Upper bound of hydrodynamic fluctuations in multiplicities

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HYDRODYNAMIC FLUCTUATIONS

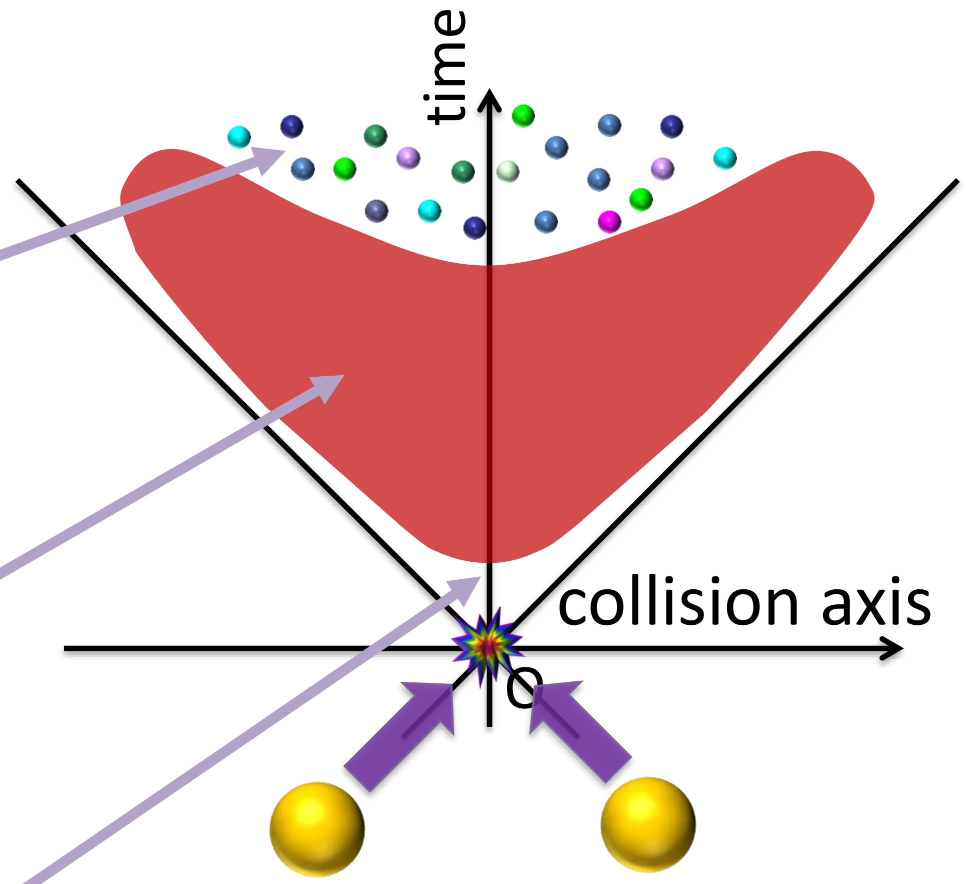
Hydrodynamic model

Final observables

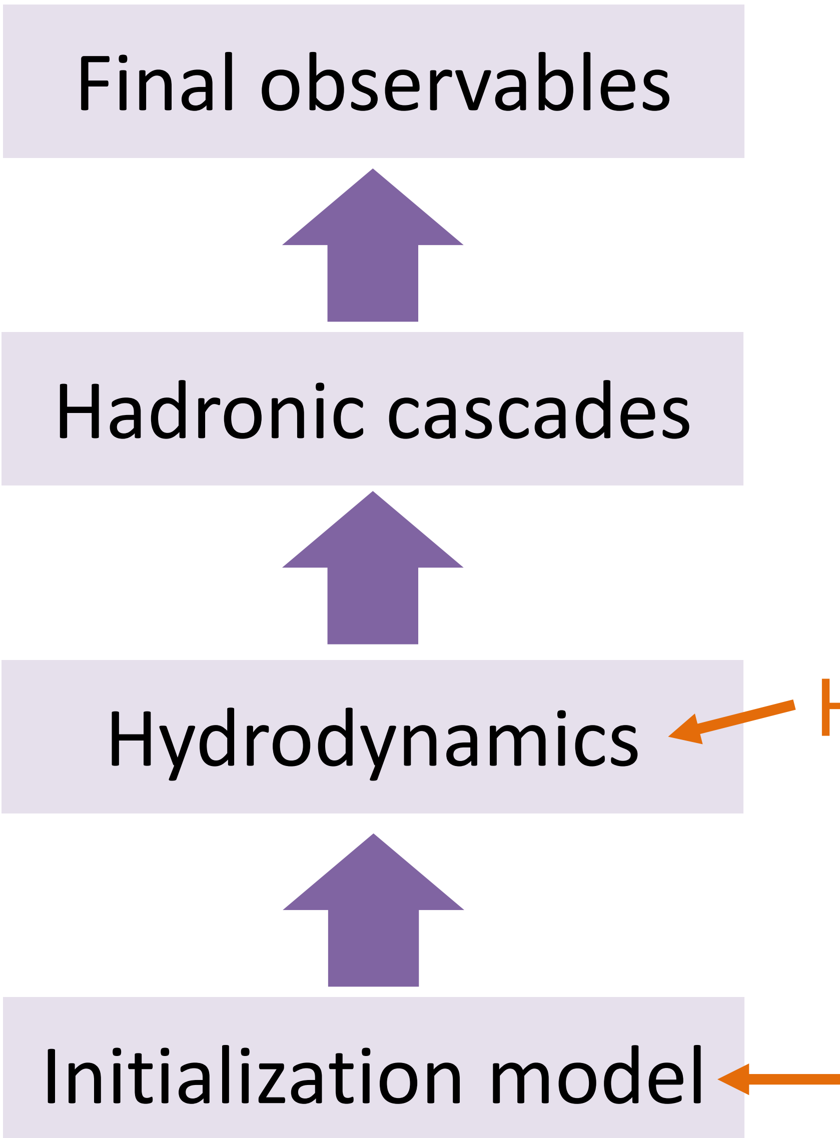
Hadronic cascades

Hydrodynamics

Initialization model



Fluctuations/Correlations



Non-flow contributions:
jets/mini-jets,
core-corona,
etc.

Hydrodynamic fluctuations:
thermal fluctuations of
hydrodynamics

Initial state fluctuations

Fluctuating hydrodynamics

= viscous hydrodynamics with thermal fluctuations

Stochastic partial differential equations (SPDE)

Conservation law $\partial_\mu T^{\mu\nu} = 0.$

Constitutive eqs.

hydrodynamic fluctuations

$$\pi^{\mu\nu} + \tau_\pi \Delta^{\mu\nu}{}_{\alpha\beta} \mathrm{D}\pi^{\alpha\beta} = 2\eta\partial^{\langle\mu}u^{\rangle\nu} + \cdots + \xi_\pi^{\mu\nu},$$
$$[1 + \tau_\Pi \mathrm{D}]\Pi = -\zeta\partial_\mu u^\mu + \cdots + \xi_\Pi.$$

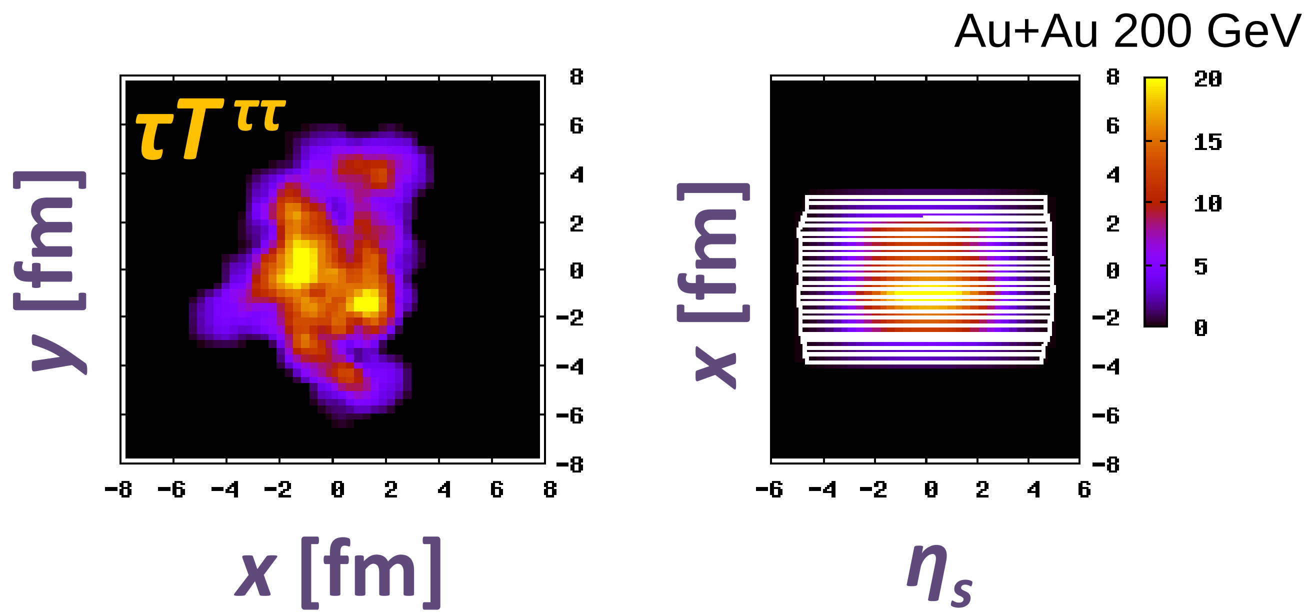
Gaussian noise

Fluctuation-dissipation relation (FDR)

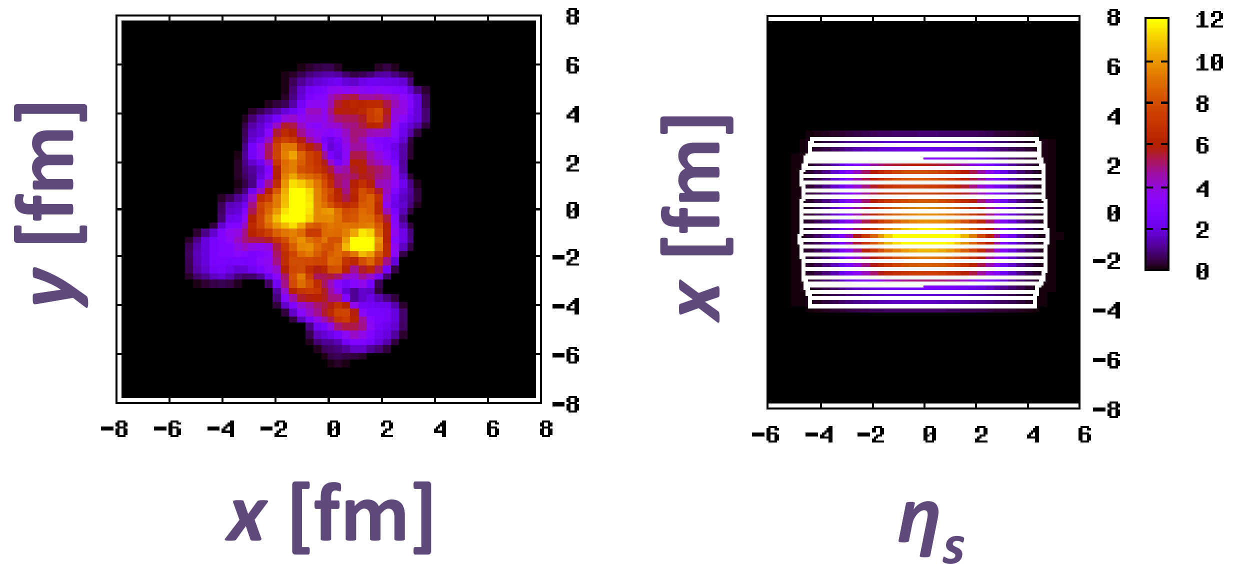
$$\langle \xi^{\mu\nu}(x)\xi^{\alpha\beta}(x')\rangle = 4T\eta\Delta^{\mu\nu\alpha\beta}\delta^{(4)}(x-x'),$$
$$\langle \xi(x)\xi(x')\rangle = 2T\zeta\delta^{(4)}(x-x'),$$

Hydrodynamic fluctuations in AA

w/o hydro fluctuations



w/ hydro fluctuations



Hydrodynamic fluctuations in AA

Pb+Pb 2.76 TeV

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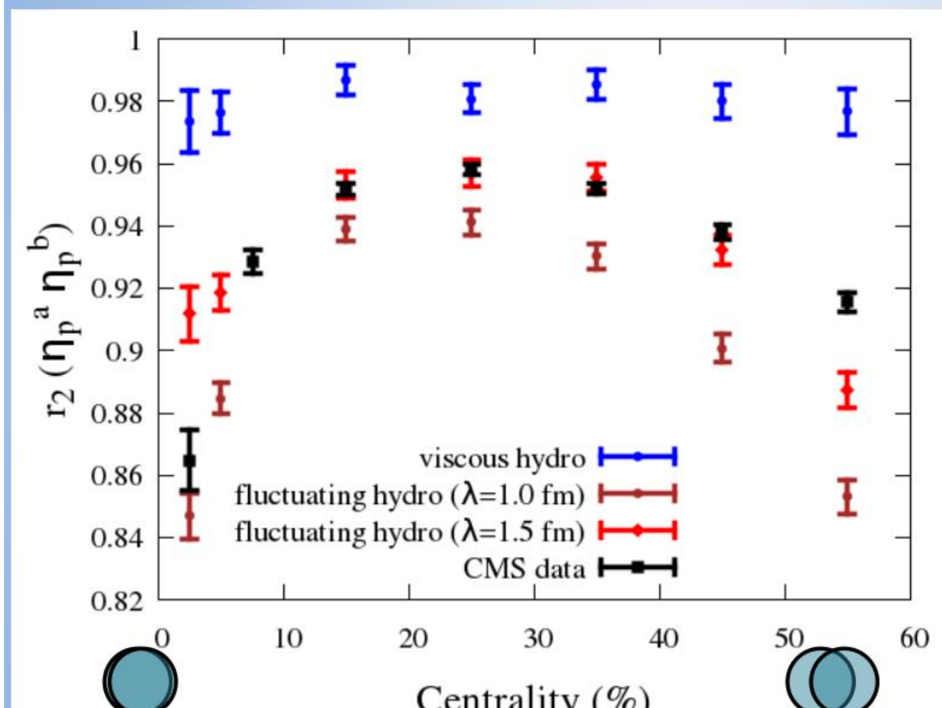
Factorization ratio $r_2(\eta_p^a, \eta_p^b)$



Viscous > CMS data

Central: Fluctuating hydro ($\lambda = 1.0$ fm) ≈ CMS data

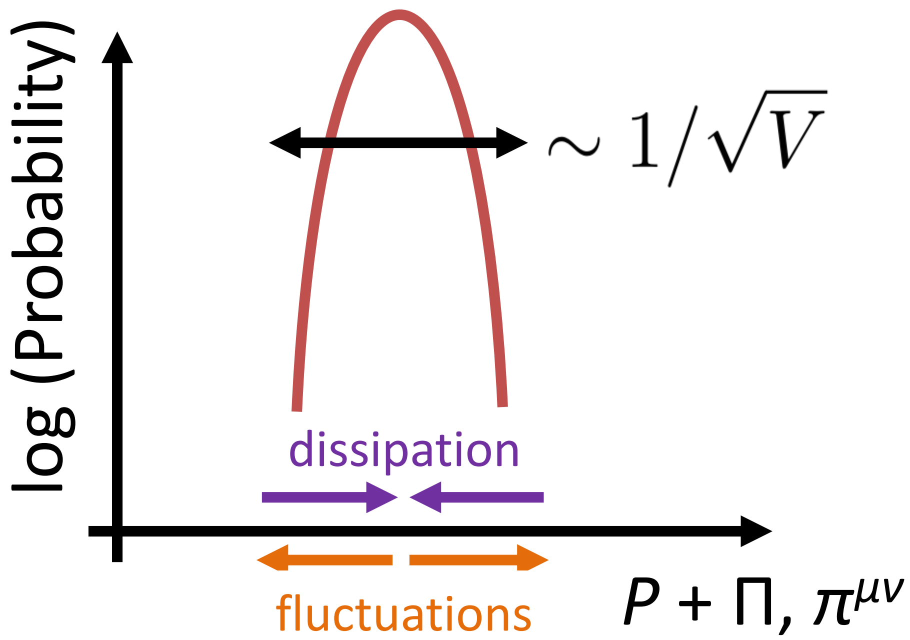
Mid-central: Fluctuating hydro ($\lambda = 1.5$ fm) ≈ CMS data

$2.0 < \eta_p^a < 2.5,\ 3.0 < \eta_p^b < 4.0$

Fluctuation-dissipation theorem

Thermal distribution of fluid fields

$$\ln \mathrm{Pr} \sim S \sim V \cdot \left(s_{\mathrm{eq}} - \frac{\tau_\Pi \Pi^2}{2T\zeta} - \frac{\tau_\pi \pi^{\mu\nu}\pi_{\mu\nu}}{4T\eta} \right)$$



FDR = Balance of fluctuations and dissipation to maintain the thermal distribution

Fluctuation-dissipation theorem

Hydrodynamic fluctuations scales as

$$\sim 1/\sqrt{V}$$

$\longrightarrow$ Hydrodynamic fluctuations
more significant in small systems?

Today: We find a qualitative understanding on the consequences of hydrodynamic fluctuations in a simple setup using Fluctuation Theorem (FT).

FLUCTUATION THEOREM

Fluctuation theorem (FT)

= Generalization of FDR
in non-equilibrium statistical mechanics

= Relation of probability distribution
of entropy production, Pr(δ*S*):

$$\ln \frac{\Pr(\delta S = \alpha)}{\Pr^{\dagger}(\delta S^{\dagger} = -\alpha)} = \alpha$$

Note: Definition of Pr(δS) and $\mathrm{Pr}^{\dagger}(\delta S^{\dagger})$ depends on the process

Fluctuation theorem (FT)

e.g. Crooks FT

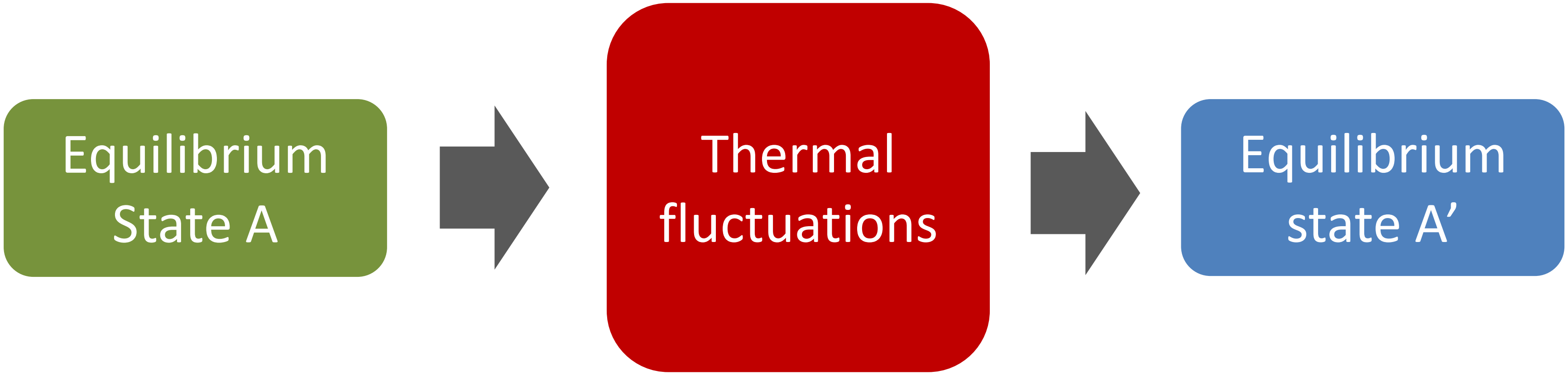
$$\ln \frac{\Pr(\delta S = \alpha)}{\Pr^{\dagger}(\delta S^{\dagger} = -\alpha)} = \alpha$$

Probability distributions of entropy production

- Pr(δS): the process A→B
- Pr†(δS†): the reverse process B→A

Fluctuation theorem (FT)

e.g. Steady-state FT (SSFT)



$$\ln \frac{\Pr(\bar{\sigma} = \alpha)}{\Pr(\bar{\sigma} = -\alpha)} = \alpha t + O(\tau_R / t)$$

σ = δS / t: entropy production rate

t: elapsed time

FLUCTUATION THEOREM IN 0+1D BJORKEN FLOW

Idealized conditions

FT in high-energy nuclear collisions?

→ **Idealized conditions**

- (1) 0+1D *fluctuating* Bjorken flow
- (2) The non-linear contribution of hydrodynamic fluctuations is negligible
- (3) The time-evolution scale is longer than relaxation time (Navier-Stokes limit)

Hydrodynamic equations

Fluctuating hydrodynamic eqs. in 0+1D Bjorken flow

Conservation

τ: proper time

$$\frac{\mathrm{d}e}{\mathrm{d}\tau} = -\frac{e+p}{\tau}\left(1-\frac{\pi-\Pi}{sT}\right)$$

Constitutive eqs

$$\pi = \pi^{00} - \pi^{33}$$

$$\left(\tau_\pi \frac{\mathrm{d}}{\mathrm{d}\tau} + 1\right)\pi = \frac{4\eta}{3\tau} + \xi_\pi,$$

$$\left(\tau_\Pi \frac{\mathrm{d}}{\mathrm{d}\tau} + 1\right)\Pi = -\frac{\zeta}{\tau} + \xi_\Pi.$$

FDR

$$V(\tau) = \tau\Delta\eta_s\Delta x\Delta y$$

$$\langle \xi_\pi(\tau)\xi_\pi(\tau')\rangle = \frac{8\eta T}{3\tau\Delta\eta_s\Delta x\Delta y}\delta(\tau-\tau'),$$

$$\langle \xi_\Pi(\tau)\xi_\Pi(\tau')\rangle = \frac{2\zeta T}{\tau\Delta\eta_s\Delta x\Delta y}\delta(\tau-\tau').$$

Entropy production rate

Definition

$$\bar{\sigma} := \frac{s(\tau)V(\tau) - s(\tau_{\rm i})V(\tau_{\rm i})}{\tau - \tau_{\rm i}}.$$

s: equlibrium entropy density
V(τ): expanding volume
$\tau_{\rm i}$: initial time

Solution (single event)

$$\bar{\sigma} = \frac{1}{\tau - \tau_{\rm i}} \int_{\tau_{\rm i}}^{\tau} {\rm d}\tau' \frac{\pi(\tau') - \Pi(\tau')}{T(\tau')} \Delta\eta_s \Delta x \Delta y.$$

$$\pi(\tau) = \int_{\tau_{\rm i}}^{\tau} {\rm d}\tau' G_\pi(\tau,\tau') \frac{4\eta}{3\tau'} + \delta\pi(\tau),$$

$$\Pi(\tau) = -\int_{\tau_{\rm i}}^{\tau} {\rm d}\tau' G_\Pi(\tau,\tau') \frac{\zeta}{\tau'} + \delta\Pi(\tau),$$

$$G_{\pi/\Pi}(\tau_2,\tau_1) := \exp\left(-\int_{\tau_1}^{\tau_2} \frac{{\rm d}\tau}{\tau_{\pi/\Pi}(\tau)}\right) \frac{1}{\tau_{\pi/\Pi}(\tau_1)}$$

$$\delta\pi(\tau) = \int_{\tau_{\rm i}}^{\tau} {\rm d}\tau' G_\pi(\tau,\tau') \xi_\pi(\tau'),$$

$$\delta\Pi(\tau) = \int_{\tau_{\rm i}}^{\tau} {\rm d}\tau' G_\Pi(\tau,\tau') \xi_\Pi(\tau').$$

Entropy production rate

Distribution

Only linear contribution is considered
→ final distribution is **Gaussian**

In the Navier Stokes limit,

Mean $$\langle \bar{\sigma} \rangle = \frac{\Delta\eta_s \Delta x \Delta y}{\tau - \tau_{\mathrm{i}}} \int_{\tau_{\mathrm{i}}}^{\tau} \frac{\mathrm{d}\tau'}{T_0(\tau')} \left(\frac{4\eta}{3\tau'} + \frac{\zeta}{\tau'} \right),$$

Variance $$a^2 = \langle \bar{\sigma}^2 \rangle - \langle \bar{\sigma} \rangle^2 = \frac{2\Delta\eta_s \Delta x \Delta y}{(\tau - \tau_{\mathrm{i}})^2} \int_{\tau_{\mathrm{i}}}^{\tau} \frac{\mathrm{d}\tau'}{T_0(\tau')} \left(\frac{4\eta}{3\tau'} + \frac{\zeta}{\tau'} \right)$$

SSFT We find

$$\frac{2\langle \bar{\sigma} \rangle}{a^2} = \tau - \tau_{\mathrm{i}} \quad \Leftrightarrow \quad \ln \frac{\Pr(\bar{\sigma} = \alpha)}{\Pr(\bar{\sigma} = -\alpha)} = \alpha t + O(\tau_R / t)$$

Entropy fluctuations

Consequences of FT in high-energy nuclear collisions?

$$\frac{\Delta S(\tau)}{\langle S(\tau)\rangle} = \frac{a(\tau-\tau_i)}{S_i + \langle\bar{\sigma}\rangle(\tau-\tau_i)}$$

$$= \frac{\sqrt{2\langle\bar{\sigma}\rangle(\tau-\tau_i)}}{S_i + \langle\bar{\sigma}\rangle(\tau-\tau_i)}$$

$$\leq \frac{1}{\sqrt{2S_\mathrm{i}}}$$

S: total entropy

S_i: initial total entropy

← used SSFT

← used mathematical inequality

$$\frac{\sqrt{2y}}{x+y} \leq \frac{1}{\sqrt{2x}}$$

upper bound of relative fluctuations of total entropy determined by the initial total entropy

Entropy fluctuations

Consequences of FT in high-energy nuclear collisions?

$$\Delta S_{\rm f} \leq \frac{\langle S_{\rm f} \rangle}{\sqrt{2 S_{\rm i}}}$$

Average over initial state fluctuations

$$\langle N^2 \rangle - \langle N \rangle^2 \sim \langle (\Delta S_{\rm f})^2 \rangle_{\rm ev} \leq \left\langle \frac{\langle S_{\rm f} \rangle^2}{2 S_{\rm i}} \right\rangle_{\rm ev}$$

Maybe it is interesting to compare with multiplicity fluctuations

NUMERICAL TESTS

Idealized conditions?

Idealized conditions used to show SSFT?

- (1) 0+1D *fluctuating* Bjorken flow
- (2) The non-linear contribution of hydrodynamic fluctuations is negligible
- (3) The time-evolution scale is longer than relaxation time (Navier-Stokes limit)

→ Numerical tests:

Simulation of **0+1D Bjorken flow**

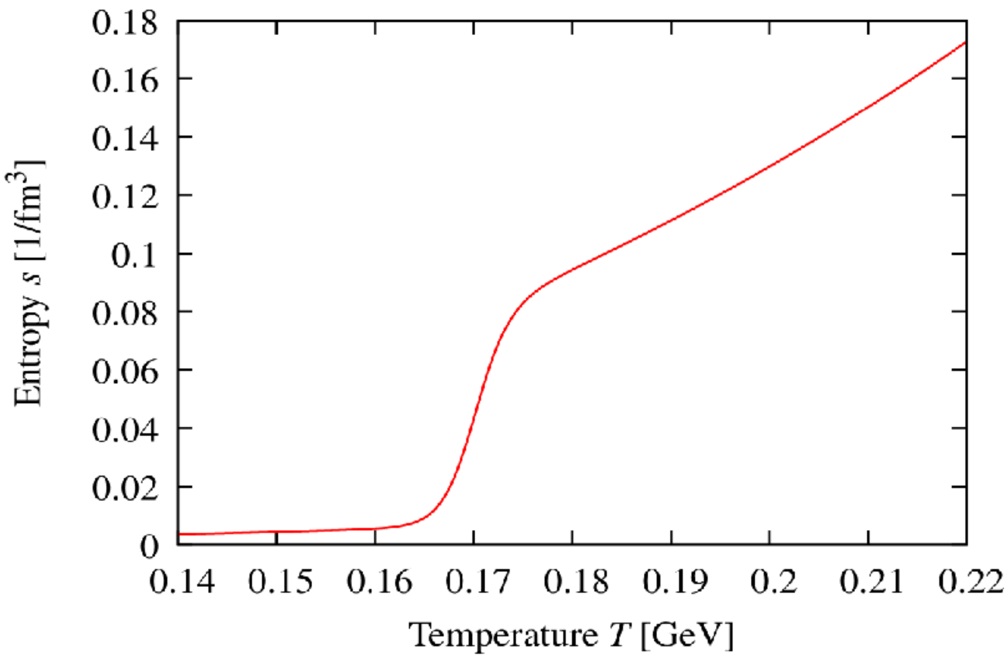
Numerical setup

- EoS given by parametrized entropy density

$$
\begin{aligned}
s(T) = &\frac{4\pi^2}{90} g_h\, T^3 \frac{1 - \tanh\left(\frac{T-T_c}{d}\right)}{2} \\
&+ \frac{4\pi^2}{90} g_q\, T^3 \frac{1 + \tanh\left(\frac{T-T_c}{d}\right)}{2}.
\end{aligned}
$$

$$g_{\mathrm{h}} = 3 \text{ and } g_{\mathrm{q}} = 37$$

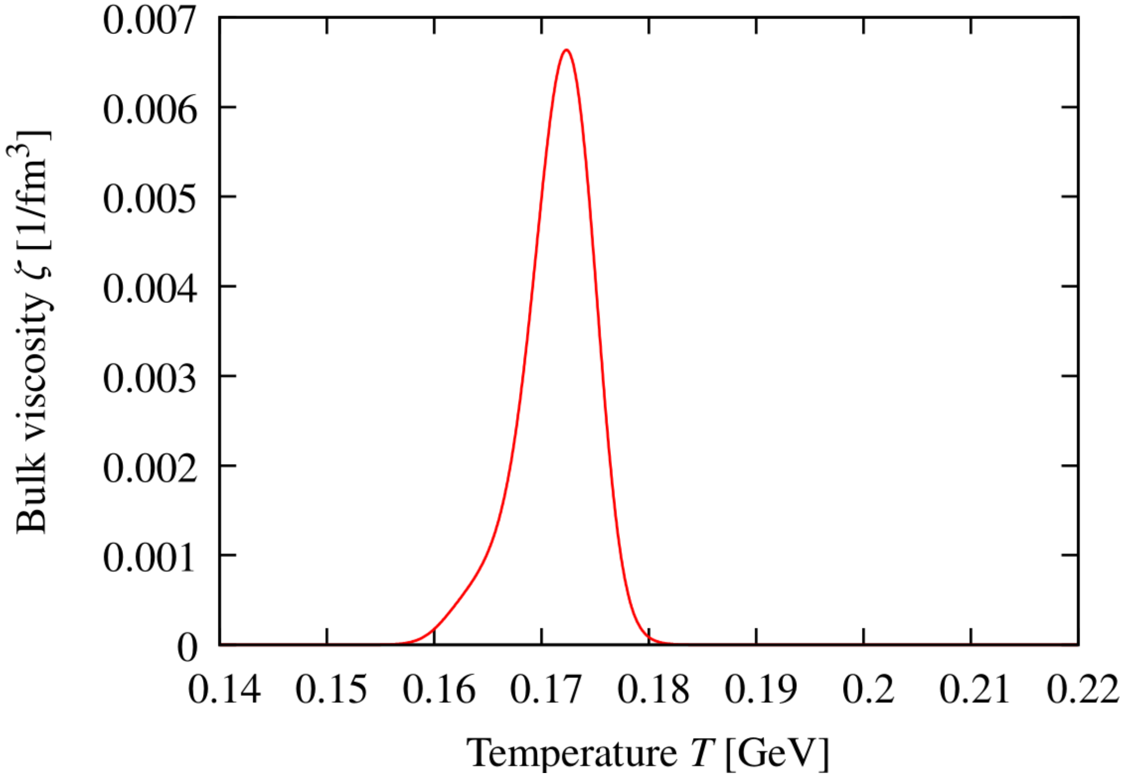
$$T_{\mathrm{c}} = 170 \text{ MeV and } d = T_{\mathrm{c}}/50 \text{ MeV}$$



Numerical setup

- Shear/bulk viscosity

$$\frac{\eta}{s} = \frac{1}{4\pi},$$
$$\frac{\zeta}{s} = 15\left(\frac{1}{3} - c_{\mathrm{s}}^2\right)^2 \frac{\eta}{s},$$



Numerical setup

- Relaxation time (three cases)

1. default

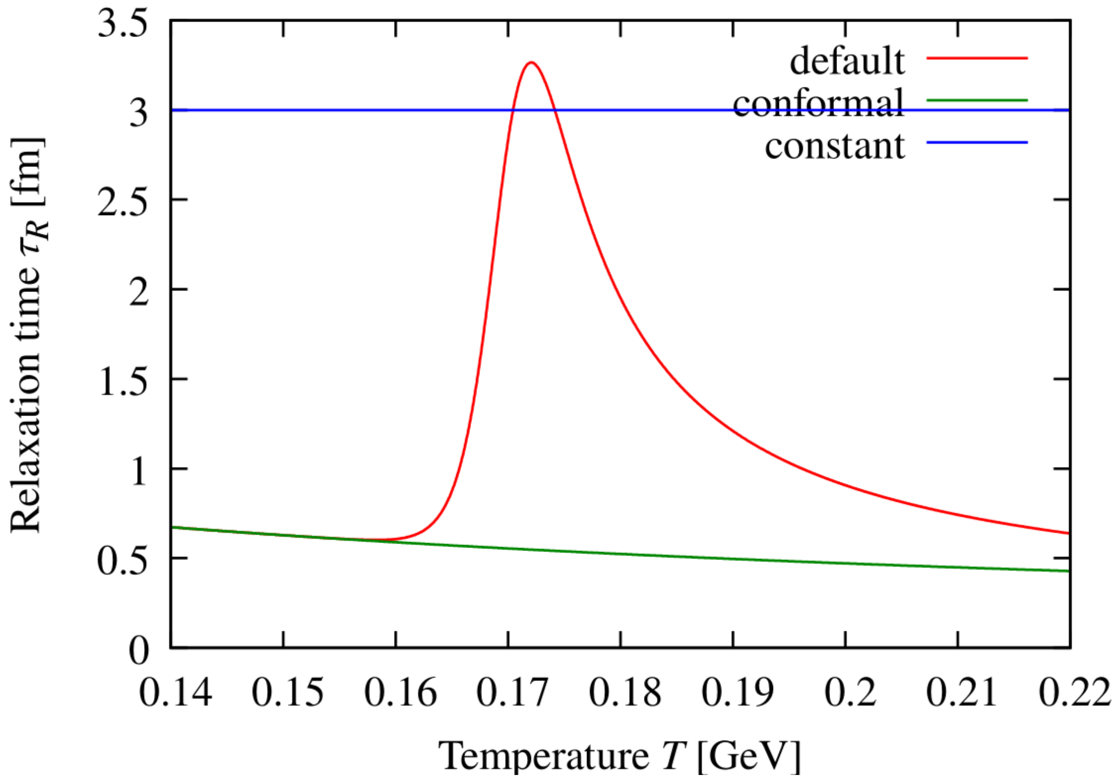
$$\tau_\pi = \tau_\Pi = \frac{3\eta}{2p}$$

2. conformal

$$\tau_\pi = \tau_\Pi = 3/2\pi T$$

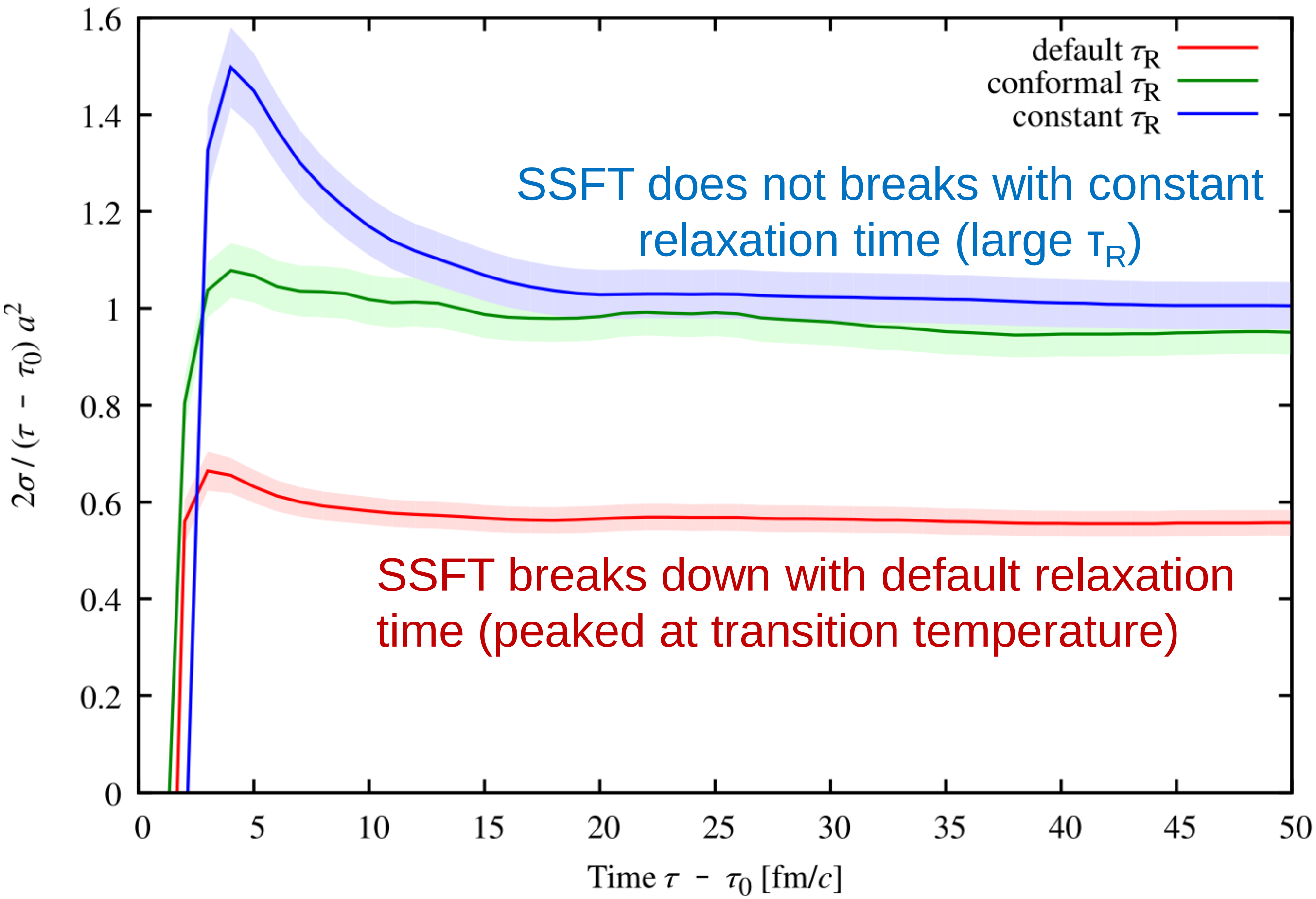
3. constant

$$\tau_\pi = \tau_\Pi = 0.3\text{fm}$$

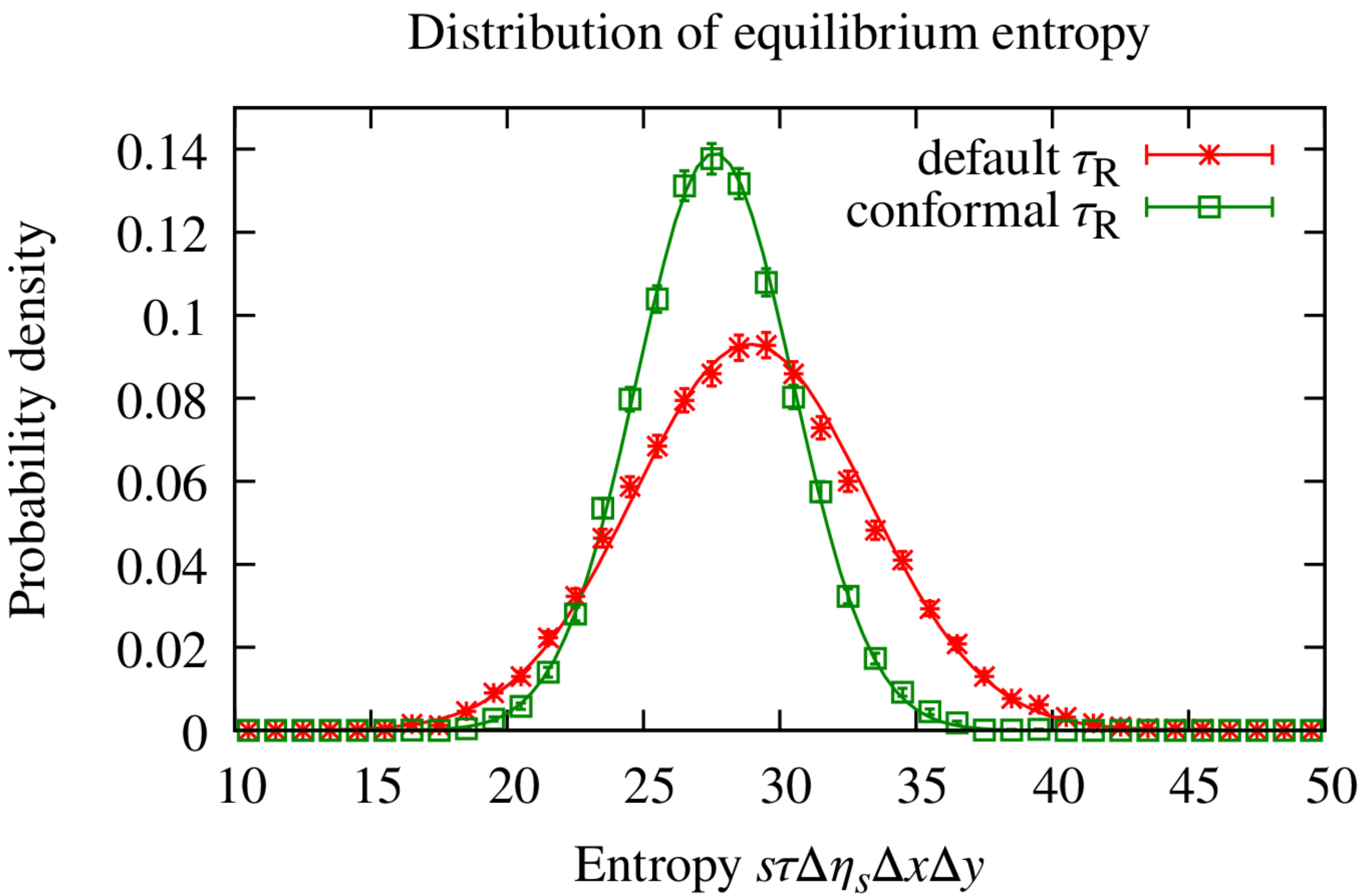


SSFT ratio

SSFT $$\frac{2\langle\sigma\rangle}{a^2(\tau - \tau_\mathrm{i})} \sim 1$$



Entropy distribution



Well fitted by Gaussian distribution ～ linear regime

Summary

- Under idealized conditions of (1) Bjorken flow, (2) Linear fluctuations, and (3) the Navier-Stokes limit, Steady-state Fluctuation Theorem is obtained for the relativistic fluctuating hydrodynamics.
- Upper bound of the entropy fluctuations caused by hydrodynamic noises is determined by the initial total entropy.
- When the relaxation time changes fast, the Steady-state fluctuation theorem breaks.

BACKUP