Renormalons and the Top quark mass measurement

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The Last GeV

Pushing the accuracy of top mass measurements with a precision below a GeV poses a hard theoretical challenge.

Main theory obstacle: QCD at low energy.

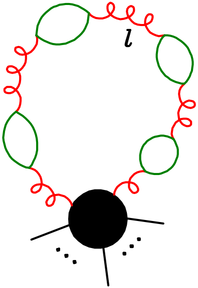
Two things to be done:

- PQCD at high energy + hadr. models at low energy.
  - It is often all what we can do.
  - It always leaves a difficult question to answer: how can we trust a model knowing that it is just a model?
- Study the problem from a theoretical point of view:
  - Perturbation theory can teach us something about non-perturbative corrections.
  - Look at simplified contexts.

ABC of I.R. Renormalons

All-orders contributions to QCD amplitude of the form

$$\int_0^m \mathrm{d}k^p\, \alpha_s(k^2) = \int_0^m \mathrm{d}k^p\, \frac{1}{b_0 \log(k^2/\Lambda^2)}$$
$$= \int_0^m \mathrm{d}k^p \frac{\alpha_s(m^2)}{1 + b_0\alpha_s(m^2)\log\frac{k^2}{m^2}}$$
$$= \alpha_s(m^2)\sum_{n=0}^{\infty}(2b_0\alpha_s(m^2))^n \int_0^m \underbrace{\mathrm{d}k^p \log^n \frac{m}{k}}_{p^n n!}.$$



Asymptotic expansion.

- Minimal term at $n_{\min} \approx \frac{1}{2pb_0\alpha_S(m^2)}$.
- Size of minimal term: $m^p\alpha_s(m^2)\sqrt{2\pi n_{\min}}e^{-n_{\min}} \approx \Lambda^p$.
- Typical scale dominating at order α_s^{n+1}: $m\exp(-np)$.

ABC of I.R. Renormalons

- Renormalon arise due to all radiative correction that build up the running of α_s.
- When $\alpha_s \approx 1$ perturbation theory breaks down. This happens when $k \approx \Lambda$. The volume of this region is

$$\int_0^{\Lambda} \mathrm{d}k^p \approx \Lambda^p. \tag{1}$$

  This corresponds to the size of the minimal term in the summation of the asymptotic expansion.
- The running of α_s is not only due to bubble insertion in the gluon propagator. However:
  In the limit of large n_f these are the dominant corrections.

Large n_f limit

In fact:

- Adding a gluon to a diagram costs a factor of α_s.
- Inserting a fermion bubble in a gluon propagator costs $\alpha_s n_f$.

So: given a process involving up to 1-gluon exchange, the diagrams obtained by inserting fermion bubbles in the gluon line are dominant in the large n_f limit.

This means that in this large n_f limit every question has a definite, calculable answer, since we know how to compute and sum these fermion bubbles.

Large n_f limit

Several applications:

- $e^+e^- \to \mathrm{hadrons}$, τ decays, Λ^4/Q^4 corrections
- DIS and DIS sum rules, Λ^2/Q^2 corrections
- b and c decays Bigi,Uraltsev, Zakharov, Shifman, Beneke, etc.
- Jets, Dokshitzer, Marchesini, Salam, etc., Λ/Q corrections.

Several of these item are summarized in a review by Beneke.

One interesting example: Beneke and Braun, arXiv:hep-ph/9506452

Abstract:

The resummed Drell-Yan cross section in the double-logarithmic approximation suffers from infrared renormalons. Their presence was interpreted as an indication for non-perturbative corrections of order $\Lambda_{\mathrm{QCD}}/(Q(1-z))$. We find that, once soft gluon emission is accurately taken into account, the leading renormalon divergence is cancelled by higher-order perturbative contributions in the exponent of the resummed cross section.

Their calculation: leading n_f one gluon correction:

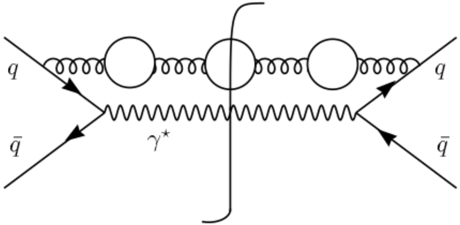


Figure 1: α_s^4-contribution to the partonic Drell-Yan cross section. $\gamma^\star$ represents a photon with invariant mass Q^2 that splits into a lepton pair.

Motivation:
High-school quiz on top mass measurements

Tick the correct statements:

- ☐ Direct top mass measurements measure the Pole Mass.
- ☐ Direct top mass measurements measure the Monte Carlo Mass.
- ☐ Direct top mass measurements measure the Monte Carlo Mass. but you can pretend that it is the pole mass, just inflate the error a bit.
- ☐ The top is the only SM particle with more than one mass.
- ☐ You should use only leptons to avoid hadronization uncertainty.
- ☐ You should use at least NLO calculations to measure the pole mass.
- ☐ The top pole mass has renormalons, you should stay away from it.
- The MC mass differs from the pole mass by
  ☐ terms of order $m\alpha_S$; ☐ terms of order $\Lambda_{\rm QCD}$; ☐ terms of order $\alpha_S\Gamma_t$.
- The Pole Mass renormalon ambiguity is
  ☐ $\approx$ 1GeV; ☐ $\approx$ 250 MeV; ☐ $\approx$ 200 MeV; ☐ $\approx$ 110 MeV.

Motivation

- We hope that a large n_f calculation can help to deal with at least some of these questions.
- Linear (i.e. Λ suppressed) renormalons may affect top mass measurements with ambiguities of order Λ (near the present experimental accuracy).
- Several sources of linear renormalons come into play in top mass measurements (for example, from jet definition). What is their structure, and what is their interplay with the pole mass renormalon?

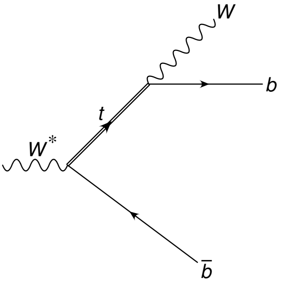
A study of linear renormalon effects in top mass measurements

Silvia Ferrario Ravasio, Carlo Oleari, P.N.

PRELIMINARY!

Set up the computation of top mass sensitive observables in leading n_f one gluon correction.

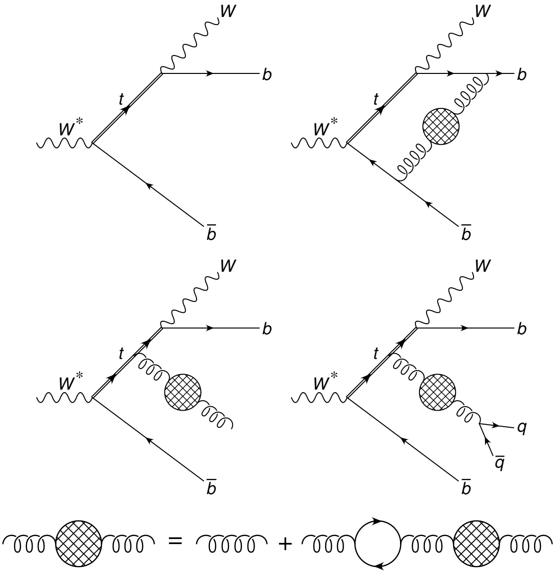
We consider a simplified production framework $W^* \to Wt\bar{b}$:



(i.e. no incoming hadrons for now).

- The b is taken massless, the W is taken stable, but the top is taken unstable, with a finite width.
- We can deal with any final state observable. In particular, we can look at the mass of the reconstructed top, i.e. a system comprising a b-jet and the W.

Diagrams up to leading N_f one gluon correction



All-order result

It turns out that the all-order result can be expressed in terms of the followimg ingredients:

- $\sigma_b(\Phi_b)$, the differential cross section for the Born process;
- $\sigma_v(k^2, \Phi_b)$, the virtual correction to the Born process due to the exchange of a gluon of mass k;
- The real cross section $\sigma_{g^*}(k^2, \Phi_{g^*})$, obtained by adding one massive gluon to the Born final state;
- The real cross section $\sigma_{q\bar{q}}(\Phi_{q\bar{q}})$, obtained by adding a $q\bar{q}$ pair, produced by a massless gluon, to the Born final state;

All-order result

Consider a (IR safe) final state observable O. Define:

$$
\begin{aligned}
N^{(0)} &= \left[\int \mathrm{d}\Phi_{\mathsf{b}}\,\sigma_{\mathsf{b}}\right]^{-1}, \quad \langle O\rangle_{\mathrm{b}} = N^{(0)}\int \mathrm{d}\Phi_{\mathsf{b}}\,\sigma_{\mathsf{b}}(\Phi_{\mathsf{b}})\,O(\Phi_{\mathsf{b}}), \\
\widetilde{V}(k^2) &= N^{(0)}\int \mathrm{d}\Phi_{\mathrm{b}}\,\sigma_{\mathrm{v}}^{(1)}(k^2,\Phi_{\mathrm{b}})\left[O(\Phi_{\mathsf{b}}) - \langle O\rangle_{\mathrm{b}}\right], \\
\widetilde{R}(k^2) &= N^{(0)}\int \mathrm{d}\Phi_{g^*}\,\sigma_{g^*}^{(1)}(k^2,\Phi_{g^*})\left[O(\Phi_{g^*}) - \langle O\rangle_{\mathrm{b}}\right], \\
\widetilde{\Delta}(k^2) &= \frac{1}{2}\frac{3}{\alpha_s T_{\mathrm{F}}}\,k^2\,N^{(0)}\int \mathrm{d}\Phi_{\mathsf{dec}}\,\mathrm{d}\Phi_{g^*}\,\sigma_{q\bar{q}}^{(2)}(\Phi_{q\bar{q}}) \times [O(\Phi_{q\bar{q}}) - O(\Phi_{g^*})]
\end{aligned}
$$

$\langle O\rangle_{\mathrm{b}} + \widetilde{V}(k^2) + \widetilde{R}(k^2)$ is the average value of O in a theory with a massive gluon with mass k^2, accurate to order α_s.

Notice: $\widetilde{V}(k^2) + \widetilde{R}(k^2)$ has a finite limit for $k^2 \to 0$, while each contribution is log divergent.

defining $\widetilde{T}(k^2) = \widetilde{V}(k^2) + \widetilde{R}(k^2) + \widetilde{\Delta}(k^2)$ our final result is

$$\langle O \rangle = \langle O \rangle_{\rm b} + \frac{3\pi}{\alpha_s T_{\rm F}} \int_0^\infty \frac{{\rm d}k}{\pi} \frac{\rm d}{{\rm d}k} \left[\widetilde{T}(k^2) \right] {\rm atan} \left(\frac{\alpha_s \pi b_0}{1 + \alpha_s b_0 log \frac{k^2}{\tilde{\mu}^2}} \right)$$

($\tilde{\mu}$ is just a constant time the scale at which α_s is evaluated)
If we thus have: $\widetilde{T}(k^2) = \alpha_s(a + b\,k + ck^2 + \ldots)$ we get

$$\langle O \rangle = \langle O \rangle_{\rm b} \frac{3\pi}{T_{\rm F}} \int_0^\infty \frac{{\rm d}k}{\pi} \ [b + 2ck + \ldots] \ {\rm atan} \left(\frac{\alpha_s \pi b_0}{1 + \alpha_s b_0 log \frac{k^2}{\tilde{\mu}^2}} \right)$$

(remember the $\int {\rm d}k \frac{\alpha_S}{1+\alpha_S b_0 log k^2/m^2}$ in the ABC of renormalons)

Thus:

a linear small-k dependence in T signals the presence of a linear renormalon in the results.

Comments

- In order to get our results, we need $\lim_{k^2\to\infty} \widetilde{T}(k^2) = 0\,.$
  This happens if we use the Pole Mass Scheme for m_t.
- The need to include the Δ term has a long story:
  - Seymour,P.N. 1995, I.R. renormalons in e^+e^- event shapes.
  - Dokshitzer,Lucenti,Marchesini,Salam, 1997-1998 Milan factor
- We compute $T(k^2)$ numerically. The $k^2 \to 0$ limit implies the cancellation of two large logs in V and R. However, the precise value at $k^2 = 0$ can also be computed directly by standard means (which we do).

Large n_f resummation leads naturally to these *massive gluon* cross sections. This is not new, and has a long history. Our novelty is the numerical formulation of a method that can be applied to any observable.

Changing the mass scheme

Also at large n_f the pole mass has a linear renormalon!
Its relation to the $\overline{\mathrm{MS}}$ mass is (Beneke, 1999)

$$\begin{aligned} m &= \bar{m}[1 + R_f(\alpha_s, \mu, \bar{m}) + R_d(\alpha_s, \mu, \bar{m})], \\ R_f &= -\frac{3\pi}{\alpha_s T_F} \int_0^\infty \frac{\mathrm{d}k}{\pi} \frac{\mathrm{d}r_f(k^2)}{\mathrm{d}k} \mathrm{atan} \frac{-\alpha_s \pi b_0}{1 + \alpha_s b_0 \log \frac{k^2}{\tilde{\mu}^2}} \\ r_f(k^2) &= -\alpha_s \frac{C_F}{2} \frac{k}{m} \,. \end{aligned} \tag{2}$$

We can easily convert our results to the $\overline{\mathrm{MS}}$ scheme:

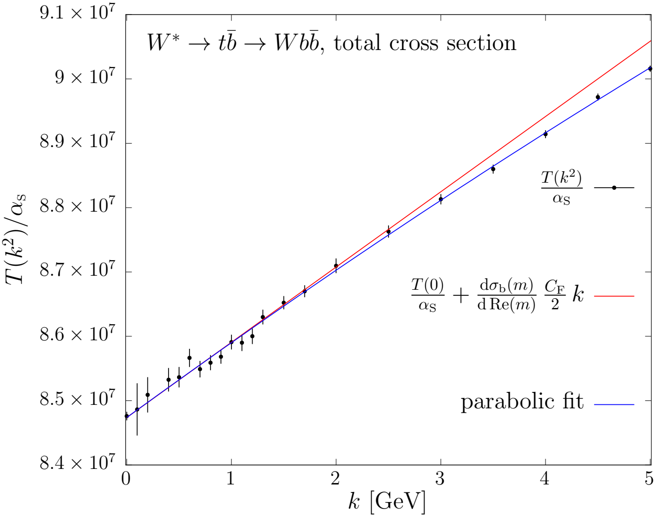
$$\langle O \rangle_{\mathrm{b}}(m, m^*) = \langle O \rangle_{\mathrm{b}}(\overline{m}, \overline{m}^*) + \left\{ \frac{\partial \langle O \rangle_{\mathrm{b}}(\overline{m}, \overline{m}^*)}{\partial \overline{m}} (m - \overline{m}) + \mathrm{cc} \right\}$$

For the leading renormalon this amounts to

$$\widetilde{T}(k^2) \to \widetilde{T}(k^2) - \frac{\partial \langle O \rangle_{\mathrm{b}}(\overline{m}, \overline{m}^*)}{\partial \mathrm{Re}(\overline{m})} \frac{C_{\mathrm{F}} \alpha_s}{2} k + \mathcal{O}(k^2) \,.$$

Selected Results

Total cross section



No linear renormalon in $\overline{\mathrm{MS}}$ scheme!

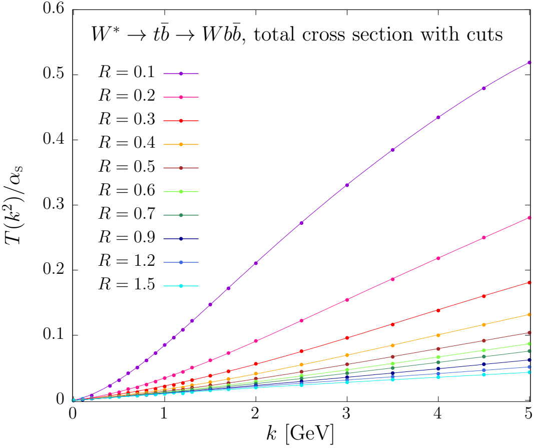
Total cross section

- For $k < \Gamma$: no renormalon in the physics! The top finite width screens the soft sensitivity of the cross section.
  The renormalon is there only if it is present in the mass counterterm; thus, it is not there in the $\overline{\mathrm{MS}}$ scheme.
- What about $k \gg \Gamma$?
  This is the narrow width limit: the cross section factorizes into a production cross section and a partial width.
  The former has no physical renormalons for obviour reasons.
  The latter is known not have them (Bigi, Zakharov, Veinshtein, Braun, Beneke...)

So, the mass from the **total** σ is free of linear power corrections, and even so in the narrow width limit.

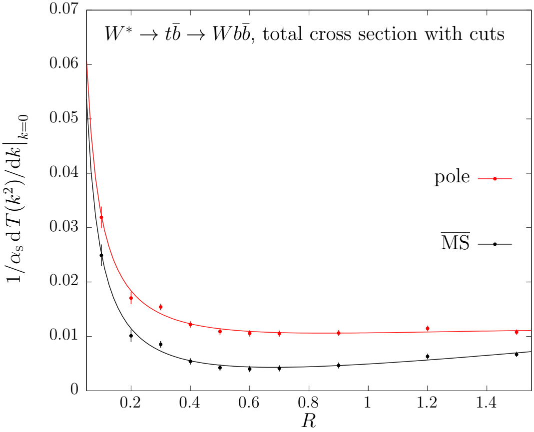
Total cross section with cuts

Cuts: two separated b-jets, energy > 30 GeV:



Well-known $1/R$ behaviour of renormalon due to jets!
(Dokshitzer, Marchesini, Salam, Dasgupta, etc.)

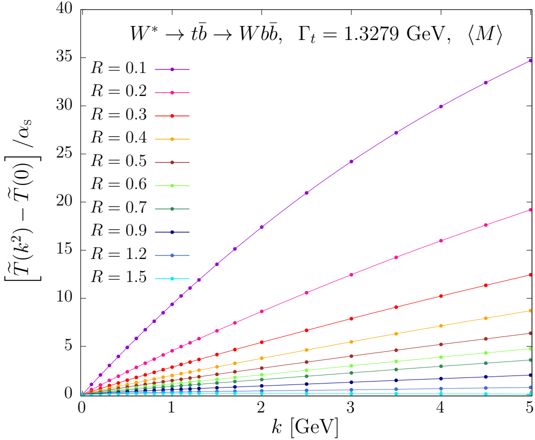
Total cross section with cuts



The requirement of *b* jets spoils it!

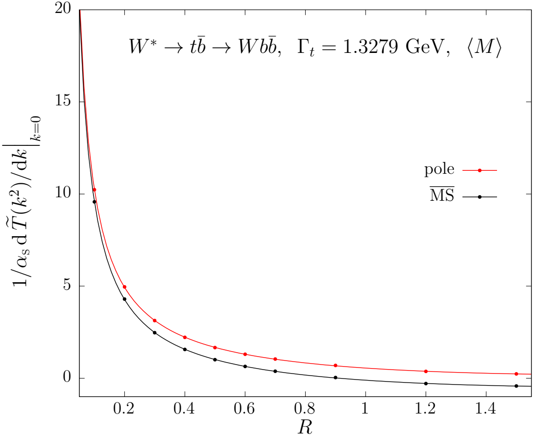
($\overline{\mathrm{MS}}$ still better, but here the acceptance is very large!)

Reconstructed top mass



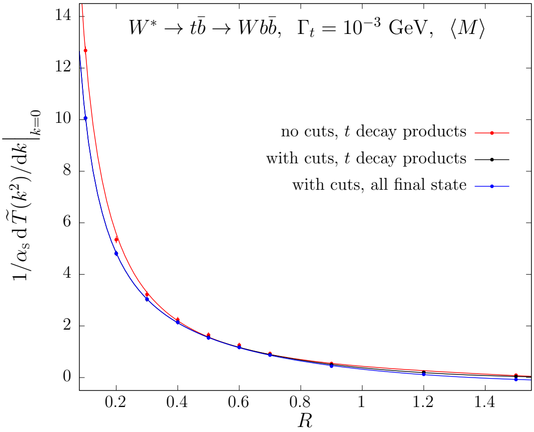
Characteristic $1/R$ behaviour of jet renormalon quite evident.

Reconstructed top mass



For large radii, m_{pole} is better!

$\Gamma \to 0$ limit: MC truth for t decay products

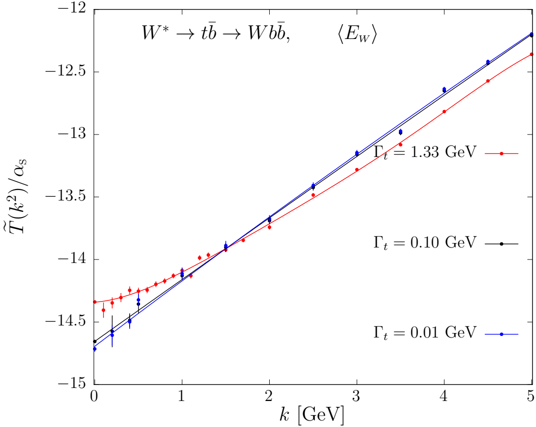


For large radii, when imposing our cuts, the reconstructed mass with the top decay products only is very close to the one with all particles, and become essentially the top pole mass!

No renormalon in the pole mass scheme!

Leptonic Observables

Choose as mass sensitive "leptonic" observable the average E_W.



For $k \gg \Gamma$, the slope is roughly 0.45.
The $\overline{\mathrm{MS}}$ conversion would add -0.067.

Leptonic Observables

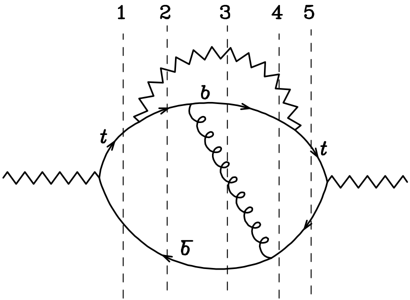
It seems that physical linear renormalons are present also in leptonic observables.

But, for $k \ll \Gamma$, the slope of $T(k)$ decreases, approaching 0.067!

So, the top finite width screens the linear renormalons!

Is this an exact statement?

Last page of the proof ...



Only 2,3,4 cuts should be considered. But:
1 and 5 denominators have opposite imaginary part of order Γ.

$$\mathrm{Im}\left[\frac{1}{E-E_W-E_{b,2}-E_{\bar{b},1}+i\epsilon}\,\frac{1}{E-E_W-E_{b,3}-E_{\bar{b},1}-E_{g,3}+i\epsilon}\,\frac{1}{E-E_W-E_{b,3}-E_{\bar{b},4}+i\epsilon}\right]$$

Analyticity is still there, but the imaginary part of q^0 cannot exceed Γ. So: soft sensitivity higher than linear below Γ.

Perturbative expansion

With our framework we can also compute the coefficients of the perturbative expansion up to very high orders, by setting b_0 equal to its $N_C = 3$, $n_f = 5$ value. The growth of the terms of the expansion reflects the presence of IR renormalons.

Perturbative expansion: total σ, no cuts

	$\sigma/\sigma_{\mathrm{b}}^{\mathrm{nocuts}}(m)$			
	pole scheme		$\overline{\mathrm{MS}}$ scheme	
i	c_i	$c_i\,\alpha_{\mathrm{S}}^i$	c_i	$c_i\,\alpha_{\mathrm{S}}^i$
0	+1.0000000	+1.000000	+0.8684133	+0.868413
1	+0.5002870	+0.054112	+1.4802245	+0.160106
2	−0.6198864	−0.007252	+0.4415864	+0.005166
3	−3.0250561	−0.003828	+0.6438394	+0.000814
4	−12.456119	−0.001704	+0.0291536	+0.000003
5	−63.620599	−0.000941	+0.7610144	+0.000011
6	−388.55285	−0.000622	−1.5072229	−0.000002
7	−2873.0283	−0.000497	−11.064898	−0.000001
8	−24596.138	−0.000460	−238.54526	−0.000004
9	−240532.31	−0.000487	−2258.4625	−0.000004
10	−2643353.1	−0.000579	−28947.803	−0.000006

Table 3: Coefficients of the α_{S} expansion of the inclusive cross section to all orders, computed in the large n_f limit, normalized to the Born cross section computed in the pole-mass scheme.

Perturbative expansion: total σ, with cuts

	$\sigma/\sigma_b^{\mathrm{nocuts}}(m)$ $R = 0.1$			
	pole scheme		$\overline{\mathrm{MS}}$ scheme	
i	c_i	$c_i\,\alpha_S^i$	c_i	$c_i\,\alpha_S^i$
0	+0.9985836	+0.998583	+0.8666708	+0.866670
1	−0.7352455	−0.079526	+0.2450440	+0.026504
2	−6.1726923	−0.072216	−5.1103336	−0.059787
3	−29.300841	−0.037078	−25.629587	−0.032432
4	−144.25410	−0.019744	−131.75934	−0.018034
5	−765.70258	−0.011336	−701.27841	−0.010382
6	−4381.0850	−0.007015	−3993.7638	−0.006395
7	−27441.415	−0.004753	−24577.501	−0.004257
8	−192222.83	−0.003601	−167848.28	−0.003144
9	−1548935.0	−0.003138	−1310497.1	−0.002655
10	−14627450.	−0.003206	−12011235.	−0.002632

	$\sigma/\sigma_b^{\mathrm{nocuts}}(m)$ $R = 0.5$			
	pole scheme		$\overline{\mathrm{MS}}$ scheme	
i	c_i	$c_i\,\alpha_S^i$	c_i	$c_i\,\alpha_S^i$
0	+0.9783310	+0.978331	+0.8511828	+0.851182
1	−0.0461524	−0.004992	+0.8972327	+0.097048
2	−2.5355012	−0.029663	−1.5205011	−0.017788
3	−10.015854	−0.012674	−6.4980027	−0.008222
4	−39.227713	−0.005369	−27.276361	−0.003733
5	−174.15102	−0.002578	−112.43573	−0.001664
6	−900.21795	−0.001441	−529.47026	−0.000847
7	−5657.0028	−0.000979	−2914.3719	−0.000504
8	−43185.123	−0.000809	−19847.983	−0.000371
9	−394759.48	−0.000799	−166443.38	−0.000337
10	−4194491.5	−0.000919	−1689471.7	−0.000370

Table 4: Coefficients c_i (see eq. (7.7)) of the α_S expansion of the cross section with cuts, to all orders, computed in the large n_f limit, normalized to the inclusive Born cross section computed in the pole-mass scheme, for two different values of the jet radius ($R = 0.1$ in the left pane and $R = 0.5$ in the right one).

Perturbative expansion: reconstructed mass

	$R = 0.1$		$R = 0.5$		$R = 1.5$	
i	pole	$\overline{\text{MS}}$	pole	$\overline{\text{MS}}$	pole	$\overline{\text{MS}}$
0	+172.828	+163.014	+172.820	+163.004	+172.753	+162.924
1	−7.5974	+0.2162	−2.7850	+5.0300	+0.4446	+8.2679
2	−4.1355	−2.8521	−1.2548	+0.0287	+0.1029	+1.3874
3	−2.3972	−1.9729	−0.5964	−0.1720	+0.0136	+0.4384
4	−1.5054	−1.3367	−0.3126	−0.1439	−0.0059	+0.1628
5	−1.0380	−0.9499	−0.1880	−0.0998	−0.0096	+0.0786
6	−0.7943	−0.7350	−0.1326	−0.0733	−0.0105	+0.0488
7	−0.6792	−0.6327	−0.1094	−0.0629	−0.0112	+0.0353
8	−0.6513	−0.6081	−0.1040	−0.0608	−0.0124	+0.0308
9	−0.6994	−0.6538	−0.1121	−0.0665	−0.0146	+0.0308
10	−0.8367	−0.7826	−0.1348	−0.0807	−0.0185	+0.0356

Table 5: Values of the $c_i\,\alpha_s^i$ terms of the perturbative expansion for the average value of the reconstructed-top mass, defined in eq. (7.8), for three different jet radii in the pole-mass and $\overline{\text{MS}}$-mass scheme.

Perturbative expansion: average E_W

	$\langle E_W \rangle$			
	pole scheme		$\overline{\text{MS}}$ scheme	
i	c_i	$c_i\,\alpha_s^i$	c_i	$c_i\,\alpha_s^i$
0	+121.5818712	+121.581	+120.8654238	+120.865
1	−14.34912289	−1.55205	−7.191792655	−0.77789
2	−49.71450711	−0.58162	−38.81964324	−0.45416
3	−178.6769037	−0.22610	−145.3978441	−0.18399
4	−689.9546087	−0.09443	−567.6082924	−0.07769
5	−2942.051619	−0.04355	−2350.851479	−0.03480
6	−14036.55837	−0.02247	−10359.68239	−0.01658
7	−75575.90816	−0.01309	−48926.90177	−0.00847
8	−465376.2431	−0.00871	−236362.5073	−0.00442
9	−3346819.349	−0.00678	−1117788.171	−0.00226
10	−28704446.32	−0.00629	−4189250.920	−0.00091
11	−295144268.5	−0.00699	+3783661.054	+0.00008

Table 6: Coefficients of the perturbative expansion of the average W-boson energy in the pole and $\overline{\text{MS}}$ schemes (see eq. (7.10)) .

Factorial growth also in $\overline{\text{MS}}$ up to the fourth-fifth order ...

What have we learned?

Mass measurement from the total cross section

Mass measurements from the total cross section are sometimes advertized as capable of accessing directly a short distance mass, bypassing the problem of the pole-mass renormalon. Our model calculation does not contradict this assumption. However:
As soon as jets are used to tag top quarks, renormalons are unavoidably introduced, and have a large impact even for considerably large acceptances.
Can we tag on leptons only?

Conclusions

Mass from direct measurements

The mass extracted from a reconstructed top is plagued by renormalons due to jets. These become less and less important for large jet radius, and, aside from the jet renormalon, address more naturally the top pole mass.
(Of course, at hadron colliders increasing the jet radius leads to more problems from radiation from ISR or other sources entering the *b*-jet cone, etc.)

Prospects: we can study jet calibration in this framework ...

Conclusions

Mass from leptonic observables

As we get rid of jets in this case, thanks to the top finite width, renormalon effects are screened. No renormalons in the physics (unless one introduces them insisting upon using the pole mass). However, the screening operates for scales below Γ_t. So, the factorial growth is still visible up to an order n such that $m_t e^{-n} \approx \Gamma_t$. This corresponds roughly to the fourth or fifth order.

Although it is unlikely that even 10-20 years from now we reach that level of accuracy in perturbative calculation, it is not absolutely impossible that we will learn how to estimate the renormalon dominated terms ...

Meta-Conclusions

The work presented here is considerably different than typical theoretical efforts for improving the accuracy of our modeling of LHC top physics. We are not proposing better and more refined calculations of top observables, neither we have addressed any urgent need in current top physics issues.

It should however be put in perspective. We have in front of us decades of LHC physics, and very challenging theoretical problems. Taking some steps back to basic issues may give us new ideas and insights for dealing with them.