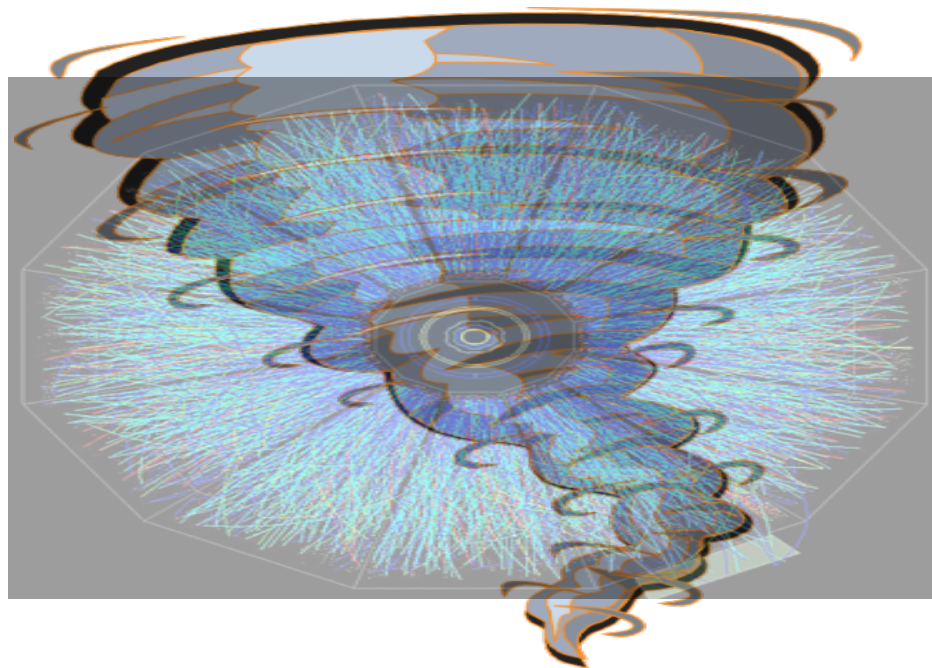


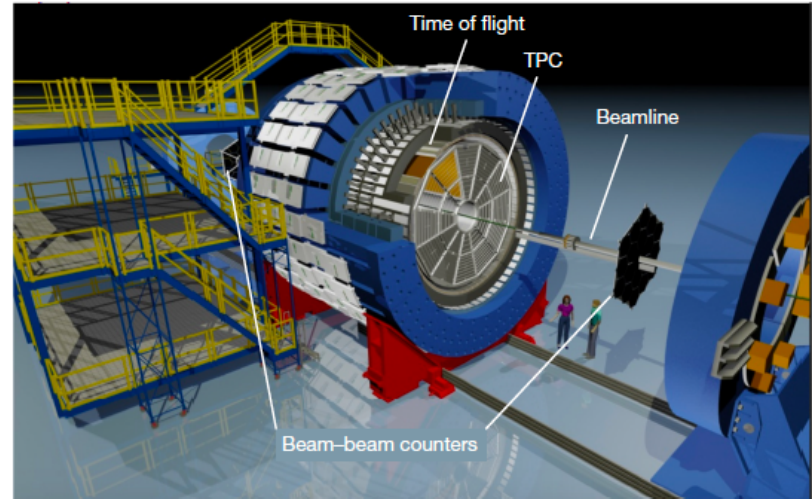
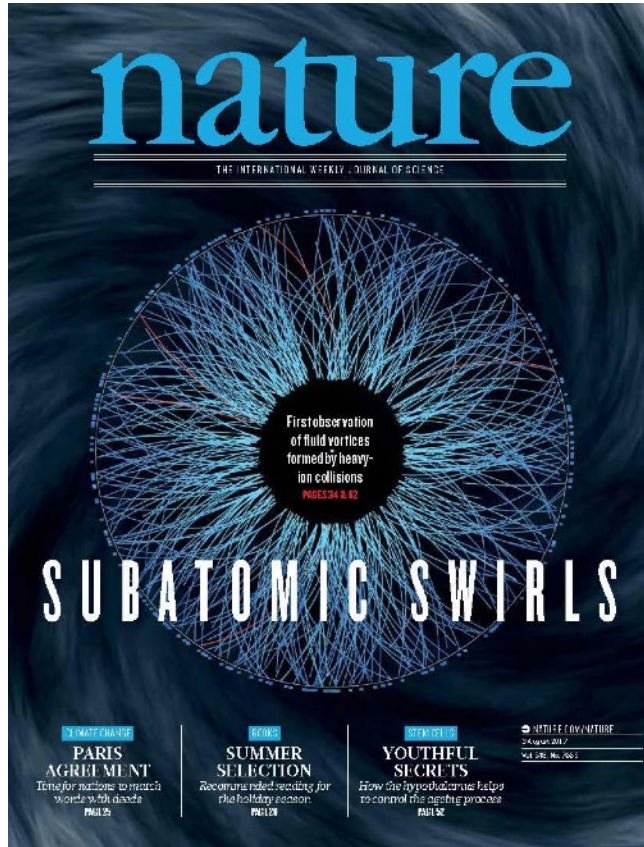
**3rd CBM-China Workshop @ Yichang Apr 16, 2018**

# **Strongly Interacting Matter Under Rotation**



**Jinfeng Liao 廖劲峰**

# Subatomic Swirls



*An exciting discovery from  
STAR Collaboration at RHIC:  
The most vortical fluid!*

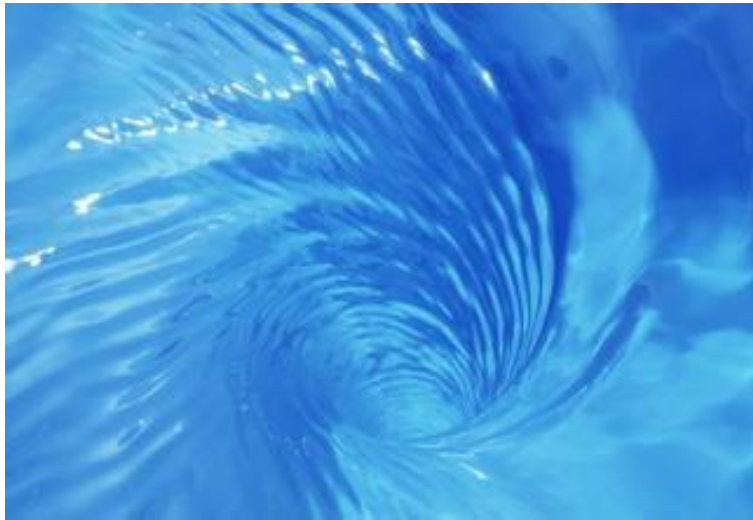
## LETTER

doi:10.1038/nature23004

## Global $\Lambda$ hyperon polarization in nuclear collisions

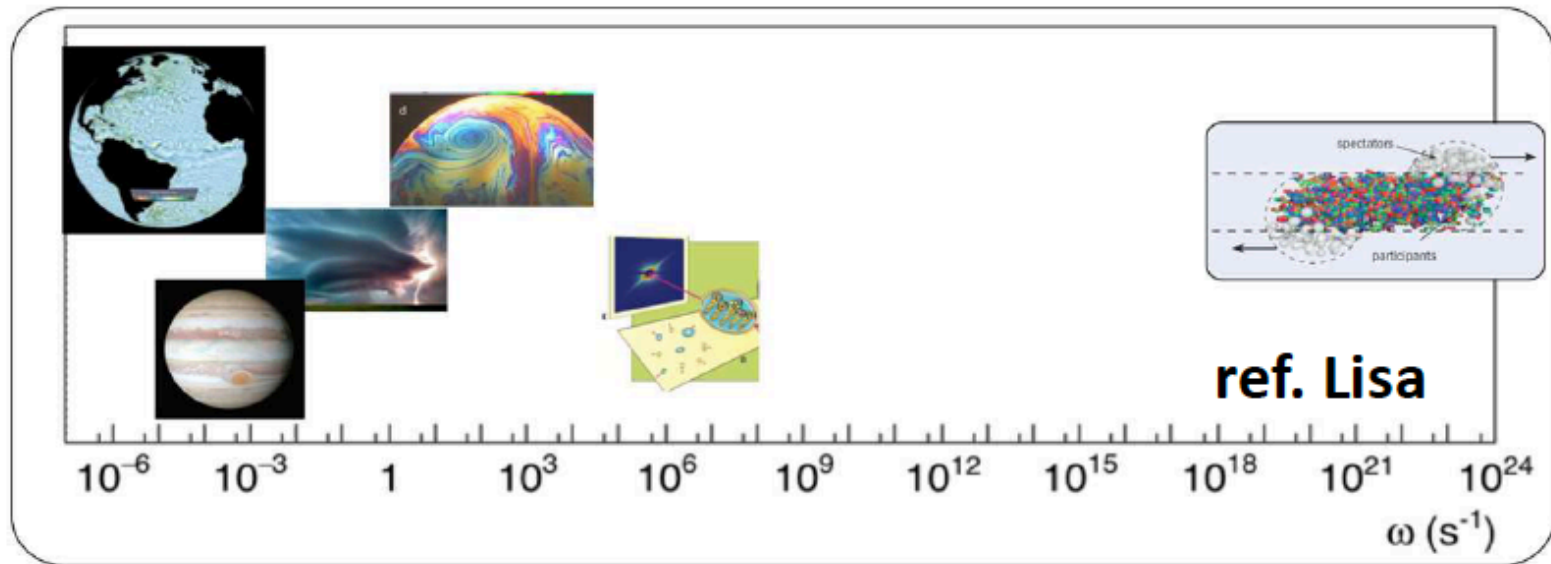
The STAR Collaboration\*

# Quantifying Fluid Rotation



$$NR \quad \vec{\omega} = \frac{1}{2} \nabla \times \vec{v}$$

$$UR \quad \Omega_{\mu\nu} = \frac{1}{2} (\partial_\nu u_\mu - \partial_\mu u_\nu)$$

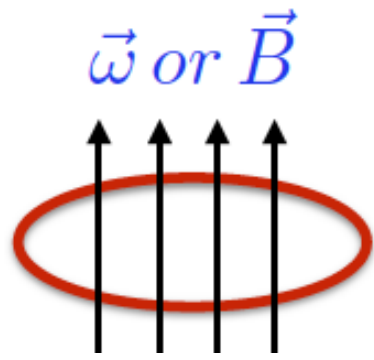


# Analogy Between Rotation & B Field

EM vector field	$\vec{A}$	Fluid velocity field	$\vec{V}$
Magnetic field	$\vec{B} = \vec{\nabla} \times \vec{A}$	Fluid vorticity	$\vec{\omega} = \vec{\nabla} \times \vec{V}$
Lorentz force		Coriolis force	
	$\vec{F}_{Lorentz} = e \vec{v} \times \vec{B}$		$\vec{F}_{cor} = 2m \vec{v} \times \vec{\omega}$

**Magnetic Polarization**

**Charged particle's spin**  
 → magnetic moment  
 → align with B field



**Rotational Polarization**

**Global rotation**  
 →  
 “equal-partition” for angular momentum  
 → individual spin & orbital angular momentum  
 align with rotation

# Description of Slowly Rotating Fermion System

*Dirac Lagrangian in rotating frame:*

$$g_{\mu\nu} = \begin{pmatrix} 1 - \vec{v}^2 & -v_1 & -v_2 & -v_3 \\ -v_1 & -1 & 0 & 0 \\ -v_2 & 0 & -1 & 0 \\ -v_3 & 0 & 0 & -1 \end{pmatrix}$$

$$\vec{v} = \vec{\omega} \times \vec{x}.$$

$$\bar{\gamma}^\mu = e_a^\mu \gamma^a$$

$$\Gamma_\mu = \frac{1}{4} \times \frac{1}{2} [\gamma^a, \gamma^b] \Gamma_{ab\mu}$$



$$\mathcal{L} = \bar{\psi} [i\bar{\gamma}^\mu (\partial_\mu + \Gamma_\mu) - m] \psi$$

*Under slow rotation:*

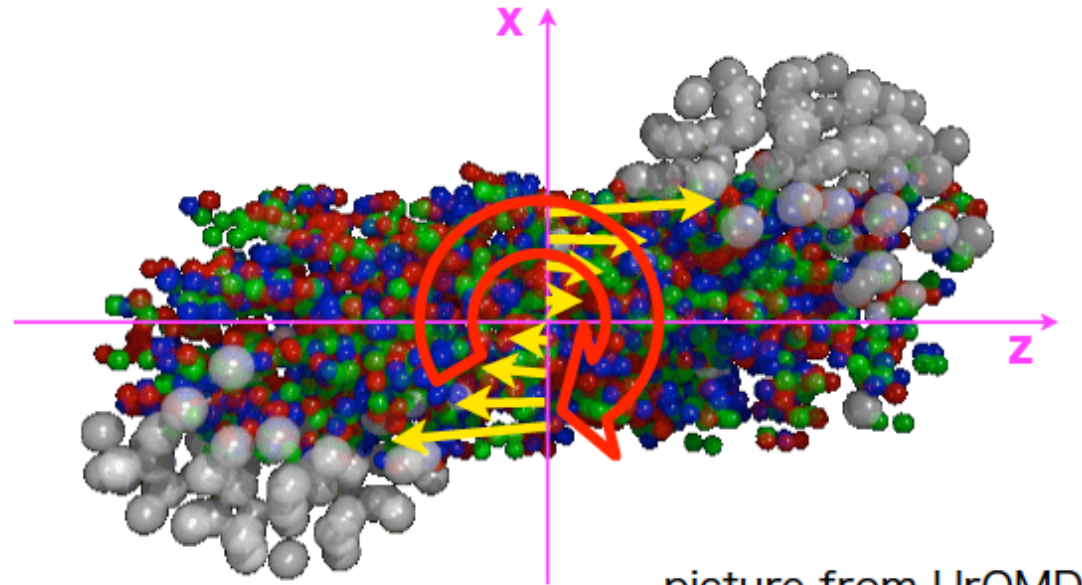
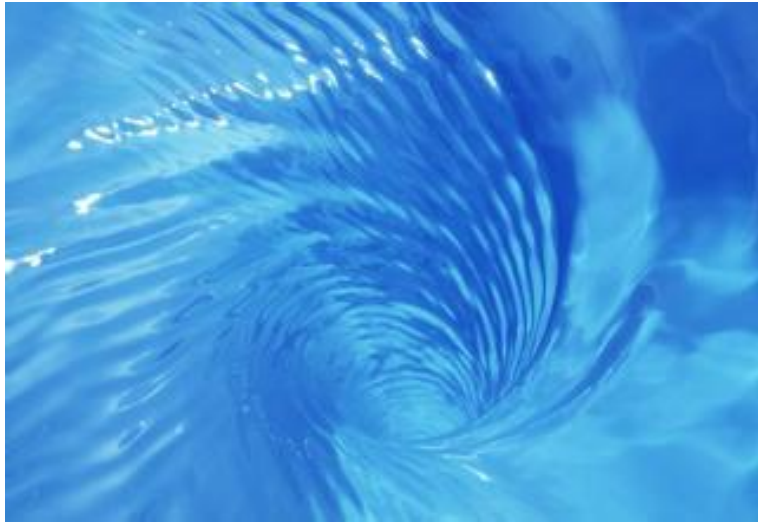
$$\mathcal{L} = \psi^\dagger \left[ i\partial_0 + i\gamma^0 \vec{\gamma} \cdot \vec{\partial} + (\vec{\omega} \times \vec{x}) \cdot (-i\vec{\partial}) + \vec{\omega} \cdot \vec{S}_{4 \times 4} \right] \psi$$

$$\hat{H} = \gamma^0 (\vec{\gamma} \cdot \vec{p} + m) - \vec{\omega} \cdot (\vec{x} \times \vec{p} + \vec{S}_{4 \times 4}) = \hat{H}_0 - \vec{\omega} \cdot \hat{\vec{J}}$$

**Rotational  
polarization effect!**



# Effects from Rotational Polarization



picture from UrQMD

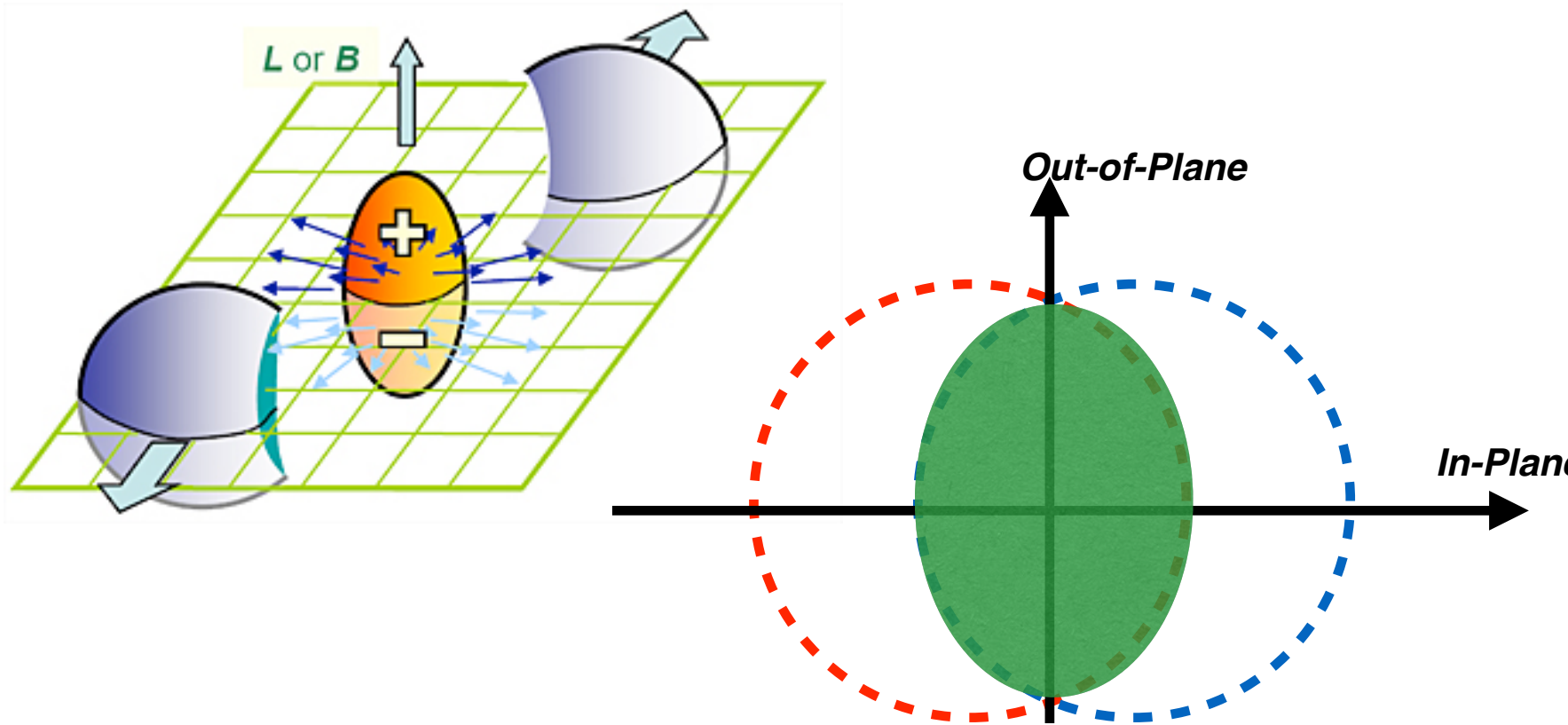
- \* **Is there strong rotational motion (vortical structures) in heavy ion collisions? YES!**
- \* **Possible physical effects due to rotational polarization? YES!**

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# Vorticity & Polarization

---

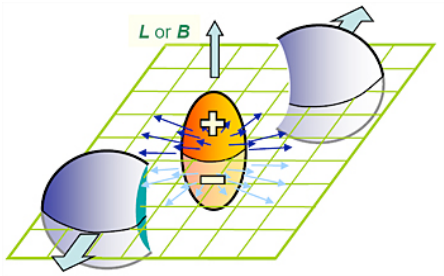
# The Setup of Heavy Ion Collision



***A typical off-central collision.***

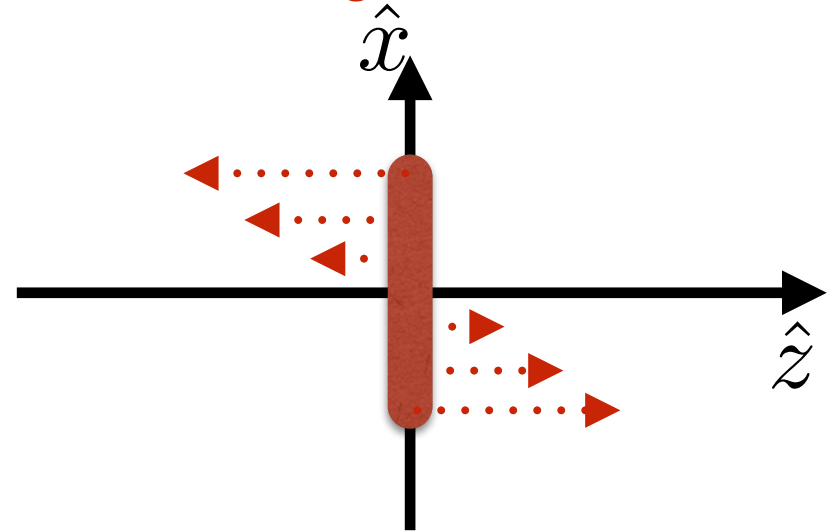
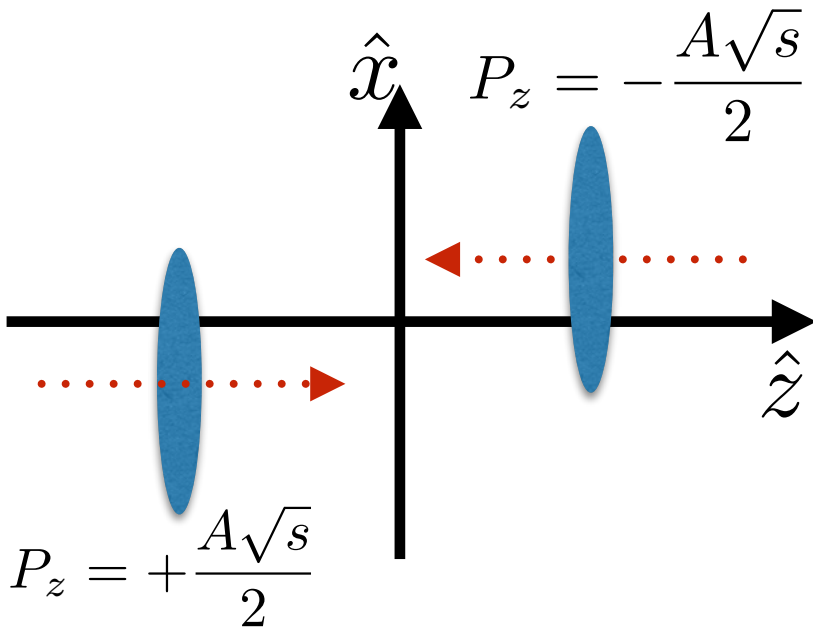


# Rotating Quark-Gluon Plasma



$$L_y = \frac{Ab\sqrt{s}}{2} \sim 10^{4\sim 5} \hbar$$

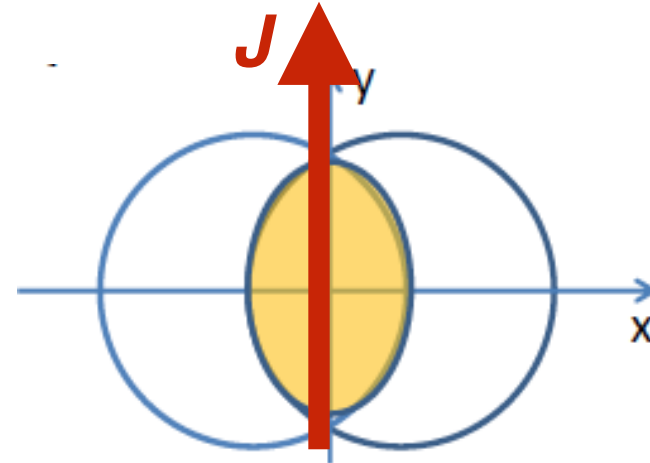
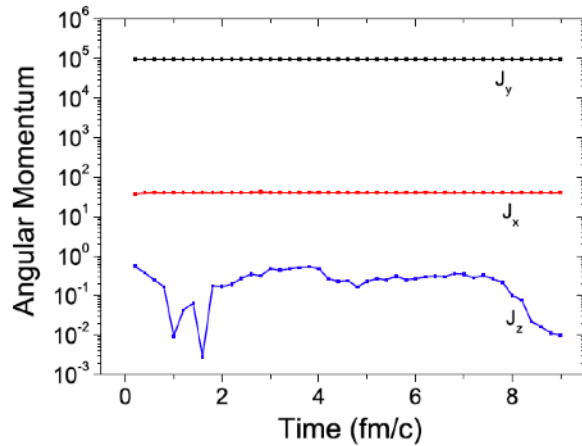
*QGP's way of accommodating this angular momentum*



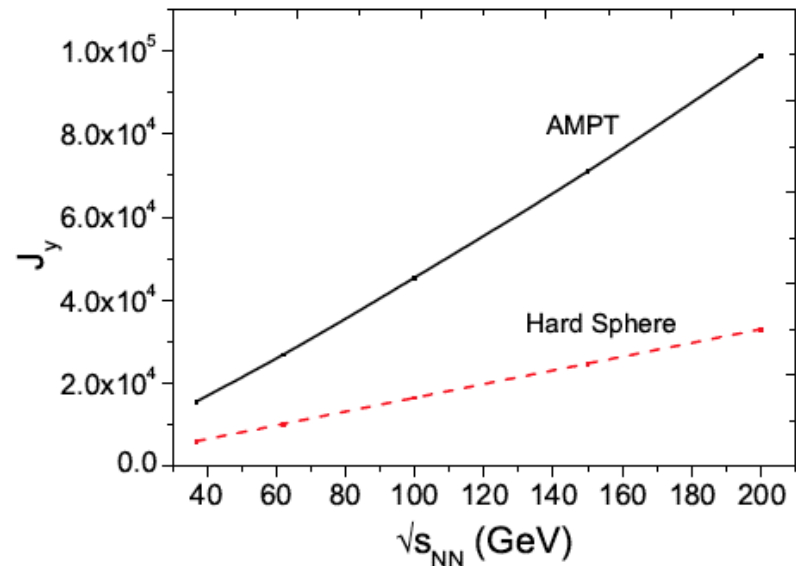
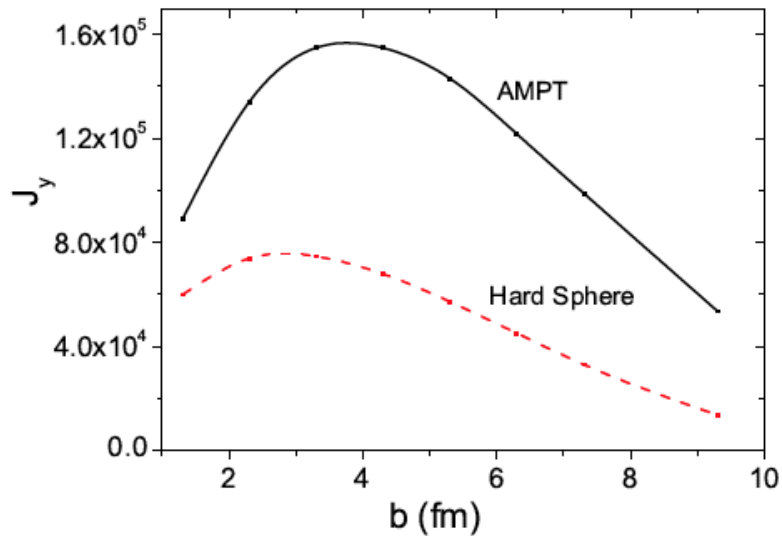
**Issues:**

- \* *Spectators taking away lots of L ?!*
- \* *Is the L residing in fireball conserved in time?*

# Rotating Quark-Gluon Plasma



**Angular momentum is conserved in time.**



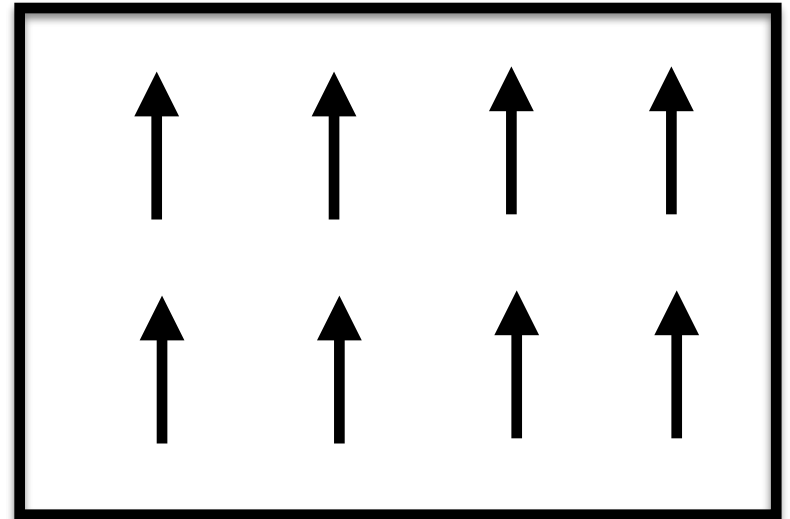
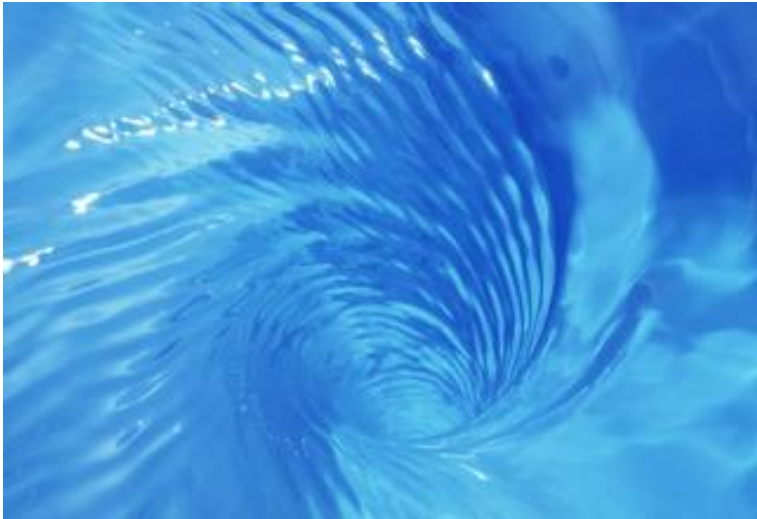
Yin Jiang, Zi-Wei Lin, JL, arXiv:1602.06580[hep-ph][PRC2016]

# Rotating Quark-Gluon Plasma

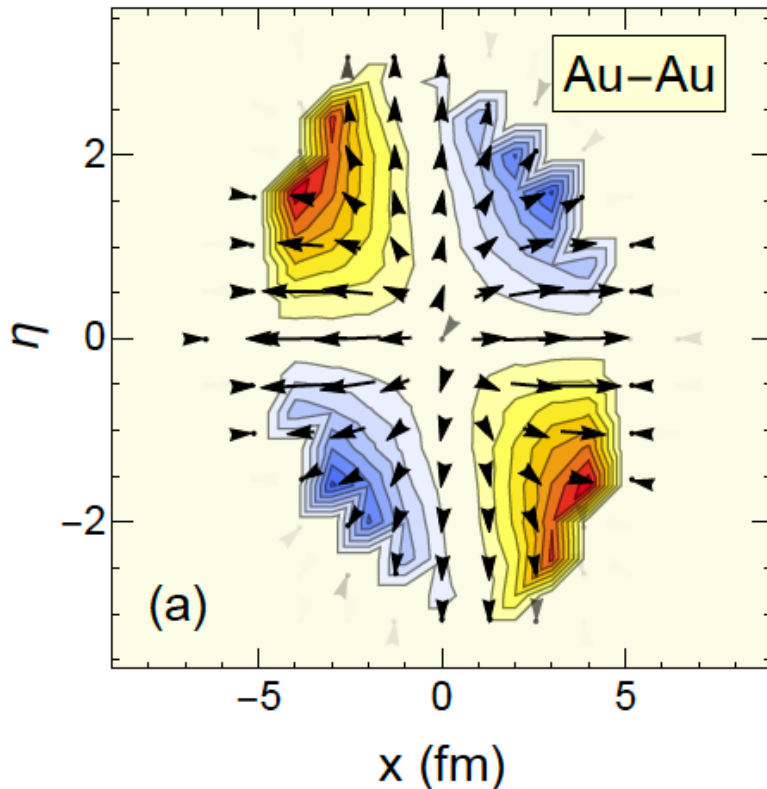
*How does this many-body system cope with a sizable angular momentum?*

*Orbital motion (vorticity);  
Spin alignment (polarization).*

*Both must occur, à la “equal-partition”*



# Rotating Quark-Gluon Plasma



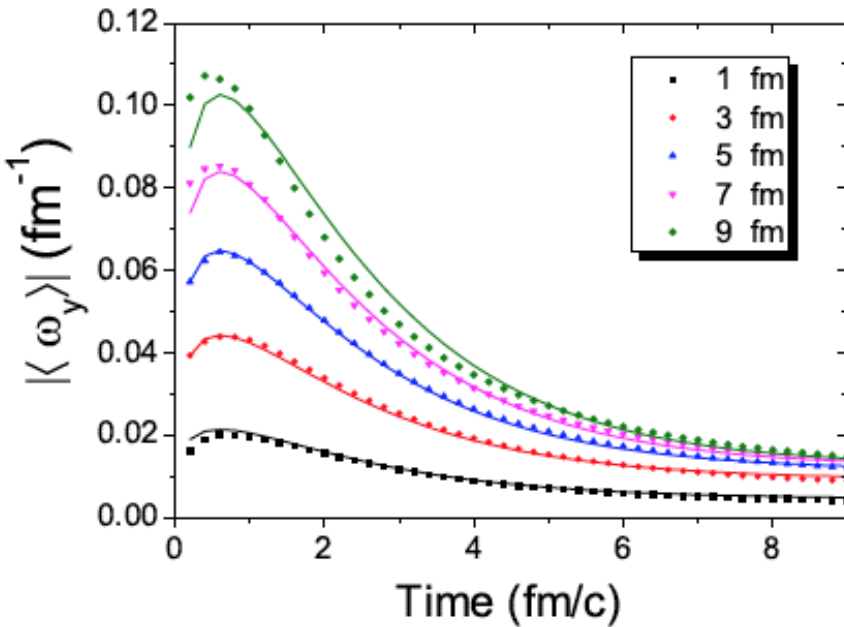
$$\vec{v}(\rho, \phi, \eta) = \hat{e}_\rho v_0(\rho, \eta) [1 + 2c_2(\rho, \eta) \cos 2\phi]$$

$$\begin{aligned} \omega_y &= \frac{\partial v_\rho}{\partial z} \cos \phi \\ &= \frac{2}{t} (ch\eta)^2 \partial_\eta (v_0 + 2v_0 c_2 \cos 2\phi) \cos \phi \\ &= \frac{2}{t} (ch\eta)^2 \left( \frac{x}{\rho} \right) \partial_\eta \left[ v_0 + 2v_0 c_2 \left( 2\frac{x^2}{\rho^2} - 1 \right) \right] \end{aligned}$$

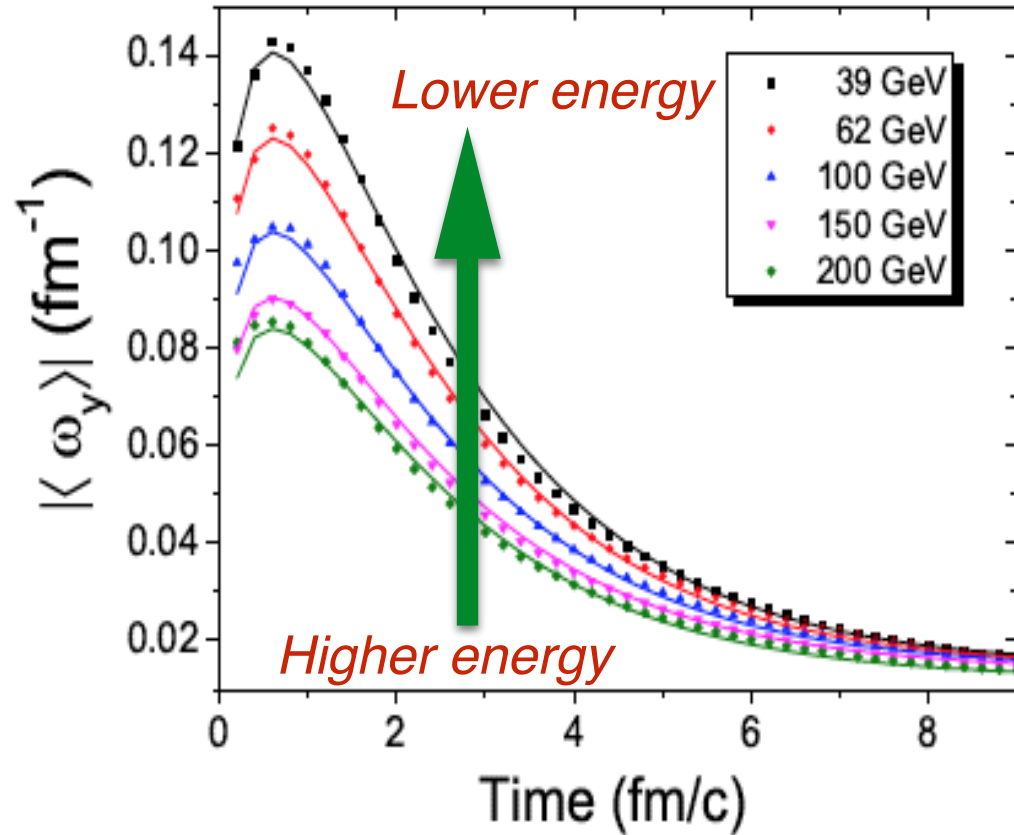
***The local vorticity pattern is strongly influenced by the bulk flow.  
The averaged vorticity reflects the global orbital angular momentum.***

**Yin Jiang, Zi-Wei Lin, JL, PRC2016**

# Rotating Quark-Gluon Plasma



**Centrality trend**



**Beam Energy  
Dependence**

Yin Jiang, Zi-Wei Lin, JL, PRC2016

# Rotational Polarization

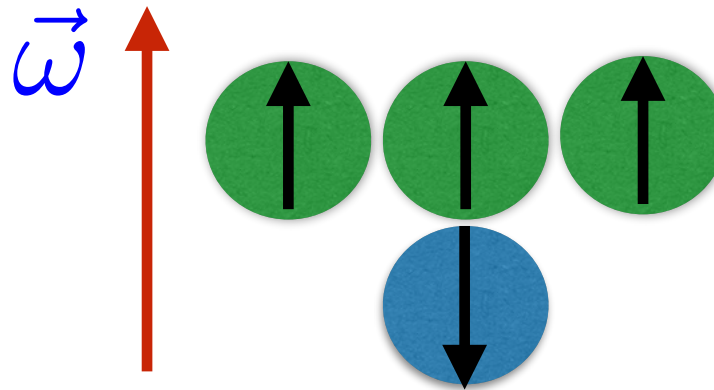
$$\hat{H} = \gamma^0(\vec{\gamma} \cdot \vec{p} + m) - \vec{\omega} \cdot (\vec{x} \times \vec{p} + \vec{S}_{4 \times 4}) = \hat{H}_0 - \vec{\omega} \cdot \hat{J}$$

**Rotational  
polarization effect!**



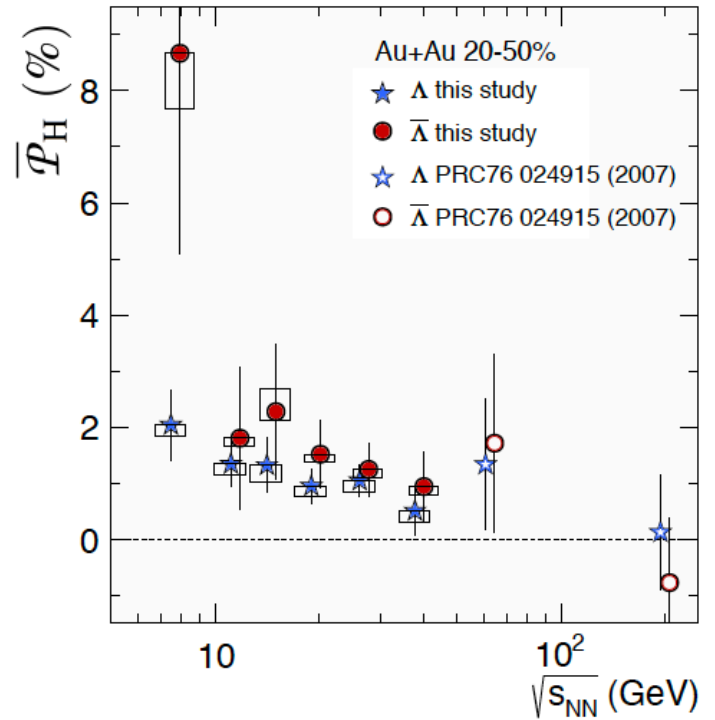
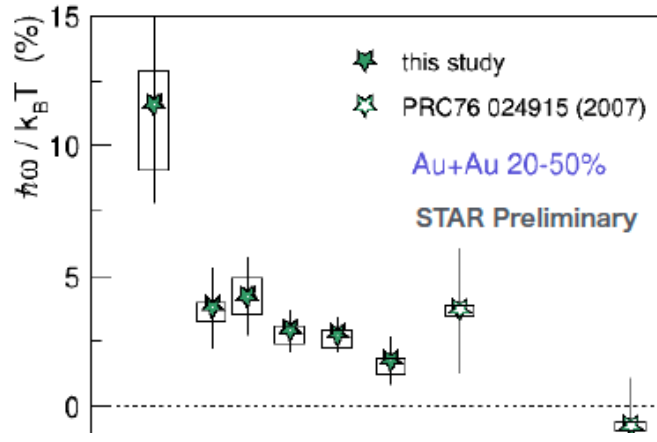
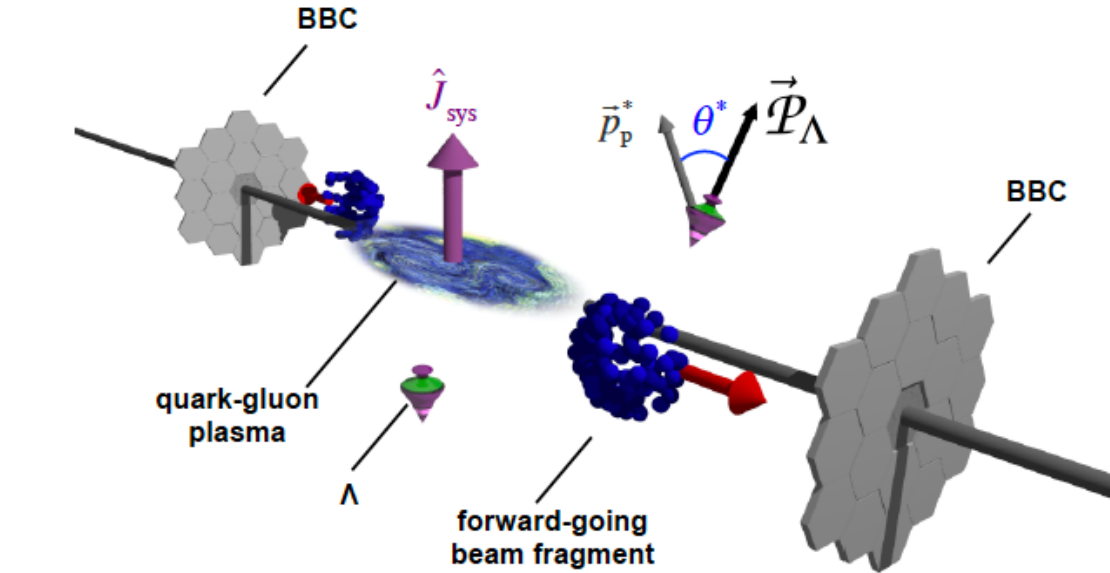
**For thermally produced particles:**

$$dN \propto e^{-\frac{\vec{\omega} \cdot \vec{J}}{T}}$$





# Rotational Polarization Observed!



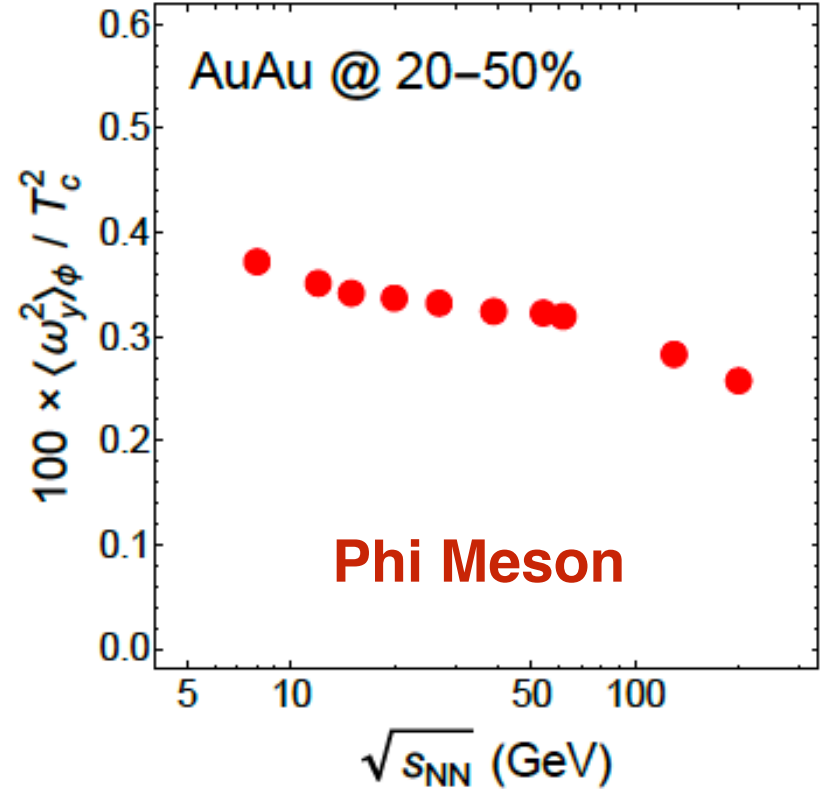
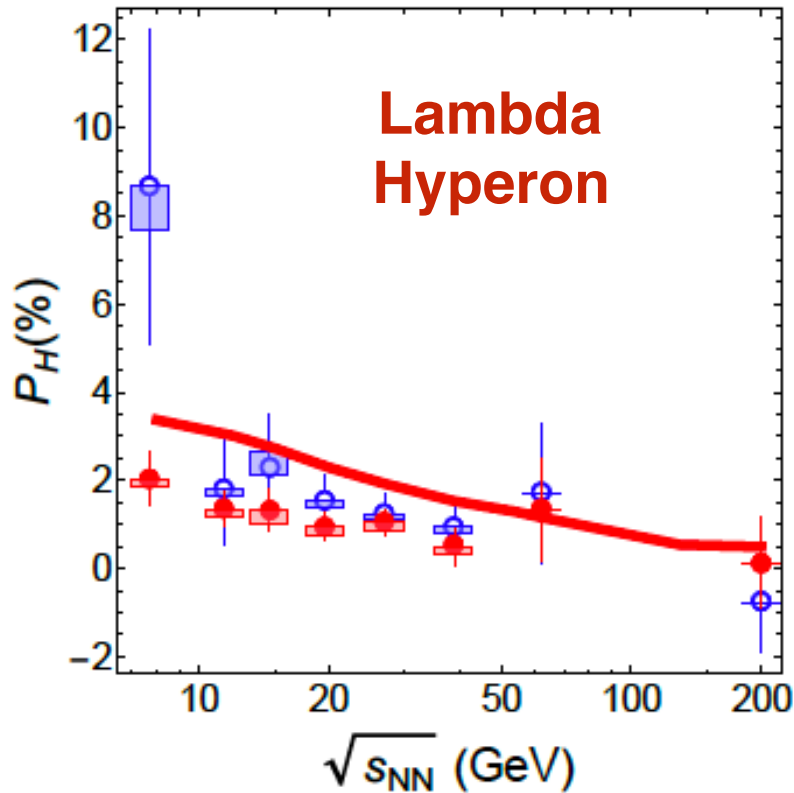
$$\omega \approx k_B T (\overline{P}_{\Lambda'} + \overline{P}_{\overline{\Lambda}'}) / \hbar$$

$$\omega \approx (9 \pm 1) \times 10^{21} \text{ s}^{-1}$$

**The most vortical fluid!**

**STAR Collaboration, Nature 2017**

# Estimating Polarization Effect

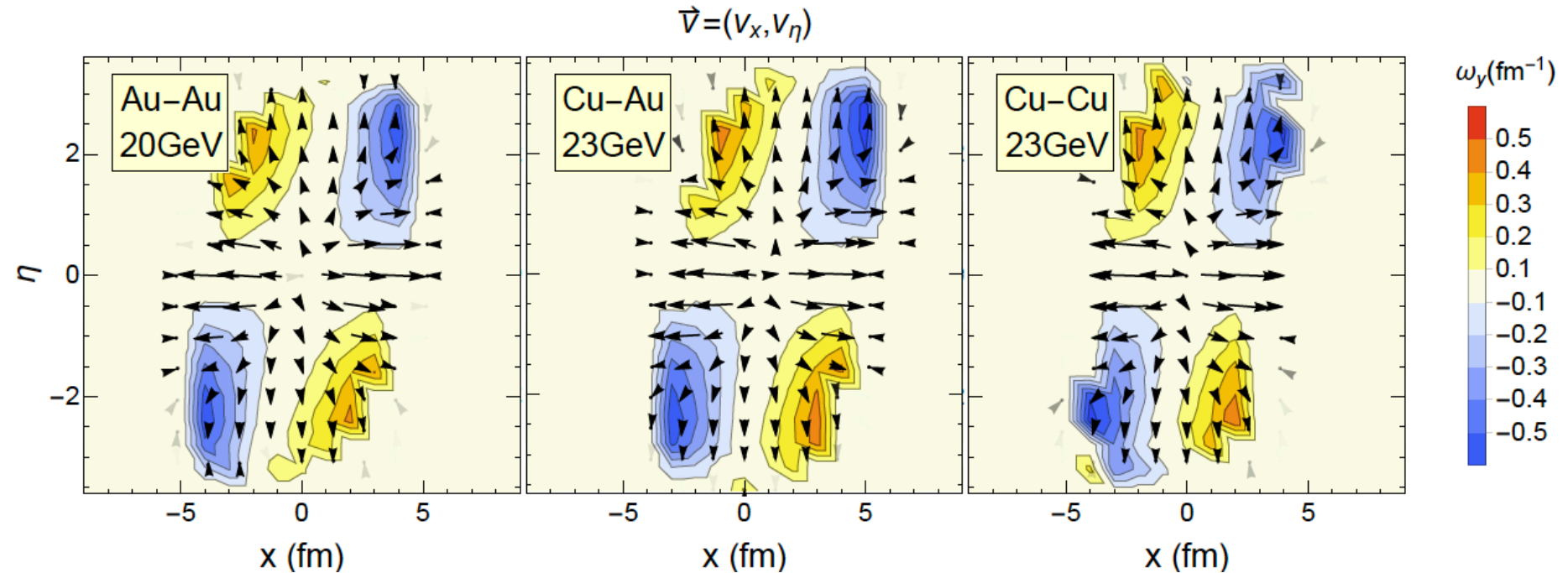


$$\rho_{00}^{\text{thermal}} = \frac{f_{BE}(E)}{f_{BE}(E + \omega_y) + f_{BE}(E) + f_{BE}(E - \omega_y)} \approx \frac{1}{3} - \frac{1}{9} \frac{\omega_y^2}{T_{fo}^2}$$

Shuzhe Shi, Kangle Li, JL, arXiv:1712.00878.

# Different Colliding Systems

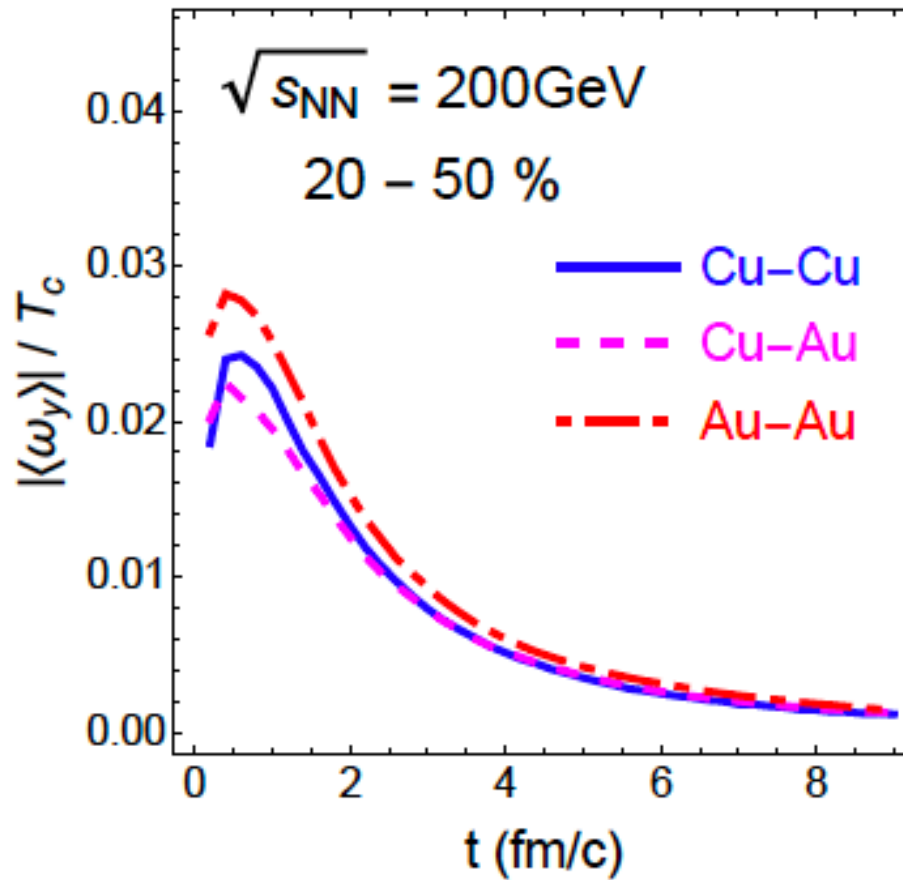
Using different colliding systems for independent check!



Shuzhe Shi, Kangle Li, JL, arXiv:1712.00878.

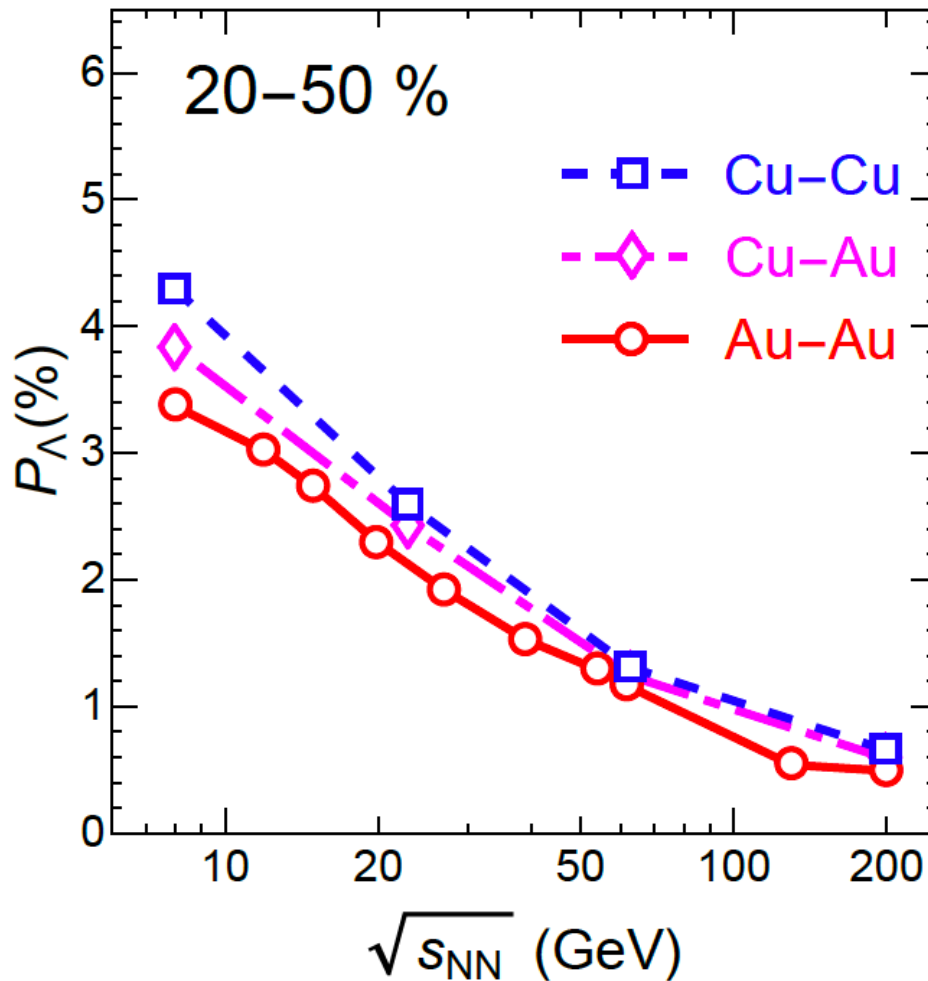
# Interesting System Size Dependence

The polarization effect in CuCu collisions should be sizable!



Shuzhe Shi, Kangle Li, JL, arXiv:1712.00878.

# Prediction for Polarization in CuCu and AuAu

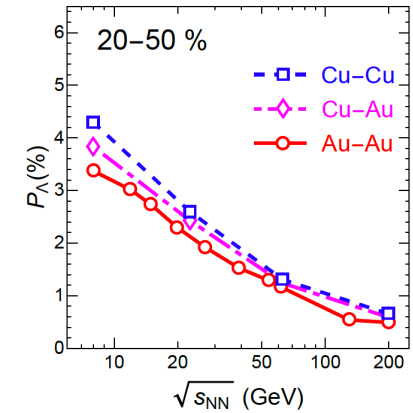
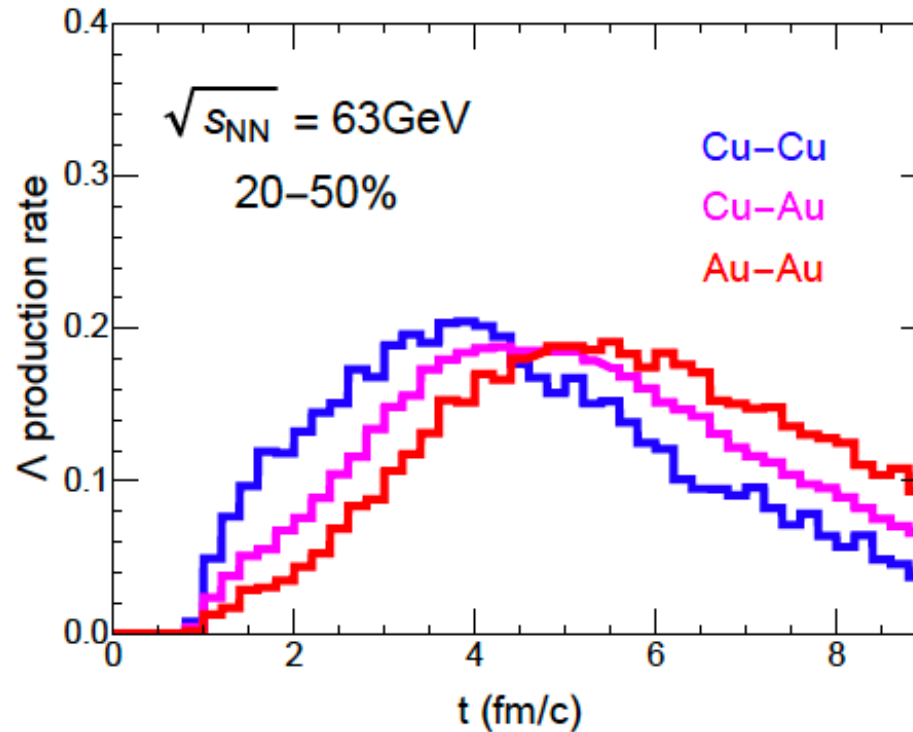
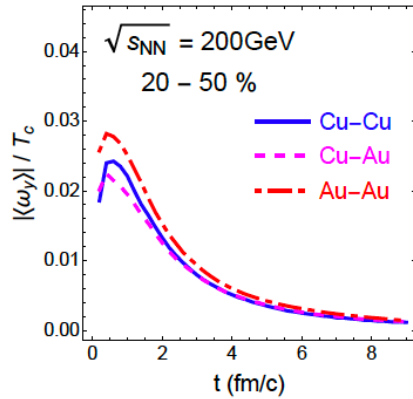


Our surprising finding of  
the signal hierarchy:  
CuCu > CuAu > AuAu !

WHY SO?

Shuzhe Shi, Kangle Li, JL, arXiv:1712.00878.

# Prediction for Polarization in CuCu and AuAu

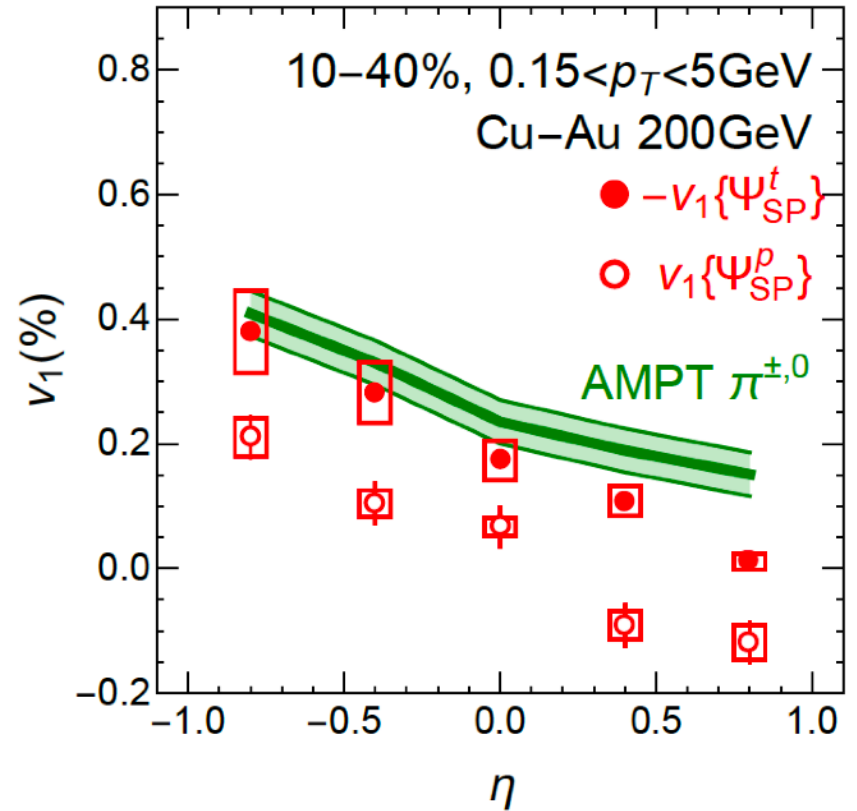
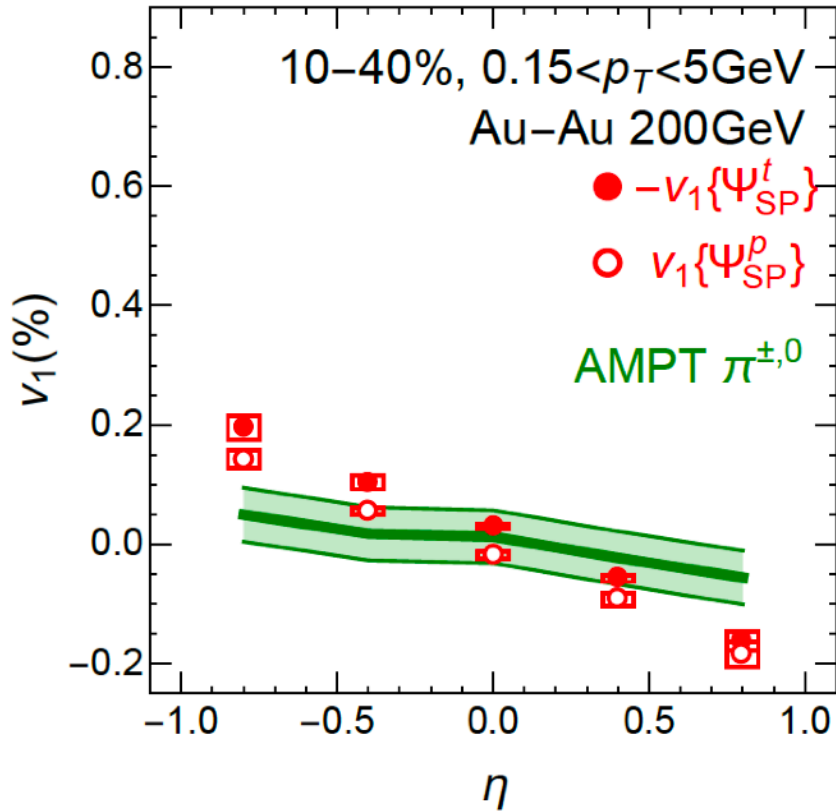


The hierarchy is due to an interesting interplay between vorticity time evolution and hyperon production timing!

Shuzhe Shi, Kangle Li, JL, arXiv:1712.00878.



# Directed Flow



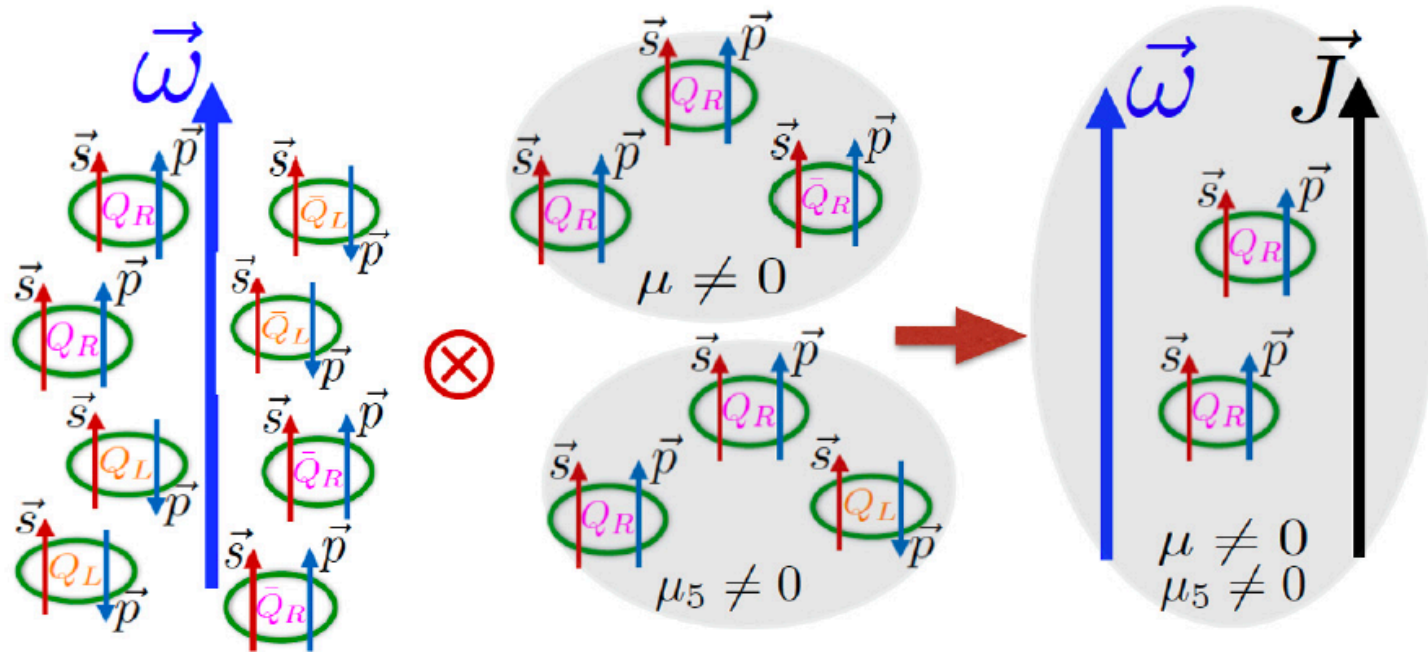
It is important to understand directed flow and vorticity simultaneously.

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# Anomalous Transport Under Rotation

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# Intuitive Picture of CVE



## Intuitive understanding of CVE:

rotational polarization  $\rightarrow$   
correlation between micro.  
**SPIN & EXTERNAL FORCE**



Chiral imbalance  $\rightarrow$   
correlation between directions of  
**SPIN & MOMENTUM**



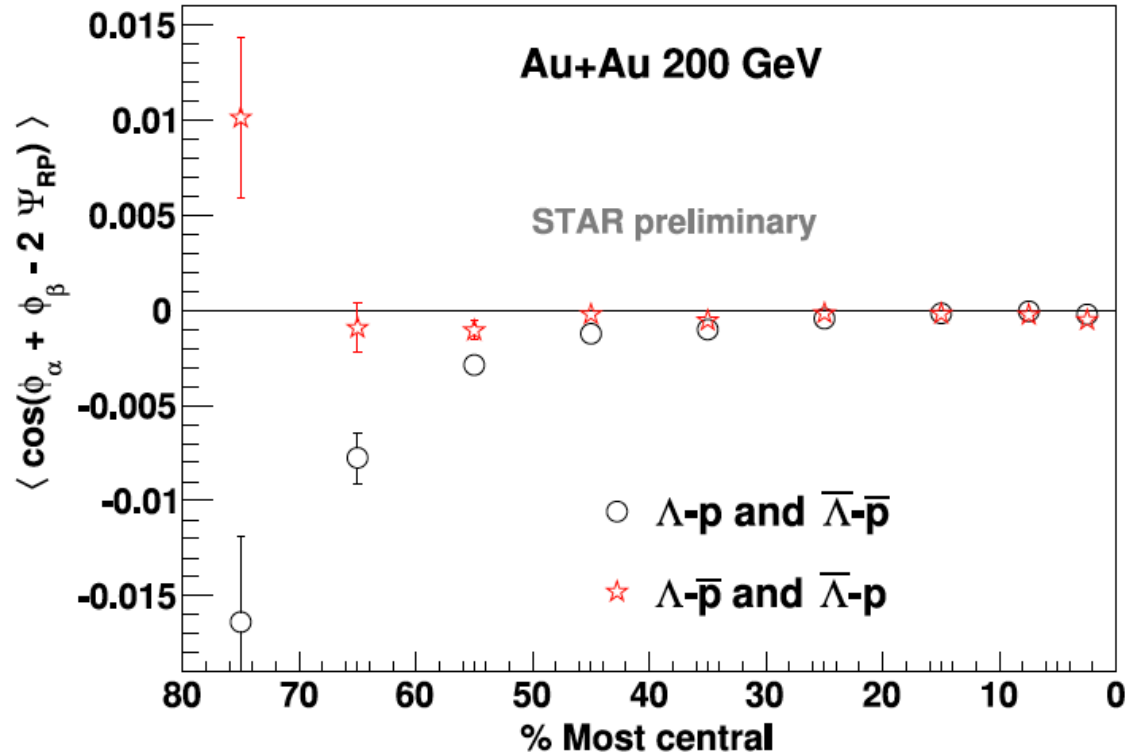
**Current along fluid rotation axis!**

$$J_V = \frac{1}{\pi^2} \mu \mu_5 \omega$$

# Potential Signal of CVE

$$\vec{J}_Q^{2f} = \frac{N_c \mu_5}{2\pi^2} \left[ \frac{5}{9} (e\vec{B}) + \frac{2}{9} (\mu_B \vec{\omega}) \right]$$

$$\vec{J}_B^{2f} = \frac{N_c \mu_5}{2\pi^2} \left[ \frac{1}{9} (e\vec{B}) + \frac{4}{9} (\mu_B \vec{\omega}) \right]$$

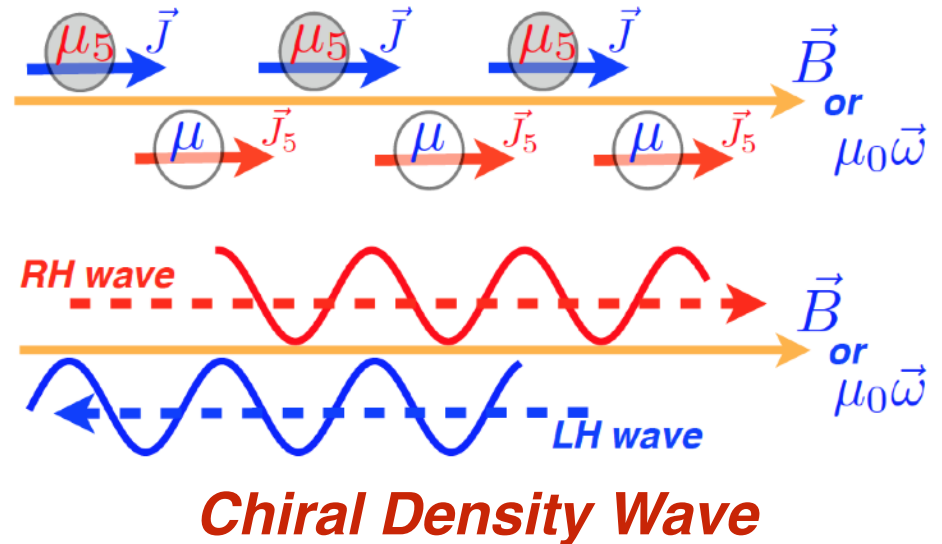
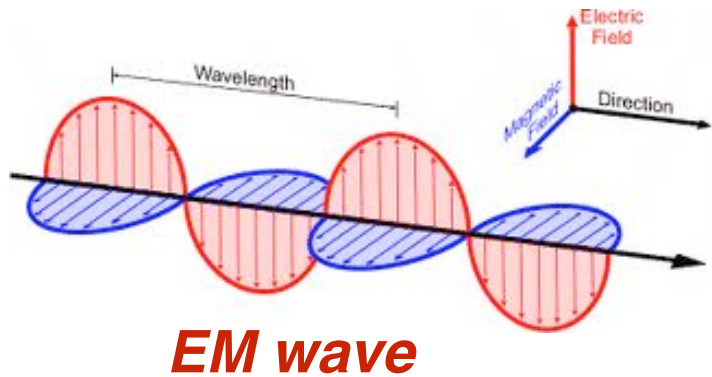


**CVE  $\rightarrow$  Baryon separation across reaction plane.**

**The lower beam energy collisions: better for CVE search!**

# Chiral Vortical Wave

*Wave: propagating “oscillations” of two coupled quantities  
e.g. sound wave (pressure & density); EM wave (E & B fields)*



## Chiral Magnetic Wave

$$\left( \partial_0 \pm \frac{(Qe)}{(4\pi^2)\chi} \vec{\mathbf{B}} \cdot \nabla \right) \delta J_{R/L}^0 = 0$$

[Kharzeev, Yee, PRD2010;  
Burnier, Kharzeev, JL, Yee, PRL2011]

## Chiral Vortical Wave

$$\left( \partial_0 \pm \frac{\mu_0}{(2\pi^2)\chi_{\mu_0}} \vec{\omega} \cdot \nabla \right) \delta J_{R/L}^0 = 0$$


[Jiang, Huang, JL, arXiv:  
1504.03201, PRD2015]

# Exciting Progress: See Recent Reviews


Progress in Particle and Nuclear Physics 88 (2016) 1–28

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Contents lists available at [ScienceDirect](#)

 **Progress in Particle and Nuclear Physics**

journal homepage: [www.elsevier.com/locate/ppnp](http://www.elsevier.com/locate/ppnp)




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Review

Chiral magnetic and vortical effects in high-energy nuclear collisions—A status report

D.E. Kharzeev<sup>a,b</sup>, J. Liao<sup>c,d,\*</sup>, S.A. Voloshin<sup>e</sup>, G. Wang<sup>f</sup>

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<sup>b</sup> Department of Physics and RIKEN-BNL Research Center, Brookhaven National Laboratory, Upton, NY 11973-5000, USA  
<sup>c</sup> Physics Department and Center for Exploration of Energy and Matter, Indiana University, 727 E Third Street, Bloomington, IN 47405, USA  
<sup>d</sup> RIKEN BNL Research Center, Bldg. 510A, Brookhaven National Laboratory, Upton, NY 11973, USA  
<sup>e</sup> Department of Physics and Astronomy, Wayne State University, 666 W. Hancock, Detroit, MI 48201, USA  
<sup>f</sup> Department of Physics and Astronomy, University of California, Los Angeles, CA 90095, USA



**Prog. Part. Nucl. Phys. 88, 1 (2016)[arXiv:1511.04050 [hep-ph]].**

**J. Liao, Pramana 84, no. 5, 901 (2015) [arXiv:1401.2500 [hep-ph]].**



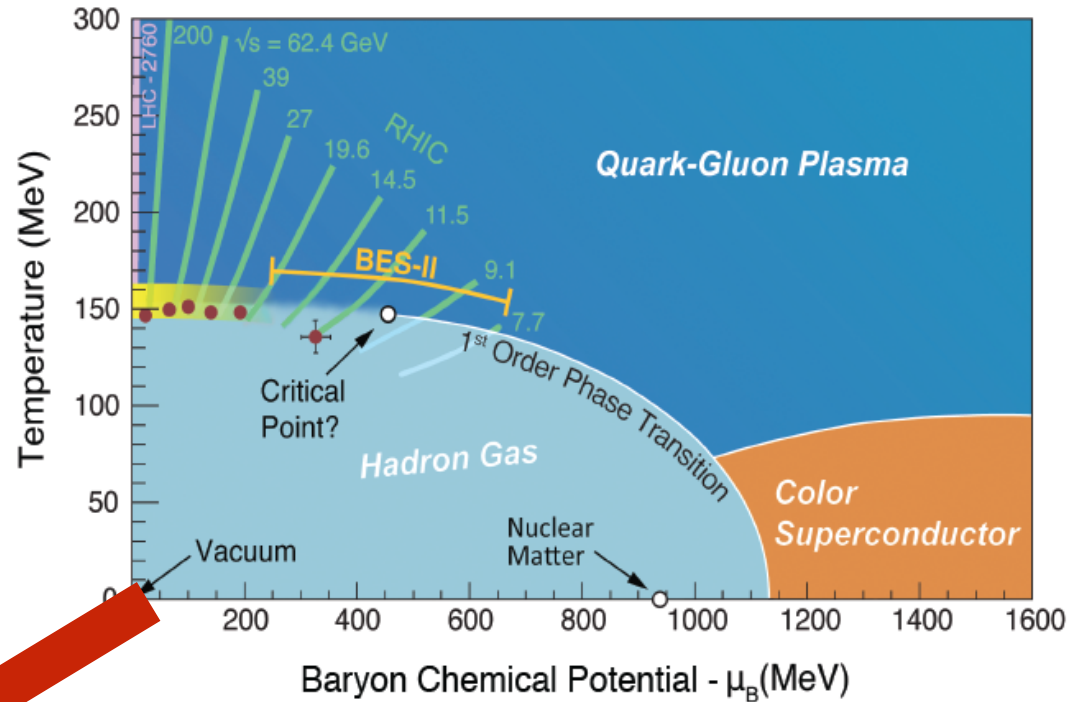
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# Phase Structure Under Rotation

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# New Dimensions of the Phase Diagram

*Putting the strongly interaction matter under strong magnetic field or fluid rotation!*

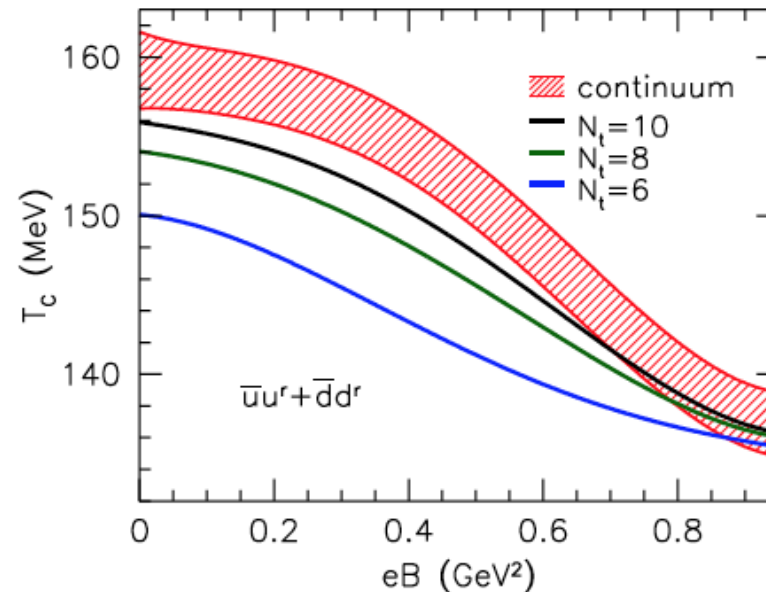
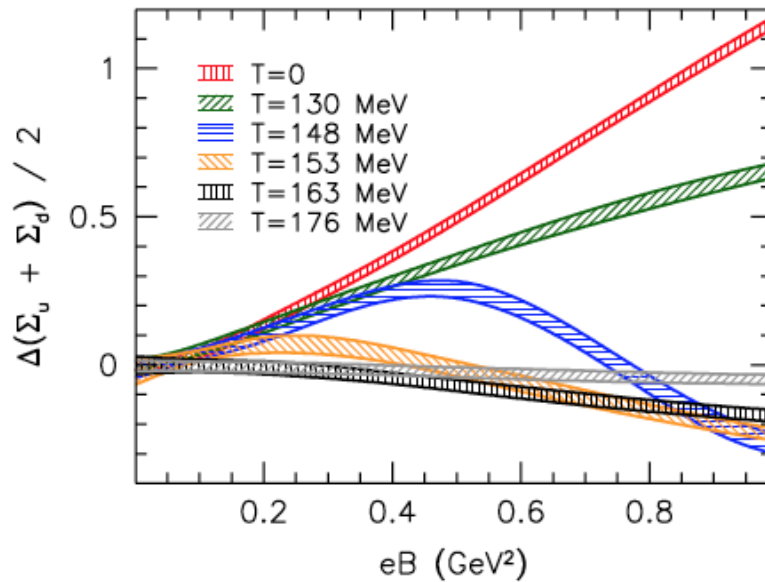


$\vec{B}$   
 $\vec{\omega}$

*Opening up new dimensions:  
Toward Hyper-Phase-Diagram!*

# Influence of Rotation on Phase Structure?

*We know that magnetic fields can change the thermodynamic properties and phase structure of QCD matter.*



**[Lattice results by Bali, et al]**

And we know the similarity between B field and rotation.

It is thus tempting to ask:

influence of rotation on phase structure?

[BTW: it could be studied on lattice,  
c.f. Yamamoto & Hirono, 2013]

# Rotational Suppression of Scalar Pairing

*Let us consider pairing phenomenon in fermion systems.*

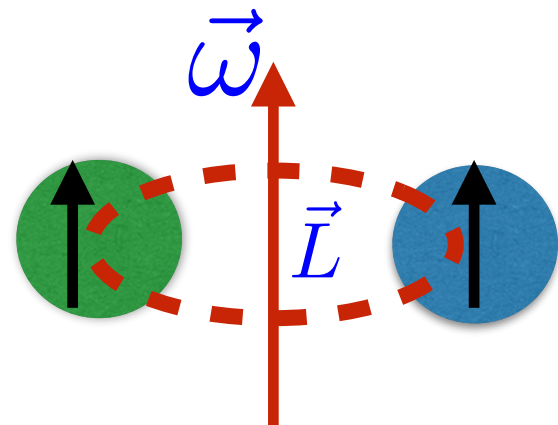
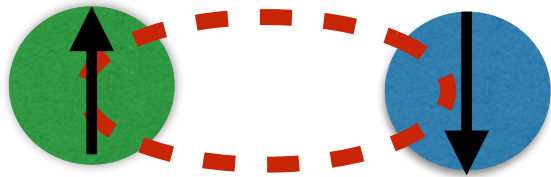
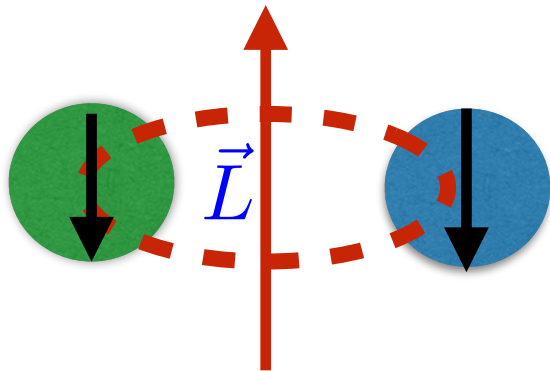
*There are many examples:*

*superconductivity, superfluidity, chiral condensate, diquark, ...*

We consider scalar pairing state, with  $J=0$ .

$$\vec{S} = \vec{s}_1 + \vec{s}_2 \quad \vec{J} = \vec{L} + \vec{S}$$

Rotation tends to polarize ALL angular momentum, both L and S, thus suppressing scalar pairing.

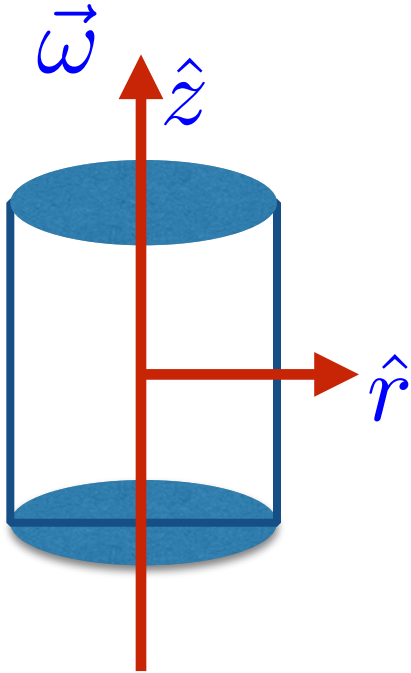


[Yin Jiang, JL, PRL2016;

See also: Chen, Fukushima, Huang, Mameda, arXiv:1512.08974]

# Description of Slowly Rotating Fermion System

**Eigenstates of free Hamiltonian:**  $\hat{H}, \hat{p}_z, \hat{p}_t^2, \hat{J}_z, \hat{h}_t \equiv \gamma^5 \gamma^3 \vec{p}_t \cdot \vec{S}$



$$u_{k_z, k_t, n, s} = \sqrt{\frac{E_k + m}{4E_k}} e^{ik_z z} e^{in\theta} \begin{pmatrix} J_n(k_t r) \\ s e^{i\theta} J_{n+1}(k_t r) \\ \frac{k_z - is k_t}{E_k + m} J_n(k_t r) \\ -\frac{s k_z + ik_t}{E_k + m} e^{i\theta} J_{n+1}(k_t r) \end{pmatrix}$$

$$v_{k_z, k_t, n, s} = \sqrt{\frac{E_k + m}{4E_k}} e^{-ik_z z} e^{in\theta} \begin{pmatrix} \frac{k_z - is k_t}{E_k + m} J_n(k_t r) \\ \frac{s k_z - ik_t}{E_k + m} e^{i\theta} J_{n+1}(k_t r) \\ J_n(k_t r) \\ -s e^{i\theta} J_{n+1}(k_t r) \end{pmatrix}$$

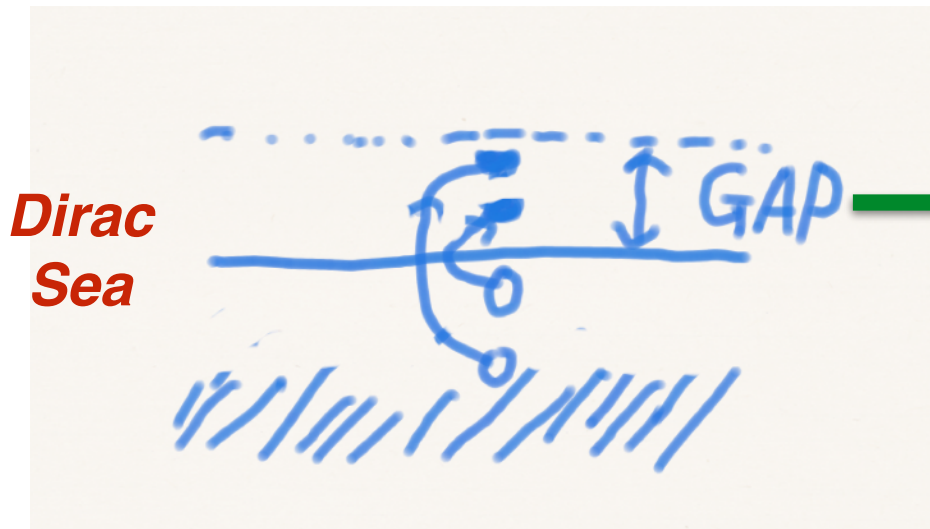
**Interaction: NJL type 4-fermion**

$$\mathcal{L}_{I_{eff}} = G(\bar{\psi}\psi)^2 + G_d(i\psi^T C \gamma^5 \psi)(i\psi^\dagger C \gamma^5 \psi^*)$$

**widely used for studying fermion pairings**

**[Yin Jiang, JL, PRL2016.]**

# The Chiral Condensate: Q-bar-Q Pairing



**Constituent  
mass**

**Lagrangian  
(SM) mass**

$$M = m - 2G \langle \bar{\psi}\psi \rangle$$

**Pairing states:  
L=1, S=1, and J=0**

$$\Omega = \int d^3\vec{r} \left\{ \frac{(M-m)^2}{4G} - \frac{1}{4\pi^2} \sum_n \int dk_t^2 \int dk_z \right. \\ \times [J_n(k_tr)^2 + J_n(k_tr)^2] \\ \times T \left[ \ln \left( 1 + e^{(\epsilon_n - \mu)/T} \right) + \ln \left( 1 + e^{-(\epsilon_n - \mu)/T} \right) \right. \\ \left. \left. + \ln \left( 1 + e^{(\epsilon_n + \mu)/T} \right) + \ln \left( 1 + e^{-(\epsilon_n + \mu)/T} \right) \right] \right\}$$

$$\epsilon_n = \sqrt{k_z^2 + k_t^2 + M^2} - (n + \frac{1}{2})\omega$$

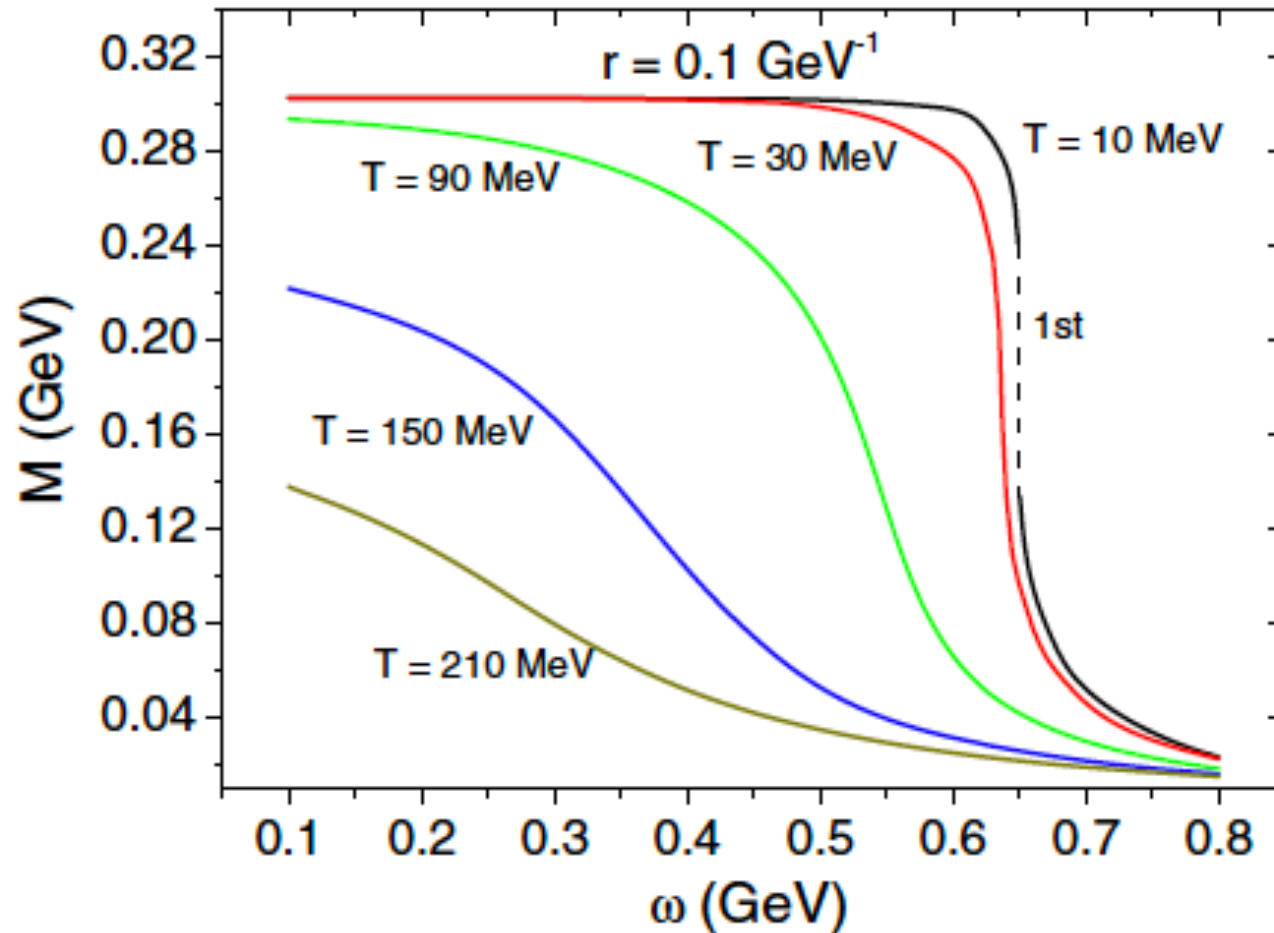
**Gap equation:**

$$\frac{\delta\Omega}{\delta M(r)} = 0$$

$$\frac{\delta^2\Omega}{\delta M(r)^2} > 0$$

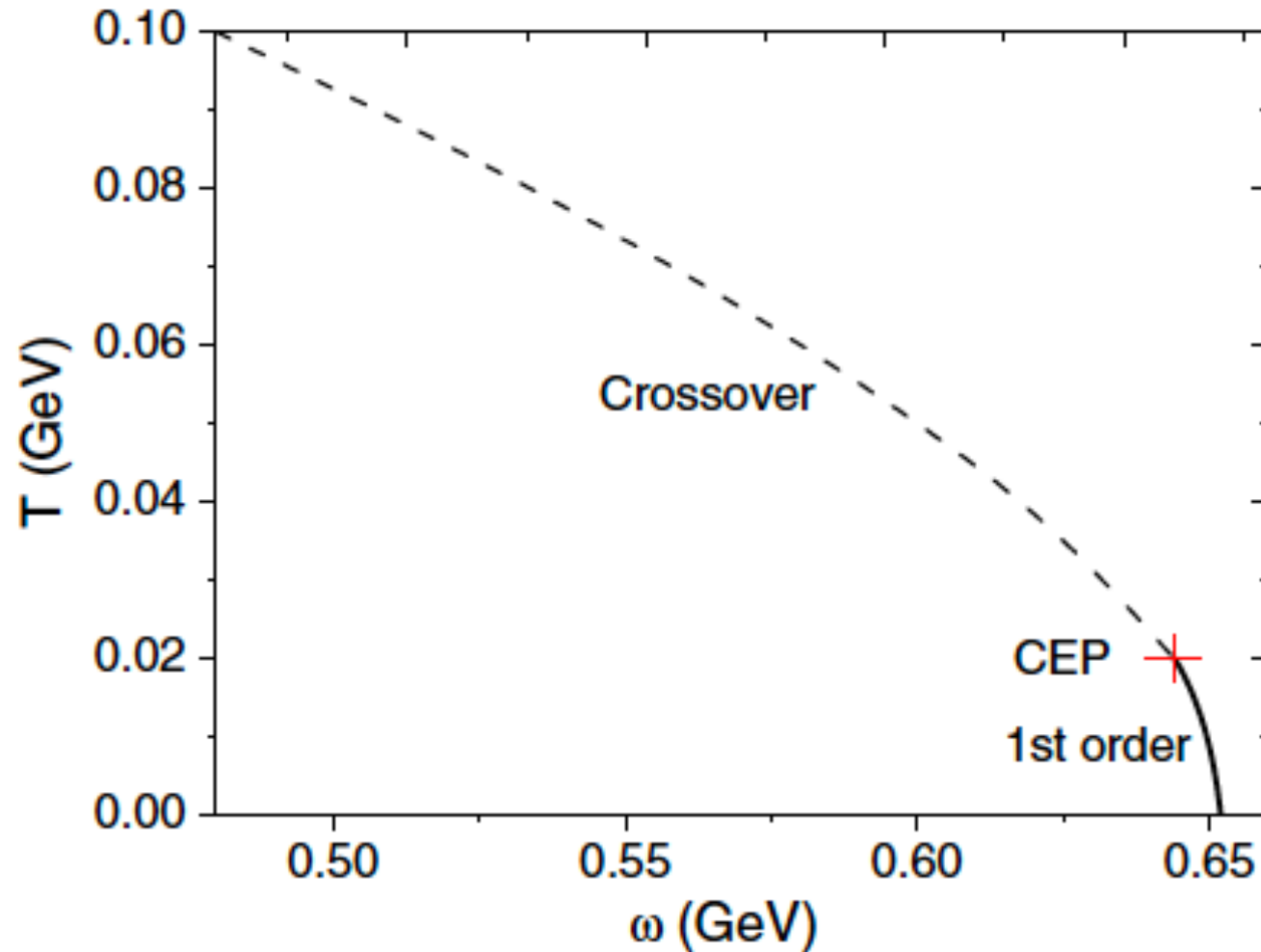
**[Yin Jiang, JL, PRL2016.]**

# Rotational Suppression of Scalar Pairing



[Yin Jiang, JL, PRL2016.]

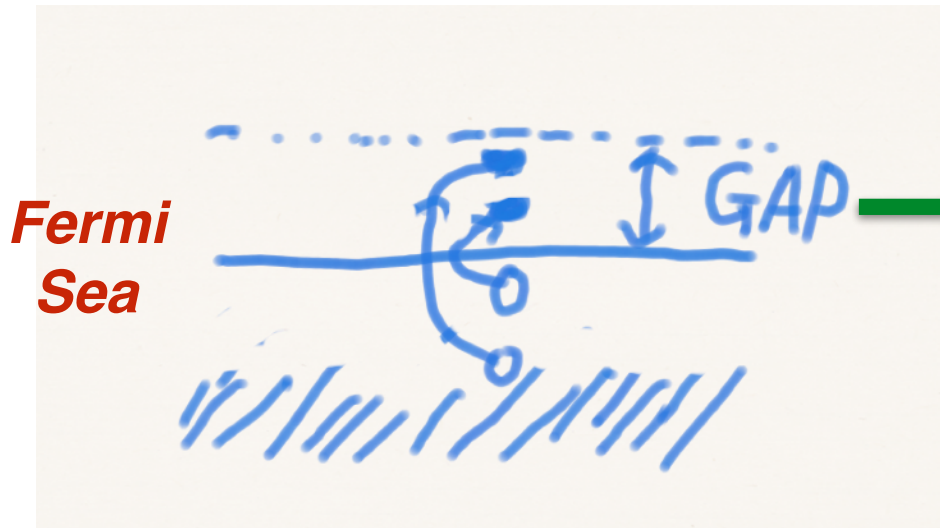
# A Possible New Critical Point



[Yin Jiang, JL, PRL2016.]



# The Diquark Condensate: Q-Q Pairing



**Color superconductivity:**

$$\Delta = -2G_d \langle i\psi^T C\gamma^5\psi \rangle$$

**Pairing states:  
L=0, S=0, and J=0**

$$\epsilon_n^{\Delta\pm} = [(\sqrt{k_z^2 + k_t^2 + m^2} \pm \mu)^2 + \Delta^2]^{\frac{1}{2}} - (n + \frac{1}{2})\omega$$

$$\Omega = \int d^3\vec{r} \left\{ \frac{\Delta^2}{4G_d} - \frac{1}{4\pi^2} \sum_n \int dk_t^2 \int dk_z \right. \\ \times [J_n(k_t r)^2 + J_n(k_t r)^2] \\ \times T \left[ \ln \left( 1 + e^{\epsilon_n^{\Delta+}/T} \right) + \ln \left( 1 + e^{-\epsilon_n^{\Delta+}/T} \right) \right. \\ \left. \left. + \ln \left( 1 + e^{\epsilon_n^{\Delta-}/T} \right) + \ln \left( 1 + e^{-\epsilon_n^{\Delta-}/T} \right) \right] \right\}$$

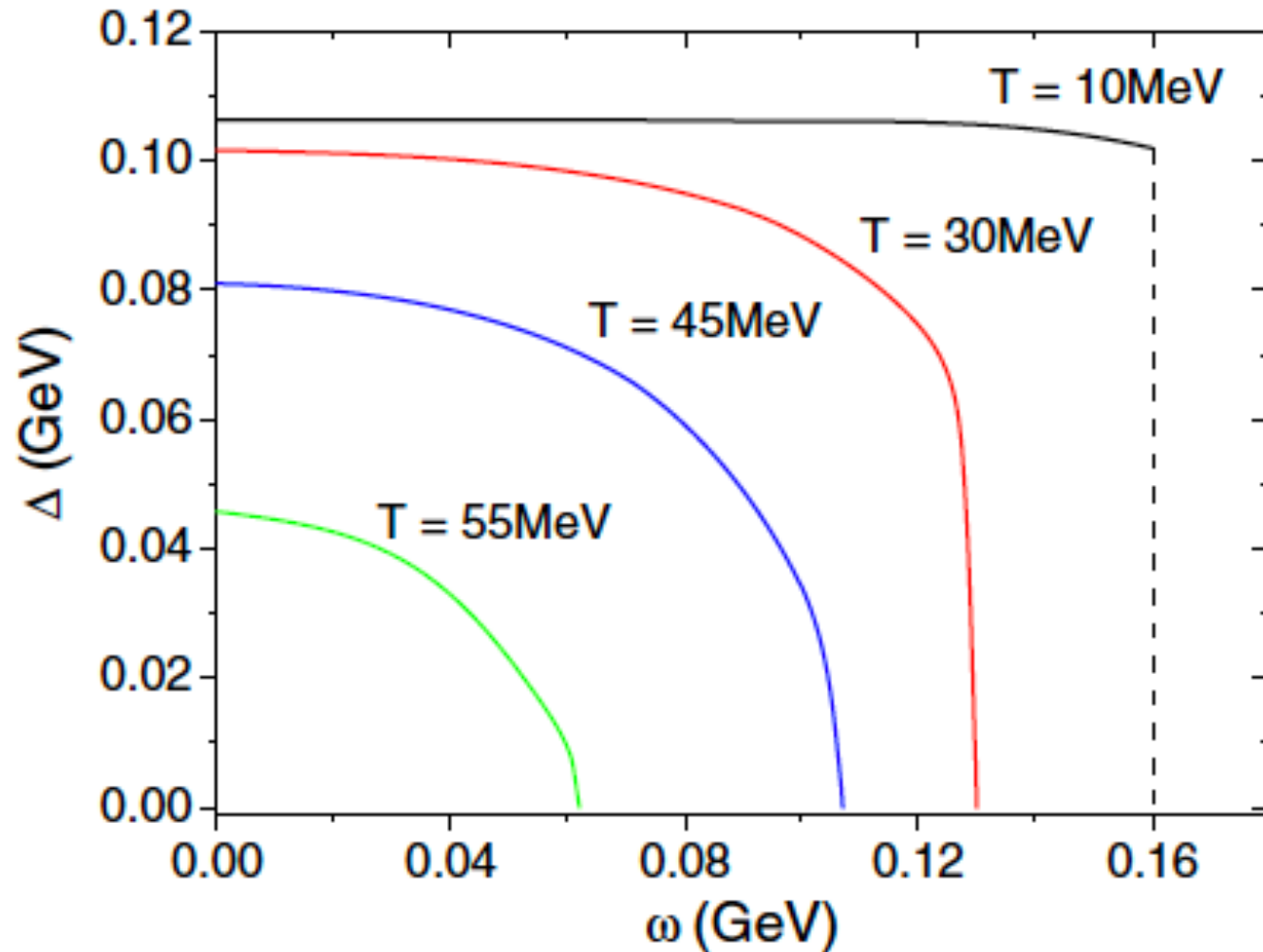
**Gap equation:**

$$\frac{\delta\Omega}{\delta\Delta(r)} = 0$$

$$\frac{\delta^2\Omega}{\delta\Delta(r)^2} > 0$$

**[Yin Jiang, JL, to appear.]**

# Rotational Suppression of Scalar Pairing



[Yin Jiang, JL, PRL2016.]

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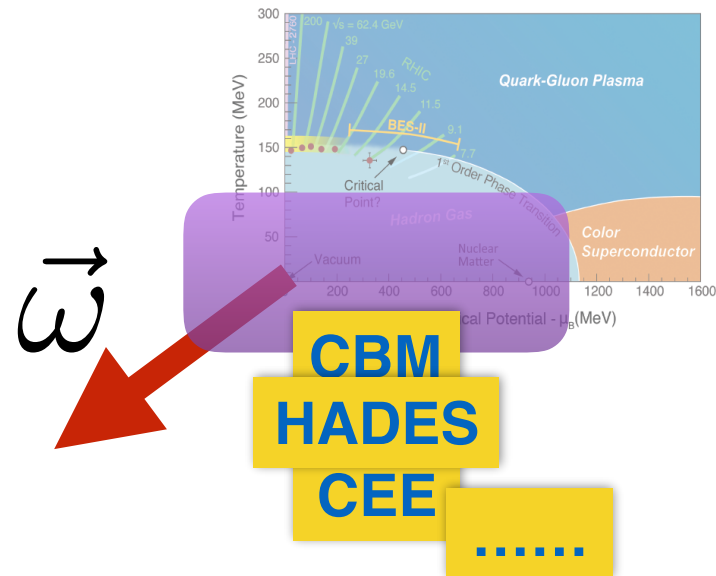
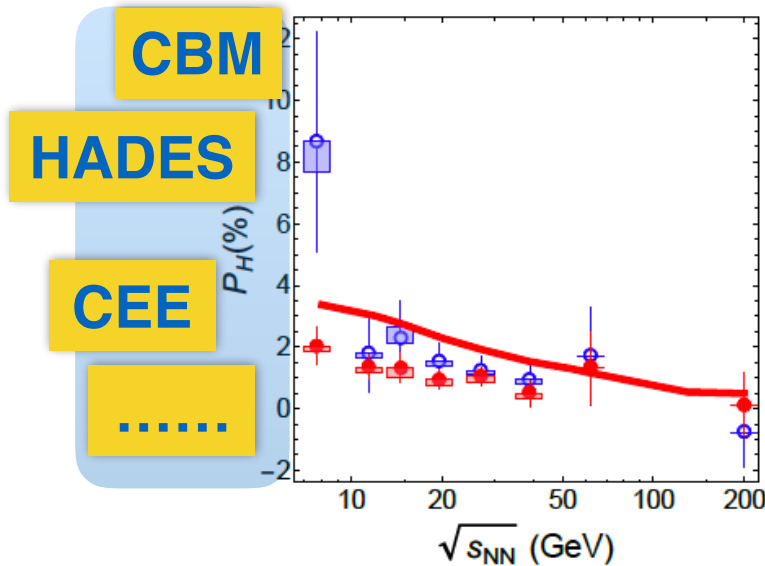
# Summary & Outlook

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# Summary & Outlook

*Properties of strongly interacting matter under rotation are interesting.*

*Rotation leads to rotational polarization:  
particle polarization; chiral vortical effects;  
change of phase structures; ...*



**Exploring rotating QCD matter  
at low beam energy heavy ion collisions!**