# Wormholes and Entanglement

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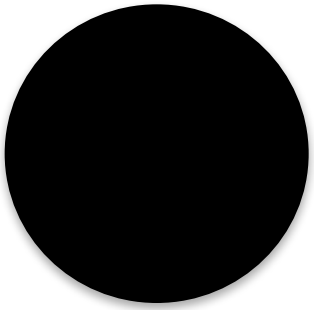
# Outline

- Black holes as quantum systems
- Schwarzschild wormhole and entangled states.
- Traversable wormholes.
- Simple dynamics in Nearly-$AdS_2$
- Traversable wormhole solution in 4d.

# Black holes as quantum systems

- A black hole seen from the outside can be described as a quantum system with S degrees of freedom (qubits).



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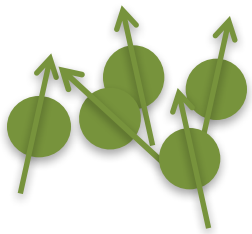
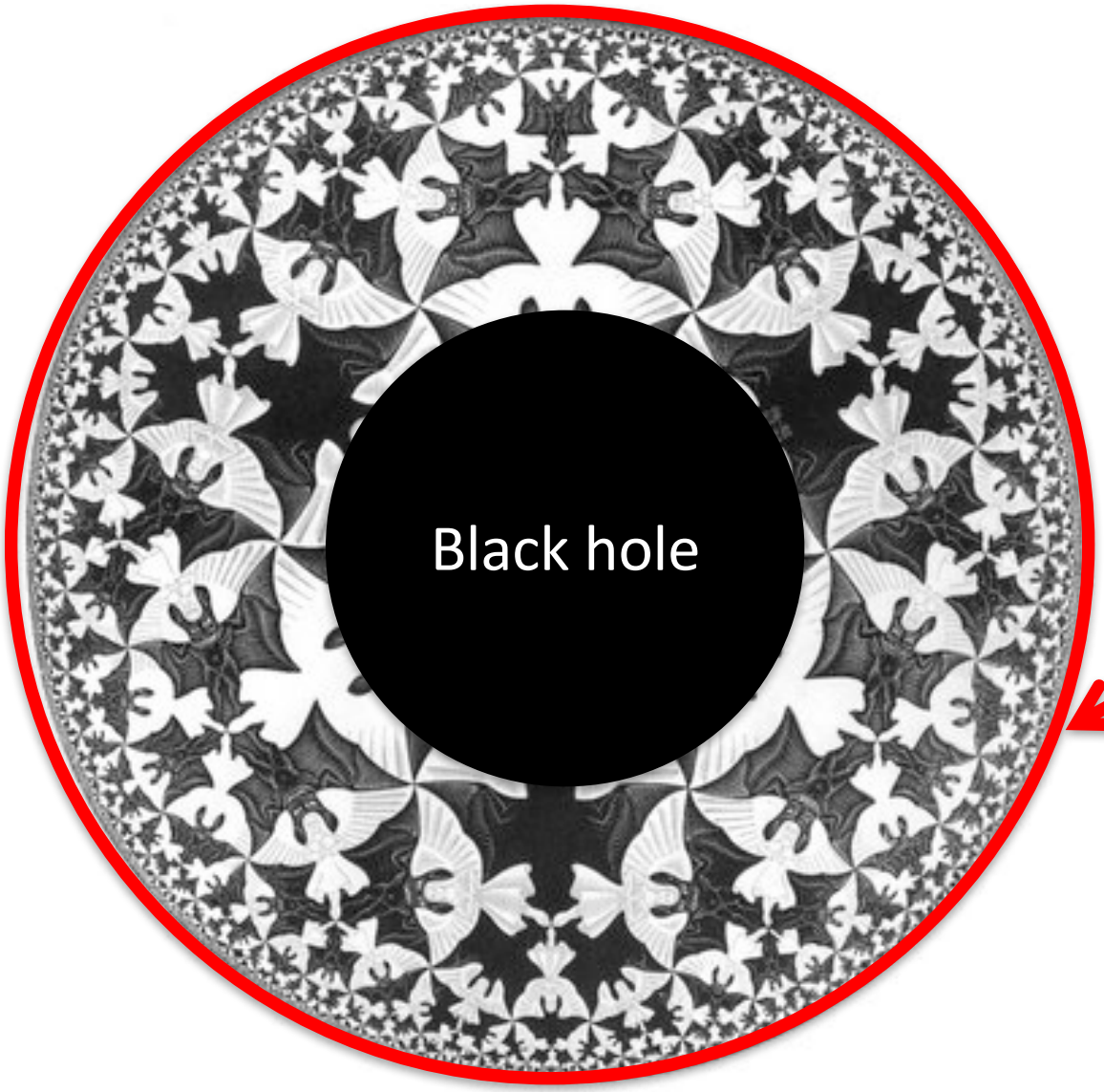
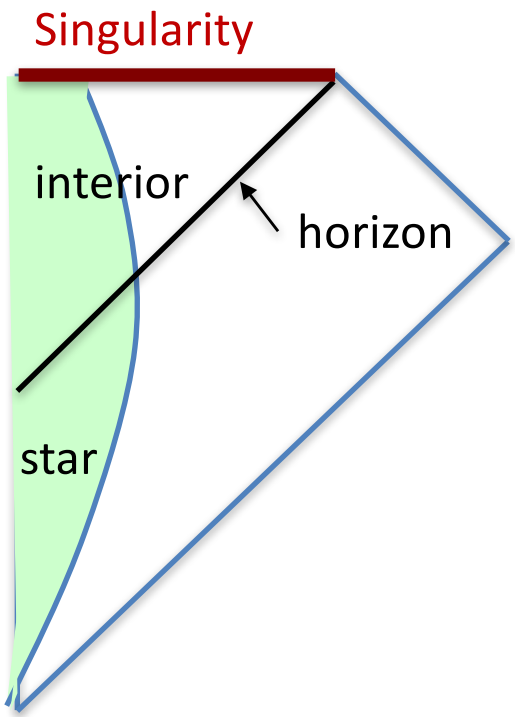


# one reason is AdS/CFT…




Hot fluid made out of
very strongly
Interacting particles.

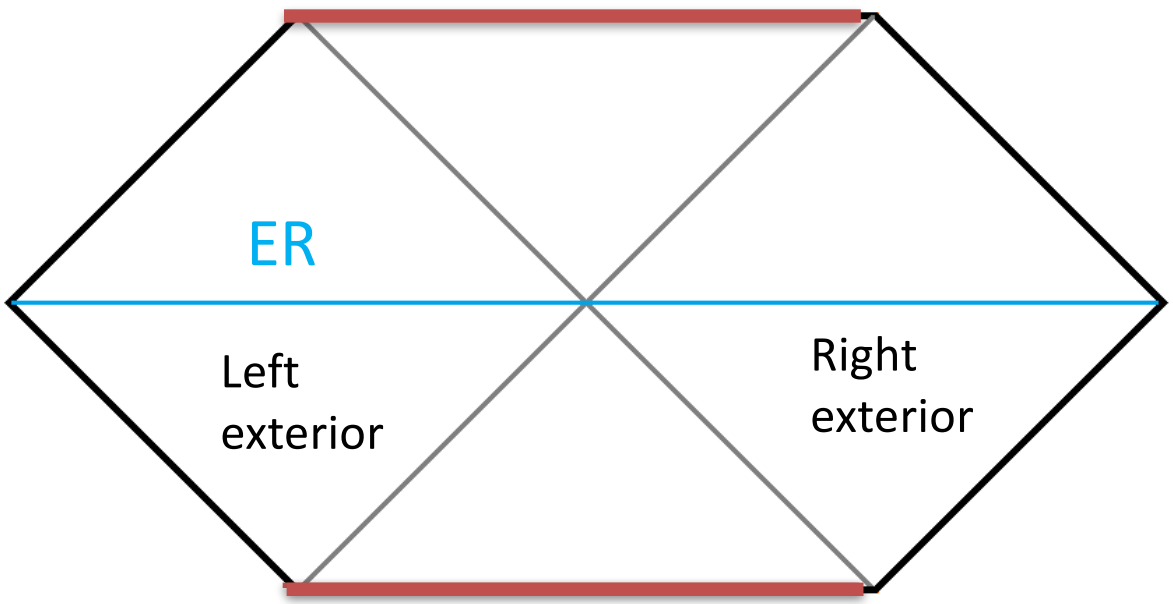
# Geometry of a Black Hole made from collapse




Oppenheimer Snyder 1939

One exterior, one interior.

# Full Schwarzschild solution

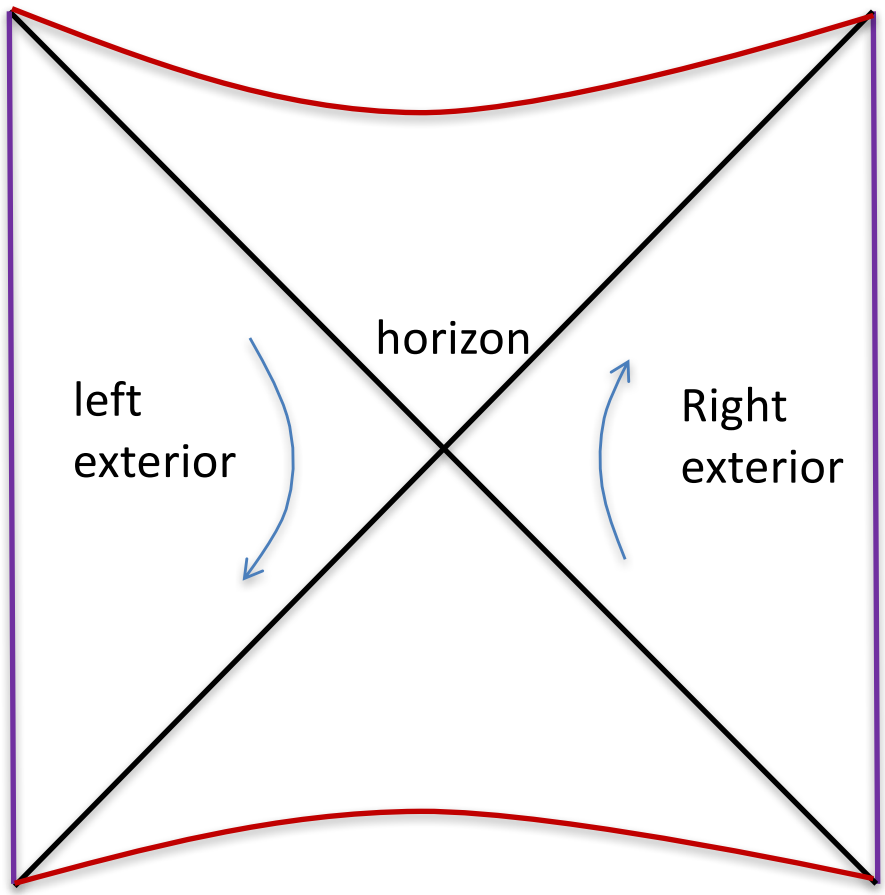



Eddington, Lemaitre, Einstein,
Rosen, Finkelstein,
Kruskal

Vacuum solution.
Two exteriors, sharing the interior.

# The full Schwarzschild wormhole

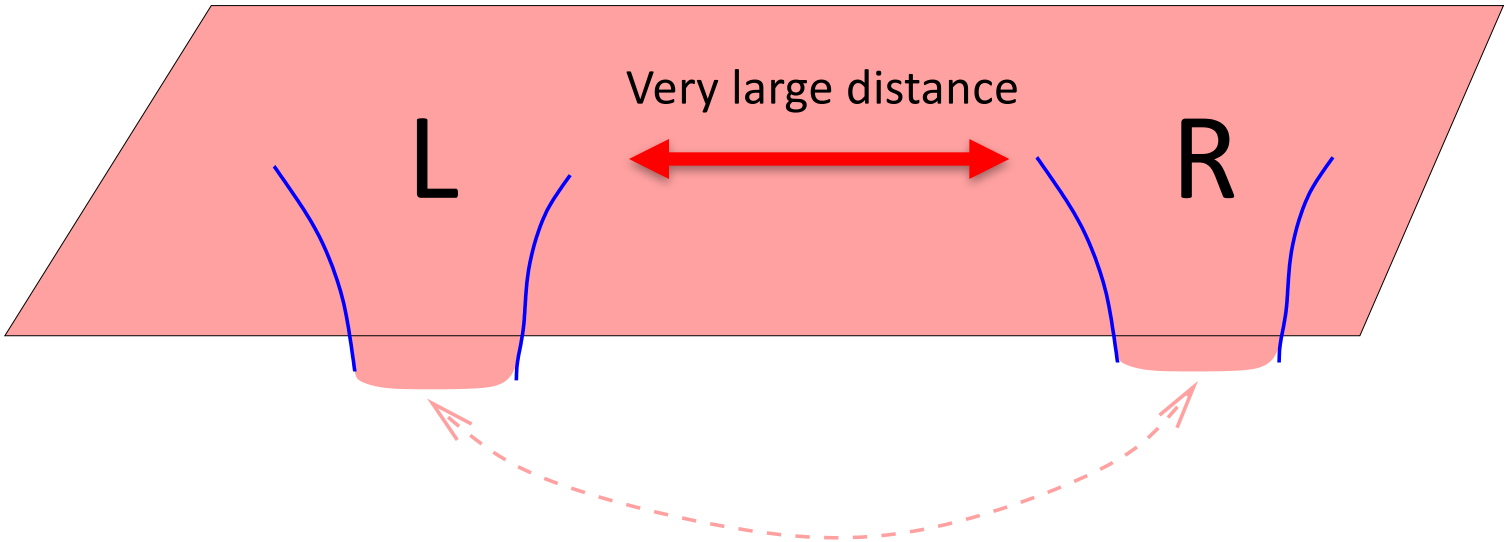
AdS asymptotics.

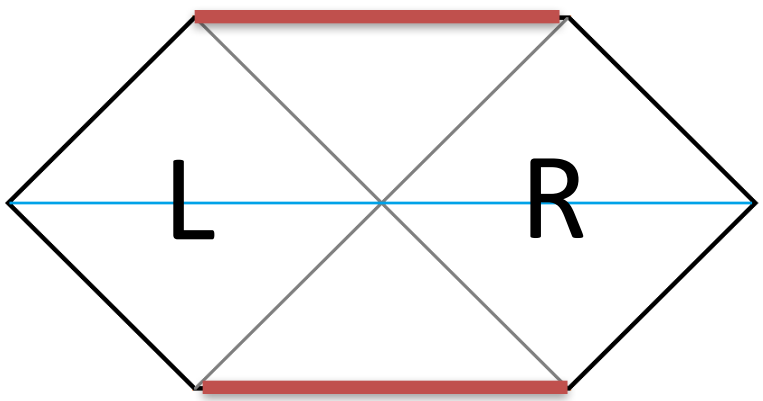



No need to postulate any exotic matter

No matter at all !

# Wormhole interpretation.



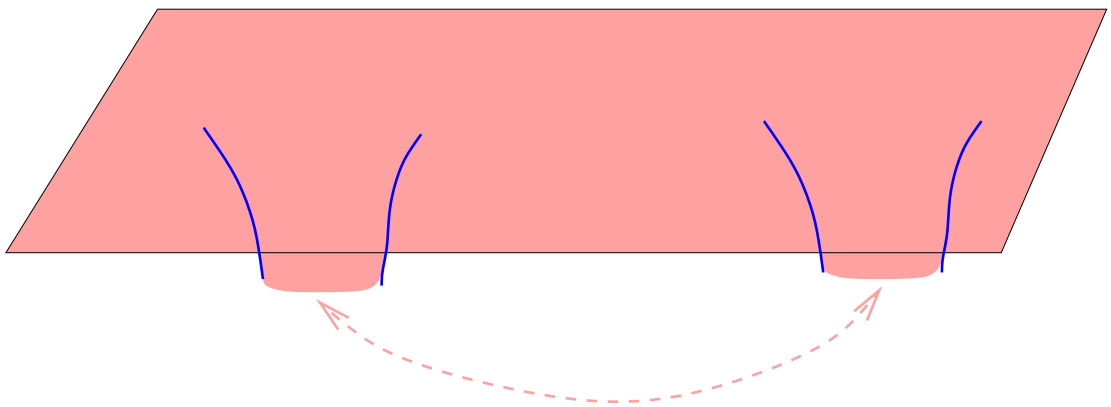

Non travesable

No signals

No causality violation

Fuller, Wheeler, Friedman, Schleich, Witt, Galloway, Wooglar

# Wormhole and entangled states



Connected through the interior

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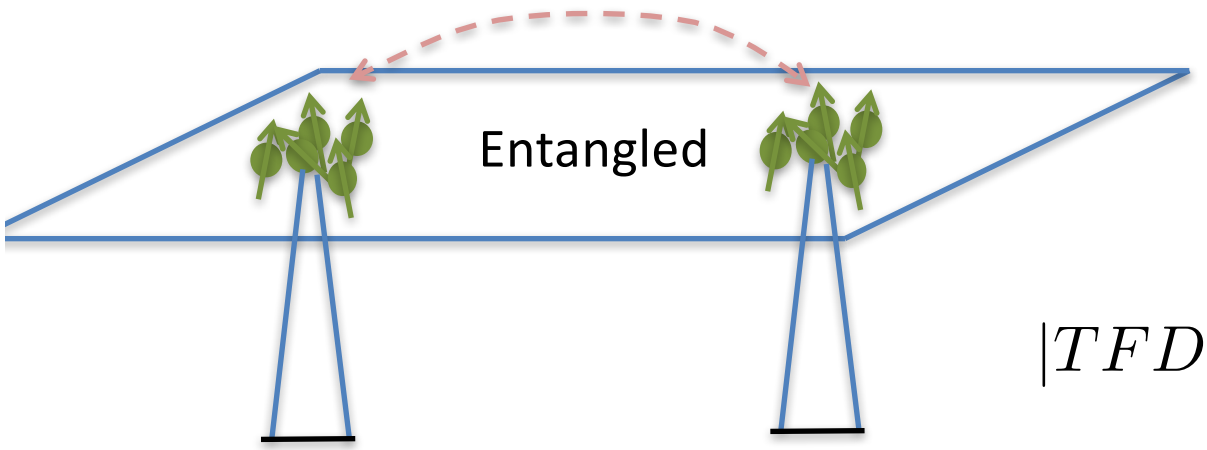



W. Israel
J.M.

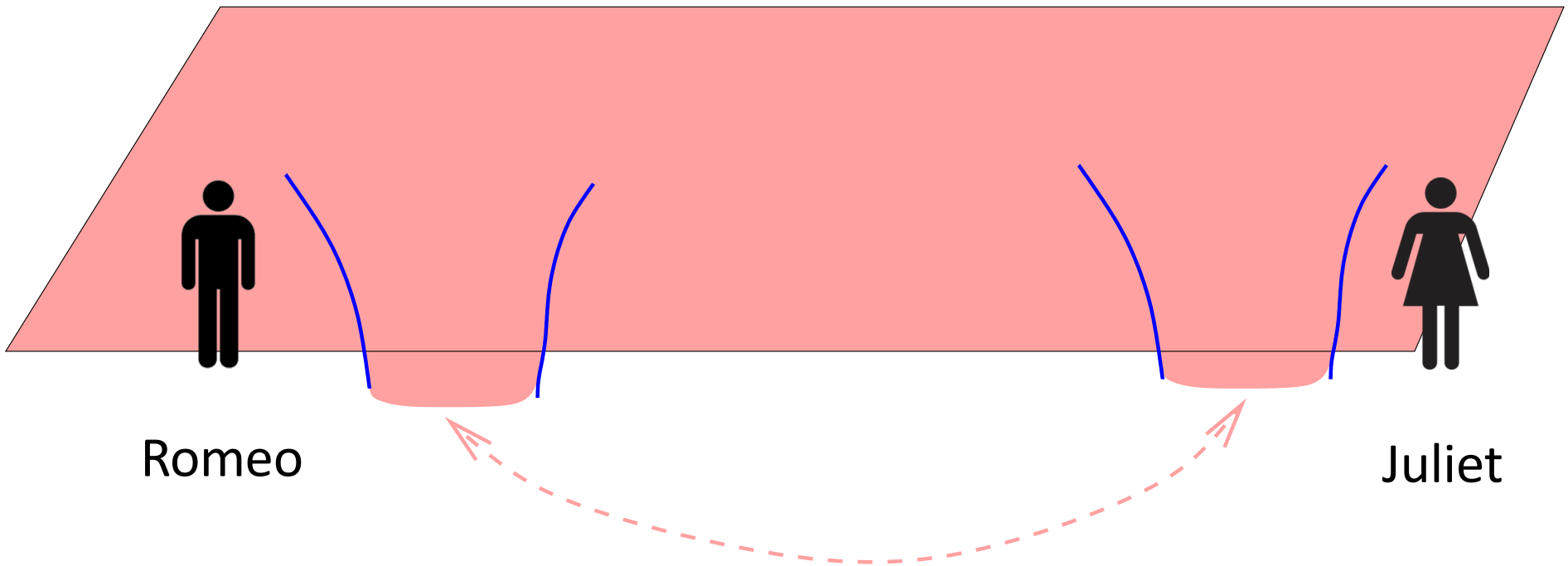
$$|TFD\rangle = \sum_n e^{-\beta E_n/2} |\bar{E}_n\rangle_L |E_n\rangle_R$$

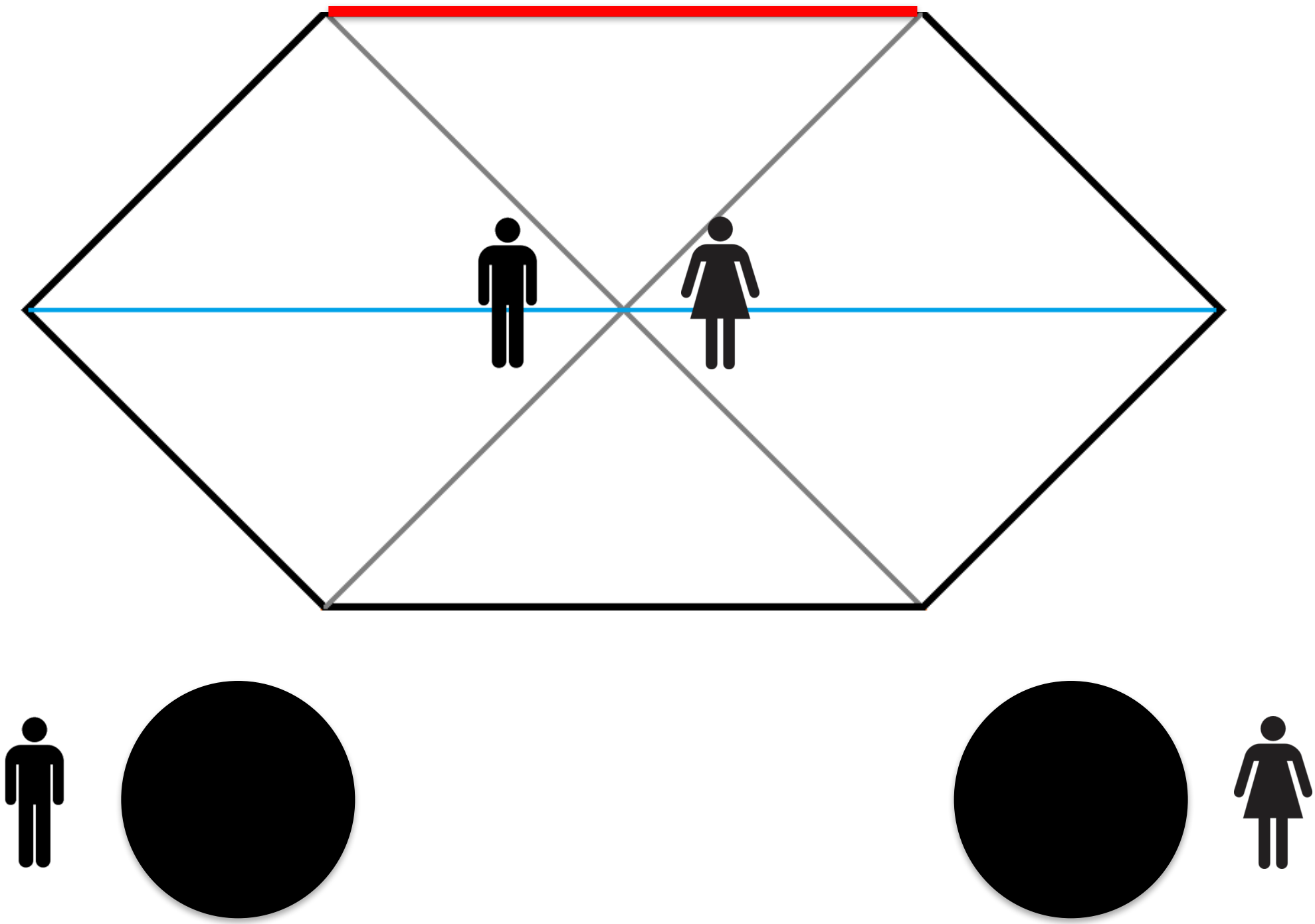
(in a particular entangled state)

# ER = EPR

J.M. , Susskind

# A forbidden meeting





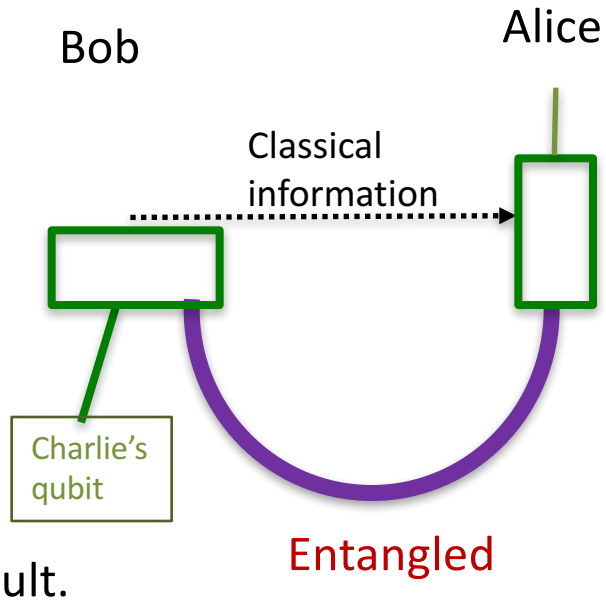
# Is there a better way ?

- Traversable wormholes
- Quantum teleportation

Gao, Jafferis, Wall, …

Bennett, Brassard, Crepeau, Jozsa, Peres, Woetters

# Quantum teleportation

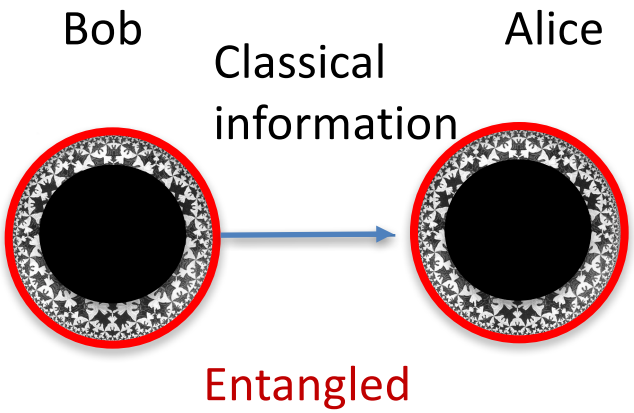


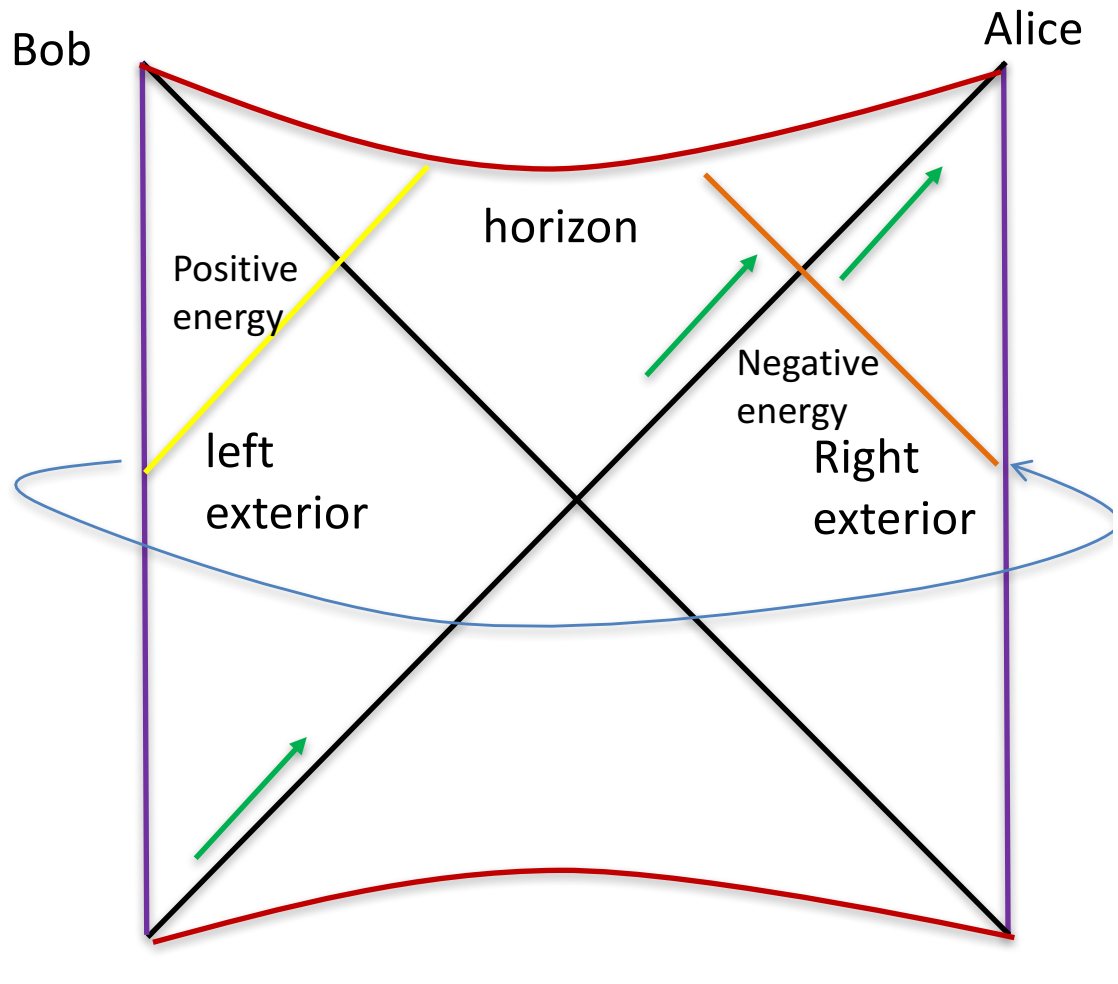

- Bob and Alice share an entagled pair of qubits.
- Charlie gives Bob a qubit and he wants to send it to Alice.
- Bob does a joint measurement of Charlie's qubit and his qubit.
- Sends the the result to Alice as classical information
- Alice does an operation on his qubit that depends on Bob's result.
- Alice gets the qubit.

- Resources needed to send a qubit: One entangled qubit and 2 bits of classical information.

How does the qubit travel ? Would you like to be teleported ?

# Teleportation through the wormhole



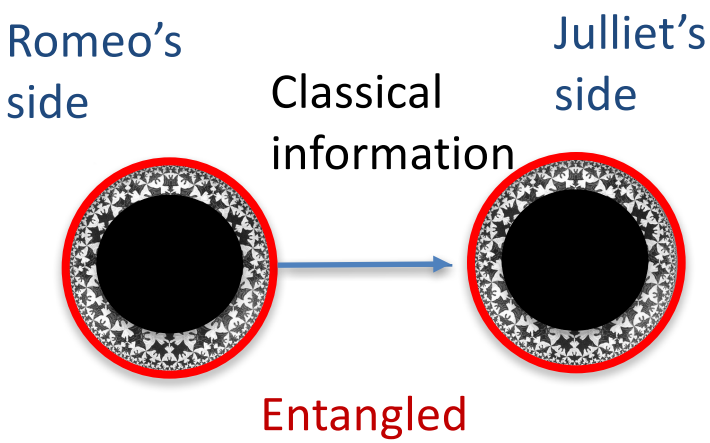



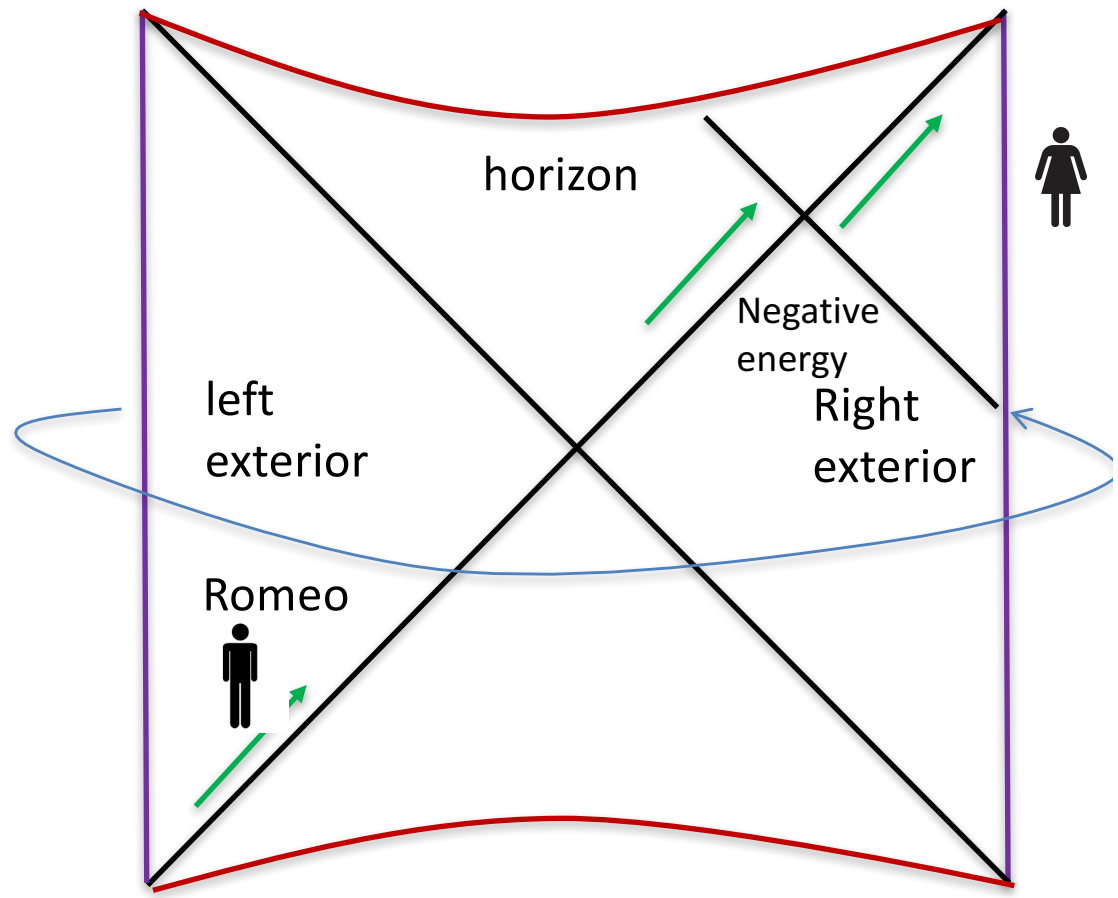

Gao, Jafferis, Wall

# Quantum teleportation through the wormhole

- What does the message feel ?
- A shock wave.
- Can be arranged to be painless if it is spherically symmetric.

# Teleportation through the wormhole




Gao, Jafferis, Wall

- Romeo can report he saw a smooth interior!

# Conclusions

- Entangled black holes can display a geometric connection.

- Entanglement is related to wormholes. ER=EPR.

- Teleportation is related to travel through wormholes.

# Challenges

- We do not really know to to “derive” the spacetime corresponding to a given state.

- What ``happens’’ at a black hole singularity or Big Bang singularity ?

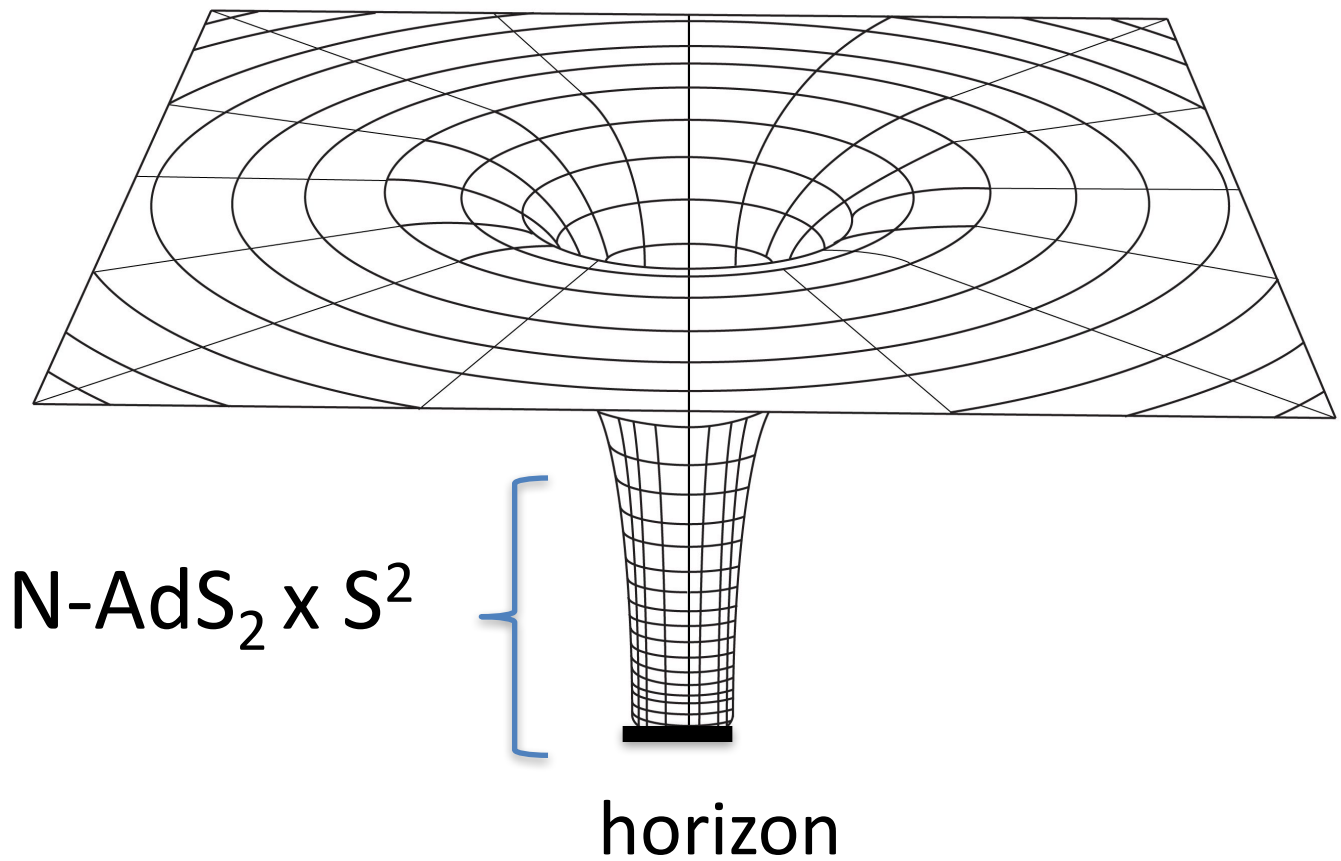
# A simple case

# Near extremal black holes
# and
# Nearly-$AdS_2$

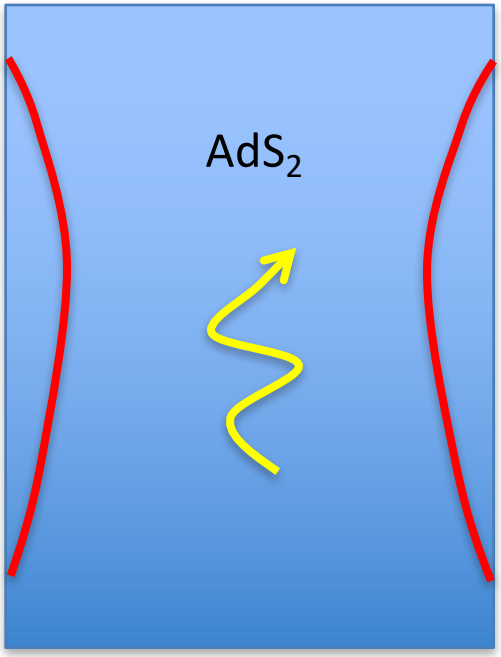
# Near extremal black holes

$$M \geq Q$$

$$M \sim Q$$



# The surprisingly simple gravitational dynamics of N-AdS$_2$



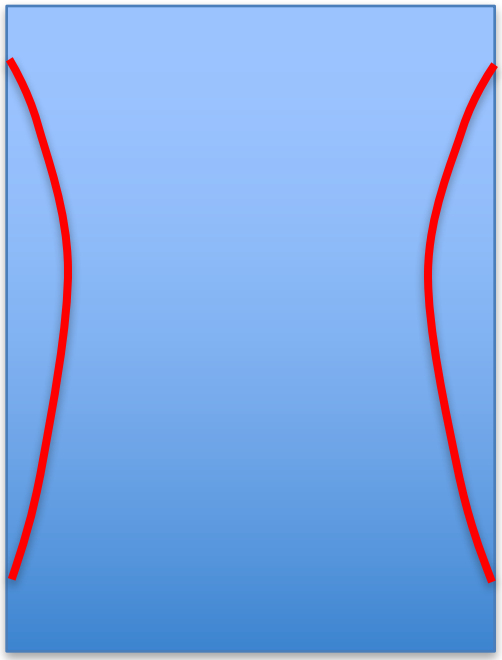

NAdS$_2$ = AdS$_2$ + location of boundary

Dynamics of the boundary is SL(2) invariant.

Proper time along the boundary = time of the asymptotically flat region = time of the quantum system

$$(H_{L\,Bdy} \times H_{\rm bulk} \times H_{R\,Bdy})/SL(2,R)$$

# Temperature



Distance between boundaries → temperature

Redshift between boundary and center is of order $\beta = 1/T$

Smaller temperature → larger redshift factor → longer wormhole

# Traversability

# Add a constant interaction between the dual quantum systems

- Nearly-AdS$_2$ gravity
- Plus matter
- Plus boundary conditions connecting the two sides

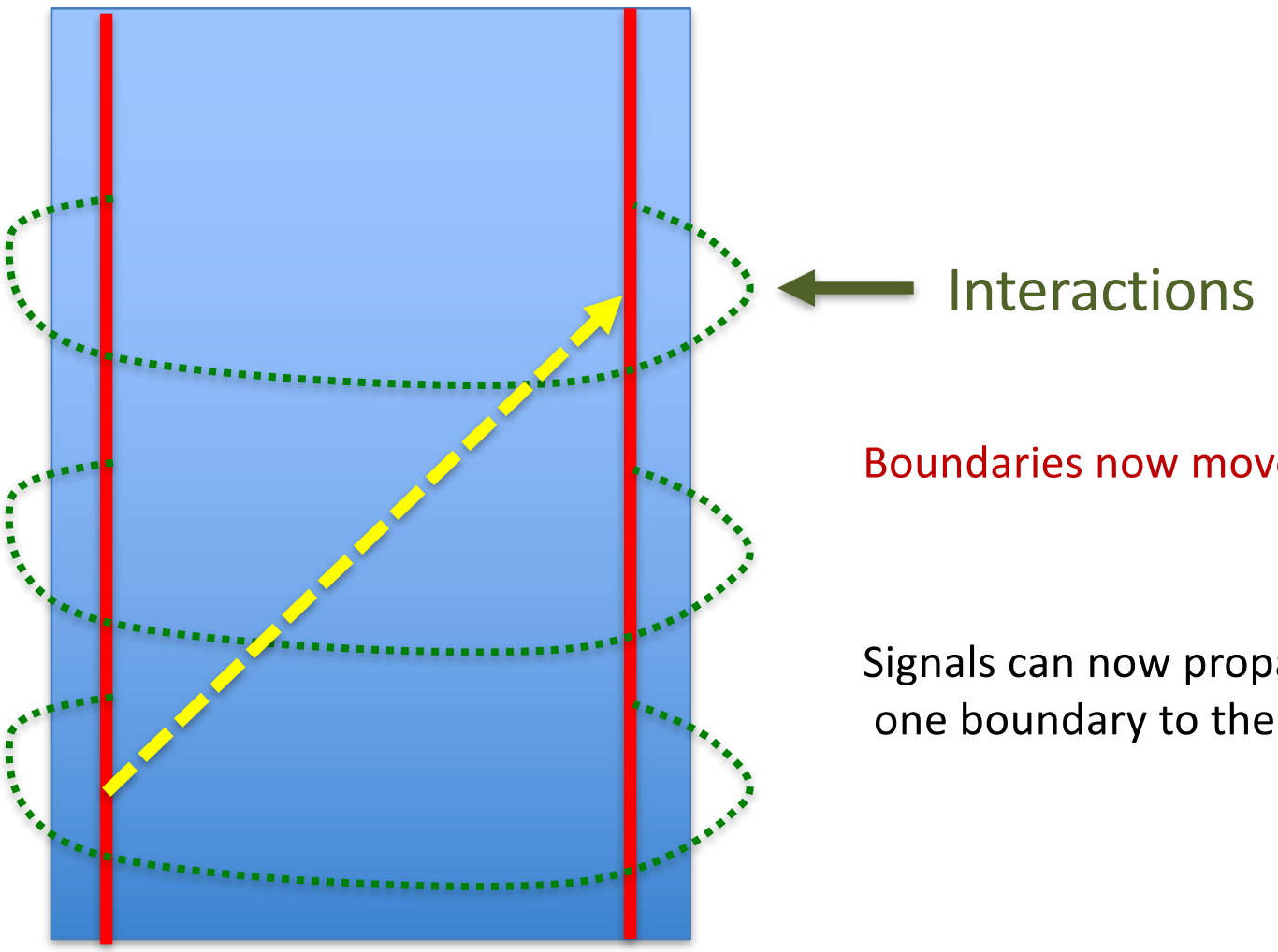
$$S_{int} = \mu \int du \chi_L(u) \chi_R(u)$$

$u$ is proper length along the boundary, or boundary time.

- This generates negative null energy and allows for an eternally traversable wormhole

$\phi$

# $NAdS_2$ gravity + Interaction



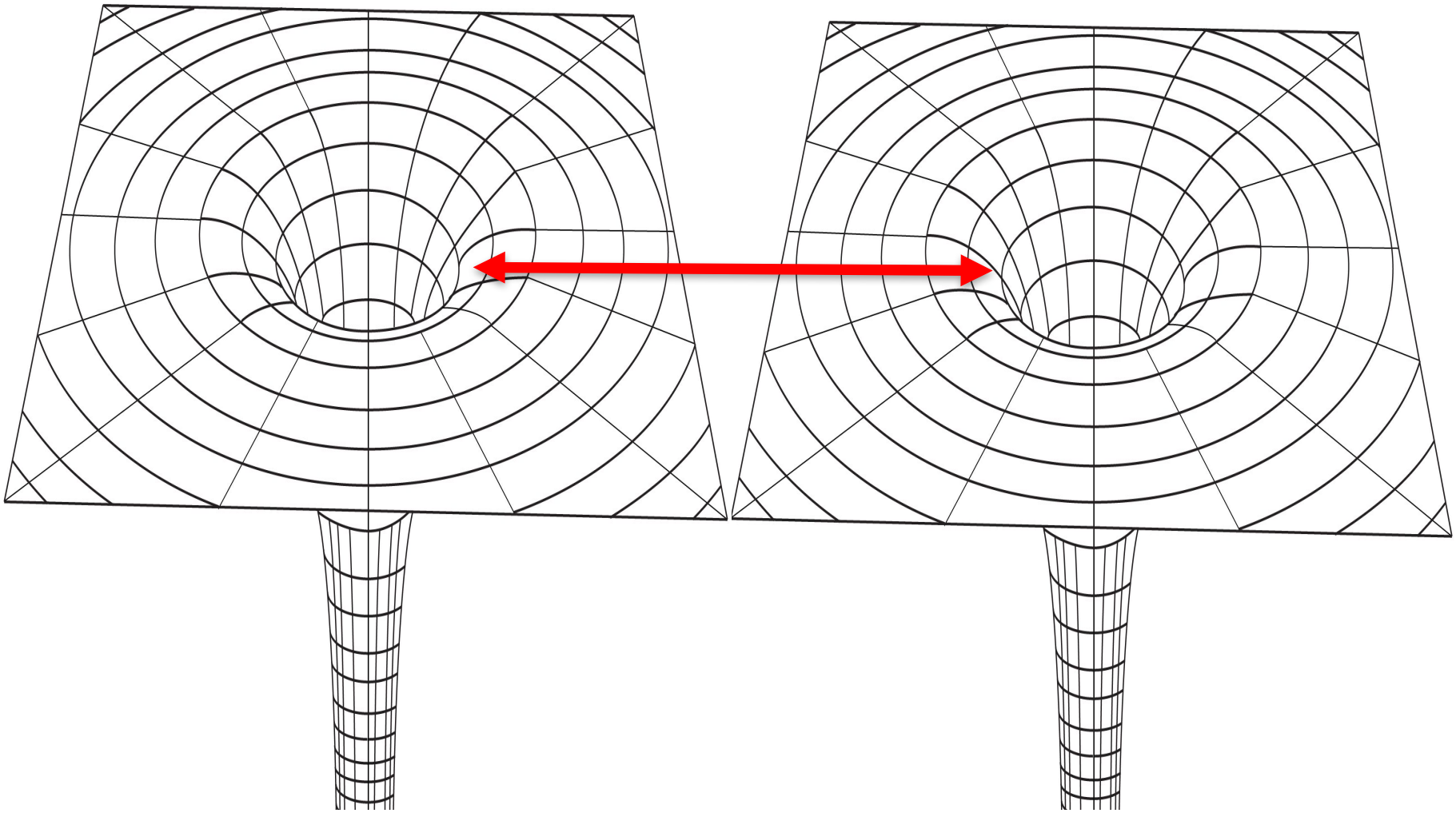

Boundaries now move “straight up”

Signals can now propagate from one boundary to the other.

- So far we have introduced the interaction by hand.
- Can we understand how it arises?

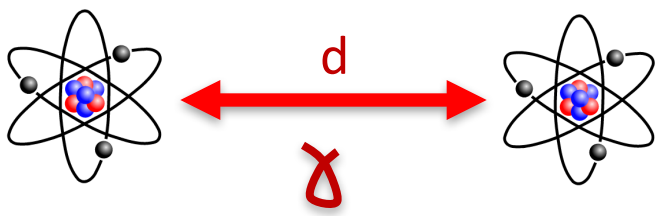
- By bringing the black holes near each other!

Exchange of bulk fields can lead to the interaction between the quantum systems the describe the black hole



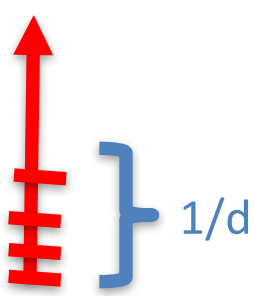
# Analogy: Van der Waals interaction

Two neutral atoms exchanging photons.

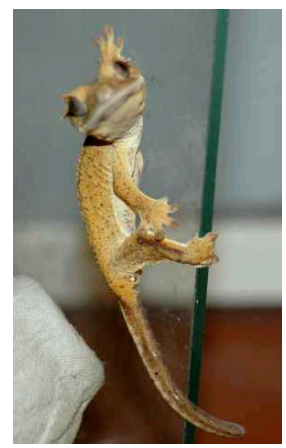



$$H_{int} \propto \frac{\vec{d}_L . \vec{d}_R}{d^3}$$

d small enough so that 1/d is larger than the gap between the ground state and the next states.




Entangle the two atoms.



This reasoning inspired the following solution

# Wormholes in 4 dimensions, in the Standard Model + gravity

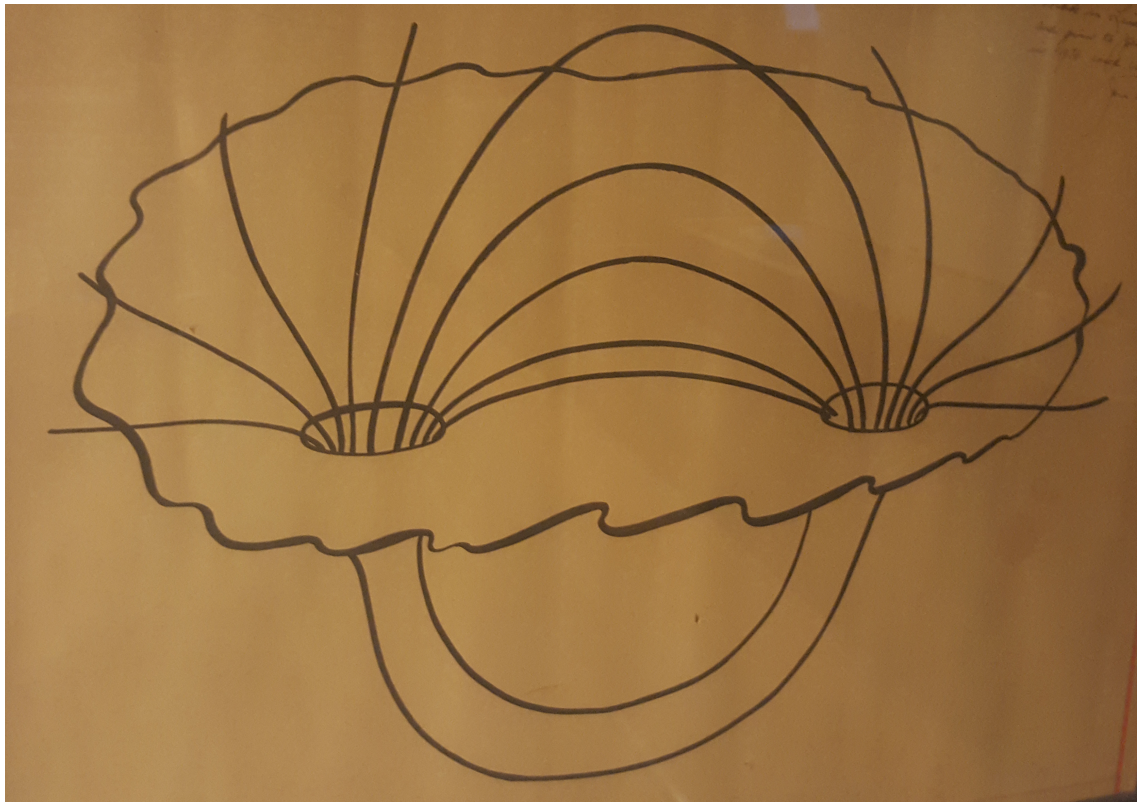
Based on work with:

Alexey Milekhin

Fedor Popov

Related to previous work with Xiaoliang Qi

# Drawing by John Wheeler, 1966



Charge without charge.
Mass without mass

Spatial geometry. Traversable wormhole

# Recall classic results

# There are no science fiction wormholes!

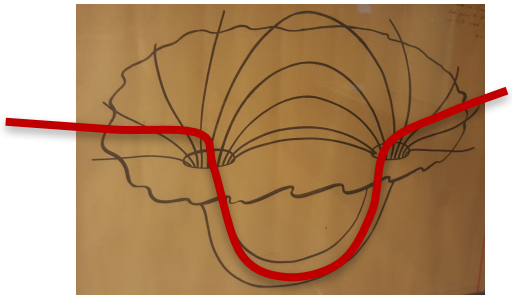
- No wormhole allows you to travel faster than the speed of light in the ambient space.

Friedman Schleich, Witt, Galloway, Woolgar
Gao Wald

- Forbidden by:
- I) Einstein equations.
- II) The Achronal Average Null Energy Condition

Not yet proven in a general spacetime, but believed to hold in QFT

$$\int dx^- T_{--} \geq 0$$



Achronal = points along null line are not timelike separated = fastest line

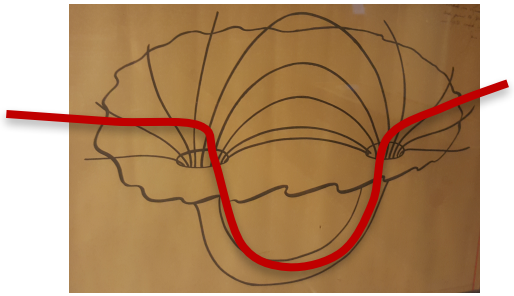
# Longer wormholes?

- What if it takes longer to go through the wormhole ?
- Not possible in classical physics due to the Null Energy Condition.

Topological censorship: Friedman Schleich, Witt, Galloway, Woolgar

- In classical physics → we can go to the covering space, which is forbidden by the previous case.

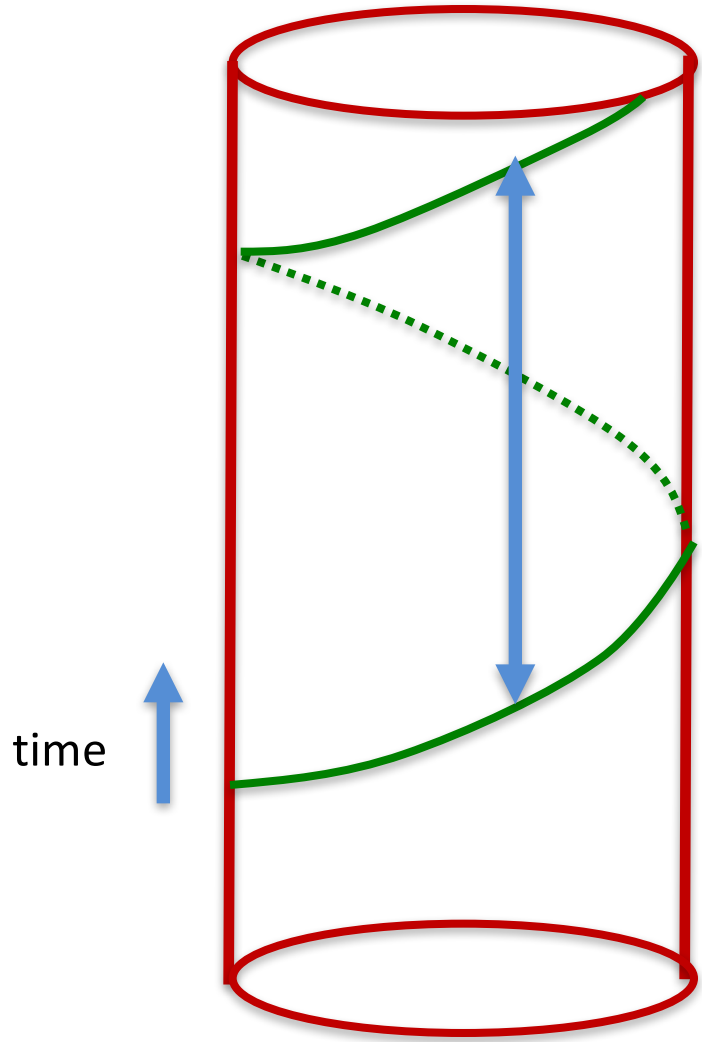
- → We need quantum effects to find a solution. Casimir-like energy.



- Can we do it in a controllable way ?

# Negative null energy in QFT

Eg. Two spacetime dimensions




Circle

$$T_{--} < 0$$

$$E \propto -\frac{c}{L}$$

Negative Casimir energy

Quantum effect

The null energy condition does not hold for null lines that are not achronal!

# Some necessary elements

- We need something looking like a circle to have negative Casimir energy.
- Large number of bulk fields to enhance the size of quantum effects.
- We will show how to assemble these elements in a few steps.

# The theory

$$S = \int d^4x \left[ R - F^2 + i\bar{\psi} \ \not{D}\psi \right]$$

Einstein + U(1) gauge field + massless charged fermion

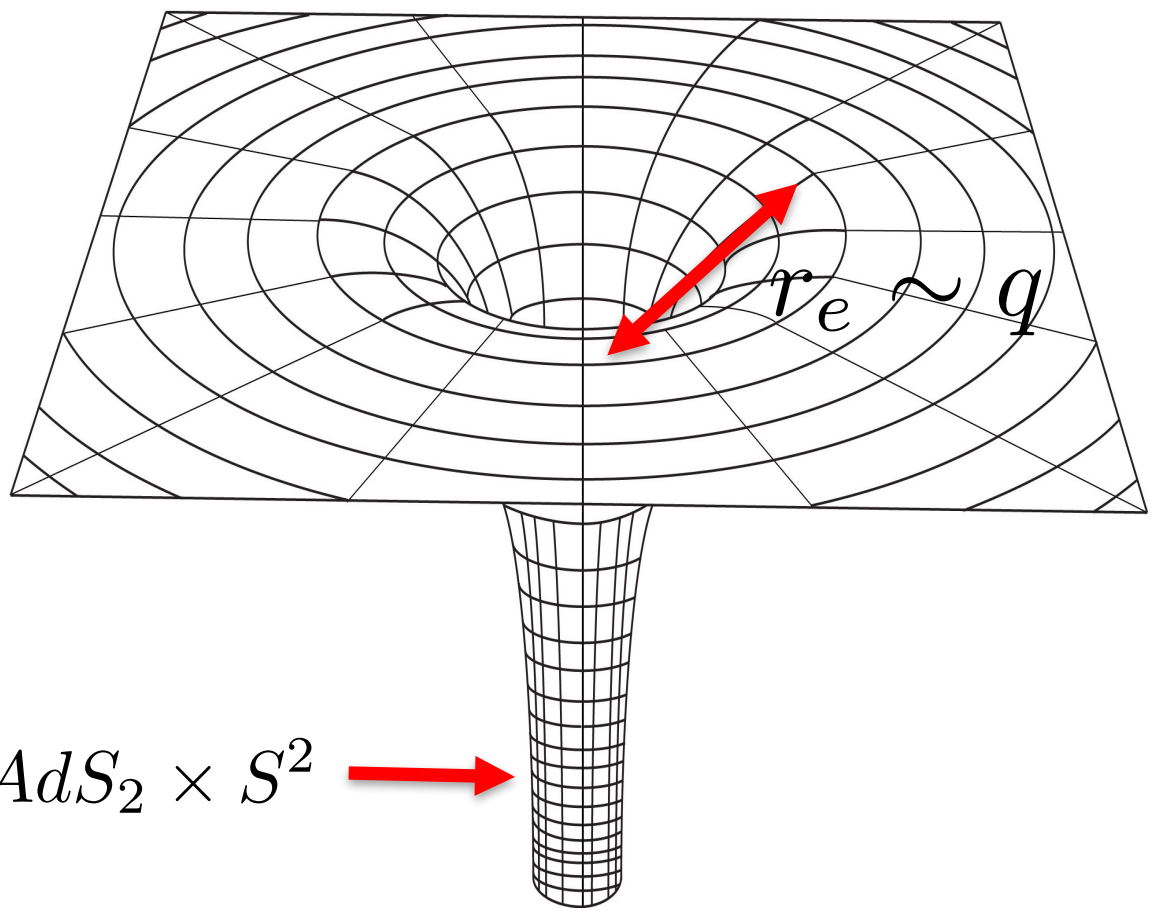
Could be the Standard Model at very small distances, with the fermions effectively massless. The U(1) is the hypercharge. SU(3) x SU(2) x U(1).

$$l_{\text{Planck}} = 1$$

# The first solution: Extremal black hole

Magnetic charge q

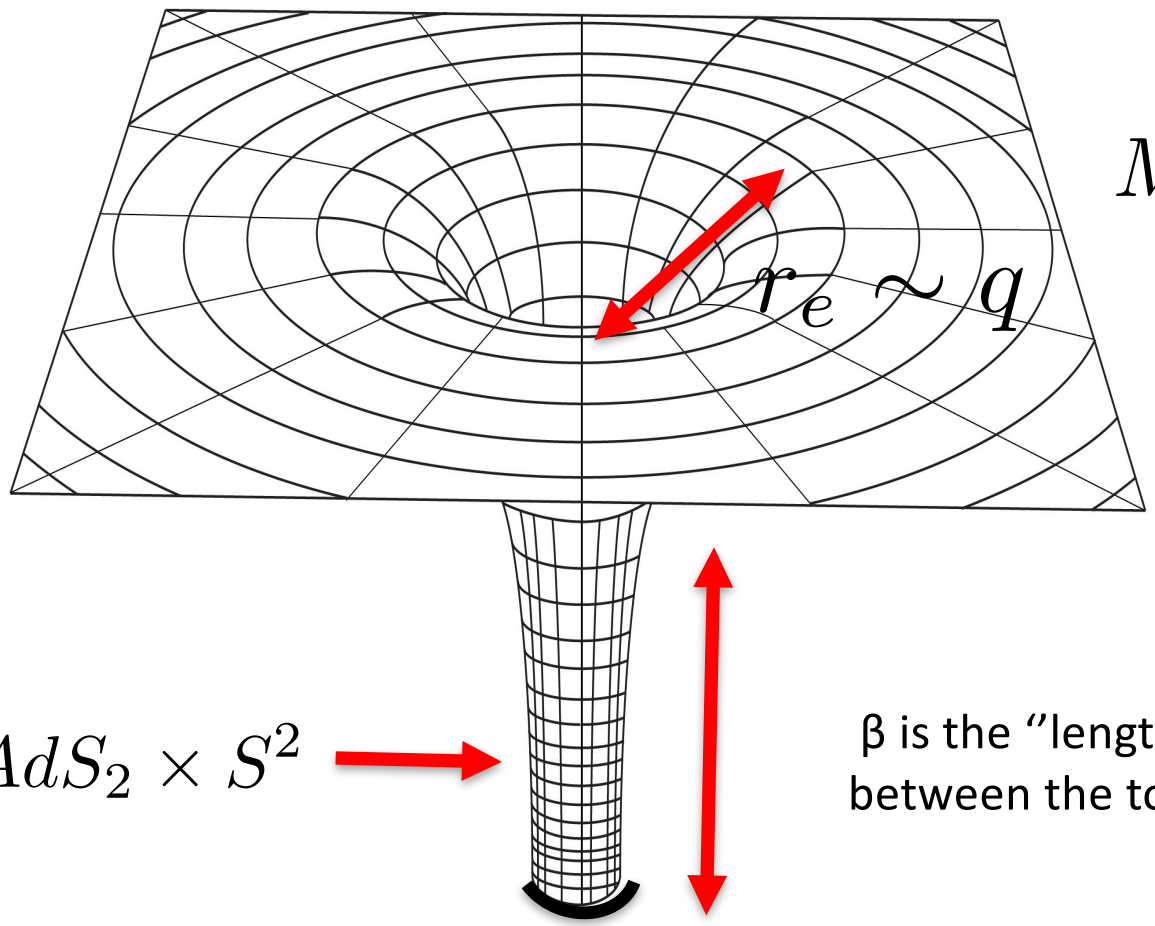
$$\int_{S^2} F = q = \text{integer}$$




$$M = q$$

$$l_{\text{Planck}} = 1$$

# The next solution: Near Extremal black hole




$$M - q = \underbrace{q^3 T^2}_{\text{Very small}} = \frac{q^3}{\beta^2}$$

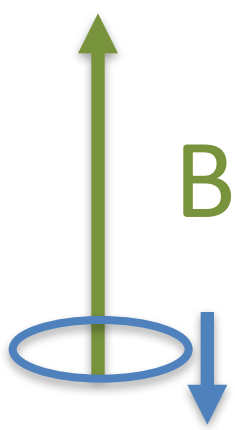
β is the "length" of the throat. Redshift factor between the top and the bottom

$l_{\text{Planck}} = 1$

# Motion of charged fermions

- Magnetic field on the sphere.
- There is a Landau level with precisely zero energy.
- Orbital and magnetic dipole energies precisely cancel.
- Explained by an anomaly argument

Ambjorn, Olesen



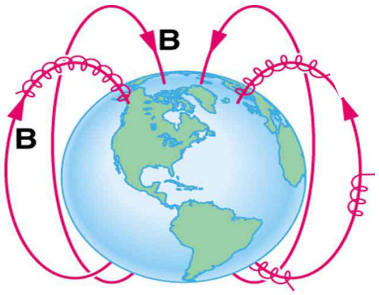

Massless fermions → U(1) chiral symmetry

4d anomaly → 2d anomaly → there should be massless fermions in 2d.

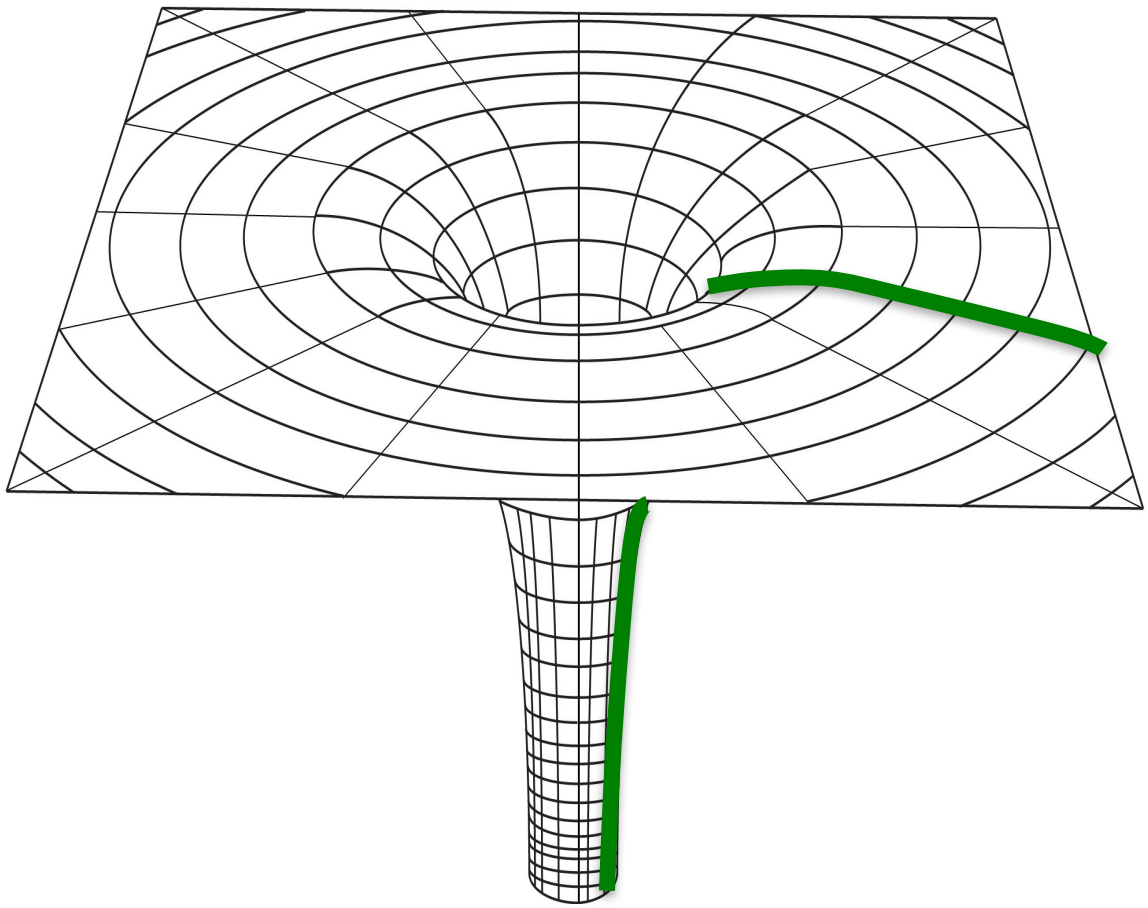
(Here we view F as non-dynamical).

# Motion of charged fermions

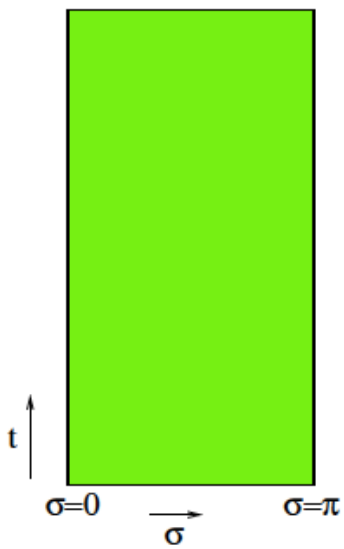
- Degeneracy = q = flux of the magnetic field on the sphere. Form a spin j, representation of SU(2), 2j +1 =q.
- We effectively get q massless two dimensional fermions along the time and radial direction.
- We can think of each of them as following a magnetic field line.

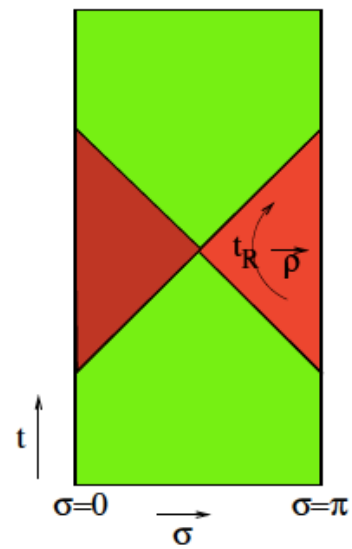


q massless two dimensional fields, along field lines.



# $AdS_2$



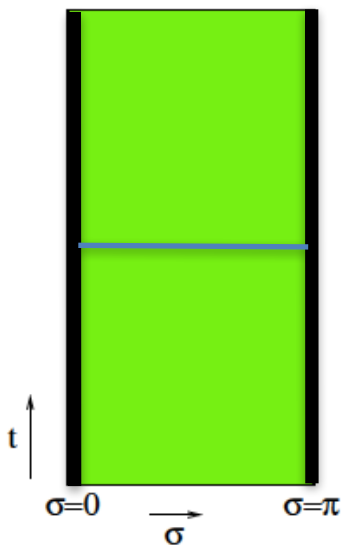

$$ds^2 = \frac{-dt^2 + d\sigma^2}{\sin^2\sigma}$$

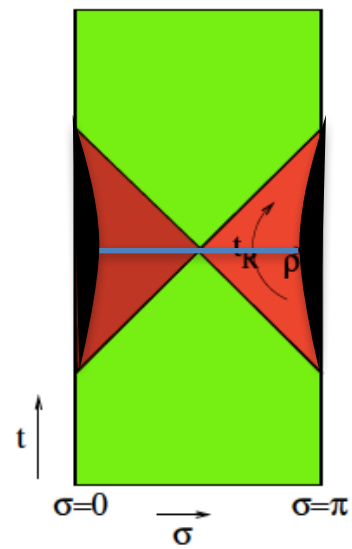
Global

$$ds^2 = -(r^2-1)d\tau^2 + \frac{dr^2}{(r^2-1)}$$

Thermal/Rindler

# Nearly $AdS_2$




$$ds^2 = \frac{-dt^2 + d\sigma^2}{\sin^2\sigma}$$

Global

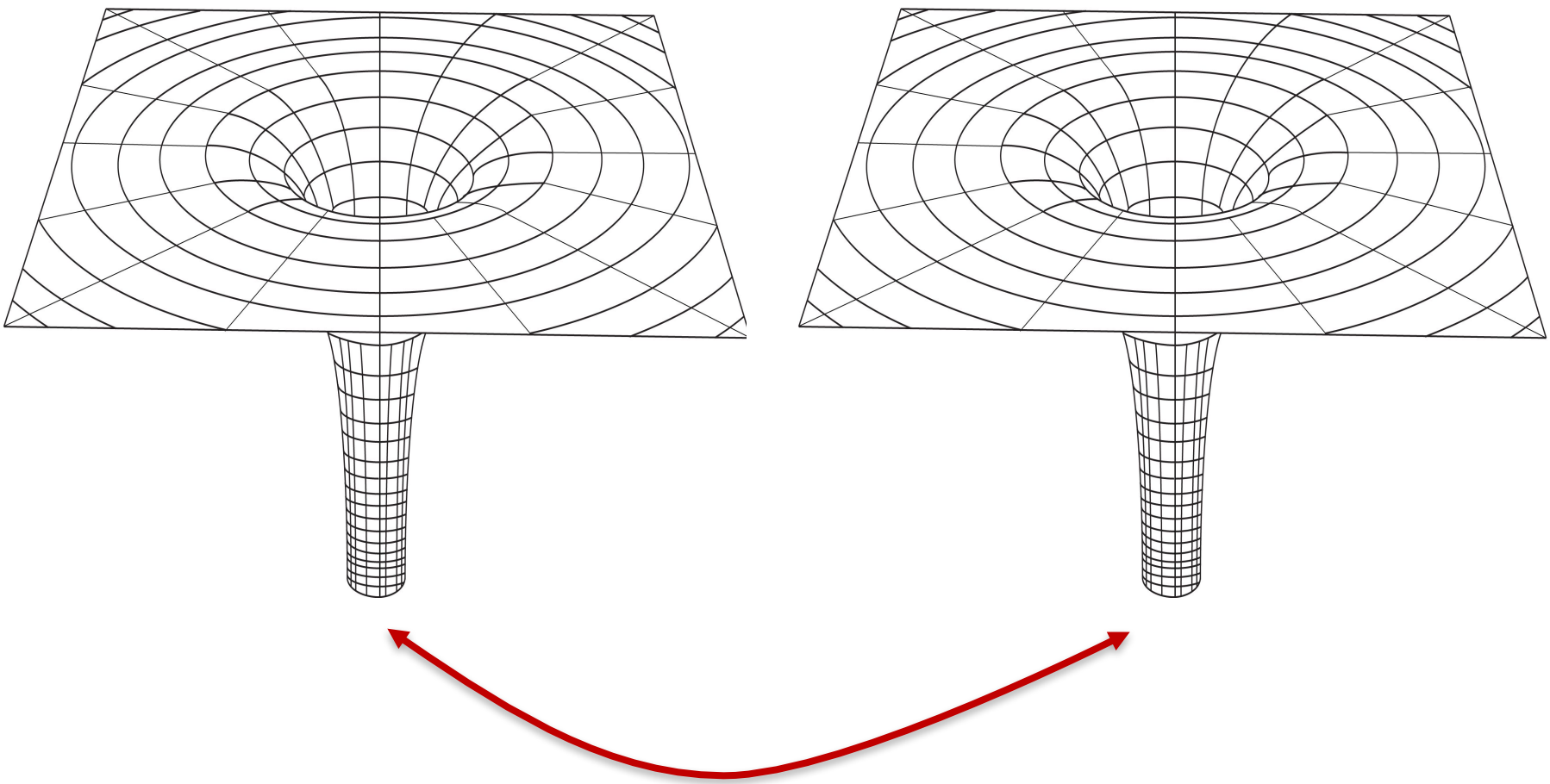
$$ds^2 = -(r^2-1)d\tau^2 + \frac{dr^2}{(r^2-1)}$$

Thermal/Rindler

Connect them to flat space, so that t is an isometry.
The acquire non-zero energy when the throat has finite length

$$M - q = q^3 T^2 = \frac{q^3}{\beta^2}$$

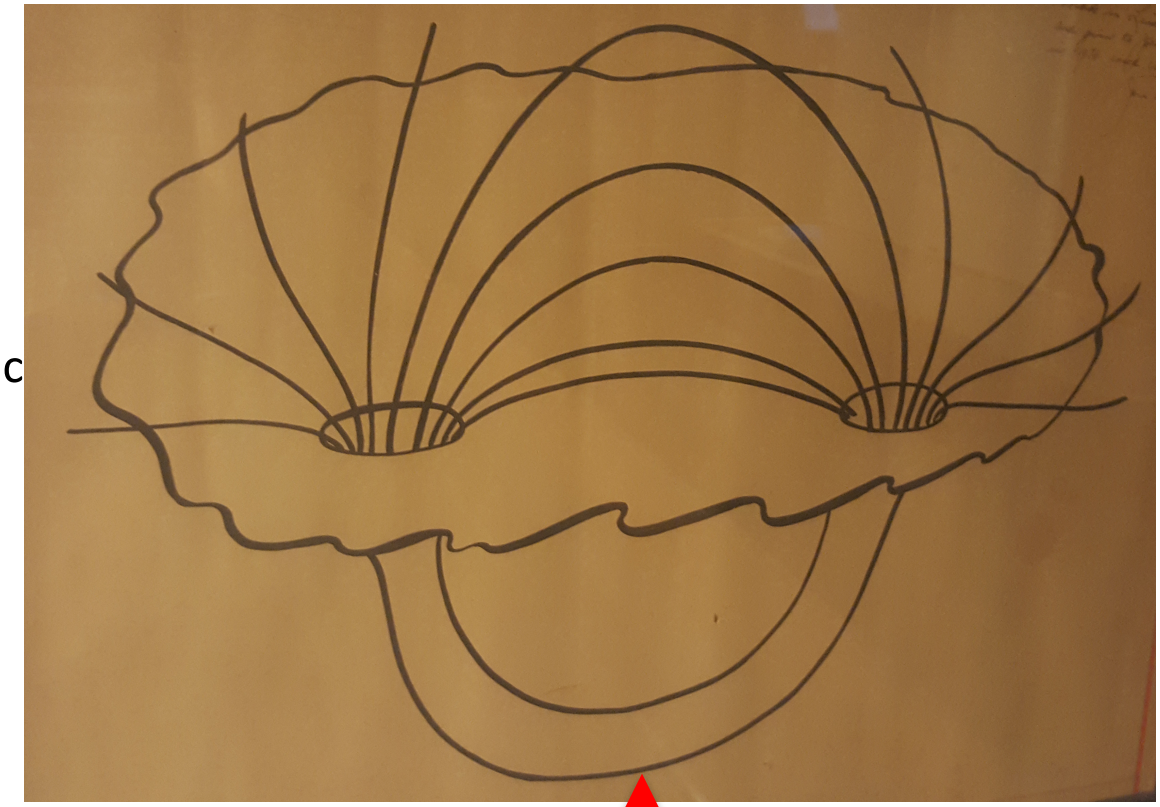
# Connect a pair black holes



connect and in global $AdS_2$

# Connect a pair black holes

Positive magnetic charge



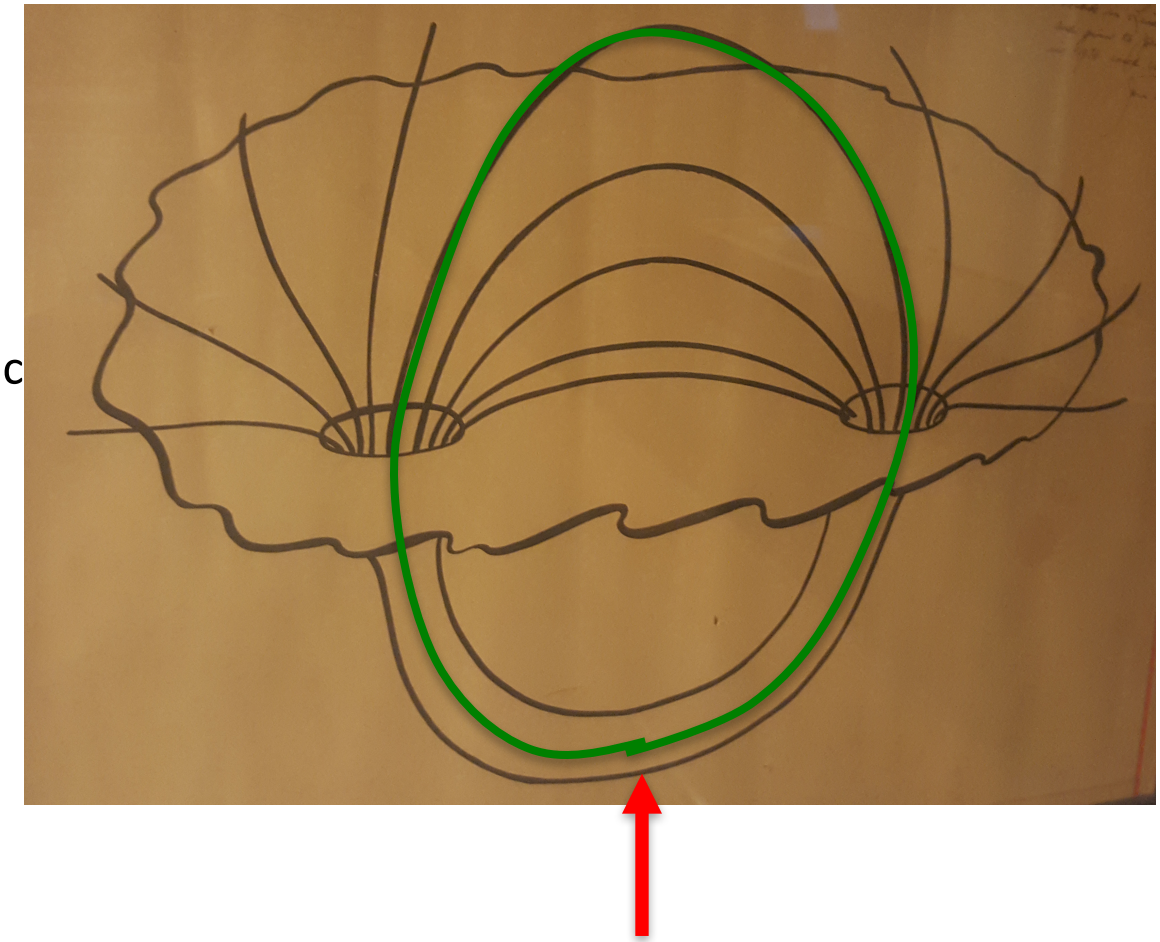
Negative magnetic charge

Not a solution yet.
Not a black hole.

Nearly $AdS_2 \times S^2$ wormhole of finite length

# Fermion trajectories

Positive magnetic charge



Negative magnetic charge

Charged fermion moves along this closed circle.

# Casimir energy

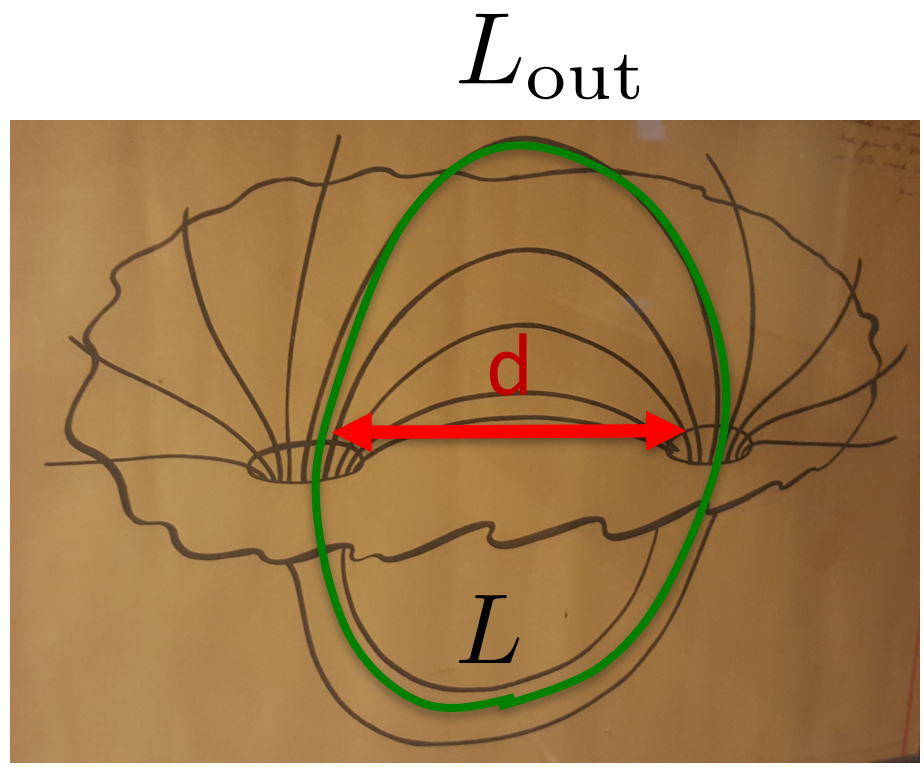
Assume: "Length of the throat" is larger than the distance.

$$L \gg L_{out} > d$$

Casimir energy is of the order of

$$E \propto -\frac{q}{L}$$

Full energy also need to take into
Account the conformal anomaly because
$AdS_2$ has a warp factor.
That just changes the numerical factor.



# Finding the solution

Solve Einstein equations in the throat region with the negative quantum stress tensor

=

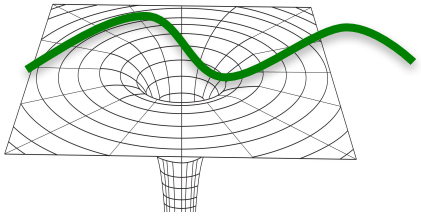
Balance the classical curvature + gauge field energy vs the Casimir energy.

$$M - q = \frac{q^3}{L^2} - \frac{q}{L}, \qquad \frac{\partial M}{\partial L} = 0 \longrightarrow L \sim q^2$$

Now the throat is stabilized. Negative binding energy.

$$E_{\text{binding}} = M - q = -\frac{1}{q} = -\frac{1}{r_s}$$

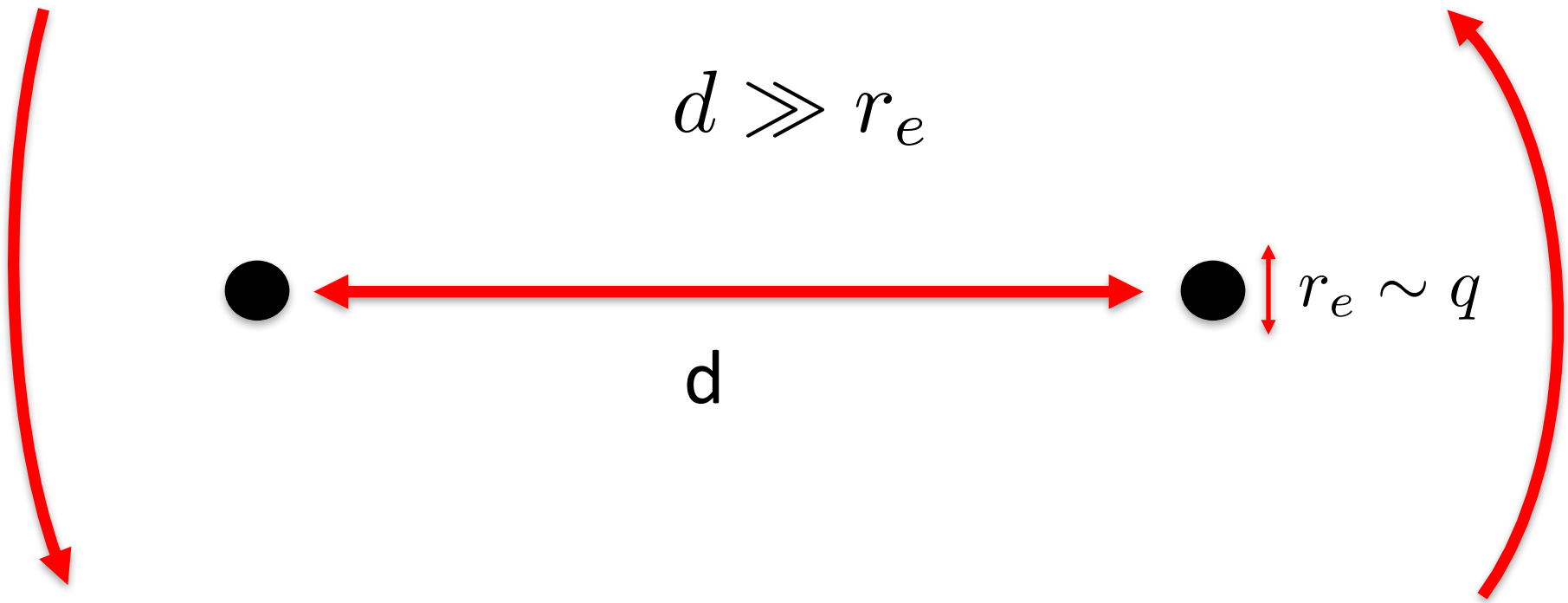
Very small. Only low energy waves can explore it



This is not yet a solution in the outside region:

The two objects attract and would fall on to each other

# Adding rotation




$$\Omega = \sqrt{\frac{r_e}{d^3}}$$

Kepler rotation frequency

# Throat is fragile

- We must make sure not to start sending matter into the throat that can accumulate there and produce a black hole.

- Rotation → radiation → effective temperature: $T \sim \Omega$

- We need that Ω is smaller than the energy gap of the throat

$$\Omega \ll \frac{1}{L}$$

- The configuration will only live for some time, until the black holes get closer..

- These issues could be avoided by going to $AdS_4$ …

# Some necessary inequalities

$$L \sim q^2$$

From stabilized throat solution

$$d \ll L \longrightarrow d \ll q^2$$

Black holes close enough to that Casmir energy computation was correct.

$$\sqrt{\frac{q}{d^3}} = \Omega \ll \frac{1}{L} \longrightarrow q^{\frac{5}{3}} \ll d$$

Kepler
rotation frequency

Black holes far enough so that they rotate slowly compared to the energy gap.

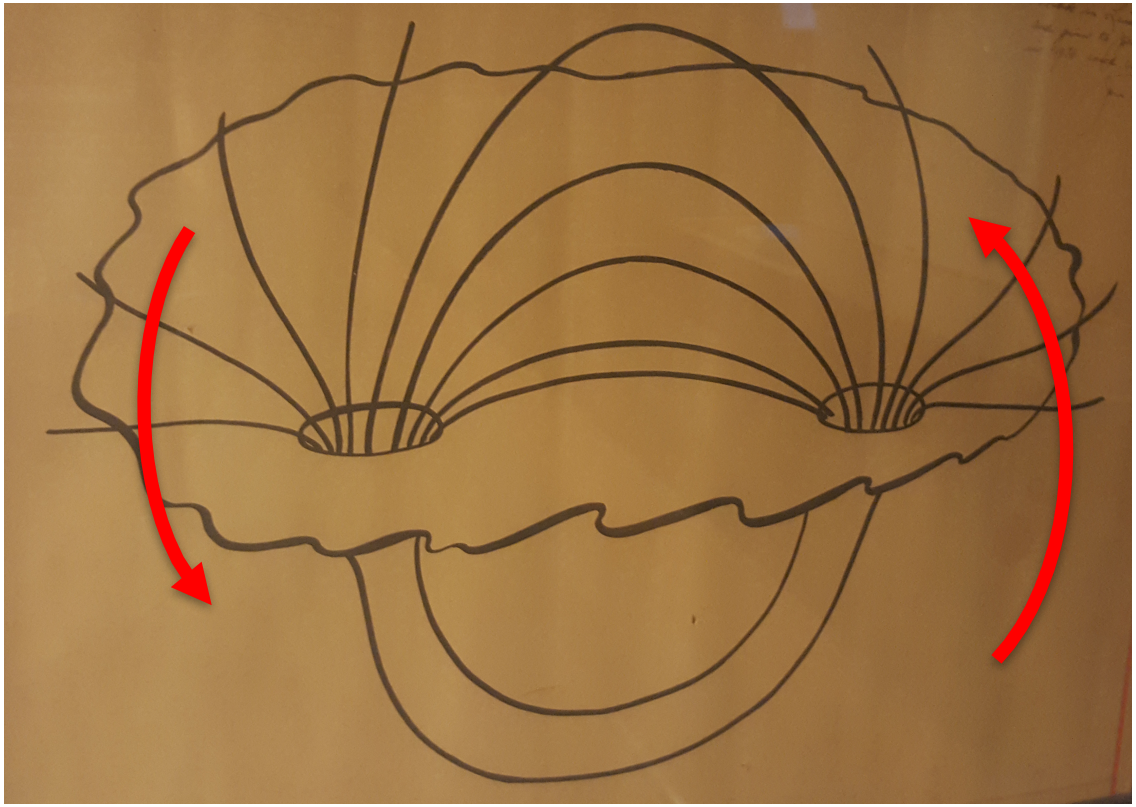
Unruh-like temperature less than energy gap

They are compatible

$$q^{\frac{5}{3}} \ll d \ll q^2$$

Other effects we could think off are also small :
can allow small eccentricity, add electromagnetic and gravitational radiation, etc. Has a finite lifetime.

# Final solution



Looks like the exterior of two near extremal black holes. But they connected.
But there is no horizon!.  Zero entropy solution.
It has a small binding energy.

# Rotation → temperature

$$T \sim \Omega \ll E_{\rm gap} \sim \frac{1}{L}$$

Temperature does not create particles in the throat

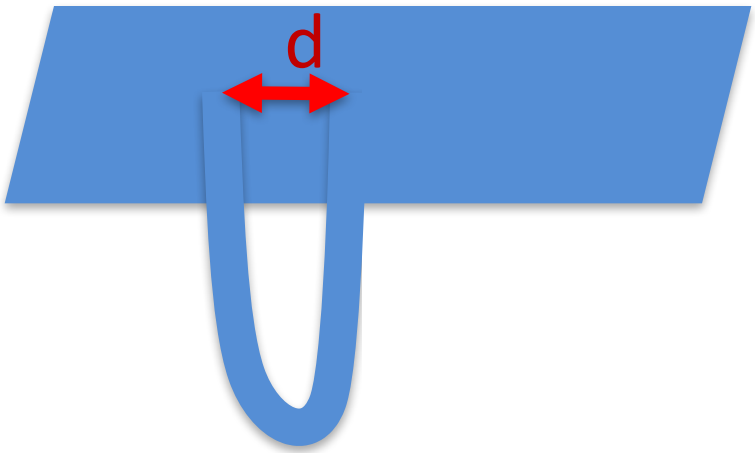
$$\text{Two Black Holes}: \quad F = -Tq^2$$

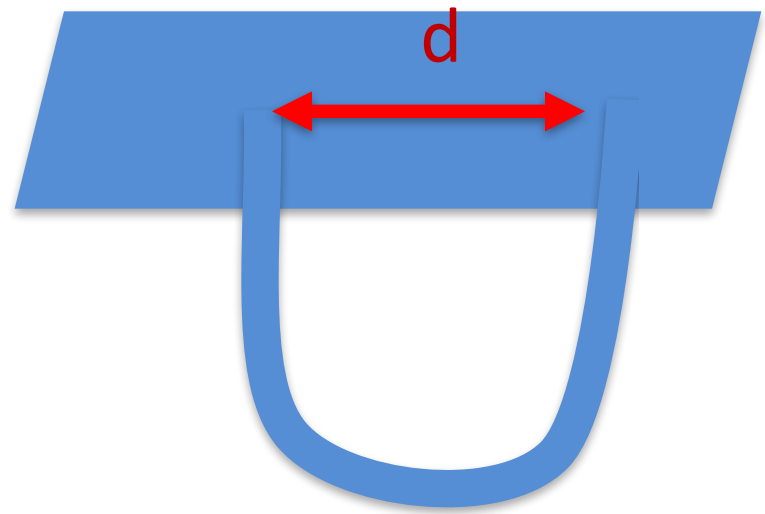
$$\text{Wormhole}: \quad F = -E_{\rm binding} = -\frac{1}{q}$$

Wormhole is the stable thermodynamic phase for $T < 1/q^3$

For the solution we described so far: Wormhole is metastable.

# Length L as d increases

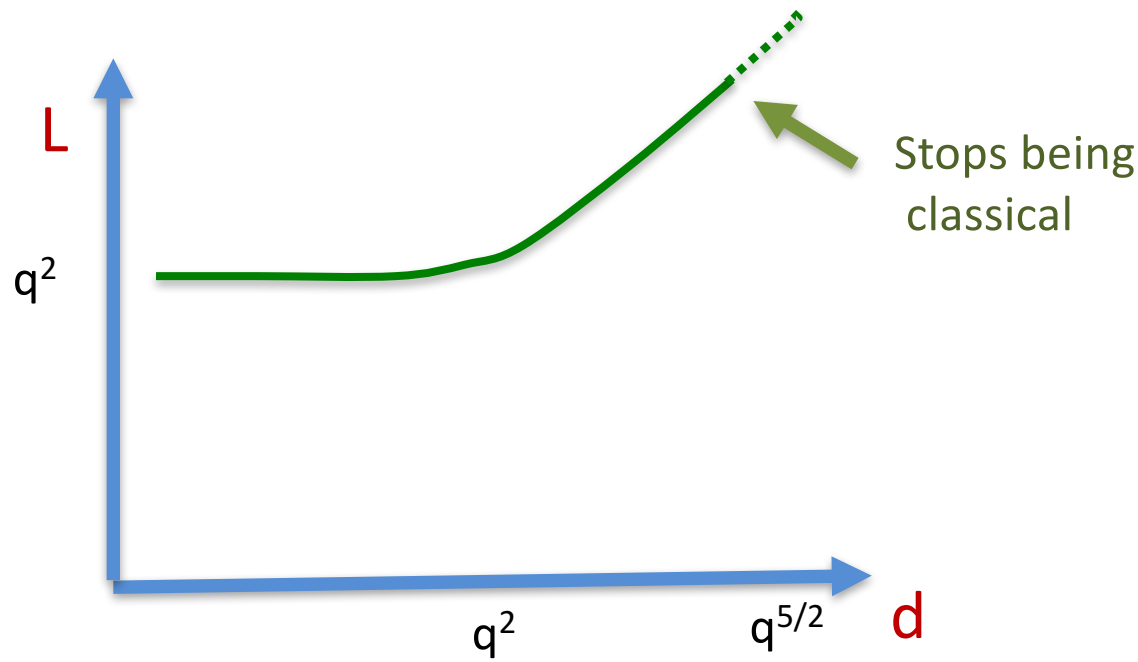



$$E_c \propto +\frac{1}{4}\frac{1}{L} - \frac{1}{L+d}$$

Conformal anomaly

Casimir cylinder



# Wormholes in the Standard Model

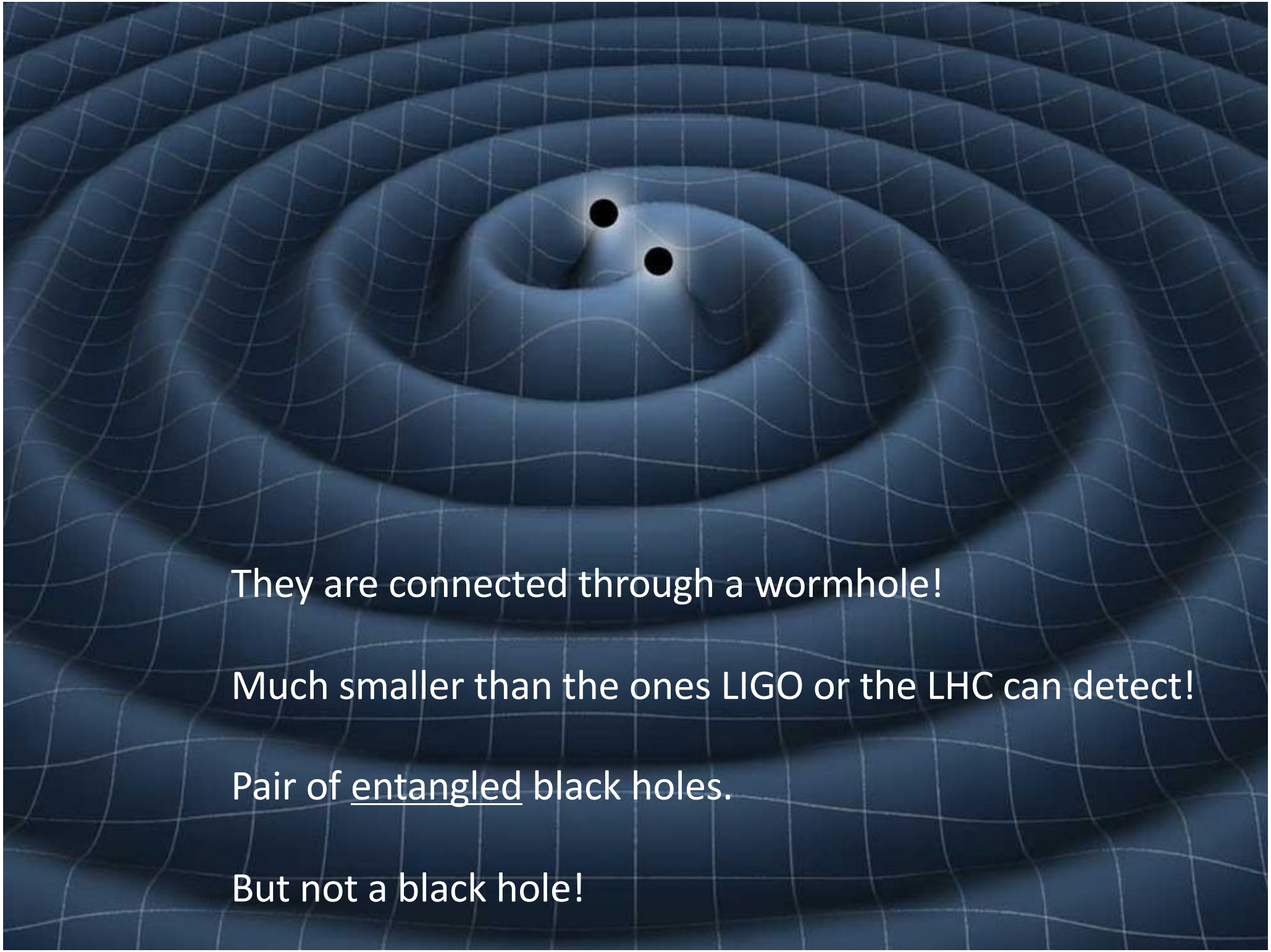
If nature is described by the Standard Model at short distances and d is smaller than the electroweak scale,

$$1 \ll q \ll 10^8$$

Distance d smaller than electroweak scale.

If the standard model is not valid → similar ingredients might be present in the true theory.

That it <u>can</u> exist, does not mean that it is <u>easily</u> produced by some natural or artificial process.



They are connected through a wormhole!

Much smaller than the ones LIGO or the LHC can detect!

Pair of <u>entangled</u> black holes.

But not a black hole!

# Entropy and entanglement

- Total spacetime has no entropy and no horizon.
- If we only look at one object → entanglement entropy = extremal black hole entropy

- Wormhole = two entangled black holes

- Total Hamiltonian $$H = H_L + H_R + H_{\rm int}$$

Generated by fermions in exterior

# Conclusions

- We displayed a solution of an Einstein Maxwell theory with charged fermions.
- It is a traversable wormhole in four dimensions and with no exotic matter.
- It balances classical and quantum effects.
- It has a non-trivial spacetime topology, which is forbidden in the classical theory.
- It has no horizon and no entropy.
- Can be viewed as a pair of entangled black holes.

# Questions

- If we start from disconnected near extremal black holes: Can they be connected quickly enough ? → topology change.
- Could we turn it into a prediction from quantum gravity ?