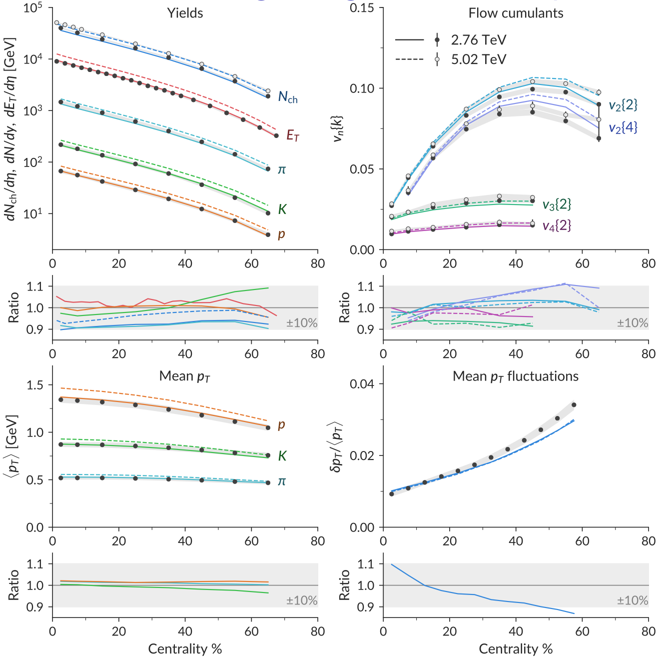
Parameters calibration



Marginalized posterior distribution

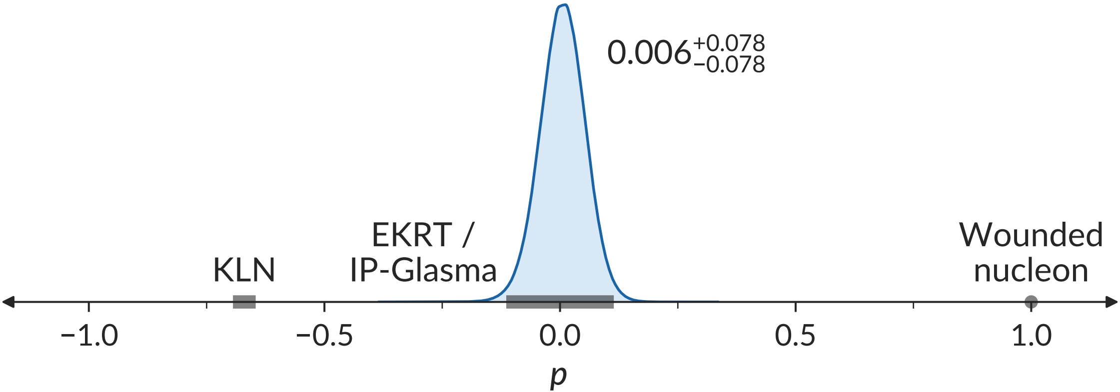
- Diagonal: single parameter distributions (integrate over $N - 1$).
- Off diagonal: pairwise correlations (integrate over $N - 2$).
- Marginalization of the other $N - 1$ parameters concentrates on the single parameter with uncertainties from other parts of the model folded in.

Calculation using a high-likelihood parameter set



A global ±10% level of description.

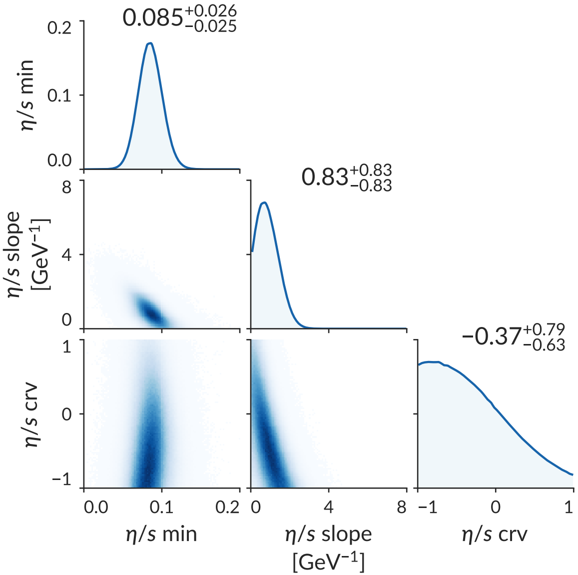
Focus on the initial condition

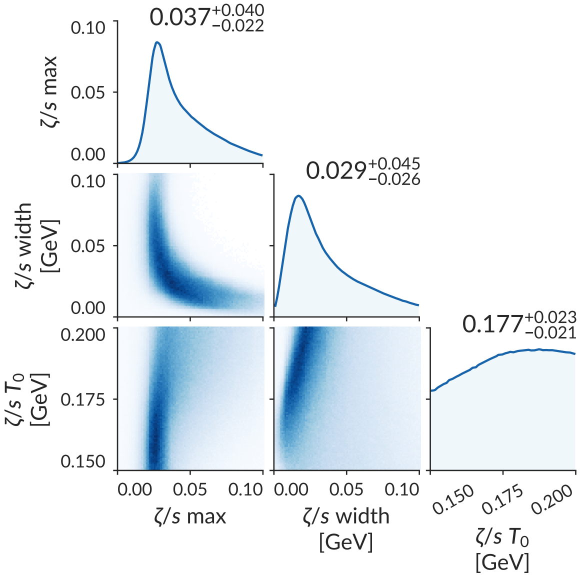


Knowledge on initial entropy deposition

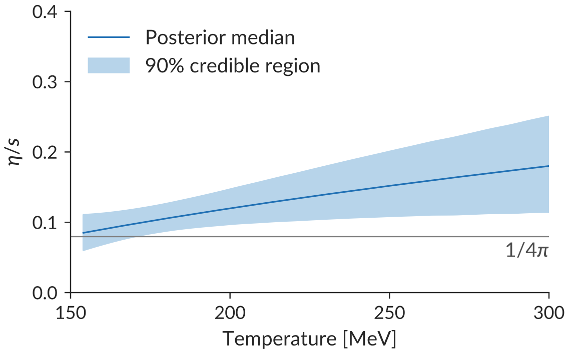
- Support the eccentricity scaling ($\epsilon_{2,3}$ as functions of b) given by saturation physics based/motivated initial condition models.

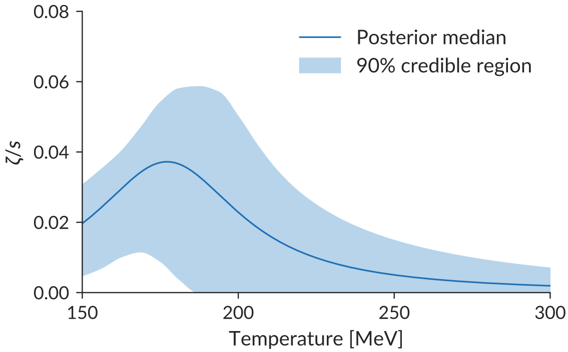
Focus on transport properties





Focus on transport properties





Knowledge on η/s

- Minimum around 0.85 ± 0.25.
- Suggest rising trend of η/s.

Knowledge on ζ/s

- The need of a non-zero ζ/s.
- Peaking T_0 is not well constrained.

Extend to small systems (from a final state point of view)

Collective features observed in small systems

- Long range pseudorapidity two-particle correlation.
- Finite v_2, v_3 from multi-particle correlations.
- Competing of two pictures,
  - Initial state effect.
  - Final state effect
  1. Pressure driven and the formation of QGP in high-multiplicity events.
  2. Escape mechanism.

Study small system from a final state perspective

- Developments of T$_R$ENTo towards a more realistic description of small system.
- Perform analysis on both pA and AA $\rightarrow$ Can we describe both in a single framework?
- Can we study other observables that may help to distinguish IS and FS effects?

Developments in TRENTo for studying possible final state effects

The TRENTo family

Original: round proton, boost-invariant

Developments in TRENTo for studying possible final state effects

The TRENTo family

Original: round proton, boost-invariant $\longrightarrow$ Add proton shape fluctuation

- A recent simultaneous calibration on both p-Pb and Pb-Pb collisions (2+1D).

Developments in TRENTo for studying possible final state effects

The TRENTo family

Original: round proton, boost-invariant ⟶ Add proton shape fluctuation

↓ ↓

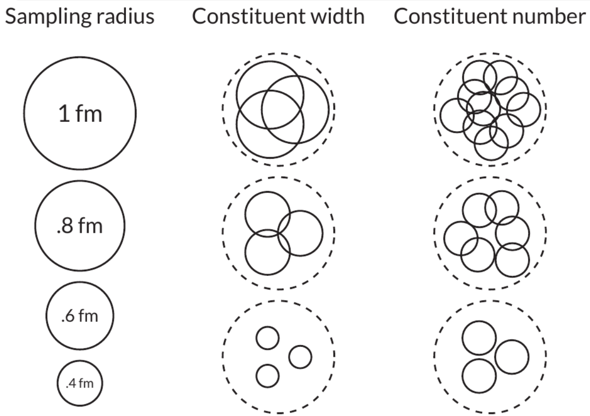
Add local rapidity dependence ⟶ Combine both features (in progress)

- A recent simultaneous calibration on both p-Pb and Pb-Pb collisions (2+1D).
- Opportunity of calibration using 3+1D simulations and predicts for possible final state effect in small systems.

Adding proton shape fluctuation to TRENTo

Subnucleonic fluctuation

- In p-A collisions, initial geometry is sensitive to proton subnucleonic fluctuations.
- Extend the default Gaussian round proton in TRENTo by adding subnucleon constituents.



Proton degrees of freedom

- A proton width parameter.
- Width of Gaussian constituents.
- Number of constituents.

Adding proton shape fluctuation to TRENTo

Collision of Gaussian round proton

- Proton-proton collision probability at fixed b,

$$P_{\mathrm{coll}}(b) = 1 - \exp(-\sigma_{\mathrm{eff}} T_{pp}(b)),\ T_{pp}(b) = \int dx_\perp^2 \rho(x + b/2)\rho(x - b/2)$$

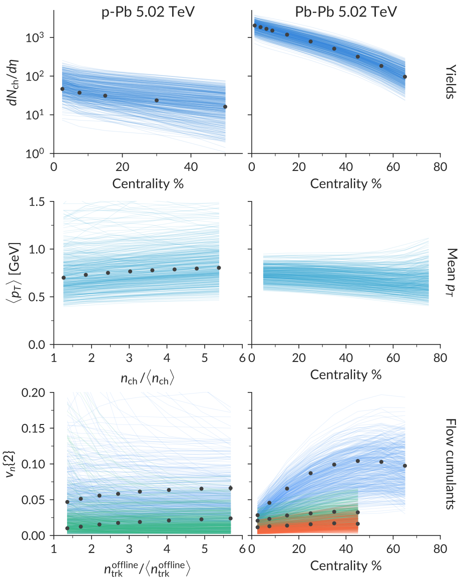
- σ_{eff} fixed by requiring $\sigma_{\mathrm{pp,\ inel}} = \int d\vec{b}^2 P_{\mathrm{coll}}(b)$.

Collision of fluctuating proton

- Proton-proton P_{coll} determined from constituent collisions,

$$P_{\mathrm{coll}}(b) = \left\langle 1 - \prod_{i,j=1}^{\#} [1 - P_{\mathrm{coll}}(b_{ij})] \right\rangle_{x_i, x_j \text{ given } b}$$

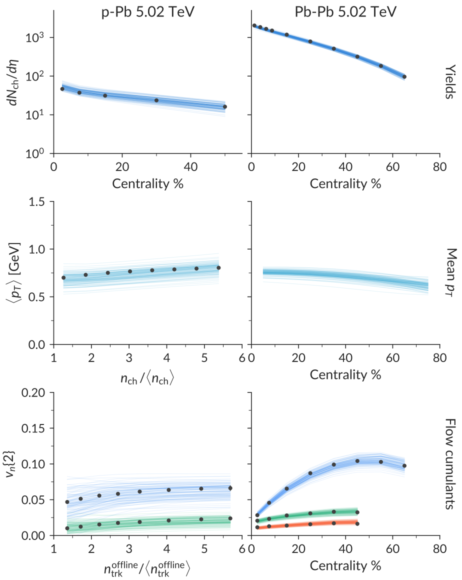
Calibration on p-Pb and Pb-Pb systems simultaneously



Calculation of design points (Prior)

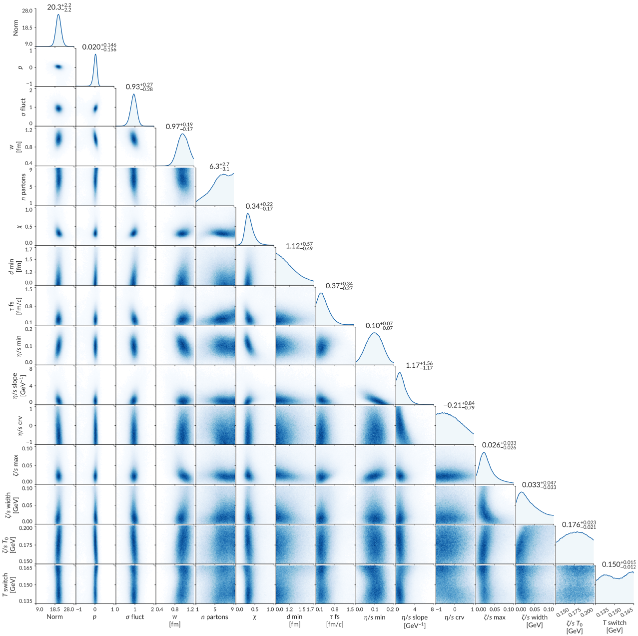
- Observables:
  Pb+Pb: multiplicity, v_2, v_3, v_4
  p+Pb: multiplicity, v_2, v_3, mean-p_T
- The fluctuating proton can generate huge eccentricity fluctuations.
- Prior range well-cover the data from both systems.

Calibration on p-Pb and Pb-Pb systems simultaneously



After compare to data (Posterior, from emulator)

- The calibrated model emulator very well predicts the observable for both p-A and A-A.

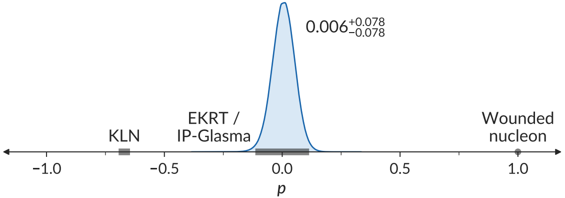


Marginalized posterior distribution

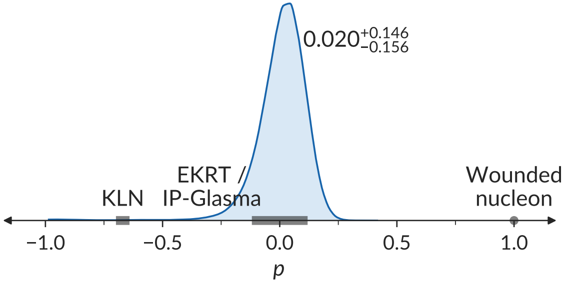
- What does it say about initial condition with proton fluctuations.
- Are the extracted transport coefficients different from using only AA data?

The constrained TRENTo with proton fluctuations

Calibrate on Pb+Pb at 2.76 TeV and 5.02 TeV



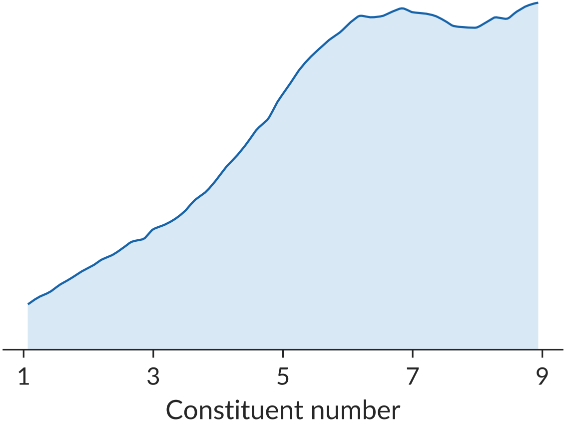
Calibrate on p+Pb and Pb+Pb at 5.02 TeV



TRENTo parameters

- The analysis still prefers entropy deposition with $p = 0$.

The constrained TRENTo with proton fluctuations



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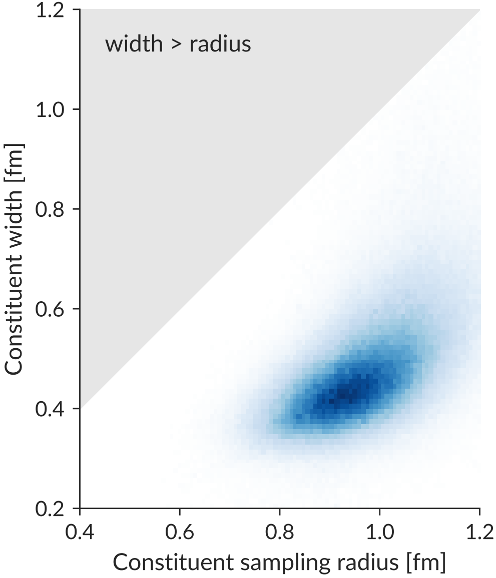
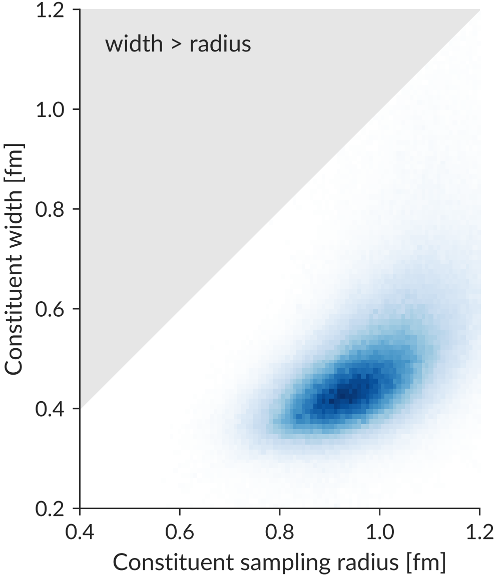


TRENTo parameters

- The analysis still prefers entropy deposition with $p = 0$.
- No particular preference of # of constituents. But a round proton ($n = 1$ case) is disfavored.
- The joint distribution of constituent width and its sampling radius within a proton is well constrained.

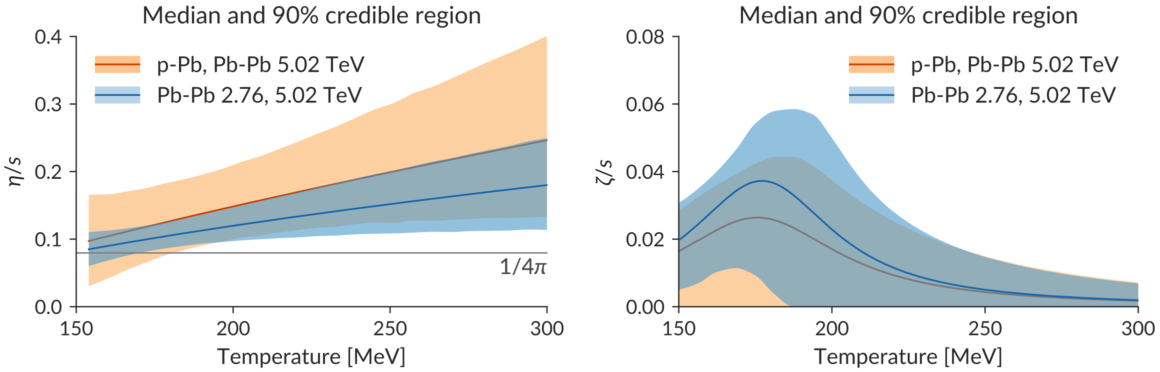
The constrained TRENTo with proton fluctuations



TRENTo parameters

- The analysis still prefers entropy deposition with $p = 0$.
- No particular preference of # of constituents. But a round proton ($n = 1$ case) is disfavored.
- The joint distribution of constituent width and its sampling radius within a proton is well constrained.
- Need to check the resulting proton eccentricities.

Do we need a different $\eta/s, \zeta/s$ to describe p-A?



Comparing transport properties from two calibration.

- The $\eta/s, \zeta/s$ that describe pA and AA @ 5.02TeV is **consistent** with those describing a lot more AA observables @ 2.76 and 5.02 TeV.
- No extra handle on $\eta/s, \zeta/s$ by including pA.

How can a 3+1D simulation help us understand the nature of pA

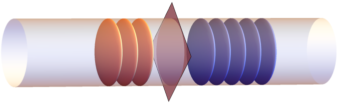
Looking at event-plane decorrelations

- In Pb-Pb collisions, the event-planes decorrelate over pseudorapidity due to initial participant plane twisting and fluctuations.
- If medium in pA also undergoes pressure driven expansion, event-plane decorrelation should be described in the same framework as AA.
- In an initial state approach, the decorrelation has a different origin.

Extending TRENTo (round proton version) to finite rapidity

$$\frac{dS}{dx_\perp^2 dy} \propto s_0(\vec{x}_\perp, y=0) \quad \times \quad f(y, x_\perp).$$

$$\text{T}_\text{R}\text{ENTo} \quad \times \quad \text{rapidity profile}$$



- $f(y, x_\perp)$ has three degrees of freedom (first 3 y-moments):

mean $\mu(x_\perp)$, std $\sigma(x_\perp)$, skewness $\gamma(x_\perp)$

$$f(y, x_\perp) \propto \mathcal{F}^{-1} \exp\left\{ i\mu k - \frac{1}{2}(\sigma k)^2 - \frac{i}{6}\gamma(\sigma k)^3 + \ldots \right\}$$

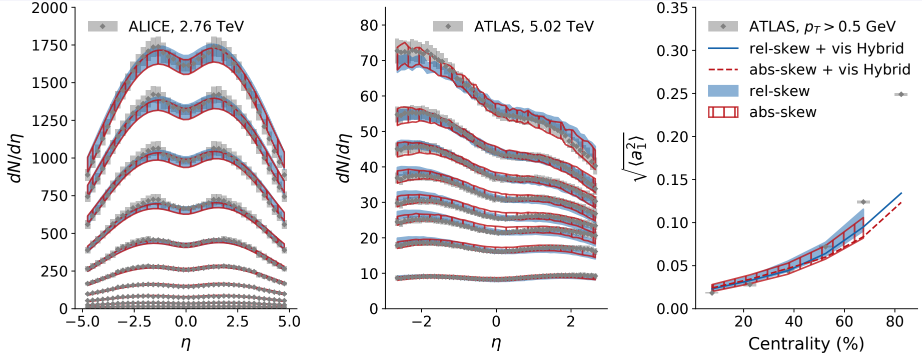
- μ, σ, γ parametrized in nuclear thickness functions $T_A(x_\perp)$, $T_B(x_\perp)$.

mean (μ)	std (σ)	absolute or relative-skew (γ)
$\frac{\mu_0}{2} \log \frac{T_A}{T_B}$	σ_0	$\gamma_0(T_A - T_B)$ or $\gamma_0 \frac{T_A - T_B}{T_A + T_B}$

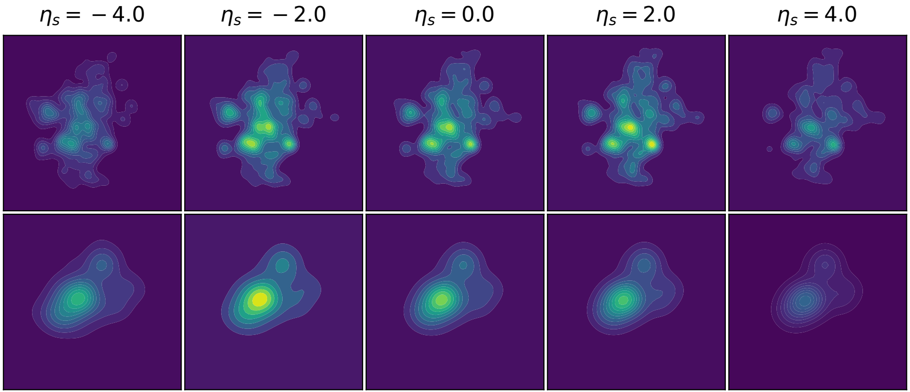
Extending TRENTo (round proton version) to finite rapidity

Calibrated to p-Pb and Pb-Pb

- This calibration did not include proton substructure.
- Parameters constrained by charged particle pseudorapidity density of Pb-Pb and p-Pb and event-by-event psuedorapidity fluctuation in Pb-Pb.



Rapidity evolution in the presence of proton shape-fluctuation.



How much of initial geometry info gets transformed into final state particles.

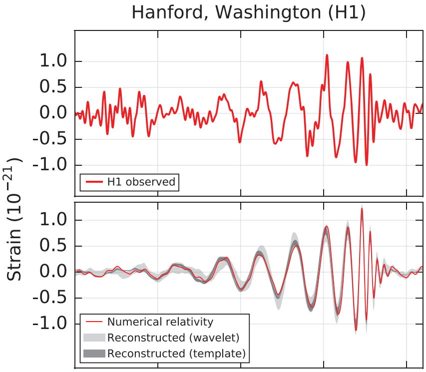
TRENTo $\rightarrow$ 3+1D Free streaming $\rightarrow$ 3+1D viscous hydrodyanmics $\rightarrow$ UrQMD.

Summary

Example application: Gravitational waves

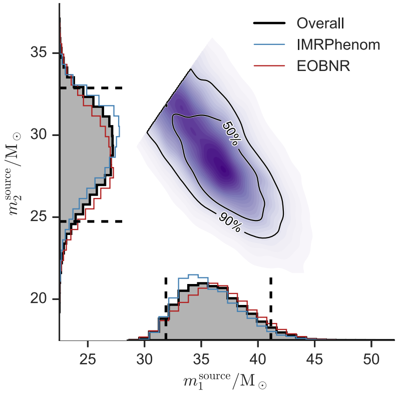
Data

LIGO gravitational wave strain



Posterior distribution

Black hole masses



Summary

- Global Bayesian model-to-data comparison is a powerful quantitative tool to learn QGP properties from experimental data.
- It has been successfully applied to large systems to infer initial condition model and QGP transport coefficients.
- Pb-Pb and p-Pb collisions can be simultaneously described at mid-rapidity by including proton shape fluctuations.
- Looking at event-plane decorrelations in future 3+1D analysis.

Outlooks

Bayes factor

- A quantitative measurement of model performance.
- Can be used to select a better-performing model.
- Introduce natural penalty for over complex model (too many parameters).
- How to calculate:

$$\frac{P(M_1)}{P(M_2)} = \frac{\int P(\vec{p}|\mathrm{Exp}, M_1) d\vec{p}}{\int P(\vec{p}|\mathrm{Exp}, M_2) d\vec{p}}$$

- It has been applied to comparing hydro+UrQMD v.s. hydro + partial chemical equilibrium EoS.

$^{16}O + ^{16}O$

Estimate fluid cell temperature distribution at $\tau = \tau_{\text{hydro}}$ using TRENTo initial condition.