



# Power efficiency vs instability in plasma accelerators

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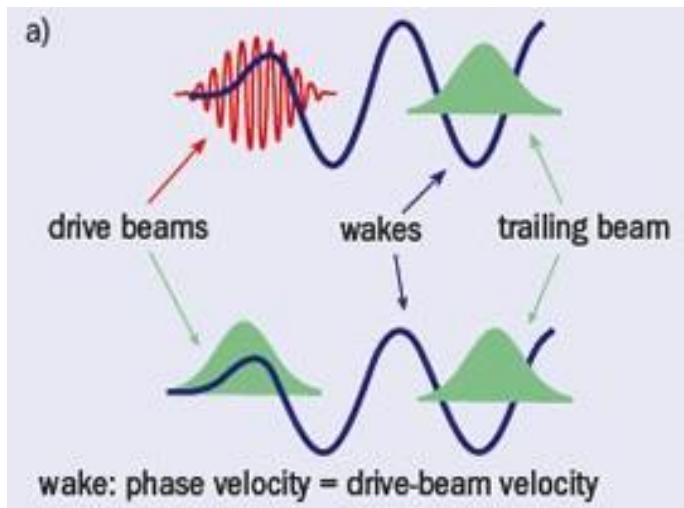
# Acknowledgements

- We would like to thank our UCLA (esp. Weiming An) and SLAC (esp. G. Stupakov) colleagues for fruitful discussions and computer simulations.
- Some of the results presented in this talk are based on our recent publication, “Efficiency versus instability in plasma accelerators”, Phys. Rev. Accel. Beams 20, 121301 – Published 20 December 2017

# Quasi-linear regime vs “Bubble” (a.k.a blow-out) regime

- These two regimes apply to the trailing beam, not the drive beam.

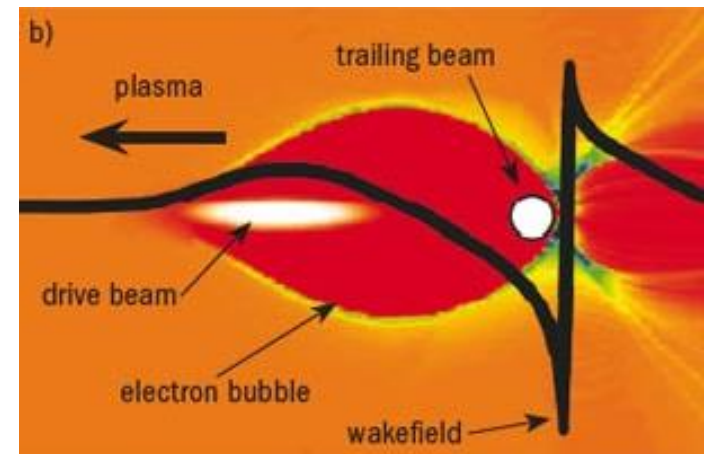
## Quasi-linear



$$n_b < n_e$$

suitable for both e- and e+

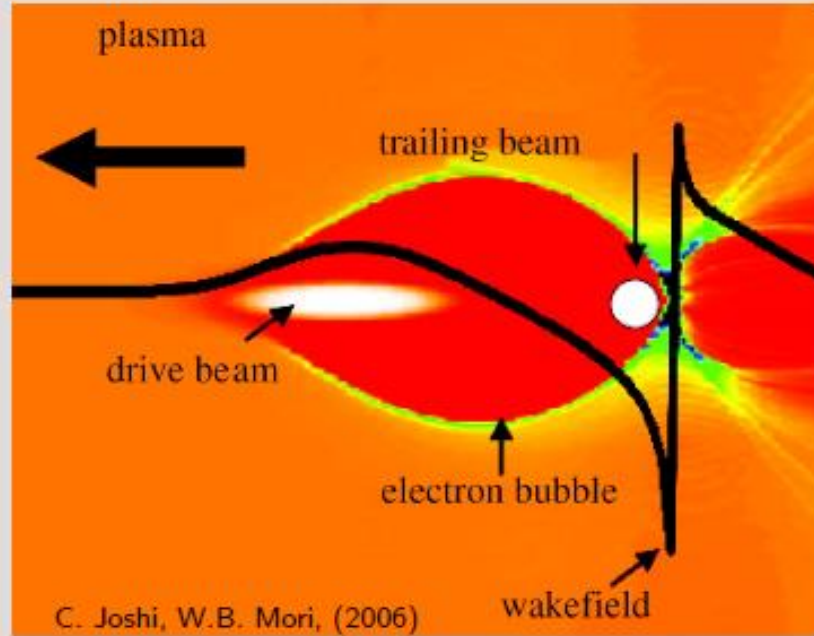
## Bubble



$$n_e = 0$$

Suitable for e-,  
not suitable for e+  
(at least with e- driver)

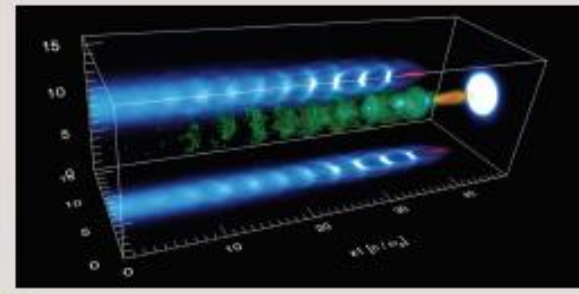
# Introduction



- Plasma wake excited by relativistic particle bunch
- "Blow-out" regime when  $n_b/n_e > 1$
- Acceleration and focusing by plasma
- Accelerating field scales as  $n_e^{1/2}$
- Typical:  $n_e \sim 10^{17} \text{ cm}^{-3}$ ,  
 $k_p^{-1} = 17 \text{ } \mu\text{m}$ ,  $E \gtrsim 10 \text{ GV/m}$ ,  $G \gtrsim \text{MT/m}$



1 m, 20 MV/m

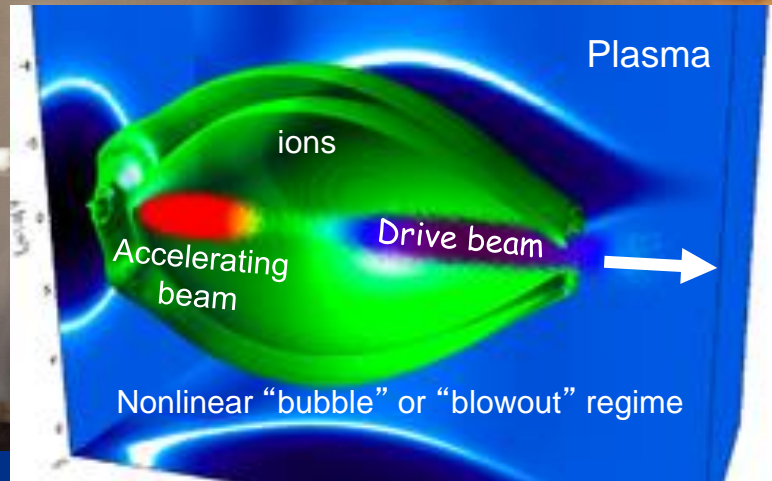


100  $\mu\text{m}$ , 20 GV/m





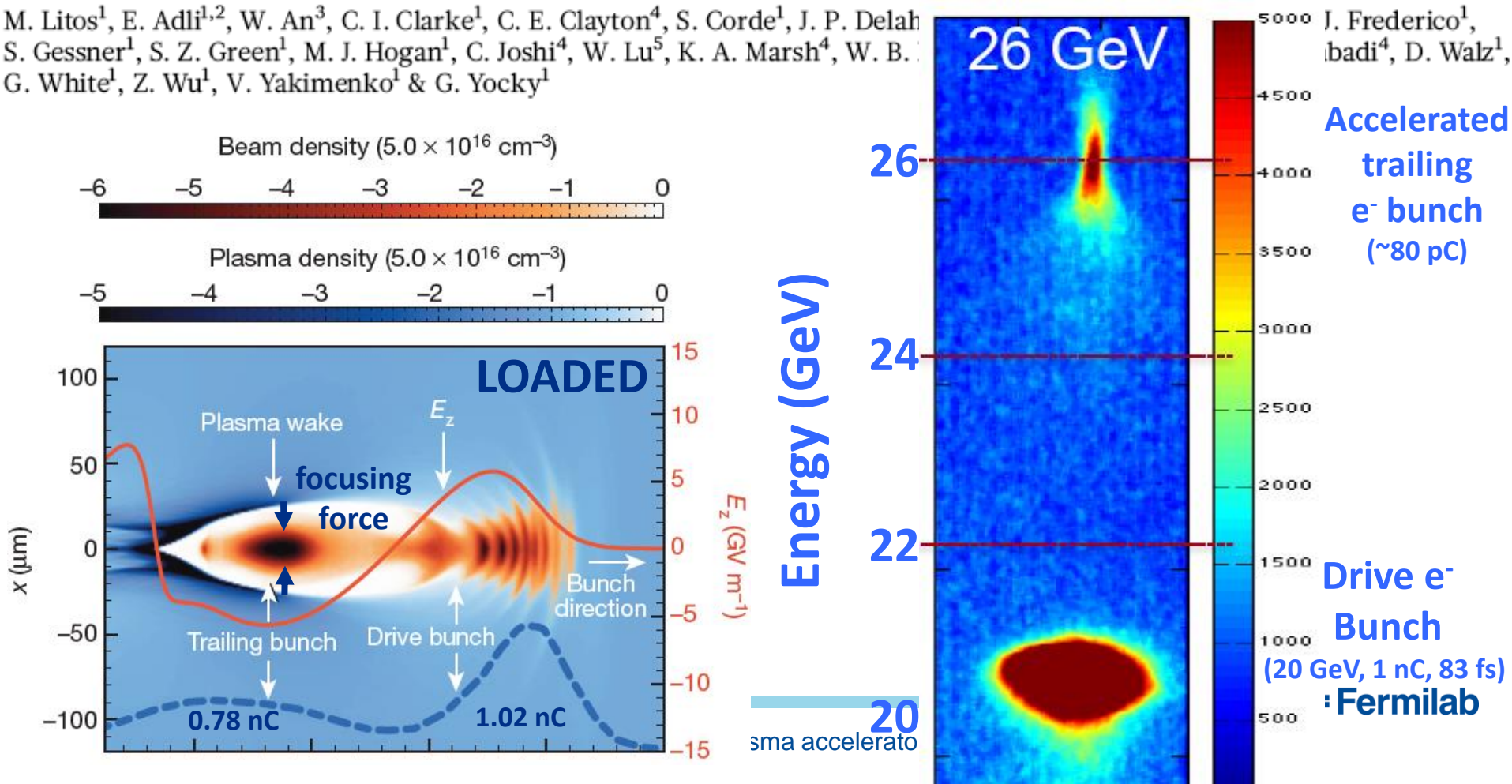
FACET used SLAC injector & 2/3 of linac to deliver synchronized 20 GeV  $e^\pm$  drive & witness bunches to a 1m plasma





# High-efficiency acceleration of an electron beam in a plasma wakefield accelerator

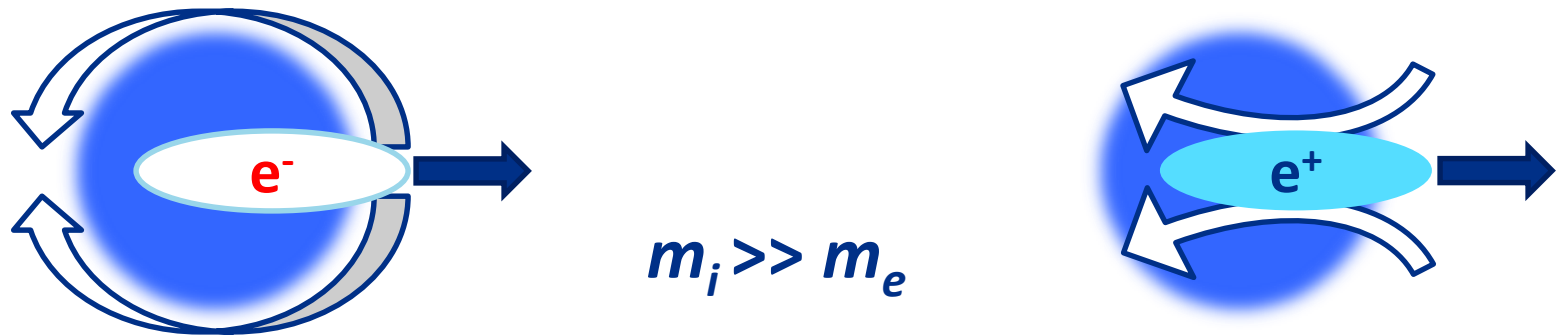
M. Litos<sup>1</sup>, E. Adli<sup>1,2</sup>, W. An<sup>3</sup>, C. I. Clarke<sup>1</sup>, C. E. Clayton<sup>4</sup>, S. Corde<sup>1</sup>, J. P. Delahaye<sup>1</sup>, S. Gessner<sup>1</sup>, S. Z. Green<sup>1</sup>, M. J. Hogan<sup>1</sup>, C. Joshi<sup>4</sup>, W. Lu<sup>5</sup>, K. A. Marsh<sup>4</sup>, W. B. Mori<sup>1</sup>, G. White<sup>1</sup>, Z. Wu<sup>1</sup>, V. Yakimenko<sup>1</sup> & G. Yocky<sup>1</sup>



# The Challenge with Positrons

Acceleration of positrons is possible (in principle)  
in a quasi-linear regime ( $n_b < n_e$ )

In a bubble regime one might try using positrons to create a wake

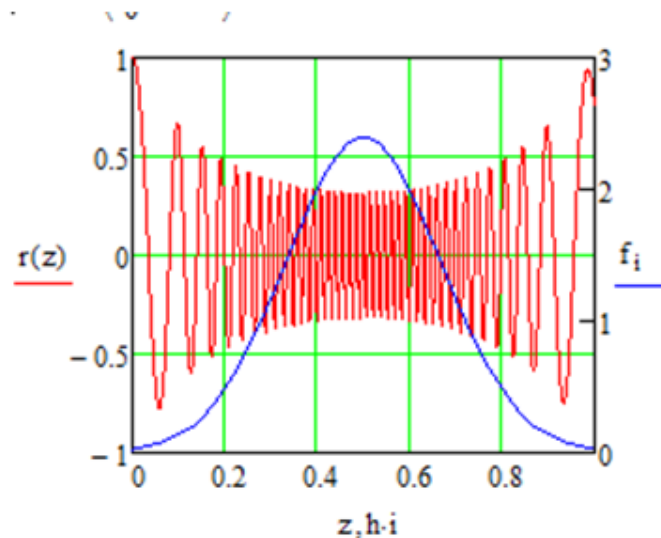


The plasma electrons are mobile but the ions are not.

The symmetry of the accelerating mechanism is broken.

# Actually, the picture is more complicated...

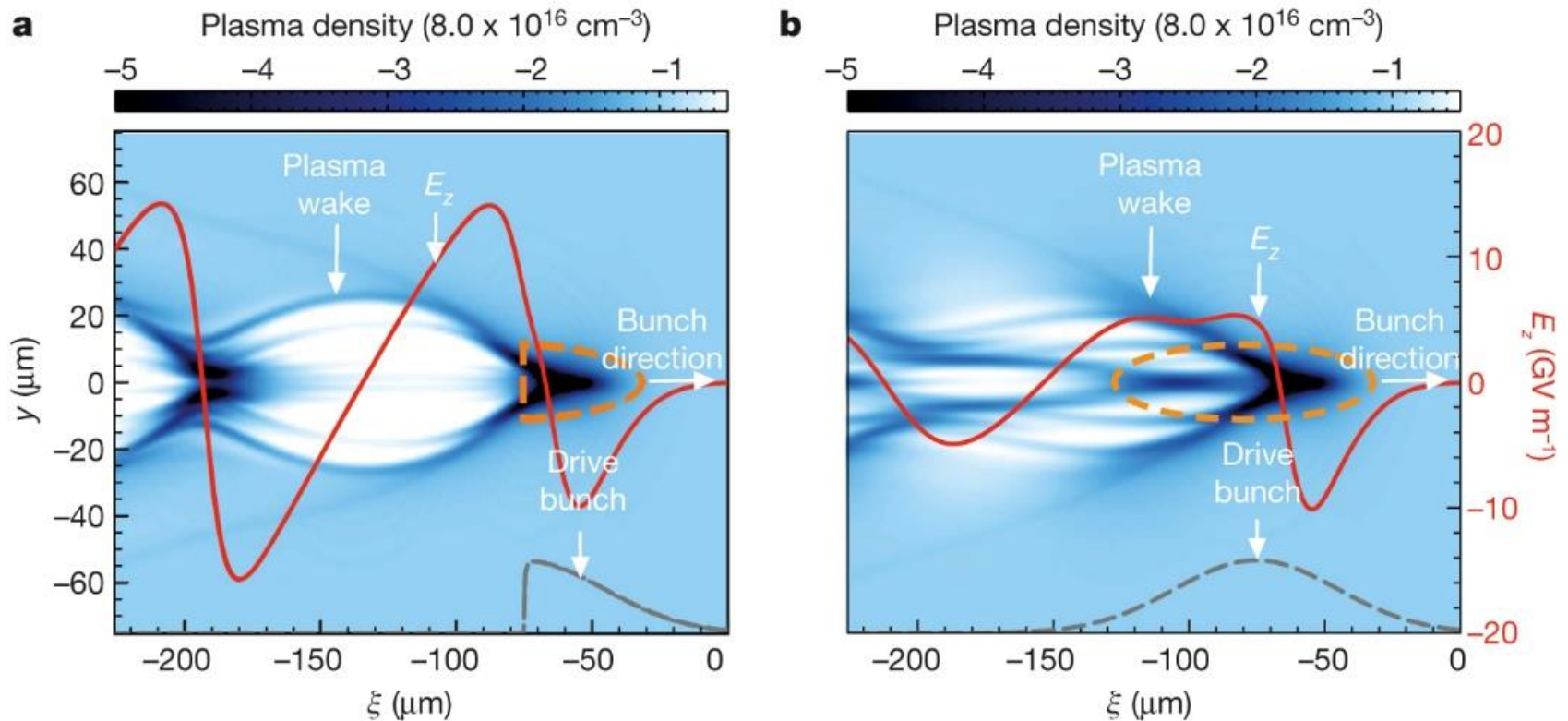
- In a regime of dense positron bunches,  $n_b > n_e$ , the plasma electrons get pulled into the positron bunch, oscillate many times, and create highly-nonlinear focusing



A trajectory of a plasma electron  
inside of the positron bunch ( $4 \times 10^9$ )



# Plasma wake, created by a positron bunch



The long positron bunch draws plasma electrons through the beam volume, which modifies the transverse shape of the beam and wake.

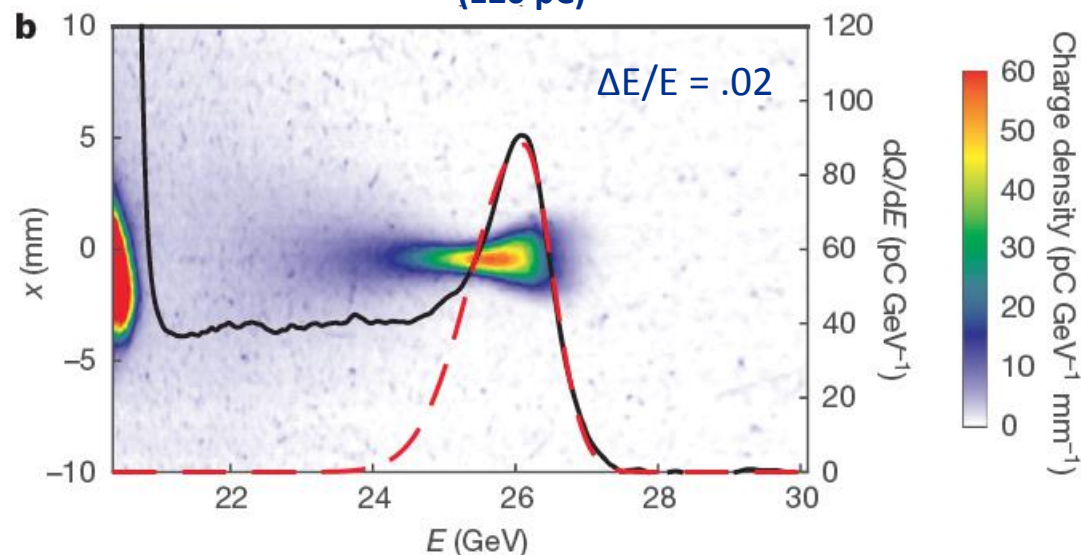
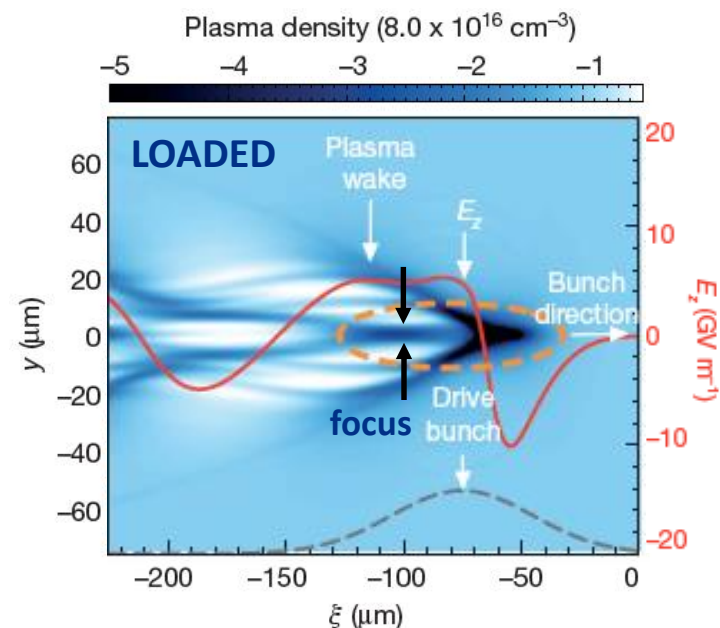
S Corde *et al. Nature* **524**, 442-445 (2015)

# Multi-gigaelectronvolt acceleration of positrons in a self-loaded plasma wakefield

S. Corde<sup>1,2</sup>, E. Adli<sup>1,3</sup>, J. M. Allen<sup>1</sup>, W. An<sup>4,5</sup>, C. I. Clarke<sup>1</sup>, C. E. Clayton<sup>4</sup>, J. P. Delahaye<sup>1</sup>, J. Frederico<sup>1</sup>, S. Gessner<sup>1</sup>, S. Z. Green<sup>1</sup>, M. J. Hogan<sup>1</sup>, C. Joshi<sup>4</sup>, N. Lipkowitz<sup>1</sup>, M. Litos<sup>1</sup>, W. Lu<sup>6</sup>, K. A. Marsh<sup>4</sup>, W. B. Mori<sup>4,5</sup>, M. Schmeltz<sup>1</sup>, N. Vafaei-Najafabadi<sup>4</sup>, D. Walz<sup>1</sup>, V. Yakimenko<sup>1</sup> & G. Yocky<sup>1</sup>

**Drive**  
**e<sup>+</sup> bunch**  
**(20.35 GeV, 2 nC, 130 fs)**

**Accelerated**  
**& focused**  
**e<sup>+</sup> bunch tail**  
**(126 pC)**



- The FACET experimental program is now finished
- We are starting to think about the next step, FACET-II



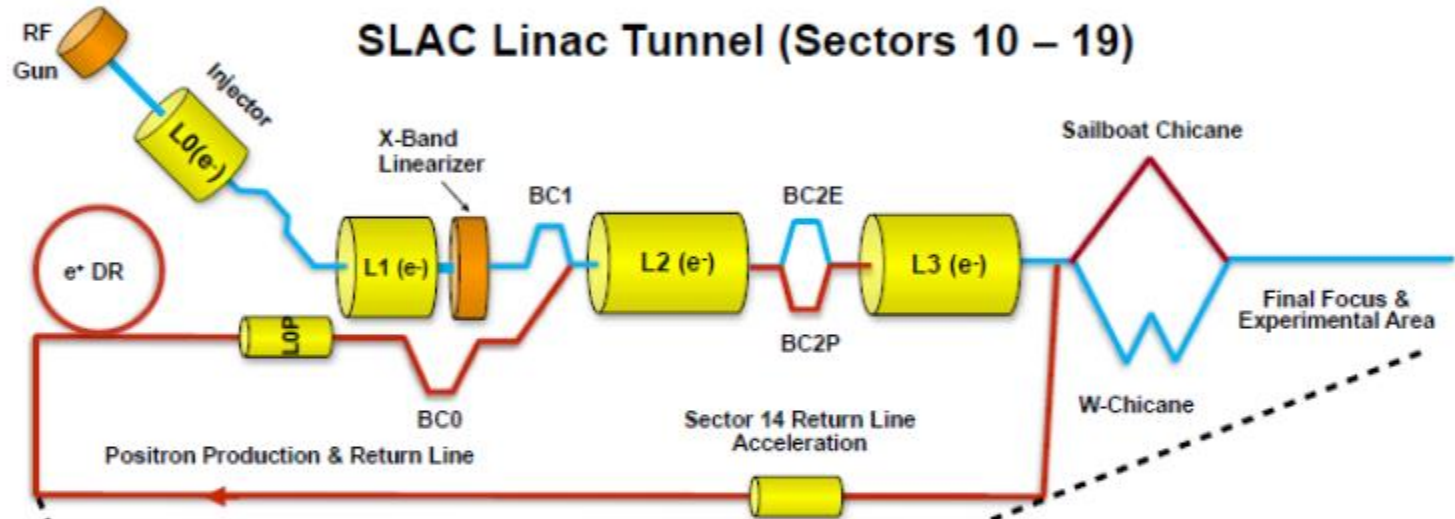
# Planning for FACET-II as a Community Resource

## FACET-II

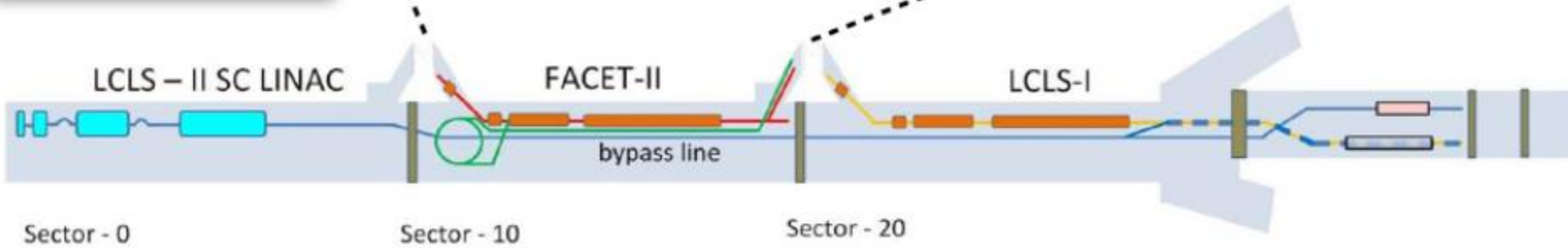
Photo injector  
(e- beam only)  
FY17-20

e+ damping ring  
(e+ or e- beams)  
FY21-22

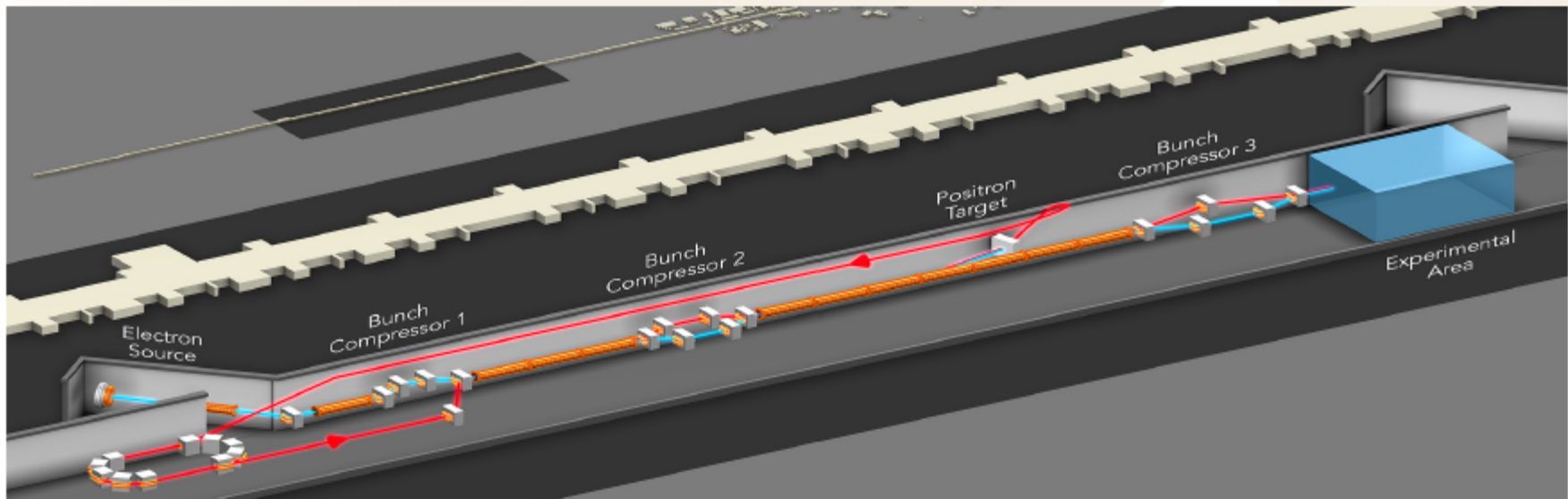
### SLAC Linac Tunnel (Sectors 10 – 19)



## SLAC Linac in 2025



# FACET-II Layout and Beams



Electron Beam Parameter	Baseline Design	Operational Ranges	Positron Beam Parameter	Baseline Design	Operational Ranges
Final Energy [GeV]	10	4.0-13.5	Final Energy [GeV]	10	4.0-13.5
Charge per pulse [nC]	2	0.7-5	Charge per pulse [nC]	1	0.7-2
Repetition Rate [Hz]	30	1-30	Repetition Rate [Hz]	5	1-5
Norm. Emittance $\gamma\epsilon_{x,y}$ at S19 [ $\mu\text{m}$ ]	4.4, 3.2	3-6	Norm. Emittance $\gamma\epsilon_{x,y}$ at S19	10, 10	6-20
Spot Size at IP $\sigma_{x,y}$ [ $\mu\text{m}$ ]	18, 12	5-20	Spot Size at IP $\sigma_{x,y}$ [ $\mu\text{m}$ ]	16, 16	5-20
Min. Bunch Length $\sigma_z$ (rms) [ $\mu\text{m}$ ]	1.8	0.7-20	Min. Bunch Length $\sigma_z$ (rms)	16	8
Max. Peak current $I_{pk}$ [kA]	72	10-200	Max. Peak current $I_{pk}$ [kA]	6	12

# Our motivation for this study came from:

## A Beam Driven Plasma-Wakefield Linear Collider: PWFA-LC From Higgs Factory to Multi-TeV

J.P Delahaye / SLAC

On behalf of the E200 Collaboration

E.Adli, M.J. Hogan, S. Corde, R.J. England, J. Frederico, S.J. Gessner, S. Li, M.D. Litos,  
T.Raubenheimer, Z. Wu, (SLAC, Stanford, USA),

C. Joshi, W. An, C.E. Clayton, K.A. Marsh, W. Mori, N. Vafaei-Najafabadi  
(UCLA, Los Angeles, USA),

W. Lu (Tsinghua Univ. of Beijing, China and UCLA)

P. Muggli (MPI, Munich, Germany)

Thanks for slides from  
M.Hogan, E.Adli, S.Gessner





# Gradient and efficiency in Linear Colliders

SLAC

- High gradient acceleration requires high peak power and structures that can sustain high fields
  - Beams and lasers can be generated with high peak power
  - Dielectrics and plasmas can withstand high fields



**Beam-driven Plasma Wake-Field Accelerator (PWFA)**

Acc. structures		Accelerating field			Acceleration efficiency		
		Limit (MV/m)	By	Comment	Wall-Plug to RF or drive (%)	RF or drive to beam (%)	Total (%)
Super-Conducting	ILC	30-40	Magnetic field	Dyn. losses Cryogenics prop $G^2$	45	45 (pulsed + Cryo)	20
Normal Conducting	CLIC Two beam	100	RF break-downs	Peak RF Power $\sim E^2$	40	30	12
Dielectric	Laser driven	1000	RF break-downs		10	50	5
	Beam driven				?	50	?
Plasma	Laser driven	10000	Laser		10	50	5
	Beam driven		Drive beam		40	50	20

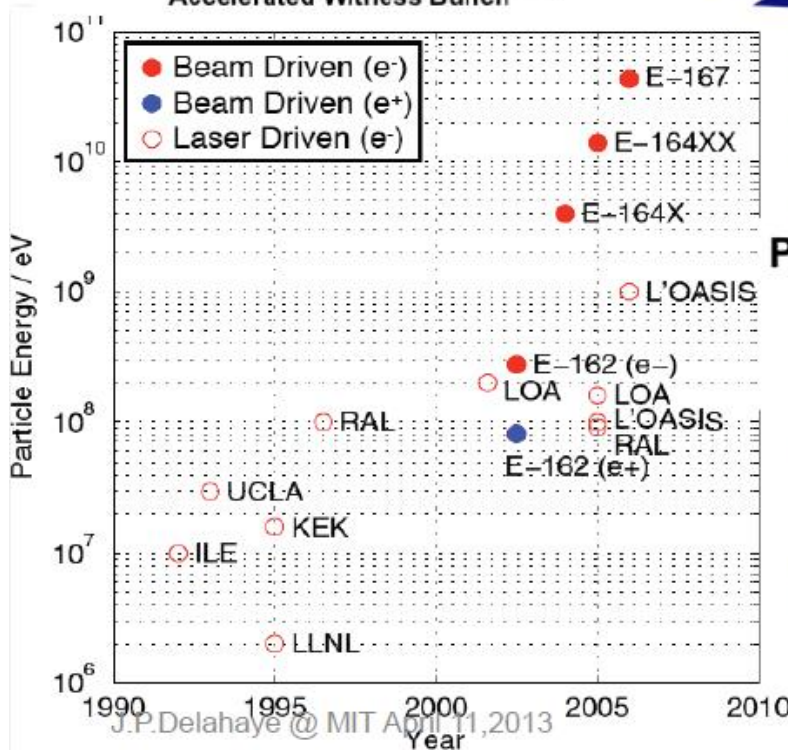
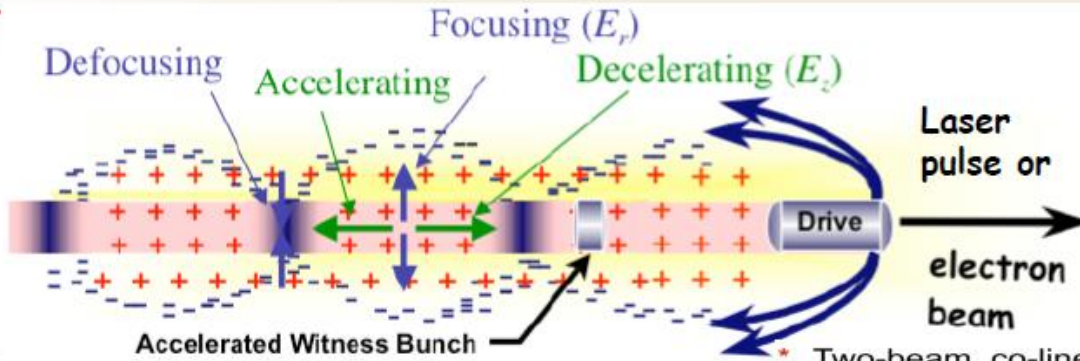
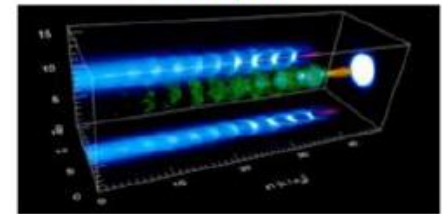
J.P.Delahaye @ MIT April 11, 2013

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# Plasma Acceleration (Beam-driven or Laser-driven)



~1m  
~100μm



- \* Two-beam, co-linear, plasma-based accelerator
- \* Plasma wave/wake excited by relativistic particle bunch
- \* Deceleration, acceleration, focusing by plasma
- \* Accelerating field/gradient scales as  $n_e^{1/2}$
- \* Typical:  $n_e \approx 10^{17} \text{ cm}^{-3}$ ,  $\lambda_p \approx 100 \mu\text{m}$ ,  $G > \text{MT/m}$ ,  $E > 10 \text{ GV/m}$
- \* High-gradient, high-efficiency energy transformer

## Peak Field For A Gaussian Bunch:

$$E = 6 \text{ GV/m} \frac{N}{2 \times 10^{10}} \frac{20 \mu}{\sigma_r} \frac{100 \mu}{\sigma_z} \Rightarrow > 10 \text{ GV/m}$$

## Extremely strong focusing:

$$B_\phi = 2\pi e n_p r \Rightarrow > \text{MT/m}$$

## Excellent power transfer efficiency:

$$\eta_{\text{drive to plasma}} \sim 76\%, \quad \eta_{\text{plasma to main}} \sim 66\% \Rightarrow \eta_{\text{drive to main}} > 50\%$$

# Is drive-to-beam 50% efficiency possible???

## *Conclusions*

SLAC

**PWFA a very promising technology:**

**Very high accelerating fields: effective 1 GV/m**

**Excellent power efficiency ( Wall-plug to beam 20%)**

**Great flexibility of time interval**

- **CW or pulsed mode of operation**
- **An alternative for ILC energy upgrade?**

**Many challenges still to be addressed;**

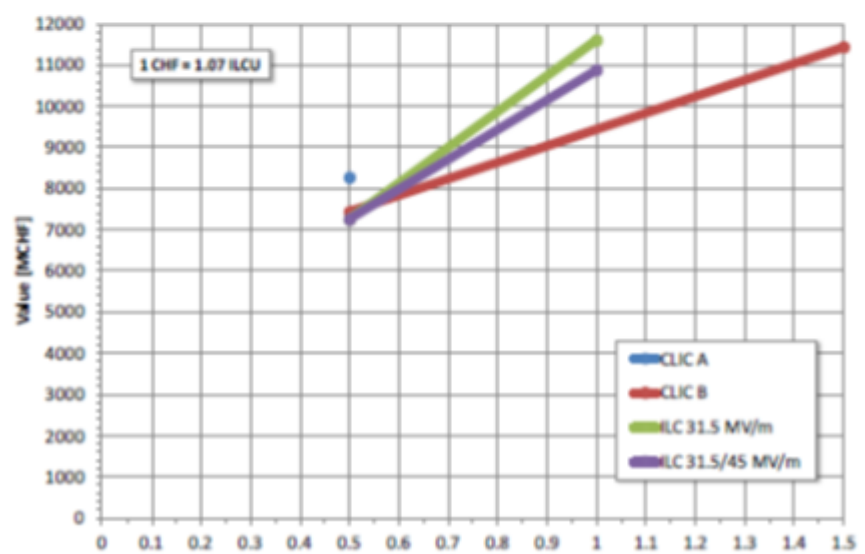
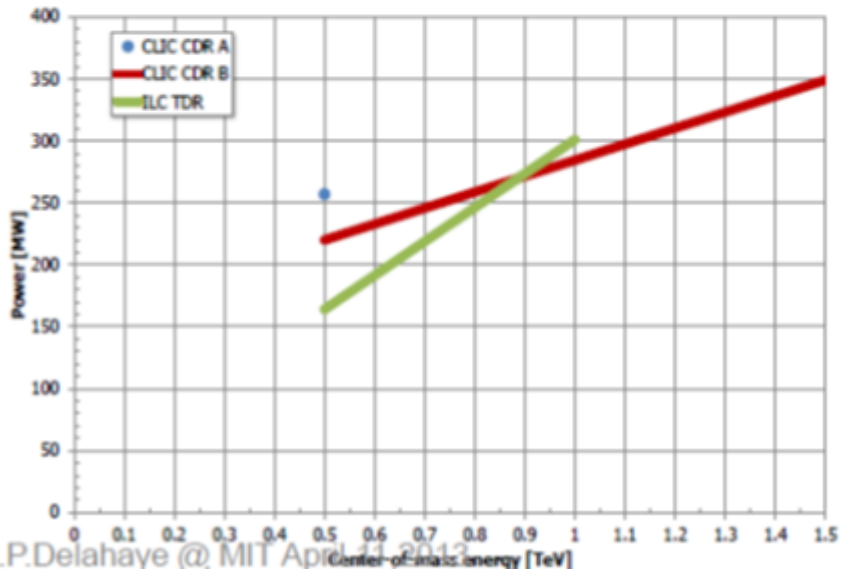
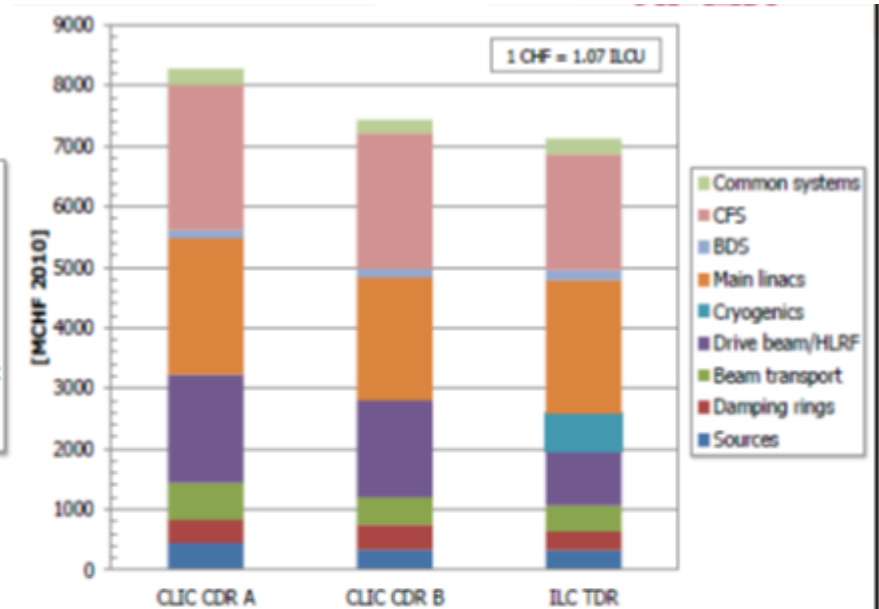
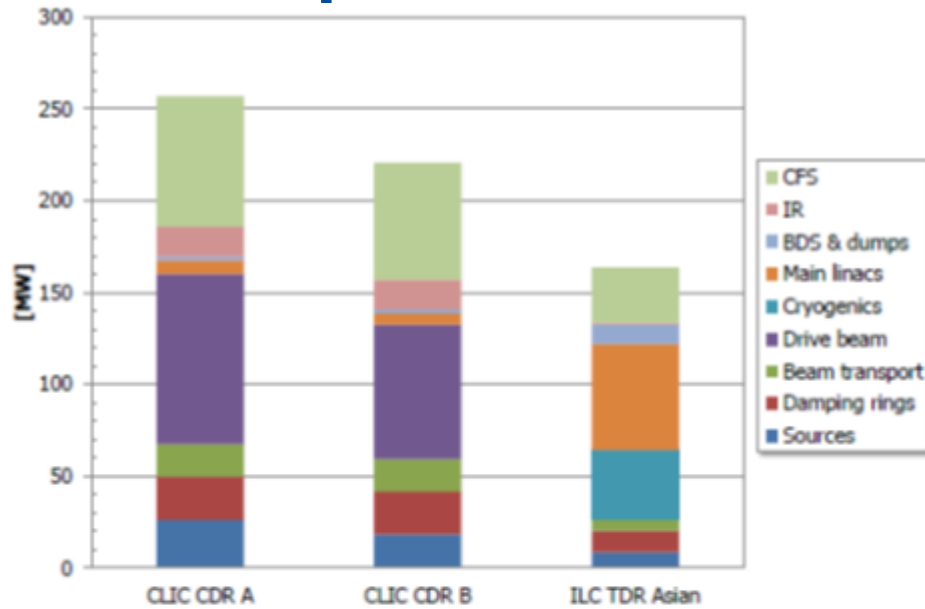
- **Beam quality preservation, efficiency, positrons?**
- **Ambitious test facilities: FACET and FACET2**
- **Feasibility addressed early next decade?**

**Thanks to excellent and expert collaboration: E200**



# Why is power efficiency important?

## Because power = cost



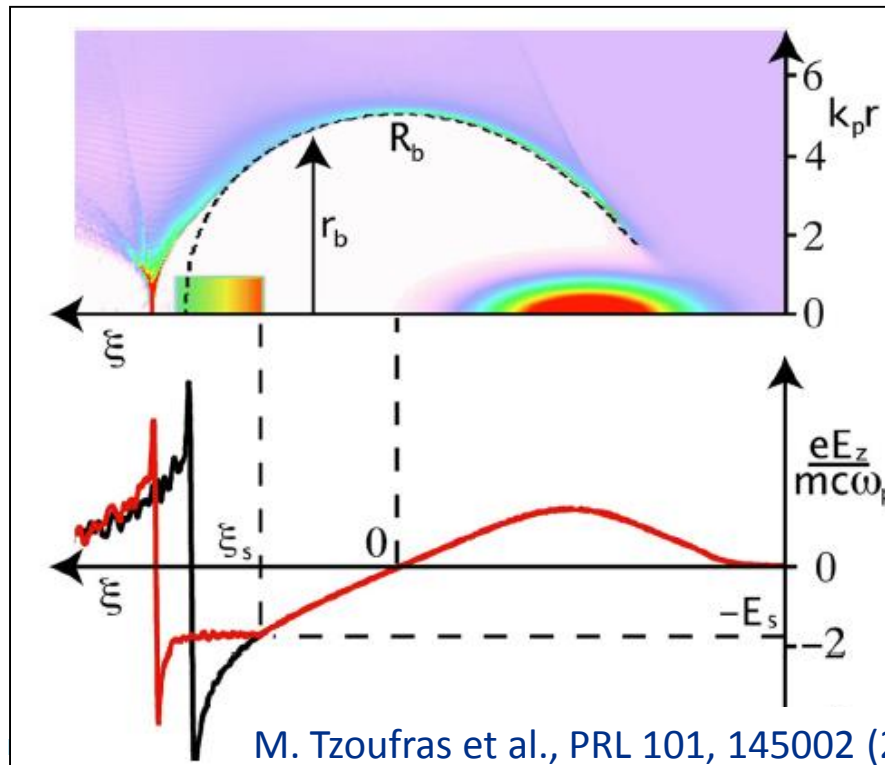
# Acceleration in ILC cavities



- The ILC cavity:  $\sim 1$  m long, 30 MeV energy gain;  $f_0 = 1.3$  GHz, wave length  $\approx 23$  cm
- The ILC beam: 3.2 nC ( $2 \times 10^{10}$ ), 0.3 mm long (rms); bunches are spaced  $\sim 300$  ns (90 m) apart
- Each bunch lowers the cavity gradient by  $\sim 15$  kV/m (beam loading 0.05%); this voltage is restored by an external rf power source (Klystron) between bunches; ( $\sim 0.5\%$  CLIC)
- Such operation of a conventional cavity is only possible because the Q-factor is  $\gg 1$ ; the RF energy is mostly transferred to the beam NOT to cavity walls.

# Acceleration in a blow-out regime

- The Q-factor is very low ( $\sim 1$ ) – must accelerate the trailing bunch within the same bubble as the driver!
- Cannot add energy between bunches, thus a single bunch must absorb as much energy as possible from the wake field.

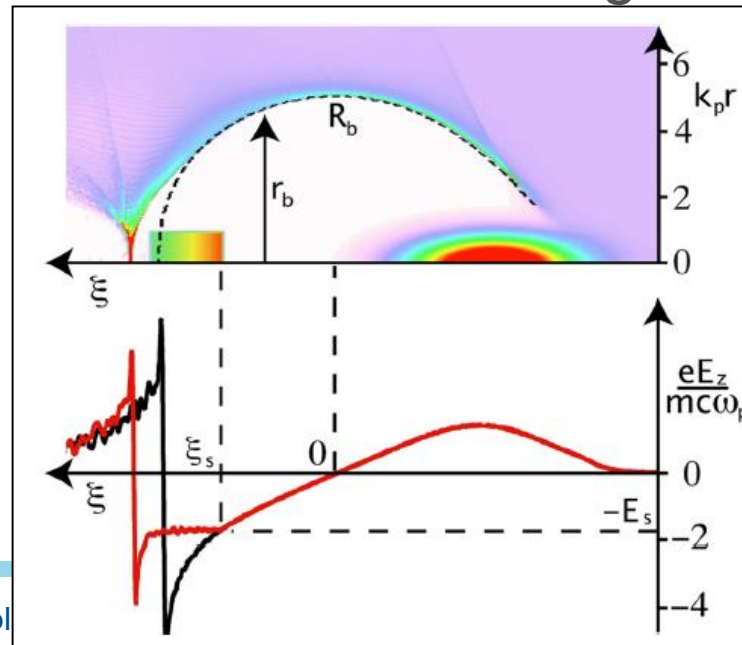
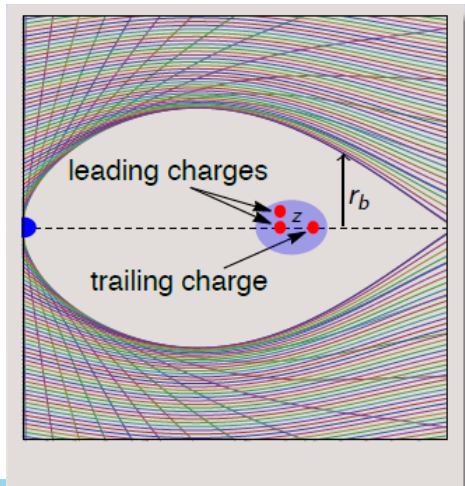


M. Tzoufras et al., PRL 101, 145002 (2008)

To achieve  $L \sim 10^{34}$ , bunches should have  $\sim 10^{10}$  particles (similar to ILC and CLIC). In principle, we can envision a scheme with fewer particles/bunch and a higher rep rate, but the beam loading still needs to be high for efficiency reasons.

# Plasma wakefields

- The terminology of wakefields in plasma can be confusing. The original meaning of the wake in plasma is the field generated by the **drive bunch**, which accelerates the **trailing bunch**. (The driver could be particle beam or laser)
- In this presentation, by wakefields I mean the fields (longitudinal and transverse) with which the trailing bunch acts on itself.

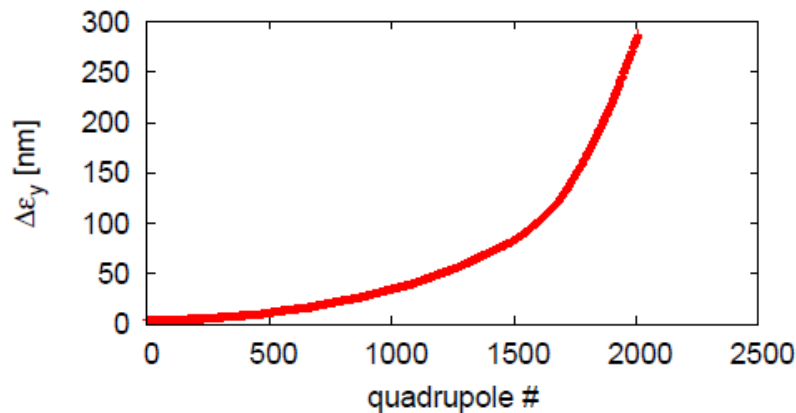
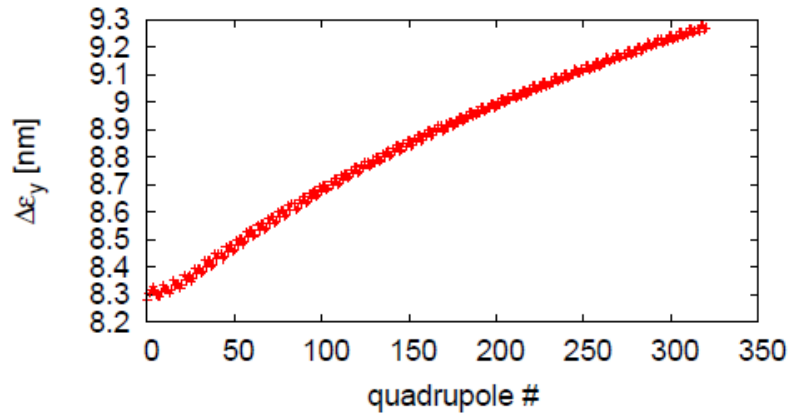




# Transverse beam break-up (head-tail instability)

- Transverse wakes act as deflecting force on bunch tail
  - beam position jitter is exponentially amplified

## Beam Stability



- Transverse stability of a beam with initial offset of  $\sigma_y$ 
    - no energy spread assumed in the beam
    - emittance with respect to the beam axis is shown
- ⇒ acceptable for ILC (top)
- ⇒ would be intolerable for CLIC (bottom)

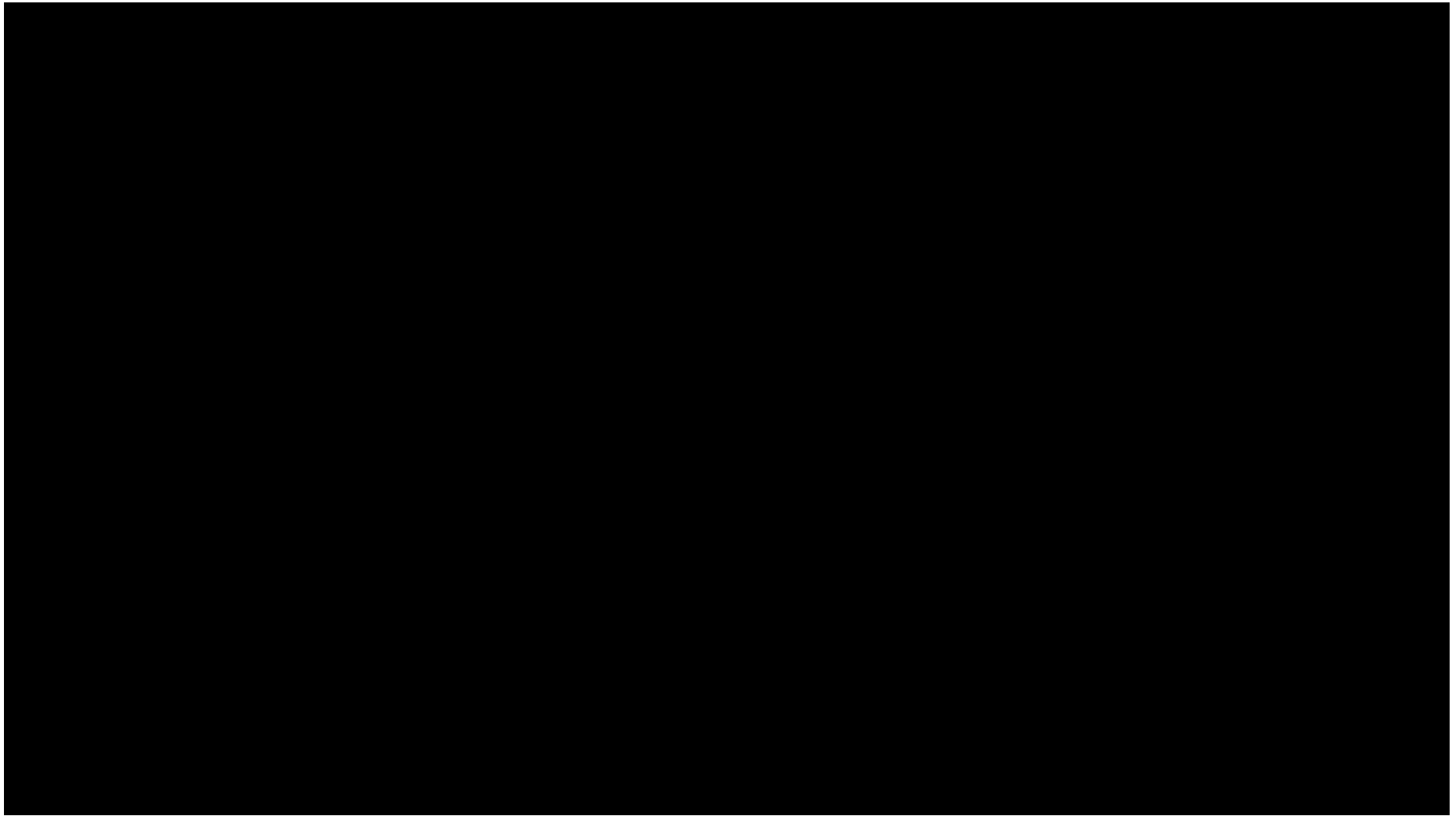
Short-range transverse wake (for solid walls)

$$W_{\perp}(z) = \frac{8z}{a^4}$$

$a \approx 35$  mm (ILC)  
 $a \approx 3.5$  mm (CLIC)

Can we use it in plasma?  
 $a \sim 0.1$  mm (PWFA)

# Case I: ~50% power efficiency



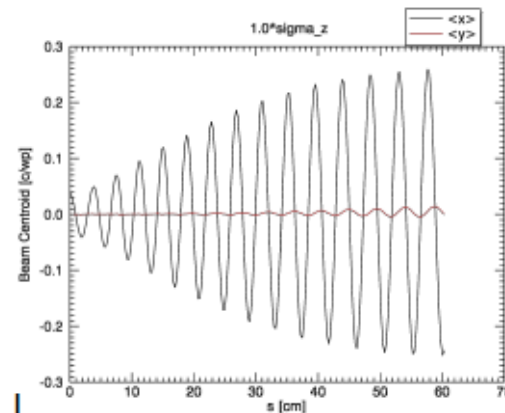
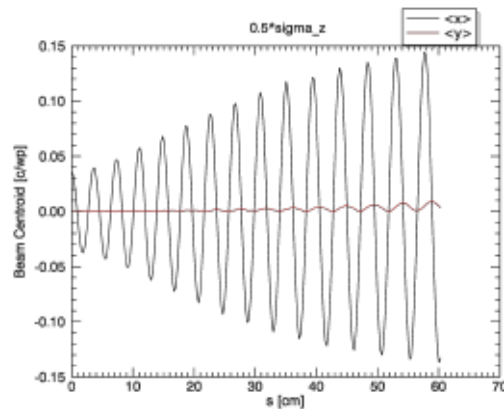
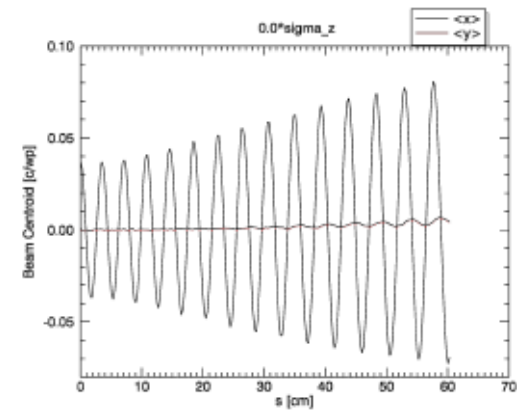
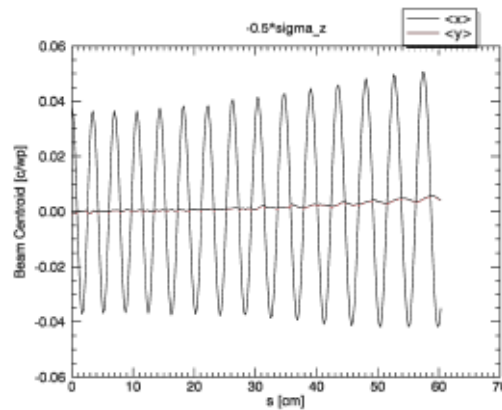
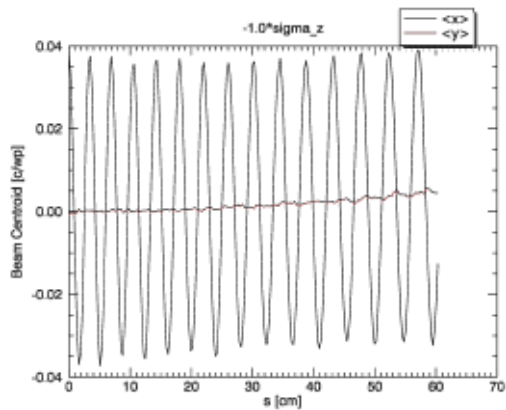
Courtesy of UCLA

**Drive Beam:**  $E = 10$  GeV,  $I_{\text{peak}} = 15$  kA  
 $\sigma_r = 3.65$   $\mu\text{m}$ ,  $\sigma_z = 12.77$   $\mu\text{m}$ ,  
 $N = 1.0 \times 10^{10}$  (1.6 nC),  $\epsilon_N = 10$   $\mu\text{m}$

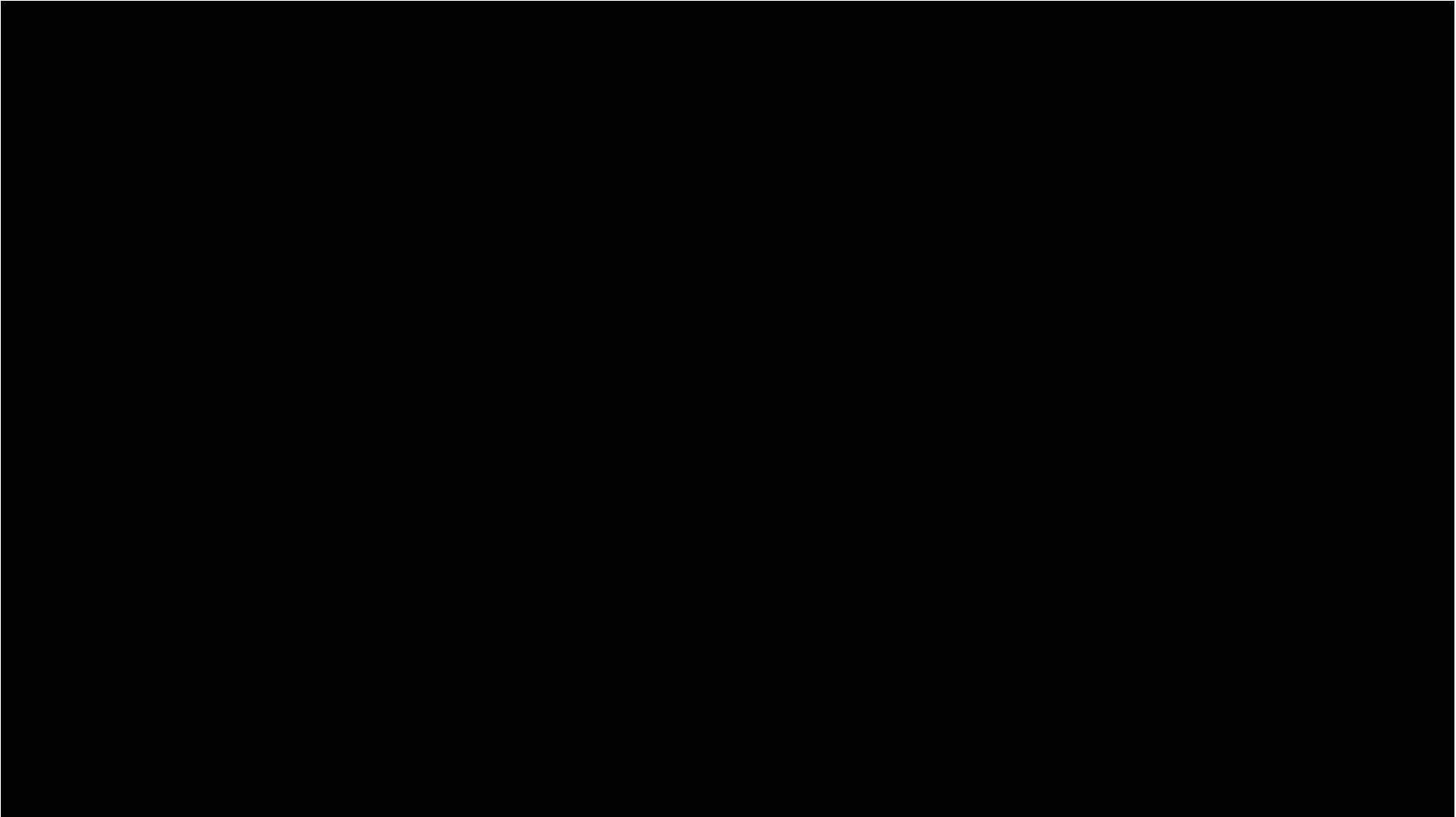
**Trailing Beam:**  $E = 10$  GeV,  $I_{\text{peak}} = 9$  kA  
 $\sigma_r = 3.65$   $\mu\text{m}$ ,  $\sigma_z = 6.38$   $\mu\text{m}$ ,  
 $N = 4.33 \times 10^9$  (0.69 nC),  $\epsilon_N = 10$   $\mu\text{m}$   
 (transversely offset by 1  $\mu\text{m}$ )

**Distance between two bunches:** 150  $\mu\text{m}$   
**Plasma Density:**  $4.0 \times 10^{16}$   $\text{cm}^{-3}$

Trailing beam centroid vs s in different slices



# Case II: ~25% power efficiency



Courtesy of UCLA



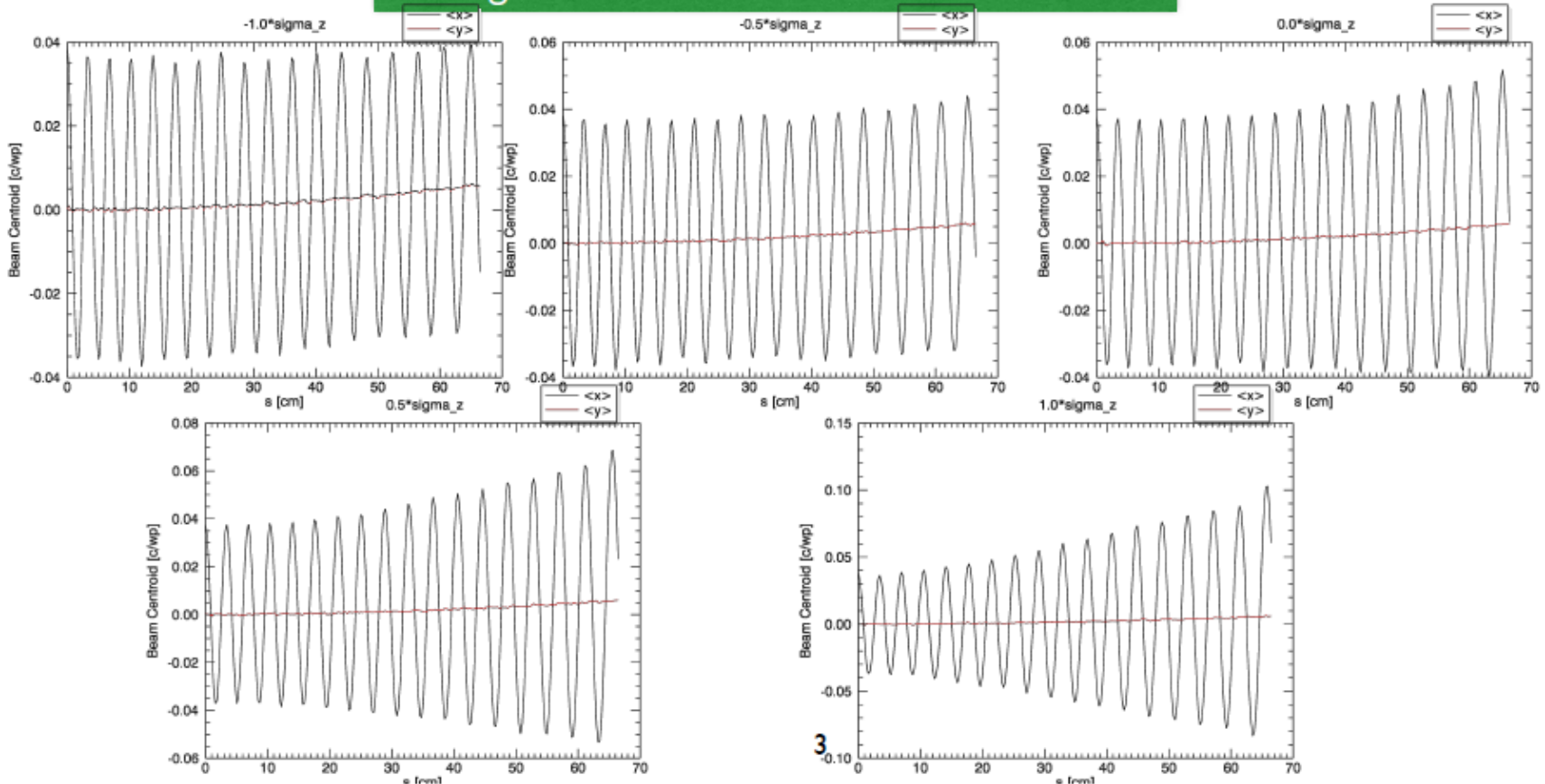


**Drive Beam:**  $E = 10$  GeV,  $I_{\text{peak}} = 15$  kA  
 $\sigma_r = 3.65$   $\mu\text{m}$ ,  $\sigma_z = 12.77$   $\mu\text{m}$ ,  
 $N = 1.0 \times 10^{10}$  (1.6 nC),  $\epsilon_N = 10$   $\mu\text{m}$

**Distance between two bunches:** 108  $\mu\text{m}$   
**Plasma Density:**  $4.0 \times 10^{16}$   $\text{cm}^{-3}$

**Trailing Beam:**  $E = 10$  GeV,  $I_{\text{peak}} = 9$  kA  
 $\sigma_r = 3.65$   $\mu\text{m}$ ,  $\sigma_z = 6.38$   $\mu\text{m}$ ,  
 $N = 4.33 \times 10^9$  (0.69 nC),  $\epsilon_N = 10$   $\mu\text{m}$   
 (transversely offset by 1  $\mu\text{m}$ )

Trailing beam centroid vs s in different slices



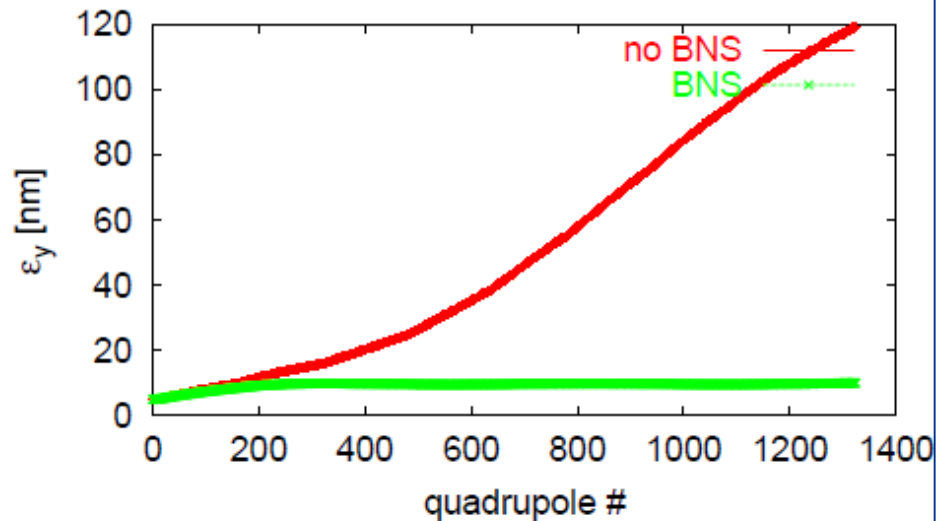
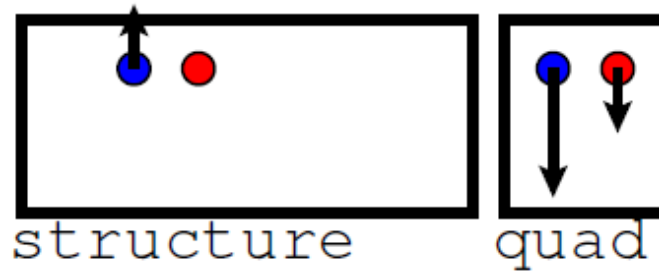
# Beam breakup in various collider concepts

- ILC
  - Not important; bunch rf phase is selected to compensate for long wake and to minimize the momentum spread
- CLIC
  - Important; bunch rf phase is selected to introduce an energy chirp along the bunch for BNS damping ( $\sim 0.5\%$  rms). May need to be de-chirped after acceleration to meet final-focus energy acceptance requirements
- PWFA – **the subject of our study**
  - Critical; BNS damping requires a large energy chirp (see below). De-chirping and beam transport is very challenging because of plasma stages (small beta-function in plasma  $\sim 1$  cm). In essence, requires a “final-focus” optics between every stage.

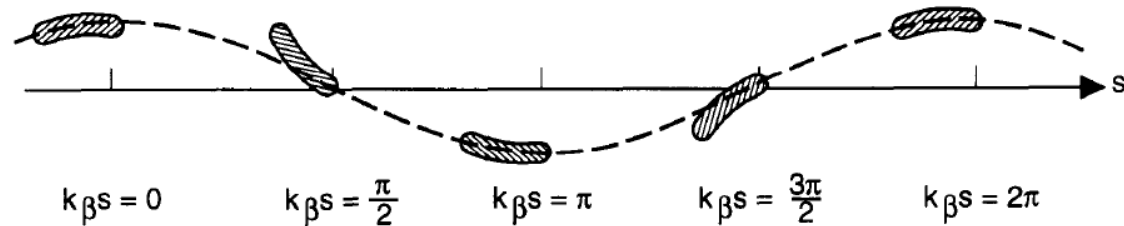
# CLIC strategy: BNS damping + $< \mu\text{m}$ alignment of cavities

## Achieving Beam Stability

- Transverse wakes act as defocusing force on tail  
⇒ beam jitter is exponentially amplified
- BNS (Balakin, Novokhatsky, and Smirnov) damping prevents this growth
  - manipulate RF phases to have energy spread
  - take spread out at end



# Strategy was also used at the SLC...



**Figure 3.3.** Sequence of snapshots of a beam undergoing dipole beam breakup instability in a linac. Values of  $k_{\beta}s$  indicated are modulo  $2\pi$ . The dashed curves indicate the trajectory of the bunch head.

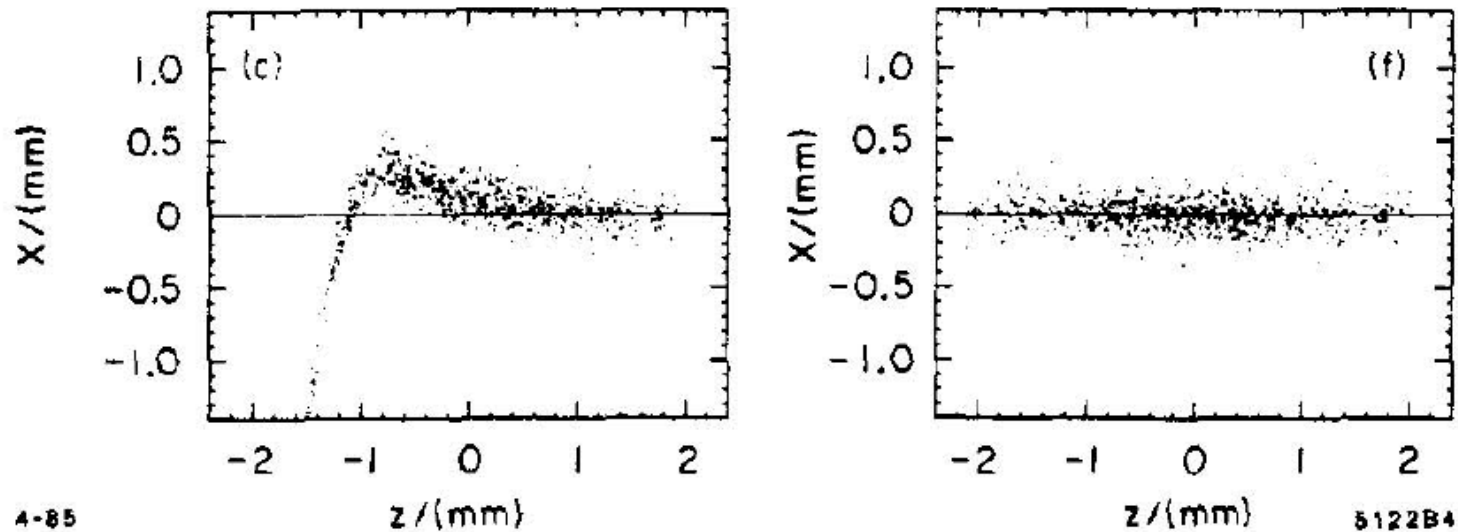


Figure 34: Multiparticle simulation of a particle bunch passing through the SLAC linac without (left) and with BNS damping (right) [36].



# We start with the Lu plasma bubble equation

- We assume the driving bunch intense enough to produce an electron-free plasma bubble with radius  $R_b \gg k_p^{-1}$ . According to Lu et al. :

$$r_b \frac{d^2 r_b}{d\xi^2} + 2 \left( \frac{dr_b}{d\xi} \right)^2 + 1 = \frac{2}{\pi n_0 r_b^2} \frac{dN_d}{d\xi} \quad E_{\parallel} = -2\pi n_0 e r_b \frac{dr_b}{d\xi}$$

$$R_b = \frac{L_d}{\sqrt[4]{2}} \sqrt[4]{\frac{8N_d}{\rho n_0 L_d^3} \left( \sqrt{\frac{8N_d}{\rho n_0 L_d^3} + 1} - 1 \right)}$$

$$R_b \approx \left( \frac{2^7 N_d^3}{\rho^3 L_d n_0^3} \right)^{1/8}, \quad \frac{N_d}{n_0 L_d^3} \gg 1$$

Example:  $N_d = 10^{10}$ ;  $n_0 = 4 \times 10^{16} \text{ cm}^{-3}$ ;  $L = 25 \text{ } \mu\text{m}$

$$R_b k_p \approx 3.2$$

# Power transfer from drive to trailing bunches

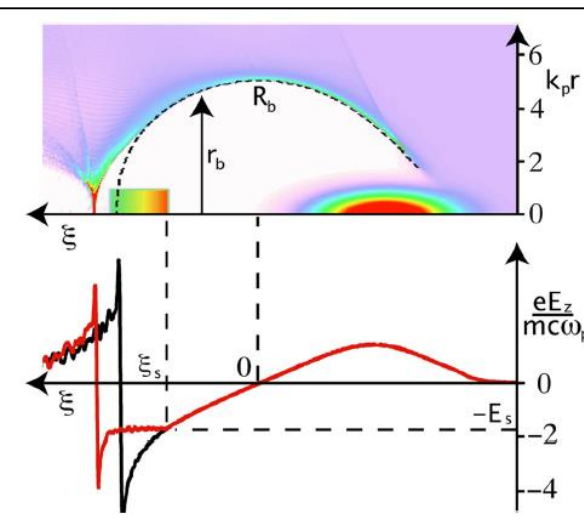
- Following M. Tzoufras et al., PRL 101, 145002 (2008)

Trapezoidal line density distribution  $\rightarrow$  constant electric field

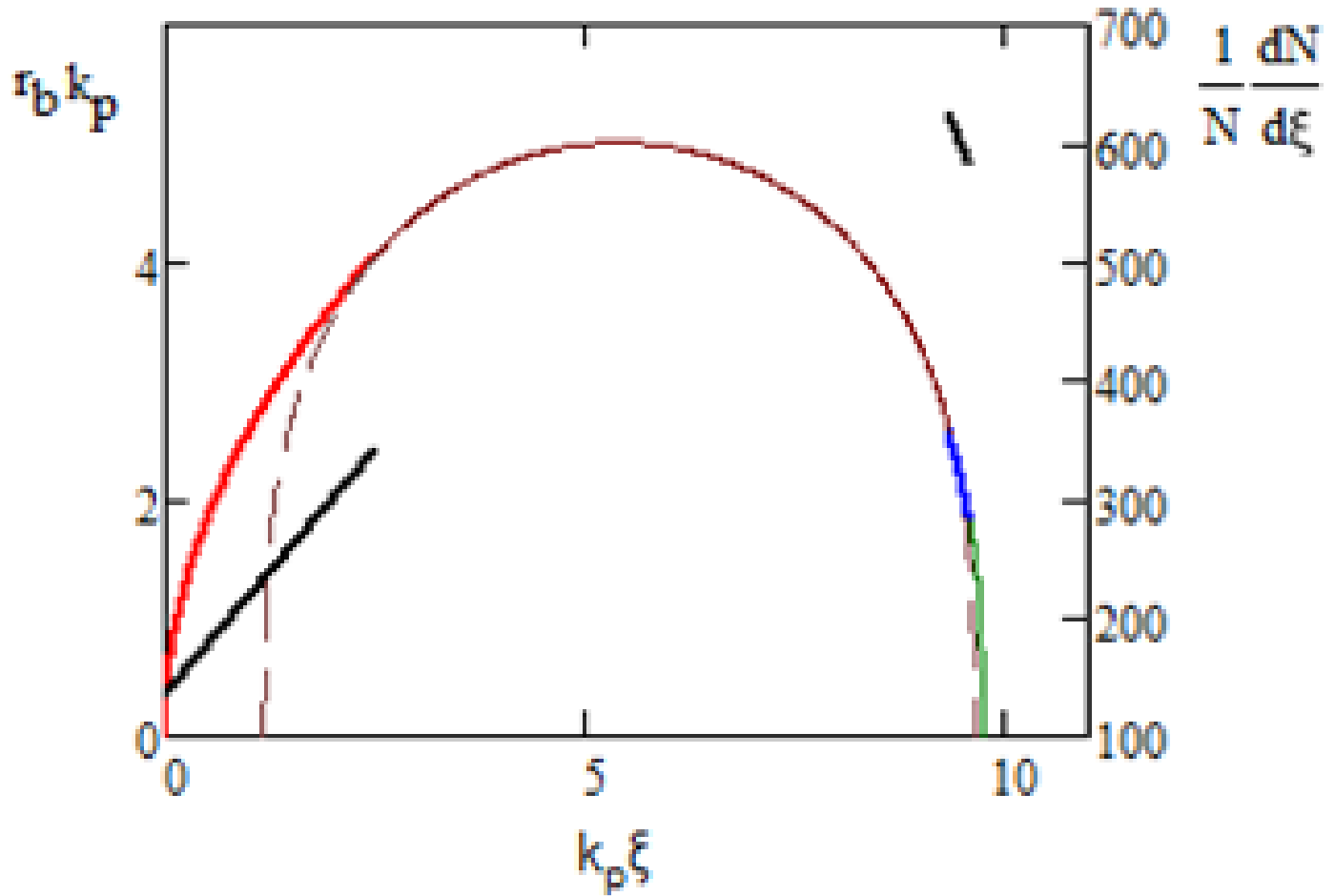
$$P = eN_d E_d c = \frac{\pi^2}{4} e^2 n_0^2 c R_b^4$$

$$P_t = ecN_t E_t = \frac{\pi^2 e^2 n_0^2 c}{4} \left( r_{t2}^2 - r_{t1}^2 \right) \left( \frac{R_b^4}{r_{t2}^2} + r_{t1}^2 \right)$$

$$\eta_P = \frac{P_t}{P} = \frac{r_{t2}^2 - r_{t1}^2}{R_b^2} \left( \frac{R_b^2}{r_{t2}^2} + \frac{r_{t1}^2}{R_b^2} \right)$$



# Example



The power transfer efficiency of 50% and the transformer ratio of 2. For  $n_0=10^{17} \text{ cm}^{-3}$  the drive bunch parameters are chosen to be  $R_b k_p=5$ ,  $L_d k_p=2.5$  yielding the decelerating field of  $E_d=50 \text{ GV/m}$  and  $N_d=3.55 \cdot 10^{10}$ . The trailing bunch parameters are:  $r_2=0.518R_b$ ,  $r_1=0.373R_b$ ,  $E_t=100 \text{ GV/m}$ ,  $N_t=8.86 \cdot 10^9$ .

# Instability of the trailing bunch

- The *Beam Break-up* (BBU) instability is characterized by the ratio of the wake deflection force to the focusing force.

$$F_r = -2\rho n_0 e^2 r \quad \text{Focusing force}$$

$$F_t \equiv F(\xi_1) = e^2 r \int_{\xi_1 - L_t}^{\xi_1} \frac{dN_t}{d\xi} W_{\perp}(\xi_1, \xi) d\xi \quad \text{Defocusing force (varies along bunch)}$$

- Need to find  $W_{\perp}(\xi)$  for the bubble regime.

- First, in a quasilinear regime,

$$W_{\perp} = 2 \frac{k_p}{\sigma_{\perp}^2} \left( \frac{\Delta n}{n} \right)_e \sin(k_p (s - s')) \ln \left( \frac{\rho_{\max}}{\rho_{\min}} \right), \quad k_p = \frac{\omega_p}{c}$$

– where  $\sigma_{\perp}$  is the rms size of plasma channel

– For a hollow channel  $\frac{\Delta n}{n} : 1$

$$W_{\perp} \approx 2k_p^3 \sin(k_p (s - s')) \ln(2), \quad \sigma_{\perp} \approx k_p^{-1}$$



# Wakes in the bubble regime

Longitudinal (from the Lu equation):  $W_{\parallel} = \frac{4}{r_b^2}; \quad (\Delta z \ll r_b, k_p^{-1})$

(similar to a dielectric channel and periodic array of cavities)

For reference, see: A. V. Fedotov, R. L. Gluckstern, and M. Venturini (PRST-AB 064401 (1999))

Transverse :

$$W_{\perp} \approx \frac{2}{r_b^2} \int W_{\parallel} dz = \frac{8\Delta z}{r_b^4}; \quad (\Delta z \ll r_b, k_p^{-1})$$

$r_b(z) \gg k_p^{-1}$  -- local bubble radius at bunch location,  $z$

(This is true for a dielectric channel, array of cavities and resistive wall)

For reference, see also: Karl Bane, SLAC-PUB-9663 and S. S. Baturin and A. D. Kanareykin, PRL 113, 214801 (2014) .

Recent findings:  $r_b(z) \rightarrow r_b(z) + k_p^{-1}$  to account for bubble wall thickness

# Recent new results

- G. Stupakov, “Short-range wakefields generated in the blowout regime of plasma-wakefield acceleration”, arXiv:1710.07371

Useful for:  $r_b(\xi) \sim k_p^{-1}$

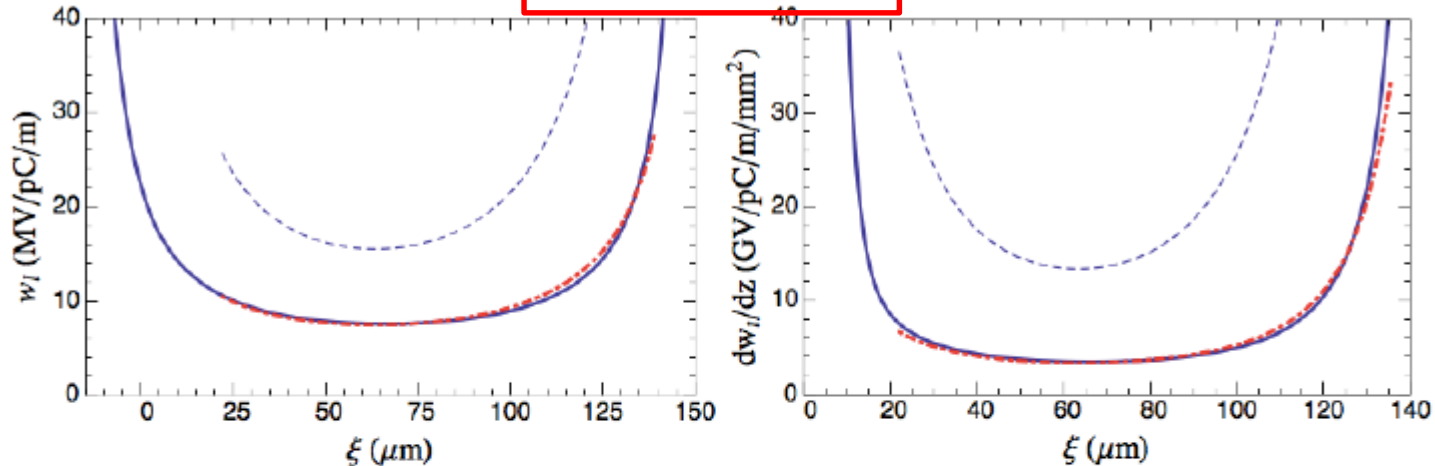


FIG. 4. Longitudinal wake (left panel) and the slope of the transverse wake (right panel) in the plasma bubble shown in Fig. 2. The dashed lines show the wakes calculated using simple formulas for the short-range wakes in a cylindrical pipe. The red dot-dashed lines are plotted using Eqs. (43).

$$w_l(\xi) = \frac{4}{(r_b(\xi) + 0.8k_p^{-1})^2}, \quad \frac{dw_t}{dz} = \frac{8}{(r_b(\xi) + 0.75k_p^{-1})^4}.$$

# Our estimate for the transverse wake

$$W_{\perp}(\xi, \xi_2) \approx \frac{8\xi^{\theta}}{r_b(\xi)r_b^3(\xi_2)}\theta(\xi^{\theta}), \quad \xi^{\theta} = \xi - \xi_2$$

$$r_b(\xi) \gg k_p^{-1}$$

- $\theta(x)$  is the Heaviside step function.
- We believe this estimate is on the “low” side. The actual wake is likely to be greater.
- Now, let’s find the ratio of the defocusing (wake) force to the focusing force:

$$\eta_t = -\frac{F_t}{F_r} = \frac{r_{t2}}{r_{t1}} \int_0^{L_t} d\xi \frac{L_t - \xi}{r_b^3(\xi)} \times \left[ r_{t2} \left( \frac{R_b^4}{r_{t2}^4} - 1 \right) - 2 \left( \xi \sqrt{2 \left( \frac{R_b^4}{r_{t2}^4} - 1 \right)} - r_{t2} \right) \right]$$

- Recall that 
$$\eta_P = \frac{P_t}{P} = \frac{r_{t2}^2 - r_{t1}^2}{R_b^2} \left( \frac{R_b^2}{r_{t2}^2} + \frac{r_{t1}^2}{R_b^2} \right)$$

# The efficiency-instability relation

$$\eta_t \approx \frac{\eta_P^2}{4(1-\eta_P)}, \quad \frac{r_{t2}}{R_b} \leq 0.7$$

- This formula does not include any details of beams and plasma, being amazingly universal!
- Note: this formula is an estimate from a “low side”. On a “high side”, we estimate it as:  $\eta_t \approx \eta_P^2 / (4(1-\eta_P)^2)$
- Example:  $\eta_P = 50\% \rightarrow 0.125 < \eta_t < 0.25$

$$\eta_P = 25\% \rightarrow 0.021 < \eta_t < 0.028$$

# Instability development

$$\frac{d^2 X}{d\mu^2} + \frac{X}{1 + \Delta p / p} = \frac{2\eta_t}{(1 + \Delta p / p)L_t^2} \int_0^{\xi} X(\xi') (\xi - \xi') d\xi'.$$

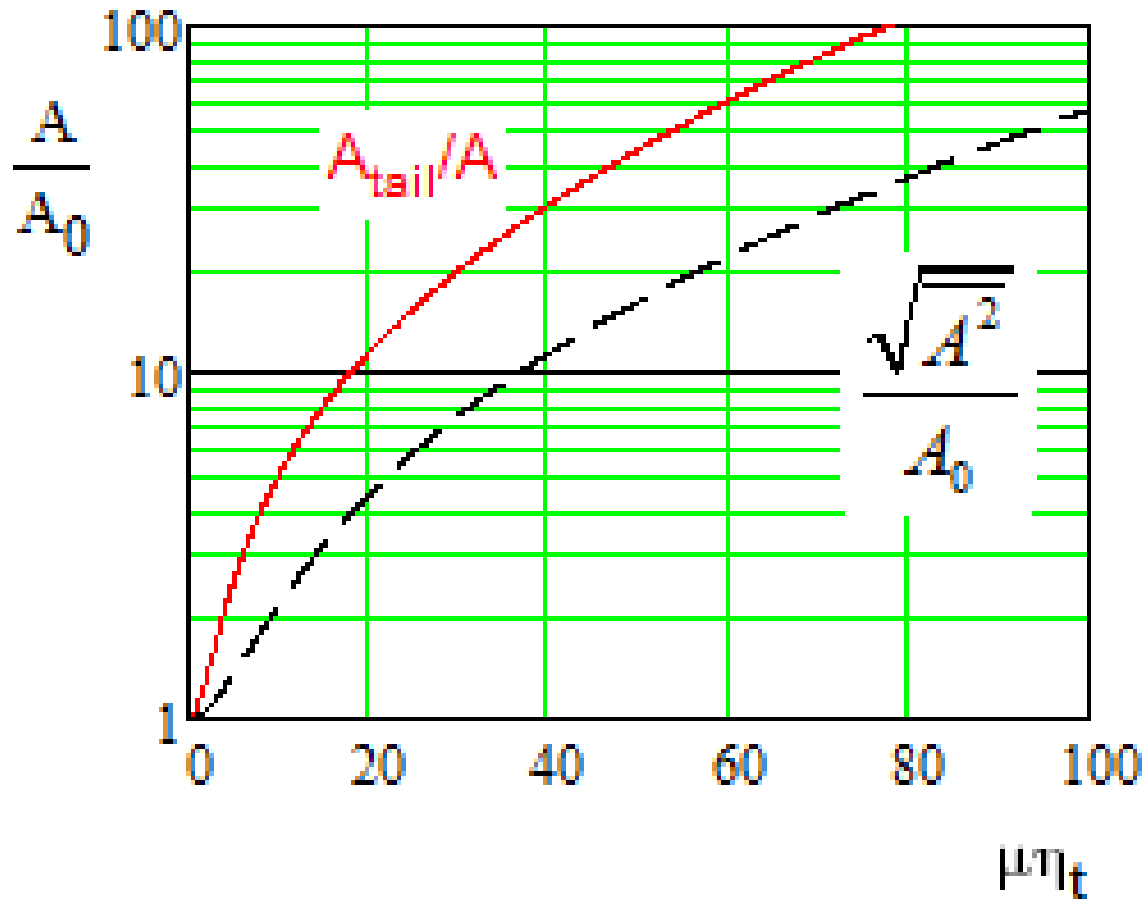
$$X = \frac{x}{\sqrt{\beta}} \sqrt{\frac{p}{p_0}}; \quad \beta = k_p^{-1} \sqrt{2\gamma} \quad d\mu = dz / \beta$$

- For  $\eta_t = 1$  and  $\Delta p / p = 0$  it was solved in:
  - C. B. Schroeder, D. H. Whittum, and J. S. Wurtele, “Multimode Analysis of the Hollow Plasma Channel Wakefield Accelerator”, Phys. Rev. Lett. **82**, n.6, 1999, pp. 1177-1180.
- Approximate solutions (it’s a very good fit, <10% deviation):

$$\frac{A}{A_0} = \exp\left\{ \frac{(\mu\eta_t)^2}{10 + 1.4(\mu\eta_t)^{1.57}} \right\} \quad \begin{array}{l} \mu\eta_t \lesssim 100 \\ \eta_t \lesssim 0.1 \end{array}$$

$$\frac{\sqrt{A^2}}{A_0} = \exp\left\{ \frac{(\mu\eta_t)^2}{60 + 2.2(\mu\eta_t)^{1.57}} \right\} \quad \begin{array}{l} \mu\eta_t \lesssim 100 \\ \eta_t \lesssim 0.1 \end{array}$$





- Note that  $A$  is a normalized particle amplitude. For a constant plasma density and without instability  $A$  would stay constant, while the initial physical amplitude  $x$  should decrease as  $1/\gamma^{\frac{1}{4}}$

## Examples (FACET-II)

Plasma:  $n_0 = 4 \times 10^{16} \text{ cm}^{-3}$ , 60 cm long channel

- $p_i = 10 \text{ GeV}/c$  for both the drive and the trailing bunches, and the final momentum of trailing bunch  $p_f = 21 \text{ GeV}/c$ ,  $N_d = 1 \times 10^{10}$  and  $N_t = 4.3 \times 10^9$

$$\eta_P = 50\%, \quad \eta_t \approx 0.12, \quad \mu\eta_t \approx 11.5 \quad \rightarrow \quad \frac{A}{A_0} \approx 5.8$$

- If one reduces the power efficiency:

$$\eta_P = 25\%, \quad \eta_t \approx 0.021, \quad \mu\eta_t \approx 2 \quad \rightarrow \quad \frac{A}{A_0} \approx 1.3$$

- Of course, the final momentum is now  $p_f = 15.5 \text{ GeV}/c$  (for the same number of particles)

$$\delta\epsilon_n = \frac{\delta x^2}{2\beta_i} \gamma_i \left( \frac{\overline{A^2}}{A_0^2} \right), \quad \beta_i = \frac{\sqrt{2\gamma_i}}{k_p}$$

# BNS damping

- Assume a constant long. density trailing bunch. Chromatic detuning of tail particles allows to keep amplitudes constant

$$\frac{1}{1 + \frac{\Delta p}{p}} - \frac{2\eta_t}{\left(1 + \frac{\Delta p}{p}\right) L_t^2} \int_0^\xi (\xi - \xi') d\xi' = 1$$

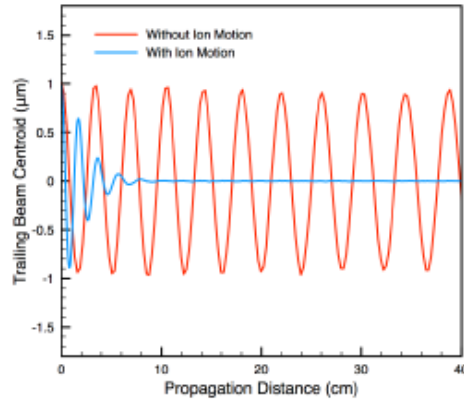
$$\frac{Dp(\xi)}{p} = - \eta_t \frac{\xi^2}{L_t^2}$$

- We believe that the collider final focus optics and transitions between stages can not tolerate  $\frac{\Delta p}{p} > 1\%$  , so  $\eta_t \leq 0.01$
- This limits the power transfer efficiency to  $< 18\%$

# The role of plasma ions

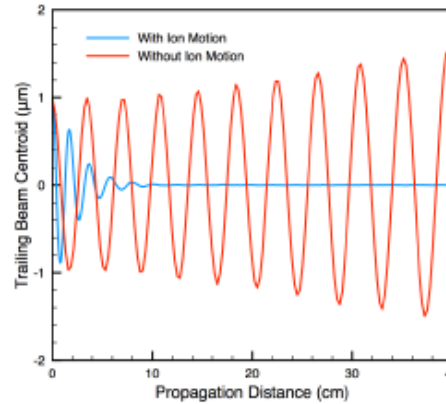
- So far, we considered plasma ions to be stationary.
- In fact, in the bunch density is high enough, the plasma ions are pulled into the electron bunch and create highly nonlinear focusing.
- Effect was considered first by J. Rosenzweig et al, PRL
- 95, 195002 (2005). Found to be detrimental because of emittance growth.
- However, nonlinear focusing might be helpful to suppress the BBU instability (by allowing some emittance growth)
- Recent simulations performed by Weiming An (UCLA)

Head



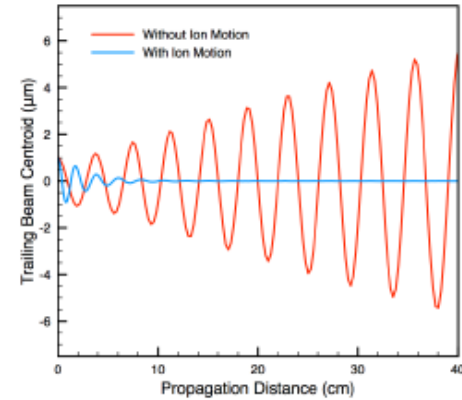
$$\xi = -\sigma_z$$

Center



$$\xi = 0$$

Tail



$$\xi = \sigma_z$$

**Drive Beam:**  $E = 10 \text{ GeV}$ ,  $I_{\text{peak}} = 15 \text{ kA}$   
 $\sigma_r = 0.516 \text{ }\mu\text{m}$ ,  $\sigma_z = 12.77 \text{ }\mu\text{m}$ ,  
 $N = 1.0 \times 10^{10}$  (1.6 nC),  $\epsilon_N = 1 \text{ }\mu\text{mrad}$

**Trailing Beam:**  $E = 10 \text{ GeV}$ ,  $I_{\text{peak}} = 9 \text{ kA}$   
 $\sigma_r = 0.516 \text{ }\mu\text{m}$ ,  $\sigma_z = 6.38 \text{ }\mu\text{m}$ ,  
 $N = 4.33 \times 10^9$  (0.69 nC),  $\epsilon_N = 1 \text{ }\mu\text{mrad}$   
 (transversely offset by  $1 \text{ }\mu\text{m}$ )

Power efficiency: 50%

Emittance growth: ~a factor of two



# Conclusions

- We have found a universal **efficiency-instability relation** for plasma acceleration. Should allow for tolerance and instability analysis without detailed computer simulations.
- We considered only the ideal “trapezoidal” distributions. Real-life distributions are worse (from the efficiency perspective).
- In a blowout regime, plasma focusing is just strong enough to keep the instability in check for low power efficiencies (<25%)
  - Even for such efficiencies, external focusing and hollow channels are not viable concepts because of transverse instability.
  - Presents obvious difficulties for positrons
- BNS damping is possible but external optical systems limit the momentum spread to ~1% max. Thus, the power efficiency (drive to trailing) can not exceed ~18%.
- Landau damping through nonlinear focusing may be based on plasma ions.

# Summary

- We now have a very interesting proposal for FACET-II to test our findings of emittance growth vs beam loading
  - Large parameter space to explore (small to large beam emittances, ion motion, plasma bubble size, etc)
- Our conclusions require confirmation by computer simulations and by experiments, especially in regimes not covered by the Lu equations (small bubble size).