CERN talk: Gravitational Collapse in the SYK model
23 August 2018.

References:

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II. J. Maldacena & Xiao-Liang Qi, 1804.00491.

III. K. Papadodimas + S. Raju, 1211.6767

IV. Jan de Boer, S. Lokhande, E. Verlinde, R. van Breukelen, Kyriakos Papadodimas - 1804.10580

to appear:

A model for gravitational collapse in the SYK model

by A. Dhar, A. Gaikwad, L. Joshi & ~~[illegible]~~ G. Mandal & SRW.

Introduction:

Soluble models within AdS/CFT that exhibit some characteristics of black holes are hard to find (e.g. for the 2-dim. bh in dilaton-gravity).

SYK model has been successful ~~in~~ in this respect.

1. It is soluble in the large N, large βJ limit.
2. the low energy dynamics is described by the time reparametrization mode $t \to f(t)$, and the action for this mode is the
4. Schwarzian : $S = \alpha \frac{N}{J} \int dt \{f,t\} + o\left(\frac{1}{\beta J}\right)$.
4. 2 dual theories of quantum gravity for the 'low energy sector'.

(4-dim reduction, JT) Polyakov (method of co-adjoint orbits of $Vir/SL(2,R)$)

only boundary degrees of freedom.

3. $t_S \cong \frac{\beta}{2\pi} \ln N$.

5. ~~Can one address other questions related to bhs?~~

$$ds^2 = \frac{dz^2 - dt^2\left(1 + \frac{\{f,t\}}{2} z^2\right)^2}{z^2}$$

Poincaré: $f(t) = t \Rightarrow ds^2 = \frac{dz^2 - dt^2}{z^2}$

Bh: $f(t) = \frac{\pi}{\beta J^2} \tanh \frac{\pi}{\beta} t \Rightarrow ds^2 = \frac{dz^2 - dt^2\left(1 - \frac{\pi^2}{\beta^2} z^2\right)^2}{z^2}$

5 Can one address other questions about bhs?

(PR)

~~Information~~

KM introduced a fine tuned quench to be able to 'see' behind the horizon (for a single SYK).

MQ discussed a pair of SYKs and wormhole - bh dynamics in the bulk.

6. We study the KM model and introduce another finely tuned quench that leads to a new asymptotic bh. with a

$$T_{bh} \propto (\epsilon_{cri} - \epsilon_2)^{1/2}$$

also the KM model can be exactly integrated to give:

$$e^{\phi} = \dot{f}(t) = \frac{a_1}{(b_1 \cos c_1 t - 1)^2} \qquad 0 < t < T$$

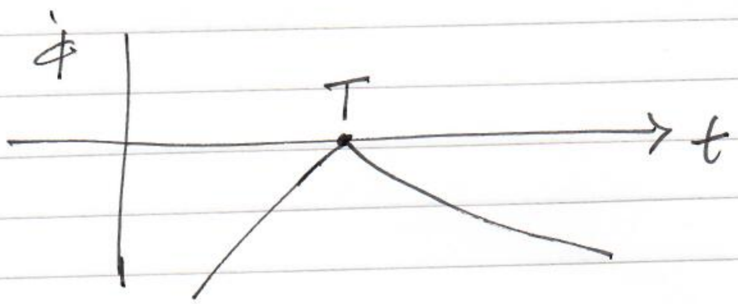
$$\cancel{e^{\phi} = \dot{f}(t) = \frac{a_2}{(b_2 \cosh c_2 t - 1)}} \qquad T < t < \infty.$$

$$e^{\phi} = \dot{f}(t) = \frac{a_2}{\left(b_2 \cosh\left(\underbrace{c_2[t-T] + d_2}\right) - 1\right)^2}$$

$\left.\begin{array}{l}\dot{f}(t) + \ddot{f}(t) \\ \phi(t) + \dot{\phi}(t)\end{array}\right\}$ continuous at $t = T$

$\ddot{\phi}$, $\dddot{f}$ have a discontinuity at $t = T$

So that $\dot{\phi}$, $\ddot{f}$ have a kink at $t=T$.



reminiscent of dynamic critical phenomena.

$\Rightarrow$ $G^{[f]}(t_1, t_2)$ has a discontinuity at

$$\bar{t} = \frac{t_1+t_2}{2} = T \quad ; \quad t = t_1 - t_2$$

$$\left[G^{[f]}(t, \bar{t}) = |t|^{-2\Delta}\left(1 - \frac{t^2 \Delta}{6}\{f, \bar{t}\} \cdots \right) \right]$$

Hilbert space of the SYK model

N Majorana fermions : ψ_i, $i=1,\dots,N$, $\{\psi_i,\psi_j\}=\delta_{ij}$, $\psi_i^2=2$

$$H=\sum_{1\le i<k<l<m\le N} J_{iklm}\,\psi^i\psi^k\psi^l\psi^m \quad , \quad \langle J^2_{iklm}\rangle = 3!\,\frac{J^2}{N^3}$$

Spin operators : $S_k \equiv 2i\,\psi^{2k-1}\psi^{2k}$, $k=1,\dots,\frac{N}{2}$, $S_k^2=1$

~~now introduce~~ the eigenstates : $|B_s\rangle$:

$$\left[\begin{array}{l} S_k|B_s\rangle = s_k|B_s\rangle,\quad s_k=\pm1,\ \text{this defines } \underbrace{\tfrac{N}{2}\ \text{states}}_{\text{basis of the hilbert space.}} \\ \Leftrightarrow\ (\psi^{2k-1}-i s_k\psi^{2k})|B_s\rangle = 0 \end{array}\right]$$

Energetics: $\langle B_s|H|B_s\rangle \sim 0$, hence $|B_s\rangle$ are high energy states compared to $E_0=-N$ (number).

[complete proof]

$\langle B_s|B_{s'}\rangle = \delta_{ss'}$

$$|B_s\rangle = \sum_\alpha c_\alpha |E_\alpha\rangle \Rightarrow \boxed{\begin{array}{c} e^{-lH}|B_s\rangle \\ \equiv \\ |B_s(l)\rangle \end{array}} = \sum_\alpha c_\alpha e^{-lH}|E_\alpha\rangle = \sum_\alpha c_\alpha e^{-lE_\alpha}|E_\alpha\rangle$$

$\langle B_s|B_s\rangle = \sum_\alpha |c_\alpha|^2 = 1$

$\Rightarrow |c_\alpha|^2 \sim \frac{1}{2^{N/2-1}}$

$|c_\alpha|^2 \sim 2^{-\frac{N}{2}+1}$

for large l, $|B_s(l)\rangle$ are over complete low energy states for large l :

$$\langle B_s(l)|B_{s'}(l)\rangle = \langle B_s|e^{-lH}e^{-lH}|B_{s'}\rangle = \langle B_s|e^{-2lH}|B_{s'}\rangle \not\propto \delta_{ss'}$$

Hence over complete.

But
$$\sum_{s_k}\langle B_s(l)|\psi\cdots\psi|B_s(l)\rangle = \mathrm{Tr}(\underbrace{e^{-\beta H}}_{Z}\psi\cdots\psi)$$
$$=\sum_{s_k}\langle B_s|e^{-lH}\psi\cdots\psi\, e^{-lH}|B_s\rangle = \mathrm{Tr}(e^{-lH}\psi\cdots\psi\, e^{-lH})$$
$$=\mathrm{Tr}(e^{-2lH}\psi\cdots\psi),$$

thermal partition function for $\beta=2l$.
This is an exact result that depends only on the completeness of the $\{|B_s\rangle\}$ basis.

Large N limit: Replica method for random coupling problem. [no spinglass]
For the replica diagonal solution (replica symmetry unbroken) interaction between replicas is down by $\frac{1}{N^3}$ $\left(\frac{1}{N^{q-1}}\right)$

Hence J_{iklm} can be treated as time independent random fields: (quenched average = annealed average) , SYK has $O(N)$ symmetry.

Flip group $\subset O(N)$: $\psi_{2k}\to-\psi_{2k}$, $S_k = 2i\,\psi_{2k-1}\psi_{2k} \to -S_k$

$$\begin{bmatrix}1&0&0&0\\0&-1&0&0\\0&0&1&0\\0&0&0&-1\end{bmatrix}\begin{bmatrix}\psi_1\\\psi_2\\\psi_3\\\psi_4\end{bmatrix} = \begin{bmatrix}\psi_1\\-\psi_2\\\psi_3\\-\psi_4\end{bmatrix}$$

$|B_s\rangle \to |B_{-s}\rangle$

$|\pm\rangle\otimes|\pm\rangle\otimes\cdots \to |\mp\rangle\otimes|\mp\rangle\cdots$

$F_k = \sqrt{2}\,\psi_{2k-1}$

Consequence of Flip symmetry:

Flip invariant operator $\rightarrow$ $\hat{O} = F\hat{O}F^{-1}$

$$\langle B_s|\hat{O}|B_s\rangle = \langle B_{s'}|\hat{O}|B_{s'}\rangle, \quad |B_{s'}\rangle = F|B_s\rangle$$

Now all of $|B_s\rangle$ can be generated by the action of the flip group on any one basic state e.g. $|+\rangle\otimes|+\rangle\otimes|+\rangle\otimes|+\rangle\cdots$ Hence ~~correlators of~~ Flip invariant operators evaluate the same for $\forall\, |B_s\rangle$, $s=\{s_1, s_2, \ldots s_{N/2}\}$

Deep consequence of this is:

$$\sum_{s_k}\langle B_s(l)|\underbrace{\psi\cdots\psi}_{\text{Flip inv.}}|B_s(l)\rangle = \sum_{\{s_\alpha\}}\langle B_{s_0}(l)|\psi\cdots\psi|B_{s_0}(l)\rangle$$

$$= 2^{N/2}\langle B_{s_0}(l)|\psi\cdots\psi|B_{s_0}(l)\rangle$$

$$\| \qquad \mathrm{Tr}\, e^{-2l}\psi\cdots\psi$$

$$\Rightarrow \quad \langle B_{s_0}(l)|\psi\cdots\psi|B_{s_0}(l)\rangle = 2^{-N/2}\,\mathrm{Tr}\, e^{-2lH}\psi\cdots\psi.$$

any set of micro-states

in particular

$$\langle B_{s_0}(l)|B_{s_0}(l)\rangle = 2^{-N/2}\,\mathrm{Tr}\, e^{-\beta H} = 2^{-N/2} Z(\beta), \quad \beta = 2l.$$

For Flip invariant operators (including $\mathbb{1}$), thermal averages are evaluated by any of the pure states $|B_s(l)\rangle$, i.e. the precise micro quantum numbers are not seen.

$\Downarrow$

Examples: $\psi_i(\tau_1)\psi_i(\tau_2)$, $i=1,\ldots N$ is Flip invariant (no sum over i)

$$\langle B_{\vec{s}_0}(l)|\psi_i(\tau_1)\psi_i(\tau_2)|B_{\vec{s}_0}(l)\rangle = 2^{-N/2}\,\mathrm{Tr}\, e^{-\beta H}(\psi_i(\tau_1)\psi_i(\tau_2))$$

$$\Rightarrow \quad \frac{\langle B_{\vec{s}_0}(l)|\psi_i(\tau_1)\psi_i(\tau_2)|B_{\vec{s}_0}(l)\rangle}{\langle B_{\vec{s}_0}(l)|B_{\vec{s}_0}(l)\rangle} = \frac{\mathrm{Tr}\, e^{-\beta H}(\psi_i(\tau_1)\psi_i(\tau_2))}{\mathrm{Tr}\, e^{-\beta H}} = \langle\psi_i(\tau_1)\psi_i(\tau_2)\rangle_\beta = G_\beta(\tau_1-\tau_2)$$

MAIN FORMULA-1

Flip non-invariant operators:

$$G_{off}(t,t') \equiv \frac{\langle B_s(l)|\psi^1(t)\psi^2(t')\hat{S}_1|B_s(l)\rangle}{\langle B_s(l)|B_s(l)\rangle} = S_1\frac{\langle B_s(l)|\psi^1(t)\psi^2(t')|B_s(l)\rangle}{\langle 1\rangle}$$

$$= 2i\frac{\langle B_s(l)|\psi^1(t)\psi^2(t')\psi_1(-il)\psi_2(-il)|B_s(l)\rangle}{\langle 1\rangle}$$

$G_{off}(t,t') = \frac{1}{N}\sum \langle B_s(l)|\sum$

$$G_{off}(t,t') = -2i \frac{\langle B_s(l)|\,\psi_1(t)\psi_1(-il)\,\psi_2(t')\psi_2(-il)\,|B_s(l)\rangle}{\langle 1 \rangle}$$

$$O(N) \Rightarrow \quad = -\frac{2i}{N^2} \frac{\langle B_s(l)|\sum_i \psi_i(t)\psi_i(-il) \,.\, \sum_j \psi_j(t)\psi_j(-il)\,|B_s(l)\rangle}{\langle 1 \rangle}$$

large N factorization.

$$= -\frac{2i}{N^2} \frac{\langle B_s(l)|\sum_i \psi_i(t)\psi_i(-il)|B_s(l)\rangle}{\langle 1 \rangle}\frac{\langle B_s(l)|\sum \psi_j(t)\psi_j(-il)|B_s(l)\rangle}{\langle 1 \rangle}$$

$$= -\frac{2i}{N^2}\, G_\beta(t+il)\, G_\beta(t'+il) \qquad , \; 2l=\beta$$

$$\therefore \quad \frac{\langle B_s(l)|\,\psi^1(t)\psi^2(t')\,|B_s(l)\rangle}{\langle 1 \rangle} = \left(\frac{1}{S_1}\right) G_{off}(t,t')_\beta .$$

MAIN FORMULA - 2

(Expectation value of non-Flip invariant operator is not thermal in $|B_s(l)\rangle$, but remembers the micro-state.)

$$G_{diag}(\tau,\tau') = G_\beta(\tau-\tau'), \quad \beta=2l \quad + o\left(\tfrac{1}{N}\right)$$

$$G_{off}(\tau,\tau') = -2i\, G_\beta(\tau)\, G_\beta(\tau') + o\left(\tfrac{1}{N}\right)$$

solution of pure state problem at large N

ONLY large N expansion used

- Diagonal correlators are independent of the state $|B_s\rangle$;
- off-Diagonal " " dependent on the state $|B_s\rangle$ through the spin values $\{S_k\}$.

Formulas:
$$G_\beta(\tau) = \frac{C_\Delta}{\left[\frac{J\beta}{\pi} \sin\frac{\pi\tau}{\beta}\right]^{2\Delta}}, \qquad C_\Delta = \left[\left(\tfrac{1}{2}-\Delta\right)\frac{\tan\pi\Delta}{\pi}\right]^\Delta$$

$$\Delta = 4, \quad C_\Delta = \left(\frac{1}{4\pi}\right)^4$$

off-diag correlator in Lorentzian time: $\tau = \frac{\beta}{2} + it$

$$G_{off}(t,t') = -2i \frac{C_\Delta^2}{\left[\left(\frac{J\beta}{\pi}\right)^2 \cosh\frac{\pi t}{\beta} \cosh\frac{\pi t'}{\beta}\right]^{2\Delta}} \quad , \; \beta = 2l$$

decays for large time $e^{-\frac{\pi}{\beta}t\,(\text{number})}$

as it should for a thermal system

Lorentzian evolution for $t \geq 0$.

$$|B_s(l)\rangle \to e^{iHt}|B_s(l)\rangle, \quad H = H_{SYK} + \epsilon H_M, \quad H_M = -Ji\sum_{k=1}^{N/2} s_k \psi^{2k-1}\psi^{2k}$$

$$= -\frac{J}{2}\, 2i \sum_k s_k \psi^{2k-1}\psi^{2k}$$

treating ϵ as small, we work in the interaction picture.

$$H_M = -\frac{J}{2}\sum_{k=1}^{N/2} s_k \hat{S}_k$$

$\epsilon = 0$, H_{SYK} low energy physics is described by the reparametrization mode $t \mapsto f(t)$.

$$S = -\frac{\alpha_s N}{J}\int dt\, \{f(t), t\}$$

{For finite temperature $f(\tau) = \tan\left(\frac{\pi}{\beta}\varphi(\tau)\right)$, $\varphi(0) = \varphi(2l)$ + $\varphi'(0) = \varphi'(2l) = 1$.}

$\varphi(\tau) = \tau$ corresponds to a black hole at temp β^{-1}.

$$H_I(t) = e^{-itH_{SYK}} H_M\, e^{+itH_{SYK}}$$

$$\sum_k \langle B_s(l)|\, s_k\, \hat{S}_k(t)\, |B_s(l)\rangle, \quad \hat{S}_k(t) = e^{-itH_{SYK}}\hat{S}_k\, e^{+itH_{SYK}}$$

$$= (2i)^2 \sum_k \langle B_s(l)|\, \psi_{2k-1}(il)\,\psi_{2k}(l)\,\psi_{2k-1}(t)\,\psi_{2k}(t)\,|B_s(l)\rangle$$

$$= (2i)^2 \sum_k \langle B_s(l)|\, \psi_{2k-1}(il)\,\psi_{2k-1}(t)\,\psi_{2k}(il)\,\psi_{2k}(t)\,|B_s(l)\rangle$$

$$\simeq \propto \quad G_\beta(t+il)\, G_\beta(t+il)$$

This is working in the interaction picture:

$$H_M^I(t) = e^{-iH_0 t} H_M\, e^{+iH_0 t}, \quad U(t,t') = T e^{i\int_t^{t'} H_I(t'')dt''}$$

and $|\Omega\rangle \cong U(T, t_0)|0\rangle$ for large T

$$U(t,t')|B_s(t)\rangle \qquad \langle\Omega|T(\phi\cdots\phi)|\Omega\rangle = \frac{\langle 0|T(\phi_I \phi_I \cdots e^{-i\int_{-T}^{T} H_I(t)})|0\rangle}{\langle 0|Te^{-i\int_{-T}^{T} H_I}|0\rangle}$$

now $|0\rangle \sim |B_s(l)\rangle$, ~~that is why~~ and since $\epsilon \ll 1$, we have

$$\langle 0|U(-T,T)|0\rangle = \langle B_s(l)|U(-T,T)|B_s(l)\rangle = e^{-i\int_{-T}^{T}\underbrace{\langle H_I(t)\rangle}_{G_{off}(t,t)}}$$

low energy dynamics: $G_{off}(t,t) \to G_{off}^{[f]}$

In summary: Computing the effective lagrangian after $t=0$ (5)

$H = H_{SYK} + \epsilon H_M$

$$\left\langle T e^{-i\epsilon\int H_M(t)\,dt} \right\rangle \underset{\epsilon\to 0}{\approx} \left\langle e^{-i\epsilon\int dt\, H_M(t)} \right\rangle = e^{-i\epsilon\int dt \langle H_M(t)\rangle}$$

$$H_M = -\frac{J}{2}\sum_{k=1}^{N/2} s_k \hat{S}_k$$

Kyriakos & Raju

Same as in $|B_s(l)\rangle$

and $\langle H_M(t)\rangle = -NJ\, G_{off}(t,t) \sim - \dfrac{2NJC_\Delta^2}{\left[\frac{J\beta}{\pi}\cosh\frac{\pi t}{\beta}\right]^2}$

low energy dynamics $t \to \hat{\varphi}(t)$ and $\frac{\pi}{\beta J^2}\tanh \varphi\frac{\pi}{\beta} = f(t)$

$$\therefore\ -i\int dt\,\langle H_M(t)\rangle = 2\epsilon J C_\Delta^2 N \int dt\, \frac{[\varphi'(t)]^{2\Delta}}{\left[\frac{\beta J}{\pi}\cosh\varphi(t)\frac{\pi}{\beta}\right]^{4\Delta}}$$

$$= 2\epsilon J C_\Delta^2 N \int dt\, (f')^{2\Delta}.$$

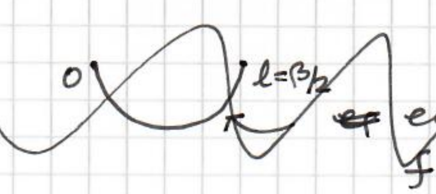
$S_{total} \cong \frac{N}{2J}\int \{f,t\} + (\ )\int (f')^{2\Delta}$

$$\therefore\ S_{total} = \frac{N\alpha}{2}\int_C dt\left[\frac{1}{J}\dot{\phi}^2 + \lambda\left(e^{\phi} - \dot{f}\right) + \hat{\epsilon} J e^{2\Delta\phi}\right]$$

$\hat{\epsilon} = \left(\frac{4C_\Delta^2}{\alpha}\right)\epsilon$, where we have defined $\dot{f} = e^{\phi}$

$\Delta = 1/4$

Boundary conditions:



euclidean solution $f(\tau) = \frac{\pi}{\beta J^2}\tan\frac{\pi}{\beta}\varphi(\tau)$

eqn of motion: 1) $-\ddot{\phi} + \frac{J\lambda}{2}e^{\phi} + \hat{\epsilon}\frac{J^2}{4}e^{\phi/2} = 0$

2) $\dot{\lambda} = 0$, $\lambda = $ const

3) $\dot{f} = e^{\phi}$

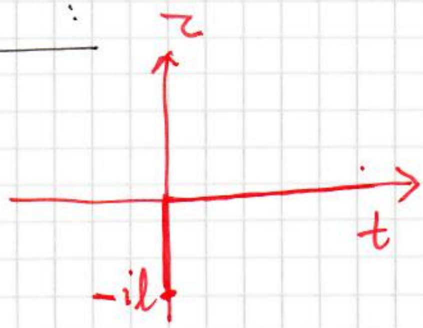
bh: $f(t) = \frac{\pi}{J^2\beta}\tanh\left(\frac{\pi t}{\beta}\right) \Rightarrow \boxed{\dot{f}(0) = e^{\phi(0)} = \left(\frac{\pi}{\beta J}\right)^2}$

Comes from $f(\tau) = \frac{\pi}{J^2\beta}\tan\left(\frac{\pi\tau}{\beta}\right)$, $\dot{f}(\tau) = \left(\frac{\pi}{J\beta}\right)^2\frac{1}{\cos\frac{\pi\tau}{\beta}}$, $\dot{f}(\tau = \beta) = \left(\frac{\pi}{\beta J}\right)^2$

$f(\tau = 2\beta)$

at $t=0$, $\dot{\phi}(t) = 0$ since it is a moment of time reflection symmetry.

Boundary conditions and eqn of motion :



1) $-\phi'' + \frac{J\lambda}{2} e^{\phi} + \hat{\epsilon}\frac{J}{4}^2 e^{\phi/2} = 0$.

2) $\lambda' = 0$, $\lambda = \text{const} = -J$

3) $\dot{f} = e^{\phi}$

BH is described by $f(t) = \frac{\pi}{J^2\beta}\tanh\left(\frac{\pi}{\beta}t\right)$

$$\dot{f}(t) = \left(\frac{\pi}{J\beta}\right)^2 \frac{1}{\cosh^2\left(\frac{\pi}{\beta}t\right)} = \dot{\phi}e^{\phi}$$

$$f(0) = e^{\phi(0)} = \left(\frac{\pi}{\beta J}\right)^2.$$

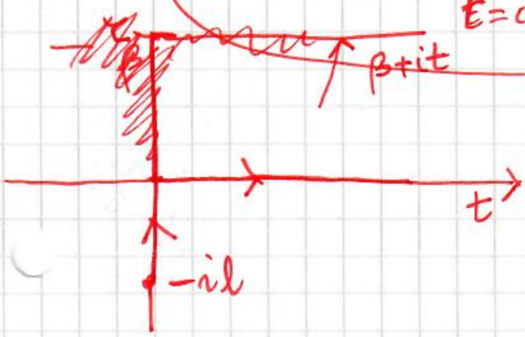
Euclidean evolution

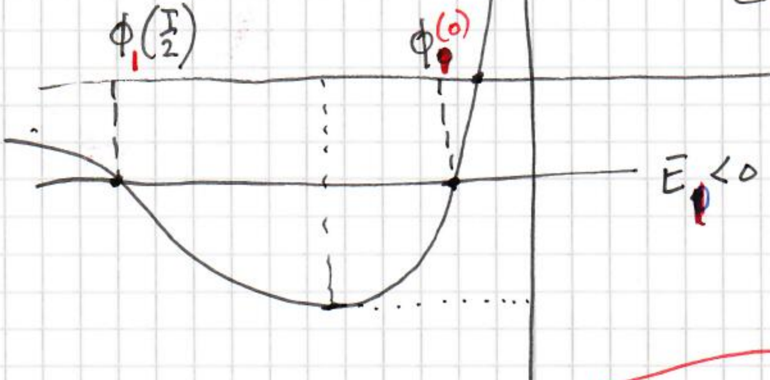
using $\hat{\epsilon} = 0$ + $\left[+\ddot{\phi} + \frac{J\lambda}{2}e^{\phi}\right] = 0$, $f = \frac{\pi}{\beta J^2}\tan\frac{\pi}{\beta}\tau$

$e^{\phi} = \dot{f} = \left(\frac{\pi}{\beta J}\right)^2 \frac{1}{\cos^2\frac{\pi}{\beta}\tau}$

$\Rightarrow \lambda = -J.$

$$\Rightarrow \quad S = \frac{N\alpha}{J}\int_{t=0}^{t} dt \left\{\frac{\phi'^2}{2} - V(\phi)\right\}, \quad V(\phi) = \frac{J}{2}^2\left(e^{\phi} - \hat{\epsilon}\, e^{\phi/2}\right)$$





Bound state motion (compact ϕ) :

$\phi_1(\frac{T}{2}) = \dot{\phi}_1(t=0) = 0$

$e^{\phi_1(0)} = \left(\frac{\pi}{\beta J}\right)^2,$

$$E_1 = V(\phi_1) = \frac{J}{2}^2\left(e^{\phi_1} - \hat{\epsilon}\, e^{\phi_1/2}\right)$$

$$= \frac{J}{2}^2\left(\frac{\pi}{\beta J}\right)\left(\frac{\pi}{\beta J} - \hat{\epsilon}\right) < 0$$

$$\Rightarrow \frac{\pi}{\beta J} < \hat{\epsilon} << 1$$

closed orbit exists.

2 - turning point

$$e^{\phi_1(0)/2} = \frac{\pi}{\beta J}$$

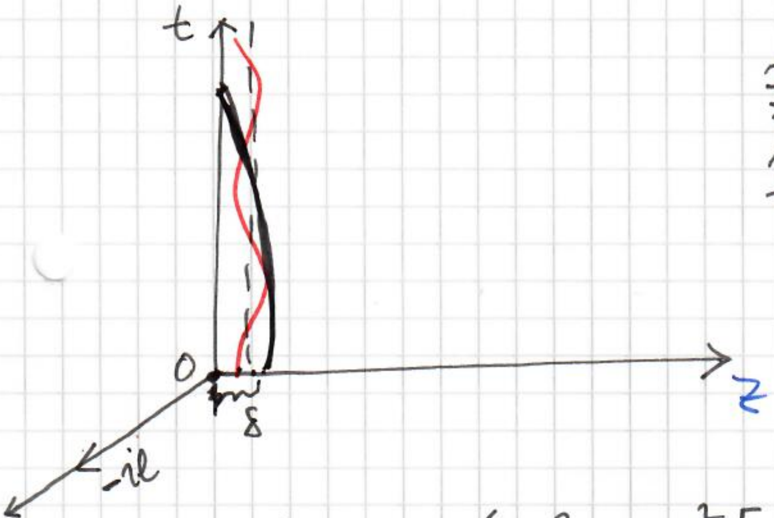
$$e^{\phi_1(\frac{T}{2})/2} = \left(\hat{\epsilon} - \frac{\pi}{\beta J}\right)$$

$e^{\phi} = \dot{f} \neq 0$ since ϕ is bounded.

In the absence $\mathrm{fn} \in H_M$,

$\phi \to -\frac{2\pi}{\beta} t$ + hence $\dot{f} \to 0$ for $t >> \frac{\beta}{2\pi}$

Geometry :



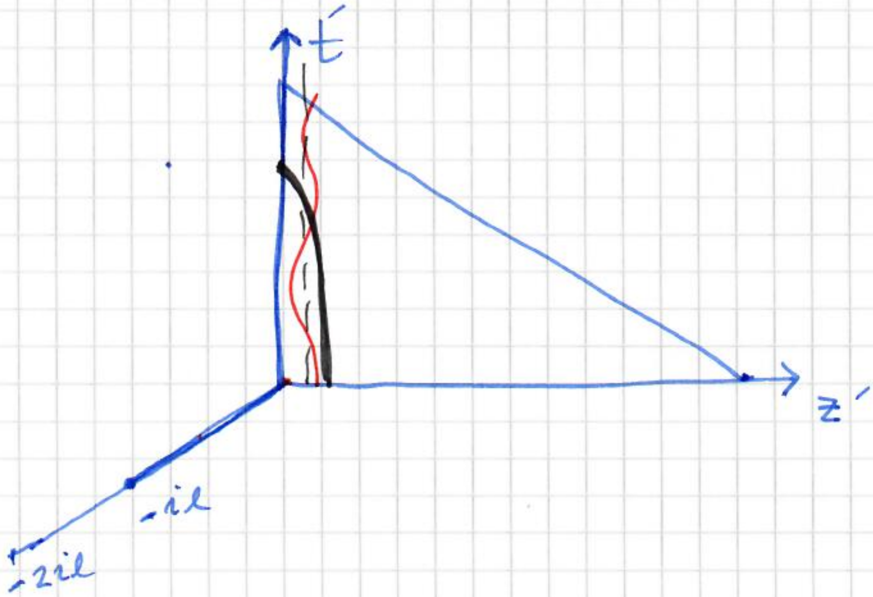
$\tilde{z}(t) \approx \delta\, f'(t)$

$\tilde{t}(t) \approx f(t)$

If $f'(t) \neq 0$ then the entire plane is visible

If $f'(t_0) = 0$ this is not true for $t > t_0$

$$ds^2 = \frac{1}{z^2}\left(dz^2 - dt^2\left[1 + \frac{z^2}{2}\{f,t\}\right]^2\right)$$



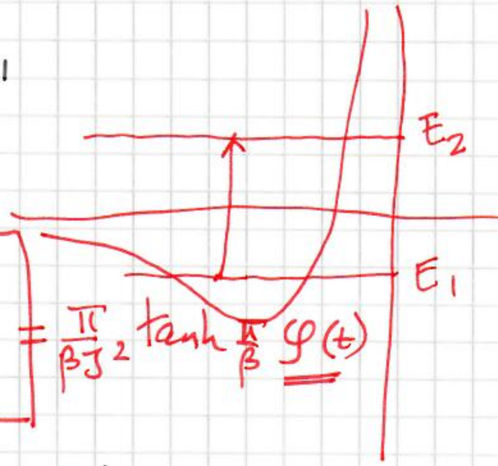
The Second quench at $t = \cancel{t_1}\, T$

Summary of the 1st quench $\longrightarrow$ at $t=0$

$$\hat{\epsilon} = \hat{\epsilon}_1, \quad \phi'(0)=0, \quad e^{\phi(0)} = \left(\frac{\pi}{\beta J}\right)^2$$

$$\hat{E}_1 = \frac{4}{\hat{\epsilon}_1^2}\left(\left(\frac{\pi}{\beta J}\right)^2 - \epsilon_1\left(\frac{\pi}{\beta J}\right)\right)$$

a_1, b_1, c_1, d_1 are function of $\hat{E}_1$ + ϵ_1



$0 < t < T$

$$\boxed{\dot{f} = e^{\phi(t)} = \frac{a_1}{(b_1 \cos c_1 t - 1)^2}} = \frac{\pi}{\beta J^2} \tanh \frac{\pi}{\beta} \underline{\varphi(t)}$$

$$\phi'(t) = e^{-\phi}(e^{\phi})' = \frac{2 b_1 c_1 \sin c_1 t}{(b_1 \cos c_1 t - 1)}$$

$t = T$ there is a sudden jump.

$\hat{\epsilon} = \epsilon_2$

$$e^{\phi(T)} = \frac{a_1}{(b_1 \cos c_1 T - 1)^2}$$

$$\phi'(T) = \frac{2 b_1 c_1 \sin c_1 T}{(b_1 \cos c_1 T - 1)}$$

$$\hat{E}_2 = \frac{4}{J^2 \hat{\epsilon}_2^2}\left[\phi'(T)^2 + J^2\left(e^{\phi(T)} - \epsilon_2 e^{\phi(T)/2}\right)\right]$$

$t > T$

$$e^{\phi(t)} = \frac{a_2}{(b_2 \cosh(c_2(t-T)+d_2) - 1)^2}$$

$$\phi'(t) = \frac{2 b_2 c_2 \sinh(c_2(t-T)+d_2)}{1 - b_2 \cosh(c_2(t-T)+d_2)}$$

a_2, b_2, c_2, d_2 chosen $\ni$ $\phi(t)$ + $\phi'(t)$ are continuous at $t=T$

Choptuik criticality

from the solution for $t > T$ we can extract the temperature of the new black hole:

$$f'(t) = e^{\phi} \sim \operatorname{sech}^2(c_2 t) \sim e^{-2c_2 t}$$

for a bh of temp β^{-1}, $f = \frac{\pi}{\beta J^2} \tanh\left(\frac{\pi}{\beta} t\right)$

$$f' = \left(\frac{\pi}{\beta J}\right)^2 \operatorname{sech}^2 \frac{\pi}{\beta} t \sim e^{-\frac{2\pi}{\beta} t}$$

$$\Rightarrow 2c_2 = \frac{2\pi}{\beta} \Rightarrow c_2 = \frac{\pi}{\beta} \quad \text{or} \quad T = \pi c_2$$

$$c_2 = \frac{1}{2}\sqrt{\hat{E}_2} \quad , \quad \boxed{\hat{E}_2 = A\left(\epsilon_{cr} - \epsilon_2\right)}$$

$$\boxed{\epsilon_{cr} = \frac{\phi'(T)^2 + J^2 e^{\phi(T)}}{J\, e^{\phi(T)/2}}}$$

(note $\phi'(T)^2 + J^2 e^{\phi(T)} - \epsilon_{cr}\, e^{\frac{\phi(T)}{2}} = 0$ is the zero energy condition).

$$\Rightarrow \quad c_2 = \frac{1}{2}\sqrt{A\left(\epsilon_{cr} - \epsilon_2\right)} = \frac{\pi}{\beta_{new}}$$

$$\Rightarrow \quad T_{bh} \sim \left(\epsilon_{cr} - \epsilon_2\right)^{\alpha}, \quad \alpha = \frac{1}{2}.$$

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choptuik $\rightarrow$ gravitational collapse of a dust ball data parametrized by a parameter P. bh forms & $M \sim (P - P_{cr})^{\gamma}$.

Dynamical criticality

$\dot f(t) = e^{\phi(t)}$ is known $\forall\, t$

$0 < t < T$ and $T \le t < \infty$.

Loschmidt amplitudes + echos!
$|\langle \psi_0 | \psi_0(t)\rangle|^2 = \mathcal{L}(t)$

at $t=T$, $\phi_1(T) = \phi_2(T)$ + $\dot\phi_1(T) = \dot\phi_2(T)$

since $\in$ jumps $\phi'''(T)$ is discontinuous

$\phi_1''(t) \sim e^{\phi_1} - \epsilon_1 e^{\phi_1}$ $\qquad 0 < t \le T$

$\phi_2''(t) \sim e^{\phi_2} - \epsilon_2 e^{\phi_2}$ $\qquad T \le t < \infty$

at $t=T$, $\left.(\phi_2'' - \phi_1'')\right|_{t=T} = -(\epsilon_2 - \epsilon_1)\, e^{\phi_1(T)}$

force jumps.

Since $\dot f = e^{\phi}$, $\ddot f = \dot\phi e^{\phi}$, $\dddot f = \ddot\phi e^{\phi} + \dot\phi^2 e^{\phi}$

$\therefore$ at $t=T$, $\dot f$ and $\ddot f$ are continuous but not $\dddot f$.

note $\{f,t\} = \left(\frac{\ddot f}{\dot f}\right)^{\cdot} - \frac{1}{2}\left(\frac{\ddot f}{\dot f}\right)^2 = \ddot\phi + \cancel{\dot\phi^2 - \dot\phi^2} - \frac12 \dot\phi^2$

$\therefore$ $\{f,t\}$ is discontinuous at $t = T$.

$$G^{[f]} = \frac{[f'(t_2)\, f'(t_1)]^{\Delta}}{|f(t_2) - f(t_1)|^{2\Delta}}$$

define $\bar t = \frac{t_1 + t_2}{2}$, $t = (t_2 - t_1)$

$$G^{[f]}(t,\bar t) = |t|^{-2\Delta}\left(1 - \frac{\Delta}{6}\{f,\bar t\}\, t^2 + o(t^4)\cdots\right)$$

now $\{f,\bar t\} = \left(\frac{\ddot f}{\dot f}\right)' - \frac12\left(\frac{\ddot f}{\dot f}\right)^2 = \left[\ddot\phi(\bar t) - \frac12 \dot\phi(\bar t)^2\right]$

$$\therefore\ G^{[f]}(t,\bar t) = |t|^{-2\Delta}\left(1 - \frac{\Delta}{6}\left\{\ddot\phi - \frac12\dot\phi^2\right\}_{\bar t} t^2 + \cdots\right).$$

discontinuous at $\bar t = T$

Conclusions :

1) $|B_s(l)\rangle$, $2l = \beta >> 1$ is a pure state that ~~we~~ can evaluate all thermal correlators of Flip invariant ~~operators~~ combination of operators.

2) Expectation value of Flip non-invariant operator combinations are not entirely thermal.

3) $t=0$ quench with $H_M = \epsilon_1 \sum s_k \hat{S}_k$. (fine tuned) for a range of parameters, the dual space time has no horizon

4) $t=T$ quench $H_M = \epsilon_2 \sum s_k \hat{S}_k$.

$\epsilon_1 \neq \epsilon_2 \Rightarrow$ a space time with a horizon.

5) $T_{bh} \sim (\epsilon_{cr} - \epsilon_2)^{1/2}$

6) $\dddot{f}$ (or $\ddot{\phi}$) has a kink at $t=T$

$\Rightarrow$ kink in the 2-pt function.

7) Need to understand gravity dual, better ~~better~~ ~~between~~ in particular. $\rightarrow$ the dual to $\sum_k s_k \hat{S}_k$

$\rightarrow \int dz (f')^{2\Delta}$

8) Maldacena - Qi

much better understanding of dual geom for 2 site SYK.

one can do a very similar analysis since the effective theory is the same.