Higher Spin de Sitter Quantum Gravity

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D Anninos, FD, R Monten and Z Sun – arXiv:1711.10037 (+ more in discussion)

Outline

Motivation

dS-CFT?

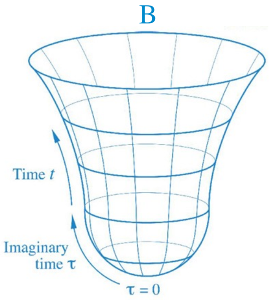
- Quantum gravity with negative c.c. $= \pm$ solved problem: AdS-CFT.
  AdS Hilbert space = CFT Hilbert space.

$$Z_{\mathrm{CFT}}\left[\mathrm{sources} = B\right] \stackrel{\checkmark}{=} Z_{\mathrm{bulk}}\left[\mathrm{asympt.\ fields} \stackrel{r\to\infty}{\sim} B\right].$$

- Our universe: positive c.c. $\rightsquigarrow$ dS-CFT? [Strominger,Witten '01,Maldacena '02]:

$$Z_{\mathrm{CFT}}\left[\mathrm{sources} = B\right] \stackrel{?}{=} Z_{\mathrm{bulk}}\left[\mathrm{as.\ fields} \stackrel{t\to\infty}{\sim} B\right] \equiv \psi_{\mathrm{HH}}(B).$$

Here $\psi_{\mathrm{HH}}(B) =$ "wave function of the universe" [Hartle-Hawking '83]



Examples?

- Although isometry group $dS_{d+1} = SO(d+1,1) = d$-dim Euclidean conformal group, no good reason in general to expect *local CFT* dual.
- No concrete candidate proposed for decade.
- Changed with [Anninos-Hartman-Strominger '11]:

$$\mathrm{dS}|_{\Lambda\sim N} \text{ higher spin gravity theories } \leftrightarrow \text{ Large } N \text{ ghost vector models}$$

e.g. free 3d fermionic scalar Sp(N) model $\leftrightarrow$ minimal 4d type A Vasiliev:

$$Z[B] \propto \int d\chi \; e^{-\int \chi D\chi + :\chi B\chi:}$$

$$\propto \det(D+B)^{+\frac{N}{2}} \, e^{-\frac{N}{2}\mathrm{Tr}(D^{-1}B)} \propto e^{-\frac{N}{2}\mathrm{Tr}\left(\frac{1}{2}(D^{-1}B)^2 - \frac{1}{3}(D^{-1}B)^3 + \ldots\right)}$$

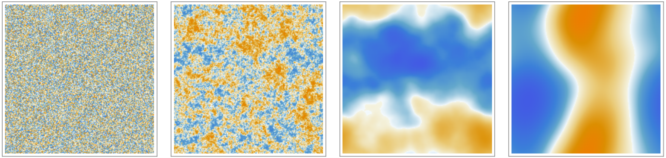
where $D = -$ Laplacian and $B =$ sources (spin $s = 0, 2, 4, \ldots$ fields).

Great, but...

- $Z[B] = \psi_{\rm HH}[B] = \langle B|vac\rangle$ generates bulk vacuum in - "Dirichlet" out correlation functions:

$$\partial_{B_1} \cdots \partial_{B_n} Z[B]|_{B=0} \; = \; \langle B\!=\!0|\hat{A}_1 \cdots \hat{A}_n|vac\rangle$$

- What we really want: vacuum-vacuum correlation functions, expectation values, probabilities, ... $\rightsquigarrow$ needed to compute, say, higher spin "CMB"



$s=0$ $s=2$ $s=4$ $s=6$

- Taking $\psi_{\rm HH}[B] = Z[B]$ seriously,

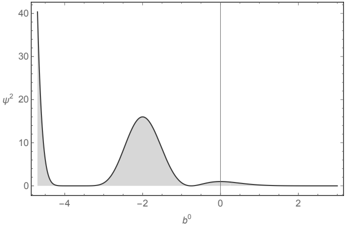
$$\langle vac|\hat{B}_1 \cdots \hat{B}_n|vac\rangle = \int [dB] \left|Z[B]\right|^2 B_1 \cdots B_n$$

- But: measure [dB] = ???, domain B = ??? Hilbert space = ??? Operator algebra of observables = ??? ... *Not* determined by "dual" CFT!

Shut up and calculate?

- Whatever. Just take [dB] = dB = flat measure; domain = all real B.
- Approach followed in [Anninos-FD-Harlow,...]. Constant scalar b_0 on S^3 slice:

$$\psi(b_0) \propto \det(1 + D^{-1} b_0)^{\frac{N}{2}} e^{-\frac{N}{2}\mathrm{Tr}(D^{-1} b_0)} \propto e^{-\frac{N}{4} \int^{b_0 - \frac{1}{4}} \pi \sqrt{u} \coth(\pi \sqrt{u})\, du}$$



Diverges exponentially: $\psi(b_0) \sim e^{N|b_0|} \times \mathrm{oscill.}$ for $b_0 \to -\infty$. Disaster ×

- Even if $\psi(b_0)$ had turned out nice for $b_0 \to \pm\infty$, result would still be pretty useless: computing actual probability $P(b_0) \propto \int dB' \left|\psi(b_0, B')\right|^2$ still requires integral over wild infinity of other vars B'. Intractable ×
- Too many d.o.f. in B to be plausibly fundamental/indep. Give up?

The Surprisingly Elegant Vasiliev Universe

We propose a different starting point, *not dS-CFT*, providing direct construction of the fundamental microscopic Hilbert space: "*Q*-model".

- Will only require match to original Sp(N) dS-CFT proposal of [AHS] in regime where integration measure and domain don't not matter, i.e. in leading $N \to \infty$ saddle point approx (= regime of validity of their work).
- Complete, non-perturbative formulation of higher spin quantum gravity in de Sitter space, capable of computing vacuum correlation functions, probabilities, expectation values, etc.
- Turns out to be surprisingly computationally powerful framework: previously seemingly intractable tasks become straightforward, e.g. computing actual probabilities for arbitrarily large scalar field excursions, exact vacuum correlation functions, including 4-point function!
- Long-standing questions about quantum gravity in dS space can in principle be addressed in a quantitatively precise, computable way.

Before stating full construction, it will be useful to consider some toy models

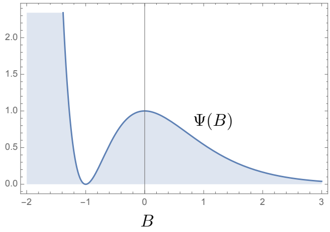
Toy models

1D QM toy model

Say we are give a single-variable QM wavefunction equal to a "partition function" for N fermionic variables χ^A, $A = 1, \ldots, N$:

$$\Psi(b) \propto \int d^N\chi \, e^{-\frac{1}{2}(\chi\chi + b:\chi\chi:)} \propto (1+b)^{+\frac{N}{2}} e^{-\frac{N}{2}b}.$$

Here $\chi\chi \equiv \epsilon_{AB}\chi^A\chi^B$, $:\chi\chi: \equiv \chi\chi - \langle\chi\chi\rangle_0 = \chi\chi + N$, and $b \in \mathbb{R}$.



Note: is $d = 0$ analog to $\Psi[B] = Z[B]$ for original $d = 3$ Sp(N) CFT:

- Hilbert space unknown at this point (domain b? measure $[db]$?)
- Shut up and calculate fails: exponential divergence at $b \to -\infty$.

Large N analysis toy model

Generating function for moments $\langle\Psi|b^n|\Psi\rangle$ is:

$$\mathcal{Z}(\lambda) \equiv \langle\Psi|e^{N\lambda b}|\Psi\rangle = \frac{1}{\mathcal{Z}_0}\int [db]\,(1+b)^N e^{-N(1-\lambda)b}.$$

Domain and measure $[db]$ unspecified, but irrelevant for leading $N \to \infty$ saddle point evaluation. Saddle point equation: $\frac{1}{1+b} = 1-\lambda \Rightarrow b = \frac{1}{1-\lambda} - 1 \Rightarrow$

$$\mathcal{Z}(\lambda) \approx (1-\lambda)^{-N} e^{-N\lambda} \qquad (N \to \infty)$$

Notice this can be viewed as partition function for $2N$ bosonic variables q^α:

$$\mathcal{Z}(\lambda) \;\approx\; \frac{1}{\mathcal{Z}_0}\int d^{2N}q\, e^{-qq+\lambda :qq:} \qquad (N \to \infty),$$

where $qq \equiv \delta_{\alpha\beta} q^\alpha q^\beta$, $:qq: := qq - \langle qq\rangle_0 = qq - N$.

Q-model Hilbert space

Thus, if we define consider Hilbert space of wave functions $\Psi(q)$ with $q \in \mathbb{R}^{2n}$ and standard inner product

$$\langle \psi_1 | \psi_2 \rangle \equiv \int d^{2N}q \, \psi_1(q)^* \psi_2(q)$$

and we define a "vacuum" state

$$\psi(q) \equiv e^{-\frac{1}{2}qq}$$

then we have the following leading large-N equivalence:

$$\langle \Psi | e^{N\lambda b} | \Psi \rangle \approx \langle \psi | e^{N\lambda b} | \psi \rangle \qquad (N \to \infty)$$

provided on the right hand side we identify

$$b \equiv \frac{1}{N} :qq: = \frac{1}{N} q^2 - 1 \, .$$

This is independent of the measure on the left hand side (for λ not too large, and assuming measure does not depend exponentially on N and is non-singular near $b = 0$: natural conditions to have reasonable large-N limit).

Exact equivalence at finite N

Alternatively we can take the Q-model Hilbert space $\mathcal{H}$ as the starting point, and declare this to be the precise definition of the vacuum wave function and its Hilbert space, under the above identification $b = \frac{1}{N} :q^2:$, turning the above large-N equivalence into an exact equivalence. This effectively amounts to a specific choice of measure and domain for b in the original formulation.

To see this, note that the O(2N)-invariant subspace of $\mathcal{H}$ has a basis of eigenkets $|h\rangle$ of $\hat{q}^2 = \hat{h} = 1 + \hat{b}$, defined by

$$\frac{1}{N}\hat{q}^2|h\rangle = h\,|h\rangle, \quad \int_0^\infty \frac{dh}{h}\,|h\rangle\langle h| = 1 \qquad \Rightarrow \quad \langle q|h\rangle \propto h^{1-N/2}\delta(\tfrac{1}{N}q^2 - h).$$

[Measure depends on normalization of $|h\rangle$; above is standard one arising from canonical quantization of the O(2N)-invariant Darboux coordinates $\{\frac{1}{2}\log q^2, pq\}$.]

With $\psi(q) \equiv \langle q|\psi_0\rangle = e^{-\frac{1}{2}q^2}$ we then have the exact equivalelence, for any N:

$$\Psi(b) \equiv \langle h|\psi\rangle = \int_q \langle h|q\rangle\langle q|\psi\rangle = \ldots \propto h^{\frac{N}{2}} e^{-\frac{N}{2}h} \propto (1+b)^{\frac{N}{2}} e^{-\frac{N}{2}b},$$

where $h = 1 + b$. This is exactly our original $\Psi(b)$! Inner product becomes:

$$\langle \Psi_1|\Psi_2\rangle = \int_{-1}^\infty \frac{db}{1+b}\,\Psi_1(b)^*\Psi_2(b).$$

[Note: basically just change of variables from Cartesian to spherical.]

Toy model II: discretized Sp(N) model with K spatial points

- This readily generalizes to discretized Sp(N) toy model:

$$\Psi_0(B) \propto \int d\chi\, e^{-\frac{1}{2}\mathrm{Tr}(\chi D\chi + :\chi B\chi:)} \propto \det(1+D^{-1}B)^{-\frac{N}{2}} e^{-\frac{N}{2}\mathrm{Tr}(D^{-1}B)},$$

  where $\chi = \chi^a_x$, $a = 1,\ldots,N$, $x = 1,\ldots,K$, D and B are symm. $K \times K$ matrices with components D^{xy}, B^{xy}. D can be thought of as a discretized Laplacian for K lattice points, and B = source. Previous toy: $K = 1$.

- To leading order in large N saddle point approximation this is equivalent to an O(2N)-invariant Q-model, where Q has $K \times 2N$ components Q^α_x, with

$$\psi_0(Q) \equiv e^{-\frac{1}{2}\mathrm{Tr}(QDQ)}, \qquad D^{xy} + B^{xy} = H^{xy} = \frac{1}{N} Q^{x\alpha} Q^{y\alpha}, \qquad Q^{x\alpha} \equiv D^{xx'} Q^\alpha_{x'}.$$

- At finite $N \geq \frac{K}{2}$, the equivalence is exact for domain and measure

$$H > 0, \qquad [dH] = \frac{dH}{(\det H)^{\frac{K+1}{2}}}.$$

  [This is unique GL(K)-invariant, volume element for natural metric $ds^2 = \mathrm{Tr}(H^{-1}dH)^2$.]

- If $K > 2N$, then $H = \frac{1}{N} QQ^T$ has reduced rank $2N < K$ and measure $[dH]$ degenerates, but Q-model remains well-defined!

Higher Spin de Sitter Hilbert Space

Summary of construction

1. Hilbert space of minimal type A dS_4 Vasiliev hs quantum gravity:
   - Not dS-CFT — we do not start from Sp(N) model CFT.
   - Fundamental d.o.f.: $2N$ *bosonic* scalar fields $Q^\alpha(x)$, $\alpha = 1, \ldots, 2N$, $x \in \mathbb{R}^3$.
   - $\mathscr{H}_0$: Wave functions $\psi(Q)$ with standard (flat measure) inner product:

$$\langle \psi_1 | \psi_2 \rangle \equiv \int dQ\, \psi_1(Q)^* \psi_2(Q)$$

   - $\mathscr{H} \equiv$ O(2N)-invariant subspace of $\mathscr{H}_0$.
   - Vacuum state $|\psi_0\rangle \in \mathscr{H}$:

$$\psi_0(Q) \equiv e^{-\frac{1}{2}\int QDQ}\,, \qquad D \equiv -\nabla^2 \quad \text{(partial gauge fixing)}$$

   - $\mathscr{H}_{phys} \equiv$ hs-invariant subspace of $\mathscr{H}$.
2. Dictionary:
   - Late time ($\eta \to 0$) asymptotics of spin-s bulk field:

$$\phi_{i_1\cdots i_s}(\eta, x) \sim \beta_{i_1\cdots i_s}(x)\, \eta^{2-2s} + \alpha_{i_1\cdots i_s}(x)\, \eta$$

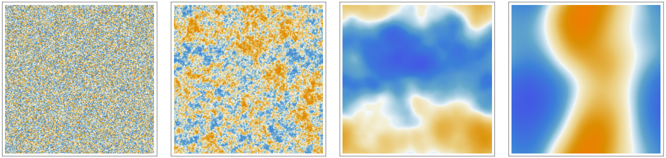
   - Identifications:

$$|\psi_{HH}\rangle = |\psi_0\rangle\,, \qquad \tilde{\beta}_{i_1\cdots i_s}(x) = Q^\alpha(x)\, \partial_{i_1} \cdots \partial_{i_s} Q^\alpha(x) + \cdots$$

   $\tilde{\beta} \equiv$ "shadow" of β; in momentum space: $\tilde{\beta}_{i_1\cdots i_s}(k) \equiv k^{2s-1} \beta_{i_1\cdots i_s}(k)$
3. Reproduces [AHS] / bulk perturbation theory at large N / tree level.

Summary of some results

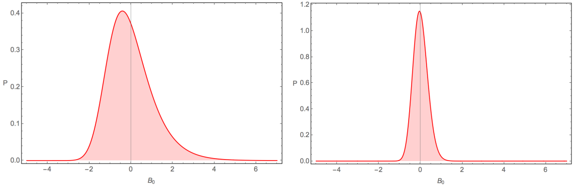
- Exact higher spin "CMB" correlation functions (momentum space):



$s = 0$ $s = 2$ $s = 4$ $s = 6$

  - Scalar-scalar-scalar, scalar-scalar-graviton 3-pt functions.
  - Scalar 4-pt function (nontrivial! – details few slides down)

- Exact probabilities for arbitrarily large field excursions:



$N = 2$ $N = 20$

Probability $P(B_0)$ for constant scalar mode on global de Sitter S^3.

Summary of some more results

- Approximate reconstruction of perturbative bulk QFT Heisenberg algebra $[\beta^I, \alpha_J] = i\,\delta^I_J$, $[\alpha_I, \alpha_J] = 0 = [\beta^I, \beta^J]$ on microscopic $\mathscr{H}$:
  - up to minimal error term $\sim e^{-\mathcal{O}(N)}$
  - operators must be coarse grained to effectively $< \mathcal{O}(N)$ spatial "pixels"

$K_{\text{eff}} = 10^6$ $K_{\text{eff}} = 10^4$ $K_{\text{eff}} = 10^2$

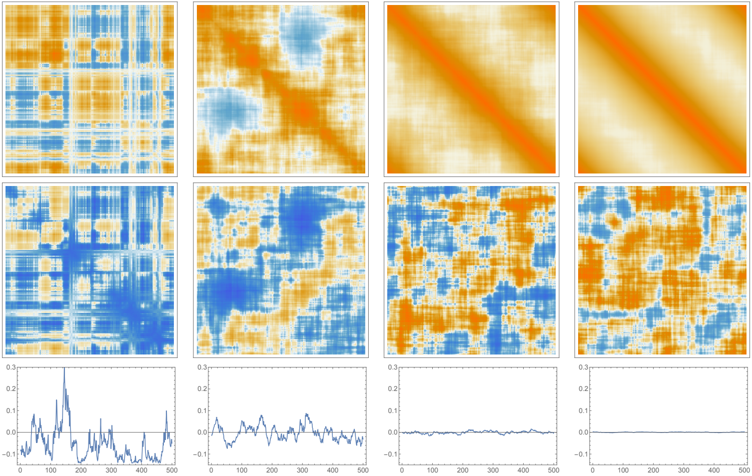
Schrödinger's cat in HSdS: resolution bounded by N

- hs-invariant $\mathscr{H}_{\text{phys}}$ quasi-topological: finite number of n-particle states; all hs-invariant quantities computed by $2N \times 2N$ matrix model
  $\rightsquigarrow$ $2N$ physical degrees of freedom.

- Suggestive of idea of cosmological complementarity, dS entropy $S_{\text{dS}} \sim N$, etc ...

A few more details on results

Sampling Vasiliev universes

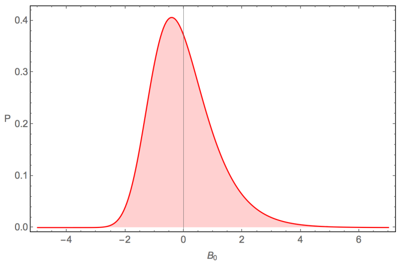


$N = 1$ $N = 10$ $N = 250$ $N = 10000$

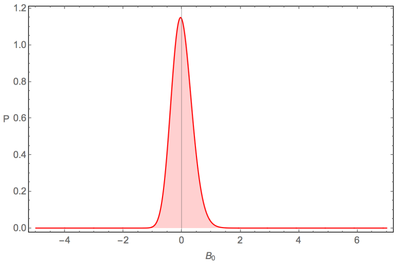
$d = 1, K = 500$. row 1: $H_{xy} = \frac{1}{N} Q_x^\alpha Q_y^\alpha$. row 2: $B_{xy} = \frac{1}{N} :Q_x^\alpha Q_y^\alpha:$. row 3: $B(x) = \frac{1}{N} :Q_x^\alpha Q_x^\alpha:$.

Note the strong non-Gaussianities at small N.

Probabilities for arbitrarily large field excursions



$N = 2$



$N = 20$

Probability $P(\beta)$ for constant scalar mode on S^3 (global dS) slice.

At large N:

$$P(\beta) \sim \begin{cases} e^{-N\,\beta} & \beta \to +\infty \\ e^{-N\,\beta^2} & \beta \to 0 \\ e^{-N\,|\beta|^3} & \beta \to -\infty \end{cases}$$

Note: seemingly intractable problem of computing $P(\beta) = \int [dB'] \left|\Psi_0(\beta, B')\right|^2$ reduced to almost trivial computation in Q-model!

Scalar 4-point function

Vacuum scalar 4-point function in momentum space

$$G_4 \equiv \langle 0|\beta(\vec{p}_1)\,\beta(\vec{p}_2)\,\beta(\vec{p}_3)\,\beta(\vec{p}_4)|0\rangle$$

gets contributions in bulk from infinite number of particle exchanges. Can be computed exactly in Q-model, using methods of [Bzowski-McFadden-Skenderis]:

$$\begin{aligned} G_4 = {} & \frac{1}{N^2}\,p_1 p_3\,\delta_{\vec{p}_1+\vec{p}_2}\,\delta_{\vec{p}_3+\vec{p}_4} \\ & + \frac{8}{N^3}\,\frac{(p_1p_2+p_3p_4)p_{21}+(p_1p_4+p_2p_3)p_{23}}{p_{21}p_{23}(p_1p_3+p_2p_4+p_{21}p_{23})}\,\delta_{\vec{p}_1+\vec{p}_2+\vec{p}_3+\vec{p}_4} \\ & + (\vec{p}_2 \leftrightarrow \vec{p}_3) + (\vec{p}_2 \leftrightarrow \vec{p}_4) \end{aligned}$$

where $p_n \equiv |\vec{p}_n|$, $p_{mn} \equiv |\vec{p}_m + \vec{p}_n|$.

- Checked numerically.
- Surprisingly simple! Equivalent to 3d 1-loop "4-mass box integral". Explicit result obtained before in amplitude literature [Lipstein-Mason '12] covers half dozen pages of nasty Mathematica output.
- Surprisingly (?) "soft" in UV, in view of [Baumann-Goon-Lee-Pimentel '17], reminiscent, perhaps, of amplitude softening in string theory by summing over infinite number of particles?

Bulk QFT reconstruction

- First step: reconstruction perturbative QFT Heisenberg algebra $[\beta^I, \alpha_J] \propto i\delta^I_J \rightsquigarrow$ definition α_J on $\mathscr{H}$? i.e. expression in terms of operators Q^α_x and $P^x_\alpha = -i\partial_{Q^\alpha_x}$?

  $\rightsquigarrow$ Let us first consider this problem in $K = 1$ toy model.

Heisenberg algebra in $K = 1$ toy model

Recall $1 + b = h = \frac{1}{N} q^2$.

Can exact Heisenberg algebra $[\hat{h}, \hat{a}] = i$ be realized in q-model?

No: because $h > 0$, no such self-adjoint $\hat{a}$ can exist. (If it did exist, $U(c) \equiv e^{ic\hat{a}}$ would be unitary translation operator mapping $h \to U(c)\, h\, U(c)^{-1} = h + c$ for arbitrary $c \in \mathbb{R}$, violating $h > 0$.)

Naively, $\hat{a} \equiv \hat{h}^{-1}\hat{d} + h.c.$, $\hat{d} \equiv \frac{1}{8}(\hat{q}\hat{p} + \hat{p}\hat{q}) = -\frac{i}{2} h\partial_h$ would seem to do the job. However this operator, while hermitian, is not self-adjoint and moreover acting more than $O(N)$ times with this $\hat{a}$ on vacuum $\psi(h) = h^{N/2} e^{-Nh/2}$ produces non-normalizable state.

Well-behaved approximate perturbative Heisenberg algebra can nevertheless be constructed (arbitrary powers $\hat{a}^n_{\rm pert}$ on vacuum produce normalizable states):

$$\hat{a}_{\rm pert} = \sum_{n=0}^{N} (-\hat{b})^n \hat{d} + h.c.,$$

but only up to $O(e^{-N})$ minimal algebra error: $[\hat{b}, \hat{a}_{\rm pert}] = i\,(1 + e^{-N}\hat{O})$.
Intuition: probability $P(b > 1)$ (exit perturbative regime) is $\sim e^{-N}$.

Bulk QFT reconstruction

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- $D + \beta = QQ^T > 0 \Rightarrow$ Exact Heisenberg algebra cannot be realized.
- Perturbative construction exists realizing approximate Heisenberg algebra up to error $> O(e^{-N})$, upon coarse-graining to effective spatial resolution $K_{\rm eff} < O(N)$ "pixels".

  Underlying reason: error $\sim$ probability of fluctuation exiting perturbative regime. This probability is always $> O(e^{-N})$, and becomes O(1) if resolution $> O(N)$ pixels.

  Can be made precise using Tracy-Widom distribution largest eigenvalue Wishart random matrix.
- Global breakdown of perturbative bulk QFT appears to violate assumptions of certain dS no-go theorems [Susskind,Kleban et al]

Physical Hilbert space and observables

- $\mathscr{H}_{\mathrm{phys}}$ = states invariant under O(2N) and $\mathcal{G}$ (hs group). In generalized toy model with K spatial points, $\mathcal{G}_K = O(K)$. One possible definition of $\mathcal{G}$ in continuum limit is simply $\mathcal{G} = \lim_{K\to\infty} \mathcal{G}_K$.
- Physical observables = operators invariant under O(2N) and $\mathcal{G}$.
- Basis of algebra of invariant operators:

$$T_n \equiv \mathrm{Tr}\, H^n = \mathrm{Tr}\, M^n\,, \qquad H_x{}^y \equiv Q_x^\alpha Q_\alpha^y\,, \qquad M^\alpha{}_\beta \equiv Q_x^\alpha Q_\beta^x$$

- $\Rightarrow$ All expectation values of physical observables can be computed by $2N \times 2N$ matrix model of symmetric real matrices M, with gauged $O(2N)$ symmetry $\Rightarrow$ reduction to $2N$ physical d.o.f.
- Concrete implementation of "group averaging" construction of physical observables by taking continuum limit of discretized model.

Ongoing work and outlook

- Generalizations? (some progress, some puzzles)
- Local bulk physics? (issues in common with AdS + more)
- Entropy? (Although some work remains to be done, there seem to be no insuperable obstacles to a precise identification and microscopic derivation of the de Sitter entropy within this framework.)
- Less deslusionally optimistic assessment: some obstacles seem still quite serious, in particular *identification* of entropy. More abstract quantum information theoretic approach may be required to overcome these.