Top quark pair production at NNLO+NNLL′

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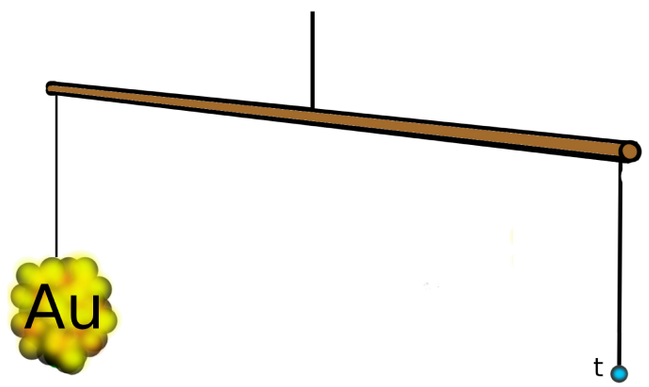
Peking University

In collaboration with: M. Czakon, A. Ferroglia, D. Heymes, A. Mitov, B. D. Pecjak, D. J. Scott and X. Wang,

11th International Workshop on Top Quark Physics (TOP2018)

Basic facts about the top

Large mass $m_t \approx 173$ GeV



Strong Yukawa coupling $y_t \sim 1$

Fermion mass origin

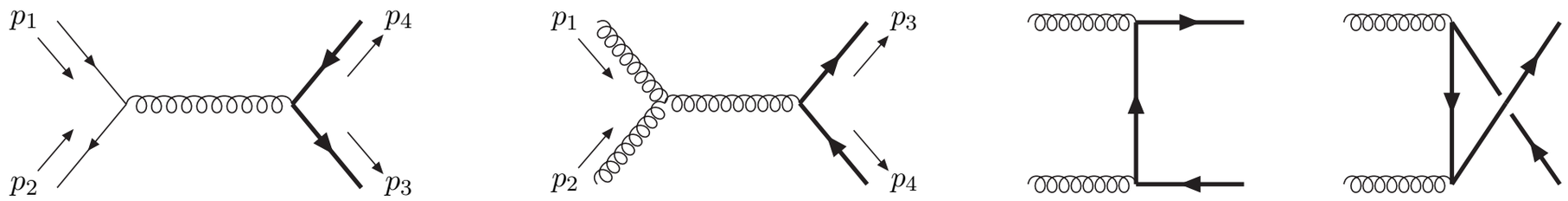
Hierarchy problem

Vacuum stability

Short lifetime $\tau \sim 5 \times 10^{-25}$ s

Decays before hadronization: pQCD dominates!

Top quark pair production

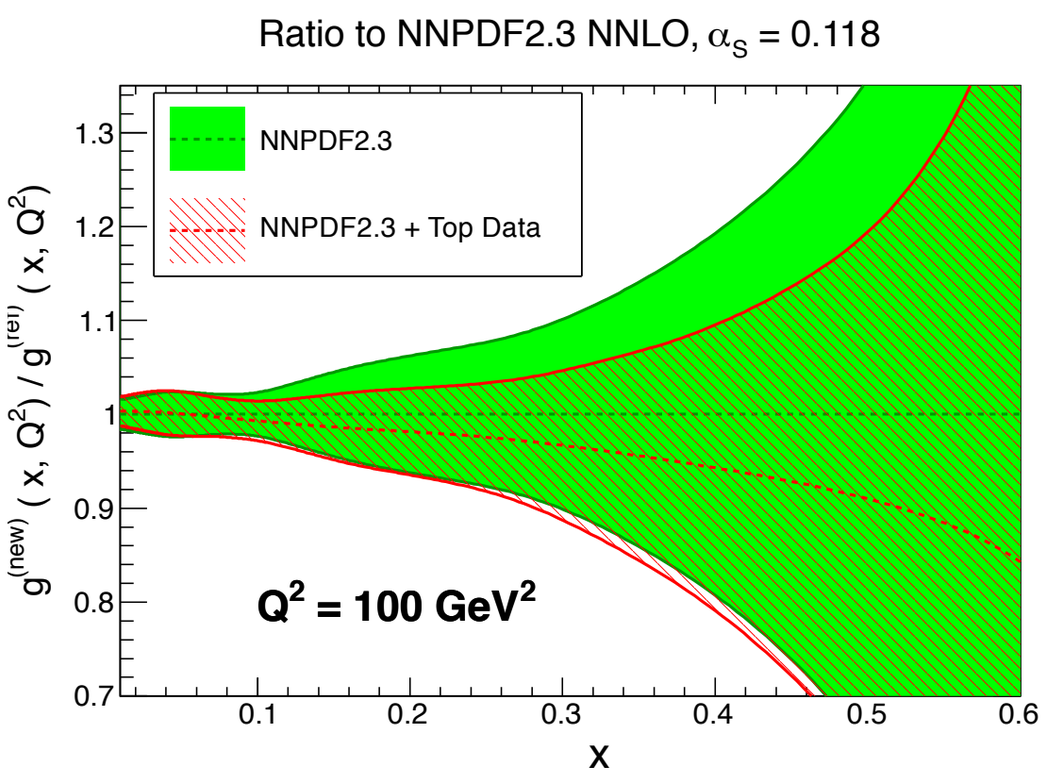


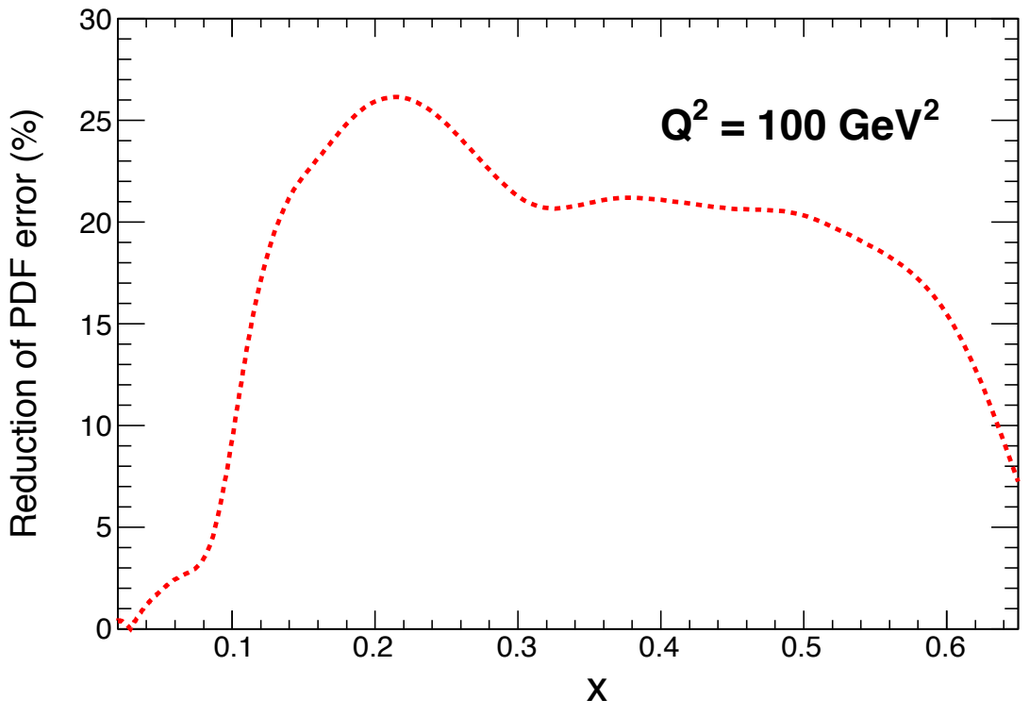
A standard candle for the LHC and future colliders

* Test of the SM at the energy frontier
* Possible signals of new physics
* Major background to many searches
* **Precise theoretical and experimental results have already enabled us to gain useful information!**

Gluon PDF

Top quark pair production can provide information about the gluon parton distribution functions



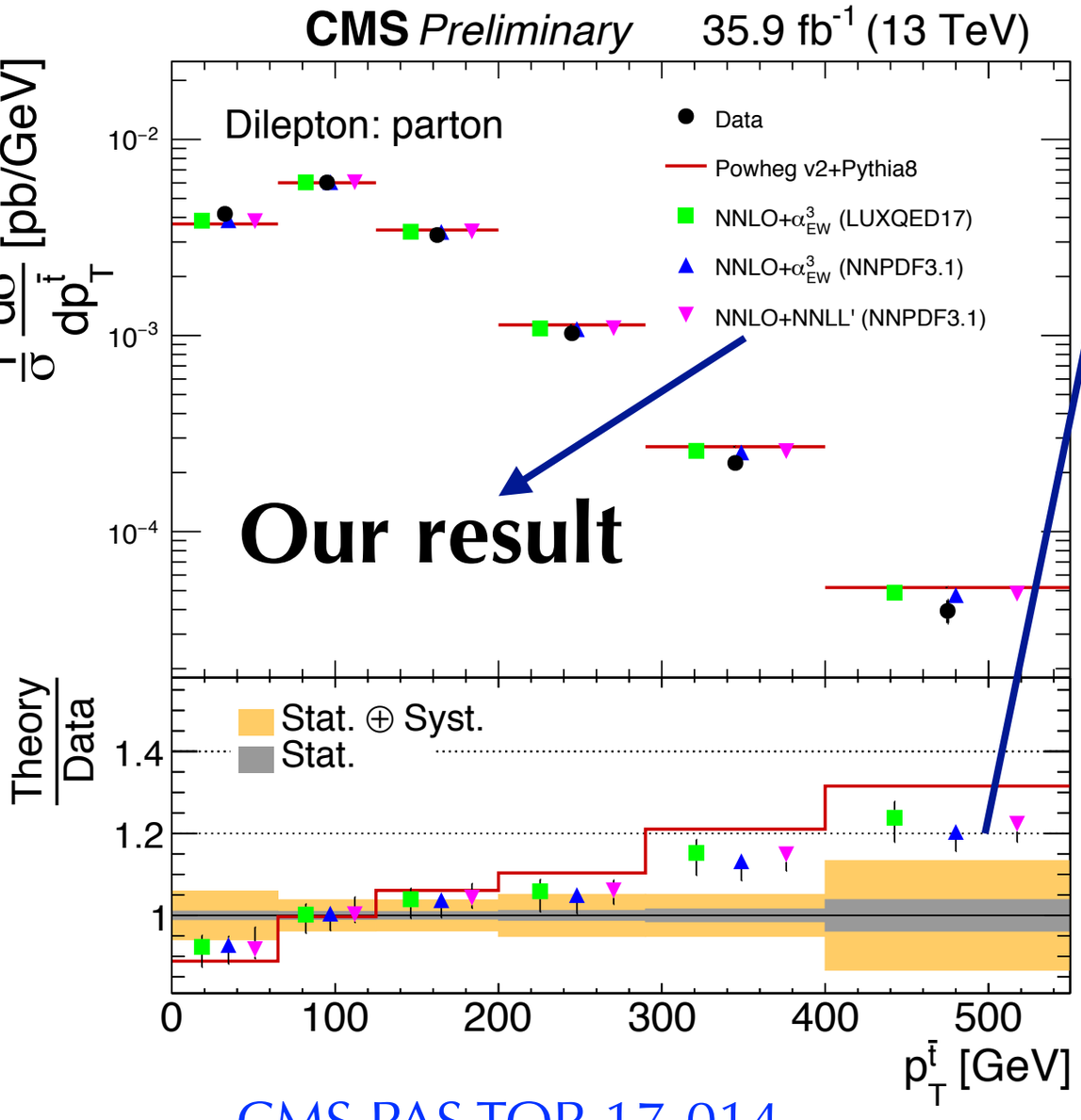


Czakon, Mangano, Mitov, Rojo: 1303.7215

Note: only used 7 and 8 TeV data!

Ongoing: CTEQ analysis with 8 and 13 TeV data

Deviation?



CMS PAS TOP-17-014

Persistent shape difference in the transverse momentum spectrum

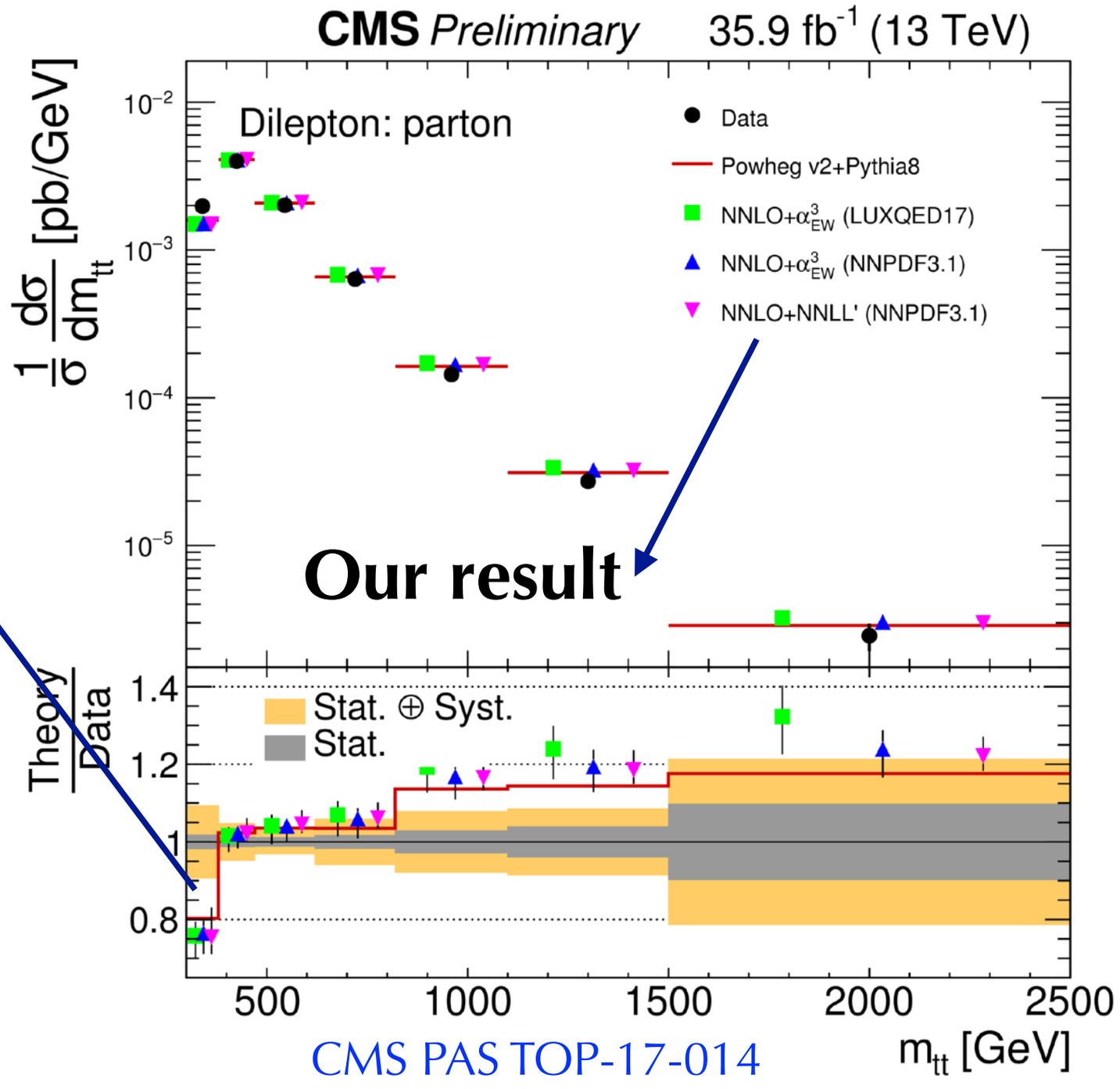
The high precision of the theoretical calculations and the experimental measurements allows to see this difference clearly!

Deviation?

What's going on here?

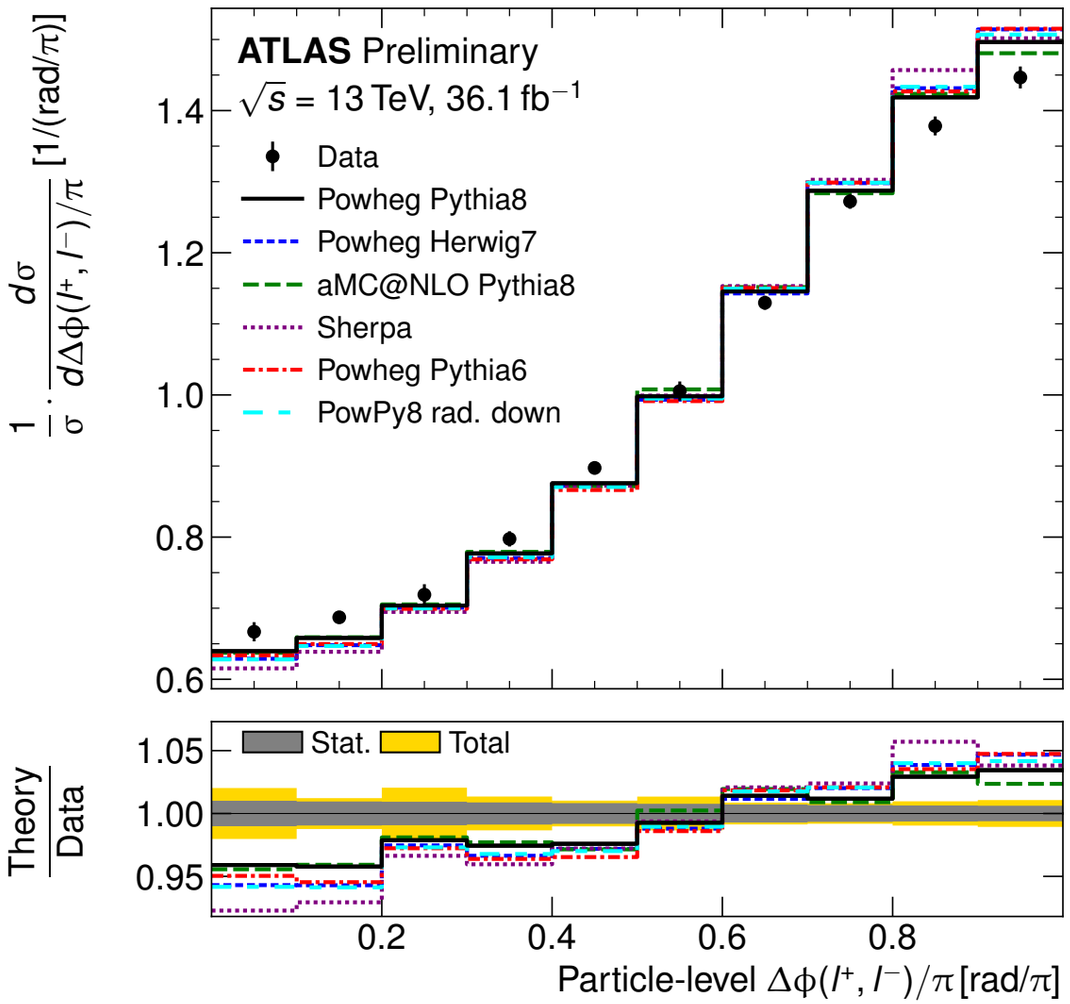
Top quark mass?

Threshold effect?

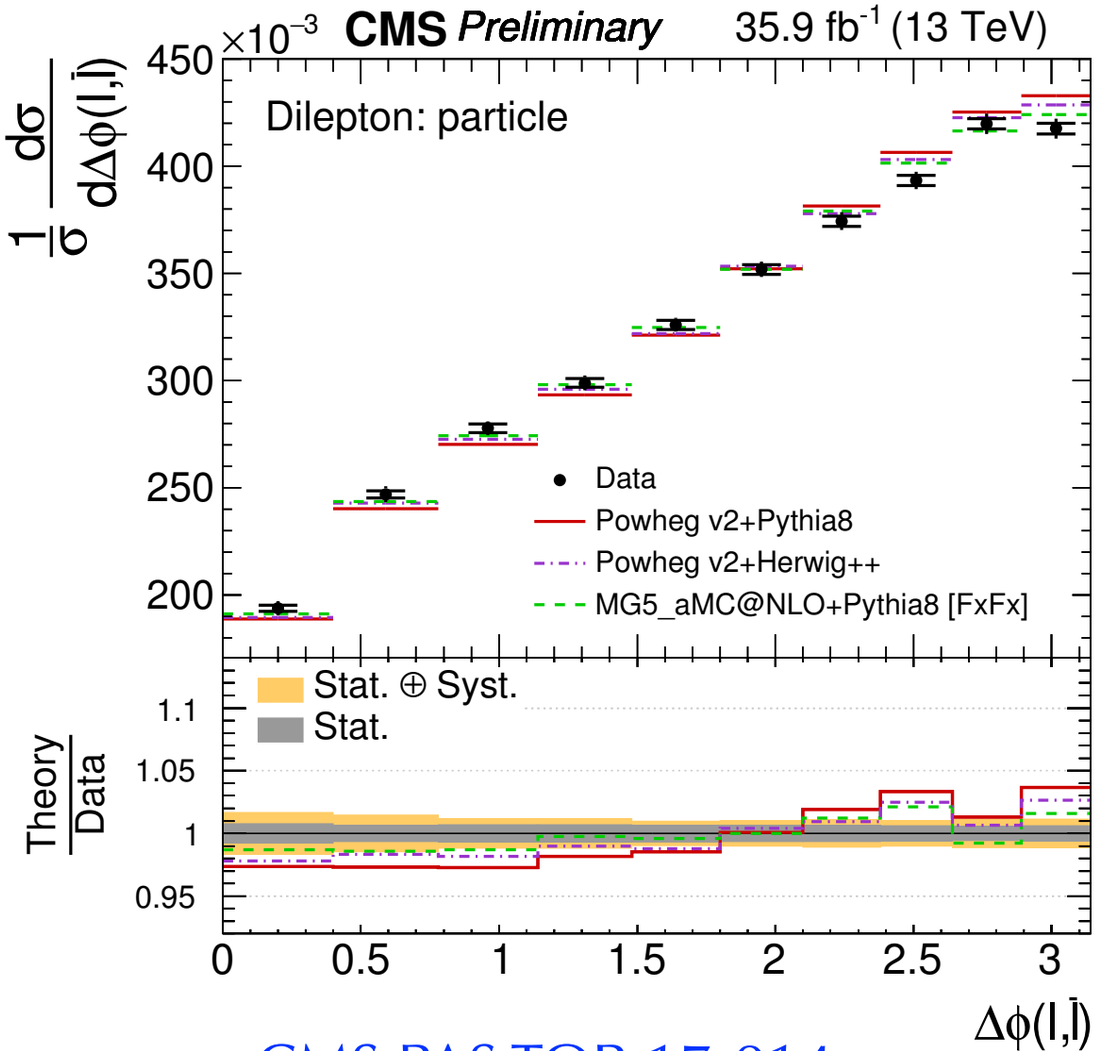


CMS PAS TOP-17-014

Deviation?



ATLAS-CONF-2018-027



CMS PAS TOP-17-014

Hoping to see more details in the talks of Federica Fabbri, Konstantinos Kousouris and Matteo Negrini

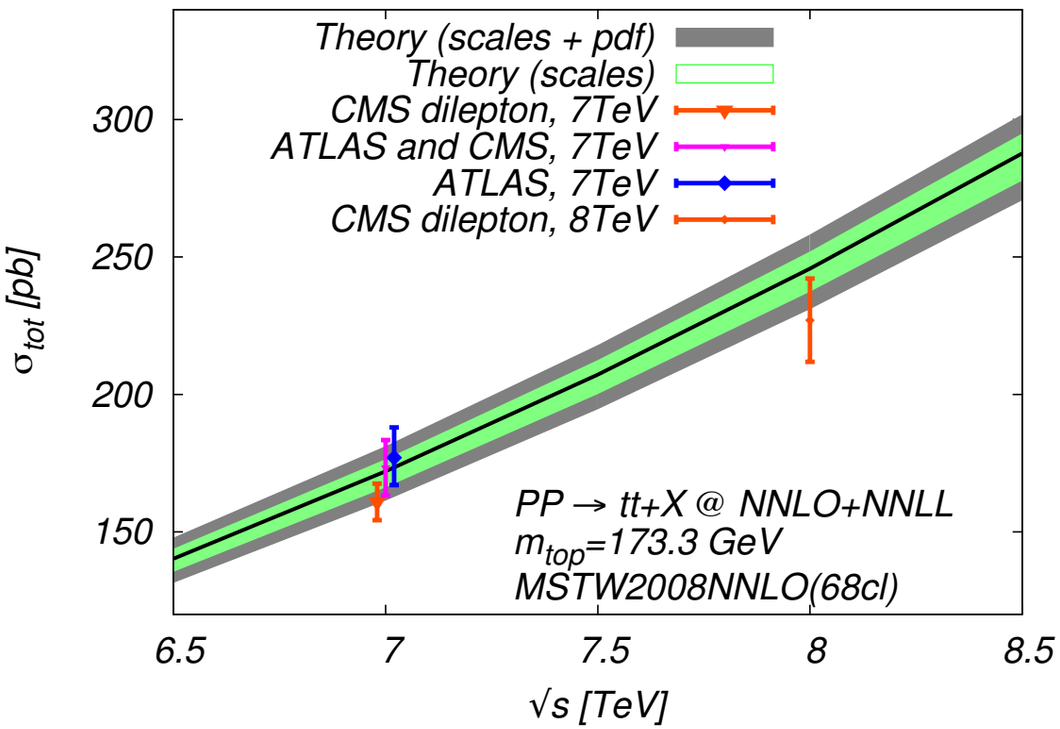
In this talk, I'm going to introduce the state-of-the-art QCD prediction for top quark pair production…

…and some ongoing developments

NNLO QCD for top pair

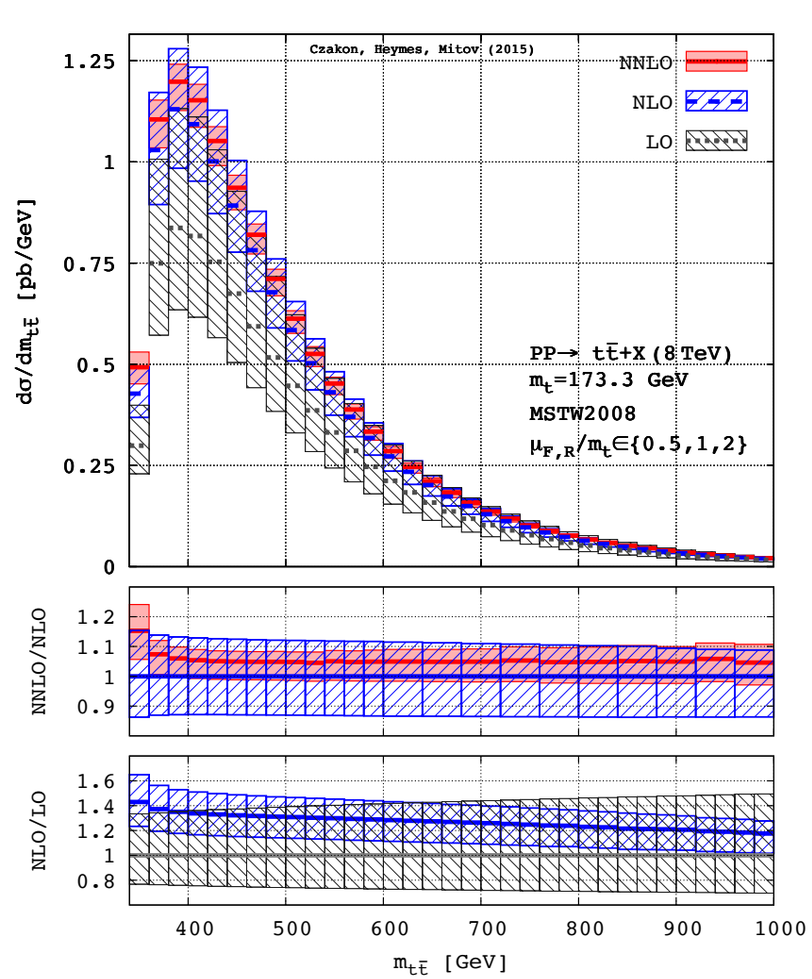
Total cross section

Baernreuther, Czakon, Mitov: 1204.5201;
Czakon, Fiedler, Mitov: 1303.6254



Differential distributions

Czakon, Heymes, Mitov: 1511.00549



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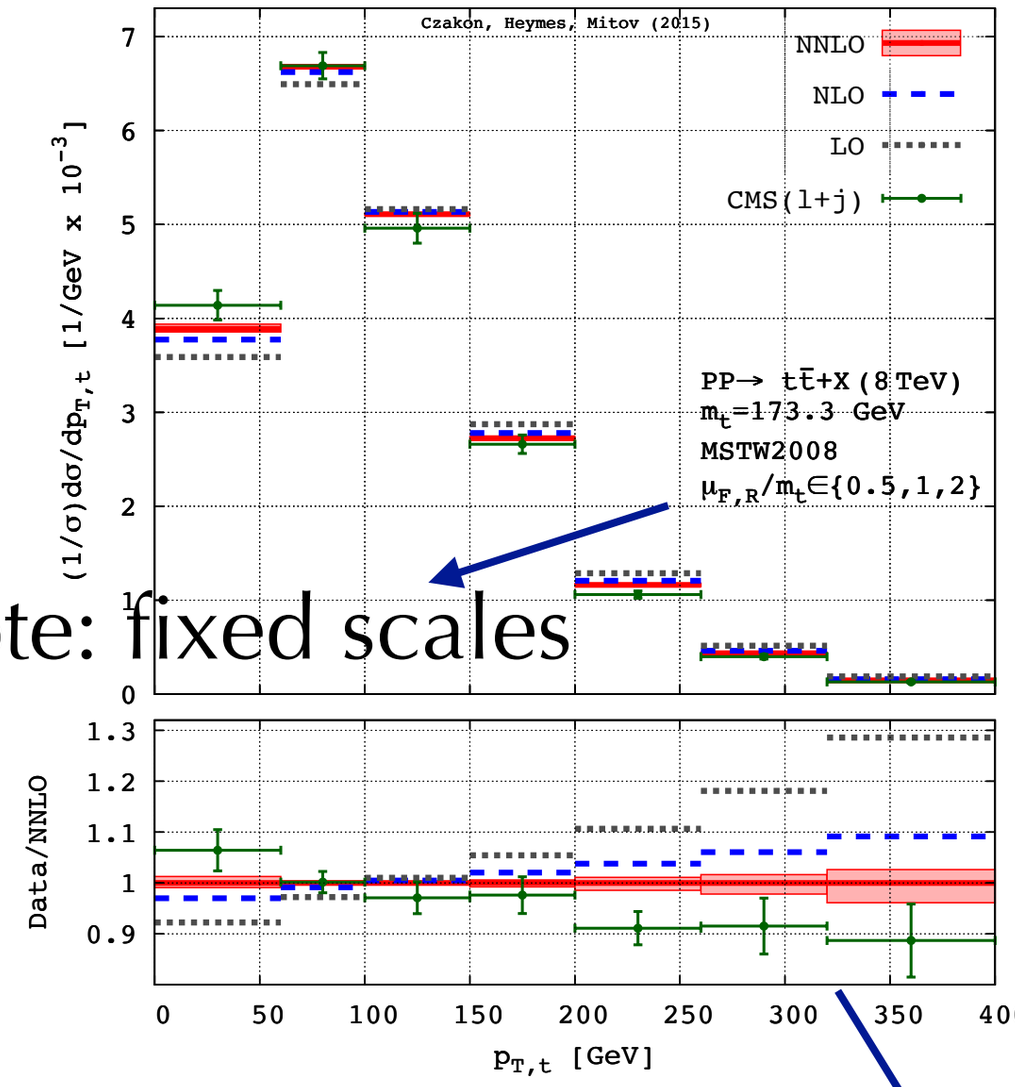
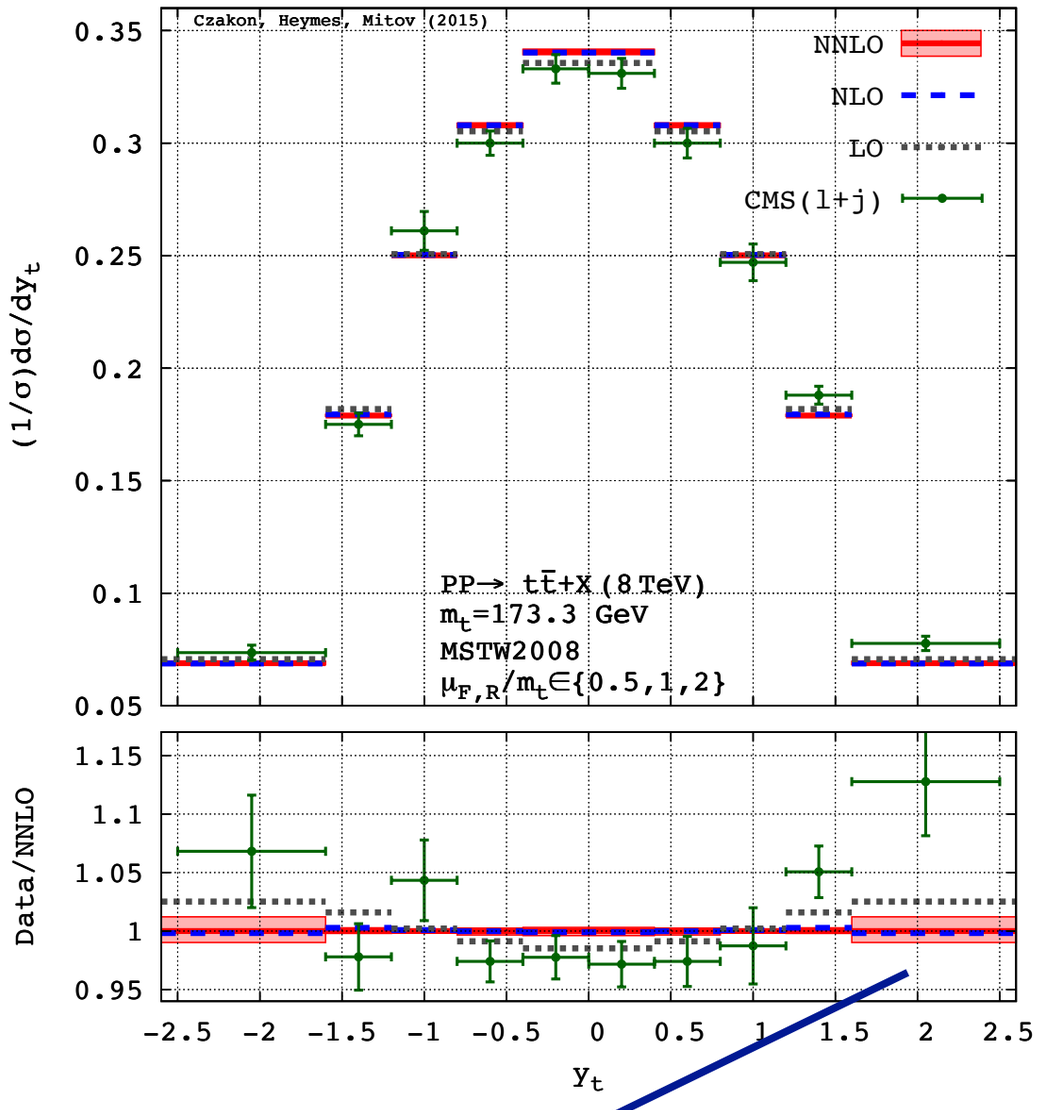


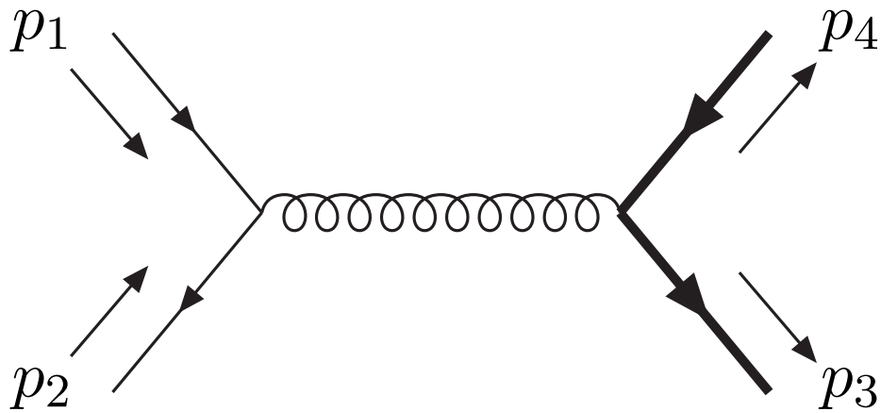


Note: fixed scales

Some tension at high energy (boosted kinematics)

Kinematics

The difficulty for fixed-order calculations: multiple-scale process with complicated kinematics!



Many kinematic variables:

top quark mass

p_T of top

p_T of anti-top

rapidity of top

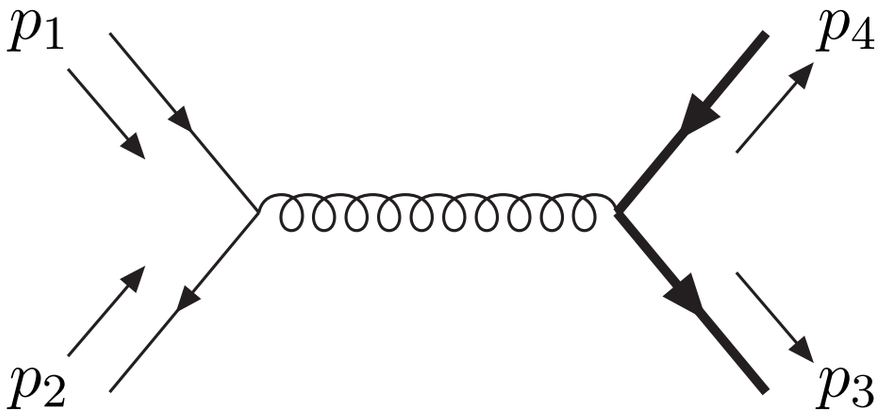
rapidity of anti-top

Invariant mass M_{tt}

…

Kinematics

The difficulty for fixed-order calculations: multiple-scale process with complicated kinematics!



Many kinematic variables:

top quark mass
p_T of top
p_T of anti-top
rapidity of top
rapidity of anti-top
Invariant mass M_{tt}
…

Which (combination) should be used for the renormalization/factorization scales?

NNLO with dynamic scale

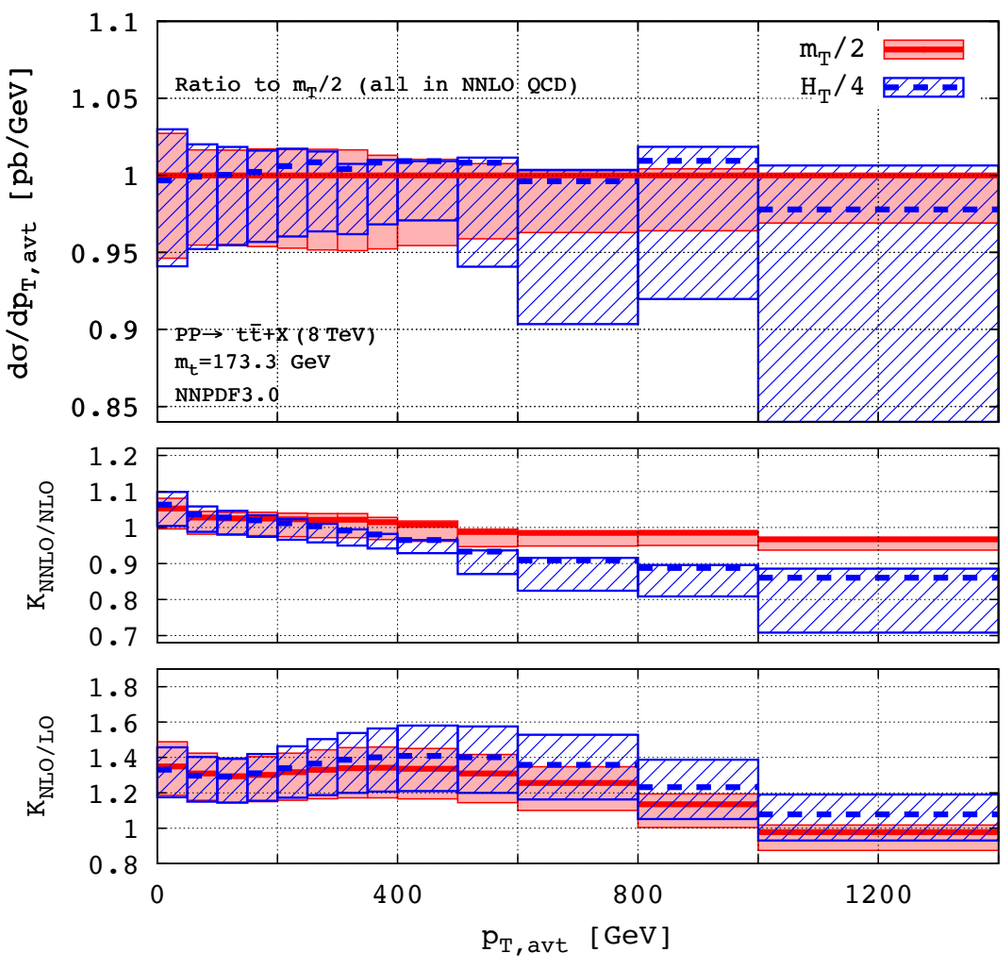
Czakon, Heymes, Mitov: 1606.03350

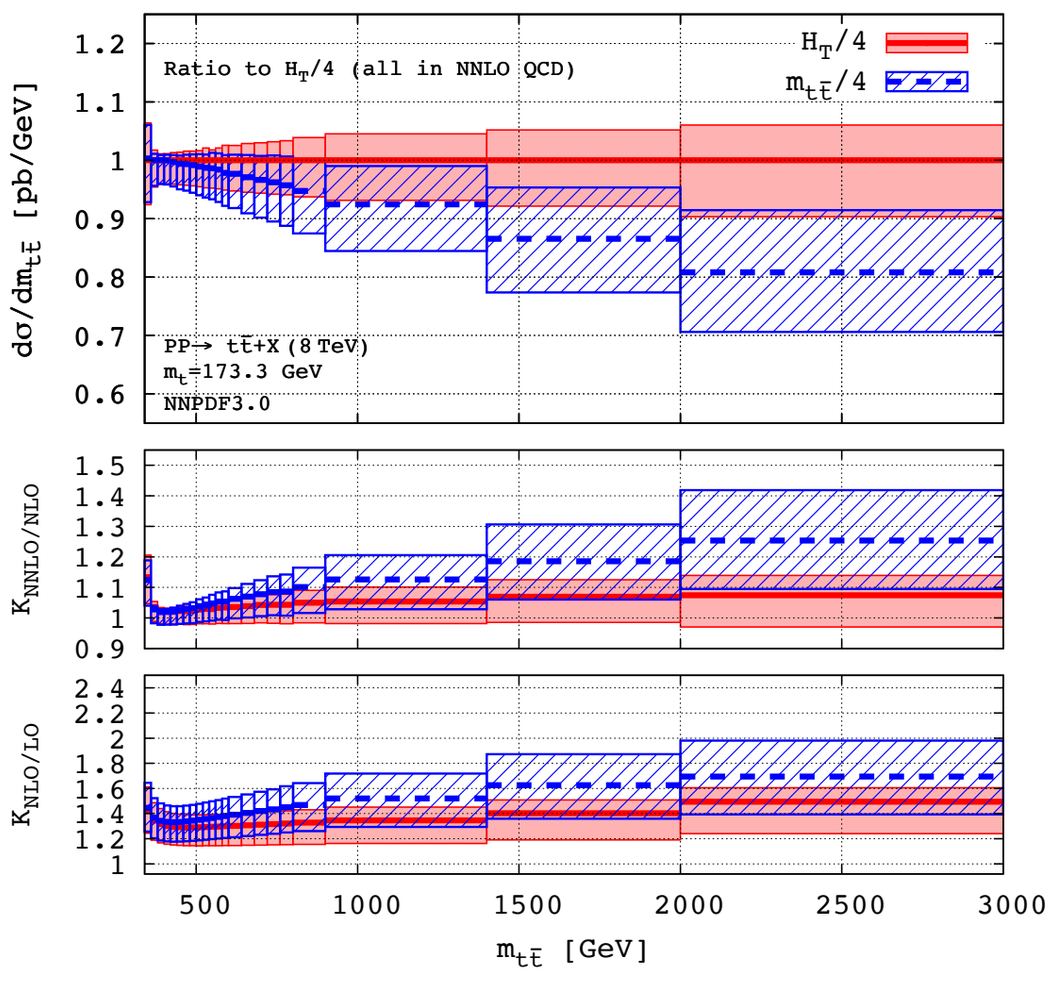
Determine optimal "scale scheme" by minimizing higher order corrections

$$
\begin{aligned}
&\mu_0 \sim m_t \,, \\
&\mu_0 \sim m_T = \sqrt{m_t^2 + p_T^2} \,, \\
&\mu_0 \sim H_T = \sqrt{m_t^2 + p_{T,t}^2} + \sqrt{m_t^2 + p_{T,\bar{t}}^2} \,, \\
&\mu_0 \sim H_T' = \sqrt{m_t^2 + p_{T,t}^2} + \sqrt{m_t^2 + p_{T,\bar{t}}^2} + \sum_i p_{T,i} \,, \\
&\mu_0 \sim E_T = \sqrt{\sqrt{m_t^2 + p_{T,t}^2}\sqrt{m_t^2 + p_{T,\bar{t}}^2}} \,, \\
&\mu_0 \sim H_{T,\mathrm{int}} = \sqrt{(m_t/2)^2 + p_{T,t}^2} + \sqrt{(m_t/2)^2 + p_{T,\bar{t}}^2} \,, \\
&\mu_0 \sim m_{t\bar{t}} \,,
\end{aligned}
\qquad \Rightarrow \qquad
\mu_0 = \begin{cases} \frac{m_T}{2} & \text{for :} \quad p_{T,t},\ p_{T,\bar{t}} \text{ and } p_{T,t/\bar{t}} \,, \\ \\ \frac{H_T}{4} & \text{for :} \quad \text{all other distributions} \,. \end{cases}
$$

NNLO with dynamic scale

Czakon, Heymes, Mitov: 1606.03350

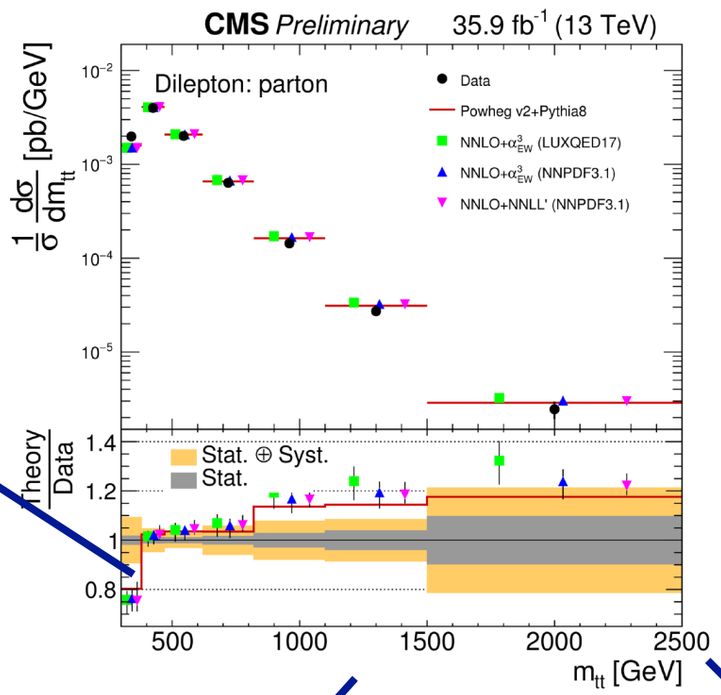




Vastly different behaviors with different scheme choices (especially in the boosted region)

Beyond NNLO

Threshold region
soft+Coulomb?



Whole region
soft emissions to all orders

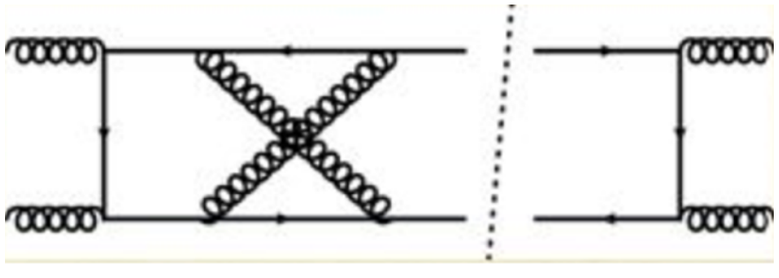
Ahrens, Ferroglia, Neubert, Pecjak, **LLY**: 1003.5827

Boosted region
soft+quasi-collinear emissions to all orders

Pecjak, Scott, Wang, **LLY**: 1601.07020
Czakon, Ferroglia, Heymes, Mitov, Pecjak, Scott, Wang, **LLY**: 1803.07623

Soft gluon resummation

Hard function



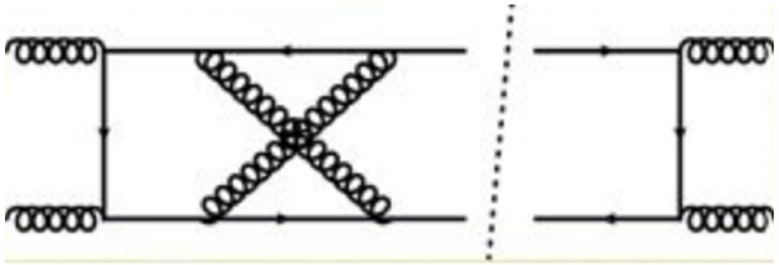
Kidonakis, Sterman: hep-ph/9705234

Ahrens, Ferroglia, Neubert, Pecjak, **LLY**: 1003.5827

Evolving from the scale of hard scatterings

Soft gluon resummation

Hard function



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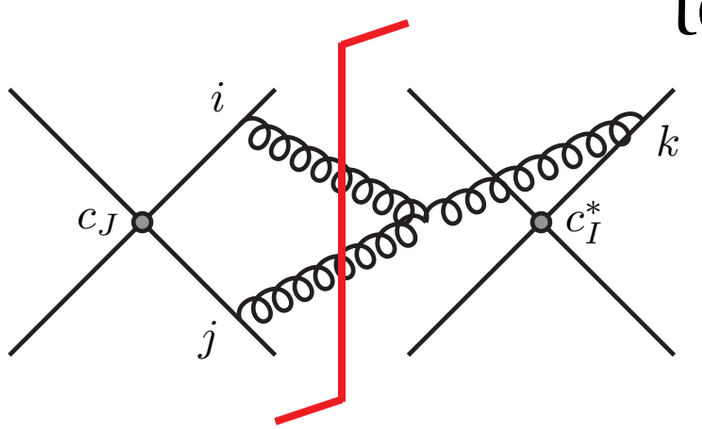
Ahrens, Ferroglia, Neubert, Pecjak, **LLY**: 1003.5827

Evolving from the scale of hard scatterings

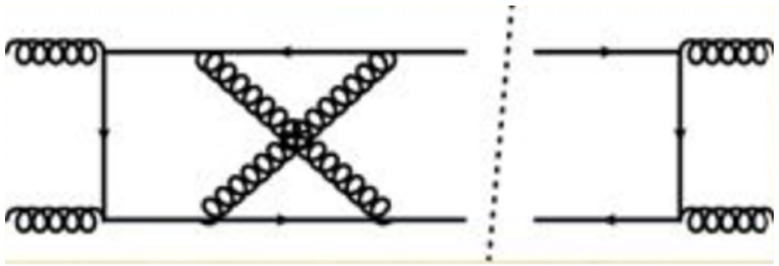
to the scale of soft interactions



Soft function

Soft gluon resummation

Hard function



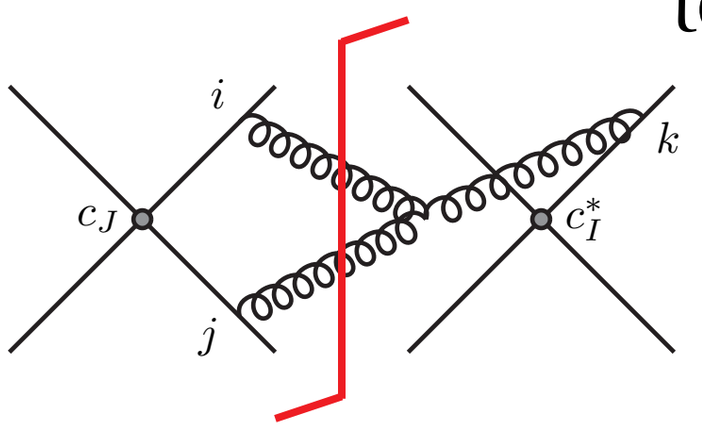
Kidonakis, Sterman: hep-ph/9705234

Ahrens, Ferroglia, Neubert, Pecjak, **LLY**: 1003.5827

Evolving from the scale of hard scatterings

Governed by IR structure

to the scale of soft interactions



Soft function

IR anomalous dimension

$$\mathbf{\Gamma} = \sum_{(i,j)} \frac{\boldsymbol{T}_i \cdot \boldsymbol{T}_j}{2} \gamma_{\rm cusp}(\alpha_s) \ln \frac{\mu^2}{-s_{ij}} + \sum_i \gamma^i(\alpha_s)$$
$$- \sum_{(I,J)} \frac{\boldsymbol{T}_I \cdot \boldsymbol{T}_J}{2} \gamma_{\rm cusp}(\beta_{IJ}, \alpha_s) + \sum_I \gamma^I(\alpha_s)$$
$$+ \sum_{I,j} \boldsymbol{T}_I \cdot \boldsymbol{T}_j \, \gamma_{\rm cusp}(\alpha_s) \ln \frac{m_I \mu}{-s_{Ij}} \qquad (2$$
$$+ \sum_{(I,J,K)} i f^{abc} \boldsymbol{T}_I^a \boldsymbol{T}_J^b \boldsymbol{T}_K^c \, F_1(\beta_{IJ}, \beta_{JK}, \beta_{KI})$$
$$+ \sum_{(I,J)} \sum_k i f^{abc} \boldsymbol{T}_I^a \boldsymbol{T}_J^b \boldsymbol{T}_k^c \, f_2\Big(\beta_{IJ}, \ln \frac{-\sigma_{Jk} \, v_J \cdot p_k}{-\sigma_{Ik} \, v_I \cdot p_k}\Big)$$

Becher, Neubert: 0904.1021

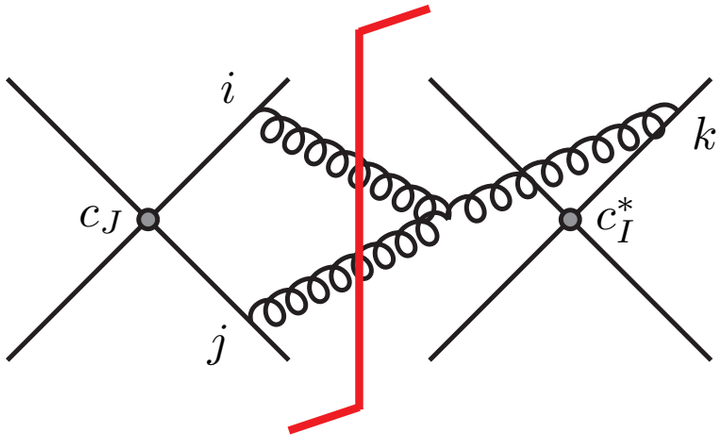
Ferroglia, Neubert, Pecjak, **LLY**: 0907.4791; 0908.3676

$$F_1(\beta_{12}, \beta_{23}, \beta_{31}) = \frac{\alpha_s^2}{12\pi^2} \sum_{i,j,k} \epsilon_{ijk} \, g(\beta_{ij}) \, r(\beta_{ki})$$

$$r(\beta) = \beta \coth \beta \,,$$
$$g(\beta) = \coth \beta \left[\beta^2 + 2\beta \ln(1 - e^{-2\beta}) - \mathrm{Li}_2(e^{-2\beta}) + \frac{\pi^2}{6}\right]$$
$$- \beta^2 - \frac{\pi^2}{6} \,. \qquad (5)$$

3-parton correlations

The soft function



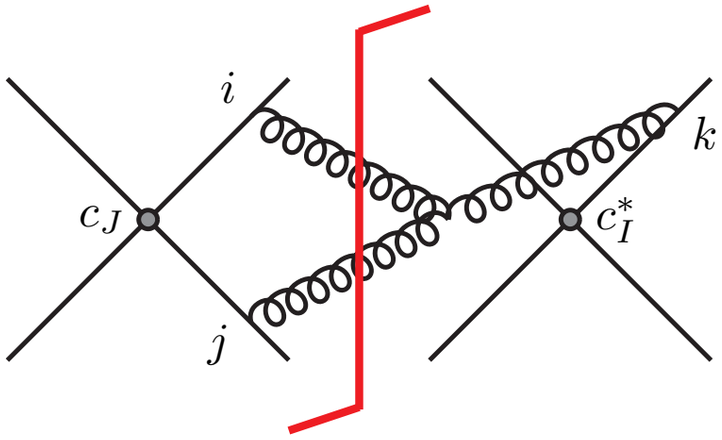
Known at NLO

Ahrens, Ferroglia, Neubert, Pecjak, **LLY**: 1003.5827

Known at NNLO in the massless limit (except an off-diagonal 3-parton piece)

Ferroglia, Pecjak, **LLY**: 1207.4798

The soft function



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Ferroglia, Pecjak, **LLY**: 1207.4798

Recent calculation at NNLO with massive tops

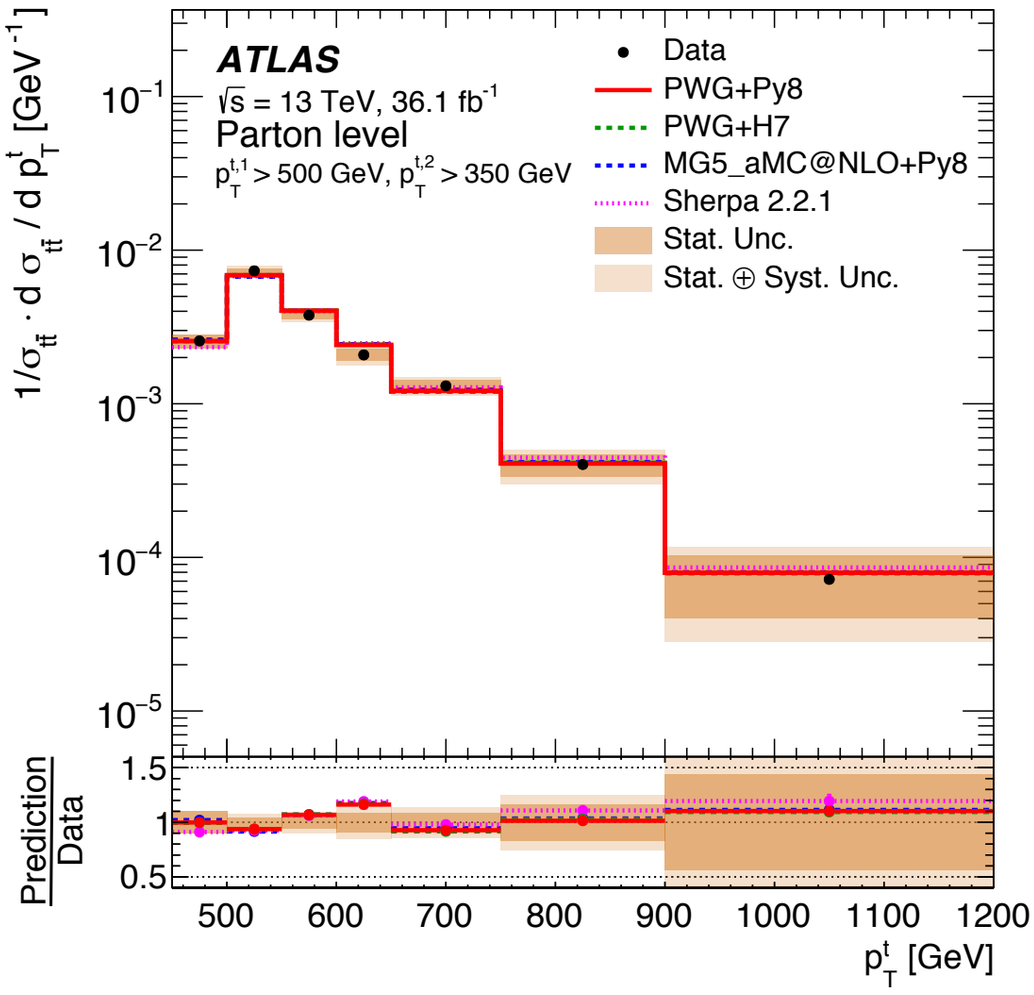
Wang, Xu, **LLY**, Zhu: 1804.05218

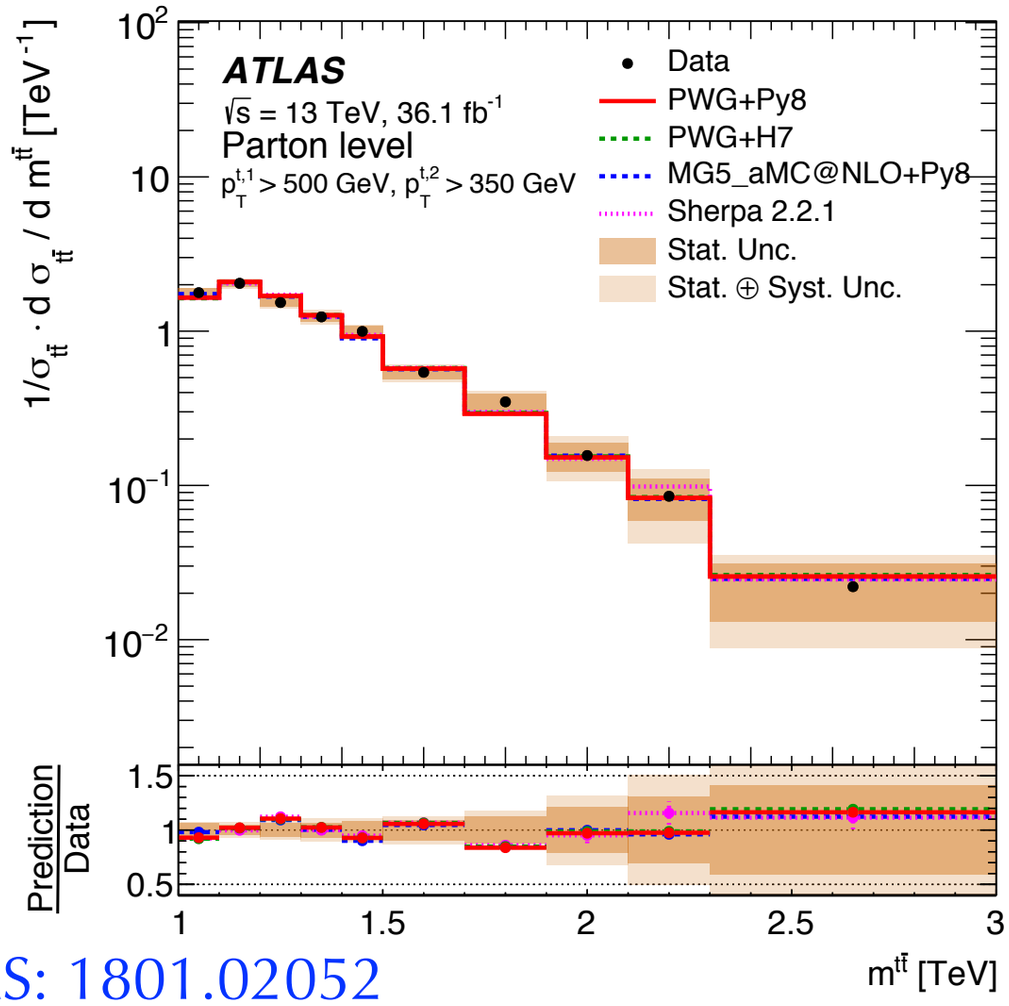
$$
\begin{aligned}
\tilde{\boldsymbol{s}}_{22}^{q\bar{q},(2)}(0,\beta,y)\Big|_{T_F N_l} &= \frac{16(7\beta^2-126\beta+127)}{243\beta}G_1 + \frac{8(5\beta^2+90\beta+53)}{81\beta}\big(G_{-1,-1}-G_{-1,1}-2G_{0,-1}\big)\\
&- \frac{16(7\beta^2+126\beta+127)}{243\beta}G_{-1} + \frac{8(5\beta^2-90\beta+53)}{81\beta}\big(G_{1,-1}-G_{1,1}+2G_{0,1}\big)\\
&+ \frac{8(\beta^2+18\beta+1)}{27\beta}\big(-G_{-1,-1,-1}+G_{-1,-1,1}+2G_{-1,0,-1}-2G_{-1,0,1}-G_{-1,1,-1}+G_{-1,1,1}\\
&+ 2G_{0,-1,-1}-2G_{0,-1,1}-4G_{0,0,-1}\big) + \frac{8(\beta^2-18\beta+1)}{27\beta}\big(4G_{0,0,1}+2G_{0,1,-1}-2G_{0,1,1}\\
&- G_{1,-1,-1}+G_{1,-1,1}+2G_{1,0,-1}-2G_{1,0,1}-G_{1,1,-1}+G_{1,1,1}\big)\\
&+ \frac{32}{243}\Big[28G_{-1/y}+98G_{1/y}+30\big(2G_{0,-1/y}+G_{-1/y,-1}+G_{-1/y,1}-2G_{-1/y,-1/y}\big)\\
&+ 105\big(2G_{0,1/y}+G_{1/y,-1}+G_{1/y,1}-2G_{1/y,1/y}\big)+18\big(4G_{0,0,-1/y}+2G_{0,-1/y,-1}+2G_{0,-1/y,1}\\
&- 4G_{0,-1/y,-1/y}-G_{-1/y,-1,-1}+G_{-1/y,-1,1}+2G_{-1/y,0,-1}+2G_{-1/y,0,1}-4G_{-1/y,0,-1/y}\\
&+ G_{-1/y,1,-1}-G_{-1/y,1,1}-2G_{-1/y,-1/y,-1}-2G_{-1/y,-1/y,1}+4G_{-1/y,-1/y,-1/y}\big)\\
&+ 63\big(4G_{0,0,1/y}+2G_{0,1/y,-1}+2G_{0,1/y,1}-4G_{0,1/y,1/y}-G_{1/y,-1,-1}+G_{1/y,-1,1}+2G_{1/y,0,-1}\\
&+ 2G_{1/y,0,1}-4G_{1/y,0,1/y}+G_{1/y,1,-1}-G_{1/y,1,1}-2G_{1/y,1/y,-1}-2G_{1/y,1/y,1}+4G_{1/y,1/y,1/y}\big)\\
&- \frac{332}{3}-\frac{5\pi^2}{2}+6\zeta_3\Big], \qquad (84)
\end{aligned}
$$

Fully analytic in terms of MPLs; fast numerics

Boosted top quarks

Sensitive to new physics, interesting in its own right!



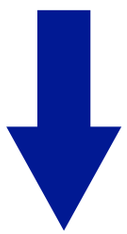


ATLAS: 1801.02052

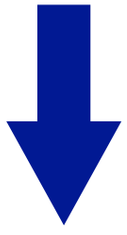
Actively being probed by LHC experiments

Producing boosted tops

Hard extra emissions suppressed



soft gluons



$$\ln \frac{\hat{s} - M^2_{t\bar{t}}}{M^2_{t\bar{t}}}$$

Producing boosted tops

Hard extra emissions suppressed

↓

soft gluons

↓

$$\ln \frac{\hat{s} - M^2_{t\bar{t}}}{M^2_{t\bar{t}}}$$

Top quark nearly massless

↓

quasi-collinear gluons

↓

$$\ln \frac{m_t^2}{M^2_{t\bar{t}}}$$

Producing boosted tops

Hard extra emissions suppressed

Top quark nearly massless

soft gluons

quasi-collinear gluons

$$\ln \frac{\hat{s} - M_{t\bar{t}}^2}{M_{t\bar{t}}^2}$$

Need to resum both!

Ferroglia, Pecjak, **LLY**: 1205.3662

$$\ln \frac{m_t^2}{M_{t\bar{t}}^2}$$

Soft and small-mass factorization

Ferroglia, Pecjak, **LLY**: 1205.3662

In Mellin space: $Q \sim \sqrt{s}, \sqrt{-t} \gg Q/N \gg m_t \gg m_t/N$

$$\hat{\sigma}(N,\mu_f) \sim \mathrm{Tr}\left[\boldsymbol{H}(L_h,\mu_f)\,\boldsymbol{S}(L_s,\mu_f)\right] C_D^2(L_c,\mu_f)\,S_D^2(L_{sc},\mu_f)$$

Soft and small-mass factorization

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$$\ln\frac{Q^2}{\mu_f^2}$$

hard log

Soft and small-mass factorization

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$$\ln\frac{Q^2}{\mu_f^2}$$

hard log

$$\ln\frac{Q^2}{\bar{N}^2\mu_f^2}$$

soft log

Soft and small-mass factorization

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$\ln \dfrac{Q^2}{\mu_f^2}$ hard log

$\ln \dfrac{Q^2}{\bar{N}^2\mu_f^2}$ soft log

$\ln \dfrac{m_t^2}{\mu_f^2}$ collinear log (small-mass)

Soft and small-mass factorization

Ferroglia, Pecjak, **LLY**: 1205.3662

In Mellin space: $Q \sim \sqrt{s}, \sqrt{-t} \gg Q/N \gg m_t \gg m_t/N$

$$\hat{\sigma}(N,\mu_f) \sim \mathrm{Tr}\left[\boldsymbol{H}(L_h,\mu_f)\,\boldsymbol{S}(L_s,\mu_f)\right] C_D^2(L_c,\mu_f)\, S_D^2(L_{sc},\mu_f)$$

$\ln \dfrac{Q^2}{\mu_f^2}$ hard log

$\ln \dfrac{Q^2}{\bar{N}^2\mu_f^2}$ soft log

$\ln \dfrac{m_t^2}{\mu_f^2}$ collinear log (small-mass)

$\ln \dfrac{m_t^2}{\bar{N}^2\mu_f^2}$ **soft-collinear log (emergent)**

Soft and small-mass resummation

Massless hard function

$$\boldsymbol{H}(L_h, \mu_h \sim Q)$$

Massless soft function

$$\boldsymbol{S}(L_s, \mu_s \sim Q/\bar{N})$$

μ_f **RG flow**

$$C_D(L_c, \mu_c \sim m_t)$$

$$S_D(L_{sc}, \mu_{sc} \sim m_t/\bar{N})$$

All ingredients known at NNLO (for NNLL′ resummation)

Matching

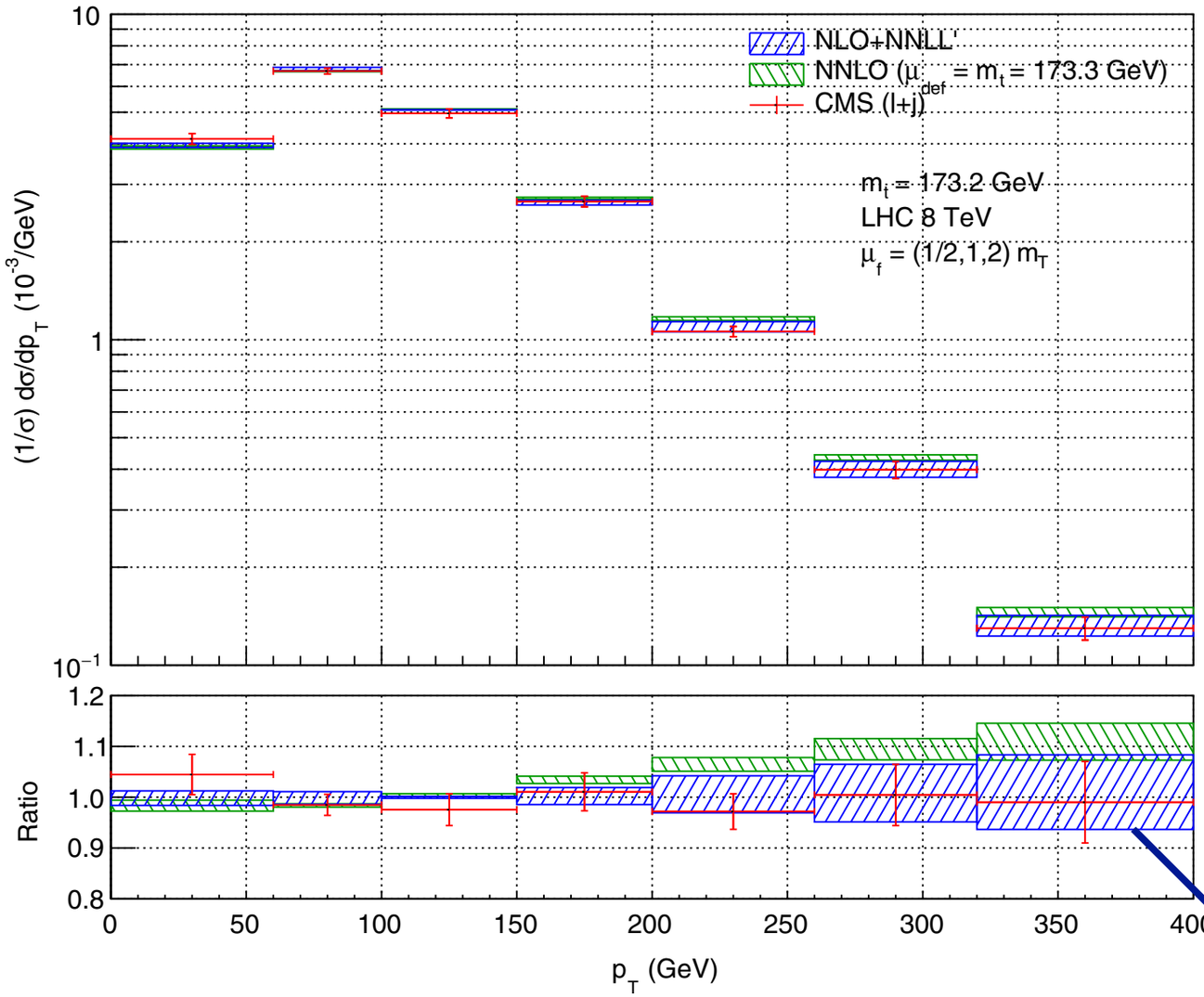
We have 3 different calculations

	NNLO	Soft resummation	Soft+small-mass resummation
✓	$\alpha_s^n (n \leq 2)$	$\alpha_s^n \ln^m N$	$\alpha_s^n \ln^m N$; $\alpha_s^n \ln^m (m_t/M_{t\bar{t}})$
✗	$\alpha_s^n (n > 2)$	1/N suppressed terms	1/N suppressed terms; m_t/M_{tt} suppressed terms

Need to remove double- and triple-counting when combining them!

NLO+NNLL′

Pecjak, Scott, Wang, **LLY**: 1601.07020



Resummation softens the spectrum

NNLO+NNLL′

A joint effort of the NNLO group and the resummation group

Czakon, Ferroglia, Heymes, Mitov, Pecjak, Scott, Wang, **LLY**: 1803.07623

$$d\sigma^{(\mathrm{N})\mathrm{NLO+NNLL'}} = d\sigma^{\mathrm{NNLL'}_{b+m}} + \left(d\sigma^{(\mathrm{N})\mathrm{NLO}} - d\sigma^{\mathrm{NNLL'}_{b+m}}\Big|_{\substack{(\mathrm{N})\mathrm{NLO} \\ \text{expansion}}} \right)$$

NNLO+NNLL′

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$$d\sigma^{\mathrm{NNLL'}_b} + \left(d\sigma^{\mathrm{NNLL}_m} - d\sigma^{\mathrm{NNLL}_m}\Big|_{m_t \to 0} \right)$$

soft & small mass resummation

match to soft resummation

NNLO+NNLL′

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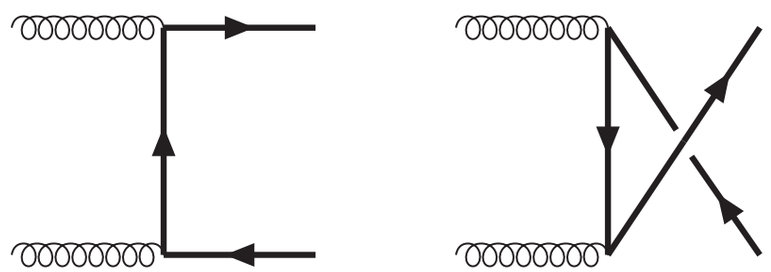
soft & small mass resummation

match to soft resummation

match to NNLO

Scale choices

In the boosted limit, t- and u-channel propagators push the effective hard scale to



$$-t_1\big|_{m_t\to 0} \approx \frac{M_{t\bar{t}}^2}{2}(1-\cos\theta) + m_t^2\cos\theta \xrightarrow{\cos\theta\to 1} p_T^2 + m_t^2 \equiv m_T^2 = H_T^2/4\,,$$

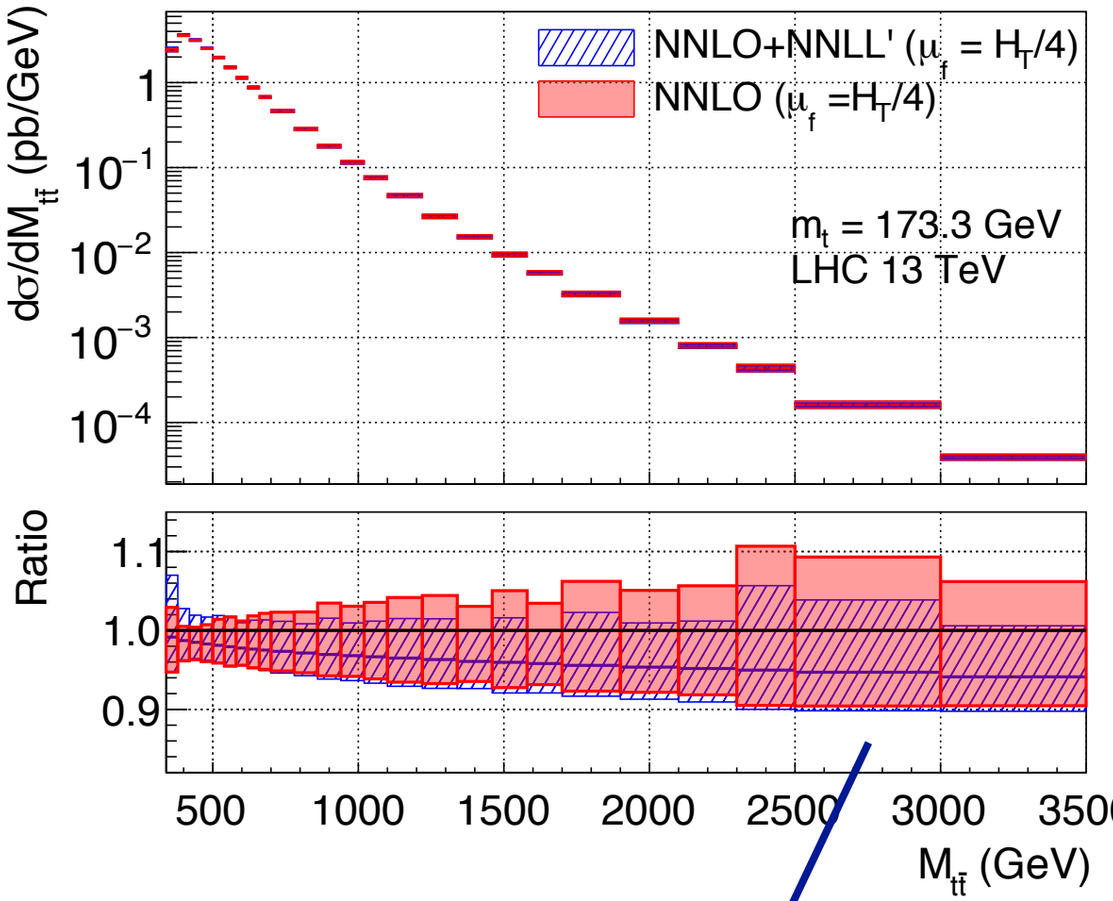
$$-u_1\big|_{m_t\to 0} \approx \frac{M_{t\bar{t}}^2}{2}(1+\cos\theta) - m_t^2\cos\theta \xrightarrow{\cos\theta\to -1} m_T^2 = H_T^2/4\,.$$

$$\left.\frac{\mathcal{H}_{gg}^{\rm NLO}(\mu_h)}{\mathcal{H}_{gg}^{\rm LO}(\mu_h)}\right|_{t_1\to 0} = 1+\frac{\alpha_s(\mu_h)}{36\pi}\left[-78\ln^2\left(\frac{-t_1}{\mu_h^2}\right)+24\ln\left(\frac{-t_1}{\mu_h^2}\right)(3+2\ln x_t)+37\pi^2-168\right]$$

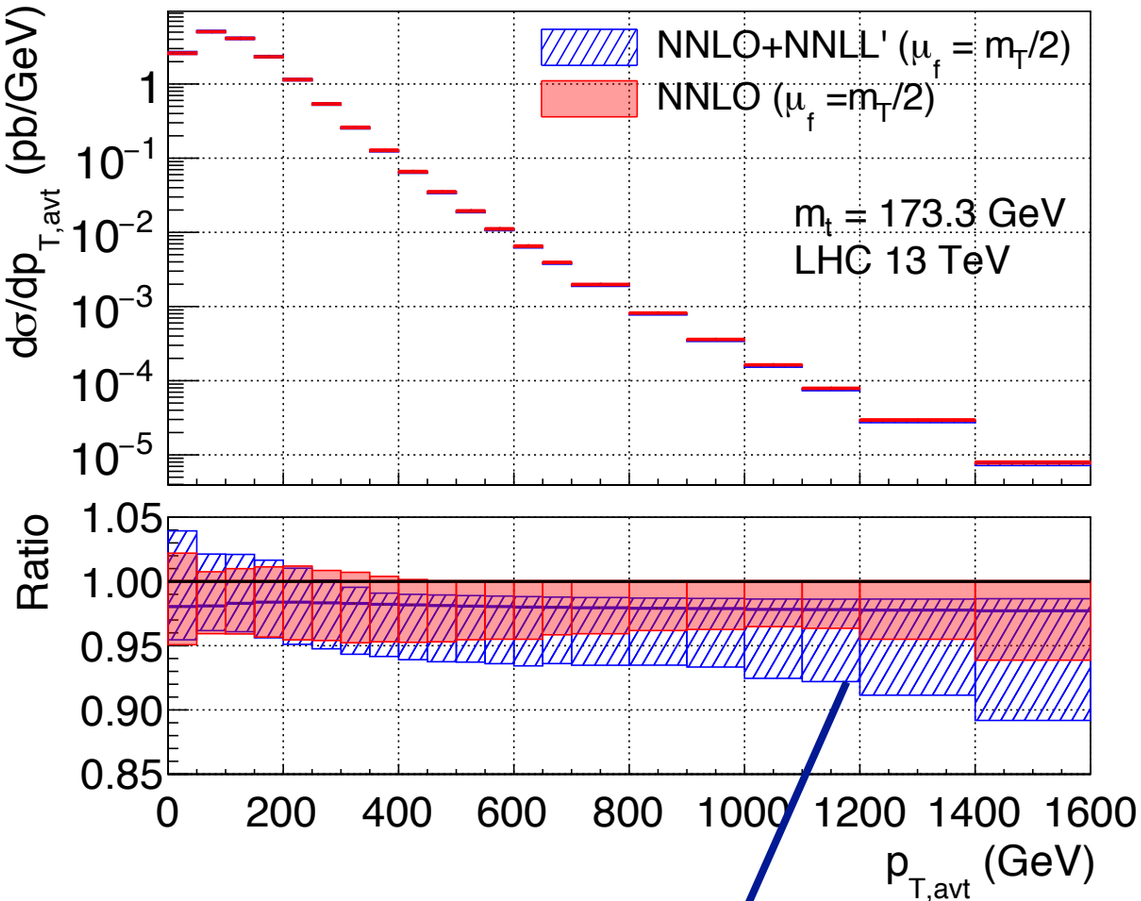
Support the findings of Czakon, Heymes, Mitov: 1606.03350

NNLO+NNLL′

Czakon, Ferroglia, Heymes, Mitov, Pecjak, Scott, Wang, **LLY**: 1803.07623



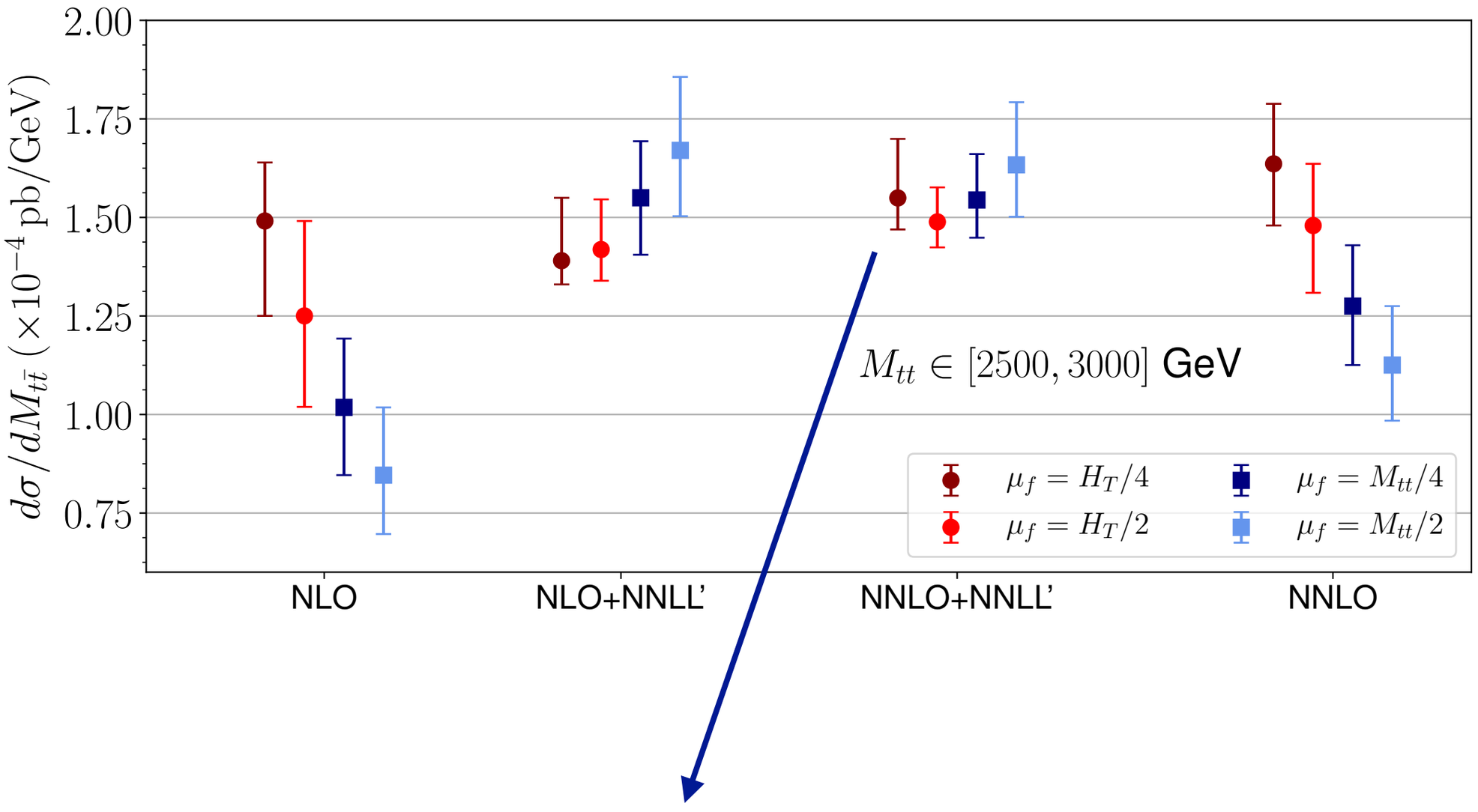
Resummation reduces scale variation



Resummation softens the spectrum

NNLO+NNLL′

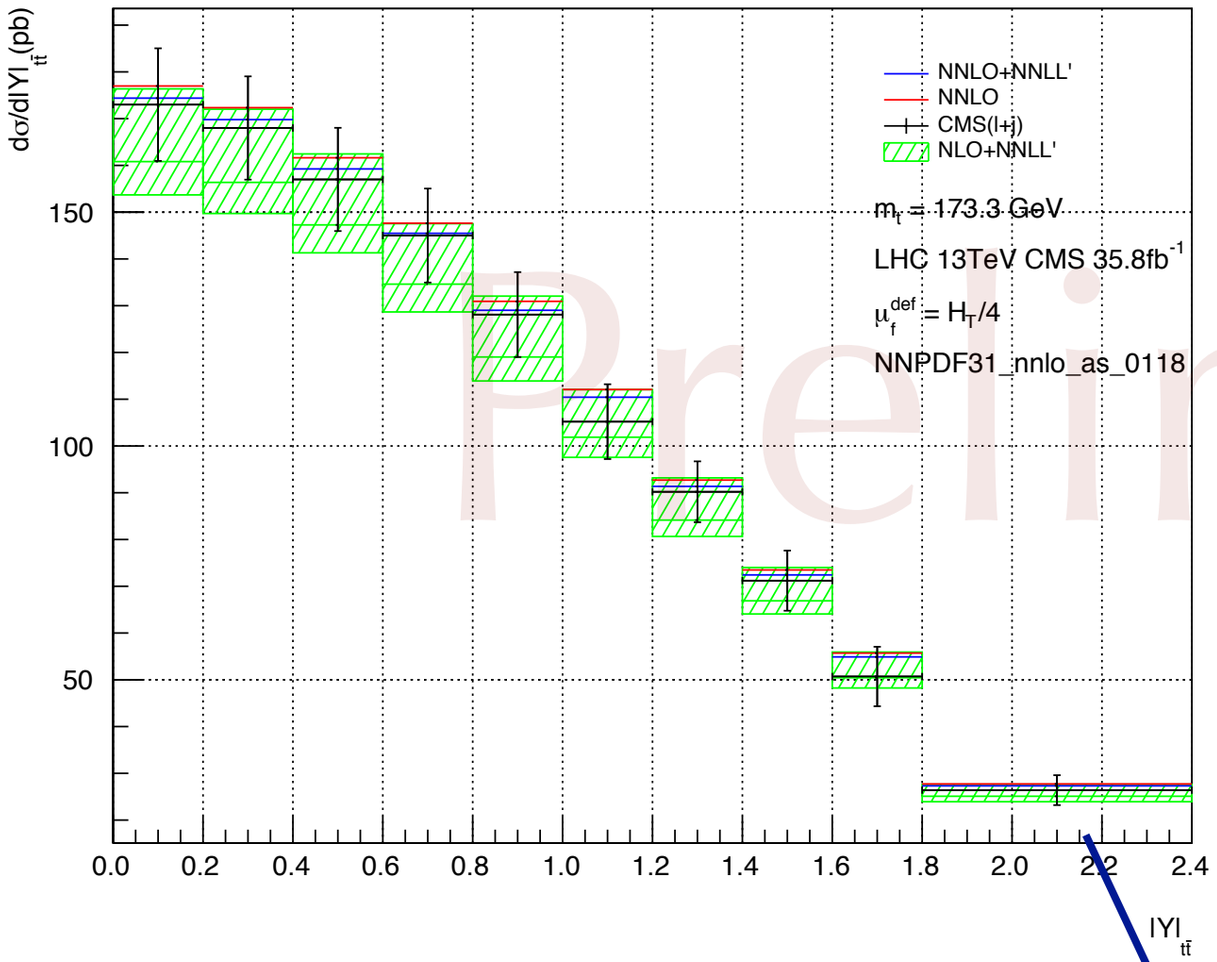
Czakon, Ferroglia, Heymes, Mitov, Pecjak, Scott, Wang, **LLY**: 1803.07623

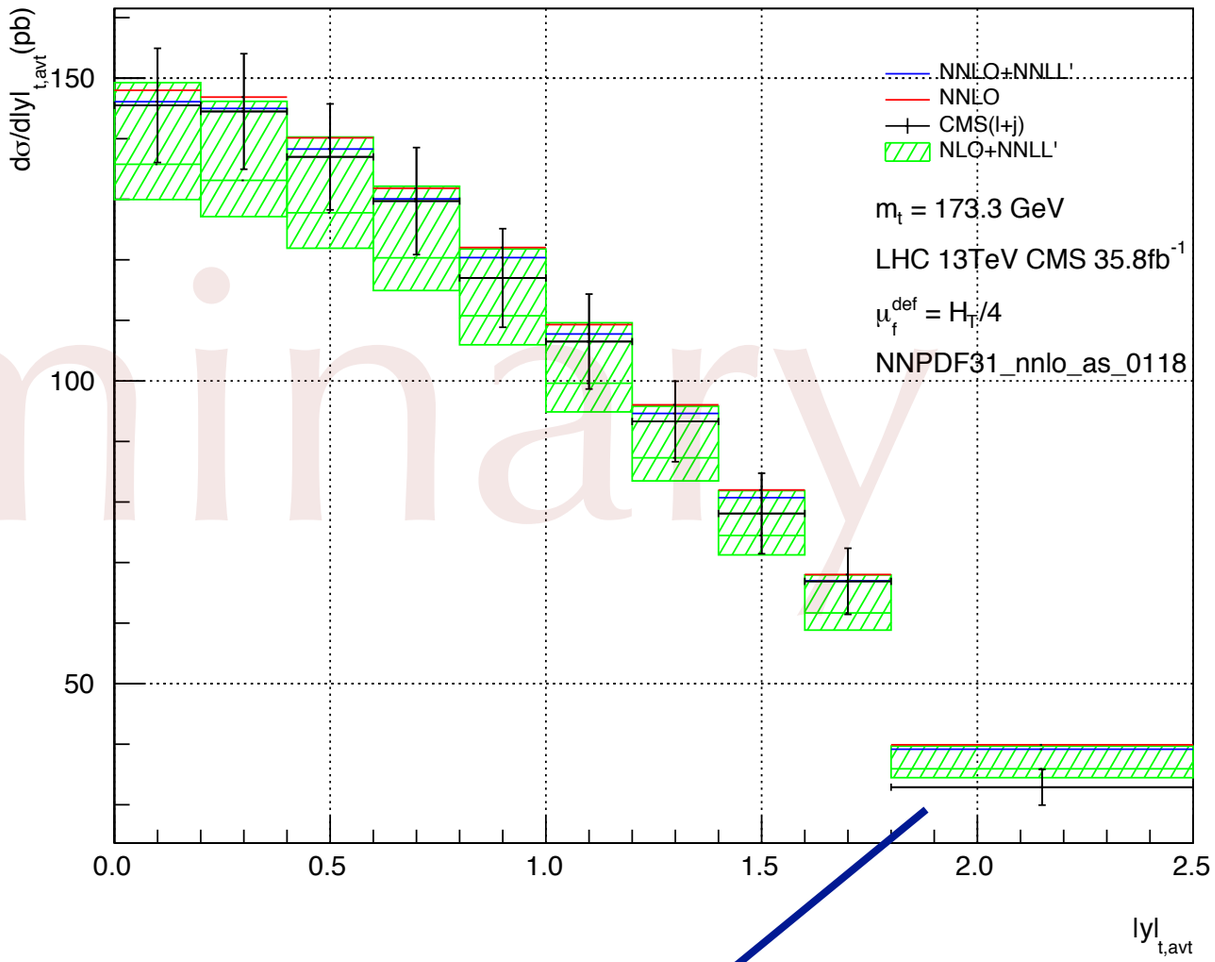


Matched result insensitive to scale scheme choices

Ongoing: rapidity distributions

Pecjak, Scott, Wang, **LLY**: to appear



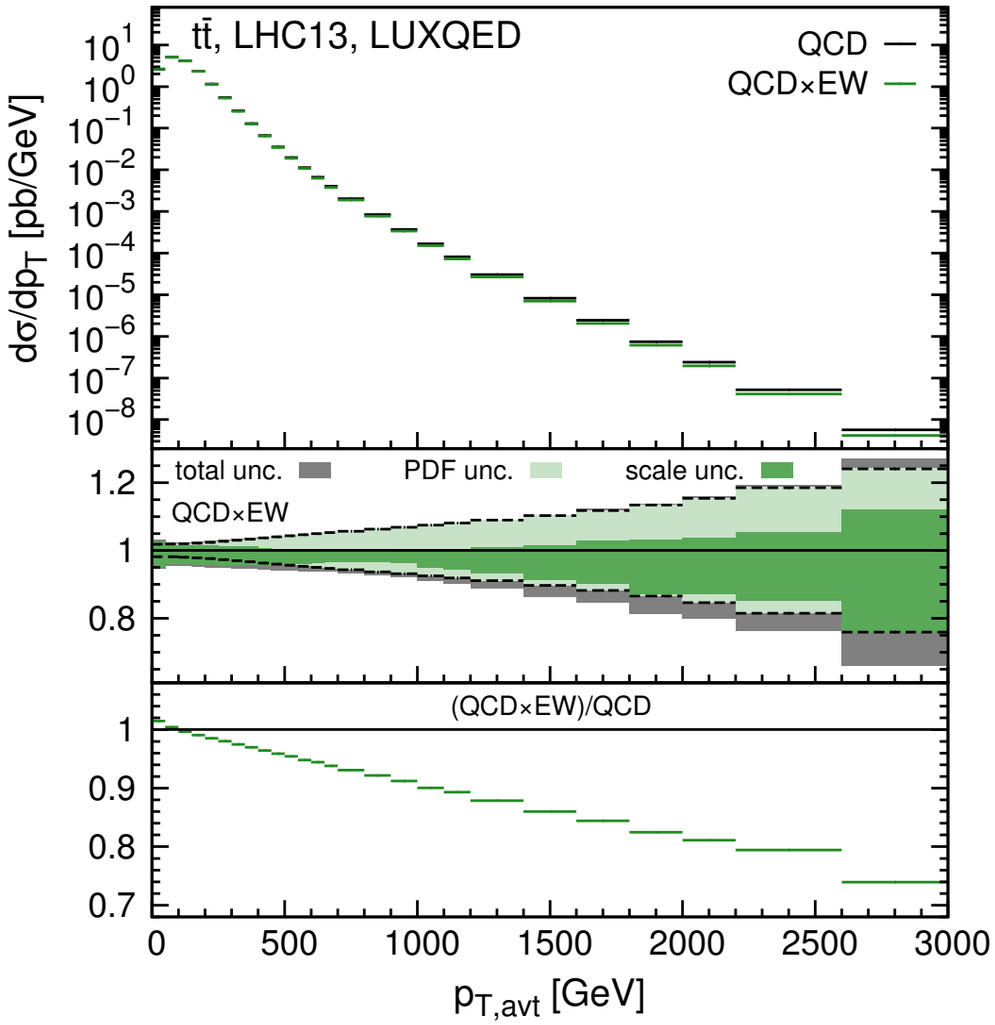


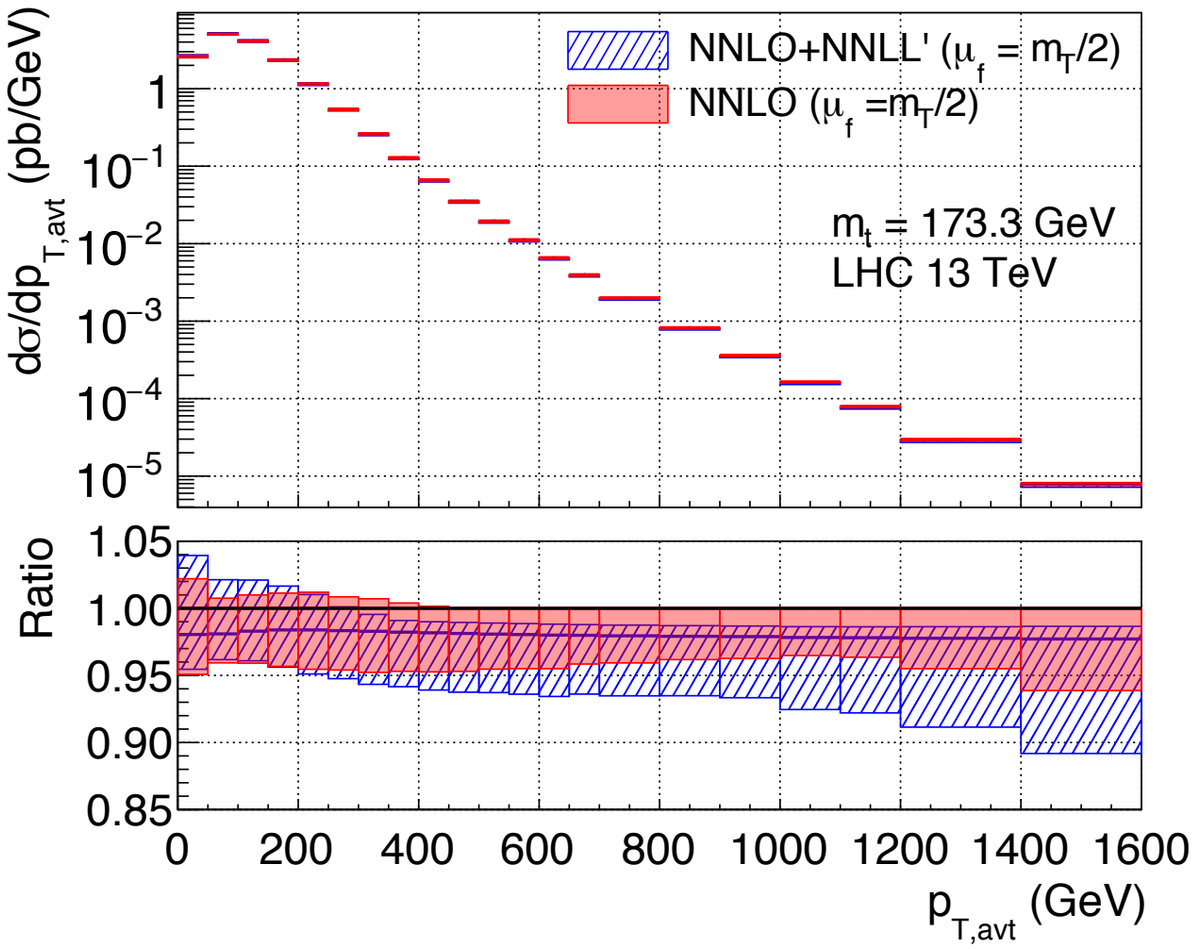
Preliminary

Sensitive to gluon PDF at large x

Ongoing: combination with electroweak corrections

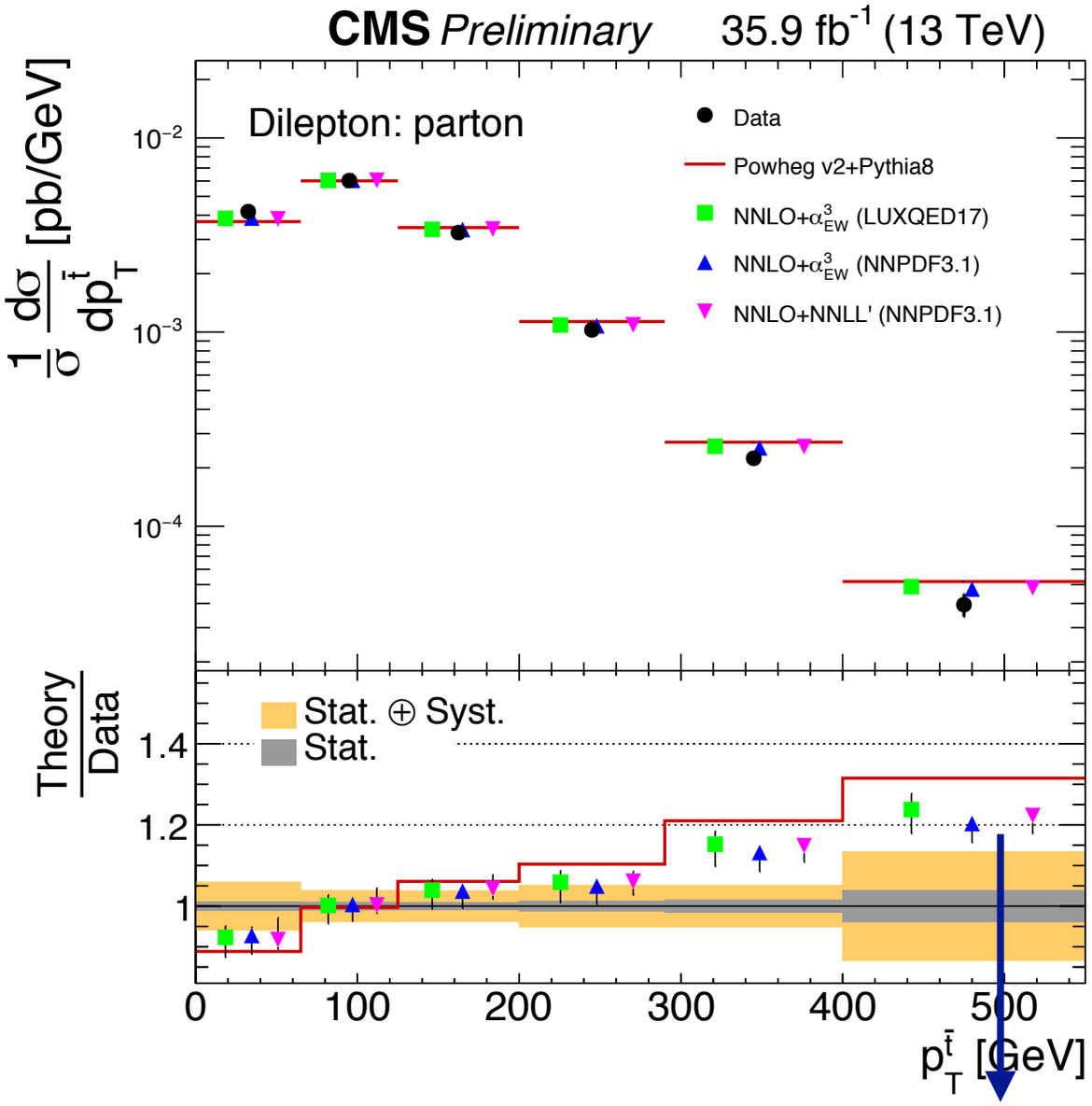
Both EW and resummation effects soften the p_T spectrum





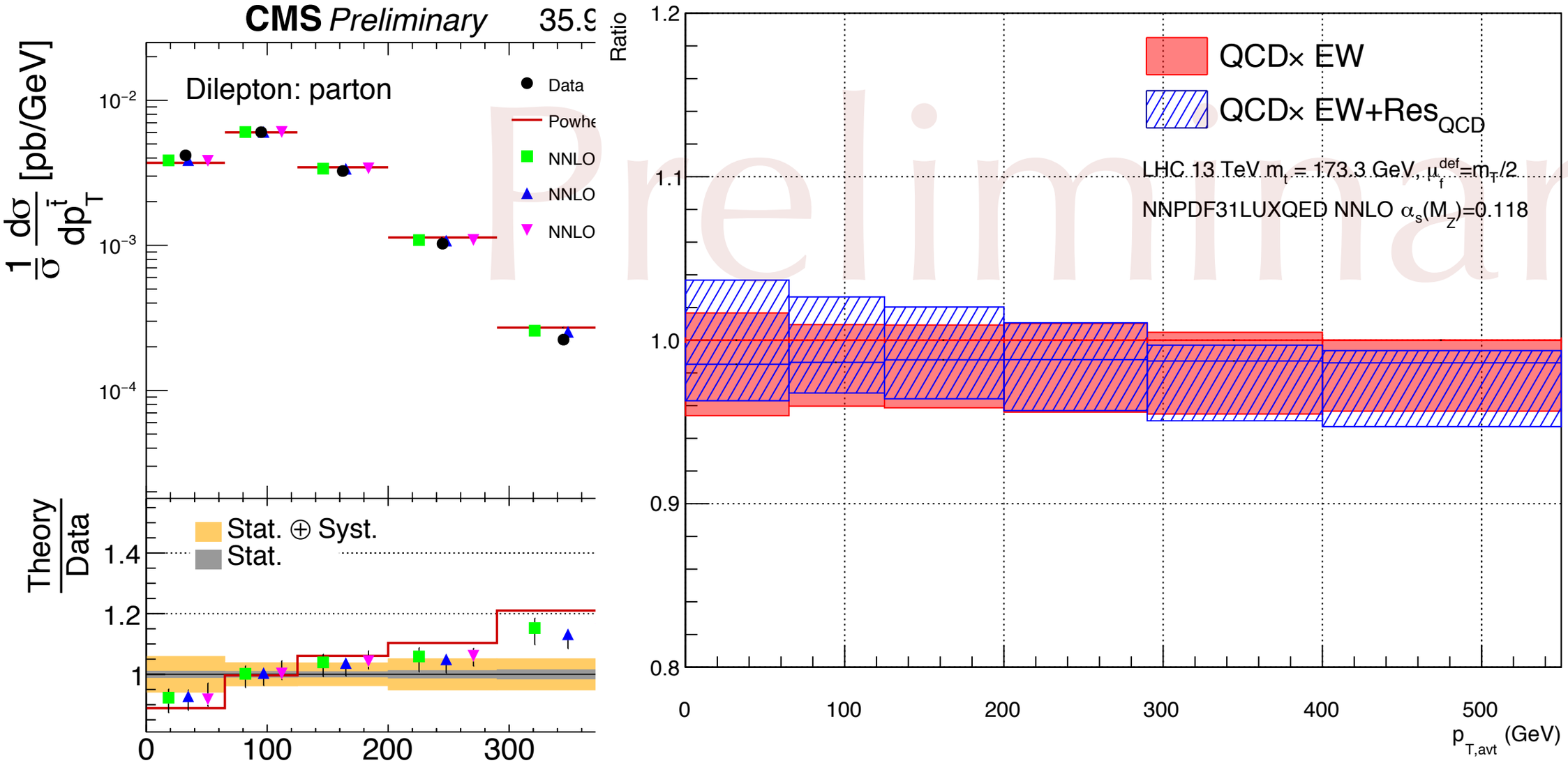
Czakon et al.: 1705.04105

Ongoing: combination with electroweak corrections



Lower

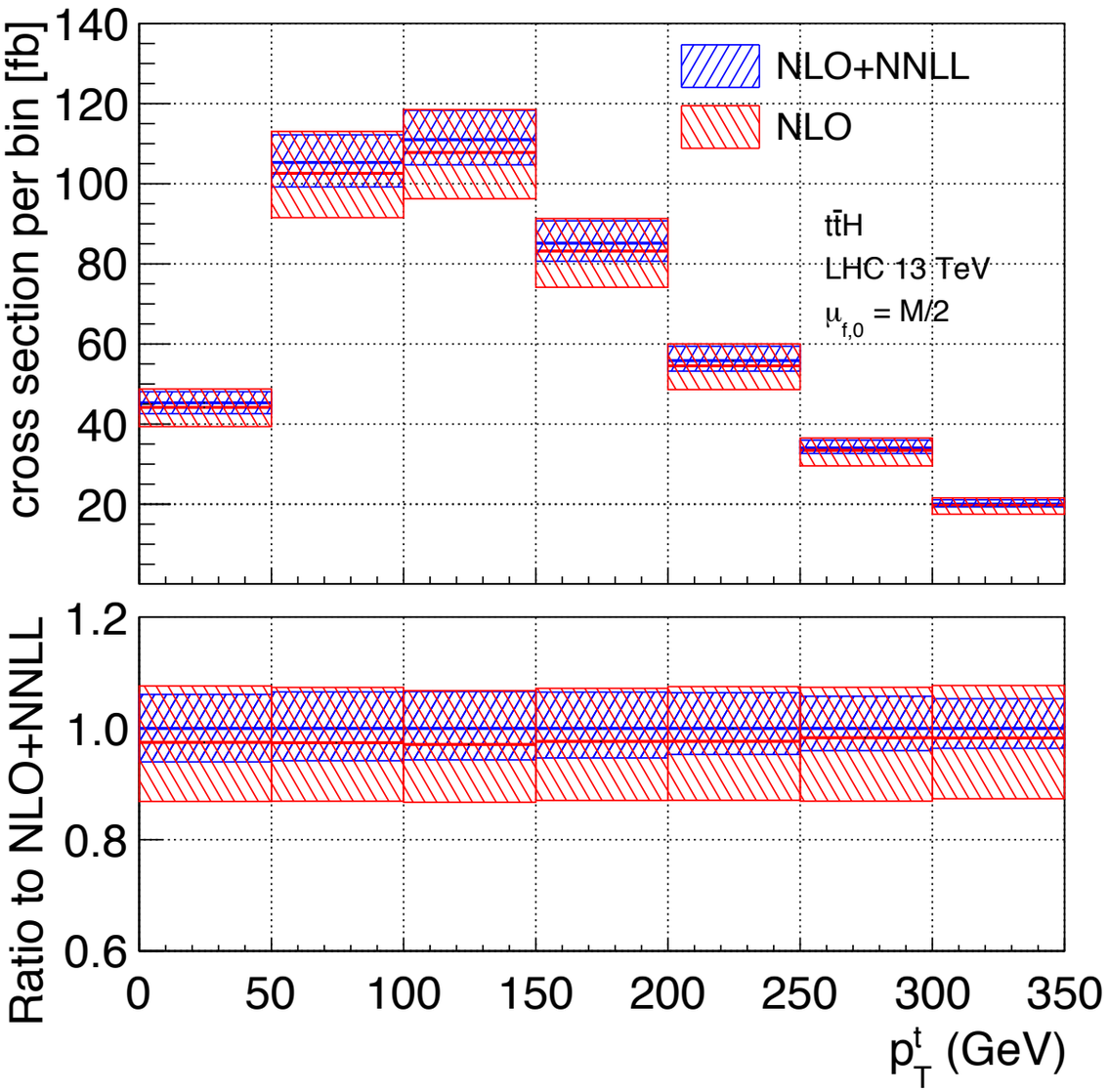
Ongoing: combination with electroweak corrections

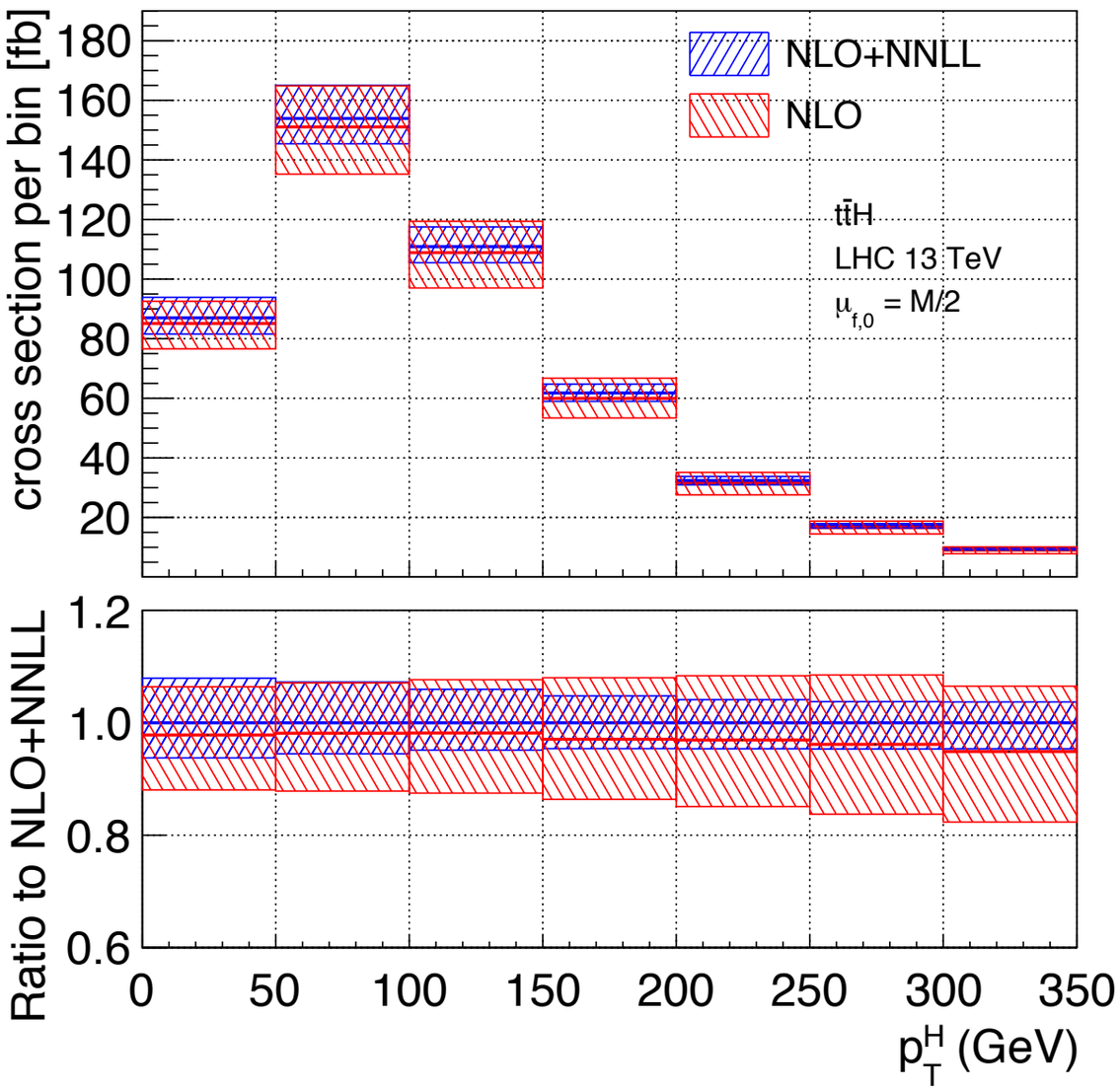


Lower

A similar process

Broggio, Ferroglia, Pecjak, **LLY**: 1611.00049





Need to go beyond? Very challenging!

Summary and outlook

* Top quark pair production is important
* The most precise QCD calculation: NNLO+NNLL′
* Ongoing:
  * Rapidity distributions
  * Combination with NLO electroweak corrections
* Outlook:
  * Combination with Coulomb resummation (first bin)
  * Top quark decays

Thank you!