$K\pi$ scattering with partial wave mixing

Ruairí Brett

Carnegie Mellon University

May 15, 2018

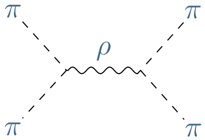
Motivations & Overview

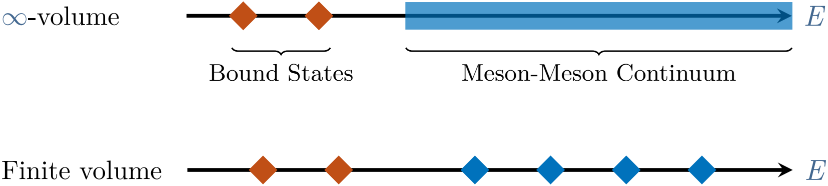
- use $2 \to 2$ Lüscher formalism to calculate hadron-hadron scattering amplitudes
- recent applications of *box matrix* formulation/software & determinant residual fitting strategy
- P-wave $K\pi$ scattering: $K^*(892)$ resonance parameters
- S-wave $K\pi$ scattering: $K_0^*(800)/\kappa$ resonance parameters

Finite Volume Spectra

Scattering process: eg.

$I = 1 \quad \pi\pi \to \pi\pi$





Momentum quantised → No continuum of scattering states

$$\boldsymbol{p} = \frac{2\pi}{L}\boldsymbol{d}$$

Lüscher Quantisation

Infinite volume physics from LQCD

Lüscher: Relationship between finite volume spectrum and infinite volume scattering matrix

- Quantisation condition (see talks by A. Hanlon & others):

$$\det[\, \widetilde{K}^{-1} - B \,] = 0$$

- For each E_{cm} in spectrum, determinant gives single relation to entire scattering matrix
  - $\Rightarrow$ Exactly solvable for single channel, single partial wave
  - $\Rightarrow$ ℓ mixing/coupled decay channels requires parameterisation of $\widetilde{K}$ and a fit (determinant residual method)

[Morningstar et al.; Nucl. Phys. B 924 (2017)]

Lüscher Quantisation

Infinite volume physics from LQCD

Lüscher: Relationship between finite volume spectrum and infinite volume scattering matrix

- Quantisation condition (see talks by A. Hanlon & others):

$$\widetilde{K}_{\ell}^{-1} = \left(\frac{q_{\rm cm}}{m_{\pi}}\right)^{2\ell+1} \cot\delta_l \qquad \det[\widetilde{K}^{-1} - B] = 0$$

- For each $E_{\rm cm}$ in spectrum, determinant gives single relation to entire scattering matrix
  - ⇒ Exactly solvable for single channel, single partial wave
  - ⇒ ℓ mixing/coupled decay channels requires parameterisation of $\widetilde{K}$ and a fit (determinant residual method)

[Morningstar et al.; Nucl. Phys. B 924 (2017)]

Lüscher Quantisation

Infinite volume physics from LQCD

Lüscher: Relationship between finite volume spectrum and infinite volume scattering matrix

- Quantisation condition (see talks by A. Hanlon & others):

$$\widetilde{K}_\ell^{-1} = \left(\frac{q_{\rm cm}}{m_\pi}\right)^{2\ell+1} \cot\delta_l \qquad \det[\widetilde{K}^{-1} - B] = 0$$

box matrix: known function of $(E_{\rm cm}, L)$

- For each $E_{\rm cm}$ in spectrum, determinant gives single relation to entire scattering matrix

  $\Rightarrow$ Exactly solvable for single channel, single partial wave

  $\Rightarrow$ ℓ mixing/coupled decay channels requires parameterisation of $\widetilde{K}$ and a fit (determinant residual method)

[Morningstar et al.; Nucl. Phys. B 924 (2017)]

Extracting finite volume spectra

- Temporal Correlator:

$$C_{\alpha\beta}(t) \equiv \langle 0|\, O_\alpha(t+t_0)\, \overline{O}_\beta(t_0)\, |0\rangle = \sum_n \langle 0|\, O_\alpha\, |n\rangle \langle n|\, \overline{O}_\beta\, |0\rangle\, e^{-E_n t}$$

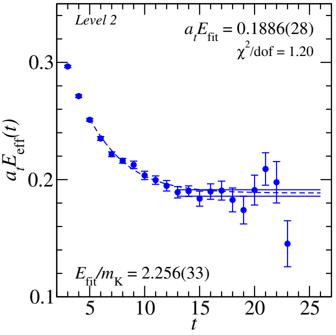
- Define new matrix

$$\widetilde{C}(t) = U^\dagger C(\tau_0)^{-1/2} C(t) C(\tau_0)^{-1/2} U$$

Columns of $U =$ eigenvectors of $C(\tau_0)^{-1/2} C(\tau_D) C(\tau_0)^{-1/2}$

- for large t, $\lambda_n(t) \to |Z'_n|^2 e^{-E_n t}$ and

$$Z_j^{(n)} \equiv \langle 0|\, O_j\, |n\rangle \approx C_{jk}(\tau_0)^{1/2} U_{kn}(t) Z'_n$$



Extracting finite volume spectra

- Signal of interest: deviation of finite-volume two hadron levels from non-interacting counterparts
  - $\Rightarrow$ Extract energy difference from

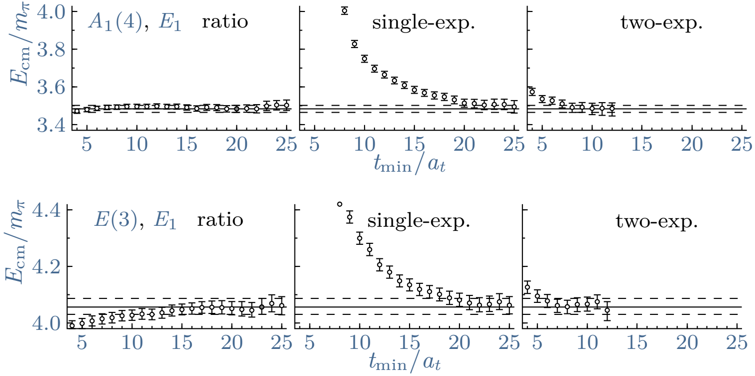
$$R_n(t) = \frac{\widetilde{C}_n(t)}{C_\pi(\boldsymbol{d}_\pi^2, t)\, C_K(\boldsymbol{d}_K^2, t)} \to A_n e^{-\Delta E_n t}$$

- Reconstruct:

$$a_t E_n = a_t \Delta E_n + \sqrt{a_t^2 m_\pi^2 + \left(\tfrac{2\pi a_s}{\xi L}\right)^2 \boldsymbol{d}_\pi^2} + \sqrt{a_t^2 m_K^2 + \left(\tfrac{2\pi a_s}{\xi L}\right)^2 \boldsymbol{d}_K^2}.$$

- Where ΔE_n is small, these ratio fits generally have smaller excited state contamination than direct fits to $\widetilde{C}_n(t)$

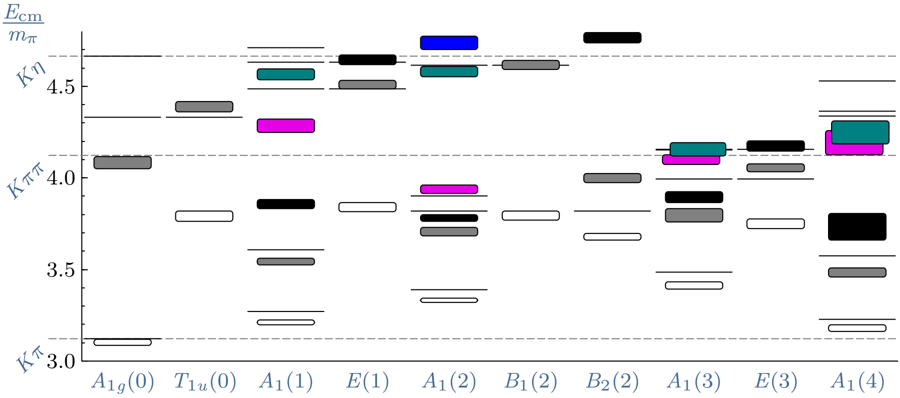
Ratio fits



Each row corresponds to the three fits for a single level specified in the left column as '$\Lambda(\boldsymbol{d}^2)$, E_n', denoting the nth level in finite volume irrep Λ with total momentum $\boldsymbol{d}^2$.

$K\pi$ energies in finite volume

- finite volume energies $32^3 \times 256$ lattice, $m_\pi \approx 230$ MeV
- basis of 13 single-hadron (K) and 33 two-hadron ($K\pi$) interpolating operators



[RB, J Bulava, J Fallica, A Hanlon, B Hörz, C Morningstar; 1802.03100]

Decay of $K^*(892)$

- large $32^3 \times 256$ anisotropic lattice, $m_\pi \approx 230$ MeV
- included $\ell = 0, 1, 2$ partial waves
- fit forms

$$
\begin{aligned}
(\widetilde{K}^{-1})_{11} &= \frac{6\pi E_{\rm cm}}{g^2 m_\pi}\left(\frac{m_{K^*}^2}{m_\pi^2} - \frac{E_{\rm cm}^2}{m_\pi^2}\right) \qquad (\widetilde{K}^{-1})_{22} = \frac{-1}{m_\pi^5 a_2} \\
(\widetilde{K}^{-1})_{00}^{\rm lin} &= a_{\rm l} + b_{\rm l} E_{\rm cm}, \quad (\widetilde{K}^{-1})_{00}^{\rm quad} = a_{\rm q} + b_{\rm q} E_{\rm cm}^2, \\
(\widetilde{K}^{-1})_{00}^{\rm ERE} &= \frac{-1}{m_\pi a_0} + \frac{m_\pi r_0}{2}\frac{\boldsymbol{q}_{\rm cm}^2}{m_\pi^2}, \qquad (\widetilde{K}^{-1})_{00}^{\rm BW}
\end{aligned}
$$

- results

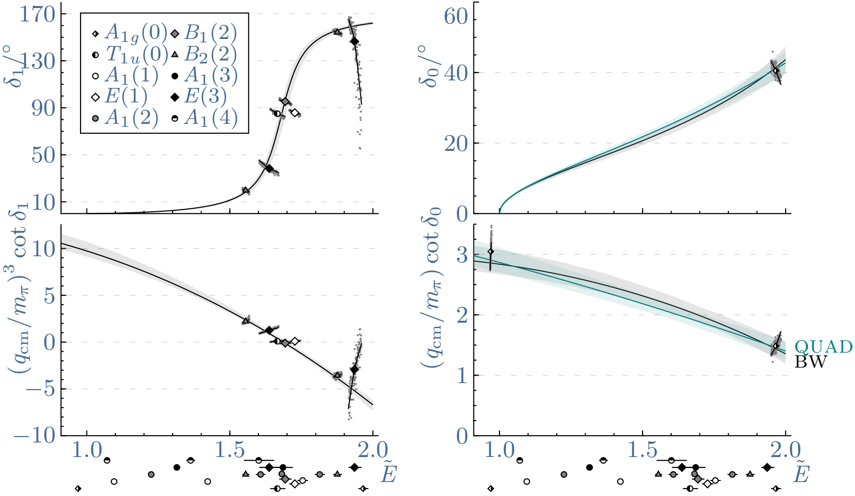
$$
\frac{m_{K^*}}{m_\pi} = 3.808(18), \quad g = 5.33(20), \quad m_\pi a_0 = -0.353(25),
$$

$$
m_\pi^5 a_2 = -0.0013(68), \qquad \chi^2/\text{dof} = 1.42
$$

[RB, J Bulava, J Fallica, A Hanlon, B Hörz, C Morningstar; 1802.03100]

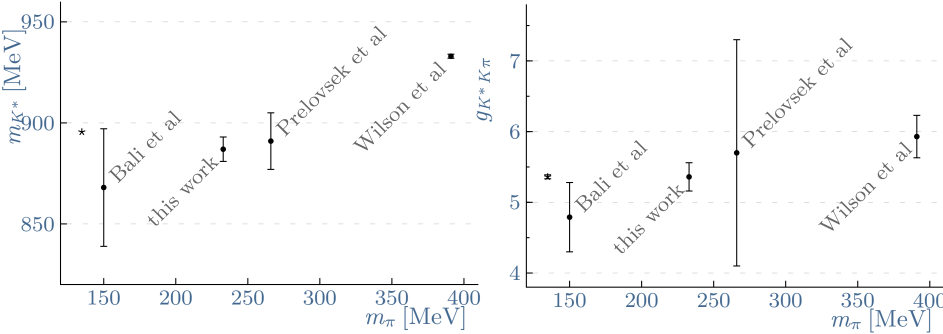
Decay of $K^*(892)$

- plots of P-wave and S-wave phase shift ($\tilde{E} = (E_{\mathrm{cm}} - m_K)/m_\pi$)
- $\kappa(\ell = 0)$ fit: Breit-Wigner or effective range



Decay of $K^*(892)$

- summary of lattice calculations of $K^*(892)$ resonance parameters to date
- phenomenological values shown as astericks



Conclusions

- $K\pi$ scattering well studied to date on the lattice
- (light/strange) meson-meson scattering at a mature stage
  - moving towards physical point results - large volumes required
- meson-baryon scattering now possible: PRD **97**, 014506 (2018), etc.
- successful results depend on time-slice to time-slice quark propagators needed for temporal correlators involving two-hadron operators
  - Stochastic LapH method (minimal volume scaling)

Decay of $\rho(770)$

- initially applied to P-wave $I = 1$ $\rho \to \pi\pi$ system
- now have included $\ell = 1, 3, 5$ partial waves
- large $32^3 \times 256$ anisotropic lattice, $m_\pi \approx 230$ MeV
- fit forms (first ever inclusion of $\ell = 5$ in lattice QCD):

$$(\widetilde{K}^{-1})_{11} = \frac{6\pi E_{\rm cm}}{g^2 m_\pi}\left(\frac{m_\rho^2}{m_\pi^2} - \frac{E_{\rm cm}^2}{m_\pi^2}\right)$$

$$(\widetilde{K}^{-1})_{33} = \frac{1}{m_\pi^7 a_3} \qquad (\widetilde{K}^{-1})_{55} = \frac{1}{m_\pi^{11} a_5}$$

- results

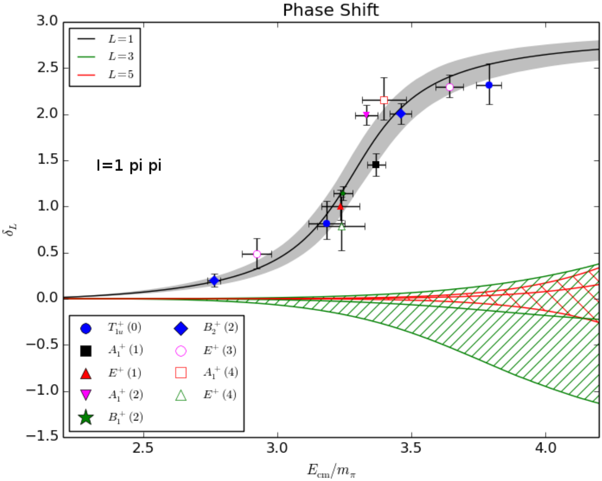
$$\frac{m_\rho}{m_\pi} = 3.349(25),\ g = 5.97(27),\ m_\pi^7 a_3 = -0.00021(100),$$
$$m_\pi^{11} a_5 = -0.00006(24),\ \chi^2/\text{dof} = 1.15$$

[J Bulava, B Fahy, B Hörz, K J Juge, C Morningstar, CH Wong; NPB 910, 842 (2016)]

[C Morningstar, J Bulava, B Singha, RB, J Fallica, A Hanlon, B Hörz; NPB 924, 477 (2017)]

Decay of $\rho(770)$

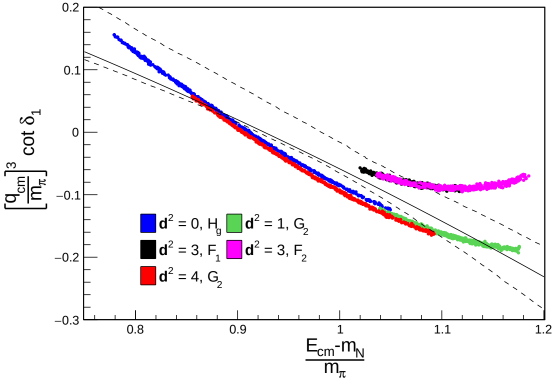
- $\ell = 1, 3, 5$ phase shifts



[J Fallica, PhD Thesis (2017)]

Decay of $\Delta(1232)$

- included $\ell = 1$ wave only (for now)
- large $48^3 \times 128$ isotropic lattice, $m_\pi \approx 280$ MeV, $a \sim 0.076$ fm
- Breit-Wigner fit gives $g_{\Delta N\pi} = 19.0(4.7)$ in agreement with experiment ~ 16.9



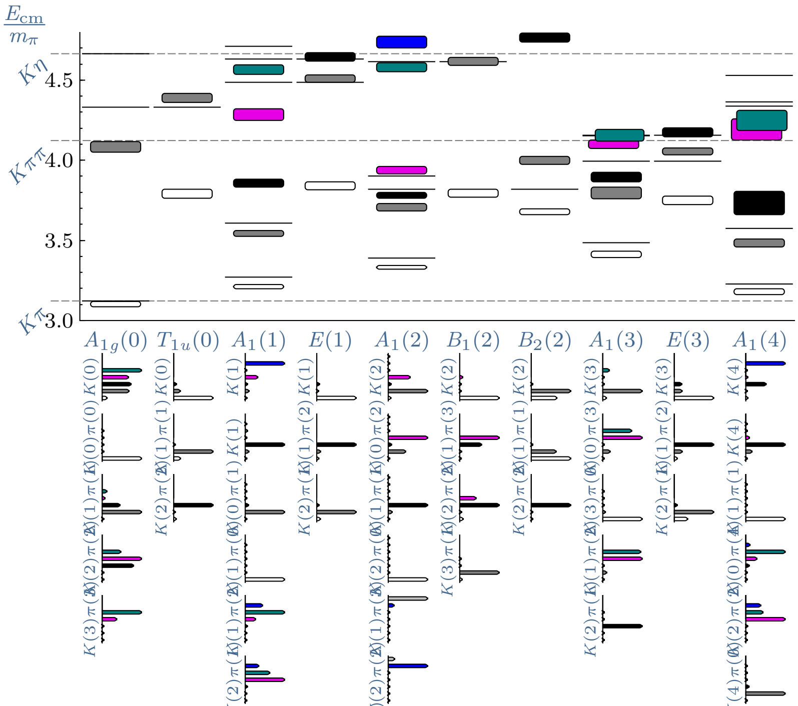
[CW Andersen, J Bulava, B Hörz, C Morningstar; PRD **97**, 014506 (2018)]

S-wave $K\pi$ amplitude: $K_0^*(800)$

$$\frac{m_{K^*}}{m_\pi} = 3.808(18), \quad g = 5.33(20), \quad m_\pi a_0 = -0.353(25),$$
$$m_\pi^5 a_2 = -0.0013(68), \qquad \chi^2/\text{dof} = 1.42$$

- based on LO ERE, $m_\pi a_0 < 0$ suggests virtual bound state
- however, NLO parameters give $1 - 2r_0/a_0 = -8.9(2.4)$ which must be > 0 for a (real or virtual) bound state
- zeros of $\boldsymbol{q}_{\text{cm}} \cot \delta_0 - i\boldsymbol{q}_{\text{cm}}$: $m_R/m_\pi = 4.66(13) - 0.87(18)i$
  - consistent with BW fit
- better energy resolution & careful analytic continuation required

[RB, J Bulava, J Fallica, A Hanlon, B Hörz, C Morningstar; 1802.03100]


$\frac{E_{cm}}{m_\pi}$
4.5
4.0
3.5
3.0
$K\eta$
$K\pi\pi$
$K\pi$
$A_{1g}(0)$ $T_{1u}(0)$ $A_1(1)$ $E(1)$ $A_1(2)$ $B_1(2)$ $B_2(2)$ $A_1(3)$ $E(3)$ $A_1(4)$