Hadron Scattering with Elongated Boxes

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Scattering from the Lattice: Applications to Phenomenology and Beyond 2018

Outline

The Research Team

- Lattice QCD
  - Andrei Alexandru, Frank Lee
  - Craig Pelissier, Mike Lujan, Dehua Guo, Chris Culver, Hossein Niyazi
- Phenomonology
  - Michael Doering
  - Maxim Mai, Raquel Molina
  - Bin Hu, Daniel Sadasivan
- Recent Papers
  - Guo et al. 1605.03993 (ρ)
  - Guo et al. 1803.02897 (σ)
  - Lee, Alexandru 1706.00262 (Lüscher in Elongated Boxes)

Methodology

- Construct operators
- Compute finite volume energy levels
- If elastic two particle scattering $\rightarrow$ Lüscher
- Otherwise parameterize scattering matrix
- Extract resonance parameter

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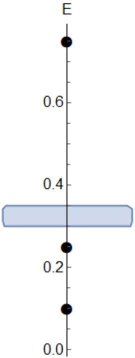
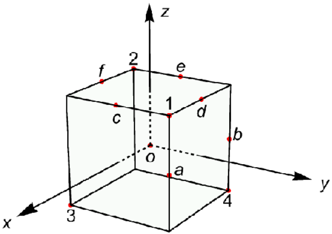
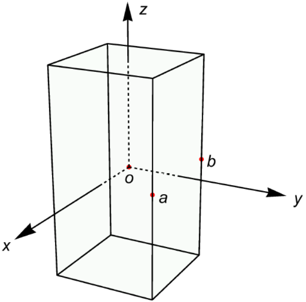


Ensembles

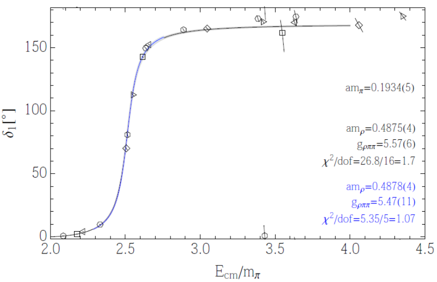


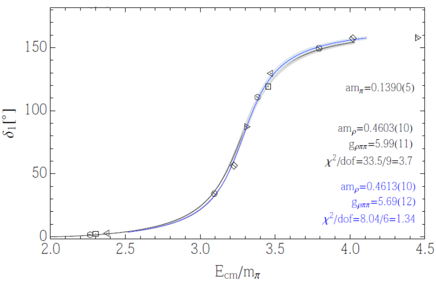


ensemble	$N_t \times N_{x,y}^2 \times N_z$	η	a[fm]	N_{cfg}	aM_π	am_N	$am_{u/d}^{pcac}$	af_π
$\mathcal{E}_1$	$48 \times 24^2 \times 24$	1.0	0.1210(2)(24)	300	0.1934(5)	0.644(6)	0.01237(9)	0.0648(8)
$\mathcal{E}_2$	$48 \times 24^2 \times 30$	1.25	–	–	–	–	–	–
$\mathcal{E}_3$	$48 \times 24^2 \times 48$	2.0	–	–	–	–	–	–
$\mathcal{E}_4$	$64 \times 24^2 \times 24$	1.0	0.1215(3)(24)	400	0.1390(5)	0.62(1)	0.00617(9)	0.060(1)
$\mathcal{E}_5$	$64 \times 24^2 \times 28$	1.17	–	–	–	–	–	–
$\mathcal{E}_6$	$64 \times 24^2 \times 32$	1.33	–	–	–	–	–	–

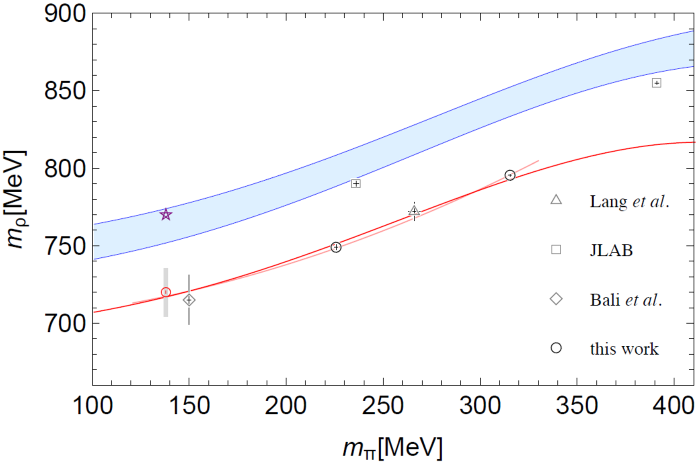
ρ Meson

ρ Phase Shifts



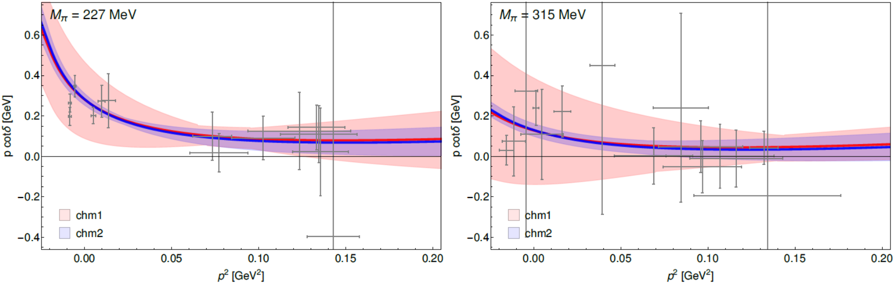


Physical Pion Mass Extrapolation

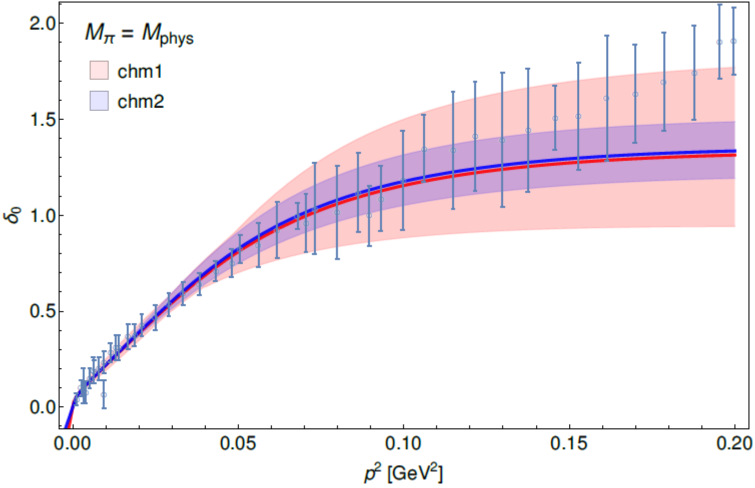


σ Meson

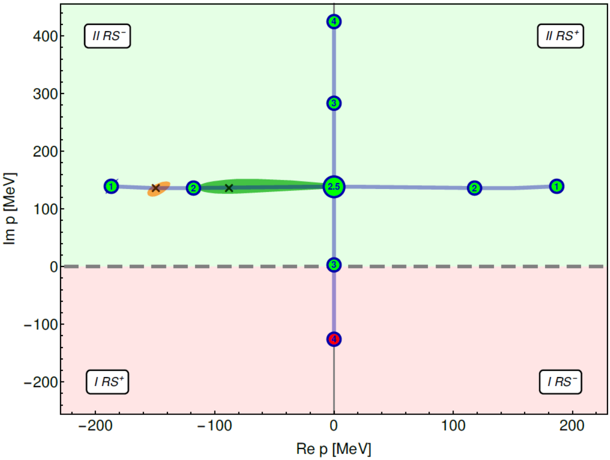
σ Phase Shifts



σ Phase Shifts



σ Resonance Position



Conclusion & Outlook

- Conclusion
  - Elongated boxes for cheap access to different momenta
  - Two flavor simulations are phenomenologically interesting
- Outlook for LQCD at GWU
  - Isoscalar channel - $f_0(980)$
  - $a_1(1260)$ with $\bar{q}q, \rho\pi, \sigma\pi, \pi\pi\pi$

	Lattice Result	Experimental Result
m_ρ	766(0.7)(11)	775.49(34)
Γ_ρ	150(0.4)(5)	146.2(7)
$Re(\sqrt{s})_\sigma$	440(13)(50)	400-550
$Im(\sqrt{s})_\sigma$	240(20)(25)	200-350

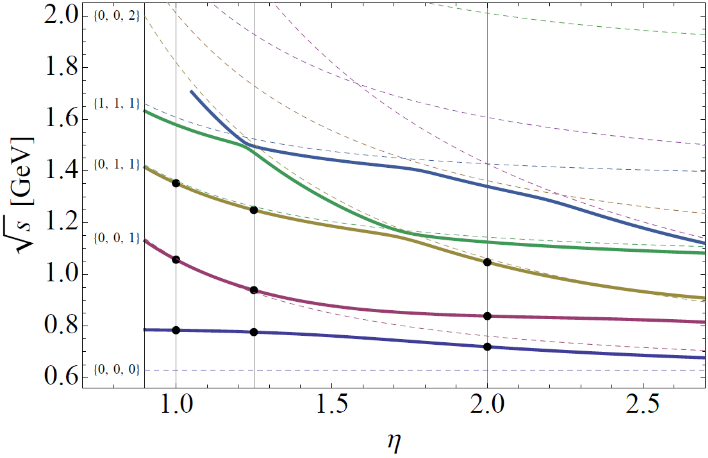
ρ Operator Basis

$$\rho^0(\Gamma_i(\boldsymbol{p}), t) = \frac{1}{\sqrt{2}}[\bar{u}(t)\Gamma_i(\boldsymbol{p})u(t) - \bar{d}(t)\Gamma_i(\boldsymbol{p})d(t)]$$

i	$\Gamma_i(\boldsymbol{p})$	$\Gamma_i'(\boldsymbol{p})$
1	$\gamma_3 e^{i\boldsymbol{p}}$	$\gamma_3 e^{-i\boldsymbol{p}}$
2	$\gamma_4\gamma_3 e^{i\boldsymbol{p}}$	$\gamma_4\gamma_3 e^{-i\boldsymbol{p}}$
3	$\gamma_3 \nabla_j e^{i\boldsymbol{p}} \nabla_j$	$-\gamma_3 \nabla_j e^{-i\boldsymbol{p}} \nabla_j$
4	$\frac{1}{2}\{e^{i\boldsymbol{p}}, \nabla_3\}$	$-\frac{1}{2}\{e^{-i\boldsymbol{p}}, \nabla_3\}$

$$\pi\pi(\boldsymbol{p}_1, \boldsymbol{p}_2) = \frac{1}{\sqrt{2}}\{\pi^+(\boldsymbol{p}_1)\pi^-(\boldsymbol{p}_2) - \pi^-(\boldsymbol{p}_1)\pi^+(\boldsymbol{p}_2)\}$$

ρ Energy Levels



σ Operator Basis

$$\sigma(\Gamma_i(\boldsymbol{p}),t) = \frac{1}{\sqrt{2}}[\bar{u}(t)\Gamma_i(\boldsymbol{p})u(t) + \bar{d}(t)\Gamma_i(\boldsymbol{p})d(t)]$$

i	$\Gamma_i(\boldsymbol{p})$	$\Gamma'_i(\boldsymbol{p})$
1	$\mathbf{1}e^{i\boldsymbol{p}}$	$\mathbf{1}e^{-i\boldsymbol{p}}$
2	$\nabla_i\mathbf{1}e^{i\boldsymbol{p}}\nabla_i$	$\nabla_i\mathbf{1}e^{-i\boldsymbol{p}}\nabla_i$
3	$\nabla_i^4\mathbf{1}e^{i\boldsymbol{p}}\nabla_i^4$	$\nabla_i^4\mathbf{1}e^{-i\boldsymbol{p}}\nabla_i^4$
4	$\gamma_i e^{i\boldsymbol{p}}\nabla_i$	$\gamma_i e^{-i\boldsymbol{p}}\nabla_i$

$$\pi\pi(\boldsymbol{p}_1,\boldsymbol{p}_2) = \frac{1}{\sqrt{3}}\{\pi^+(\boldsymbol{p}_1)\pi^-(\boldsymbol{p}_2) + \pi^-(\boldsymbol{p}_1)\pi^+(\boldsymbol{p}_2) + \pi^0(\boldsymbol{p}_1)\pi^0(\boldsymbol{p}_2)\}$$

σ Subtracting Vacuum

- Sigma has Quantum Numbers of Vacuum
- Need to remove vacuum contribution to correlators
- Direct Subtraction
- $\langle O(t_2) O^\dagger(t_1) \rangle_{\text{sub}} = \langle O(t_2) O^\dagger(t_1) \rangle - \langle O(t_2) \rangle \langle O^\dagger(t_1) \rangle$
- Implicit Subtraction
- $\tilde{C}_{ij}(t) = C_{ij}(t+d) - C_{ij}(t)$

σ Enery Levels