How 1/D expansion simplifies BH physics

Work in 2013-2018 w/ R. Suzuki + K Tanabe

+ Andrade, Grumiller, Izumi, Licht, Luna, Martínez, Shiromizu, Tanaka

A different formulation: S. Bhattacharyya + Minwalla

Saha, Thakur, Dandekar, Mohan, Mandlik
Dc, Mazumdar, A Mandal, Mehta, Mishra, Kundu
Nandi

Same concepts but different implementation

Others: Withers, Gentle, Pantelidou, Herzog, Yarom, Rozali....
mostly AdS/CFT-CMT Chen et al

Early: B. Kol et al: did one example, got some notions right, didn't develop into a full picture

$R_{\mu\nu}=0$: too perfect Theory! No scale
$(R_{\mu\nu}=\Lambda g_{\mu\nu})$ No parameter

$\downarrow$

D

D as a parameter

$D<4$: UV regulator IR very different

$D=3,2$: no local dof's
no UV divergences
May learn how to quantize a Theory w/ diff-invce

But: lose BHs of $R_{\mu\nu}=0$
BHs of $R_{\mu\nu}=-\Lambda g_{\mu\nu}$ are rather different

No GWs

$D > 4$: IR regulator

(Bad UV?)

Keep BHs & GWs

But Theory becomes more complicated!

$D = 5, 6, \ldots$: many other bhs

More complication may be interesting

But we were thinking of simplifying

Large N: SU(2), SU(3) ... s

$N \to \infty$ gives a highly non-trivial reformulation

dof's reorganize into worldsheets

gluons — Meson sector

Maybe $D \to \infty$ gravity (or some sector) dof's

reorganize into something else non-trivially

It does, at least for bh sector

Large D and other large # : 1/# expansions

Two different classes of #

\# ~ local dof's at a point

Large N, eg SU(N)

also vector models, Potts models, SYK...

Large c in 2D CFT

: ~ connections between nearby points
~ directions out of a point : large D

Large coordination number in a lattice
quantum fluctuations average out ~ MFT

CFT_D : conformal blocks are solvable (good down to $D=2$)

Hydro? Burgers Turbulence?

Large D gravity has both aspects

dof's at a point ~ D^2 (graviton polarizations)

directions out of a point ~ D

Which one is more useful? Large-N-like?
or MFT-like?

Exploit large radial gradients of gravitational potential

Large-D expansion and quantum entanglement?

- Short-distance qu fluctuations strongly enhanced
- Long-distance " " average out

Dual to behavior of grav'l field at large D

$\boxed{\text{BHS}}$ (other aspects of gravity: for questions)

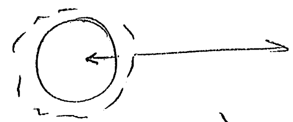
Large-D limit of BH dynamics: main results

$\exists$ well-defined near-horizon region (universal)

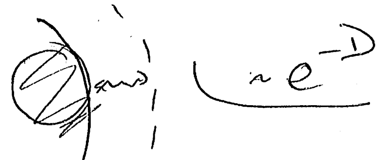
~~BH~~ -

Fluctuations split into two sectors ~~regimes~~ depending on frequency $\to$ fast/slow and on whether they're localized in NH or not non-decoupled / decoupled

Fast, non-decoupled: $\omega \sim D/r_0$



Slow; decoupled: $\omega \sim D^0/r_0 \sim 1/r_0$



Non-dec: most modes $O(D^2)$ of them

natural frequency $\omega \sim \kappa$: characteristic time to cross NH region

universal spectrum

Dec: few modes, $O(D^0)$ 3 modes

"unnaturally" small frequency

non-universal spectrum: depends on bh

characterizes the bh

~~Near-horizon solution~~ Elementary observations

$$ds^2 = -\left(1-\left(\frac{r_0}{r}\right)^{D-3}\right)dt^2 + \frac{dr^2}{1-\left(\frac{r_0}{r}\right)^{D-3}} + r^2 d\Omega_{D-2}$$

$$\Phi(r) \sim \left(\frac{r_0}{r}\right)^{D-3} \qquad \nabla\Phi\Big|_{r_0} \sim \frac{D}{r_0} \gg \frac{1}{r_0}$$

Two scales: $r_0 \gg \frac{r_0}{D}$

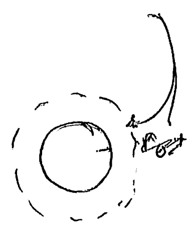
other scales for black branes $\lambda_{instab} \sim \frac{r_0}{\sqrt{D}}$

$v_{sound} \sim \frac{1}{\sqrt{D}}$

$r > r_0$: $D \to \infty$ $\Phi(r) \to 0$: flat space
force lines infinitely diluted

$r - r_0 \lesssim \frac{r_0}{D} \Rightarrow \left(\frac{r_0}{r}\right)^{D-3} = O(1)$

non-trivial gravitational field



$\frac{r_0}{D}$

Near-horizon geometry $\left(\frac{r}{r_0}\right)^{D-3} \equiv \cosh^2\rho$

$$ds^2_{nh} \to \frac{4r_0^2}{D^2}\left(-\tanh^2\rho\, dt^2 + d\rho^2\right) + r_0^2(\cosh\rho)^{4/D}\, d\Omega^2_{D-2}$$

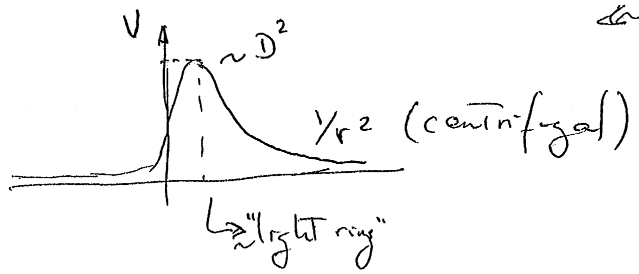
$\hookrightarrow$ dilaton $e^{-4\Phi/D}$

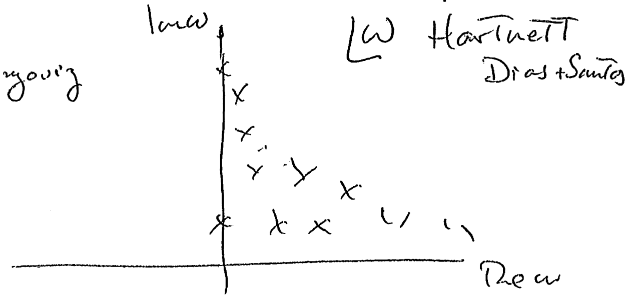
2D string bh [Wadia et al, Rabinovici et al, Witten] $\longrightarrow$

Fluctuations of BH horizon

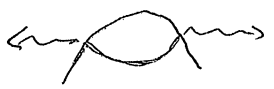
QNMs $\left(\frac{d^2}{dr_*^2} + \omega^2 - V_s\right)\Psi_s = 0$ $s = S, V, T$

$r_* \in (-\infty, \infty)$ no outgoing down $\rightsquigarrow$ No ingoing

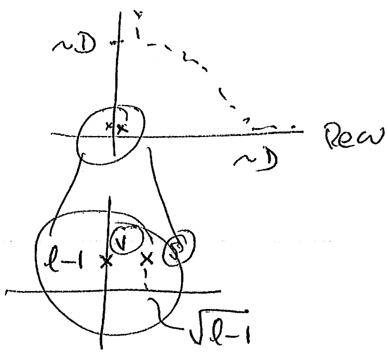




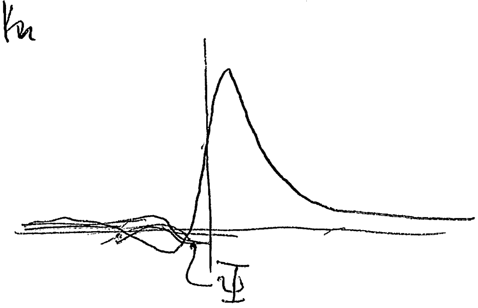
QNMs

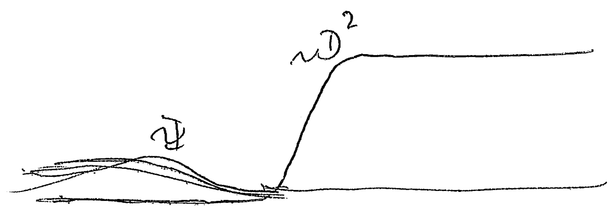


$\omega \sim D/r_0$



Im

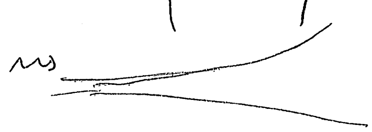




Decoupled modes $\sim e^{-D}$ outside

These modes are <u>static</u> at LO $\frac{\omega}{\kappa} \Rightarrow 0$ at $D \to \infty$

No infinitely long throat



Here it is a very short region that is crossed in very short time $\sim 1/D$

Decoupled modes must be static to LO:

LO geometry is universal LO decoupled modes appear universally

But their frequency $\frac{\omega}{\kappa} = \underset{\text{universal}}{0} + \underbrace{\frac{\omega_1}{D} r_0}_{\text{not-universal}}$ $\kappa \sim \frac{D}{r_0}$

$\omega \sim \omega_1 + \frac{\omega_2}{D} + \dots$

$\hookrightarrow$ not-universal but easily calculable LO

Analytic calculations of QNMs

Schw-(A)dS BHs

Black branes: GL instability

sound + shear in AdS branes

Rotating bhs: ultraspinning instability

Large D can be a very good approx for moderate, even small D

4D vector mode purely imaginary agrees (up to $1/D^3$) w/ 6% in $\underline{D=4}$

GL Threshold mode in $D=6$ $k_{GL} r_0 = 1.238$ $(1/D^4)$

1.269 numerical 2.4%

Ultraspinning MP zero mode

Numerical: $(D=8)$ $\frac{a}{r_+} = 1.77$ 2.27 2.72

Leading large D $\sqrt{3}$ $\sqrt{5}$ $\sqrt{7}$ error $\lesssim 2.7\%$

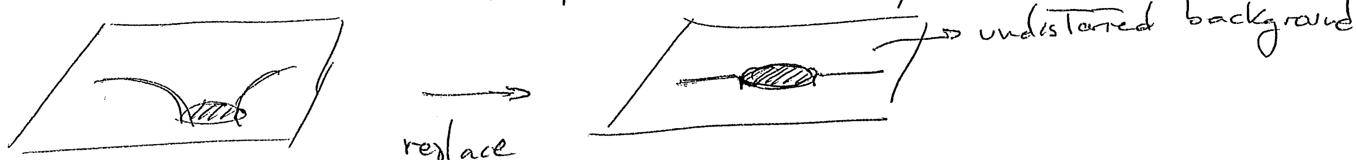
Series in $1/D$ is not convergent but asymptotic: zero-convergence radius

Far-region effects enter like $\frac{1}{r^D} \sim e^{-D}$

This limits the accuracy at relatively low D: higher terms in the series don't improve accuracy

Non-linear fluctuations: effective Theory

Construct non-linear Theory of slow, decoupled QNMs

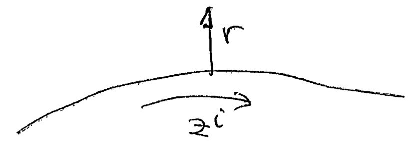


replace
bh w/ membrane
shape and motion of membrane obtained by
solving Einstein equations near The horizon

Gradient hierarchy

$\perp$ Horizon : $\partial_r \sim D$

$\parallel$ Horizon : $\partial_\theta,\ \partial_z \sim 1$ (or $\sim \sqrt{D}$)



~~Comparison To fluid/gravity correspondence~~

Integrate (solve) radial dependence
Obtain integration constants-in-r, functions in $(\overbrace{t, z^i}^{z^\mu})$ $\begin{cases} \rho(t, z^i) \\ v^i(t, z^i) \end{cases}$
Obtain constraints on ρ, v^i : effective ~~...~~ membrane equations
(from Einstein eqns)

membrane collective fields

$$ds^2 = \frac{N^2(r, z^a)}{D^2} dr^2 + g_{ab}(r, z^c)\, dz^a dz^b$$

$\Rightarrow$ Einstein eqns : r-index: radial "evolution" eqn : solve explicitly
μ-index: r-independent scalar and vector constraints
express in Terms of "integration functions"

Comparison To fluid / gravity:

- In both cases one integrates a radial direction, obtain constraint effective equations for $\rho(z^a)$, $v^i(z^a)$

- The modes That are captured are The same:
  scalar (sound)
  vector (shear)

- Validity regime is essentially equivalent $|\partial_a| \ll |\partial_r|$
  fluid/gravity $|\partial_a| \ll T \sim \frac{D-3}{L^2} r_H$

  large D $|\partial_a| \ll T \sim \underset{\downarrow \text{AF}}{\frac{D}{r_H}}$, $\underset{\downarrow \text{AdS}}{D \frac{r_H}{L^2}}$

- But They are not equivalent - They capture The non-linear dynamics of sound and shear in different ways
  In short: leading fluid order is perfect fluid
  leading large-D order is never just perfect fluid
  leading large-D: IT ~~captures~~ includes higher Transport coeffs (viscosity + next order)

- ~~Large D applies To~~ Fluid/gravity requires extended (black brane) horizon so That $\partial_a \ll \partial_r \sim O\left(\frac{1}{r_0}\right)$
  otherwise no separation of scales

  Large-D applies for (essentially) all bhs, where $\frac{\partial_a}{\partial_r} \sim O(1), O(\sqrt{D})$

  Eg can apply To Schw$_D$ fluctuations

Fluid gravity makes ∂_a small, while ∂_r remains $O(1)$

Large D makes ∂_r large, while ∂_a remains $O(1)$

Effective Theory

Stationary configurations

Soap-bubble equation

$$\boxed{K = 2\gamma\kappa}$$

K: Trace of extrinsic curvature of membrane embedding

γ: redshift factor on membrane $\gamma^{-1} = \sqrt{|g_{tt}|(1-v^2)}$

Lorentz boost from rotation

gravtl redshift from background

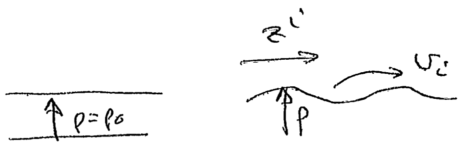
κ: surface gravity

Static soap-bubble : sphere = Schw (A)dS

Rotating soap-bubble : spheroid : ~~MP~~ Myers-Perry rotating bh

Effective Theory for black branes (AF or AdS)

Eff fields $\rho(t, z^i)$ $\quad v_i(t, z^i)$



$$\partial_t \rho + \nabla_i(\rho v^i) = 0 \quad : \text{continuity}$$

ρ = mass-density
~~area~~ horizon-radius

$$\partial_t(\rho v_i) + \nabla^j\Big(\underbrace{\pm \rho\, \delta_{ij}}_{\text{pressure}} + \rho v_i v_j \underbrace{- 2\rho \nabla_{(i} v_{j)}}_{\text{viscosity}} - \rho \nabla^2_{ij} \log \rho\Big) = 0$$

$+ \rightarrow$ AdS : +ve pressure
$- \rightarrow$ AF : −ve pressure: unstable

- "Hydro" truncates exactly.

  Why? For stationary configs the previous eqn can be rewritten as

  $$\sqrt{1 - v^2}\, K = \text{const}$$

  $\downarrow$ includes crucially the term $\nabla^2_{ij} \log \rho$ but not more, to LO in $1/D$

- Hydro is non-relativistic.

  Why? Sound speed is $c^2_{\text{sound}} \sim \frac{1}{D} \quad \left(\frac{1}{D-1}, -\frac{1}{D-3}\right)$

  $\ll 1$

Other problems:

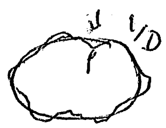
- Critical collapse
- BH collisions
- SYK-type models (Ferrari 2017)

Limitations:

Gradients along horizon $\partial_z < O(D)$

In many situations this doesn't happen, as the evolution drives these

Amplitude of non-stationary fluctuations $\sim O(1/D)$



S-wave reduction

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu + r_0^2 e^{-4\phi/n+1} d\Omega_{n+1}^2 \qquad n = D-3$$

$$I = \frac{\Omega_{n+1} r_0^{n+1}}{16\pi G} \int d^2x \sqrt{-g}\, e^{-2\Phi} \left(R + \frac{4n}{n+1} (\nabla \Phi)^2 + \frac{n(n+1)}{r_0^2} e^{4\Phi/n+1} \right)$$

$$\overset{\shortparallel}{\frac{1}{16\pi G_2}}$$

$n \to \infty$ w/ $\lambda = \frac{n}{2 r_0}$ (finite surface gravity $\equiv$ Temperature) $T = \frac{\lambda}{2\pi}$

$$\to I = \frac{1}{16\pi G_2} \int d^2x \sqrt{-g}\, e^{-2\Phi} \left(R + 4(\nabla\Phi)^2 + 4\lambda^2 \right)$$

$$\sqrt{\alpha'} \sim \frac{r_0}{D}$$

$$S_{BH} \simeq M^{1 + \frac{1}{D-3}} \qquad T_H \sim \frac{1}{\sqrt{\alpha'}}$$

In far region we keep r_0 fixed so $\alpha' \to 0$: massless gravitons

Near-horizon is stringy region

Planck scale effects can be suppressed parametrically