πN P-wave resonant scattering from lattice QCD

Srijit Paul

Scattering from the lattice: applications to phenomenology and beyond.

Dublin, May 14, 2018

Overview

Aspects of π-N study

- $\pi\pi$ study

π-N study

Expected Outcomes

- $\pi - N$ results

section 1

Aspects of π-N study

- $\pi\pi$ study

π-N study

Expected Outcomes

- $\pi - N$ results

$\pi\pi$ Calculation

Gauge Ensemble Setup

- $N_f = 2 + 1$ Clover fermions.
- isotropic lattice.$(32^3 \times 96)$
- $m_\pi L = 5.865(32)$
- m_π is low enough: ρ is unstable.

a(fm)	L(fm)	m_π(MeV)	m_K(MeV)	N_{config}
$0.11403(77)$	$3.649(25)$	317	530	1041

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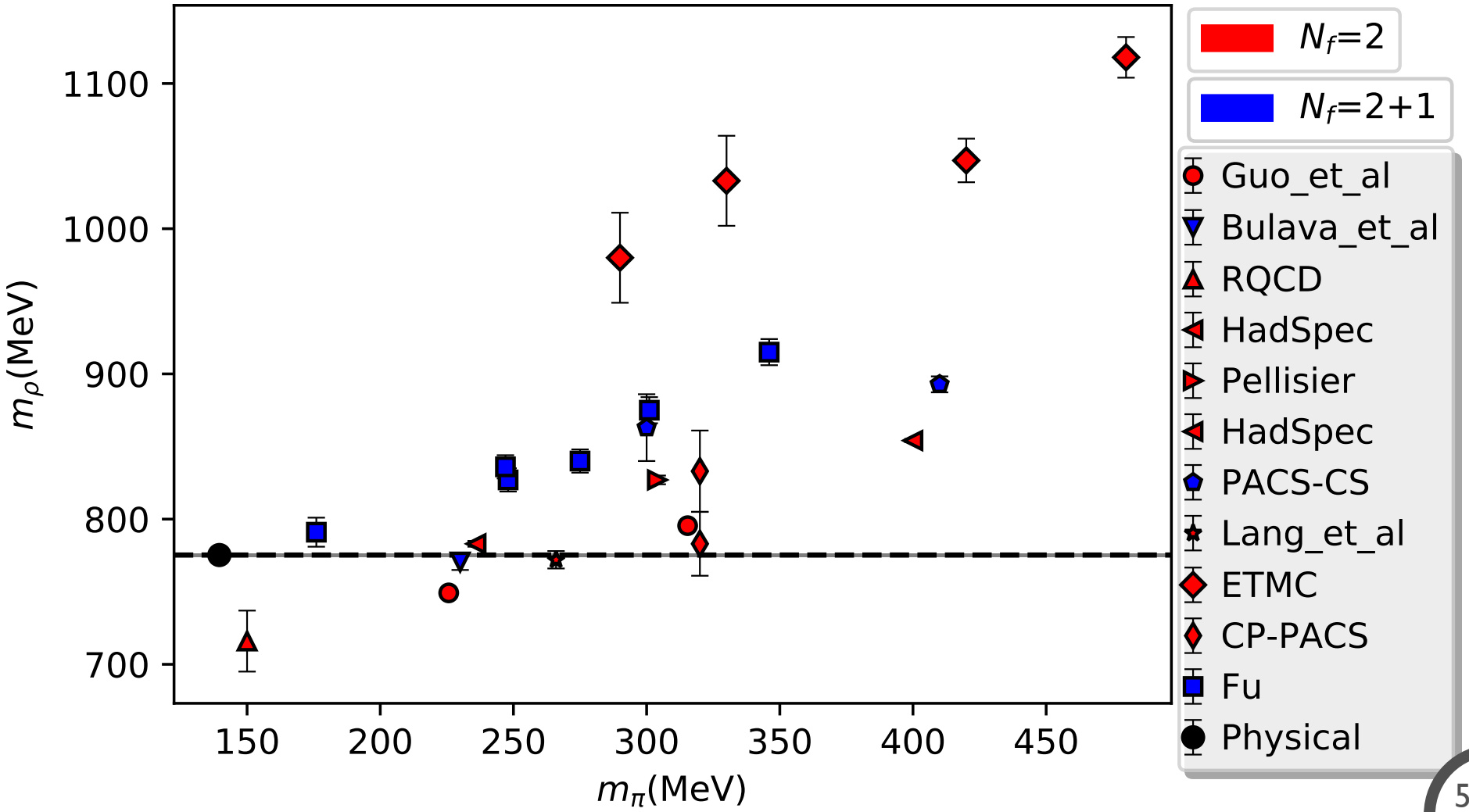
P -wave $\pi\pi$ scattering and the ρ resonance from lattice QCD authored by C. Alexandrou, L. Leskovec, S. Meinel, J. Negele, S. Paul, M. Petschlies, A. Pochinsky, G. Rendon, S. Syritsyn.(arXiv:1704.05439v2 [hep-lat])

$\pi\pi$ Calculation

Our results in modern context

ρ meson mass comparison



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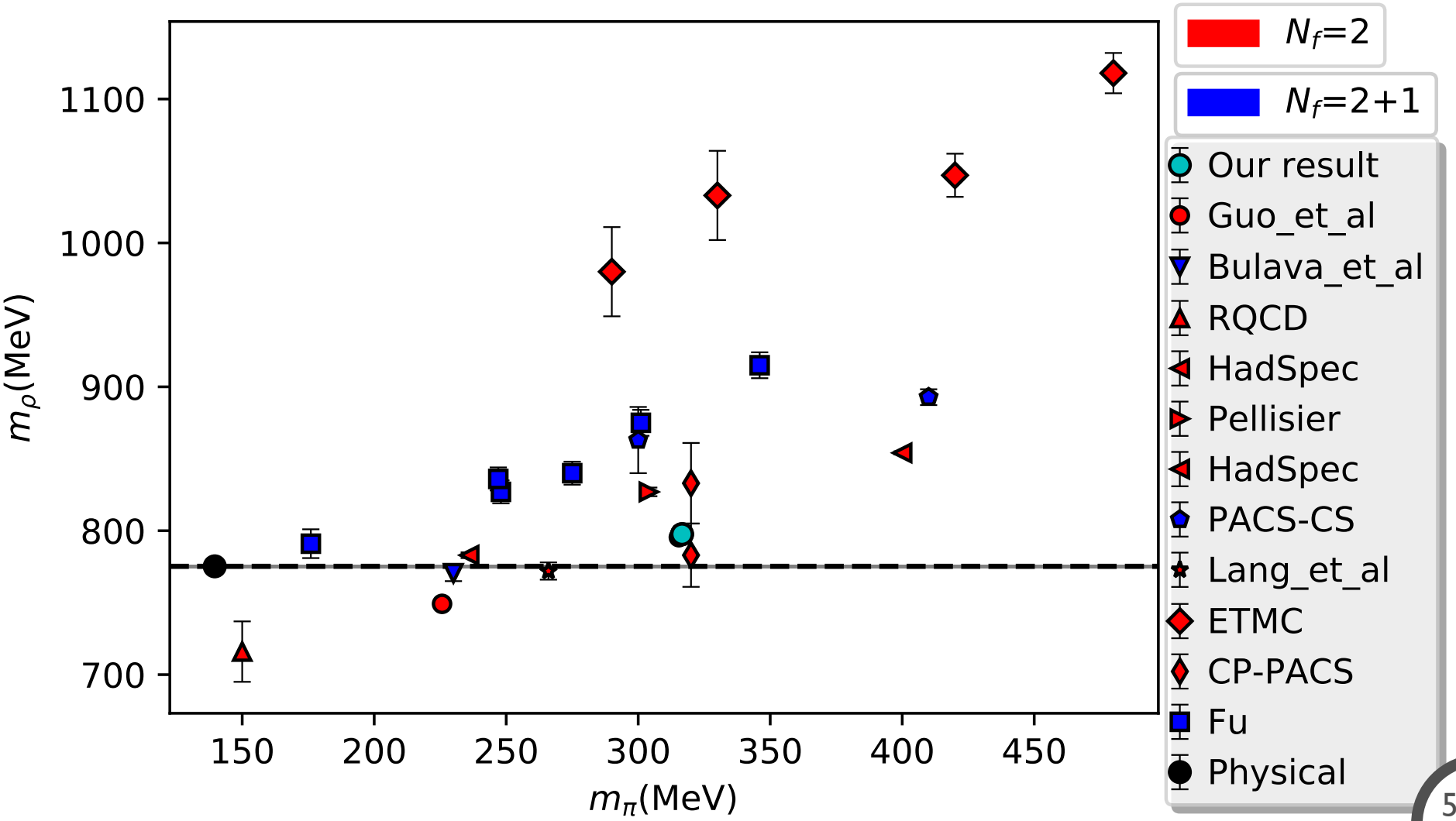

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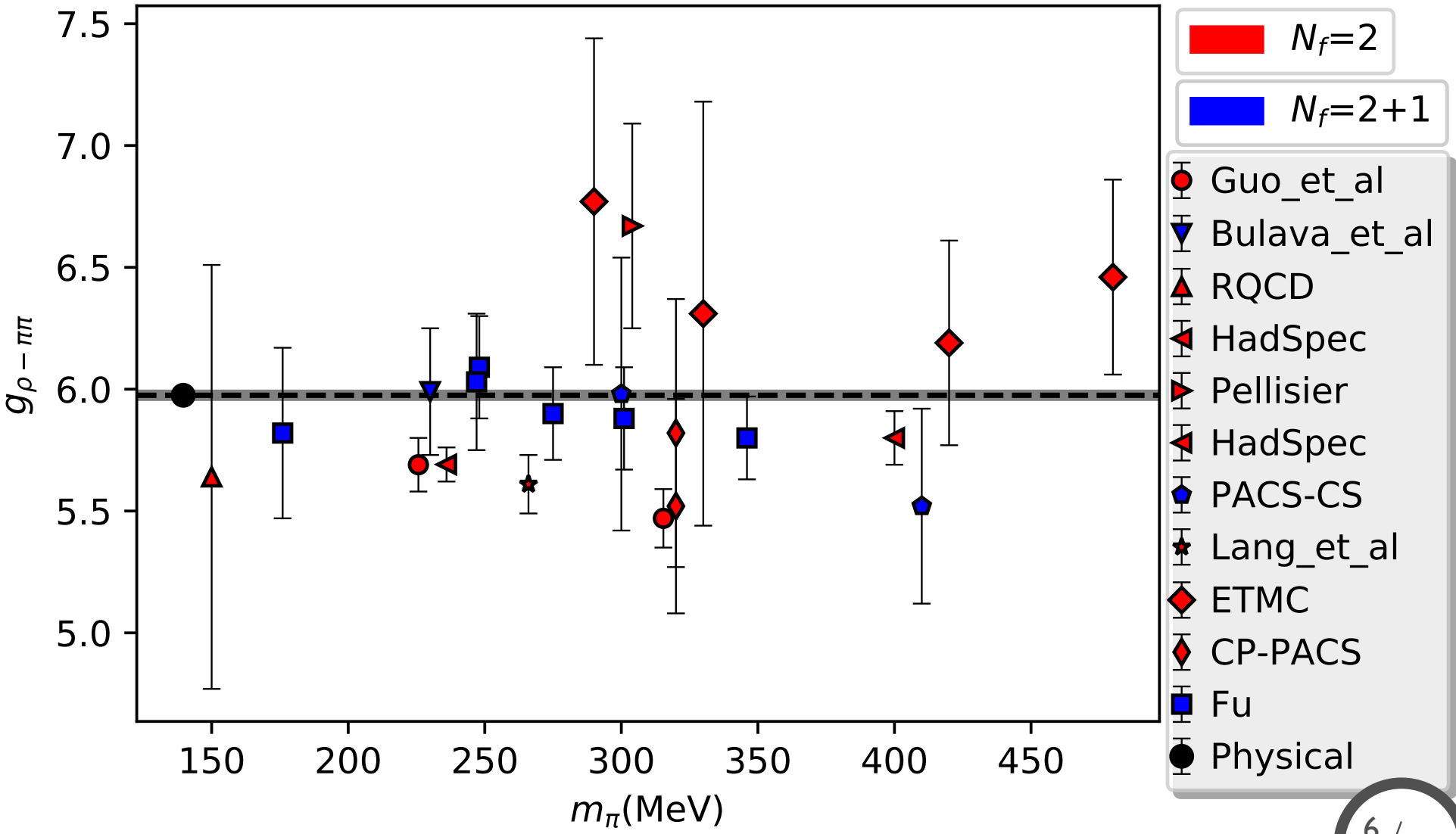
$g_{\rho-\pi\pi}$ coupling comparison



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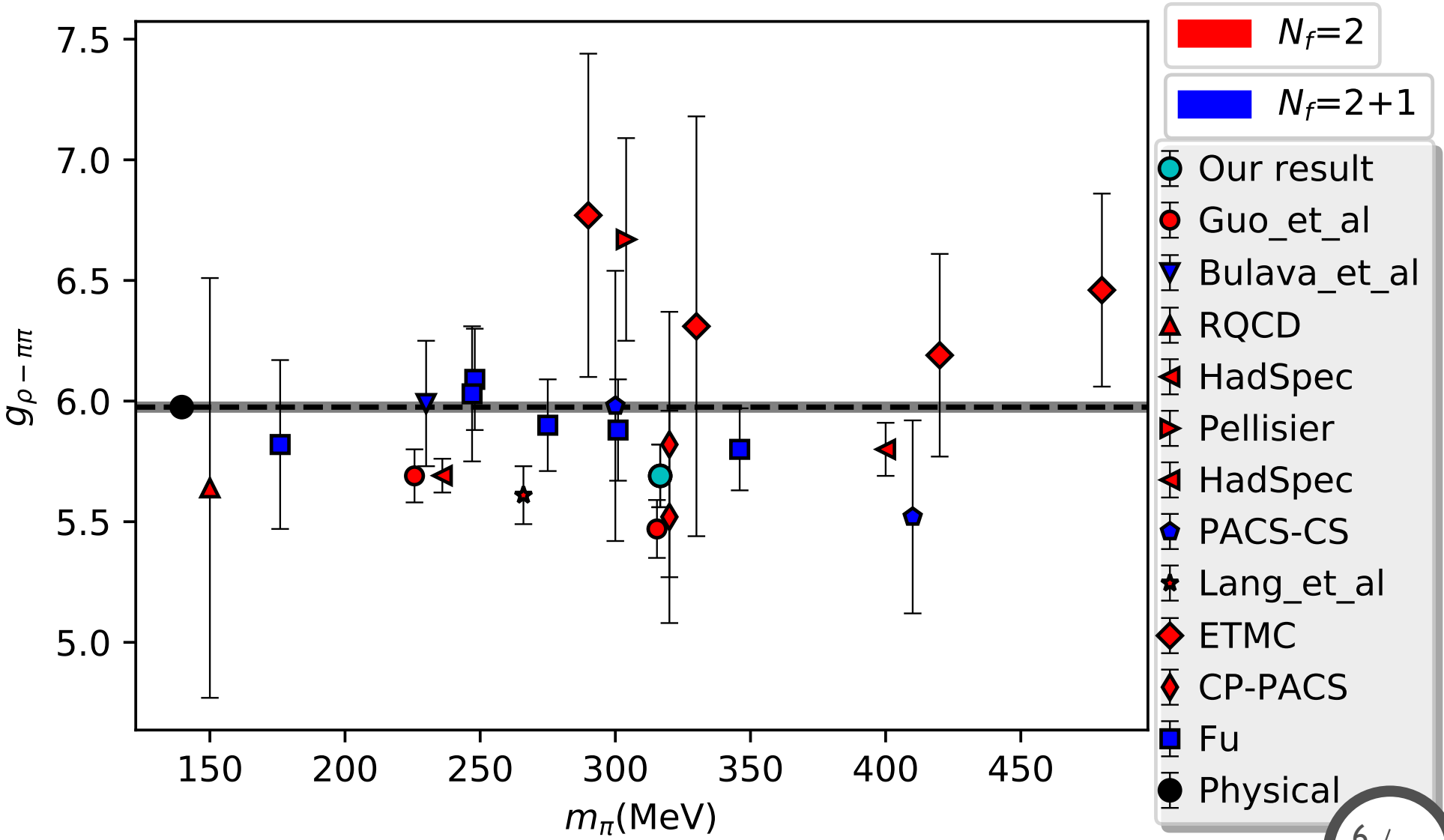

section 2

Brief History

- First calculation setup using Lüscher method, and some unpublished results.($m_\pi \approx 250$ MeV)

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- Elastic $I = 3/2$ p-wave resonance, using Lüscher method, with distillation.($m_\pi \approx 280$ MeV)

  [Andersen(2017)]

π-N Calculation

Lüscher Analysis

Quantization condition For Baryons

$$\det\left(M^{\Delta}_{Jl\mu,J'l'\mu'} - \delta_{JJ'}\delta_{ll'}\delta_{\mu\mu'} \cot \delta_{Jl} \right) = 0$$

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π-N Calculation

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$$M^{\Delta}_{Jl\mu,J'l'\mu'} = \sum_{\substack{m,\sigma \\ m',\sigma'}} \left\langle lm, \frac{1}{2}\sigma \middle| J\mu \right\rangle \left\langle l'm', \frac{1}{2}\sigma' \middle| J'\mu' \right\rangle M^{\Delta}_{lm,l'm'}$$

where

$$M^{\Delta}_{lm,l'm'} = \frac{(-1)^l \gamma^{-l}}{\pi^{3/2}} \sum_{j=|l-l'|}^{l+l'} \sum_{s=-j}^{j} \frac{i^j}{q^{j+1}} Z_{js}(1,q^2) C_{lm,js,l'm'}$$

π-N Calculation

Common Problems

- The exponential degradation in the S-n ratio.
- The additional valence quark $\Uparrow$ Wick contractions, $\Uparrow$ computational and storage costs.

π-N Calculation

Gauge Ensemble Setup

- $N_f = 2 + 1$ Clover fermions.
- isotropic lattice.($24^3 \times 48$)
- $m_\pi L = 3.6$
- m_π is low enough: Δ is unstable.

a(fm)	L(fm)	m_π(MeV)	m_N(GeV)	N_{config}
0.116	2.8	254(1)	1.072(7)	600

π-N Calculation

Lattice setup for scattering

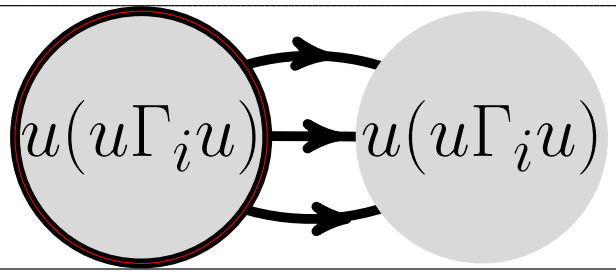
P	Little group	Irreps that contains the Δ (dim)	Spin content J
$(0,0,0)$	O_h^D	$H_g(4)$	$H_g = 3/2, 5/2..$
$(0,0,1)$	C_{4v}^D	$G_1(2) \oplus G_2(2)$	$G_1 = 1/2, 3/2, ...$ $G_2 = 3/2, 5/2, ...$
$(0,1,1)$	C_{2v}^D	$2G(2)$	$G = 1/2, 3/2, ...$
$(1,1,1)$	C_{3v}^D	$F_1(1) \oplus F_2(1) \oplus G(2)$	$F_1 = 3/2, 5/2, ...$ $F_2 = 3/2, 5/2, ...$ $G = 1/2, 3/2, ...$

Specific Problem

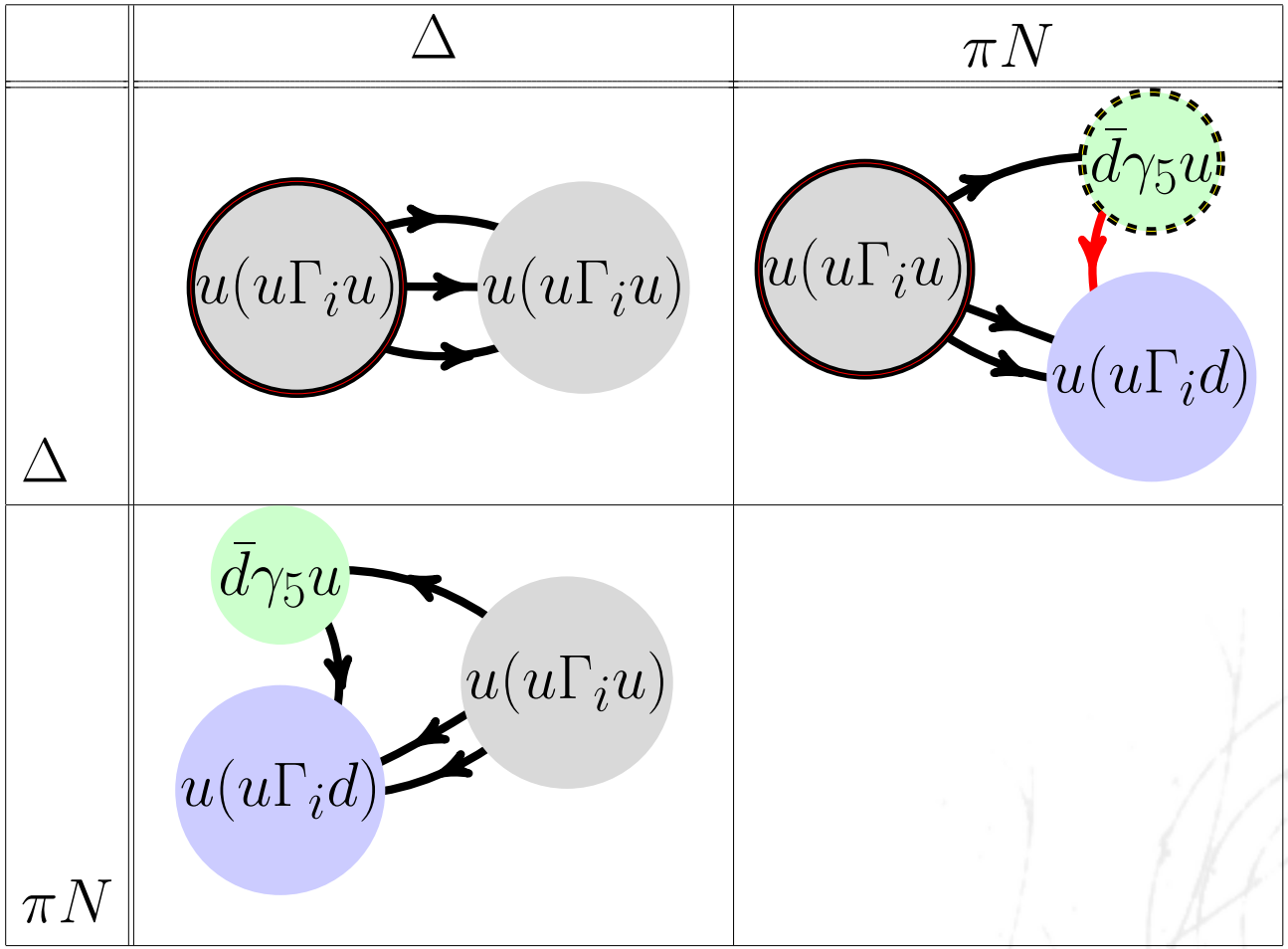
- Mixing of S and D wave channel to the $\Delta(1232)$ channel

[Addressed in Goeckler 2012, Roper 1965]

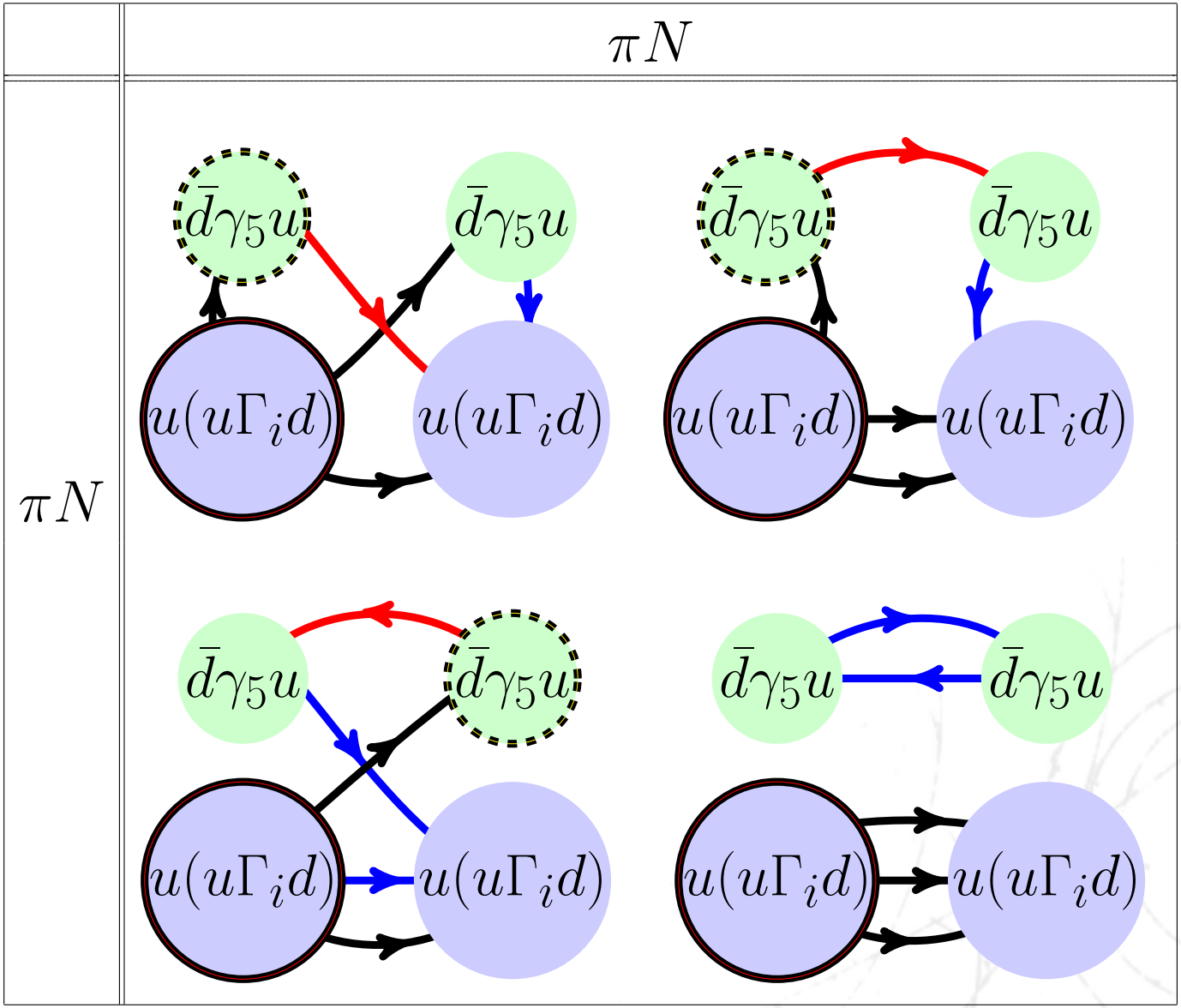
Wick contractions

	Δ	$N\pi$
Δ		
$N\pi$		

Wick contractions



$N\pi - N\pi$ contraction



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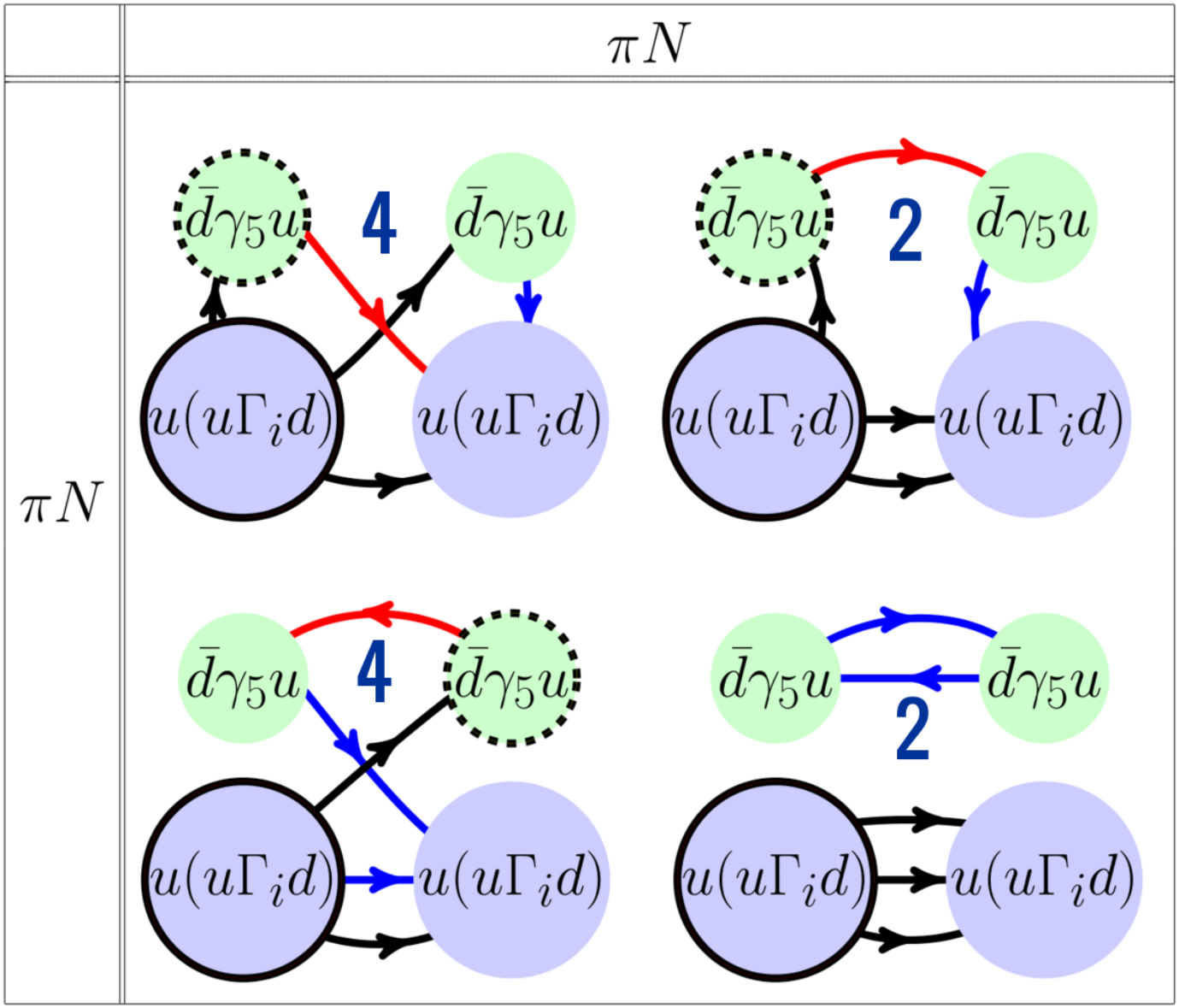

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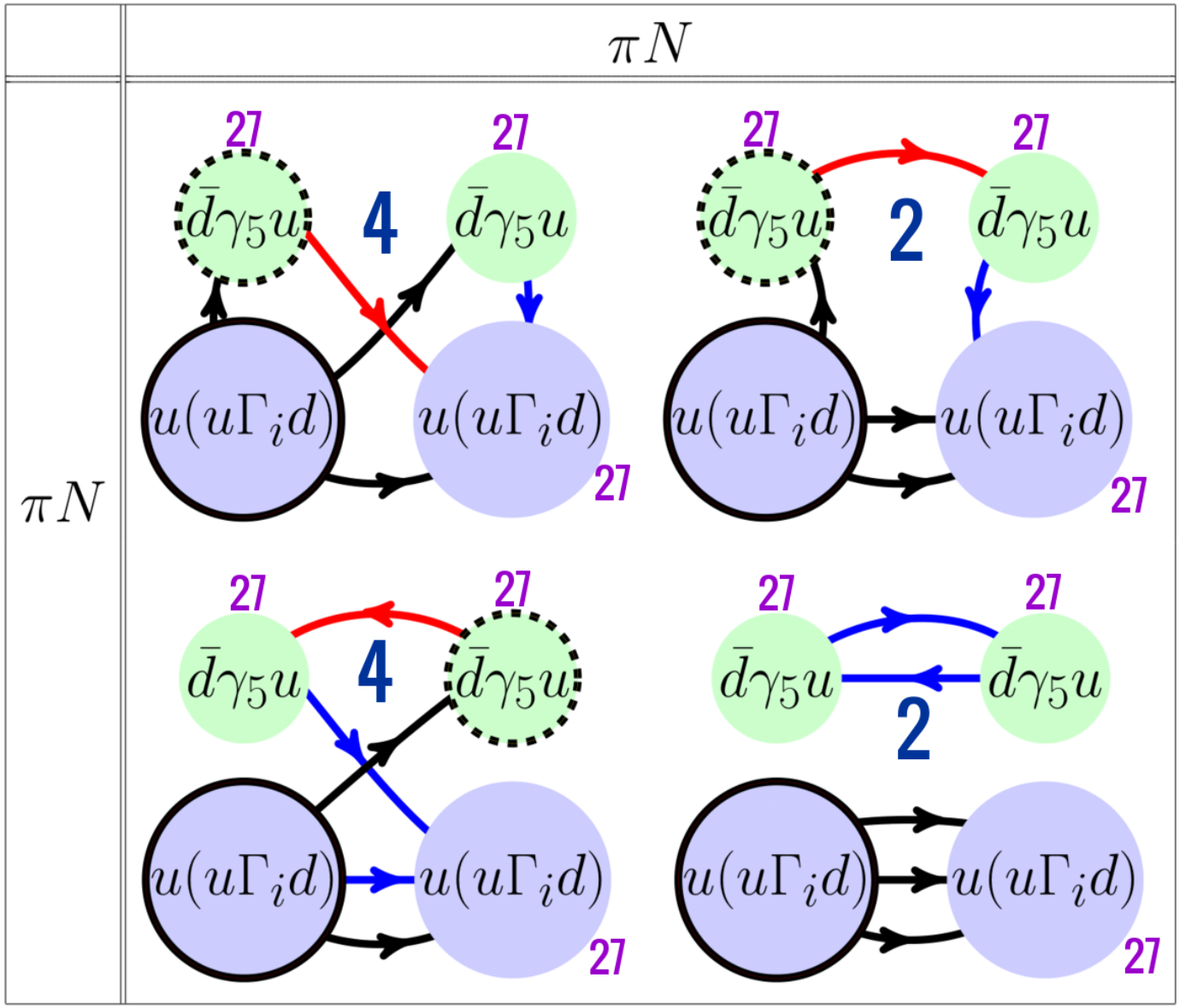

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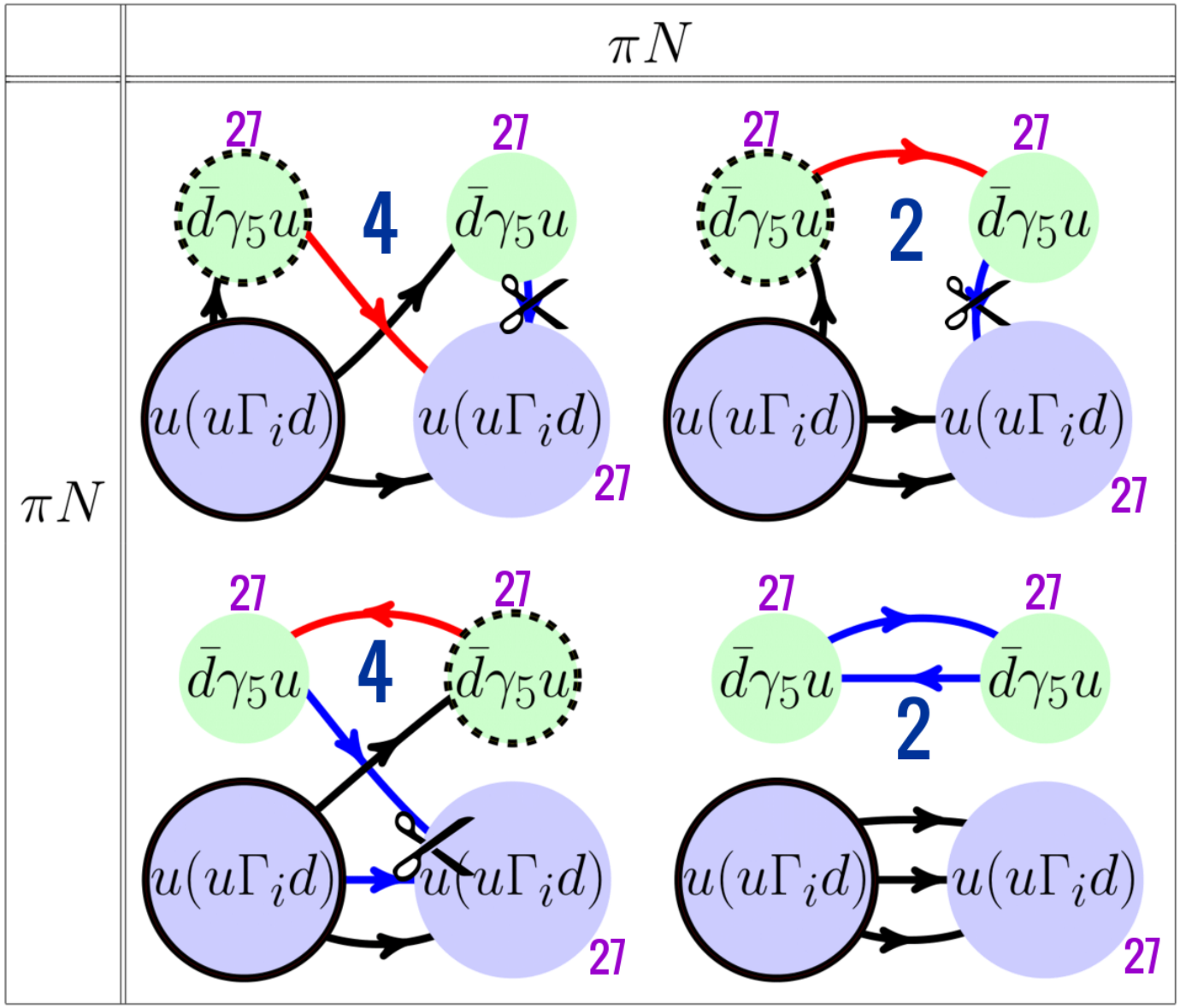

Delta and Nucleon Interpolators

$$\chi_{N1}(x) = \epsilon_{abc} \left[u_a^T(x)\, C\gamma_5\, d_b(x) \right] u_c(x)\,, \tag{1}$$

$$\chi_{N2}(x) = \epsilon_{abc} \left[u_a^T(x)\, C\, d_b(x) \right] \gamma_5 u_c(x)\,, \tag{2}$$

$$\chi_{N3}(x) = \epsilon_{abc} \left[u_a^T(x)\, C\gamma_5\, \gamma_t\, d_b(x) \right] u_c(x) \tag{3}$$

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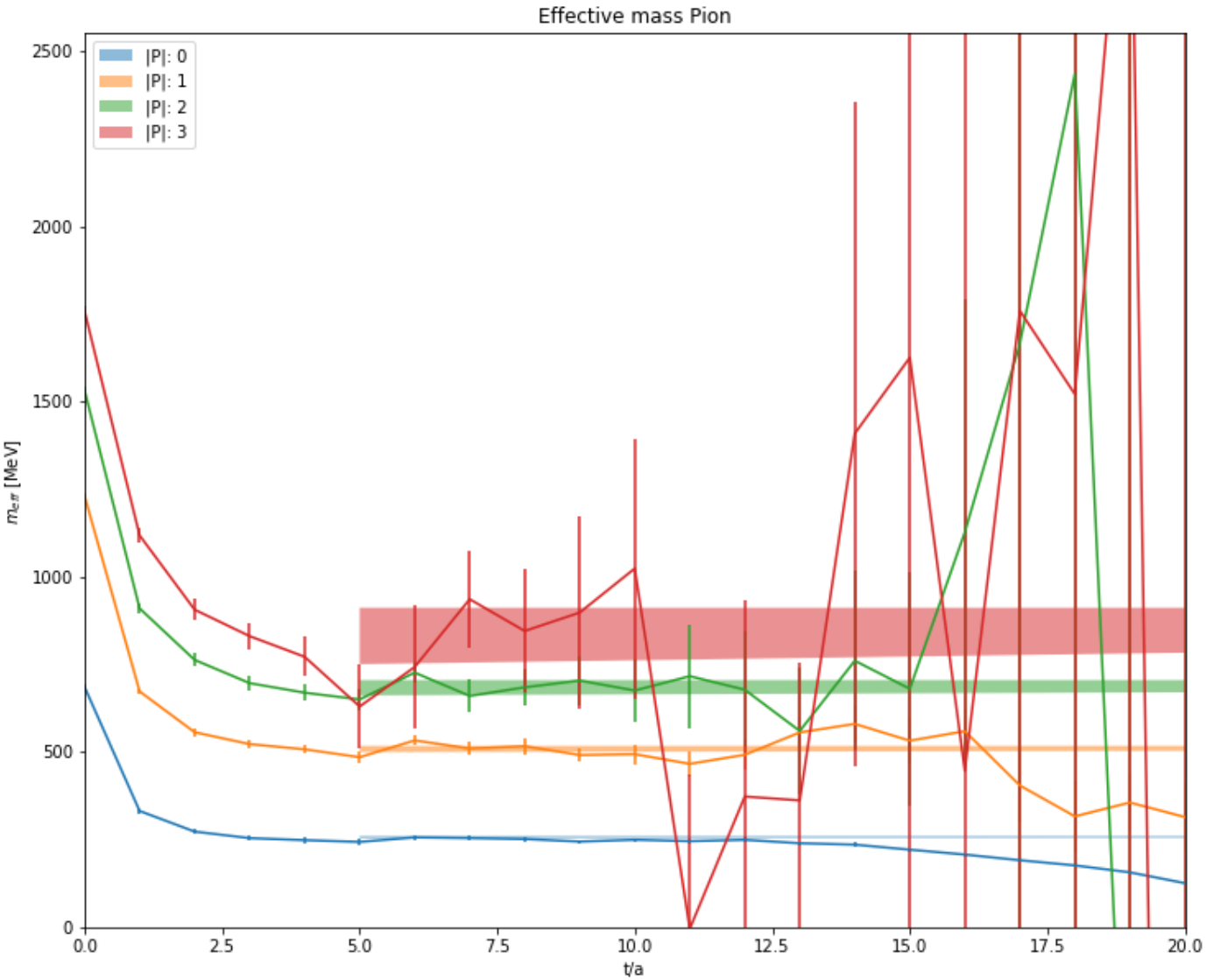
$$\chi_{\Delta 1}(x) = \epsilon_{abc} \left[u_a^T(x)\, C\gamma_\mu\, u_b(x) \right] u_c(x) \tag{4}$$

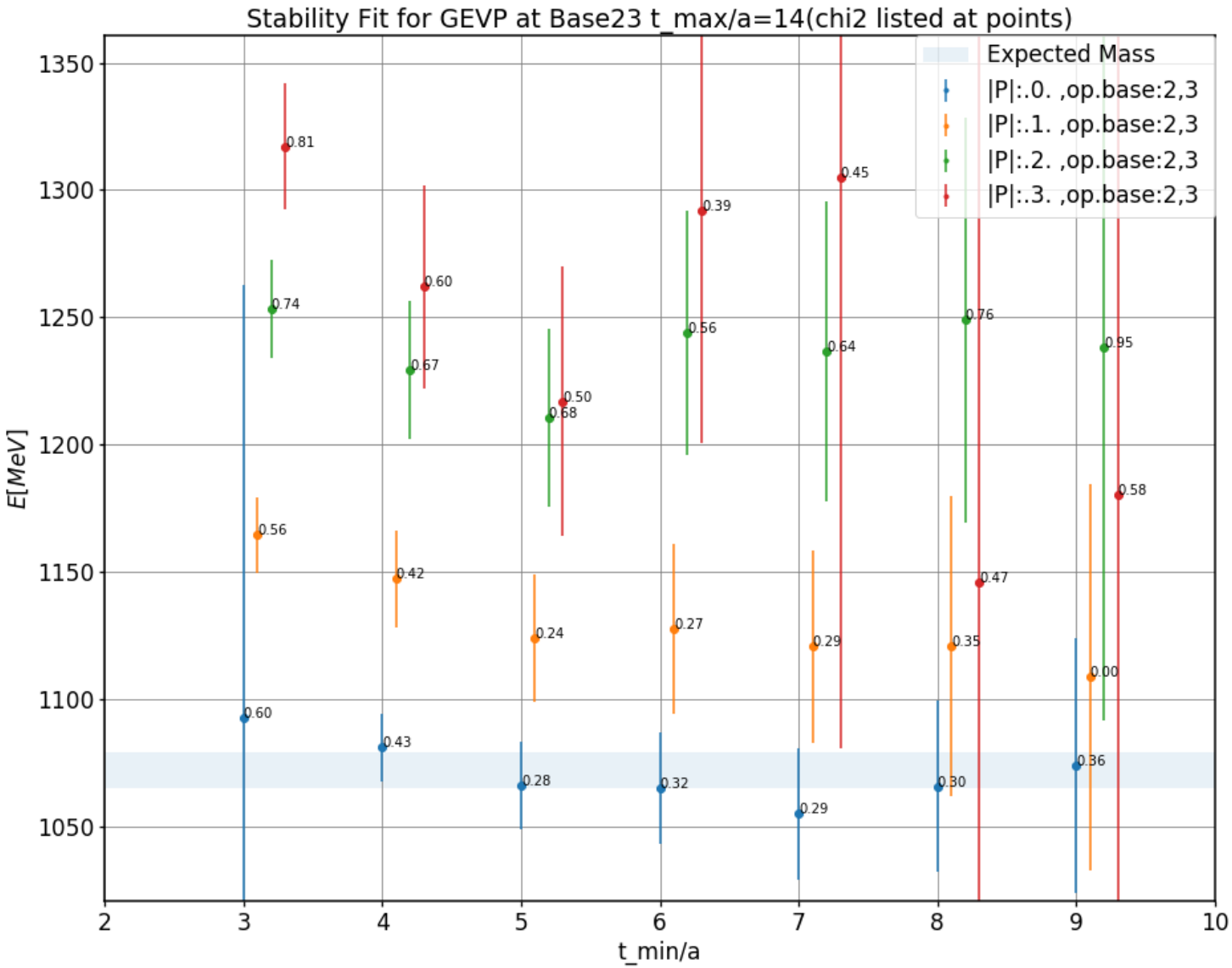
$$\chi_{\Delta 2}(x) = \epsilon_{abc} \left[u_a^T(x)\, C\gamma_\mu\gamma_t\, u_b(x) \right] u_c(x)\,, \tag{5}$$

$$\chi_{\Delta 3}(x) = \epsilon_{abc} \left[u_a^T(x)\, C\gamma_\mu\gamma_t\gamma_5\, u_b(x) \right] \gamma_5 u_c(x) \tag{6}$$

Pion energy spectrum



Proton energy spectrum



section 3

Aspects of π-N study

$\pi\pi$ study

π-N study

Expected Outcomes

$\pi - N$ results

π-N status and results

- Correlation functions have been computed for each Irrep.
- To do a GEVP on the correlation matrices for each Irrep.
- Using the Lüscher quantization condition, we need to extract to the scattering phase shifts for irrep and moving frame.
- Using the Breit Wigner parametrization of the decay width, we need to extract the mass of $\Delta(1232)$ and the $g_{\Delta-N\pi}$
- Using the Briceno-Hansen-Walker-Loud formalism for extract scattering matrix elements of $N\pi \to N\gamma$.

Thank you