Scattering of hadrons in lattice QCD: some applications

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Outline

➢ **bound states in mesonic systems:**

- $D_{s0}^*(3217)$: possibly cleanest example in meson systems
  [Mohler, Lang, Leskovec, S.P. , Woloshyn, PRL 2013, PRD 2014; RQCD PRD 2017; HSC, C. Thomas @ Lattice 2016]
- charmonium-like X(3872) : very interesting, but why theoretically less clean than Ds0

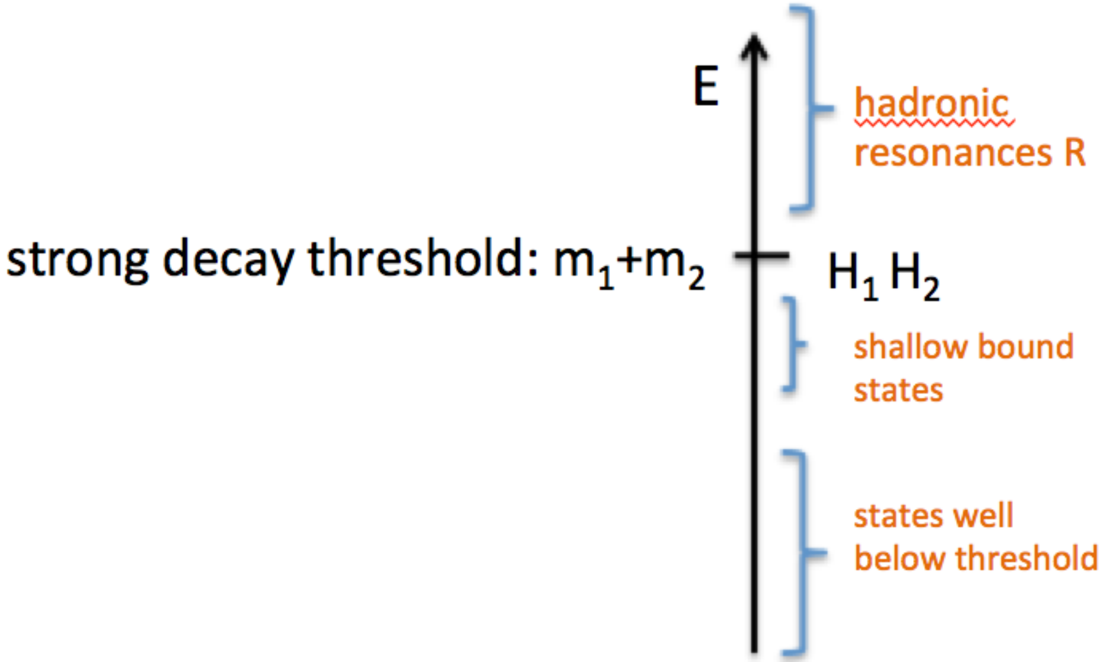
➢ **charmonium resonances** (breifly)

➢ **Scattering of two particles with spin on the lattice**

- motivation
- the relation to extract the scattering matrix from energies is known
- construction of operators (interpolators)
  by three different methods that give consistent results: reassuring
- example: Nucleon-pion scattering in p-wave, $J^P=1/2^+$

  lattice results and implications for the Roper resonance

Clasification of hadron states



decay strongly

can not decay strongly

Shallow bound states of two mesons

Mesonic bound states in s-wave ? (analogues of deuterium)

- ❑ states well below threshold
- ❑ strongly decay: resonances
- ❑ candidates for shallow bound st.

$\bar{u}u$

$\pi^\pm$, π^0, η

$f_0(500)$ or σ was $f_0(60$, $\rho(770)$, $\omega(782)$, $\eta'(958)$, $f_0(980)$, $a_0(980)$, $\phi(1020)$, $h_1(1170)$, $b_1(1235)$, $a_1(1260)$, $f_2(1270)$, $f_1(1285)$, $\eta(1295)$, $\pi(1300)$, $a_2(1320)$, $f_0(1370)$, $h_1(1380)$, $\pi_1(1400)$, $\eta(1405)$, $f_1(1420)$, $\omega(1420)$, $f_2(1430)$, $a_0(1450)$, $\rho(1450)$

$\bar{s}u$

$K^\pm$, K^0, K_S^0, K_L^0

$K_0^*(800)$ or κ, $K^*(892)$, $K_1(1270)$, $K_1(1400)$, $K^*(1410)$, $K_0^*(1430)$, $K_2^*(1430)$, $K(1460)$, $K_2(1580)$, $K(1630)$, $K_1(1650)$, $K^*(1680)$, $K_2(1770)$, $K_3^*(1780)$, $K_2(1820)$, $K(1830)$

$\bar{c}u$

$D^\pm$, D^0

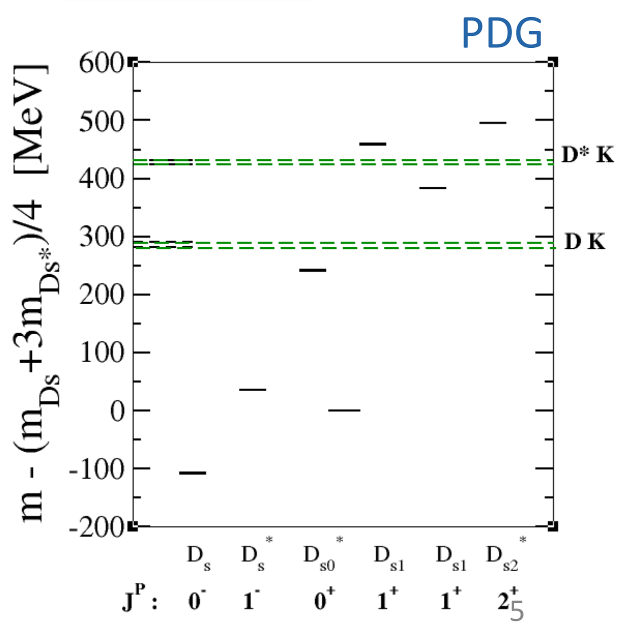
$D^*(2007)^0$, $D^*(2010)^\pm$, $D_0^*(2400)^0$, $D_0^*(2400)^\pm$, $D_1(2420)^0$, $D_1(2420)^\pm$, $D_1(2430)^0$, $D_2^*(2460)^0$, $D_2^*(2460)^\pm$, $D(2550)^0$, $D(2600)$, $D^*(2640)^\pm$, $D(2750)$

$\bar{c}s$

$D_s^\pm$, $D_s^{*\pm}$

$D_{s0}^*(2317)^\pm$, $D_{s1}(2460)^\pm$

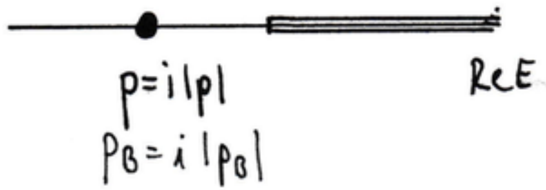
$D_{s1}(2536)^\pm$, $D_{s2}^*(2573)$, $D_{s1}^*(2700)^\pm$, $D_{s1}^*(2860)^\pm$, $D_{s3}^*(2860)^\pm$, $D_{sJ}(3040)^\pm$



How to search for shallow s-wave bound state on the lattice?

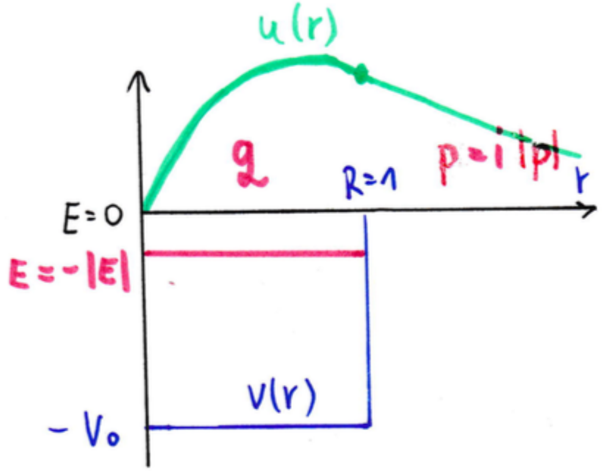
Example: $D_{s0}^*(2317)$ bound state in DK scattering, s-wave

To be shown on next slides:



- extract scattering matrix, phase shift δ
- bound state : pole in scattering matrix S or T for p=i |p|
- signature: negative and large a_0 $$p \cot\delta = \frac{1}{a_0} + \frac{1}{2} r_0 p^2 + O(p^4)$$

non-relativistic QM: spherical potential well with one bound state



$$A\sin qr \qquad B\,e^{-|p|r}$$

$$\psi(r) = R(r) = \frac{u(r)}{r} \qquad (l=0)$$

$$\frac{1}{\sqrt{C^2-|p|^2}}\tan(\sqrt{C^2-|p|^2}R) = -\frac{1}{|p|}$$

$$u(R) = A\sin qR = B\,e^{-|p|R}$$
$$u'(R) = qA\cos qR = -|p|B\,e^{-|p|R}$$

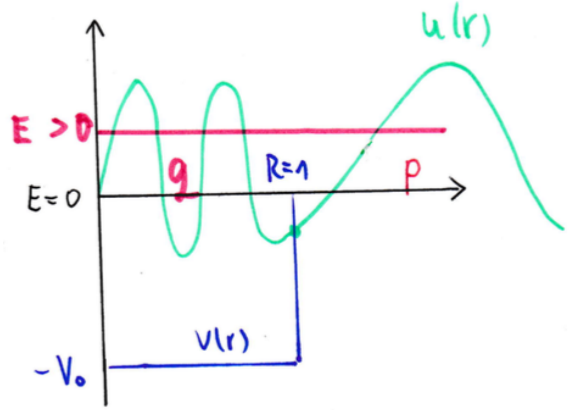
$$\frac{1}{q}\tan qR = -\frac{1}{|p|} \rightarrow \quad |p_B| \approx 0.2, \quad p_B = i|p_B|$$

momentum of the bound st.

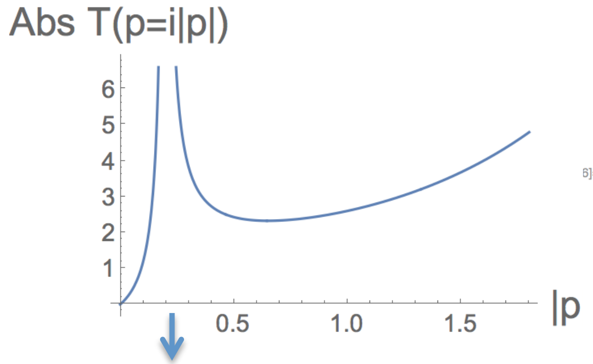
$$q = \sqrt{2\mu(V_0-|E|)} = \sqrt{2\mu V_0 - |p|^2}$$

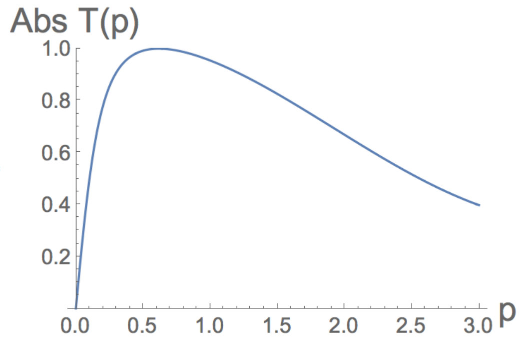
$C^2 = 1.7^2$ (to ensure one shallow bound st.)

s-wave scattering on spherical potential well



$A\sin qr \qquad B\sin(pr+\delta_0)$





$|p|=|p_B|=0.2$: pole of T at the position of the bound st.

$$u(R)=A\sin qR=B\sin(pR+\delta_0)$$
$$u'(R)=qA\cos qR=kB\cos(pR+\delta_0)$$

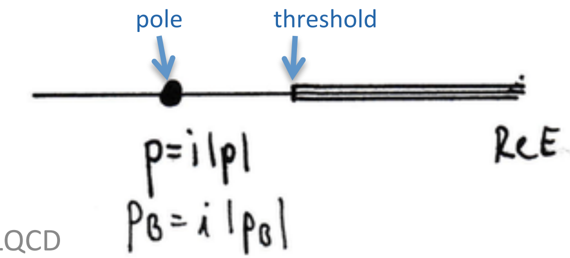
$$\frac{1}{q}\tan qR=\frac{1}{k}\tan(pR+\delta_0)$$

$$\delta_0(p)=\arctan\left(\frac{p}{q}\tan(pR)\right)-pR+n\pi$$

$$q=\sqrt{2\mu(V_0+E)}=\sqrt{2\mu V_0+p^2}$$

$C^2=1.7^2$ (as before)

$$S=e^{2i\delta(p)}=1+2iT(p) \qquad T(p)=\frac{1}{\cot\delta(p)-i}$$



s-wave scattering on spherical potential well

$$\delta_0(p) = \arctan\left(\frac{p}{q}\tan(pR)\right) - pR + n\pi \qquad q = \sqrt{2\mu V_0 + p^2} = \sqrt{C^2 + p^2}$$

Taylor expanding

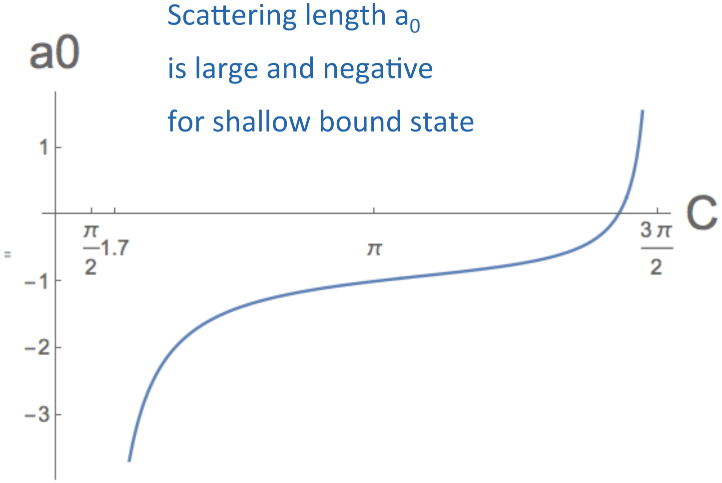
$$p\cot\delta_0(p) = \underbrace{\frac{C}{-C + \mathrm{Tan}[C]}}_{1/a_0} + \frac{1}{6}\left(3 - \frac{C^2}{(C - \mathrm{Tan}[C])^2} - \frac{3}{C^2 - C\,\mathrm{Tan}[C]}\right)p^2 + O[p]^4 = \frac{1}{a_0} + \frac{1}{2}r_0\,p^2$$

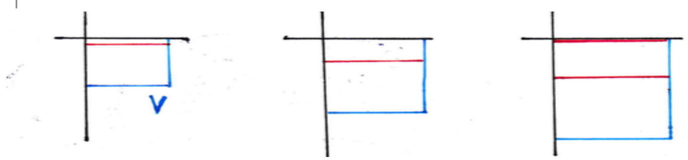
1/a_0

Scattering length a_0

is large and negative

for shallow bound state





for general potential and general partial wave l

$$\lim_{p\to 0}\tan\delta_l(p) \propto p^{2l+1}, \qquad p^{2l+1}\cot\delta_l(p) = C + O(p^2)$$

Landau: QM, p -> k $\qquad \psi(r) = R_l(r)Y_{lm}$

$$R_l = c_1(-1)^l \frac{(2l+1)!!}{k^{2l+1}} r^l \left(\frac{d}{r\,dr}\right)^l \frac{\sin kr}{r} + c_2(-1)^l \frac{r^l}{(2l-1)!!}\left(\frac{d}{r\,dr}\right)^l \frac{\cos kr}{r}.$$

kr >>1 : $R_l \approx \frac{c_1(2l+1)!!}{rk^{l+1}}\sin\left(kr - \frac{\pi l}{2}\right) + \frac{c_2 k^l}{r(2l-1)!!}\cos\left(kr - \frac{\pi l}{2}\right)$

$$R_l \approx \mathrm{const}\cdot\frac{1}{r}\sin\left(kr - \frac{\pi l}{2} + \delta_l\right)$$

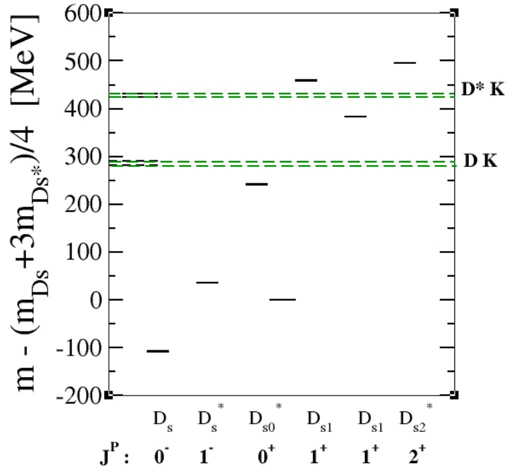
$$\mathrm{tg}\,\delta_l \simeq \delta_l = \frac{c_2}{c_1(2l-1)!!(2l+1)!!}k^{2l+1}$$

D_{s0}^* shallow bound state in DK scattering: I=0, $J^P=0^+$

Mode		Fraction (Γ_i/Γ)
Γ_1	$D_s^+\gamma$	(93.5 ± 0.7)%
Γ_2	$D_s^+\pi^0$	(5.8 ± 0.7)%
Γ_3	$D_s^+e^+e^-$	(6.7 ± 1.6) × 10^{-3}

		PACSCS
$N_L^3 \times N_T$	$16^3 \times 32$	$32^3 \times 64$
N_f	2	2+1
a[fm]	0.1239(13)	0.0907(13)
L[fm]	1.98(2)	2.90(4)
m_π	266 MeV	156 MeV

results for this ensemble shown

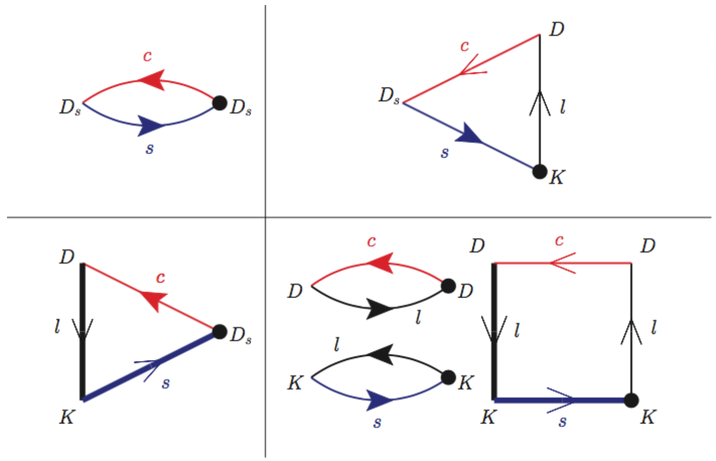


Interpolators in A_1^+ of Oh: P_{tot} =0 is simulated by our and Regensburg group

$$O^{qq} = \bar{s}c$$
$$\bar{s}\gamma_i\nabla_i c$$
$$\bar{s}\gamma_t\gamma_i\nabla_i c$$
$$\bar{s}\nabla_i\nabla_i c$$

$$O_1^{DK} = [\bar{s}\gamma_5 u]\,(\vec{p}=0)\,[\bar{u}\gamma_5 c]\,(\vec{p}=0) + \{u \to d\}\ ,$$
$$O_2^{DK} = [\bar{s}\gamma_t\gamma_5 u]\,(\vec{p}=0)\,[\bar{u}\gamma_t\gamma_5 c]\,(\vec{p}=0) + \{u \to d\}$$
$$O_3^{DK} = \sum_{\vec{p}=\pm e_{x,y,z}\ 2\pi/L} [\bar{s}\gamma_5 u]\,(\vec{p})\,[\bar{u}\gamma_5 c]\,(-\vec{p}) + \{u \to d\}\ .$$

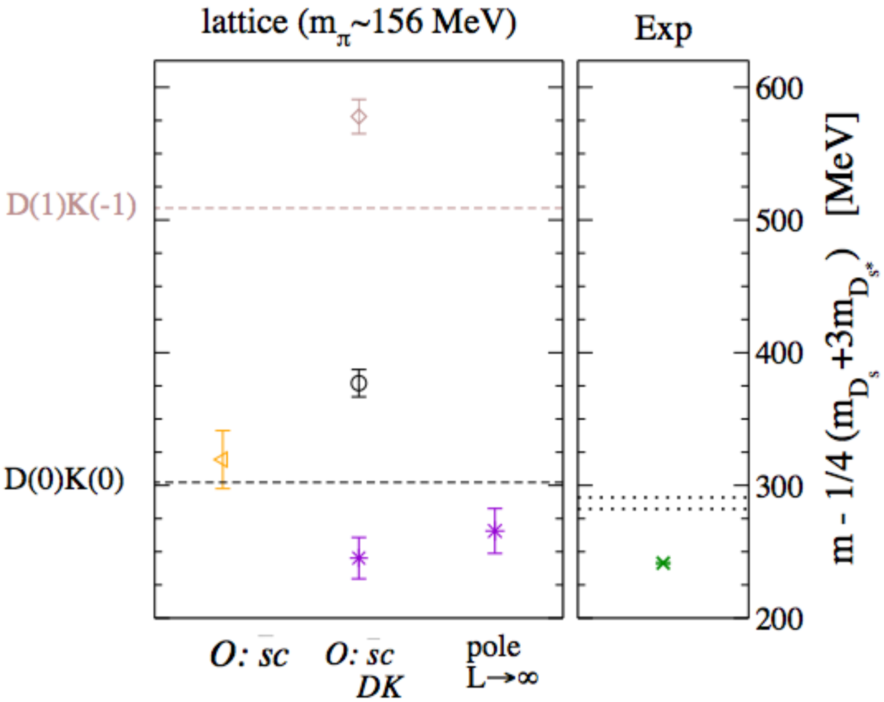
$$C_{ij}(t) = \langle 0|O_i(t)\,O_j^\dagger|0\rangle = \sum_k Z_{ik}^\dagger Z_{kj} e^{-E_k t}$$



C evaluated using distillation method [Peardon et al.]

Figure by RQCD

$D_{s0}^{*}(2317)$



D. Mohler, C. Lang, L. Leskovec, S.P. , R. Woloshyn:

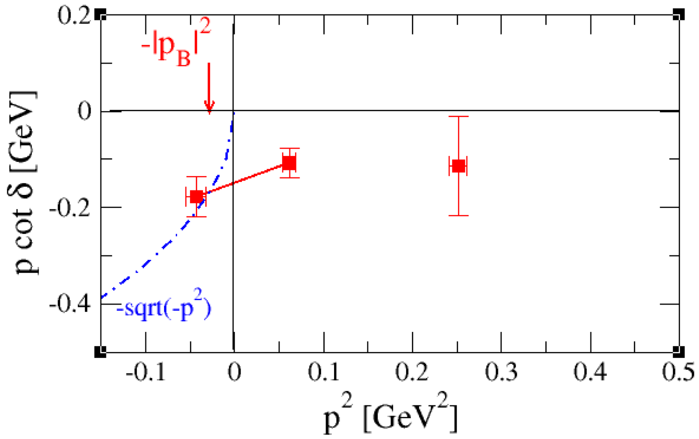
Phys. Rev. Lett. 2013: $m_\pi \approx 156$ MeV, $L \approx 2.9$ fm, Nf=2+1, PACSCS

mesonic bound st. established on lattice for the first time

$D_{s0}^{*}(2317)$	m - ¼ (m_{Ds}+3m_{Ds^*})	mD+mK-m
lat	266 ± 16±4 MeV	36 ± 17 MeV
exp	241.45 ± 0.6 MeV	45 MeV

$$E = \sqrt{m_D^2 + p^2} + \sqrt{m_K^2 + p^2} \qquad p\cot\delta(p) = \frac{2}{\sqrt{\pi}L} Z_{00}\left(1, \left(\tfrac{pL}{2\pi}\right)^2\right)$$

Luscher 1991



$$p\cot\delta(p) = \frac{1}{a_0} + \frac{1}{2} r_0 p^2$$

$$a_0 = -1.33 \pm 0.20 \text{ fm} \qquad r_0 = 0.27 \pm 0.17 \; fm$$

$$T \propto \frac{1}{\cot\delta - i} = \infty, \qquad \cot\delta(p_B) = i, \quad p_B = i\,|\,p_B\,|$$

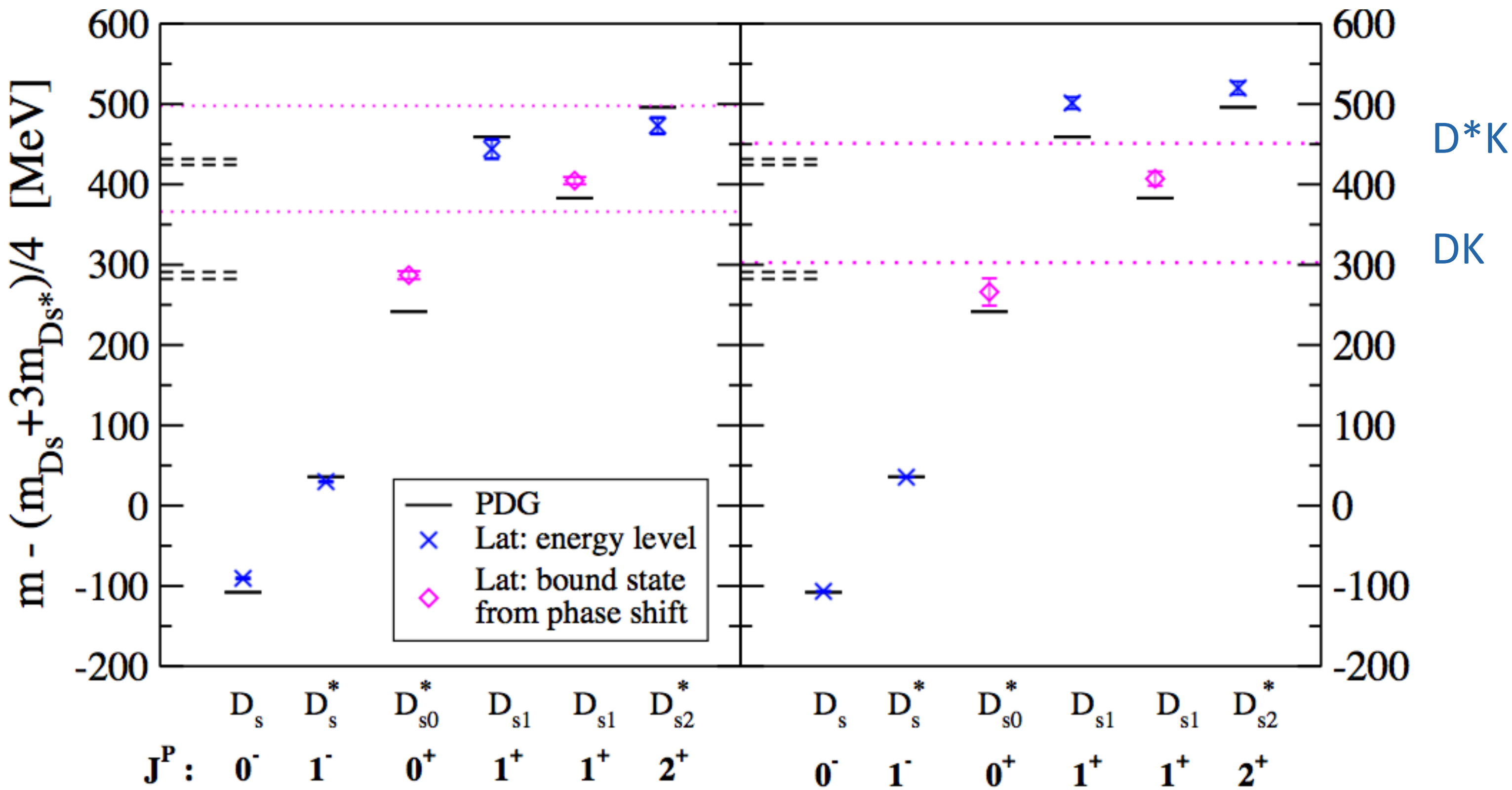
$$i\,|\,p_B\,|\,*\,i = \frac{1}{a_0} - \frac{1}{2} r_0\,|\,p_B\,|^2 \quad \rightarrow \quad |\,p_B\,|^2 = 0.028 \pm 0.012\; GeV^2$$

$$m_{D_{s0}}^{lat,\,L\to\infty} = \sqrt{m_D^2 - |\,p_B\,|^2} + \sqrt{m_K^2 - |\,p_B\,|^2}$$

Spectrum of Ds mesons

m_π=266 MeV, Nf=2, L≈2 fm

m_π=156 MeV, Nf=2+1, L≈2.9 fm



Lang, Leskovec, Mohler, S.P. , Woloshyn: PRD 2014

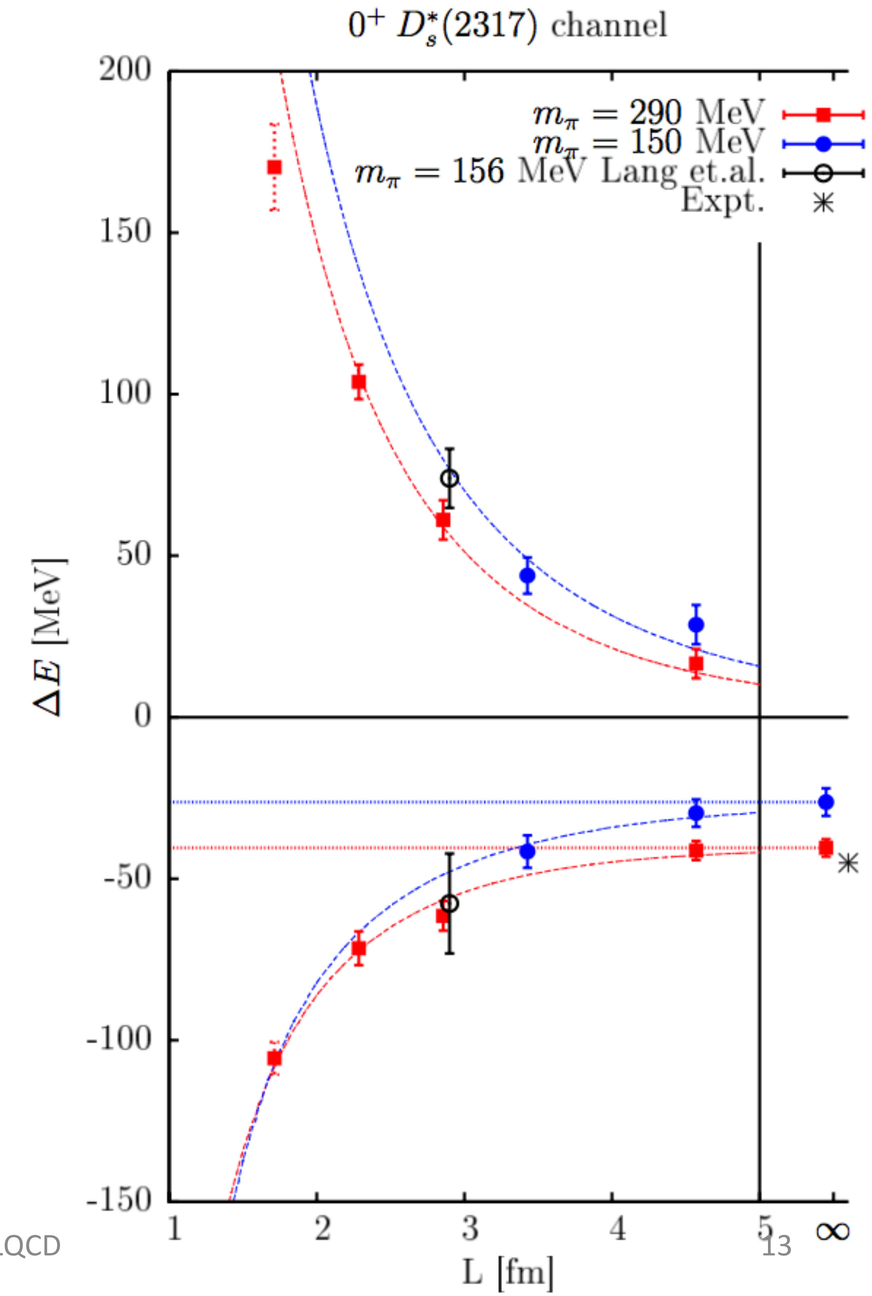
$D_{s0}^*(2317)$

Bali, Collins, Cox, Schafer (RQCD):

PRD (2017) 074501

κ_l	a [fm]	V	am_π	m_π [MeV]
0.13632	0.071	$24^3 \times 48$	0.1112(9)	306.9(2.5)
	0.071	$32^3 \times 64$	0.10675(52)	294.6(1.4)
	0.071	$40^3 \times 64$	0.10465(38)	288.8(1.1)
	0.071	$64^3 \times 64$	0.10487(24)	289.5(0.7)
0.13640	0.071	$48^3 \times 64$	0.05786(55)	159.7(1.5)
	0.071	$64^3 \times 64$	0.05425(49)	149.7(1.4)

$$O: \quad \bar{s}c,\ D(0)K(0)\ ,\ D(1)K(-1)$$



$D_{s0}^*(2317)$

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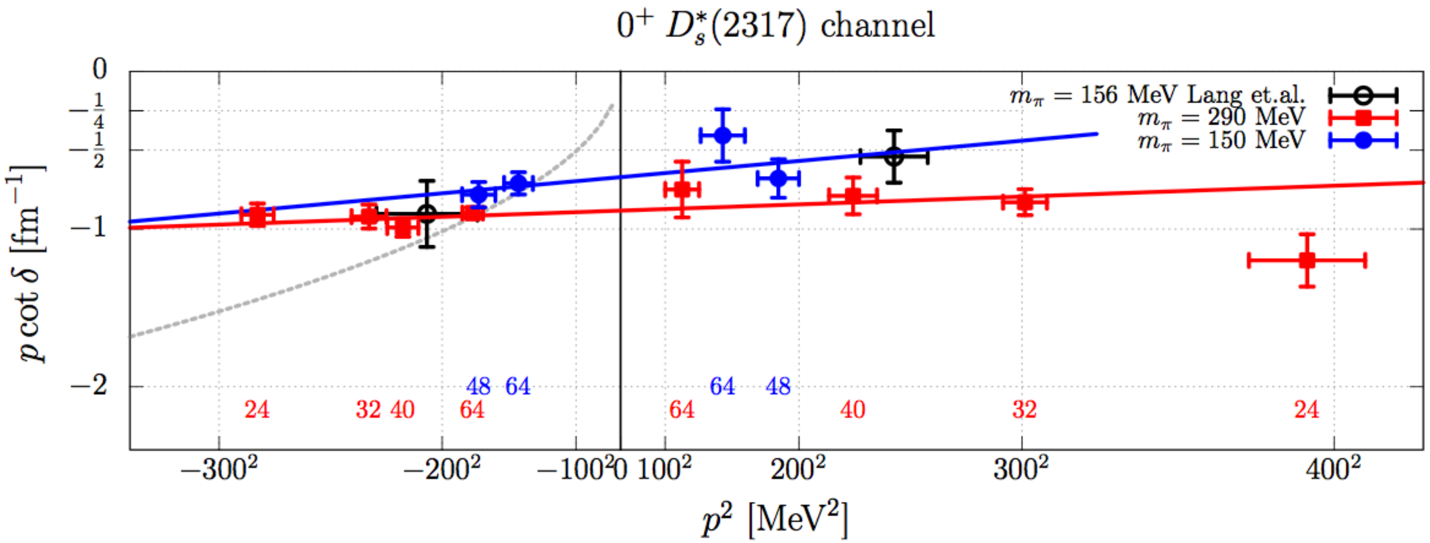


$$p\cot\delta(p) = \frac{1}{a_0} + \frac{1}{2} r_0 p^2$$

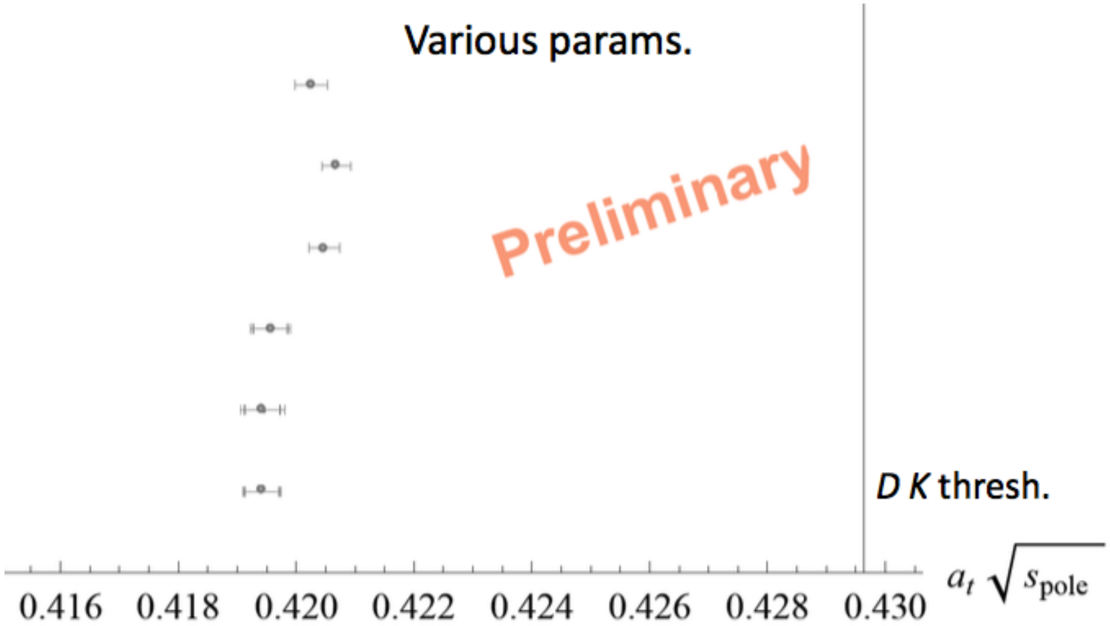
0^+ channel

	$m_\pi = 290$ MeV	$m_\pi = 150$ MeV	Expt.	m_π=156 MeV
a_0 [fm]	$-1.13(0.04)(+0.05)$	$-1.49(0.13)(-0.30)$		-1.33(20)
r_0 [fm]	$0.08(0.03)(+0.08)$	$0.20(0.09)(+0.31)$		0.27(17)
$\|p_B\|$ [MeV]	$180(6)(0)$	$142(11)(-9)$		
Δm [MeV]	$40(3)(0)$	$26(4)(-3)$	$42.6(0.7)(2.0)$	36 (17)
m_{D_s} [MeV]	$2384(2)(-1)$	$2348(4)(+6)$	$2317.7(0.6)(2.0)$	

RQCD (first two result columns) — Lang et al. (m_π=156 MeV column)

$D_{s0}^*(2317)$ C. Thomas @ lattice 2016

location of extracted bound-state pole for various parametrizations



Bound state, below *DK* threshold:
$a_t \Delta m \approx 0.010 \rightarrow \Delta m \approx 55$ MeV

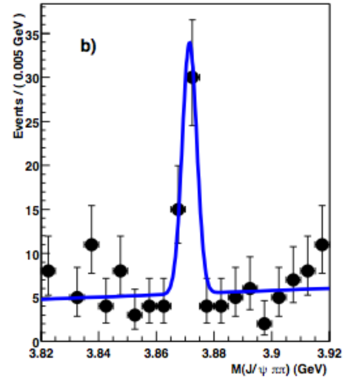
X(3872) , $J^{PC}=1^{++}$, charmonium-like

- First charmonium-like state discovered [Belle, PRL, 2003]
- sits within 1 MeV of $D^0\underline{D}^{0*}$ threshold
  8 MeV below D^+D^{*-} threshold

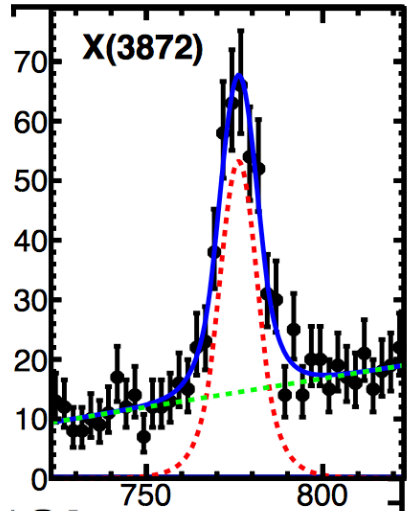
  } isospin breaking effects my be important
- believed to have a large molecular $D^0\underline{D}^{0*}$ Fock component
- $\Gamma < 1.2$ MeV
- decays to I=0, 1 equally important

  X(3872) → J/Ψ ω (I=0)

  X(3872) → J/Ψ ρ (I=1)

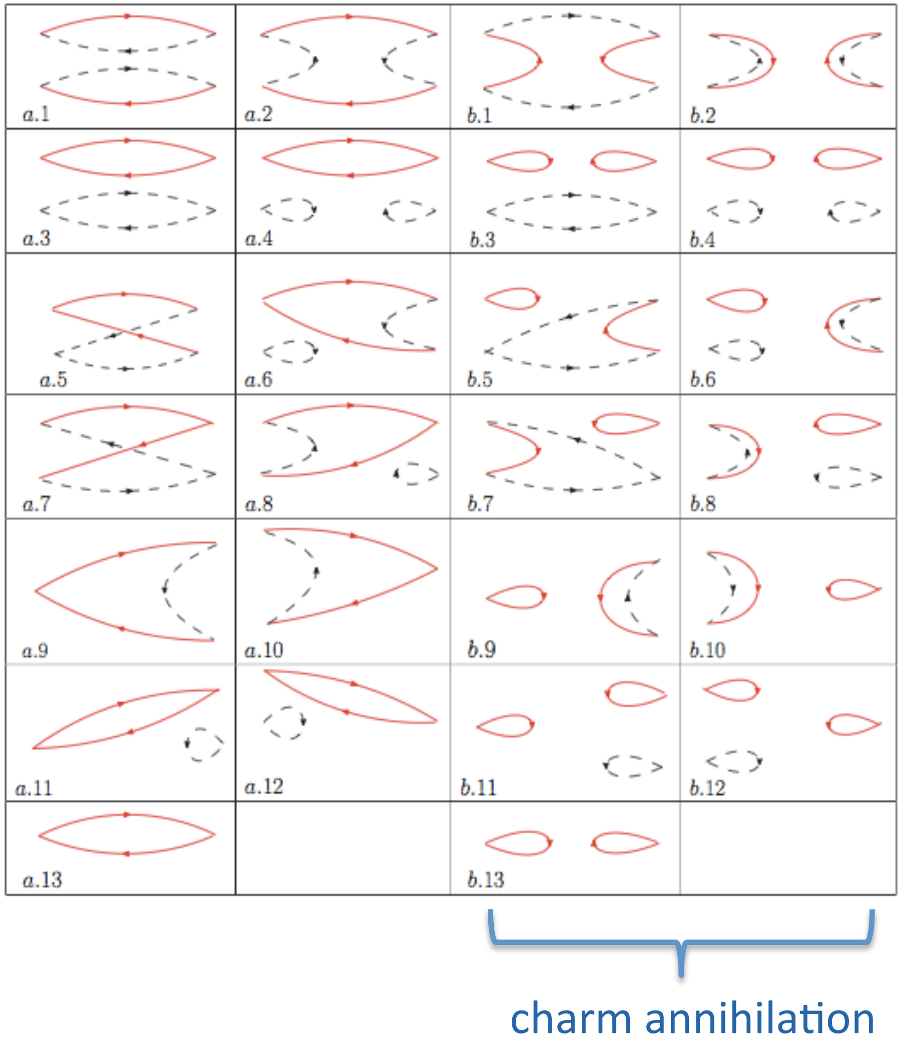


[LHCb, PRL 2013]



X(3872), 1^{++}, I=0

$$\mathcal{O}:\ \bar{c}\,c,\quad D\bar{D}^{*} = (\bar{c}u)(\bar{u}c) + (\bar{c}d)(\bar{d}c),\quad J/\psi\,\omega = (\bar{c}c)(\bar{u}u + \bar{d}d)$$



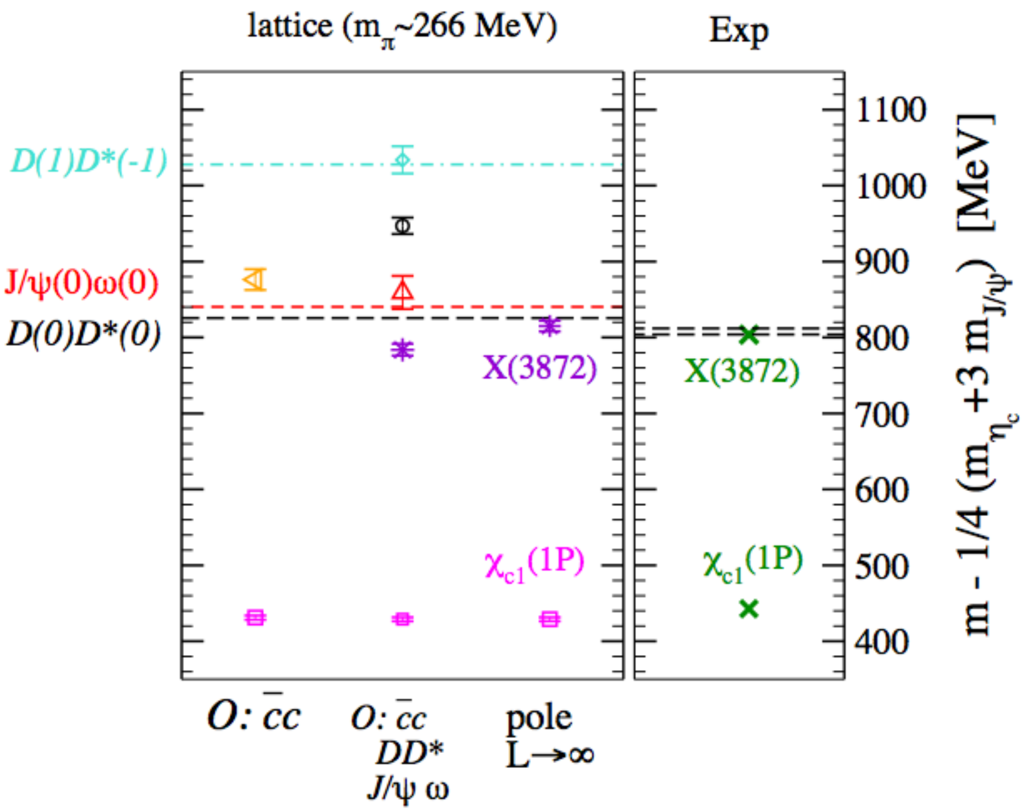
- all Wick contractions calculated using distillation method [Peardon et al. 2009]

- charm annihilation contractions not used in analysis

[S.P. and L. Leskovec, Phys.Rev.Lett. 2013]

X(3872) below DD* threshold, I=0

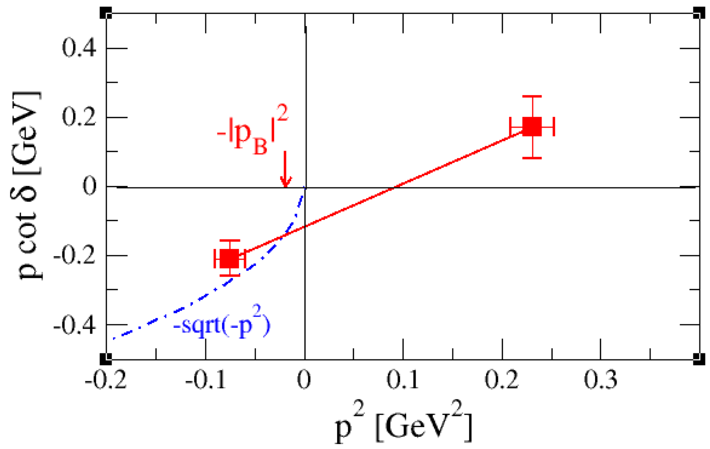
$\mathcal{O}: \bar{c}\, c,\quad D\bar{D}^*,\quad J/\psi\omega$ ← treated as decoupled



[S.P. , Leskovec : Phys. Rev. Lett. 2013]

$m_\pi \approx 266$ MeV, Nf=2

[Padmanath, Lang, S.P., PRD 2015]



$$p\cot\delta(p) = \frac{2}{\sqrt{\pi}L} Z_{00}(1,(\tfrac{pL}{2\pi})^2) = \frac{1}{a_0} + \frac{1}{2} r_0 p^2$$

$$a_0^{DD^*} = -1.7 \pm 0.4 \text{ fm}, \quad r_0^{DD^*} = 0.5 \pm 0.1 \text{ fm}$$

$$T \propto [\cot\delta - i]^{-1} = \infty, \quad \cot\delta(p_B) = i$$

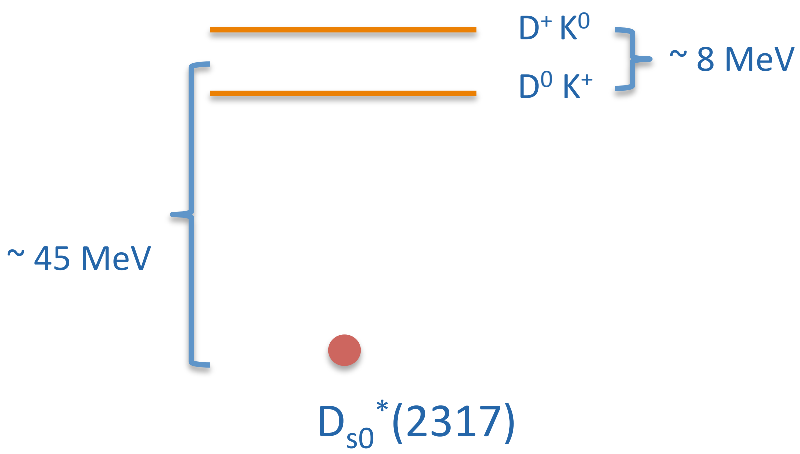
$$m_X^{lat,L\to\infty} = \sqrt{m_D^2 - |p_B|^2} + \sqrt{m_{D^*}^2 - |p_B|^2}$$

X(3872)	m - $(m_{D0}+m_{D0^*})$
lat	- 11 ± 7 MeV
exp	- 0.14 ± 0.22 MeV

X(3872) appears only if both cc and DD* interp. used.

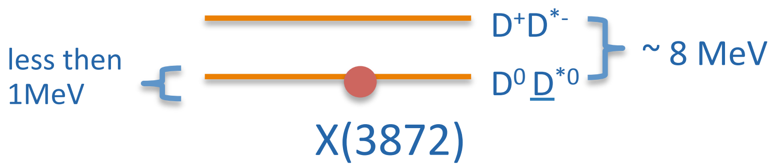
$D_{s0}^*(2317)$ in DK vs. X(3872) in $D\underline{D}^*$

- very narrow: width not measured
- theoretically cleaner
- no Wick contraction omitted
- no other nearby threshold
- isospin breaking less relevant
- only s-wave contributes to $J^P=0^+$



- same
- currently theoretically less clean -> more to be done
- charm annihilation omitted
- I=0 state in isospin limit:
  J/Ψ ω (I=0) threshold at 3879 MeV , J/Ψ πππ
  J/Ψ ππ (I=0) threshold formally below X(3872)
- isospin breaking more relevant (not considered on lat)

  another threshold for broken I
  J/Ψ ρ (I=1) threshold 3873 MeV , J/Ψ ππ
- s and d-wave in $J^P=1^+$ (d-wave not considered on lat)
- nevertheless: the lattice result obtained is believed to be rather solid (X has width much less than MeV)
- more work to be done



Resonances
.. in charmonium sector

[Lang, Leskovec, Mohler, S.P., JHEP 2015]

[the physics conclusions from on-going study

on CLS ensembles with Regensburg group not finalized yet]

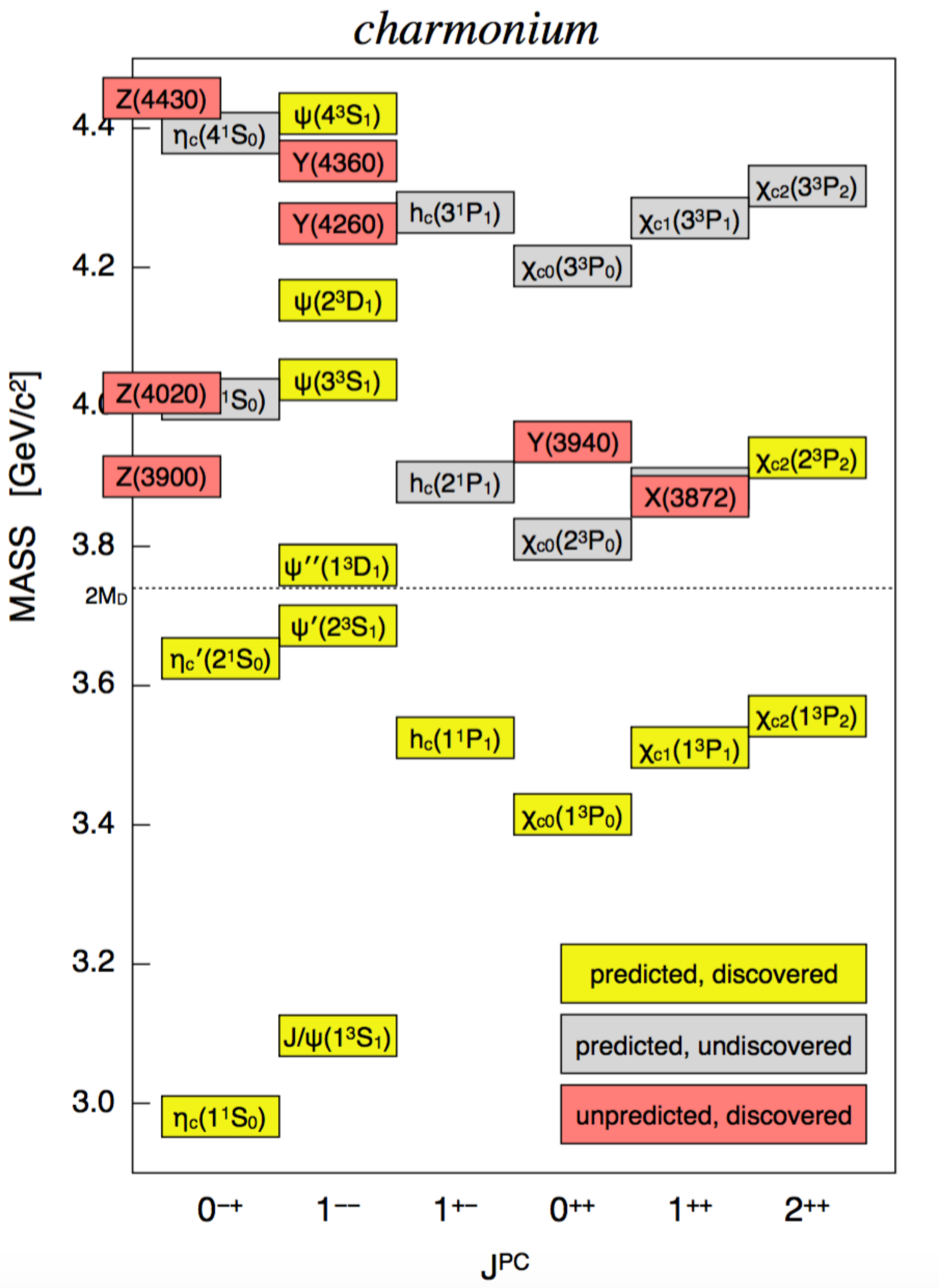


Fig: Mitchell @ Hadrons 2017

Resonances vs. bound states

for one channel scattering dominated by single partial wave l

$$S(E) = e^{2i\delta(E)}, \quad S(E) = 1 + 2iT(E), \quad T(E) = \frac{1}{\cot\delta(E) - i}$$

simplest E-dependence expected in a region near a relatively narrow reson.

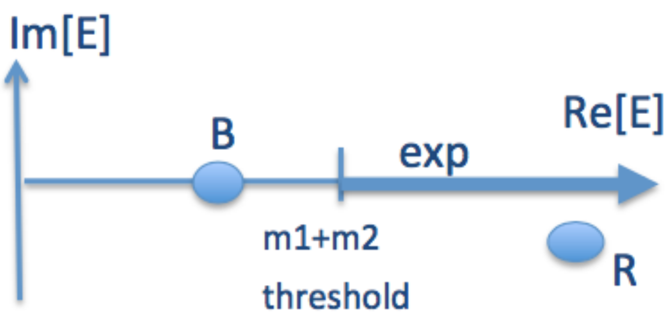
Bound state (B):

$\cot[\delta(E_B)] = i\,, \quad E_B < m_1 + m_2$

Resonance (R) (of Breit-Wigner type):

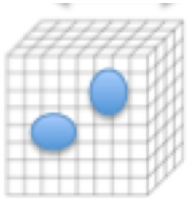
$$T(E) = \frac{E\,\Gamma(E)}{m_R^2 - E^2 - i\,E\,\Gamma(E)} \qquad \Gamma(E) = g^2\,\frac{p^{2l+1}}{E^2}$$

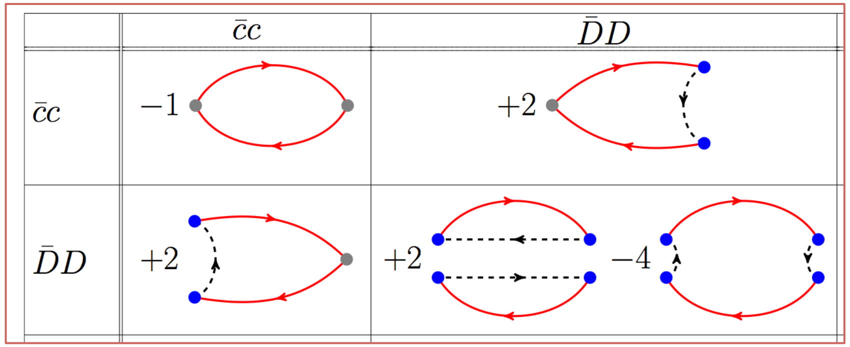
Locations of poles of T(E) for res. and bound st.



Resonance ψ(3770) and bound st. ψ(2S) from D$\underline{\text{D}}$ scattering in p-wave

$$\mathcal{O}:\ \bar{c}\,c,\ D\bar{D},\quad J^{PC}=1^{--}$$

$D\bar{D}$ 



$\eta_c(1S)$
$J/\psi(1S)$
$\chi_{c0}(1P)$
$\chi_{c1}(1P)$
$h_c(1P)$
$\chi_{c2}(1P)$
$\eta_c(2S)$
$\psi(2S)$
$2m_D$
$\psi(3770)$
$X(3872)$
$\chi_{c0}(2P)$wa
$\chi_{c2}(2P)$
$X(3940)$
$\psi(4040)$
$X(4050)^{\pm}$
$X(4140)$
$\psi(4160)$
$X(4160)$
$X(4250)^{\pm}$

$$D\bar{D}\rightarrow\psi(3770)\rightarrow D\bar{D}$$

Charm annihilation Wick contractions omitted
(like in all charmonium-like lattice simulations)

$$E_n\ \rightarrow\ \delta(E_n)$$

First lattice simulation of a charmonium resonance above open-charm threshold
taking into account its strong decay $\psi(3770)\rightarrow D\bar{D}$

Lang, Leskovec, Mohler, S.P.,
1503.05363, JHEP 2015

Resonance ψ(3770) and bound st. ψ(2S) from DD scattering in p-wave

$$E_n \;\rightarrow\; \delta(E_n) \;\rightarrow\; p^3 \cot\delta(p)/\sqrt{s}$$

- BW fit (i): $$T_l(s) = \frac{\sqrt{s}\,\Gamma(s)}{m_R^2 - s - i\sqrt{s}\,\Gamma(s)} = \frac{1}{\cot\delta_l(s) - i}$$

$$\Gamma(s) = \frac{g^2}{6\pi}\frac{p^3}{s} \qquad \frac{p^3\cot\delta(s)}{\sqrt{s}} = \frac{6\pi}{g^2}4(p_R^2 - p^2)$$

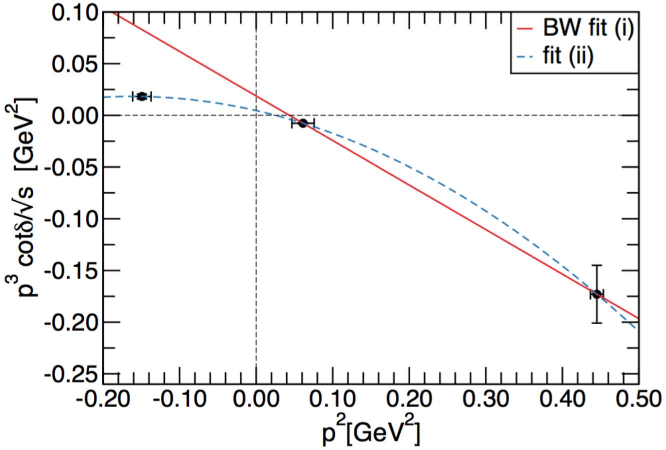
- fit (ii): includes bound st.: $$\frac{p^3}{\sqrt{s}}\cot\delta_1(s) = A + Bp^2 + Cp^4$$

R : m_R: zero, Γ_R : slope near zero

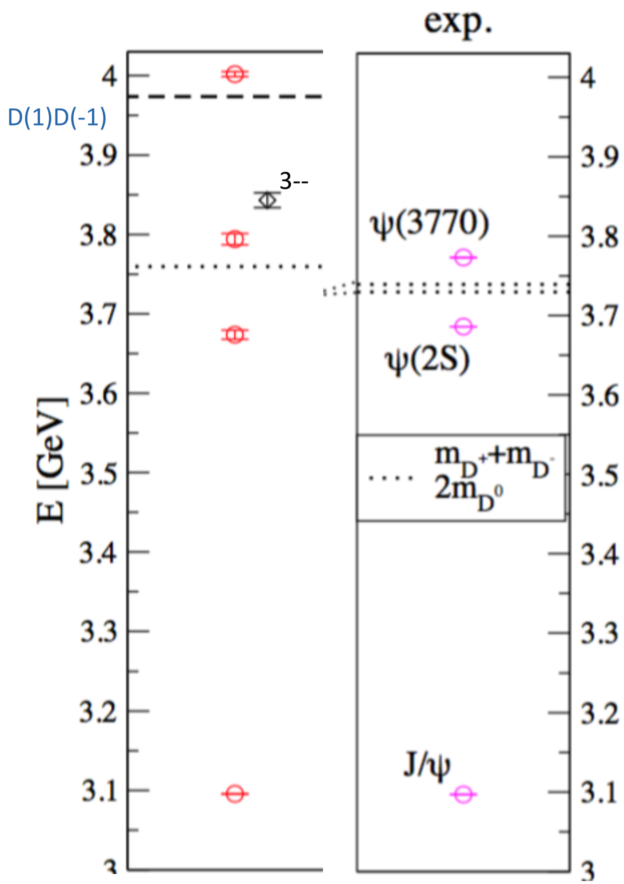
B: cot δ = i

$$p = i|p| \;\rightarrow\; p^3 cot\delta = (i|p|)^3 i = |p|^3 > 0$$

m_π=266 MeV



$$\frac{p^3\cot\delta}{\sqrt{s}}$$

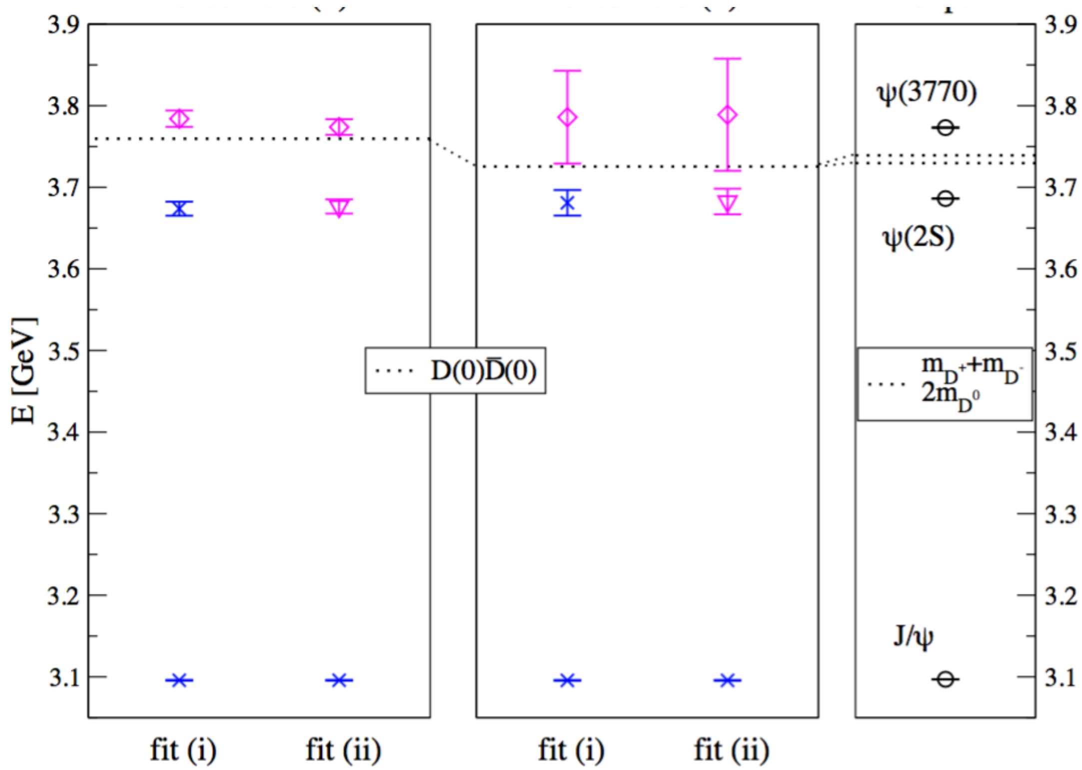


$\psi(2S)$ DD threshold

Lang, Leskovec, Mohler, S.P., 1503.05363, JHEP 2015

Resonance ψ(3770) and bound st. ψ(2S) from DD scattering in p-wave

m_π=266 MeV m_π=156 MeV



BW resonance ψ(3770):

m_R (magenta diamonds)

Γ (given below)

Bound state ψ(2S) from pole in T:

m_B (magetna triangles)

ψ(3770) , fit (ii)	Mass [MeV]	g (no unit)
Lat (m_π=266 MeV)	3774 ±6±10	19.7 ±1.4
Lat (m_π=156 MeV)	3789 ±68±10	28 ± 21
Exp.	3773.15± 0.33	18.7 ± 1.4

$$\Gamma = \frac{g^2}{6\pi}\frac{p^3}{s}$$

Lang, Leskovec, Mohler, S.P., 1503.05363, JHEP 2015]

Lang, Leskovec, Mohler, S.P., JHEP 2015

Scalar charmonia from DD scattering in s-wave, $J^{PC}=0^{++}$: puzzles remain to be solved

DD scattering phase shift

$$E_n \to \delta(E_n) \to p\cot\delta/\sqrt{s}$$

real also below threshold

Near simple BW resonance:

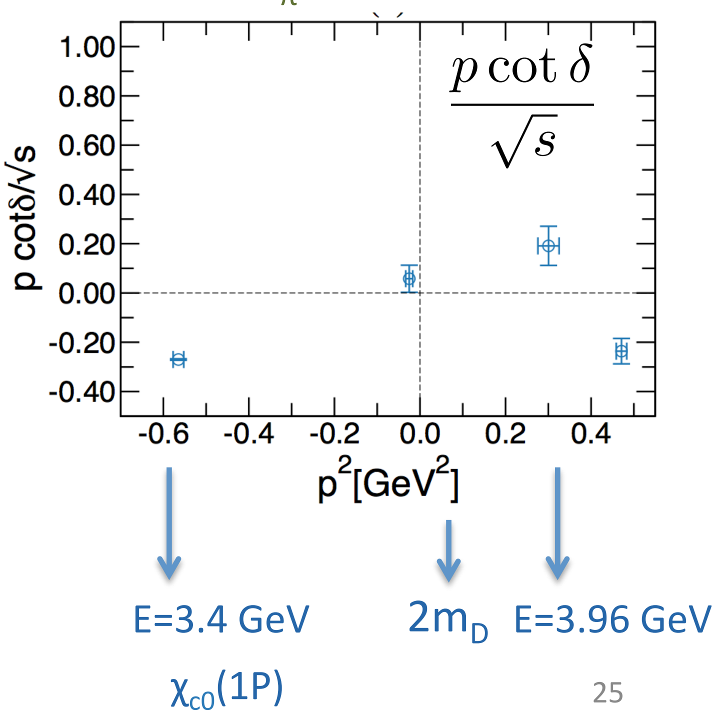
$$\frac{p\cot\delta(s)}{\sqrt{s}} = \frac{4}{g^2}(p_R^2 - p^2)\ , \quad \Gamma(s) = g^2\frac{p}{s}$$

At the bound state pole:

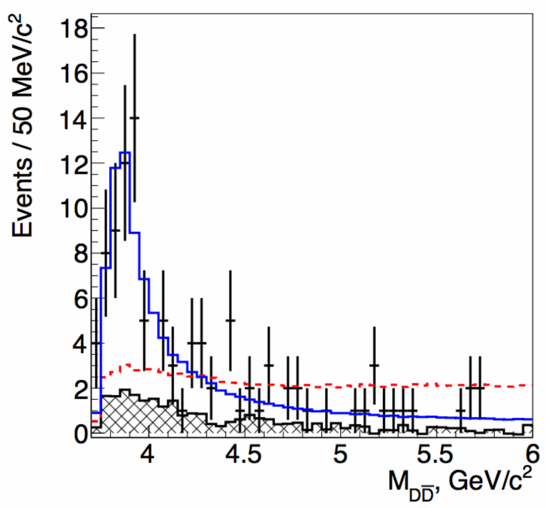
$$p = i|p| \to p\cot\delta = (i|p|)i = -|p| < 0$$

This curious shape seems to suggest narrow resonance and influence from the bound state pole at $\chi_{c0}(1P)$

Lat: m_π=266 MeV



Compatibility of
X*(3860) [Belle 2017] & lattice [Lang et al, 2015] ?



$m = 3862^{+26+40}_{-32-13}$ MeV $\Gamma=201^{+154+88}_{-67-82}$ MeV

- $J^{PC}=0^{++}$ hypothesis favored over 2^{++} at 2.5 σ
- experimental paper compares with our lattice result and indicates compatibility at 2.7 σ level

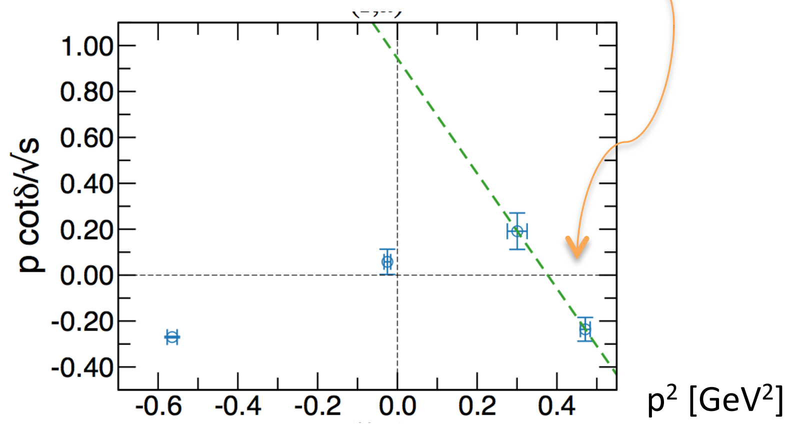
Chilikin et al, Belle,

1704.01872, PRD 2017

$$\frac{p\cot\delta(s)}{\sqrt{s}} = \frac{4}{g^2}(p_R^2 - p^2)\ , \quad \Gamma(s) = g^2\frac{p}{s}$$

Hypothesis: one "narowish" resonance

- BW fit in vicinity of the resonance
- omitting the points away from it
- does not describe our results near threshold



m= 3966 ± 20 MeV

Γ= 67 ± 18 MeV

"pre-dicted"

Lang, Leskovec, Mohler, S.P.,

1503.05363, JHEP 2015, $m_\pi \approx$ 266 MeV

agreement not ideal, but not completely incompatible

clearly more work needed, at least on ab-inito theory side

Conclusions

- bound states in mesonic sector:
  - corresponds to pole in T for p=i|p|, large and negative a0
  - not many candidates
  - D_{s0}*(3217): getting mature on the lattice
  - X(3872): rather solid lattice evidence, but currently less theoretically clean

- resonances in mesonic sector:
  - very mature is several PP scattering channels: ππ, πK
  - studies barely started in others which are particularly interesting: charmonium sector,...

Tomorrow: scattering of hadrons with spin

Scattering of particles with spin

- motivation
- the relation to extract the scattering matrix from energies is known
- construction of operators (interpolators)
  by three different methods that give consistent results: reassuring
- example: Nucleon-pion scattering in p-wave, $J^P=1/2^+$

  lattice results and implications for the Roper resonance

Current status
of hadron-hadron scattering from lattice

- most detailed scattering results exist only for spin-less particles

  ππ , Kπ, KK, DK, Dπ, ...

- $H^{(1)}$ $H^{(2)}$: where one or both H carry spin was explored mostly only for L=0

  many interesting channels still unexplored, particularly for L>0

  only few simulations available for L>0 using Luscher-type method:

  * NN scattering (L=0, L>0), *CALLAT,* Phys. Lett. B 2017
  * Nπ scattering (L=1, I=1/2, Roper Channel), Lang, Leskovec, Padmanath, S.P., PRD 2017
  * Nπ scattering (L=1, I=3/2, Delta Channel), Andersen, Bulava, Hortz, Morningstar PRD 2018
  * ρπ scattering (L=0,2, I=2, non-resonant), HSC, 1802.05580
  * something else ?

Motivation for lattice simulation and building ops

- in lattice QCD:

  hadron-hadron scattering $H^{(1)} H^{(2)}$

  where H is one of P,V,N hadrons, which is (almost) stable with respect to strong decay:

P=psuedoscalar ($J^P=0^{\wedge -}$) = π , K, D, B, η_c , ...

V=vector ($J^P=1^{\wedge -}$) = D*, B*, J/ψ, Υ_b , B_c^*,... (but not ρ as is unstable...)

N=nucleon ($J^P=1/2^{\wedge +}$) = p, n, Λ, Λ_c, Σ, ... (but not N^* as is unstable...)

All combinations of two-hadron scattering are interesting and we will consider building ops for those :

PV: meson resonances and exotics (for example X(3872) in D$\underline{D}$*; Z_c in π J/ψ, D $\underline{D}$* ..)

PN, VN: baryon resonances (e.g. in π N, K N ...) and pentaquarks (e.g. P_c in J/ψ N channel)

NN: nucleon-nucleon and deuterium, baryon-baryon

- in any lattice field theory (beyond SM)

  scattering channels with vector bosons and fermions

The need for interpolators

$$\langle O_i(t) | O_j^+(0) \rangle \rightarrow E_n \rightarrow \text{scattering matrix M}$$

O=HH needed to create/annihilate HH

Relation between scattering matrix M and energies E_n are known

- two spinless particles Luscher (1991):

- two particles with arbitrary spin
  Briceno, PRD89, 074507 (2014)
  (other authors: some specific cases)

scattering matrix: $S = I + i\,M = e^{2i\delta}$

$$\det_{oc}\left[\det_{lSJm_J}\left[\mathcal{M}^{-1} + \delta\mathcal{G}^V\right]\right] = 0$$

$$[\delta\mathcal{G}_j^V]_{Jm_J,lS;J'm_{J'},l'S'} = \frac{ik_j^*\delta_{SS'}}{8\pi E^*} n_j \left[\delta_{JJ'}\delta_{m_J m_{J'}}\delta_{ll'} + i\sum_{l'',m''}\frac{(4\pi)^{3/2}}{k_j^{*l''+1}} c^{\mathbf{d}}_{l''m''}(k_j^{*2};L)\right.$$

$$\left.\times \sum_{m_l,m_{l'},m_S}\langle lS,Jm_J|lm_l,Sm_S\rangle\langle l'm_{l'},Sm_S|l'S,J'm_{J'}\rangle\int d\Omega\, Y^*_{l,m_l}Y^*_{l'',m''}Y_{l',m_{l'}}\right]$$

$$c^{\mathbf{d}}_{lm}(k_j^{*2};L) = \frac{\sqrt{4\pi}}{\gamma L^3}\left(\frac{2\pi}{L}\right)^{l-2}\mathcal{Z}^{\mathbf{d}}_{lm}[1;(k_j^*L/2\pi)^2].$$

related to eigen-energy E_n

Some other analytic work on lattice HH operators for hadrons with spin and L≠0

Partial-wave method for HH:
Berkowitz, Kurth, Nicolson, Joo, Rinaldi, Strother, Walker-Loud, 1508.00886
Wallace, Phys. Rev. D92, 034520 (2015), [arXiv:1506.05492]

Projection method for HH:
M. Göckeler et al., Phys.Rev. D86, 094513 (2012), [arXiv:1206.4141].

Helicity operators for single-H:
Thomas, Edwards and Dudek, Phys. Rev. D85, 014507 (2012), [arXiv:1107.1930]

Some aspects of helicity operators for HH:
Wallace, Phys. Rev. D92, 034520 (2015), [arXiv:1506.05492].
Dudek, Edwards and Thomas, Phys. Rev. D86, 034031 (2012), [arXiv:1203.6041].

Which CG of H_1 and H_2 to H_1H_2 irreps are nonzero; values of CG not published:
Moore and Fleming, Phys. Rev. D 74, 054504 (2006), [arXiv:hep- lat/0607004].

Tetraqurak operators (appeared after our paper on operators)
Cheung, Thomas, Dudek, Edwards [1709.01417, JHEP 2017]

etc ...

Constructing HH operators for scattering with spin: outline

based on S. P., U. Skerbis, C.B. Lang: arXiv:1607:06738, JHEP 2017

- three different methods to construct operators
- illuminate the proofs (given in the paper)
- verify they lead to consistent operators (that gives confidence in each one of them)
- they lead to complementary physics info
- explicit ops for PV, PN, VN, NN for lowest two momentum shells.

Ops with P_{tot}=0 are considered

$H^{(1)}(p)\ H^{(2)}(-p)$, $P_{tot}=0$

Advantage of $P_{tot}=0$:

- parity P is a good number
- channels with even and odd L do not mix in the same irrep

} not true for $P_{tot}\neq 0$

For $P_{tot}\neq 0$: projection method can be applied as here.

Building blocks H: required transformation properties of H

to prove correct transformation properties of HH

rotations R WignerD[{j, m1, m2}, ψ, θ, φ]

inversion I

$|p,s,m_s\rangle \equiv H^\dagger_{m_s}(p)|0\rangle$

$$R|p,s,m_s\rangle = \sum_{m_s'} D^s_{m_s' m_s}(R)|Rp,s,m_s'\rangle\ , \qquad I|p,s,m_s\rangle = (-1)^P|-p,s,m_s\rangle \qquad \text{state}$$

$$RH^\dagger_{m_s}(p)R^{-1} = \sum_{m_s'} D^s_{m_s' m_s}(R) H^\dagger_{m_s'}(Rp)\ , \qquad IH^\dagger_{m_s}(p)I = (-1)^P H^\dagger_{m_s}(-p)\,. \qquad \text{creation field}$$

note: D → D*

$$RH_{m_s}(p)R^{-1} = \sum_{m_s'} D^s_{m_s' m_s}(R)^* H_{m_s'}(Rp)\ , \qquad IH_{m_s}(p)I = (-1)^P H_{m_s}(-p) \qquad \text{annihilation field}$$

$D^s_{m_s m_s'}(R^{-1})$

m_s is a good quantum number at p=0: $S_z H_{m_s}(0) S_z^{-1} = m_s H_{m_s}(0)$

m_s is not good quantum number in general for p≠0: in this case it denotes eigenvalue of S_z of corresponding $H_{ms}(p=0)$

Non-practical choice of $H_{ms}(p)$: canonical fields $H^{(c)}$

with correct transformation properties under R and I

$$H^{(c)}_{m_s}(p) \equiv L(p) H_{m_s}(0)$$

L(p) is boost from 0 to p; drawback: $H^{(c)}(p)$ depend on m, E,..

$$V_{m_s=1}(0) = \tfrac{1}{\sqrt{2}}[-V_x(0) + iV_y(0)] \;\to\; V^{(c)}_{m_s=1}(p_x) = \tfrac{1}{\sqrt{2}}[-\gamma V_x(p_x) + iV_y(p_x)]$$

$$\begin{pmatrix}-1\\ i\\ 0\end{pmatrix} \xrightarrow{\Lambda^1(p_x)} \begin{pmatrix}\gamma & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}-1\\ i\\ 0\end{pmatrix} = \begin{pmatrix}-\gamma\\ i\\ 0\end{pmatrix}$$

$$N_{m_s=1/2}(0) = \mathcal{N}_1(0) \;\to\; N^{(c)}_{m_s=1/2}(p_x) \propto \mathcal{N}_1(p_x) + \tfrac{p_x}{E+m}\mathcal{N}_4(p_x)$$

$$\begin{pmatrix}1\\ 0\\ 0\\ 0\end{pmatrix} \xrightarrow{\Lambda^{1/2}(p_x)} \begin{pmatrix}1\\ 0\\ 0\\ \frac{p_x}{E+m}\end{pmatrix}$$

$\mathcal{N}_{\mu=1,..,4}$ are Dirac components in Dirac basis

Non-practical choice of H: canonical fields $H^{(c)}$

with correct transformation properties under R and I

$$H^{(c)}_{m_s}(p) \equiv L(p) H_{m_s}(0)$$

L(p) is boost from 0 to p; drawback: $H^{(c)}(p)$ depend on m, E,..

$$V_{m_s=1}(0) = \tfrac{1}{\sqrt{2}}[-V_x(0) + iV_y(0)] \;\rightarrow\; V^{(c)}_{m_s=1}(p_x) = \tfrac{1}{\sqrt{2}}[-\gamma V_x(p_x) + iV_y(p_x)]$$

$$\begin{pmatrix} -1 \\ i \\ 0 \end{pmatrix} \xrightarrow{\Lambda^1(p_x)} \begin{pmatrix} \gamma & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ i \\ 0 \end{pmatrix} = \begin{pmatrix} -\gamma \\ i \\ 0 \end{pmatrix}$$

$$N_{m_s=1/2}(0) = \mathcal{N}_1(0) \;\rightarrow\; N^{(c)}_{m_s=1/2}(p_x) \propto \mathcal{N}_1(p_x) + \tfrac{p_x}{E+m}\mathcal{N}_4(p_x)$$

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \xrightarrow{\Lambda^{1/2}(p_x)} \begin{pmatrix} 1 \\ 0 \\ 0 \\ \frac{p_x}{E+m} \end{pmatrix}$$

$\mathcal{N}_{\mu=1,..,4}$ are Dirac components in Dirac basis

Practical choice of $H_{ms}(p)$

with correct transformation properties under R and I

$$V_{m_s=\pm 1}(p) = \tfrac{1}{\sqrt{2}}[\mp V_x(p) + iV_y(p)], \quad V_{m_s=0}(p) = V_z(p)$$

$$N_{m_s=1/2}(p) = \mathcal{N}_{\mu=1}(p)\ , \quad N_{m_s=-1/2}(p) = \mathcal{N}_{\mu=2}(p)$$

$$P(p) = \sum_x \bar{q}(x)\gamma_5 q(x) e^{ipx}$$

$$V_i(p) = \sum_x \bar{q}(x)\gamma_i q(x) e^{ipx}, \quad i = x, y, z$$

$$\mathcal{N}_\mu(p) = \sum_x \epsilon_{abc}[q^{aT}(x) C\gamma_5 q^b(x)]\ q^c_\mu(x)\ e^{ipx}, \quad \mu = 1, .., 4$$

These H are employed as building block in our HH operators

simple examples

Required transformation properties of O=HH

$$RO^{J,m_J}(P_{tot}=0)R^{-1}=\sum_{m'_J} D^J_{m_J m'_J}(R^{-1})O^{J,m'_J}(0) \qquad IO^{J,m_J}(0)I=(-1)^P O^{J,m_J}(0)$$

continuum R

good parity since P_{tot}=0 !

relevant rotations: $R \in O^{(2)}$ O with 24 el. for J=integer ; O^2 with 48 elements for J=half-integer

The group including inversion I: O_h with 48 el. for J=integer ; O^2_h with 96 elements for J=half-integer

The representation O^J is irreducible under continuum R, but it is reducible under discrete R in $O^{(2)}$.

The operators should transform according to certain irreducible representation Γ and its row r.

$$R|\Gamma,r\rangle=\sum_{r'} T^{\Gamma}_{r',r}(R)|\Gamma,r'\rangle \quad R\in O^{(2)}, \qquad I|\Gamma,r\rangle=(-1)^P|\Gamma,r\rangle\ ,$$

$$RO_{\Gamma,r}R^{-1}=\sum_{r'} T^{\Gamma}_{r,r'}(R^{-1})O_{\Gamma,r'} \quad R\in O^{(2)}, \qquad IO_{\Gamma,r}I=(-1)^P O_{\Gamma,r}$$

discrete R

T(R) given for all irreps in
Bernard, Lage, Meißner, Rusetsky, JHEP 2008, 0806.4495
We use same conventions for rows.

J	Γ (dim$_\Gamma$)
0	$A_1(1)$
$\frac{1}{2}$	$G_1(2)$
1	$T_1(3)$
$\frac{3}{2}$	$H(4)$
2	$E(2)\oplus T_2(3)$
$\frac{5}{2}$	$H(4)\oplus G_2(2)$
3	$A_2(1)\oplus T_1(3)\oplus T_2(3)$

Method I: Projection operators

$$O_{|p|,\Gamma,r,n} = \sum_{\tilde{R}\in O_h^{(2)}} T^{\Gamma}_{r,r}(\tilde{R})\ \tilde{R}\ H^{(1),a}(p)\ H^{(2),a}(-p)\ \tilde{R}^{-1}\ , \qquad n = 1,..,n_{max}$$

T(R) given for all irreps in
Bernard, Lage, Meißner, Rusetsky, JHEP 2008, 0806.4495

"seed": Each H^a can have any polarization m_s and direction p with given |p|. Different choices lead to different linearly independent O_n

Some examples for |p|=1:

PV in T_1^+ , n_{max}=2:

$$O_{T_1^+,r=3,n=1} = \mathrm{P}(e_z)V_z(-e_z) + \mathrm{P}(-e_z)V_z(e_z)$$
$$O_{T_1^+,r=3,n=2} = \mathrm{P}(e_x)V_z(-e_x) + \mathrm{P}(-e_x)V_z(e_x) + \mathrm{P}(e_y)V_z(-e_y) + \mathrm{P}(-e_y)V_z(e_y)$$

PN in H^+ , n_{max}=1:

$$O_{H^+,r=1} = -iN_{\frac{1}{2}}(-e_x)\mathrm{P}(e_x) + iN_{\frac{1}{2}}(e_x)\mathrm{P}(-e_x) - N_{\frac{1}{2}}(-e_y)\mathrm{P}(e_y) + N_{\frac{1}{2}}(e_y)\mathrm{P}(-e_y)$$

VN in H^- , n_{max}=3:

$$O_{H^-,r=1,n=1} = iN_{\frac{1}{2}}(e_x)V_x(-e_x) + iN_{\frac{1}{2}}(-e_x)V_x(e_x) + N_{\frac{1}{2}}(e_y)V_y(-e_y) + N_{\frac{1}{2}}(-e_y)V_y(e_y)$$
$$O_{H^-,r=1,n=2} = \dots$$
$$O_{H^-,r=1,n=3} = \dots$$

Disadvantage:

not informative which continuum numbers (partial wave L or helicity) each O_n corresponds

This is remedied in next two methods

Method II: Partial-wave operators

Starting annihilation operator
(before sobduction to irreps)

Clebsch-Gordans — Spherical Harmonics — building blocks H mentioned on slide 10 (bottom)

$$O^{|p|,J,m_J,S,L} = \sum_{m_L,m_S,m_{s1},m_{s2}} C^{Jm_J}_{Lm_L,Sm_S} C^{Sm_S}_{s_1m_{s1},s_2m_{s2}} \sum_{R\in O} Y^*_{Lm_L}(\widehat{Rp})\; H^{(1)}_{m_{s1}}(Rp)\; H^{(2)}_{m_{s2}}(-Rp)$$

Proposed for NN in [Berkowitz, Kurth, Nicolson, Joo, Rinaldi, Strother, Walker-Loud, CALLAT, 1508.00886] There Y_{lm}^* appears where we have Y_{lm}

Proof (in our paper and next slide): the correct transformation properties

$$R_a O^{J,m_J,S,L} R_a^{-1} = \sum_{m'_J} D^J_{m_J m'_J}(R_a^{-1}) O^{J,m'_J,S,L}$$

follow from transformations of H (slide 8) and properties of C, Y_{lm} and D.

Example of PV operators

$$O^{|p|=1,J=1,m_J=0,L=0,S=1} = \sum_{p=\pm e_x,\pm e_y,\pm e_z} \mathrm{P}(p)V_z(-p)\;,$$

$$O^{|p|=1,J=1,m_J=0,L=2,S=1} = \sum_{p=\pm e_x,\pm e_y} \mathrm{P}(p)V_z(-p) - 2\sum_{p=\pm e_z} \mathrm{P}(p)V_z(-p)$$

Subduction to irreps discussed later on.

Proof: partial-wave operators

$$O^{|p|,J,m_J,S,L} = \sum_{m_L,m_S,m_{s1},m_{s2}} C^{Jm_J}_{Lm_L,Sm_S} C^{Sm_S}_{s_1m_{s1},s_2m_{s2}} \sum_{R\in O} Y^*_{Lm_L}(\widehat{Rp})\ H^{(1)}_{m_{s1}}(Rp)\ H^{(2)}_{m_{s2}}(-Rp)$$

Proof of correct transformation properties:

$$R_a O^{J,m_J,S,L} R_a^{-1} = \sum_{m_L,m_S,m_{s1},m_{s2}} C^{Jm_J}_{Lm_L,Sm_S} C^{Sm_S}_{s_1m_{s1},s_2m_{s2}} \sum_{R\in O^{(2)}} Y^*_{Lm_L}(\hat{Rp})\ \underbrace{R_a H_{m_{s1}}(Rp) H_{m_{s2}}(-Rp) R_a^{-1}}$$

$$= \sum_{m_L,m_S,m_{s1},m_{s2}} C^{Jm_J}_{Lm_L,Sm_S} C^{Sm_S}_{s_1m_{s1},s_2m_{s2}} \sum_{R\in O_h} Y^*_{Lm_L}(\hat{Rp})$$

$$\times \sum_{m'_{s1}} D^{s_1}_{m_{s1}m'_{s1}}(R_a^{-1}) H_{m'_{s1}}(R_aRp) \sum_{m'_{s2}} D^{s_2}_{m_{s2}m'_{s2}}(R_a^{-1}) H_{m'_{s2}}(-R_aRp)\ ,$$

$$Y^*_{Lm_L}(Rp) = Y^*_{Lm_L}(R_a^{-1}(R'p)) = \sum_{m'_L} D^L_{m_Lm'_L}(R_a^{-1}) Y^*_{Lm'_L}(R'p) \qquad R' \equiv R_a R \qquad Y^*_{Lm_L}(R_1p) = \textstyle\sum_{m'_L} D^L_{m_Lm'_L}(R_1) Y^*_{Lm'_L}(p)$$

$$D^{s_1}_{m_{s1}m'_{s1}}(R_a^{-1}) D^{s_2}_{m_{s2}m'_{s2}}(R_a^{-1}) = \sum_{\tilde{S},\tilde{m}_S,m'_S} C^{\tilde{S},\tilde{m}_S}_{s_1m_{s1},s_2m_{s2}} C^{\tilde{S},m'_S}_{s_1m'_{s1},s_2m'_{s2}} D^{\tilde{S}}_{\tilde{m}_Sm'_S}(R_a^{-1}) \qquad \sum_{m_{s1},m_{s2}} C^{Sm_S}_{s_1m_{s1},s_2m_{s2}} C^{\tilde{S},\tilde{m}_S}_{s_1m_{s1},s_2m_{s2}} = \delta_{m_S,\tilde{m}_S}\delta_{S,\tilde{S}}$$

$$D^L_{m_Lm'_L}(R_a^{-1}) D^{\tilde{S}}_{\tilde{m}_Sm'_S}(R_a^{-1}) = \sum_{\tilde{J},\tilde{m}_J,m'_J} C^{\tilde{J},\tilde{m}_J}_{Lm_L,\tilde{S}\tilde{m}_S} C^{\tilde{J},m'_J}_{Lm'_L,\tilde{S}m'_S} D^{\tilde{J}}_{\tilde{m}_Jm'_J}(R_a^{-1}) \qquad \sum_{m_L,m_S} C^{Jm_J}_{Lm_L,Sm_S} C^{\tilde{J},\tilde{m}_J}_{Lm_L,Sm_S} = \delta_{m_J,\tilde{m}_J}\delta_{J,\tilde{J}}$$

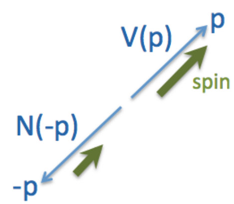
$$R_a O^{J,m_J,S,L} R_a^{-1} =$$

$$= \sum_{m'_J} D^J_{m_Jm'_J}(R_a^{-1}) \sum_{m'_L,m'_S,m'_{s1},m'_{s2}} C^{Jm'_J}_{Lm'_L,Sm'_S} C^{Sm'_S}_{s_1m'_{s1},s_2m'_{s2}} \sum_{R'\in O^{(2)}} Y^*_{Lm'_L}(\hat{R'p}) H_{m'_{s1}}(R'p) H_{m'_{s2}}(-R'p)$$

$$= \sum_{m'_J} D^J_{m_Jm'_J}(R_a^{-1}) O^{J,m'_J,S,L}$$

Method III: helicity operators

[HH in continuum: Jacob, Wick (1959)]
[for single H on lattice: HSC, Thomas et al. (2012)]



- building blocks in partial-wave operators are $H_{ms}(p)$ and m_s is not good for p≠0:
- Helicity λ is projection of S to p. It is good also for particles in flight $$h \equiv S \cdot p \,/\, |p|$$
- Definition of single-hadron helicity operator (denoted by superscript h) $$H^h_\lambda(p) \equiv R^p_0 \; H_{m_s=\lambda}(p_z) \; (R^p_0)^{-1}$$ (rotation from p_z to p; good m_s)
- Helicity is not modified under R (p and S transform the same way) $$R H^h_\lambda(p) R^{-1} = e^{i\varphi(R)} H^h_\lambda(Rp)$$

p is arbitrary momentum in given shell |p|; R does not modify $\lambda_{1,2}$, so $H_{1,2}$ have chosen $\lambda_{1,2}$ in all terms

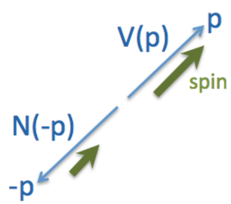
- Two-hadron O: $$O^{|p|,J,m_J,\lambda_1,\lambda_2,\lambda} = \sum_{R\in O^{(2)}} D^J_{m_J,\lambda}(R) \; R \; H^{(1),h}_{\lambda_1}(p) \; H^{(2),h}_{\lambda_2}(-p) \; R^{-1}$$
- Proof:

$$\begin{aligned} R_a O^{J,m_J,\lambda_1,\lambda_2} R_a^{-1} &= \sum_{R\in O^{(2)}} D^J_{m_J,\lambda}(R) \; R_a R \; H^h_{\lambda_1}(p) H^h_{\lambda_2}(-p) \; R^{-1} R_a^{-1} \\ &= \sum_{R\in O^{(2)}} D^J_{m_J,\lambda}(R_a^{-1} R') \; R' \; H^h_{\lambda_1}(p) H^h_{\lambda_2}(-p) \; R'^{-1} \\ &= \sum_{R'\in O^{(2)}} \sum_{m'_J} D^J_{m_J,m'_J}(R_a^{-1}) D^J_{m'_J,\lambda}(R') \;\; R' \; H^h_{\lambda_1}(p) H^h_{\lambda_2}(-p) \; R'^{-1} \\ &= \sum_{m'_J} D^J_{m_J,m'_J}(R_a^{-1}) \; O^{J,m'_J,\lambda_1,\lambda_2}, \end{aligned}$$

$R' = R_a R$

$$D(R_1 R_2) = D(R_1) D(R_2)$$

Method III: helicity operators (continued)



Using definitions of $H^h_\lambda(p) \equiv R_0^p\ H_{m_s=\lambda}(p_z)\ (R_0^p)^{-1}$ and parity projection $\frac{1}{2}(\mathcal{O} + PI\mathcal{O}I)$

p is arbitrary momentum in given shell |p|

$$O^{|p|,J,m_J,P,\lambda_1,\lambda_2,\lambda} = \frac{1}{2}\sum_{R\in O^{(2)}} D^J_{m_J,\lambda}(R)\ RR_0^p\ [H^{(1)}_{m_{s_1}=\lambda_1}(p_z)H^{(2)}_{m_{s_2}=-\lambda_2}(-p_z)$$

$$+ P\ I\ H^{(1)}_{m_{s_1}=\lambda_1}(p_z)H^{(2)}_{m_{s_2}=-\lambda_2}(-p_z)\ I]\ (R_0^p)^{-1}R^{-1}$$

$\lambda = \lambda_1 - \lambda_2$

- H are building blocks from slide 10 (bottom): actions of R and I on $H_{ms}(p)$ are given in slide 8
- p is arbitrary momentum in given shell |p|; there are several choices of R_0^p which rotate from p_z to p:
  - these lead to different phases in definition of H_λ^h : inconvenience
  - but they lead to the same O above (modulo irrelevant overall factor): so no problem for such construction
- Simple choice for momentum shell |p|=1 : $p=p_z$ and R_0^p=Identity
- paper provides details how to use functions from Mathematica for construction, also since Mathematica uses non-conventional defnition of D

$$D^j_{m,m'}[R^\omega_{\alpha\beta\gamma}] = F\cdot \texttt{WignerD}[\{j,m,m'\},-\alpha,-\beta,-\gamma], \qquad F = \begin{cases} 1\ :\ j = \text{integer} \\ \pm 1:\ j = \text{halfinteger},\ \mathrm{F}(\omega+2\pi) = -\mathrm{F}(\omega) \end{cases}$$

, choice of sign in our paper

$$\{\alpha,\beta,\gamma\} = \underbrace{\texttt{EulerAngles}}_{\textsc{Mathematica}}[T] \quad T = \exp(-i\vec{n}\vec{J}\omega) \text{ and } (J_k)_{ij} = -i\epsilon_{ijk}$$

one last step before reaching the results ...

Subduction of $O^{J,mJ}$ to irreducible representations

subduction

continum R

discrete R in discrete group $O^{(2)}$

Partial-wave operators $O^{J,\, mJ,\, L,\, S}$

Helicity operators $O^{J,\, mJ,\, \lambda1,\, \lambda2}$

$$O^{[J,S,L]}_{|p|,\Gamma,r} = \sum_{m_J} \mathcal{S}^{J,m_J}_{\Gamma,r} O^{|p|,J,m_J,S,L}$$

$$O^{[J,P,\lambda_1,\lambda_2,\lambda]}_{|p|,\Gamma,r} = \sum_{m_J} \mathcal{S}^{J,m_J}_{\Gamma,r} O^{|p|,J,m_J,P,\lambda_1,\lambda_2,\lambda}$$

The representation O^J is irreducible under continuum R.

But it is reducible under R in discrete group lattice $O^{(2)}$.

Operators that transform according to irrep Γ and row r obtained via subduction.

Subduction matrices S

[Dudek et al., PRD82, 034508 (2010)

Edwards et al, PRD84, 074508 (2011)]

Single-hadron operators H: experience by Hadron Spectrum collaboration Phys. Rev. D 82, 034508 (2010)

- subduced operators $O^{[J]}_{\Gamma}$ carry memory of continuum spin and dominantly couple to states with this J

Expectation for partial-wave and helicity operators HH obtained by subduction :

- $O^{[J,S,L]}_{|p|,\Gamma,r}$ would dominantly couple to eigen-states with continuum (J,L,S)
- $O^{[J,P,\lambda_1,\lambda_2,\lambda]}_{|p|,\Gamma,r}$ would dominantly couple to eigen-states with continuum (J,λ1,λ2)

valuable for simulations

give physics intuition on quant. num.

Example: P(p)V(-p) operators

J	Γ (dim$_\Gamma$)
0	$A_1(1)$
$\frac{1}{2}$	$G_1(2)$
1	$T_1(3)$
$\frac{3}{2}$	$H(4)$
2	$E(2) \oplus T_2(3)$
$\frac{5}{2}$	$H(4) \oplus G_2(2)$
3	$A_2(1) \oplus T_1(3) \oplus T_2(3)$

row=1 provided

Conventions for row
Bernard et al. , 0806.4495

rows of T1: (x,y,z)

rows of T2: (yz,xz,xy)

$|p| = 0$

T_1^+ :

$$O_{T_1^+,r=1} = \mathrm{P}(0)V_x(0)$$

$$O^{[J=1,L=0,S=1]}_{T_1^+,r=1} = O_{T_1^+,r=1}$$

other irreps: O=0

$|p| = 1$

A_1^- :

$$O_{A_1^-,r=1} = \mathrm{P}(e_x)V_x(-e_x) - \mathrm{P}(-e_x)V_x(e_x) + \mathrm{P}(e_y)V_y(-e_y) - \mathrm{P}(-e_y)V_y(e_y) + \mathrm{P}(e_z)V_z(-e_z) - \mathrm{P}(-e_z)V_z(e_z)$$

$$O^{[J=0,m_J=0,P=-,\lambda_V=0,\lambda_P=0]}_{A_1^-,r=1} = O^{[J=0,m_J=0,L=1,S=1]}_{A_1^-,r=1} = O_{A_1^-,r=1}$$

T_1^+ :

$$O_{T_1^+,r=1,n=1} = \mathrm{P}(e_x)V_x(-e_x) + \mathrm{P}(-e_x)V_x(e_x)$$
$$O_{T_1^+,r=1,n=2} = \mathrm{P}(e_y)V_x(-e_y) + \mathrm{P}(-e_y)V_x(e_y) + \mathrm{P}(e_z)V_x(-e_z) + \mathrm{P}(-e_z)V_x(e_z)$$

$$O^{[J=1,P=+,\lambda_V=\pm1,\lambda_P=0]}_{T_1^+,r=1} = O_{T_1^+,r=1,n=2}$$
$$O^{[J=1,P=+,\lambda_V=0,\lambda_P=0]}_{T_1^+,r=1} = O_{T_1^+,r=1,n=1}$$

$$O^{[J=1,L=0,S=1]}_{T_1^+,r=1} = O_{T_1^+,r=1,n=1} + O_{T_1^+,r=1,n=2}$$
$$O^{[J=1,L=2,S=1]}_{T_1^+,r=1} = -2\ O_{T_1^+,r=1,n=1} + O_{T_1^+,r=1,n=2}$$

T_1^- :

$$O_{T_1^-,r=1} = -\mathrm{P}(e_y)V_z(-e_y) + \mathrm{P}(-e_y)V_z(e_y) + \mathrm{P}(e_z)V_y(-e_z) - \mathrm{P}(-e_z)V_y(e_z)$$

$$O^{[J=1,P=-,\lambda_V=\pm1,\lambda_P=0]}_{T_1^-,r=1} = O^{[J=1,L=1,S=1]}_{T_1^-,r=1} = O_{T_1^-,r=1}$$

T_2^+ :

$$O_{T_2^+,r=1} = \mathrm{P}(e_y)V_x(-e_y) + \mathrm{P}(-e_y)V_x(e_y) - \mathrm{P}(e_z)V_x(-e_z) - \mathrm{P}(-e_z)V_x(e_z)$$

$$O^{[J=2,P=+,\lambda_V=\pm1,\lambda_P=0]}_{T_2^+,r=1} = O^{[J=2,L=2,S=1]}_{T_2^+,r=1} = O_{T_2^+,r=1}$$

T_2^- :

$$O_{T_2^-,r=1} = \mathrm{P}(e_y)V_z(-e_y) - \mathrm{P}(-e_y)V_z(e_y) + \mathrm{P}(e_z)V_y(-e_z) - \mathrm{P}(-e_z)V_y(e_z)$$

$$O^{[J=2,P=-,\lambda_V=\pm1,\lambda_P=0]}_{T_2^-,r=1} = O^{[J=2,L=1,S=1]}_{T_2^-,r=1} = O^{[J=2,L=3,S=1]}_{T_2^-,r=1} = O_{T_2^-,r=1}$$

E^- :

$$O_{E^-,r=1} = \mathrm{P}(e_x)V_x(-e_x) - \mathrm{P}(-e_x)V_x(e_x) + \mathrm{P}(e_y)V_y(-e_y) - \mathrm{P}(-e_y)V_y(e_y) - 2\mathrm{P}(e_z)V_z(-e_z) + 2\mathrm{P}(-e_z)V_z(e_z)$$

$$O^{[J=2,P=-,\lambda_V=0,\lambda_P=0]}_{E^-,r=1} = O^{[J=2,L=1,S=1]}_{E^-,r=1} = O^{[J=2,L=3,S=1]}_{E^-,r=1} = O_{E^-,r=1}$$

$$O_{A_1^+} = O_{A_2^+} = O_{A_2^-} = O_{E^+} = 0\ .$$

Example: P(p)V(-p) operators

$|p| = 1$

A_1^- :

$$O_{A_1^-,r=1} = \mathrm{P}(e_x)V_x(-e_x) - \mathrm{P}(-e_x)V_x(e_x) + \mathrm{P}(e_y)V_y(-e_y) - \mathrm{P}(-e_y)V_y(e_y) + \mathrm{P}(e_z)V_z(-e_z) - \mathrm{P}(-e_z)V_z(e_z)$$

$$O^{[J=0,m_J=0,P=-,\lambda_V=0,\lambda_P=0]}_{A_1^-,r=1} = O^{[J=0,m_J=0,L=1,S=1]}_{A_1^-,r=1} = O_{A_1^-,r=1}$$

T_1^+ :

projection operators

$$O_{T_1^+,r=1,n=1} = \mathrm{P}(e_x)V_x(-e_x) + \mathrm{P}(-e_x)V_x(e_x)$$

$$O_{T_1^+,r=1,n=2} = \mathrm{P}(e_y)V_x(-e_y) + \mathrm{P}(-e_y)V_x(e_y) + \mathrm{P}(e_z)V_x(-e_z) + \mathrm{P}(-e_z)V_x(e_z)$$

provides lin. combination of projection operators O_n that enhances the coupling to state with continuum (J^P, λ_V) — helicity operators

$$O^{[J=1,P=+,\lambda_V=\pm1,\lambda_P=0]}_{T_1^+,r=1} = O_{T_1^+,r=1,n=2}$$ $J^P=1^+$, $\lambda_V=0$

$$O^{[J=1,P=+,\lambda_V=0,\lambda_P=0]}_{T_1^+,r=1} = O_{T_1^+,r=1,n=1}$$ $J^P=1^+$, $\lambda_V=1$

provides lin. combination of projection operators O_n that enhances the coupling to state with continuum (J^P, S,L) — partial-wave operators

$$O^{[J=1,L=0,S=1]}_{T_1^+,r=1} = O_{T_1^+,r=1,n=1} + O_{T_1^+,r=1,n=2}$$ $J^P=1^+$, S=1,L=0

$$O^{[J=1,L=2,S=1]}_{T_1^+,r=1} = -2\ O_{T_1^+,r=1,n=1} + O_{T_1^+,r=1,n=2}$$ $J^P=1^+$, S=1,L=2

T_1^- :

$$O_{T_1^-,r=1} = -\mathrm{P}(e_y)V_z(-e_y) + \mathrm{P}(-e_y)V_z(e_y) + \mathrm{P}(e_z)V_y(-e_z) - \mathrm{P}(-e_z)V_y(e_z)$$

$$O^{[J=1,P=-,\lambda_V=\pm1,\lambda_P=0]}_{T_1^-,r=1} = O^{[J=1,L=1,S=1]}_{T_1^-,r=1} = O_{T_1^-,r=1}$$

T_2^+ :

$$O_{T_2^+,r=1} = \mathrm{P}(e_y)V_x(-e_y) + \mathrm{P}(-e_y)V_x(e_y) - \mathrm{P}(e_z)V_x(-e_z) - \mathrm{P}(-e_z)V_x(e_z)$$

$$O^{[J=2,P=+,\lambda_V=\pm1,\lambda_P=0]}_{T_2^+,r=1} = O^{[J=2,L=2,S=1]}_{T_2^+,r=1} = O_{T_2^+,r=1}$$

T_2^- :

$$O_{T_2^-,r=1} = \mathrm{P}(e_y)V_z(-e_y) - \mathrm{P}(-e_y)V_z(e_y) + \mathrm{P}(e_z)V_y(-e_z) - \mathrm{P}(-e_z)V_y(e_z)$$

$$O^{[J=2,P=-,\lambda_V=\pm1,\lambda_P=0]}_{T_2^-,r=1} = O^{[J=2,L=1,S=1]}_{T_2^-,r=1} = O^{[J=2,L=3,S=1]}_{T_2^-,r=1} = O_{T_2^-,r=1}$$

E^- :

$$O_{E^-,r=1} = \mathrm{P}(e_x)V_x(-e_x) - \mathrm{P}(-e_x)V_x(e_x) + \mathrm{P}(e_y)V_y(-e_y) - \mathrm{P}(-e_y)V_y(e_y) - 2\mathrm{P}(e_z)V_z(-e_z) + 2\mathrm{P}(-e_z)V_z(e_z)$$

$$O^{[J=2,P=-,\lambda_V=0,\lambda_P=0]}_{E^-,r=1} = O^{[J=2,L=1,S=1]}_{E^-,r=1} = O^{[J=2,L=3,S=1]}_{E^-,r=1} = O_{E^-,r=1}$$

$$O_{A_1^+} = O_{A_2^+} = O_{A_2^-} = O_{E^+} = 0\,.$$

J	Γ (dim$_\Gamma$)
0	$A_1(1)$
$\frac{1}{2}$	$G_1(2)$
1	$T_1(3)$
$\frac{3}{2}$	$H(4)$
2	$E(2)\oplus T_2(3)$
$\frac{5}{2}$	$H(4)\oplus G_2(2)$
3	$A_2(1)\oplus T_1(3)\oplus T_2(3)$

$|p| = 0$

T_1^+ :

$$O_{T_1^+,r=1} = \mathrm{P}(0)V_x(0)$$

$$O^{[J=1,L=0,S=1]}_{T_1^+,r=1} = O_{T_1^+,r=1}$$

other irreps: O=0

row=1 provided

Conventions for row
Bernard et al. , 0806.4495

rows of T1: (x,y,z)

rows of T2: (yz,xz,xy)

P(1)V(-1) operators, T_1^+, row=r=1

projection op.

$$T_1^+ :$$
$$O_{T_1^+,r=1,n=1} = \mathrm{P}(e_x)V_x(-e_x) + \mathrm{P}(-e_x)V_x(e_x)$$
$$O_{T_1^+,r=1,n=2} = \mathrm{P}(e_y)V_x(-e_y) + \mathrm{P}(-e_y)V_x(e_y) + \mathrm{P}(e_z)V_x(-e_z) + \mathrm{P}(-e_z)V_x(e_z)$$

partial-wave op.

$$O^{[J=1,L=0,S=1]}_{T_1^+,r=1} = O_{T_1^+,r=1,n=1} + O_{T_1^+,r=1,n=2}$$ $J^P=1^+$, S=1,L=0

$$O^{[J=1,L=2,S=1]}_{T_1^+,r=1} = -2\ O_{T_1^+,r=1,n=1} + O_{T_1^+,r=1,n=2}$$ $J^P=1^+$, S=1,L=2

helicity op.

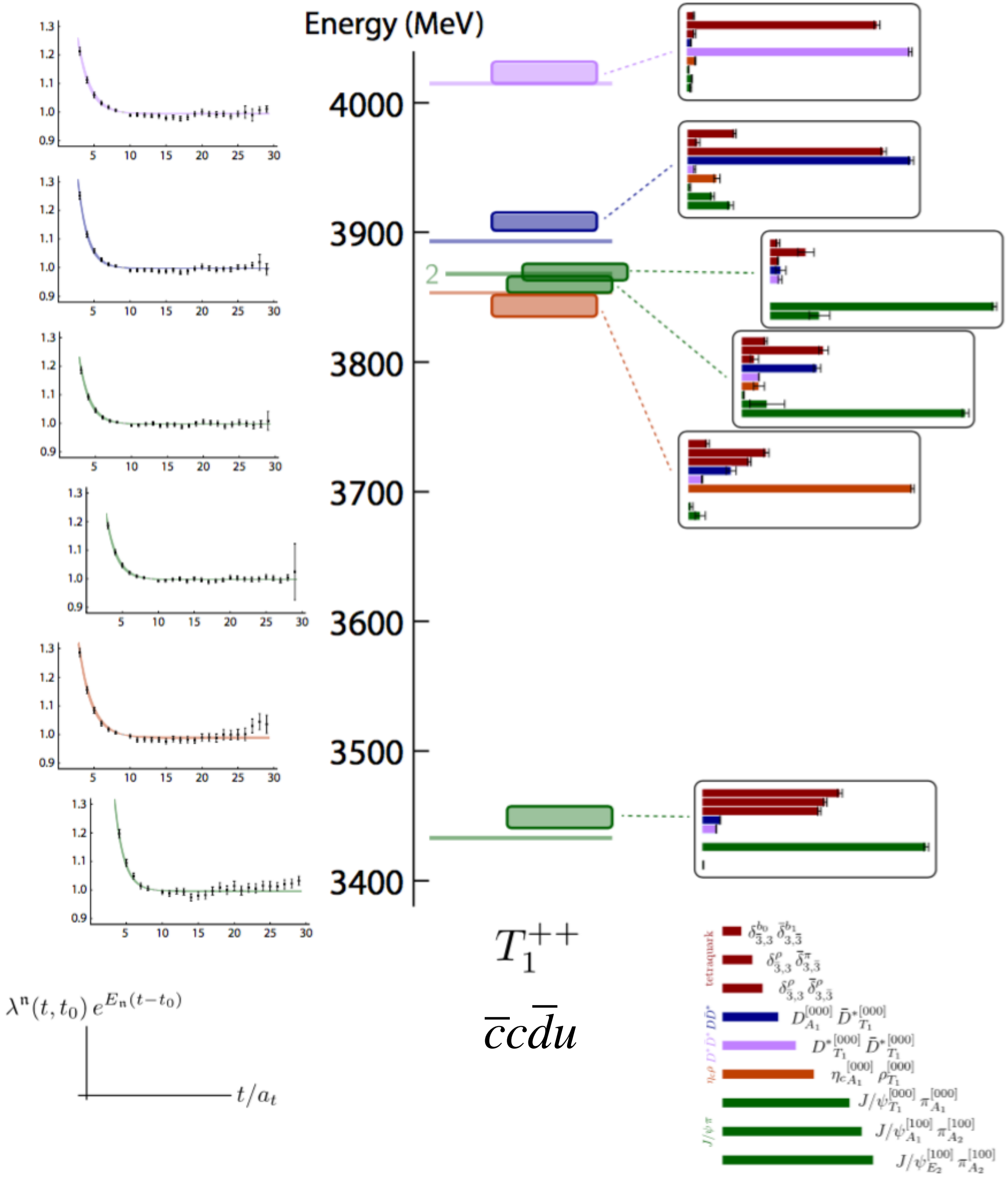
$$O^{[J=1,P=+,\lambda_V=0,\lambda_P=0]}_{T_1^+,r=1} = O_{T_1^+,r=1,n=1}$$ $J^P=1^+$, $\lambda_V=0$

$$O^{[J=1,P=+,\lambda_V=\pm1,\lambda_P=0]}_{T_1^+,r=1} = O_{T_1^+,r=1,n=2}$$ $J^P=1^+$, $\lambda_V=1$

Partial-wave and helicity operators expressed in terms of projection operators throughout and consistency is found.

Two levels P(1) V(-1)=π(1) J/Ψ(-1) observed in T_1^+

HSC, Gavin Cheung et al, JHEP 2017



Sasa Prelovsek

Results for operators

Explicit expressions all for $H^{(1)}(p)H^{(2)}(-p)$

- PV, PN, VN, NN

- in three methods

- all irreps, |p|=0,1

given in [S. P., U. Skerbis, C.B. Lang, arXiv:1607:06738, JHEP 2016]

operators from three methods are consistent (not equal) with each other

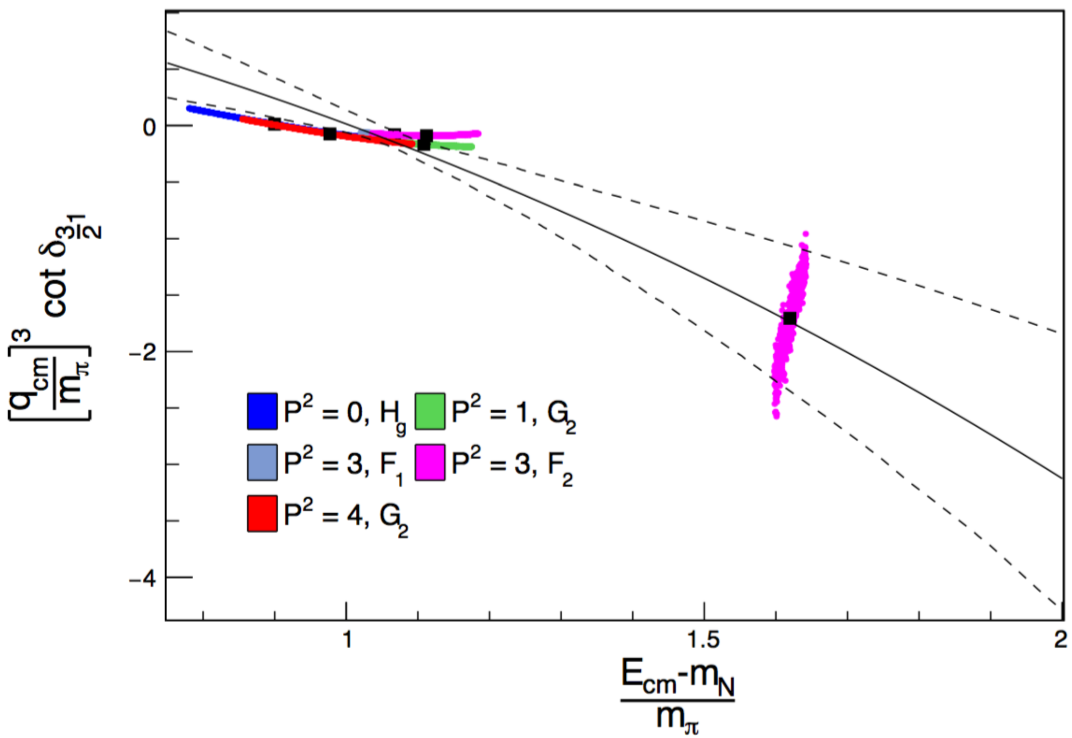
Relation between partial-wave and helicity operators is derived

$$O^{|p|,J,m_J,S,L} = \sqrt{\frac{2L+1}{4\pi}} \sum_{\lambda=-S}^{S} \sum_{\lambda_1,\lambda_2} \sum_{\lambda'} D^J_{\lambda',\lambda}(R_0^p) C^{J\lambda}_{L0,S\lambda} C^{S\lambda}_{s_1\lambda_1,s_2\,-\lambda_2}\; O^{|p|,J,m_J,\lambda',\lambda_1,\lambda_2}$$

Nπ scattering in the Δ(1232) channel

L=1, I=3/2, $J^P=3/2^+$

Andersen, Bulava, Hortz, Morningstar, PRD 2018



$$\frac{m_\Delta}{m_\pi} = 4.738(47), \quad g^{\rm BW}_{\Delta N\pi} = 19.0(4.7),$$

$$g^{\rm BW,exp}_{\Delta N\pi} \approx 16.9$$

m_Δ~ 1.327 (13) GeV

$$\left(\frac{q_{\rm cm}}{m_\pi}\right)^3 \cot\delta_{\frac{3}{2}1} = \left(\frac{m_\Delta^2}{m_\pi^2} - \frac{E_{\rm cm}^2}{m_\pi^2}\right)\frac{6\pi E_{\rm cm}}{(g^{\rm BW}_{\Delta N\pi})^2 m_\pi}$$

CLS

ID	β	a(fm)	$L^3 \times T$	m_π, m_K(MeV)	$N_{\rm conf}$	N_{t_0}
N401	3.46	0.0765	$48^3 \times 128$	280, 460	275	2

L=1, I=1/2

Nπ scattering in $J^P=½^+$ and the Roper resonance

C.B. Lang, L. Leskovec, M. Padmanath, S.P.

Phys. Rev. D 95 (2017) 014510; hep-lat:1610.01422

Brief intro to Roper resonance

uud

p
n
N(1440) $1/2^+$
N(1520) $3/2^-$
N(1535) $1/2^-$
N(1650) $1/2^-$
N(1675) $5/2^-$
N(1680) $5/2^+$
N(1685) $?^?$
N(1700) $3/2^-$
N(1710) $1/2^+$
N(1720) $3/2^+$
N(1860) $5/2^+$
N(1875) $3/2^-$
N(1880) $1/2^+$
N(1895) $1/2^-$
N(1900) $3/2^+$
N(1990) $7/2^+$
N(2000) $5/2^+$
N(2040) $3/2^+$
N(2060) $5/2^-$
N(2100) $1/2^+$

$N(1440)$ BREIT-WIGNER MASS	1410 to 1450 (≈ 1430) MeV
$N(1440)$ BREIT-WIGNER WIDTH	250 to 450 (≈ 350) MeV

$N\pi$	$55-75\%$
$N\eta$	$< 1\%$
$N\pi\pi$	25–50%
$\Delta(1232)\pi$	20–30%
$\Delta(1232)\pi$, P–wave	$13-27\%$
$N\sigma$	$11-23\%$

Puzzling since its discovery in 1964 by L.D. Roper.

In particular: why is it lighter than N(1535) with $½^-$?

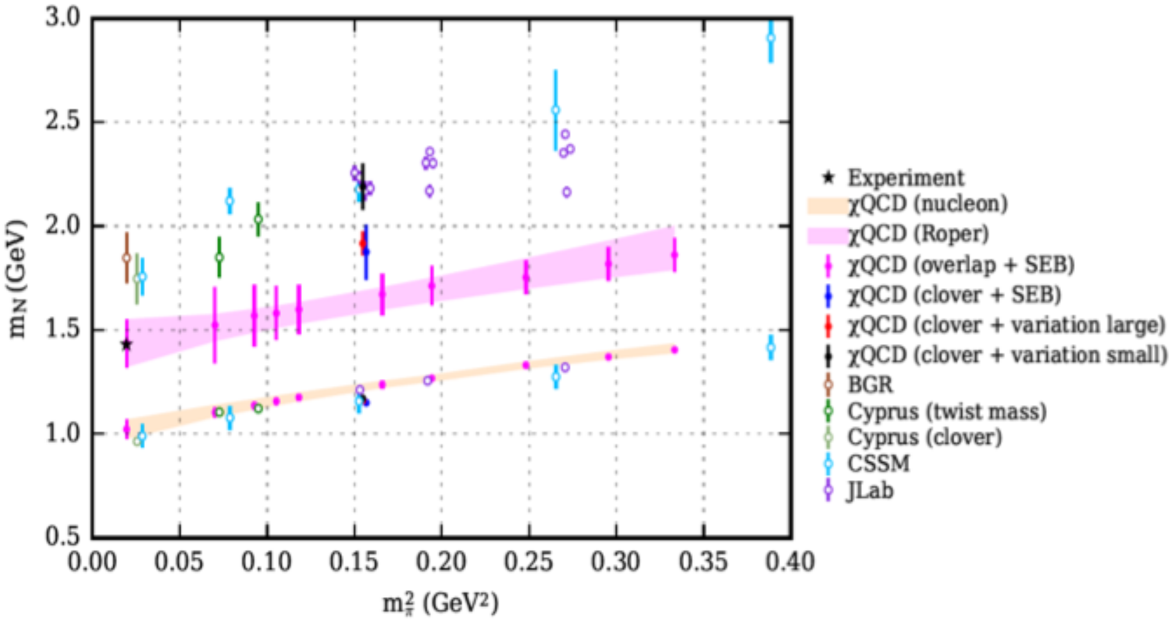
Some previous simulations of the proton/Roper channel: $J^P=1/2^+$

- all used just O=qqq interpolators (with exception of Adelaide 1608.03051 which did not find two-hadron state in spite of that)
- ignored that Roper is strongly decaying resonance
- assumed that $E_1=m_N$ (correct)

  $E_2=m_R$ (not correct); E_2 could in principle be energy of Nπ eigenstate

E_2

$E_1=m_N$



χQCD : Liu *et al.*, arXiv:1403.6847[hep-ph]

BGR : Engel *et al.*, PRD, arXiv:1301.4318[hep-lat]

Cyprus : Alexandrou *et al.*, PRD, arXiv:1411.6765[hep-lat]

JLab : Edwards *et al.*, PRD, arXiv:1104.5152[hep-lat]

CSSM : Adelaide group, PLB, arXiv:1011.5724[hep-lat] Figure courtesy : K. F. Liu, arXiv:1609.02572

Lattice simulation

Lattice size	N_f	$N_{\rm cfgs}$	m_π [MeV]	a [fm]	L [fm]
$32^3 \times 64$	$2+1$	197(193)	156(7)(2)	0.0907(13)	2.9

$m_\pi L \approx 2.3$ (!)

PACS-CS lattices, Aoki *et al.*, PRD, arXiv:0807.1661.

- Wilson clover fermions

- Lowest non-interacting N(1)π(-1) states in p-wave expected at

$$E \approx \sqrt{(\tfrac{2\pi}{L})^2 + m_\pi^2} + \sqrt{(\tfrac{2\pi}{L})^2 + m_N^2} \approx 1.5\,\mathrm{GeV}$$

This is in the Roper resonance region: favorable

Implementing nucelon-pion interpolators in $J^P=1/2^+$ channel

(for the first time in this channel)

- only total momentum P=0 is simulated
- P≠0 not used (since p-wave mixes with s-wave in all irreps where p-wave appears)

momenta of hadrons in units of 2π/L

$$O^{N\pi}_{1,2} = -\sqrt{\tfrac{1}{3}}\ [p^{1,2}_{-\frac{1}{2}}(-e_x)\pi^0(e_x) - p^{1,2}_{-\frac{1}{2}}(e_x)\pi^0(-e_x) - ip^{1,2}_{-\frac{1}{2}}(-e_y)\pi^0(e_y) + ip^{1,2}_{-\frac{1}{2}}(e_y)\pi^0(-e_y) + p^{1,2}_{\frac{1}{2}}(-e_z)\pi^0(e_z) - p^{1,2}_{\frac{1}{2}}(e_z)\pi^0(-e_z)] + \sqrt{\tfrac{2}{3}}\ [\{p\to n, \pi^0\to\pi^+\}] \quad [narrower]$$

$$O^{N_w}_{3,4,5} = p^{1,2,3}_{\frac{1}{2}}(0) \quad [wider]$$

$$O^{N_n}_{6,7,8} = p^{1,2,3}_{\frac{1}{2}}(0) \quad [narrower]$$

$$O^{N\sigma}_{9,10} = p^{1,2}_{\frac{1}{2}}(0)\sigma(0) \quad [narrower]$$

Nπ in p-wave: irrep G_1^+

all three methods give one and the same O

"Lattice operators for scattering of particles with Spin", JHEP 2017, S. P., U. Skerbis, C.B. Lang

Nσ in s-wave

$$N^i_{m_s=1/2}(\mathbf{n}) = \mathcal{N}^i_{\mu=1}(\mathbf{n}),\ N^i_{m_s=-1/2}(\mathbf{n}) = \mathcal{N}^i_{\mu=2}(\mathbf{n})$$

$$\mathcal{N}^i_\mu(\mathbf{n}) = \sum_{\mathbf{x}} \epsilon_{abc}[u^{aT}(\mathbf{x},t)\Gamma^i_2 d^b(\mathbf{x},t)]\ [\Gamma^i_1 q^c(\mathbf{x},t)]_\mu\ e^{i\mathbf{x}\cdot\mathbf{n}\frac{2\pi}{L}}$$

$$i=1,2,3: \quad (\Gamma^i_1,\Gamma^i_2) = (\mathbf{1}, C\gamma_5),\ (\gamma_5, C),\ (i\mathbf{1}, C\gamma_t\gamma_4)$$

$$\pi^+(\mathbf{n}) = \sum_{\mathbf{x}} \bar{d}(\mathbf{x},t)\gamma_5 u(\mathbf{x},t)e^{i\mathbf{x}\cdot\mathbf{n}\frac{2\pi}{L}}$$

$$\sigma(0) = \frac{1}{\sqrt{2}}\sum_{\mathbf{x}}[\bar{u}(\mathbf{x},t)u(\mathbf{x},t) + \bar{d}(\mathbf{x},t)d(\mathbf{x},t)]\ .$$

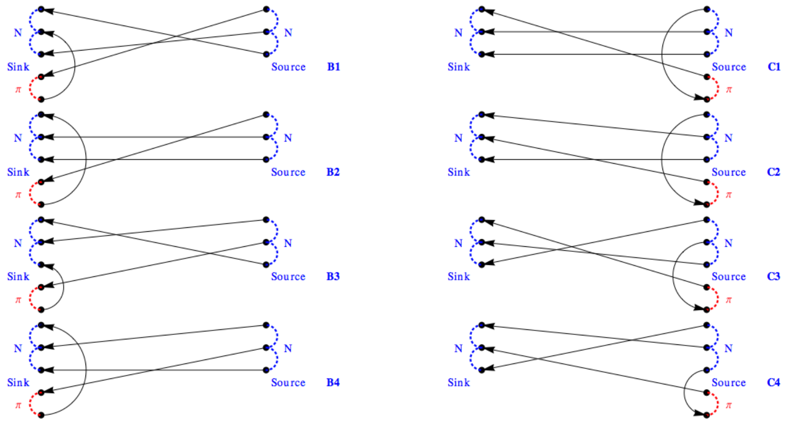
Computing 10x10 matrix C: Wick contractions

part of them are similar as in Nπ in s-wave
[Verduci, Lang, PRD 2013] plots taken from there

$$C_{ij}(t) = \langle \Omega | O_i(t + t_{\rm src}) \bar{O}_j(t_{\rm src}) | \Omega \rangle$$

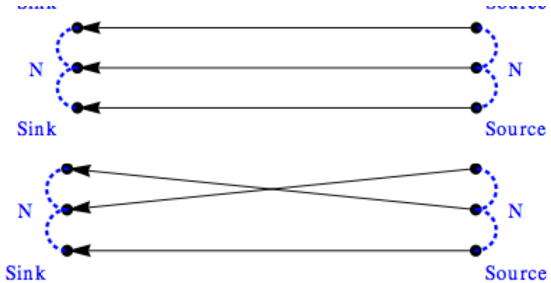
TABLE III. Number of Wick contractions involved in computing correlation functions between interpolators in Eq. (7).

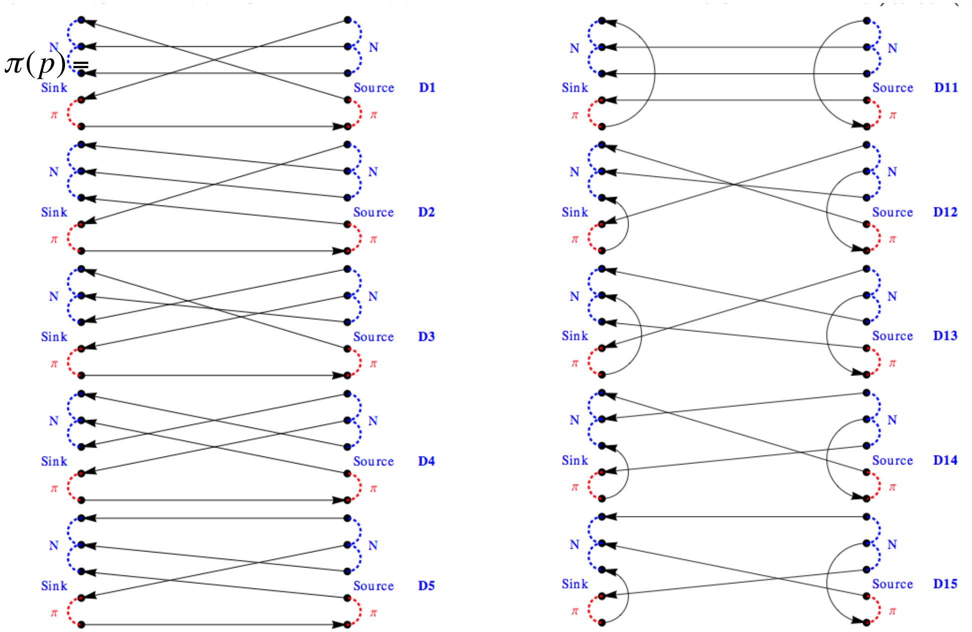
$O_i \backslash O_j$	O^N	$O^{N\pi}$	$O^{N\sigma}$
O^N	2	4	7
$O^{N\pi}$	4	19	19
$O^{N\sigma}$	7	19	33



- just part of Wick contractions plotted
- computational challenge:

- all-to-all quark propagators needed;
- full distillation employed [Peardon et al, 2009]





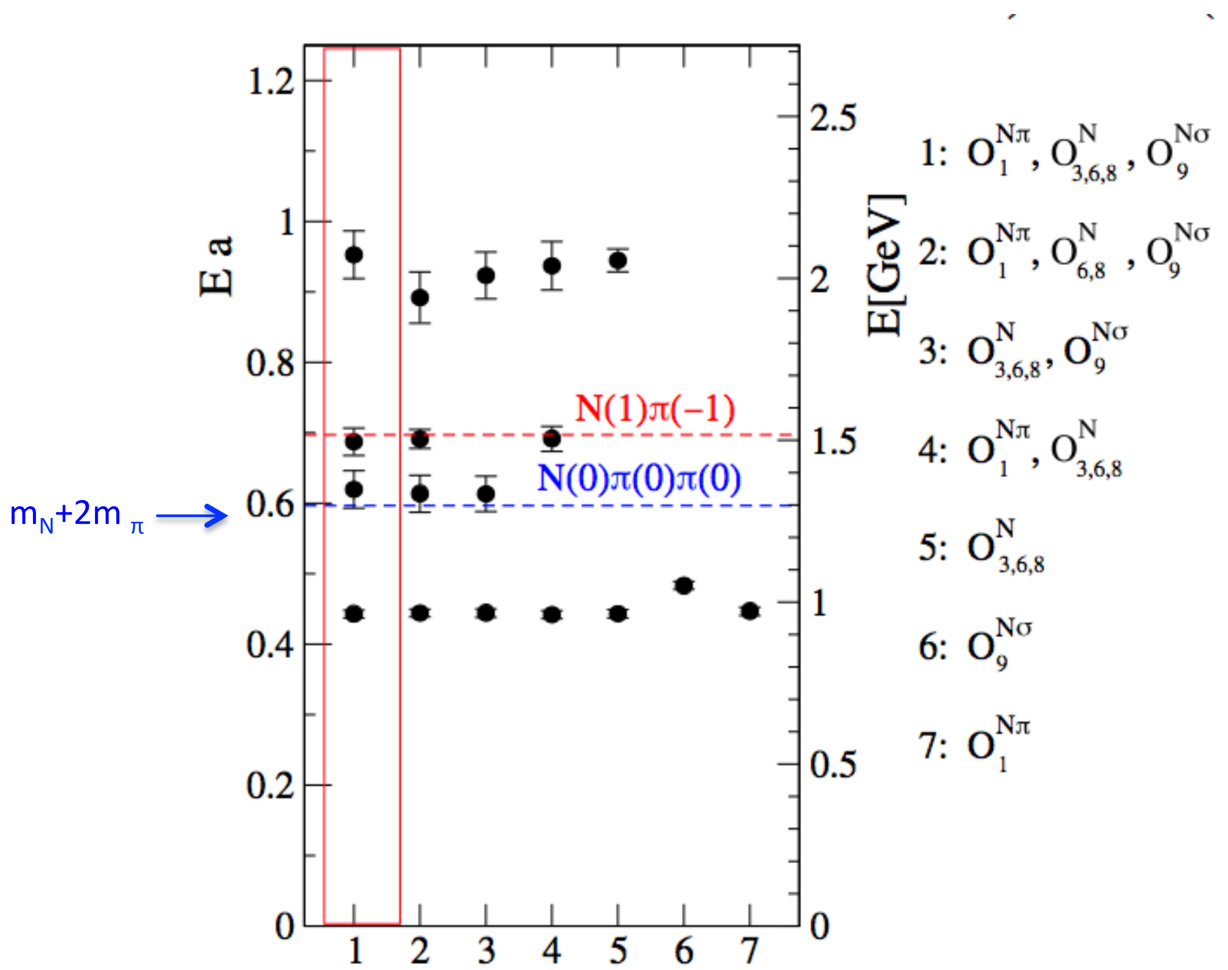
$$\pi(p) = \sum_x e^{ipx} \bar{d}(x) \gamma_5 u(x)$$

$$C_{ij}(t) = \sum_n Z_i^n Z_j^{n*} e^{-E_n t}$$

$$C(t) v^{(n)}(t) = C(t_0) \lambda(t) v^{(n)}(t)$$

[Luscher & Wolf 1991, Blossier et al 2009]

E_n: dependence on the interpolators used



note again:

total momentum P=0

N(0)π(0) not possible

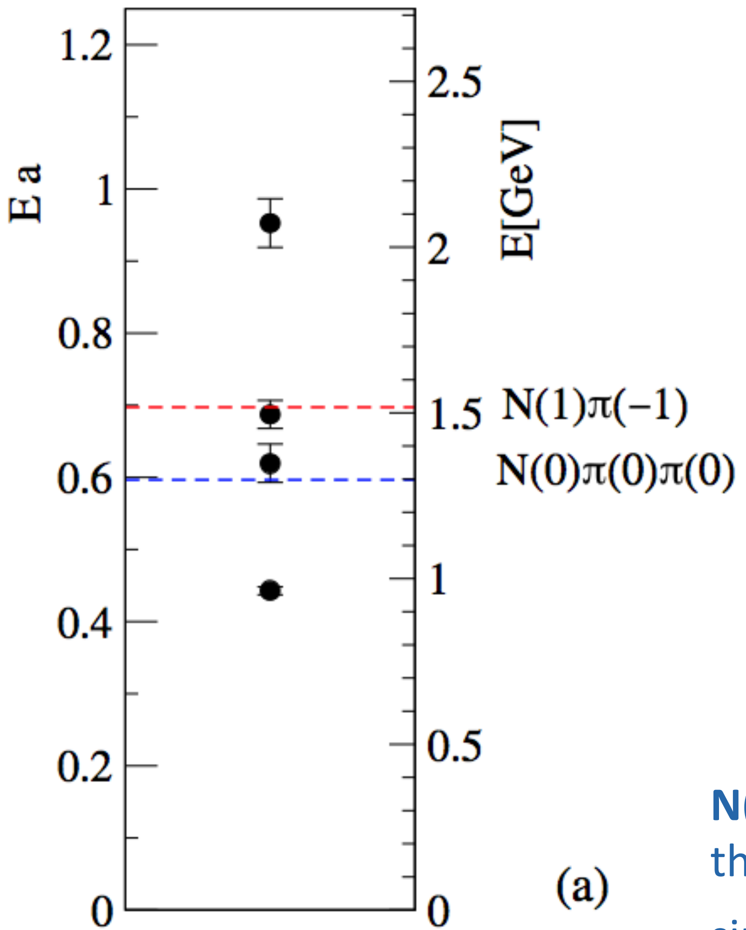
for p-wave

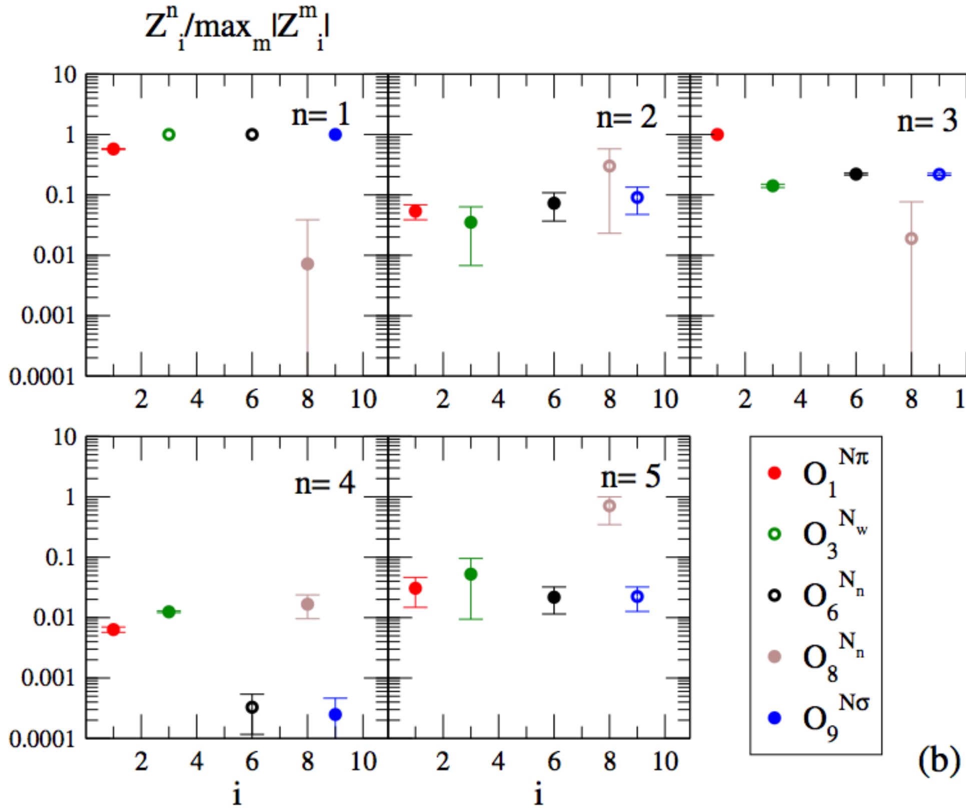
lowest non-interacting

Nπ state is N(1)π(-1)

p= 1 * 2π/L

Final E_n and overlaps $Z_i^n = <O_i|n>$





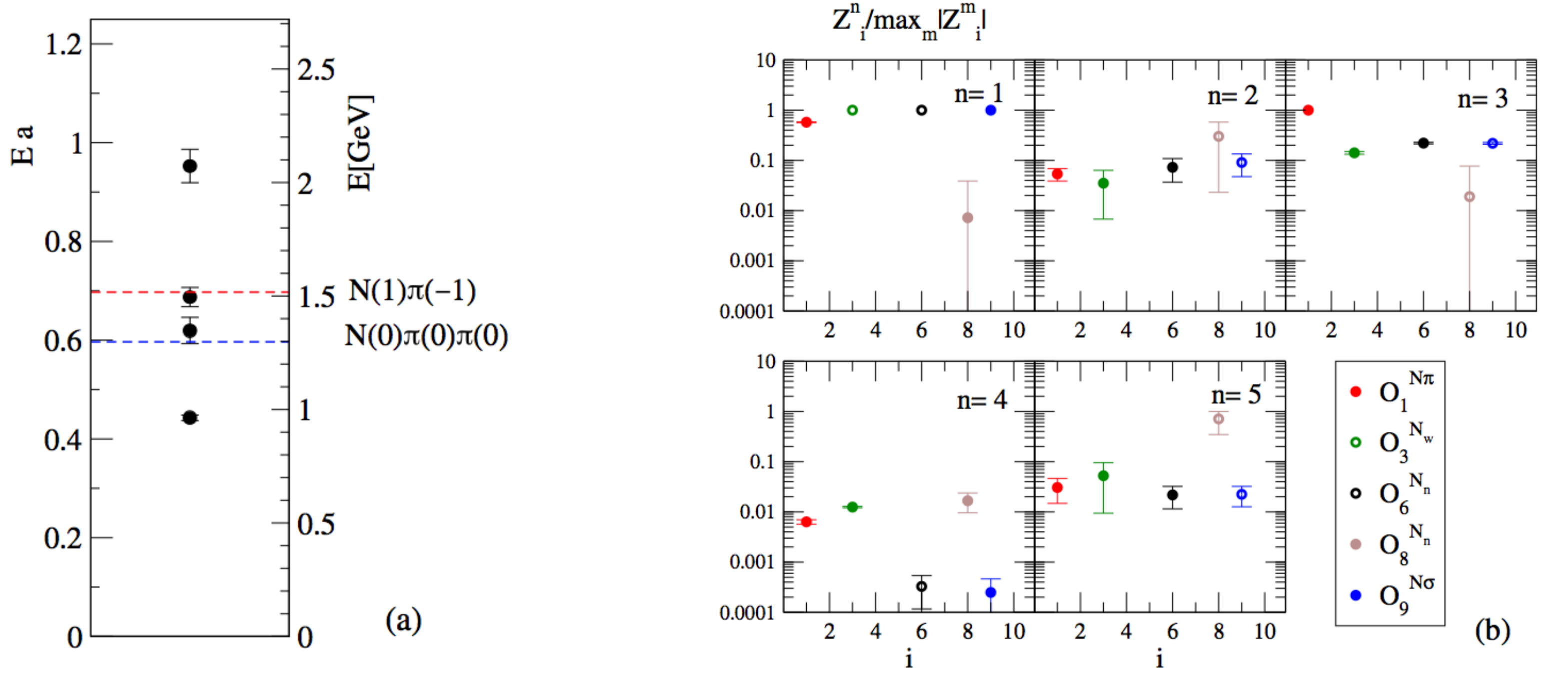
Lang, Leskovec, Padmanath, S.P. PRD 2017

N(1)π(-1) lattice eigenstate established in ½⁺ channel for the first time;

similar applies to **Nππ** eigenstate

analytic work by O. Bar within ChPT

Nπ and Nππ pollution of Nucleon observables



ChPT based on local O(qqq) applies if $r_{smear}\, m_\pi$ =small and $L\, m_\pi$ =sizable

not strictly satisfied in our simulation, so comparison not expected to work perfectly

O. Bar, 1503.03649, 1802.10442, private com.

lat

$N(p_n)\pi(-p_n)$: $\langle O^{qqq} | O^{qqq}\rangle = 2|\tilde{\alpha}|^2 e^{-M_N t}\left[1+\sum_{p_n} c_n^+ e^{-(E_{\mathrm{tot},n}-M_N)t}\right]$

$$0.42 \approx \sqrt{c_1^+} \leftrightarrow \frac{\langle O^{qqq} | N(1)\pi(-1)\rangle}{\langle O^{qqq} | N(0)\rangle} = \frac{Z_{i=6}^{n=3}}{Z_{i=6}^{n=1}} \approx 0.2$$

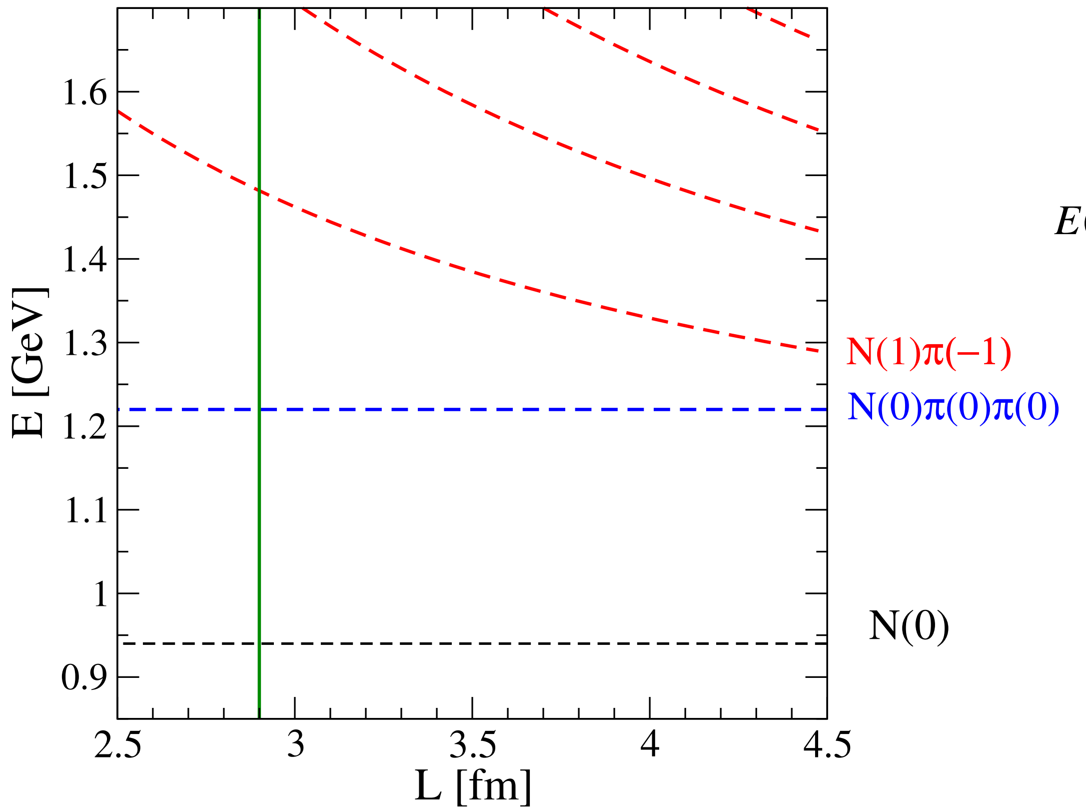
N π π: $\langle O^{qqq} | O^{qqq}\rangle = C\,[1+ c_{0,0}\, e^{-[E(N\pi\pi) - E(N)]}]$

$$0.036 \approx \sqrt{c_{0,0}} \leftrightarrow \frac{\langle O^{qqq} | N(0)\pi(0)\pi(0)\rangle}{\langle O^{qqq} | N(0)\rangle} = \frac{Z_{i=6}^{n=3}}{Z_{i=6}^{n=1}} \approx 0.07(4)$$

E not precise enough to reliably determine ΔE and δ: not unexpected for $m_\pi \approx 156$ MeV !!

Alternative path to reach physics conclusions from the results.

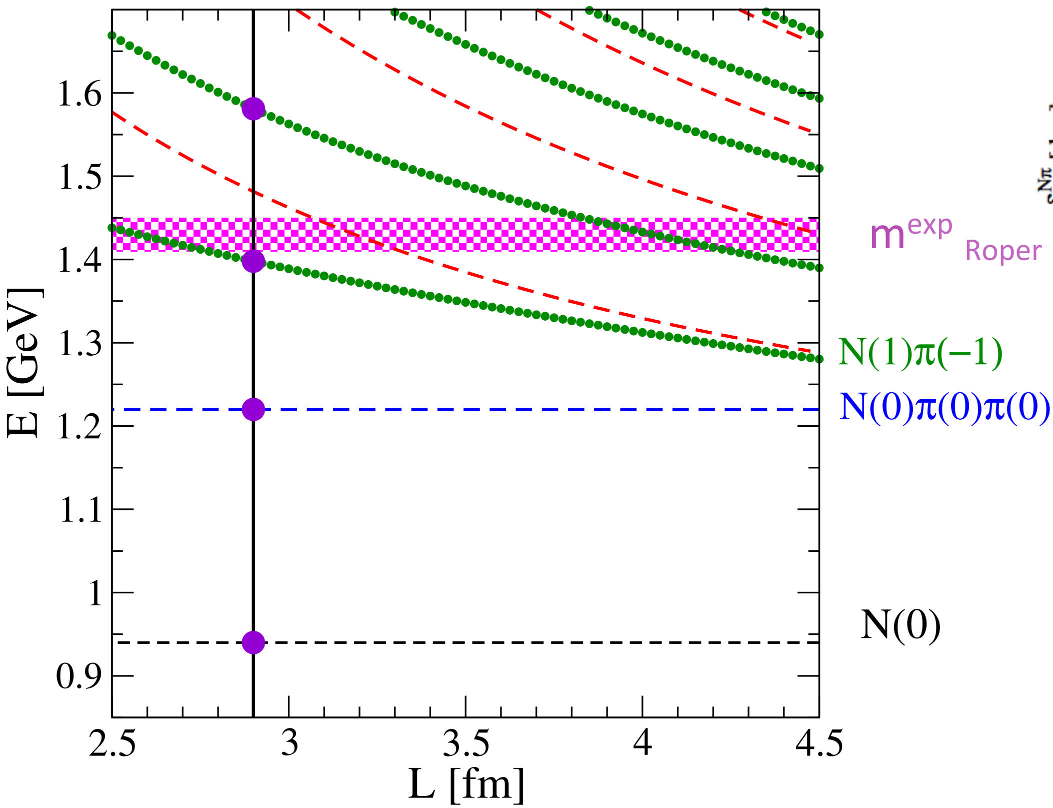
(A) Expectation from elastic Nπ scattering based on low-lying Roper (from experimental $\delta_{N\pi}$)

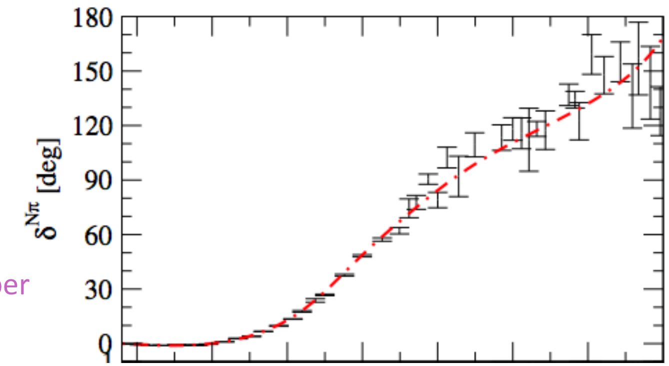


non-interacting case

$$E(L) = \sqrt{m_N^2 + \left(\frac{2\pi}{L}n\right)^2} + \sqrt{m_\pi^2 + \left(\frac{2\pi}{L}n\right)^2}$$

(A) Expectation from elastic Nπ scattering based on low-lying Roper (from experimental $\delta_{N\pi}$)





Luscher's relation

$$\cot \delta(p) = \frac{1}{pL\pi} \sum_{\vec{n} \in N^3} \frac{1}{\vec{n}^2 - (\frac{pL}{2\pi})^2}$$

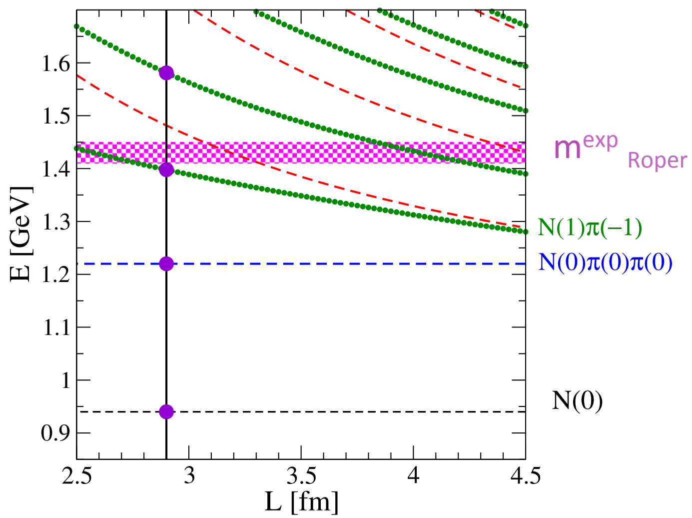
$$E(p) = \sqrt{p^2 + m_1^2} + \sqrt{p^2 + m_2^2}$$

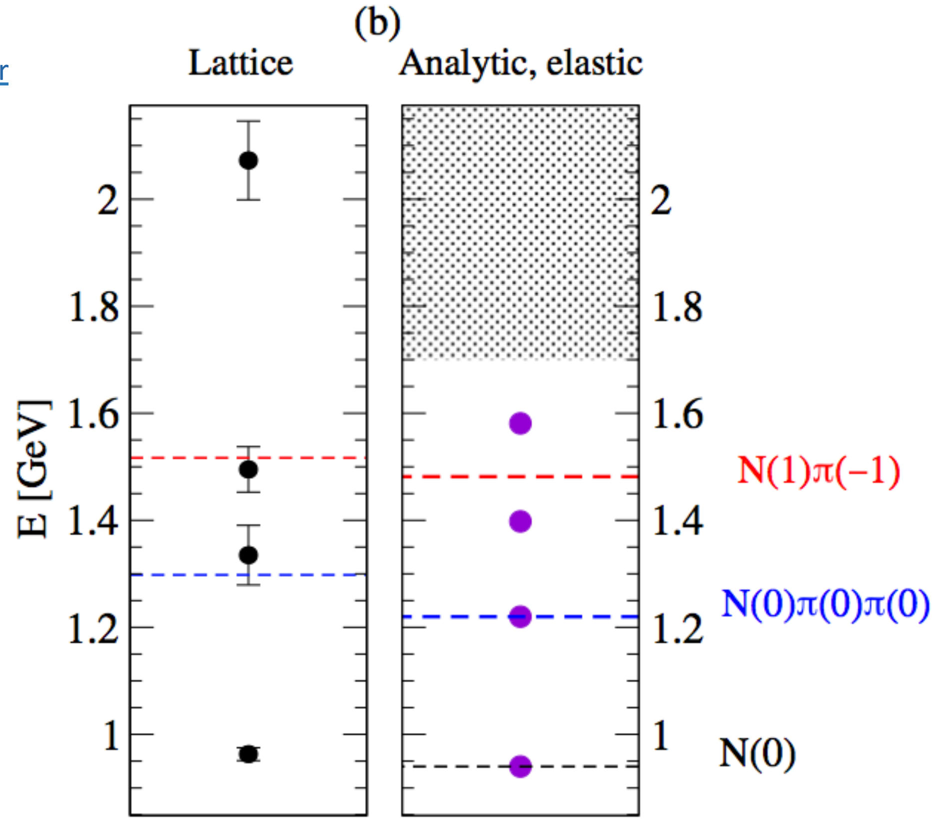
(A) Expectation from elastic Nπ scattering
based on low-lying Roper (from experimental $\delta_{N\pi}$)

- lattice data is qualitatively different from the prediction of the decoupled Nπ channel with resonant phase
- the scenario of mainly elastic low-lying Roper is not supported by the lattice data
- this calls for other possibilities for experimental state: one possibility is that <u>the coupling of Nπ with other channels (Nσ or Nππ) is essential for low-lying Roper resonance in experiment</u>:

  this is dubbed dynamically generated Roper resonance

  [Krehl, Hanhart, Krewald, Speth, PRC 62 025207 (2000), many other follow up-works]





(B) 3 scenarios with coupled Nπ – Nσ – Δπ scattering

Hamiltonian EFT study of Roper
Adelaide group, Leinneweber et al,
PRD 2017, 1607.04536

$$H = H_0 + H_I.$$

$$H_0 = \sum_{B_0} |B_0\rangle m_B^0 \langle B_0| + \sum_\alpha \int d^3\vec{k} |\alpha(\vec{k})\rangle \times \left[\sqrt{m_{\alpha_1}^2 + \vec{k}^2} + \sqrt{m_{\alpha_2}^2 + \vec{k}^2}\right] \langle \alpha(\vec{k})|$$

bare baryon

α= Nπ , Nσ, Δπ

$$H_I = g + v,$$

$$g = \sum_{\alpha B_0} \int d^3\vec{k} \{ |\alpha(\vec{k})\rangle G^\dagger_{\alpha,B_0}(k) \langle B_0| + |B_0\rangle G_{\alpha,B_0}(k) \langle \alpha(\vec{k})| \},$$

$$v = \sum_{\alpha,\beta} \int d^3\vec{k} d^3\vec{k}' |\alpha(\vec{k})\rangle V^S_{\alpha,\beta}(k,k') \langle \beta(\vec{k}')|.$$

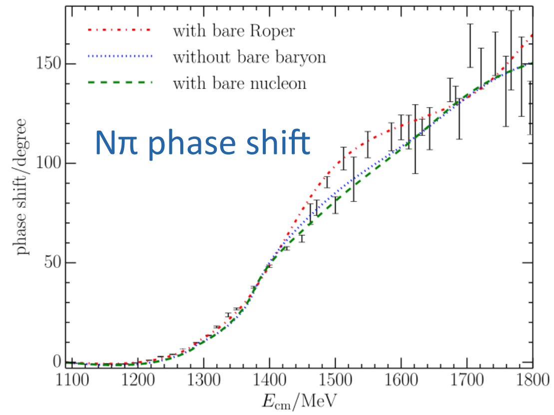
caveat: σ treated as stable

3 scenarios, which all fit experimental Nπ scattering well

I : with bare Roper B0
without bare nucleon
no coupling between Nπ – Nσ

II : without bare Roper B0
without bare nucleon N;
with strong Nπ – Nσ coupling

III: without bare Roper B0
with bare nucleon N;
with strong Nπ – Nσ coupling



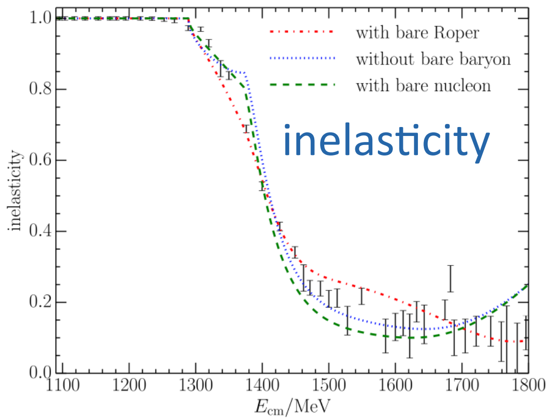


TABLE I. Best-fit parameters and resultant pole positions in the three scenarios: I, the system with the bare Roper; II, the system without a bare state; and III, the system with a bare nucleon. Underlined parameters were fixed in the fitting of that scenario. The experimental pole position for the Roper resonance is $(1365 \pm 15) - (95 \pm 15)i$ MeV [4].

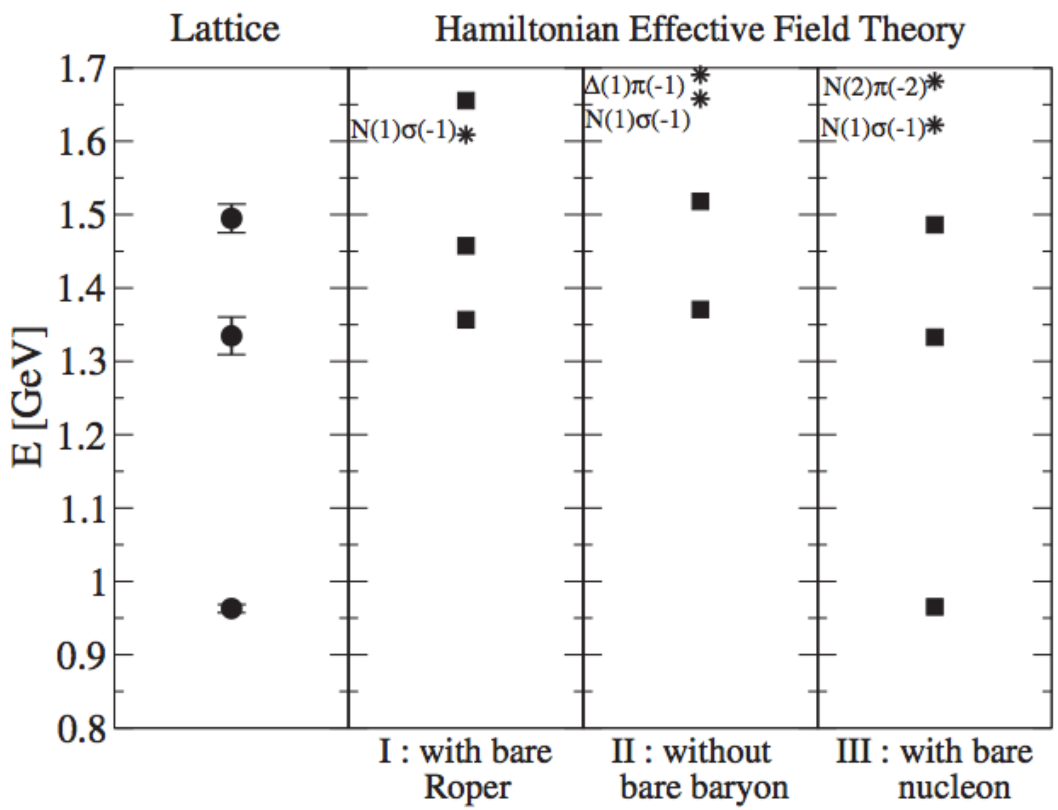
Parameter	I	II	III
$g^S_{\pi N}$	0.161	0.489	0.213
$g^S_{\pi\Delta}$	−0.046	−1.183	−1.633
$g^S_{\pi N,\pi\Delta}$	0.006	−1.008	−0.640
$g^S_{\pi N,\sigma N}$	$\underline{0}$	2.176	2.401
$g^S_{\sigma N}$	$\underline{0}$	9.898	9.343
$g_{B_0\pi N}$	0.640	$\underline{0}$	−0.586
$g_{B_0\pi\Delta}$	1.044	$\underline{0}$	1.012
$g_{B_0\sigma N}$	2.172	$\underline{0}$	2.739
m_B^0/GeV	2.033	$\underline{\infty}$	1.170
$\Lambda_{\pi N}$/GeV	$\underline{0.700}$	0.562	$\underline{0.562}$
$\Lambda_{\pi\Delta}$/GeV	$\underline{0.700}$	0.654	$\underline{0.654}$
$\Lambda_{\sigma N}$/GeV	$\underline{0.700}$	1.353	$\underline{1.353}$
Pole (MeV)	$1380 - 87i$	$1361 - 39i$	$1357 - 36i$

(B) 3 scenarios with coupled Nπ − Nσ − Δπ scattering

Hamiltonian EFT study of Roper resonance
Adelaide group, Leinneweber et al,
PRD 2017, 1607.04536

lattice results

predicted E_n for PACS-CS lattice with L=2 fm in 3 scenarios on previous page



3 scenarios, which all fit experimental Nπ scattering well

<u>I : with bare Roper B0</u>
without bare nucleon
no coupling between Nπ − Nσ

<u>II : without bare Roper B0</u>
with bare nucleon N;
with strong Nπ − Nσ coupling

III: without bare Roper B0
<u>with bare nucleon N;</u>
with strong Nπ − Nσ coupling

comparing analytic predictions and lattice data:

- scenario I disfavoured
- scenarios II, III favoured

- Roper as dynamically generated resonance favoured

Structure of the Roper resonance from Lattice QCD constraints

Leineweber et al. 1703.10715

If experimental low-lying Roper resonance results from strong rescattering between coupled meson-baryon channels ...

$N(1440)$ BREIT-WIGNER MASS	1410 to 1450 ($\approx$ 1430) MeV
$N(1440)$ BREIT-WIGNER WIDTH	250 to 450 ($\approx$ 350) MeV
$N\pi$	$55-75\%$
$N\eta$	$< 1\%$
$N\pi\pi$	25–50%
$\Delta(1232)\pi$	20–30%
$\Delta(1232)\pi$, $P\text{−}wave$	$13-27\%$
$N\sigma$	$11-23\%$

If this is the case, the prospects of rigorous lattice treatment will be challenging:

- coupled channel scattering (doable if both hadrons HH are stable)
- three-body $N\pi\pi$ decay: relation of E and scattering matrix under development

    [Sharpe, Hansen, Briceno, Rusetsky, Doring, Mai,.]

    scattering matrix has never been extracted within QCD

Conclusions

H_1H_2 scattering where one or both carry spin:

- a number of simulations at L=0; only few for L>0
- generalized Luscher's relation between E and S exists

(1) $H_1(p)H_2(-p)$ operators constructed for scattering of particles with spin

- Consistent results found in three methods: PV, PN, VN, NN

✧ Projection operators O_n: gives little guidance on underlying quantum numbers

✧ Partial-wave operators: provides linear combinations O_n to enhance coupling to (J, S, L)

✧ Helicity operators: provides linear combinations O_n to enhance coupling to (J, P, λ1, λ2)

$$O \to E_n \to \delta, S$$

(2) simulation of Nπ scattering in p-wave

J^P=3/2$^+$, I=3/2: Δ(1232) resonance : "vanilla" baryon resonance confirmed by LQCD

J^P=1/2$^+$, I=1/2: N(1440) resonance (Roper)

- meson-baryon eigenstates (Nπ and Nππ) are identified for the first time in this channel
- the scenario of the low-lying Roper that is mainly elastic in Nπ is not supported by lattice data
- coupling of Nπ with other channels (Nσ or Nππ) seems important to render low-lying Roper in exp
- this step was only the first on in more to follow ....