The correction to the Fermi's golden rule: particle physics and cosmology

Kenzo Ishikawa : Hokkaido University

Collab. Tobita、Shimomura,

Exp : Sloan,Jinnouchi,Kubota,Ushioda,

Theo:Tajima,Oda,Nakatsuka,Tatsuishi, -

Transition probability

$$P = \Gamma T + P^{(d)}$$

golden rule correction

"Basis of the Universe with Revolutionary Ideas 2018 (BURI2018)"

1-1.Quantum mechanics

1. Superposition principle : $|\Psi\rangle = c_1|\Psi_1\rangle + c_2|\Psi_2\rangle$
2. Commutation relation : $[q_i, p_j] = i\hbar\delta_{i,j}$
3. Schroedinger equation : $i\hbar\frac{\partial}{\partial t}\psi(t) = H\psi(t)$
4. Probability principle : $P = |\langle\beta|\alpha\rangle|^2, \langle\beta|\beta\rangle = \langle\alpha|\alpha\rangle = 1$

$$P \geq 0, \sum_{\beta} P_{\beta,\alpha} = 1$$

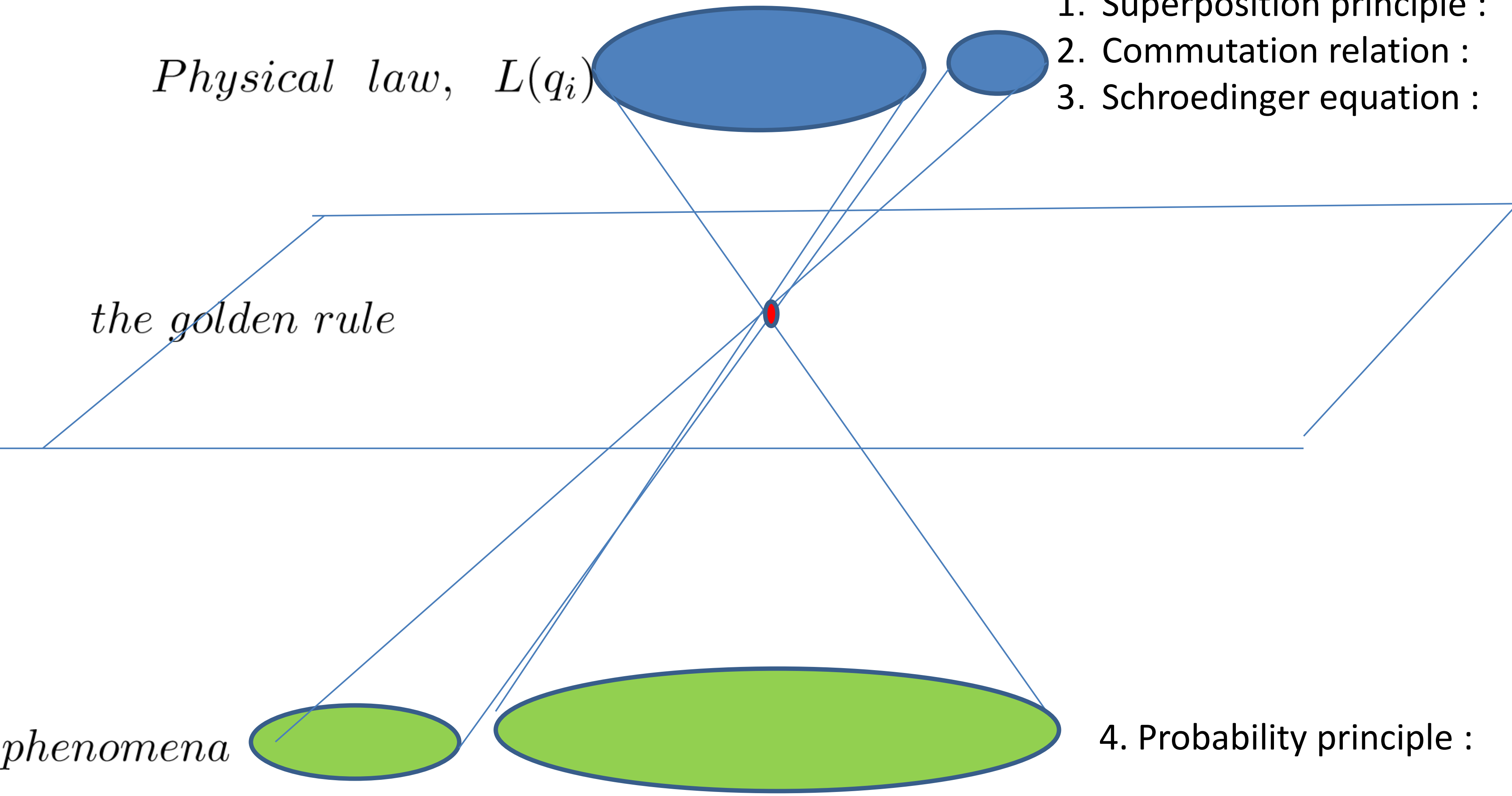
$$P = \left|\langle\Psi_o(T)|\Psi_i(0)\rangle\right|^2 = \Gamma T + P^{(d)}$$

Golden rule

Correction

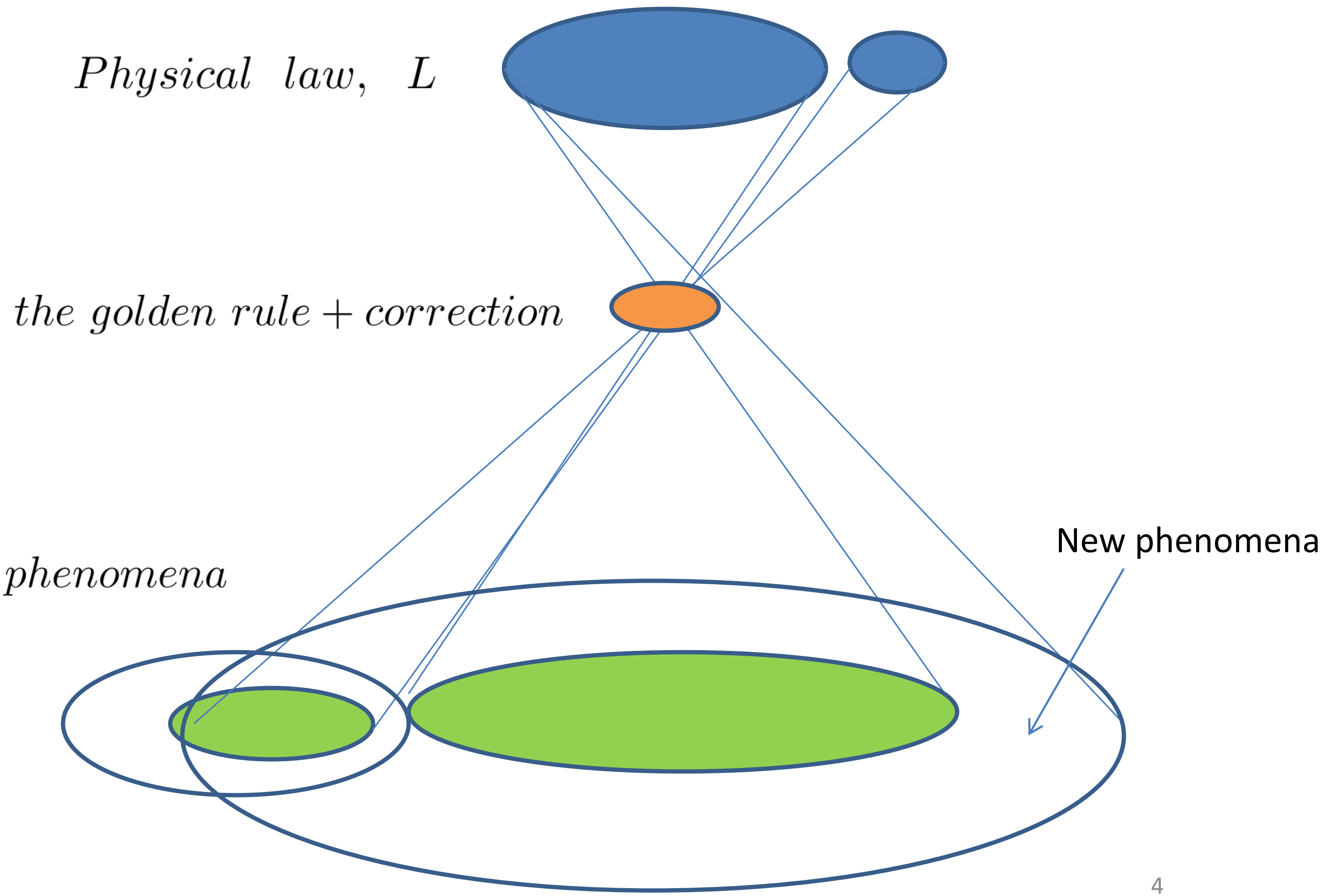
Physical Law vs phenomena

1. Superposition principle :
2. Commutation relation :
3. Schroedinger equation :



4. Probability principle :

Physical Law vs phenomena



2-1. Scattering probability

1. Non-normalizable states, $P = \infty$

   Dirac's prescription: (Dirac textbook)

   [Take the ratio of the fluxes of the in and out states.]

   good for cross section for particles but not for P.

2. Use the normalized wave functions (wave packets).

   LSZ takes plane wave approximation, and compute ΓT

3. We compute the probability without plane wave approximation. (KI and Tobita)

$$P = \Gamma T + P^d$$

2-2. Physical implication of P^d

(A)Rigorous unitarity , (B)absolute branching ratio, (C)selection rules, (D) Fields interaction energy .

2-2

Many-body Schroedinger equation

$$Interaction\ picture$$

$$i\hbar\frac{\partial}{\partial t}|\Psi(t)\rangle_i = H_{int}(t)|\Psi(t)\rangle_i$$

$$|\Psi(t)\rangle_s = e^{\frac{H_0}{i\hbar}t}|\Psi(t)\rangle_i, H_{int}(t) = e^{-\frac{H_0 t}{i\hbar}}H_{int}e^{\frac{H_0}{i\hbar}t}$$

solution

$$|\Psi(t)\rangle_{int} = [1 + \int_0^t \frac{dt'}{i\hbar}H_{int}(t') + \int_0^t \frac{dt'}{i\hbar}\int_0^{t'}\frac{dt''}{i\hbar}H_{int}(t')H_{int}(t'') + \cdots]|\Psi(0)\rangle_{int}$$

$$Study\ the\ wavefunction:$$

$$\langle\Psi_\beta(0)|\Psi(t)_\alpha\rangle_{int} = \langle\Psi_\beta(0)|\Psi_\alpha(0)\rangle_{int} + \langle\Psi_\beta(0)|\int_0^t \frac{dt'}{i\hbar}H_{int}(t')|\Psi_\alpha(0)\rangle_{int}$$

$$+ \langle\Psi_\beta(0)|\int_0^t \frac{dt'}{i\hbar}\int_0^{t'}\frac{dt''}{i\hbar}H_{int}(t')H_{int}(t'')|\Psi_\alpha(0)\rangle_{int} + \cdots,$$

$$|\Psi_{\beta(\alpha)}(0)\rangle = |\Psi_{\beta(\alpha)}(0)\rangle_s = |\Psi_{\beta(\alpha)}(0)\rangle_{int}$$

$$(1) First\ order:$$

$$\langle\Psi_\beta(0)|\int_0^t \frac{dt'}{i\hbar} H_{int}(t')|\Psi_\alpha(0)\rangle_{int} = \int_0^t \frac{dt'}{i\hbar} e^{-\frac{E_\alpha - E_\beta}{i\hbar}t'} \langle\Psi_\beta|H_{int}|\Psi_\alpha\rangle$$

$$= D(\omega,t)\langle\Psi_\beta|H_{int}|\Psi_\alpha\rangle,$$

$$D(\omega,t) = \int_0^t \frac{dt'}{i\hbar} e^{-\frac{E_\alpha - E_\beta}{i\hbar}t'} \qquad \omega = (E_i - E_f)/\hbar$$

$$(2) Second\ order:$$

$$\langle\Psi_\beta(0)|\int_0^t \frac{dt'}{i\hbar}\int_0^{t'} \frac{dt''}{i\hbar} H_{int}(t')H_{int}(t'')|\Psi_\alpha(0)\rangle$$

$$= \int_0^t \frac{dt'}{i\hbar} e^{-\frac{(E_\beta - E_\alpha)t'}{i\hbar}} \int_0^{t'} \frac{dt''}{i\hbar} e^{-\frac{E_\alpha(t'-t'')}{i\hbar}} \langle\Psi_\beta|H_{int} e^{\frac{H_0}{i\hbar}(t'-t'')} H_{int}|\Psi_\alpha\rangle$$

$$= D(\omega,t)\int_0^\infty \frac{dt''}{i\hbar} e^{-\frac{E_\alpha(t'-t'')}{i\hbar}} \langle\Psi_\beta|H_{int} e^{\frac{H_0}{i\hbar}(t'-t'')} H_{int}|\Psi_\alpha\rangle$$

2-3. Transition probability (interval T)

Large T

$$\int_0^T dt e^{i\omega t} = \frac{1}{i\omega}(e^{i\omega T} - 1) = e^{i\omega T/2}\frac{2\sin\omega T/2}{\omega}$$

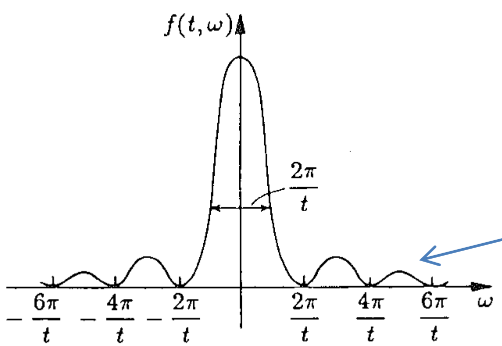
$$P = g^2 \int d\omega (\frac{2\sin\omega T/2}{\omega})^2 |f_{\alpha,\beta}|^2$$

$$= g^2 T \int dx (\frac{2\sin x}{x})^2 [g(0) + \frac{x}{T} g'(0) + \cdots]$$

$$= g^2 2\pi T g(0)[1 + \frac{1}{T} \times \infty + \cdots],$$

$$x = \omega T/2, g = |f_{\alpha,\beta}|^2 \qquad \omega = (E_i - E_f)/\hbar$$

Schiff



11.8：関数 $f(t,\omega)$ は $t \to \infty$ において $2\pi t\delta(\omega)$ となる．すなわち $\omega = 0$ のピークは鋭くる．

1. Large T

$$\omega \approx 0 \quad P = T\Gamma_0$$

Golden rule (Dirac(1927),Fermi(1949))

2. $(a).\ \omega \neq 0 (= \infty)$

$$P = \Gamma_0 T + P^{(d)}$$

$$(b).\ g(0) = 0, g(x) \neq 0$$

$$P = P^{(d)}$$

$$Tails\ give\ P^d$$

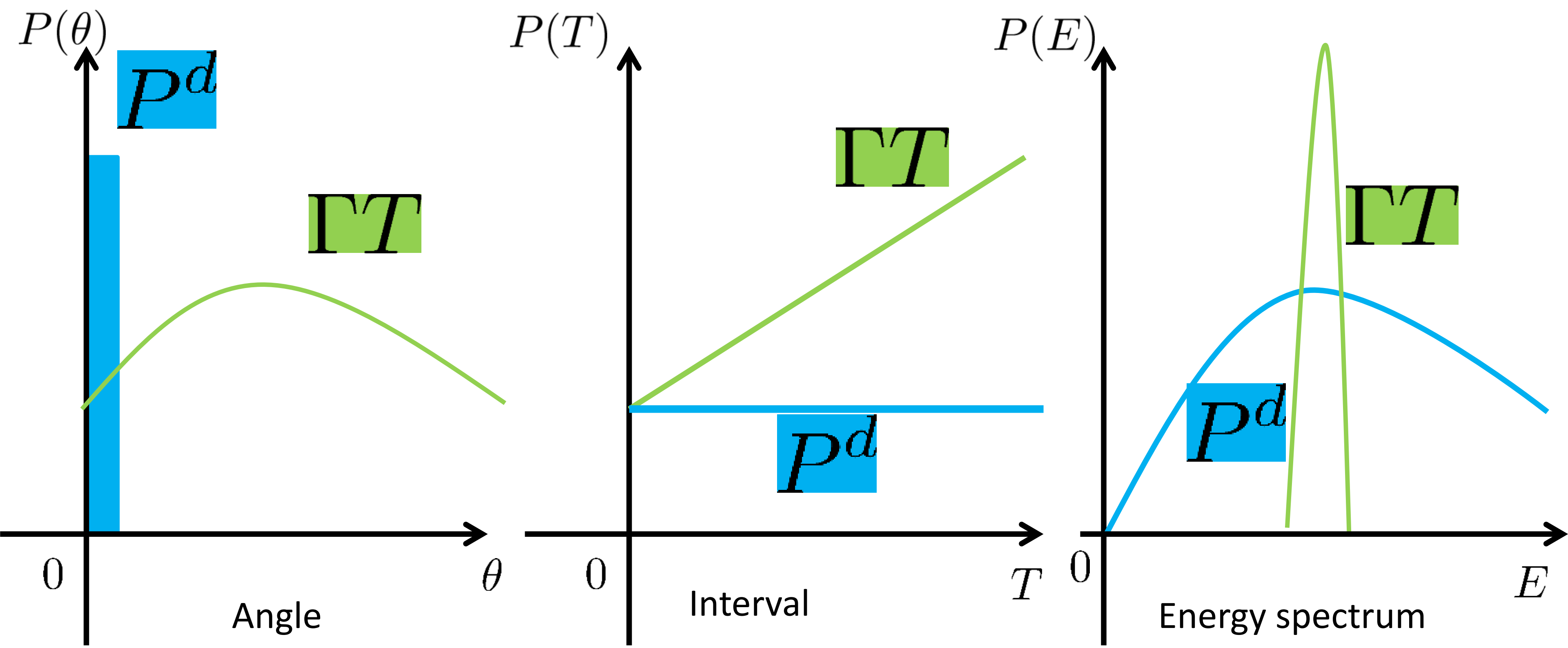
Kinetic-energy non-conservation

$$E_k = E_{total} - E_{int} \neq E_{total},\ \text{for}\ E_{int} \neq 0$$

Interaction energy due to the overlaping waves

T-dependence, angle dependence, energy spectrum

$$P = \Gamma T + P^d$$



high energy scattering

$$\Gamma T \gg P^d : normal\ case\ \ Gibbs \approx Bolzman$$

The field theory for the finite time interval

Stueckelberg Phys.Rev .1951

the probability for the plane waves is divergent

Wave packets
is a complete set of normalized states and gives a representation of one-body and many-body states. As one-particle state is normalized, the transition probability is defined uniquely following the principle of the quantum mechanics.
K.I and T. Shimomura, PTP(2005)

$Wave\ packets$

$$\langle t,\vec{p}|\vec{P},\vec{X},T_0\rangle = (\frac{\sigma}{\pi})^{3/4} e^{-iE_l(\vec{p})(t-T_0)-i\vec{p}\cdot\vec{X}-\frac{\sigma}{2}(\vec{p}-\vec{P})^2},$$

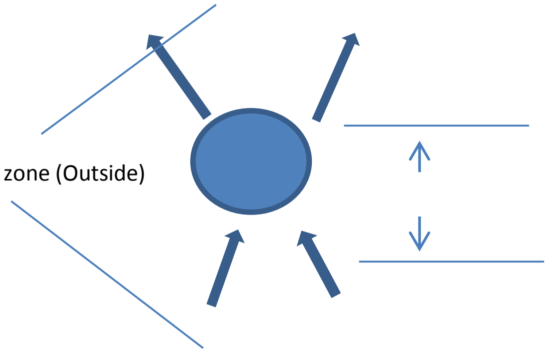
$$w(\vec{P},\vec{x},t;\vec{X}) = N e^{-\frac{1}{2\sigma}(\vec{x}-\vec{X}-\vec{v}_l(t-T_0))^2} e^{-iE_l(\vec{P})(t-T_0)+i\vec{P}\cdot(\vec{x}-\vec{X})},$$

$$\int d\chi|\chi,T_0\rangle\langle\chi,T_0| = 1,\ d\chi = \frac{d\vec{X}d\vec{P}}{(2\pi)^3},$$

$S[T]$

2-6 A: Particle zone

Standard S-matrix (Heisenberg, others) $S[\infty]$,



$$(1)[S[\infty], H_0] = 0$$

$$(2) Kinetic\ energy\ is\ conserved. \quad (kinetic\ energy = total\ energy)$$

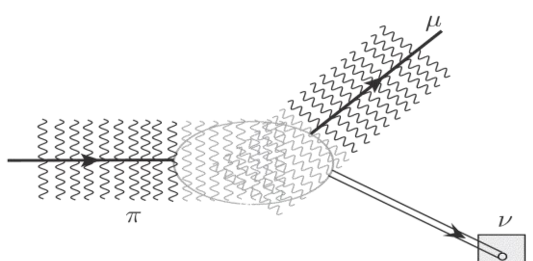
$$(3) Fermi's\ golden\ rule, S - matrix, e^{-\epsilon|t|} H_{int}$$

2-3 B: Wave zone

Waves overlap（KI-Shimomura,KI-Tobita,KI-tajima-tobita, KI-Oda-Nakatsuka)

$$S[T], L = cT$$

Amplitude of the events that ν is detected at $(\vec{X}_\nu, T_\nu)$



Waves overlap in very large area

$$(1)\ [S[T], H_0] \neq 0$$

$$(2)\ kinetic\ energy\ non - conservation,$$

$$(3)\ Fermi's\ golden\ rule\ has\ a\ correction$$

$$e^{i(E(p)t - \vec{p}\vec{x})}|_{\omega=\infty} = e^{i(E(p)-pc)t}$$

Rigorous calculation : light-cone singularity

$$\int \frac{d^3p_l}{(2\pi)^3}\sum_{spin}|T|^2 = \frac{N_3}{E_\nu}\int d^4x_1 d^4x_2 e^{-\frac{1}{2\sigma_\nu}\sum_i(\vec{x}_i-\vec{X}_\nu-\vec{v}_\nu(t_i-T_\nu))^2}$$

$$\times\, \Delta_{\pi,l}(x_1-x_2)e^{ip_\nu\cdot(x_1-x_2)}$$

Universal finite-size corrections

$$\Delta_{\pi,l}(x_1-x_2) = \frac{1}{(2\pi)^3}\int \frac{d^3p_l}{E(p_l)}\left[(2p_\pi\cdot p_\nu)(p_\pi\cdot p_l) - m_\pi^2(p_l\cdot p_\nu)\right]$$

$$\times\, e^{-i(p_\pi-p_l)\cdot(x_1-x_2)}$$

$$= \delta((x_1-x_2)^2) + short - range$$

Light-cone singularity

$$e^{ip_\nu(x_1-x_2)}\delta((x_1-x_2)^2) = e^{i(E_\nu-p_\nu)(x_1-x_2)^0}$$

Thomson scattering at small T

$$\frac{d\Gamma_{Thom}}{d\vec{k}_2}T = T\frac{1}{2E_{e,1}E_{\gamma,1}E_{\gamma,2}E_{e,2}\pi^2}\delta(E_{e,1}+E_{\gamma,1}-E_{e,2}-E_{\gamma,2})(e^2)^2 2,$$

$$\frac{dP^{(d)}_{Thom,\gamma}}{d\vec{k}_2} = \sigma_\gamma\frac{1}{E_{e,1}E_{\gamma,1}E_{\gamma,2}(2\pi)^3}e^4 2T\tilde{g}(\omega_\gamma T)\theta(s-m_e^2-2k_2(p_{e_1}+k_1)),$$

3. Physical implication of P^d

(A,B) unitarity: 0- -> two photons
(neutrino reactions)
(C) Selection rule : 1+ -> two photons
total derivative interaction,
Landau-Yang theorem
(helicity suppression, multipole expansions)
(D) Interaction energy
Where is the interaction energy?

$3-1\ \pi^0 \to \gamma\gamma$ *decay*

π^0 *is the lightest hadron*

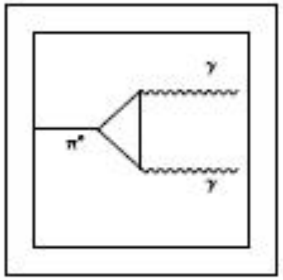
PrimEx Collaboration Jefferson Lab Hall B

Physics Motivation

- π^0 decay rate is a fundamental prediction of confinement scale QCD.

Chiral Anomaly

Presence of closed loop triangle diagram results in nonconserved axial vector current, even in the limit of vanishing quark masses.



→In the leading order (chiral limit), the anomaly leads to the decay amplitude:

$$A_{\pi^0\to\gamma\gamma} = \frac{\alpha_{em}}{4\pi F_\pi}\varepsilon_{\mu\nu\rho\sigma}k^\mu k'^\nu \varepsilon^{*\rho}\varepsilon'^{*\sigma}, \quad (1)$$

or the reduced amplitude,

$$A_{\gamma\gamma} = \frac{\alpha_{em}}{4\pi F_\pi} = 0{,}02513\ \mathrm{GeV}^{-1} \quad (2)$$

where $F_\pi = 92.42 \pm 0.25$ MeV is the pion decay constant.

Dustin McNulty November 1, 2006, Hall A Post Doctoral Interview Seminar 3

$(1)\,Fukuda-Miyamoto(1949)$

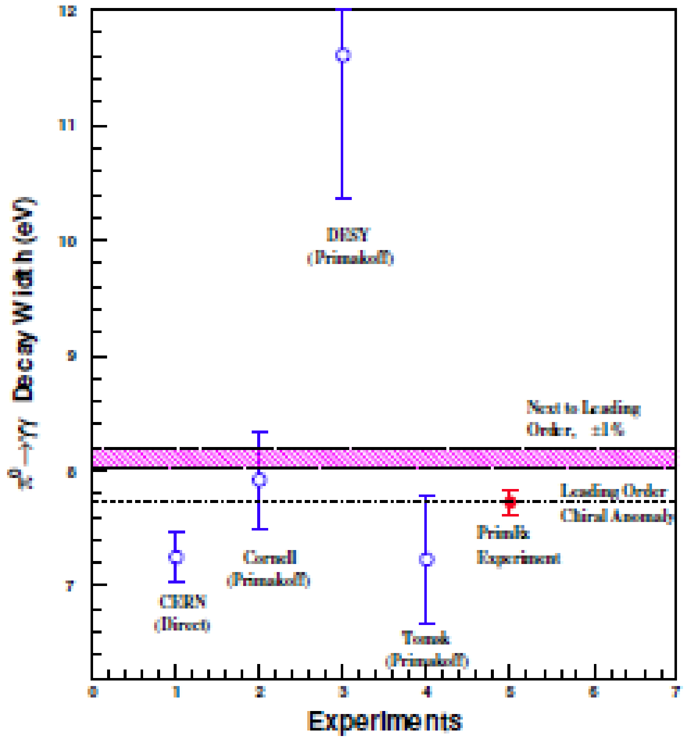
$Steinberger(1949)$

$(2)\,Adler, Bell-Jackiw(1969)$

$$N_c = 3$$

Physics Goal

- Use the Primakoff effect to measure $\Gamma_{\pi^0 \to \gamma\gamma}$ to within 1,5% uncertainty



Experiments

$$Uncertainty \;\; \tau_\mu : 10^{-5}, \tau_\tau : 10^{-3}, \tau_{\pi^+} : 10^{-4}, \tau_{\pi^0} : 10^{-1}, \tau_\eta : \tau_\rho 10^{-3}$$

$$Why\ is\ the\ experimental\ uncertainty\ so\ large\ still\ now?$$

Our answer is : because $P^{(d)}$ has not been included.

$Effective\ Lagrangian$

$$L = L_0 + L_{int},$$

$$L_0 = \frac{1}{2}\partial_\mu \varphi \partial^\mu \varphi - \frac{1}{2} m_\pi^2 \varphi\varphi + \frac{1}{4} F_{\mu\nu} F^{\mu\nu},$$

$$L_{int} = g \epsilon_{\mu\nu\rho\sigma} \varphi(x) F^{\mu\nu} F^{\rho\sigma}, \ g = \frac{\alpha}{4\pi f_\pi}$$

Tri-angle diagram

KI-Nakatsuka-Oda, in preparation

0 → photon_1 +photon_2,

EPR :Spin of 1 and of 2 has a long range correlation at a large L, after the time T, where L=vT. This Is expressed precisely with the wave packets.

EPR(energy) : Each energy also has the correlation derived from the many-body interaction at L.

$$\pi^0(e^+ + e^-) \to \gamma + \gamma$$

$$M = i\int_{T_{Ps}}^{T_\gamma} dt \int d\vec{x} \langle \vec{k}_{\gamma_1}, \vec{X}_{\gamma_1}; \vec{k}_{\gamma_2}, \vec{X}_{\gamma_2} | H_{int}(x) | \vec{p}_{Ps}.\vec{X}_{Ps} \rangle$$

$$= igN_1 \int_{T_{Ps}}^{T_\gamma} dt \int d\vec{x} e^{-\frac{1}{\sigma_s}(\vec{x}-\vec{x}_0(t))^2 - \frac{1}{2\sigma_t}(t-T_0)^2 - \frac{\sigma_s}{2}(\delta\vec{p})^2 - \frac{\sigma_t}{2}(\delta\omega)^2 + R + i\Phi}$$

$$\frac{dP}{d\vec{k}_{\gamma_1} d\vec{k}_{\gamma_2}} = 2P_0(k_{\gamma_1} \cdot k_{\gamma_2})^2 e^{-\sigma_s(\delta\vec{p})^2} |G(\omega)|^2$$

$$|G(\omega)|^2 \begin{cases} = (2\sigma_t)e^{-\sigma_t(\delta\omega)^2}; bulk \\ = \frac{2}{(\delta\omega)^2 + \frac{1}{\sigma_t}} : boundary \end{cases}$$

$$\pi^0 decays \qquad P(T) = \Gamma T + P^d \tag{1}$$

$$\begin{cases} \Gamma T = g^2 \sigma_\gamma T \frac{1}{E_\pi} \int \frac{d^3 p_\gamma}{E_\gamma (2\pi)^3} 2(p_\pi \cdot p_\gamma)^2 G_0 \\ P^{(d)} = g^2 \sigma_\gamma T_1 \frac{1}{E_\pi} \int \frac{d^3 p_\gamma}{E_\gamma (2\pi)^3} 2(p_\pi \cdot p_\gamma)^2 [\tilde{g}(0) \theta(m_\pi^2 - m_\gamma^2 - 2p_\pi \cdot p_\gamma)], \end{cases} \tag{2}$$

$$\begin{cases} G_0 = \frac{2\pi}{\sigma_\gamma} \delta(E_\pi - E_\gamma - |p_\pi - p_\gamma|) \\ p_\pi \approx p_\gamma + p_\gamma, (m_\pi^2 - 2p_\pi \cdot p_\gamma) = m_\gamma^2, \\ \omega T \approx 0, \tilde{g}(0) = \frac{\pi}{2} \end{cases} \tag{3}$$

$$\begin{cases} \Gamma = g^2 \frac{m_\pi^3}{2\pi} \frac{m_\pi}{E_\pi} \\ \frac{1}{T} P^{(d)} = g^2 \sigma_\gamma \frac{1}{\pi 2^6 3} \frac{m_\pi^8}{E_\pi p_\pi (E_\pi - p_\pi)} \end{cases} \Rightarrow \quad \begin{array}{l} P^{(d)} \approx \dfrac{1}{10}, \dfrac{T_1}{\tau} = 10^{-3} \\ \dfrac{10}{100} \ uncertainty \end{array}$$

A large conctribution of $P^{(d)}$!!

Proposal to KLEO(Sloan and Ishikawa)

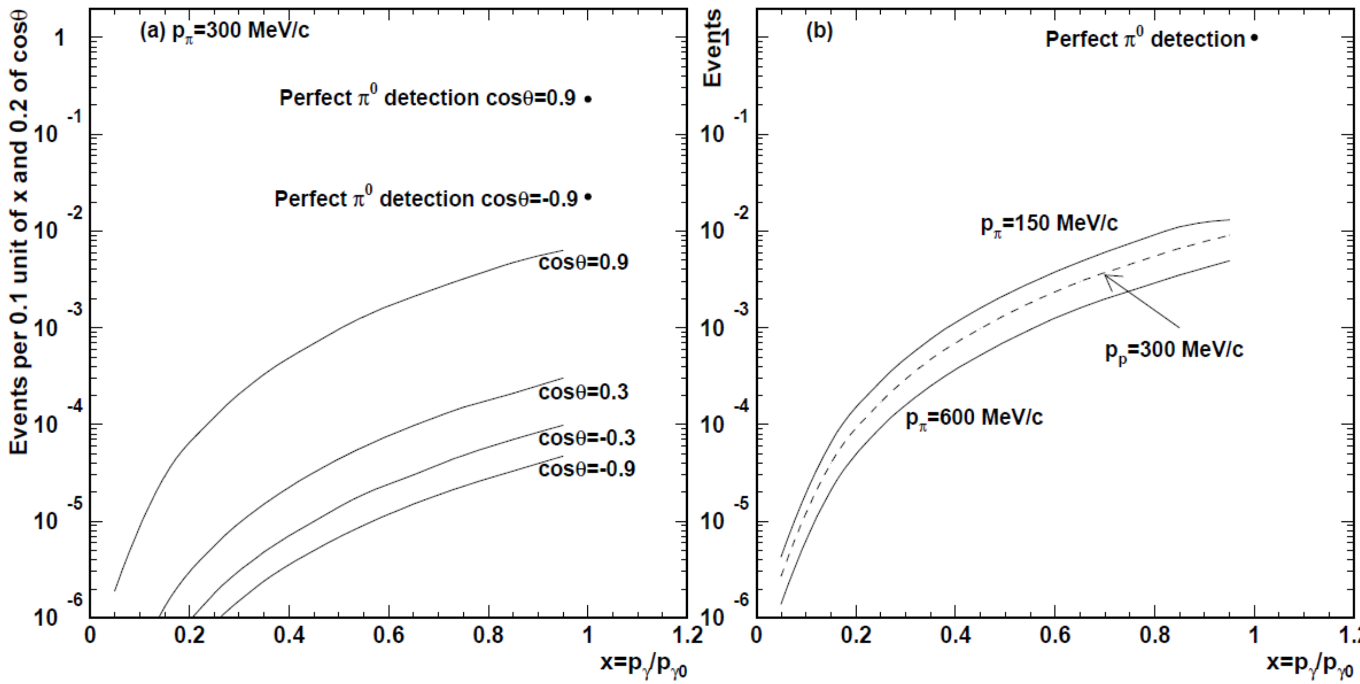


Figure 1: The differential distribtions of P^d in $x = p_\gamma/p_{\gamma 0}$; (a) the distributions for a π^0 momentum of 300 MeV/c for different values of $\cos\theta$, the angle of the photon in the laboratory frame relative to the π^0 flight direction; (b) the distribution of events per 0.1 bin in x integrated over $\cos\theta$ but for different π^0 momenta. The solid points labelled "perfect π^0 detection" are the values expected from standard model π^0 decay detected in a perfect calorimeter. The curves show the relative number of events per 0.1 bin in x and 0.2 bin in $\cos\theta$ normalised to the total value of P^d quoted in the text (equation 3).

$Ps \to \gamma\gamma$ Positronium decay

$$P^d/\Gamma\tau_{Ps} = (T_1/\tau_{Ps})\left(m_{Ps}^2\sigma_\gamma\right)1/64$$

$$= (10^{-18} - 10^{-17})/10^{-10}MeV^2(10^{-10} - 10^{-9}meter)^2/(2\cdot 10^{-13})^2 1/$$

$$= 10^{-4} - 10^{-1}$$

Experiment J Cizek et al (journnal. Physics:confeence series 505(2014)012043),

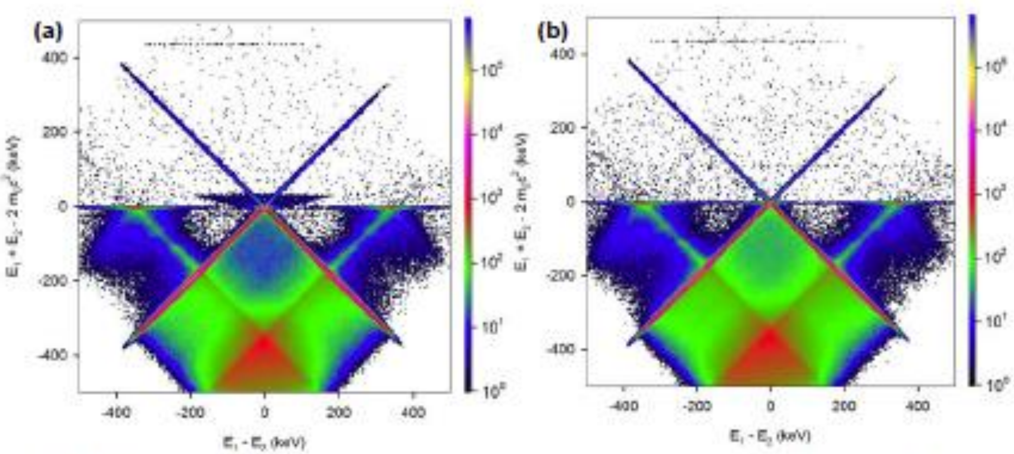
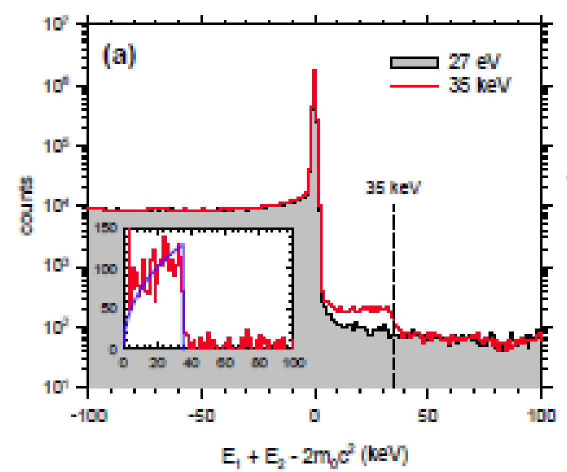


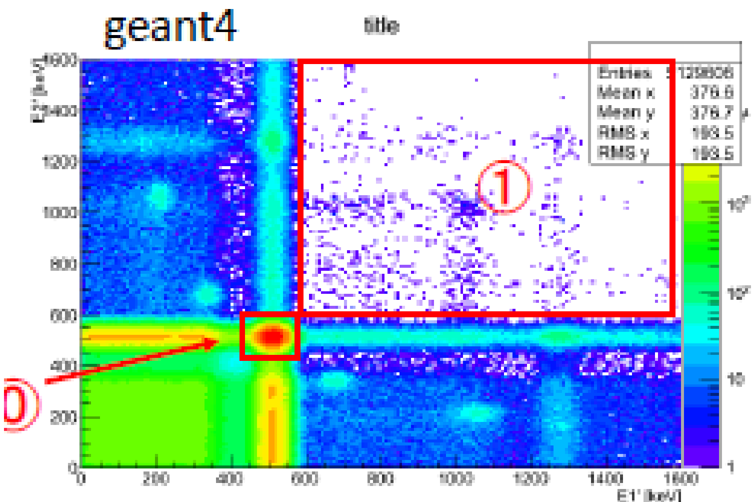
Figure 1. CDB spectra for Fe target obtained using positrons with energy of 35 keV (a) and 27 eV (b).



TIT experiment(jinnouchi ,Kubota,Ushioda)

実験

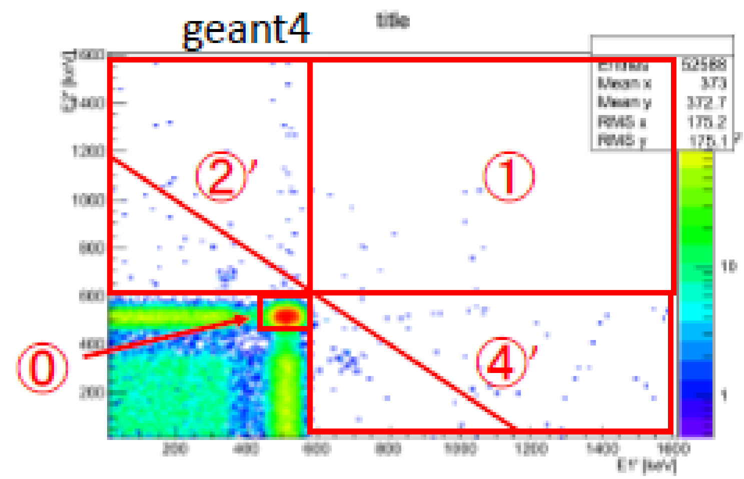
back-to-backの2コインシデンス全て



N0/N1～715

実験2

back-to-back+1本の3コインシデンス全て
(back-to-backでないPMTでのenergy dep



N0/(N1+N2'+N4') ～ 99

(C) Selection rule , Landau-Yang's theorem(KI-Tajima-Tobita)

1^{+} -> 2 photons is forbidden from the golden rule

$$\Gamma_0 = 0 \quad (Landau - Yang)$$

$$(j_{z1}, j_{z2}) = |+1,+1\rangle,\ |-1,-1\rangle, |+1,-1\ \rangle \pm |-1,+1\rangle$$

$$(J, J_z) = (2,+2),\quad (2,-2),\qquad (2,0),\ \pm\ (0,0)$$

Momentum 1; $(0,0,p)$
Momentum 2; $(0,0,-p)$

Effective action :axialvector-photon-photon "tri-angle"

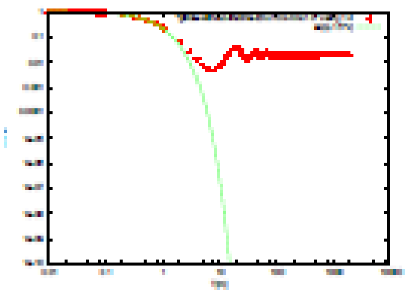
$$S_{int} = \int dx \partial_\mu (g\phi^\mu \epsilon_{\alpha\beta\gamma\delta} F^{\alpha\beta} F^{\gamma\delta})$$

The surface term gives no bulk effect, $\Gamma = 0$, but $P^{(d)} \neq 0$.

Experiment in Charmonium $(C\bar{C}), \gamma \to$ gluon

$\alpha_s \approx 0.2$

$$P^{(d)} = \frac{1}{1152\pi^5}\left(\frac{2g\alpha}{\pi}\right)^2 \frac{Im(A\vec{B}\vec{v})}{\sqrt{\pi\sigma_\gamma}}\left(\frac{m_e}{m_\gamma}\right)^2$$



decreases with T as $e^{-T f(m,T)}$

Experiments

$P - state(l = 1)$

$$\Gamma^{(0)}_{\xi_{C_0} \to gg} = \frac{6\alpha_s^2}{m_Q^4}|R'_p(0)|^2[1 + 9.5\alpha_s/\pi] \qquad 10.3\text{MeV}$$

$$\Gamma^{(0)}_{\xi_{C_2} \to gg} = \frac{8\alpha_s^2}{5m_Q^4}|R'_p(0)|^2[1 - 2.2\alpha_s/\pi] \qquad 1.97\text{MeV}$$

$$\Gamma^{(0)}_{\xi_{C_1} \to gg} = \left(\frac{\alpha_s^2}{m_Q^4} \times 0 + \alpha_s^4\right)|R'_p(0)|^2[1 + O(\alpha_s/\pi)] \qquad 0.86\text{MeV}\left(= 0.15 \times \frac{10.3+1.97}{2}\right)$$

Landau-Yang's theorem

Coefficient

$S - state(l = 0)$

$$\Gamma^{(0)}_{J/\Psi \to ggg} = \frac{\alpha_s^3}{m_Q^2}|R_p(0)|^2 \times numerical\ factor \qquad 92.9\text{KeV}$$

$$\Gamma^{(0)}_{\eta_c \to gg} = \frac{\alpha_s^2}{m_Q^2}|R_p(0)|^2 \times numerical\ factor \qquad 28.6\text{MeV}$$

Thanks to Landau-Yang's theorem, p^{(d)} can be directly observed. in Charmonium.
In Positronium ? Anomaly in e+ annihilation.

M → photon + photon

1+

ΓT =0 ,

P^d finite.

0- ΓT finite

P^d finite

P^d is universal but shows the internal structure

M → lepton +neutrino (KI-Tobita)

ΓT helicity suppressed

P^d universal

SN1987A Neutrino : Galaxy effect
nu -> nu+gamma

Ishikawa, sloan, tobita

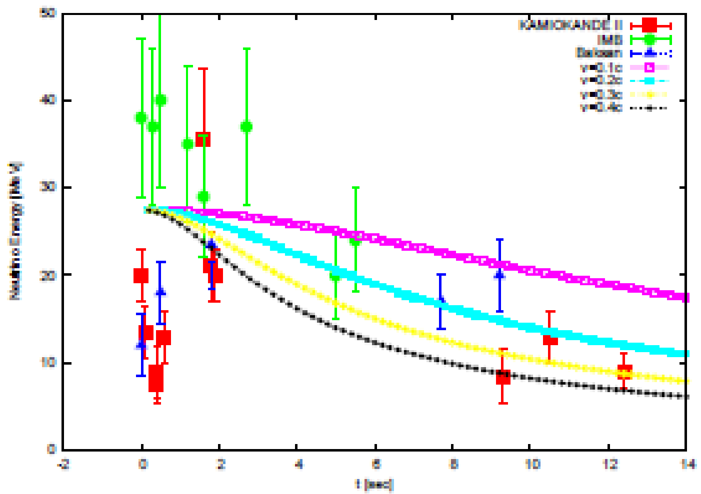


FIG. 1. Time dependence of neutrino energy form SN1987A

Excitation energy transfer (maeda,yabuki,tobita,KI,PTEP,2017)

$$Eitation\ energy\ transfer$$

$$t = 0 : \Psi^*_D(X_D)\Psi^0_A(X_A)$$

$$\rightarrow excnange\ of\ massless\ scalar$$

$$t = T : \Psi^0_D(X_D)\Psi^*_A(X_A)$$

$$
\begin{aligned}
H &= H_0 + H_{\rm int},\\
H_0 &= H_0^A + H_0^D + H_0^\phi,\\
H_0^A &= \frac{p_A^2}{2m} + V_A(\boldsymbol{r}_A, q_A) + H^A_{\rm nucl}(q_A),\\
H_0^D &= \frac{p_D^2}{2m} + V_D(\boldsymbol{r}_D, q_D) + H^D_{\rm nucl}(q_D),\\
H_0^\phi &= \int \frac{d^3k}{(2\pi)^3} E_k a_k^\dagger a_k,\ \ H_{\rm int} = g\phi(\boldsymbol{r}_A) + g\phi(\boldsymbol{r}_D),
\end{aligned}
$$

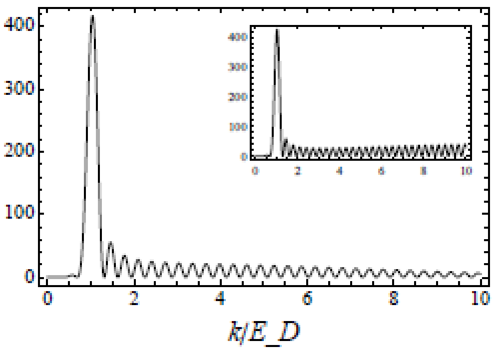


Fig. 1. $f_{\rm rad}(k,T)$ for $TE_D = 20$, $\sigma_D/E_D = 5$ (15 inset), and $0 < k/E_D < 10$.

$$A^* \rightarrow A + \gamma$$

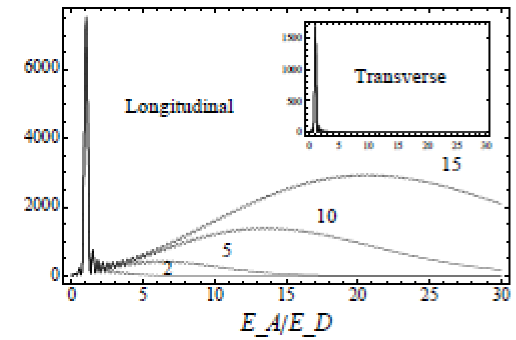


Fig. 3. Longitudinal component $|f_\parallel|^2 E_D^2$ for $TE_D = 20$, $RE_D = 2$, $\sigma/E_D = 2$, 5, 10 15, and $0 < E_A/E_D < 30$. Inset shows transverse component $|f_\perp|^2 E_D^2$ for the same parameters.

$$A^* + D^0 \rightarrow A^0 + D^*$$

Rapid transitoin by $P^{(d)}$ *and slow transition by* ΓT

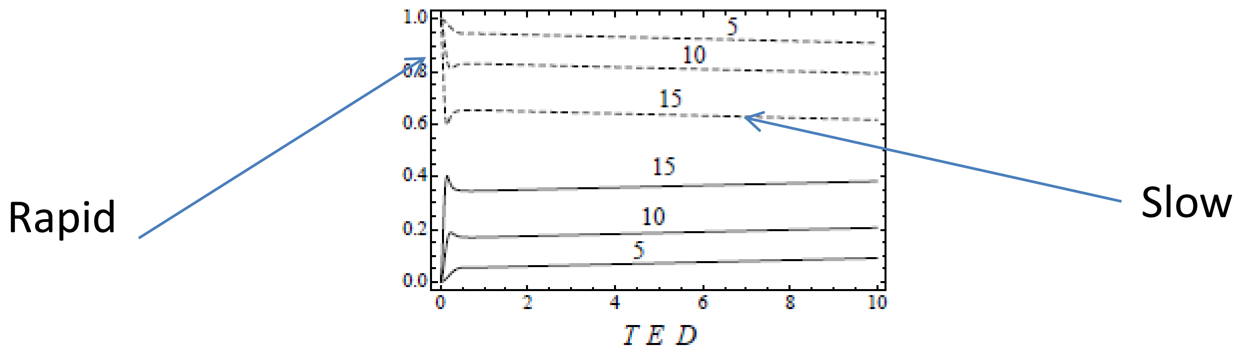


Fig. 2. $P_{\rm rad}(T)$ (Solid lines) and $1 - P_{\rm rad}(T)$ (Dashed lines) are plotted for $\sigma_D/E_D = 5, 10, 15$, $E_D/\Gamma_{\rm rad} = 250$, and $0 < TE_D < 10$.

Table I. Characteristics of the resonance peak and the bump in $|f_{||}|^2$. The resonance peak and the bump correspond to Fermi's golden rule and the finite size correction respectively.

	Energy-distribution	σ-dependence	Dominant R-range	T-dependence
Resonance peak	Narrow	Small	Short-range	Linear
Bump	Broad	Large	Long-range	Constant

(D) Interaction energy of the fields

$$\langle H\rangle = \langle H_0\rangle + \langle H_{int}\rangle$$

$\langle H_0\rangle$: Kinetic energy

$\langle H_{int}\rangle$: Interaction energy

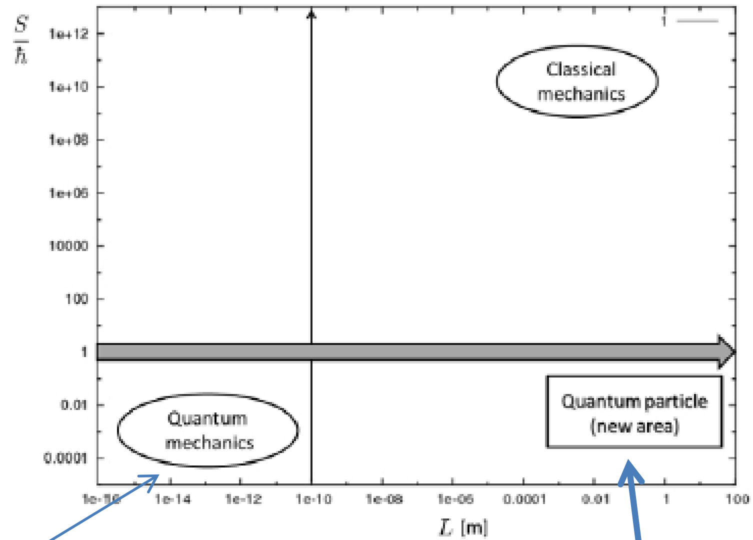
P^d finite $\leftrightarrow$ $\langle H_{int}\rangle$ finite

4. Conclusion

- The correction to the Fermi's golden rule ,P^d, is finite and relevant to particle physics and cosmology.
- P^d is not included in the standard S-matrix calculation, similar to background and has been ignored in most previous analysis.
- P^d has many intriguing properties.

$P^{(d)}$ causes reactions extended to macroscopic areas:

1-1 quantum vs classical
length L vs action S



1-2 electron mass and Bohr radius

$$m_e = 0.5MeV/c^2$$

$$a_0 = \frac{4\pi\epsilon_0\hbar^2}{m_e e^2} = r_e\alpha^{-2} = 0.5 \times 10^{-10} M(Bohr\ radius)$$

ΓT

Forward anomaly
P^d

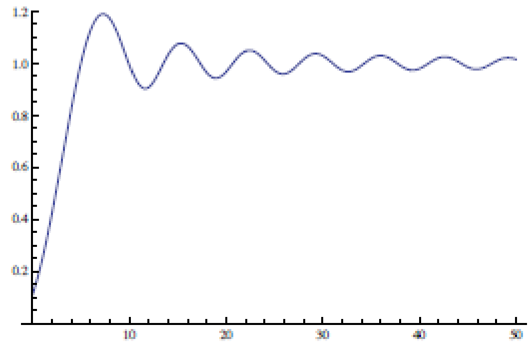
Back up

Coulomb wave function

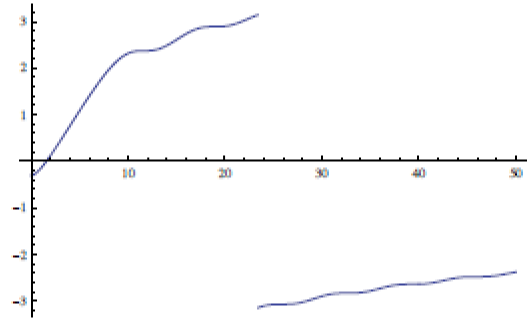
C F(n,k,xi)

Repulsive n>0

```
n = 1; k = 1; Plot[Abs[fn[n, k, xi]], {xi, 0, 50}, PlotRange → {0, 1.2}]
```



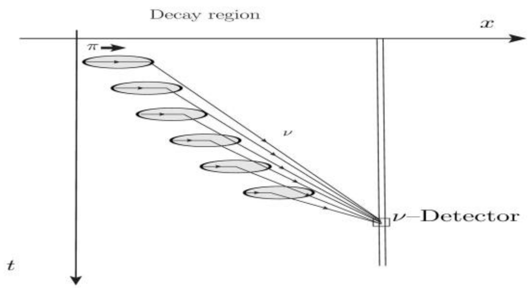
```
n = 1; k = 1; Plot[Arg[fn[n, k, xi]], {xi, 0, 50}, PlotRange → All]
```



Computation

(II)Pion decay amplitude : neutrino is detected at L=cT $S[T]$

T’



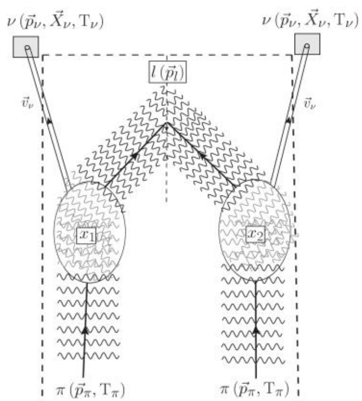
By using wave packets, amplitude to detect ν is given [LSZ]. The finite-size correction is caused by the kinetic-energy non-onservation.

$$T = \int d^4x \langle l,\nu|H_w(x)|\pi\rangle$$
$$= \int d^4x N_1 \langle 0|J^{\mu}_{V-A}|\pi\rangle \bar{u}(\vec{p}_l)\gamma_\mu(1-\gamma_5)\nu(p_\nu) e^{ip_l\cdot x + ip_\nu\cdot(x-\mathrm{X}_\nu)} e^{-\frac{1}{2\sigma_\nu}(\vec{x}-\vec{\mathrm{X}}_\nu-\vec{v}_\nu(t-\mathrm{T}_\nu))^2}$$

$$= N_1\langle 0|J^{\mu}_{V-A}|\pi\rangle \bar{u}(\vec{p}_l)\gamma_\mu(1-\gamma_5)\nu(\vec{p}_\nu) e^{-\frac{\sigma_\nu}{2}\delta\vec{p}^2} \int_0^T dt e^{-i\omega t}$$
$$\omega = \delta E - \vec{v}_\nu\cdot\delta\vec{p}$$
$$\delta E = E_\pi - E_l - E_\nu, \delta\vec{p} = \vec{p}_\pi - \vec{p}_l - \vec{p}_\nu$$

Neutrino velocity

important (moving frame)



$$|T|^2 = |N_1|^2|\bar{u}(\vec{p}_l)\gamma_\mu(1-\gamma_5)\nu(\vec{p}_\nu)\langle 0|J^{\mu}_{V-A}(0)|\pi\rangle|^2 e^{-\sigma_\nu\delta\vec{p}^2}|2\frac{\sin\omega T/2}{\omega}|^2$$

Decay of heavier particle

$$|\Psi_1^h(T)\rangle = (N_1^h(T)A_2^\dagger(\chi) + |\Psi_2(T)\rangle$$

$$|\Psi_2(T)\rangle = \int d\chi (N_2^h(\chi,T)A_1^\dagger(\chi_1)A_1^\dagger(\chi_2) + \cdots)|0\rangle,$$

The disconnected part $= 0$ by the energy gap.

$N_1^h(T)$ and $N_2^h(\chi,T)$ finite. $(\vec{P},\vec{X})$ for χ, $F_{\beta,\varphi_2}(\omega) = \langle\beta|H_{int}(0)|\varphi_2\rangle$, $\omega = E_{\varphi_2} - E_\beta$. The initial state is wave packets.

$t = T$ and $t = T_1$ of $T \gg T_1$. At T_1, $|\Psi_{(2,p)}(T_1)\rangle$ of $|\omega| \leq \epsilon_1, \epsilon_1 \leq \frac{b}{T_1}$, b is a constant, and $|\Psi_{(2,w)}(T_1)\rangle$ of $|\omega| \geq \epsilon_1$ satisfy,

$$|\Psi_2(T_1)\rangle = |\Psi_{(2,p)}(T_1)\rangle + |\Psi_{(2,w)}(T_1)\rangle,$$

$$\langle\Psi_{(2,p)}(T_1)|\Psi_{(2,p)}(T_1)\rangle = \Gamma T_1, \Gamma = \int_{|\omega|\leq\epsilon_1} d\beta\delta(\omega)|F_{\varphi_2,\beta}(0)|^2 2\pi\tilde{\rho}(0),$$

$$\langle\Psi_{(2,w)}(T_1)|\Psi_{(2,w)}(T_1)\rangle = P^{(d)} = \int_{|\omega|\geq\epsilon_1} d\vec{p}_1 d\vec{p}_2 |F_{\varphi_2,\beta}(\omega)|^2 |D(\omega,T_1)|^2.$$

Quasi-stationary compositie-state(QCS)