Information Loss and Bulk Reconstruction in AdS_3/CFT_2

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in collaboration with
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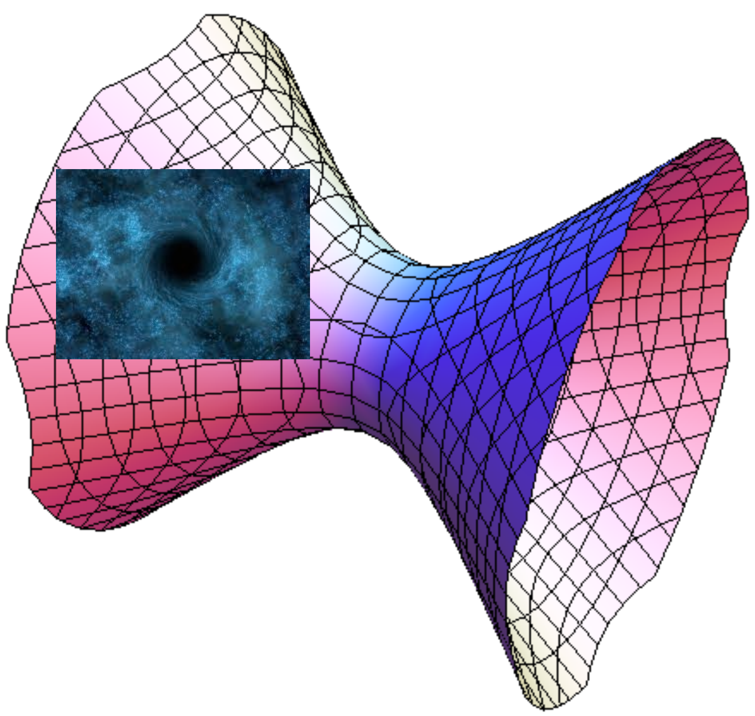
CFTs and Quantum Gravity

Gravity in Anti de Sitter in d+1 dimensions

Conformal Field Theory in d dimensions



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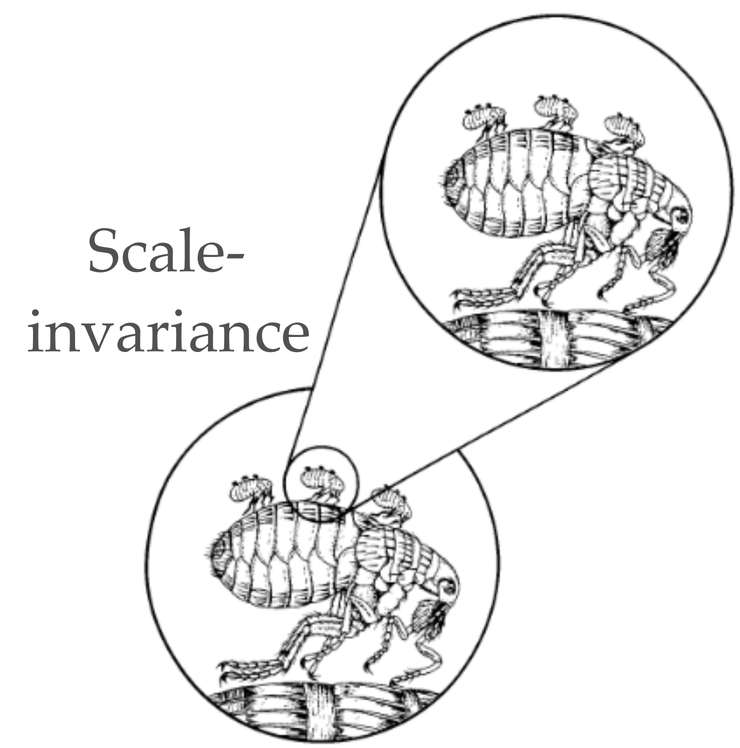


Studying CFTs teaches us about gravity, and vice versa

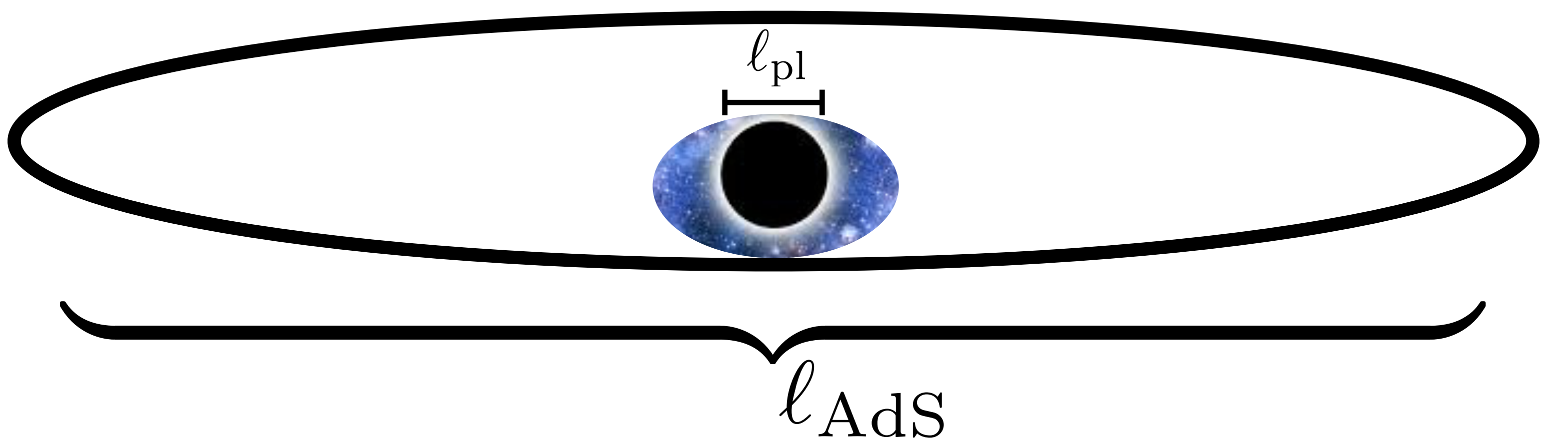
Large C Expansion

Consider large CFT central charge : essentially, large number of degrees of freedom. Like a classical limit.

Brown, Henneaux, '86

$$c = \frac{3\ell_{\rm AdS}}{2G_N}$$

"Semi-classical" gravity limit



"Perturbative" corrections $\sim \dfrac{1}{c^n}$

"Non-perturbative" corrections $\sim e^{-c}$

Some Motivation

Want to be able to calculate how information escapes from black hole, hidden in non-perturbative effects

E.g.: - late-time decay of correlators,
- physics near and across horizons.

In AdS_3/CFT_2, many non-perturbative effects are controlled by conformal symmetry; we want to calculate them.

EFT vs Entropy Growth

EFT near horizon predicts Entropy Growth

$$\langle \phi\phi \rangle_{\rm BH} \xrightarrow{\text{Bogoliubov}} |\Psi\rangle_{\rm pair} \sim e^{\gamma b^\dagger \hat{b}^\dagger} |0\rangle_{\rm infall} |0\rangle_{\rm out}$$

$$\sim \left(\frac{1}{\sqrt{2}} |0\rangle_{\rm inf} |0\rangle_{\rm out} + \frac{1}{\sqrt{2}} |1\rangle_{\rm inf} |1\rangle_{\rm out} \right)$$

At each step, roughly:

$$|\Psi\rangle \to |\Psi\rangle \otimes |\Psi\rangle_{\rm pair} + \epsilon$$

$$S_{\rm out} \to S_{\rm out} + \log 2 \ + \epsilon$$

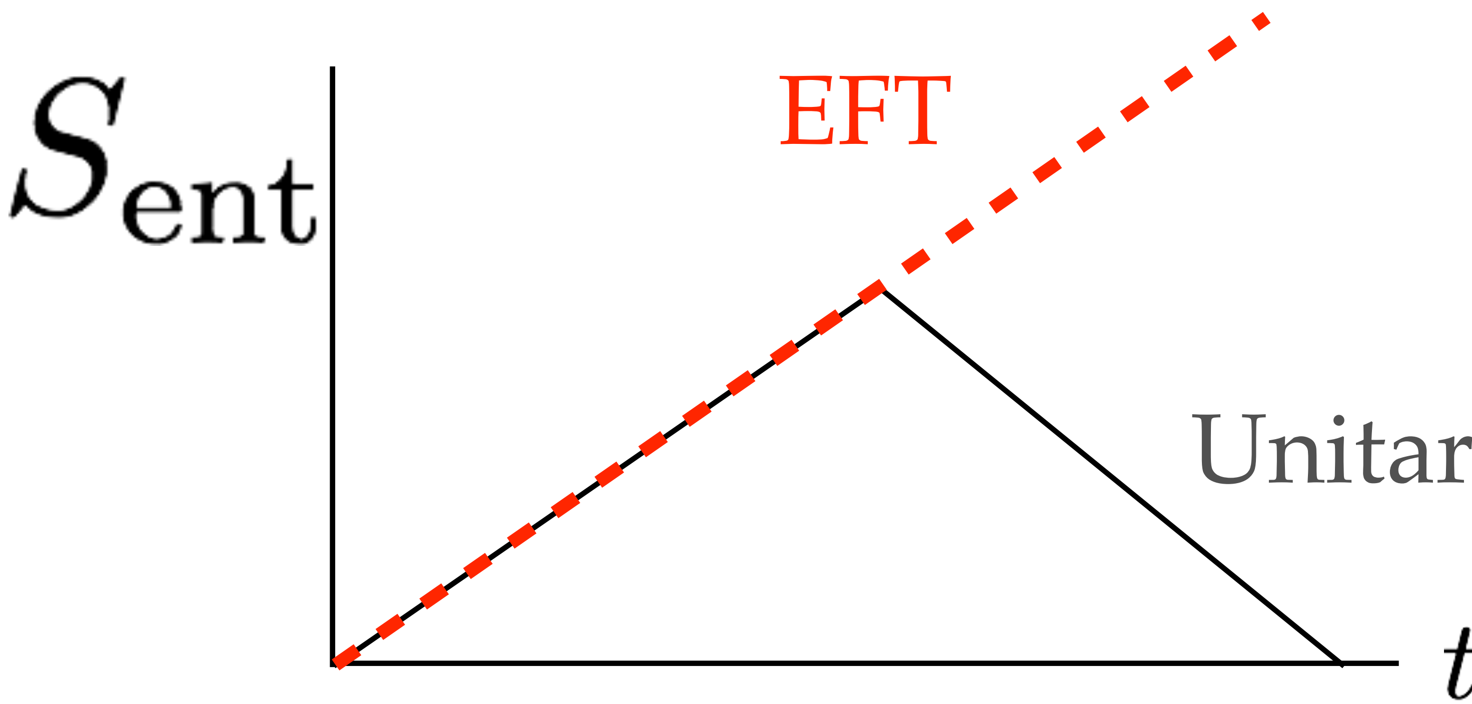
EFT vs Entropy Growth

Strong Subadditivity $\longrightarrow$ Small corrections aren't enough

Mathur '09

$$|\Psi\rangle \to |\Psi\rangle \otimes |\Psi\rangle_{\text{pair}} + \epsilon$$

$$S_{\text{out}} \to S_{\text{out}} + \log 2 \;\; + \epsilon$$



So some part of the EFT argument breaks down

We want the CFT to point to which cherished assumption should be discarded

Large Non-perturb Corrections

Trivial comment: non-perturbative corrections can be large

e.g. $e^{-(c-t)}$ invisible in 1/c pert theory
large at t>c

Less trivial example: Stokes phenomena

$$Z(g) \equiv \frac{1}{\sqrt{g}} \int_0^{\pi} d\theta e^{-\frac{1}{2g}\sin^2(\theta)}$$

has two saddle points $\theta = 0, \frac{\pi}{2} \longrightarrow Z(g) \sim \sum_k p_{k,0} g^k + e^{-\frac{1}{2g}} \sum_k p_{k,1} g^k$

"subleading" saddle dominates at $\arg(g) > \frac{\pi}{2}$

We will see explicit examples of large non-pert effects in CFT

Focusing on 2d

Useful toy model: conformal symmetry is much bigger!

AdS_3: no gravity waves, but there are still black holes.

Ideally, can play a similar role to 2d QCD at large N: the gluon has no DOFs, and the theory is solvable.

Algebraic Gravity

Power of AdS_3/CFT_2: gravitons are algebraic

AdS		CFT
$h_{\mu\nu}$	$\longleftrightarrow$	$T_{\mu\nu}$
multi-grav		products of T

Virasoro

CFT_2 $\quad T(z) = \sum_n \frac{L_n}{z^{n+2}}$ generators of Conf. Alg.

Algebra knows about General Relativity!

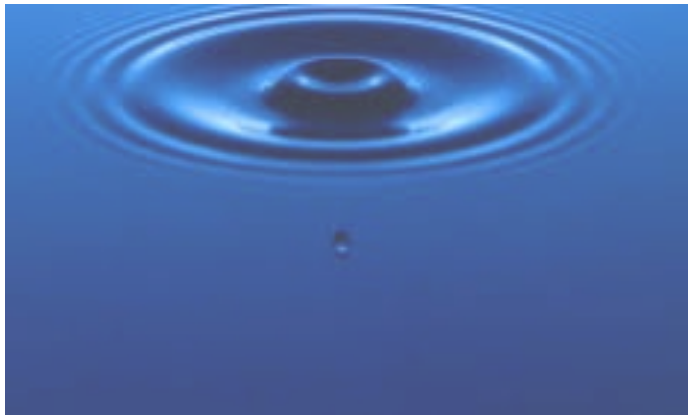
Operators and States

Every operator creates a unique state, and vice versa:

$$\rho(x)|0\rangle \leftrightarrow |\rho\rangle$$

By "measuring" ρ, we perturb the vacuum and put it in a new state.

$$\rho(x)$$



Operator Product Expansion

Start with insertion of two operators

$Y_{\ell m}$

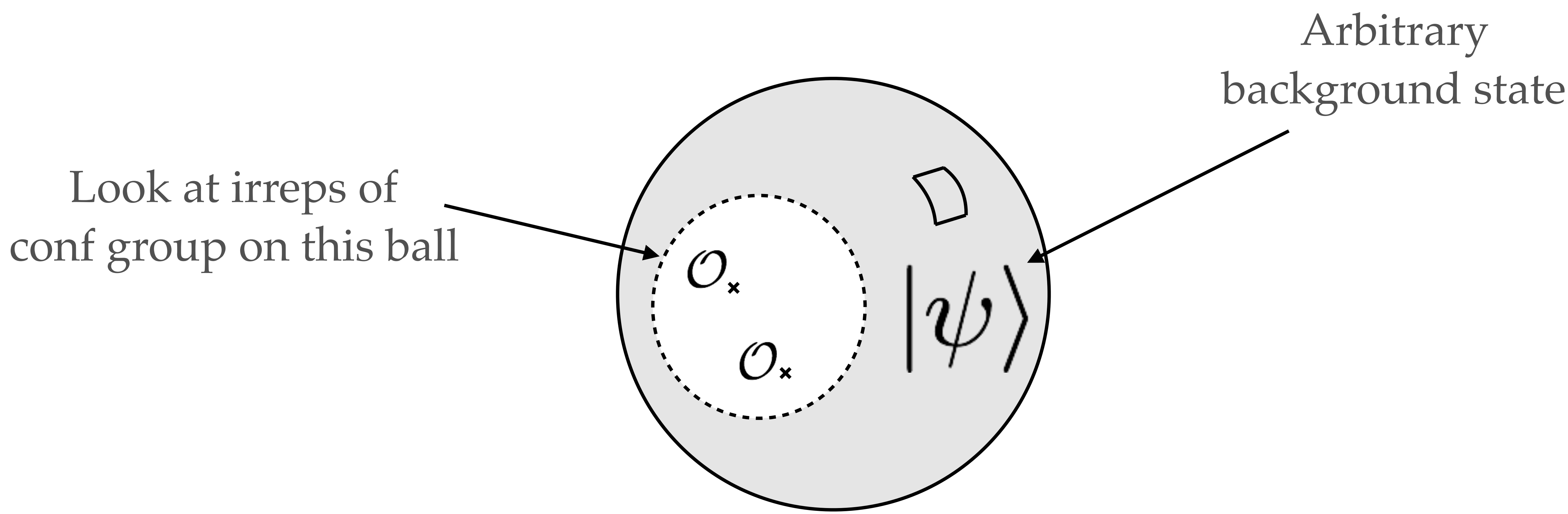
Decompose into a convenient basis at a fixed radius.

E.g. ~~Spherical harmonics~~

Conformal Blocks

Conformal Irreps

"OPE blocks"= contribution to OPE from a single irrep



OPE block is an operator (can be evaluated in any state)

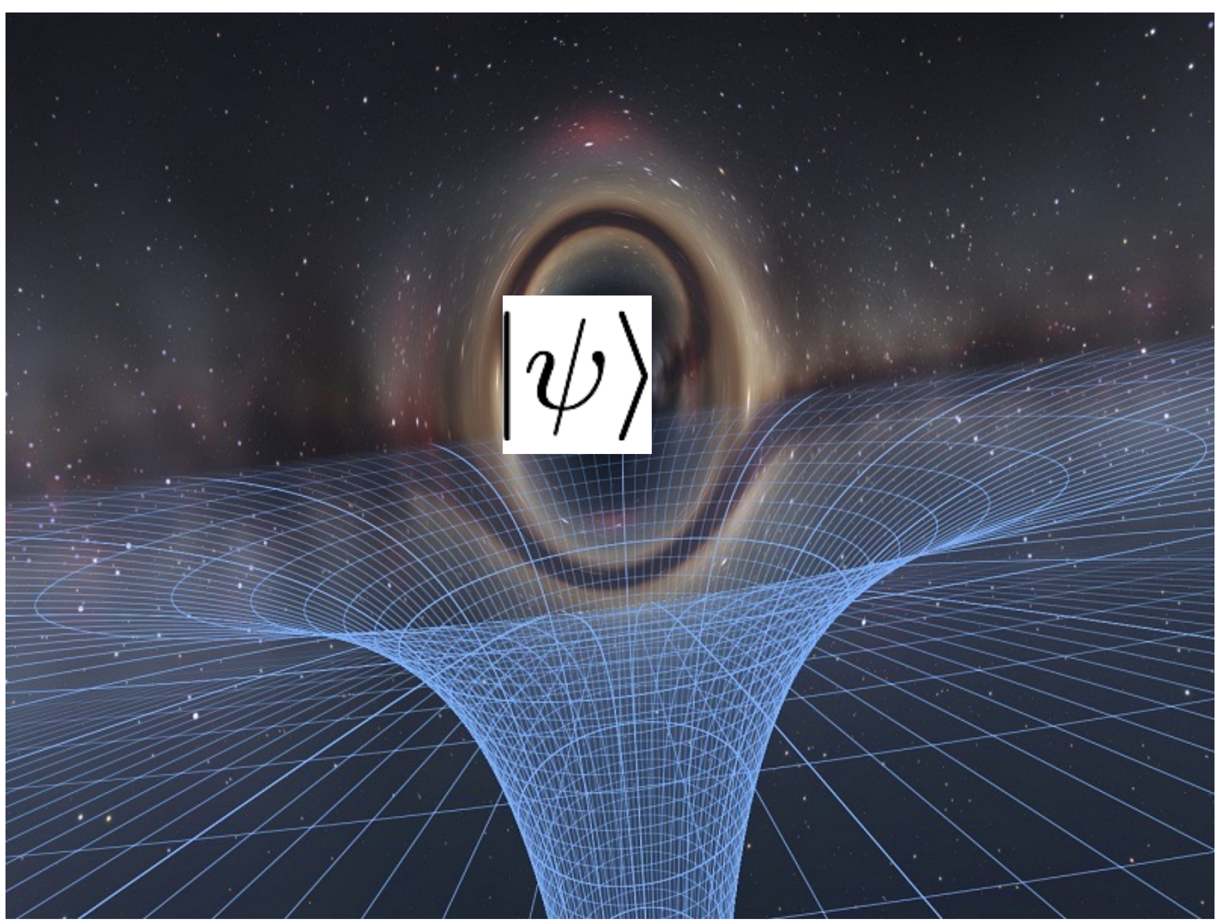
"Vacuum OPE block": $[\mathcal{O}(z_1)\mathcal{O}(z_2)]_{\text{vac}} = \sum_n C_{\mathcal{O}\mathcal{O}T^n}(z_1, z_2) T^n(z_2)$

$\alpha = 1, T, T^2, \ldots$

Large c and "Heavy" states

How do we get interesting effects in gravity at $G_N \to 0$? Keep $G_N M \sim R$ fixed

Heavy state $|\psi\rangle$: $\dfrac{\Delta_\psi}{c}$ fixed, $c \to \infty$



"BH microstate": $G_N \leftrightarrow \dfrac{1}{c}$

$$M_\psi \leftrightarrow \Delta_\psi$$

$$\frac{\Delta_\psi}{c} \leftrightarrow G_N M_\psi \sim R_S$$

Fixed geometry

Large c and "Heavy" states

Example: a heavy primary state $|\psi\rangle$

OPE block at large c:

Exactly thermal!

$$\langle\psi|[\mathcal{O}(z_1)\mathcal{O}(z_2)]_{\text{vac}}|\psi\rangle = \left(\frac{1}{\sinh(\pi T_\psi t)}\right)^{h_\mathcal{O}} + \mathcal{O}(\frac{1}{c}) \qquad z_{12} = 1 - e^{-t}$$

~Eigenstate Thermalization

$|\psi\rangle =$ 

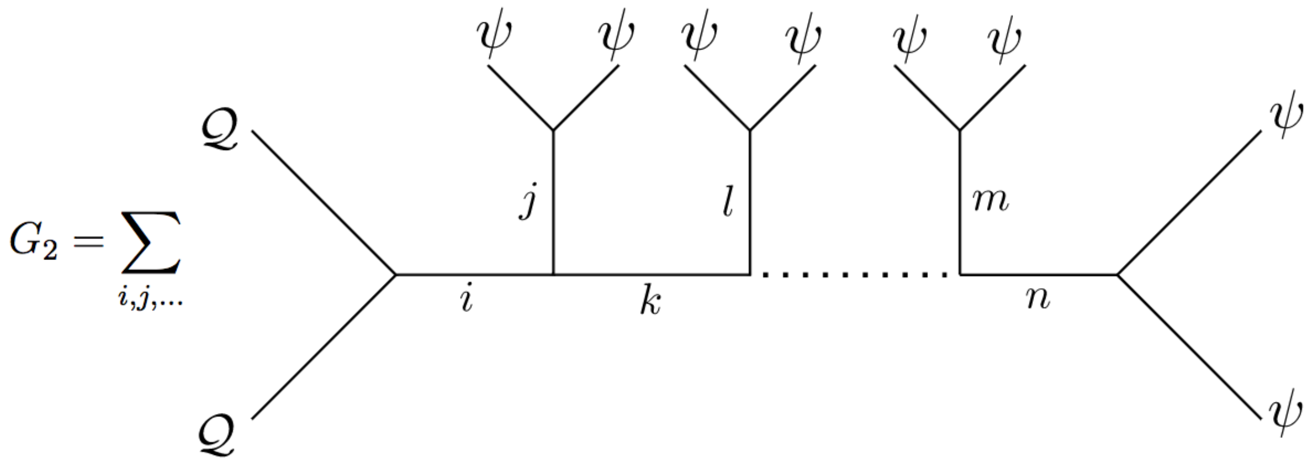
$$T_\psi = \frac{1}{2\pi}\sqrt{\frac{12\Delta_\psi}{c} - 1}$$

$$t \to \infty : \left(\frac{1}{\sinh(\pi T_\psi t)}\right)^{h_\mathcal{O}} \sim e^{-\pi h_\mathcal{O} T_\psi t}$$

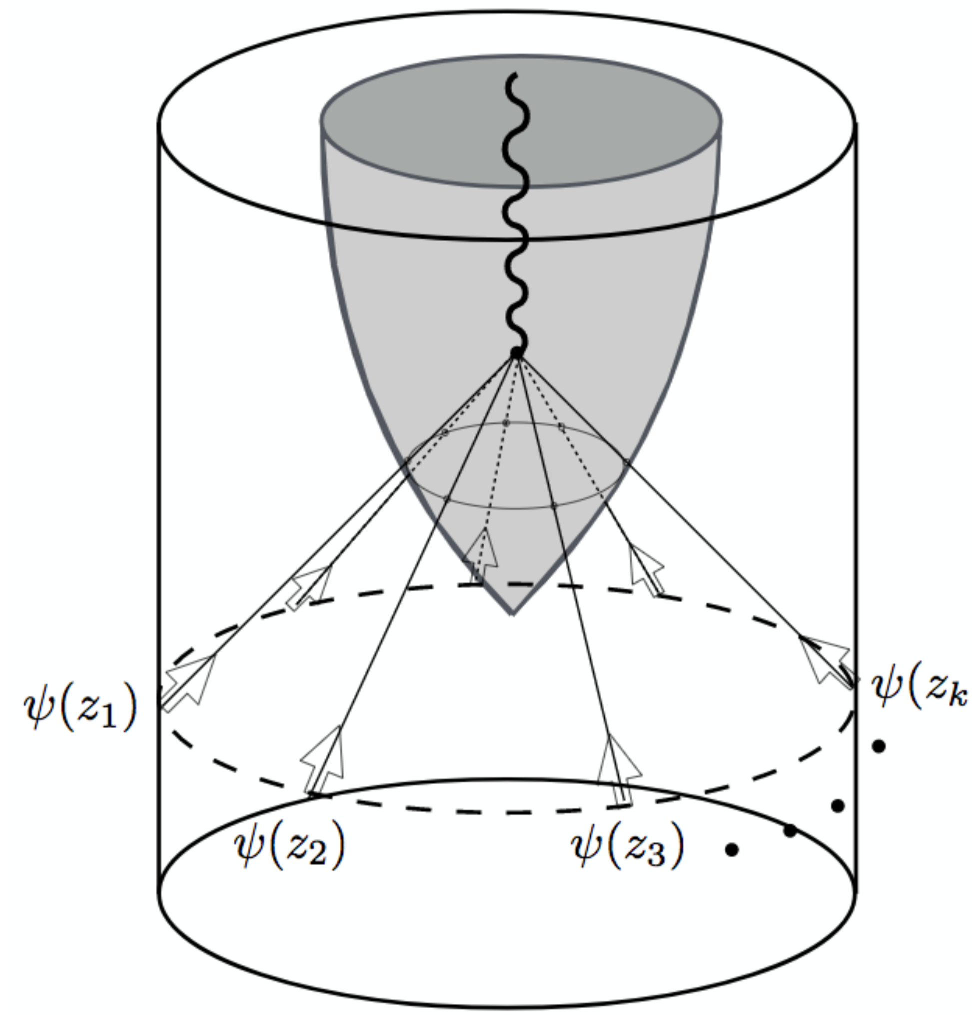
Info loss at large c

Large c and “Heavy” states

Anous, Hartman, Rovai, Sonner, ‘16



From blocks to black hole collapse



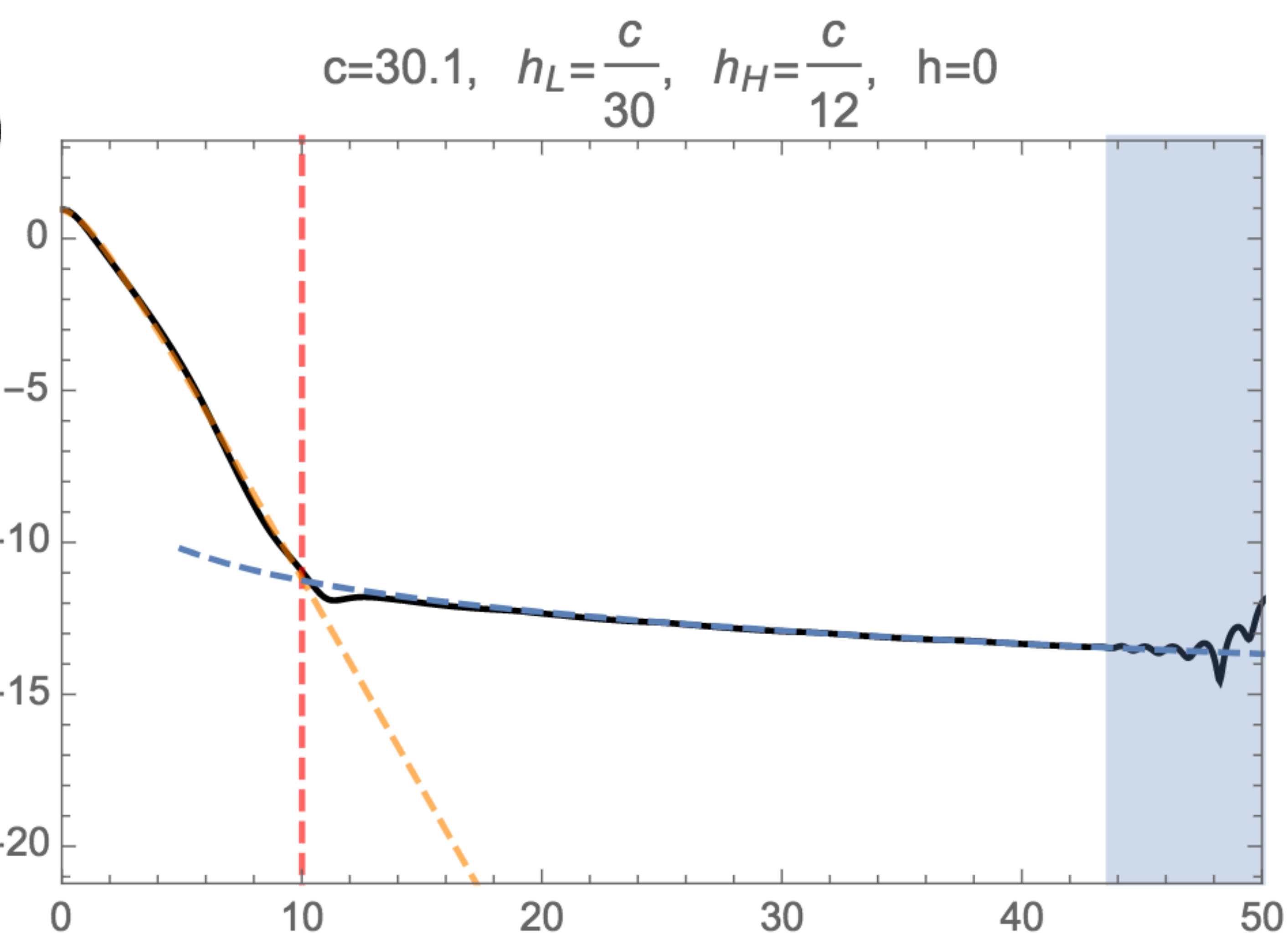
Bin Chen & Jie-qiang Wu 1605.06753

From blocks to geometry in any CFT state

Exact Behavior of $\langle\psi|[\mathcal{O}(z_1)\mathcal{O}(z_2)]_{\rm vac}|\psi\rangle$

In the exact block, late-time exponential decay becomes power-law $t^{-3/2}$ at t ≳ c

$\log\left(\langle\psi|\mathcal{O}\mathcal{O}|\psi\rangle\right)$



$t_D = \frac{\pi c}{6h_L}$ t

Chen, Hussong, Kaplan, Li, ‘17

Euclidean time periodicity and forbidden singularities

$$\frac{1}{\sin(\pi T_\psi t_E)}$$ Periodic in Euclidean time (KMS condition):

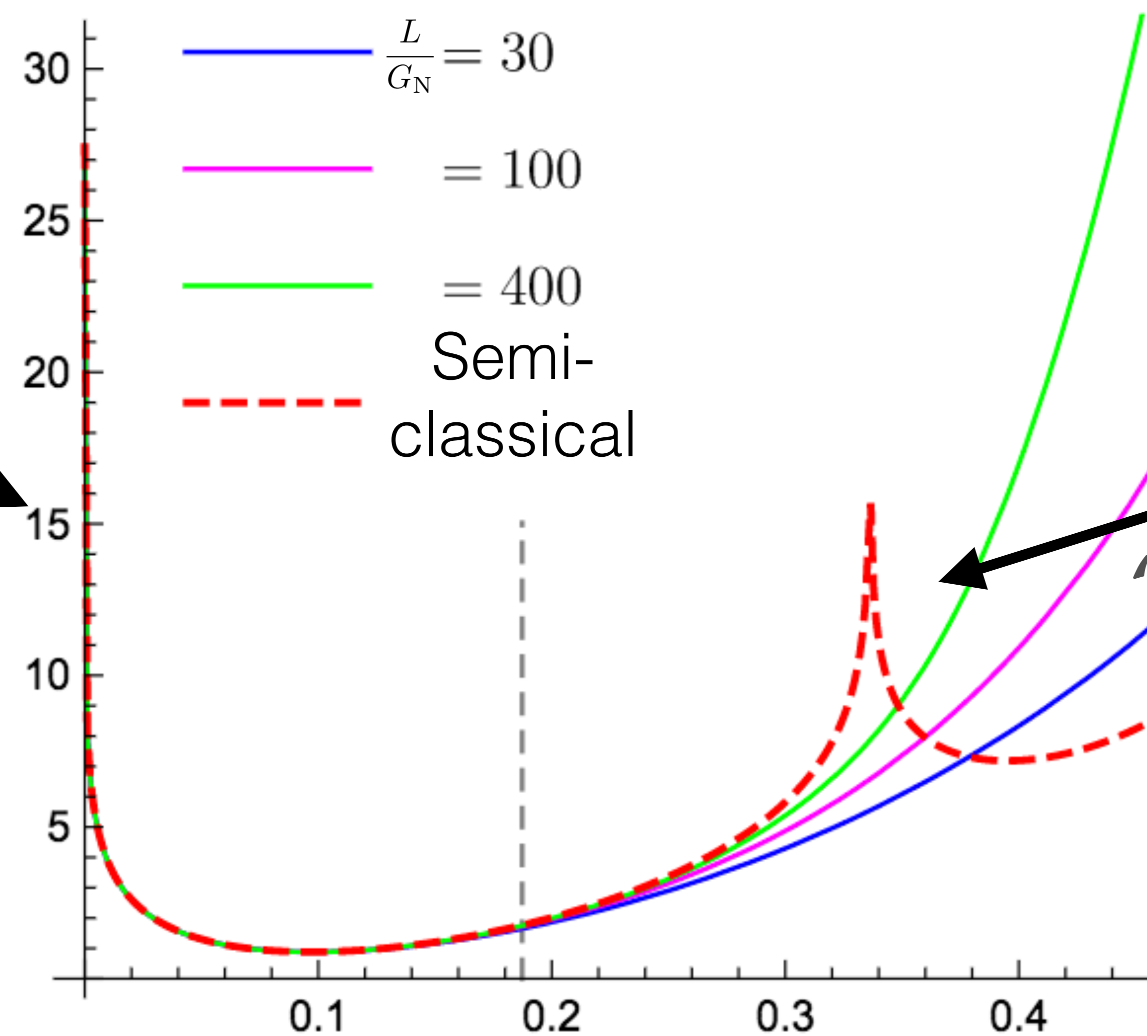
$\mathcal{O}(t_E)$

But the black hole is really a *pure* state not a thermal state, so this can't be true exactly

Going beyond the Semi-classical Limit

Unitarity restoration can be seen in the exact quantum theory

"True" OPE singularity



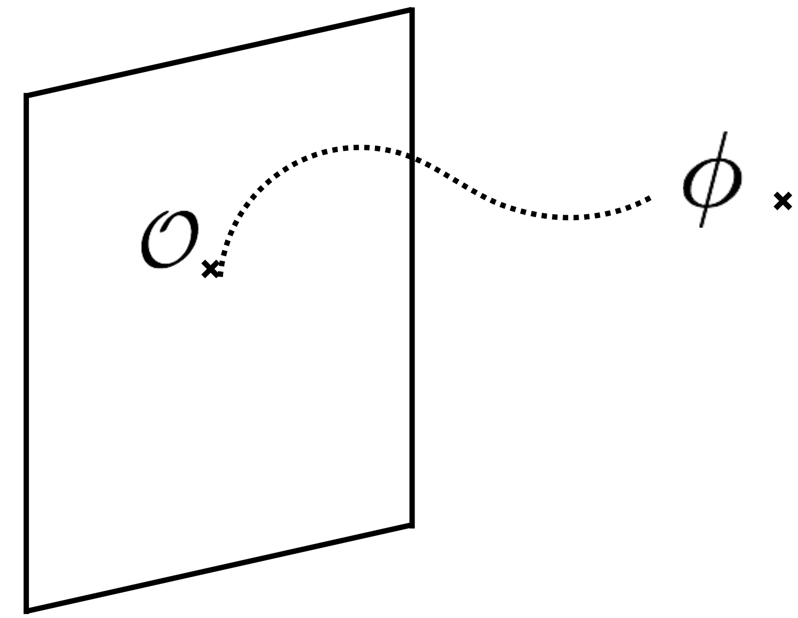
Approximate "thermal image"

Chen, Hussong, Kaplan, Li, '17

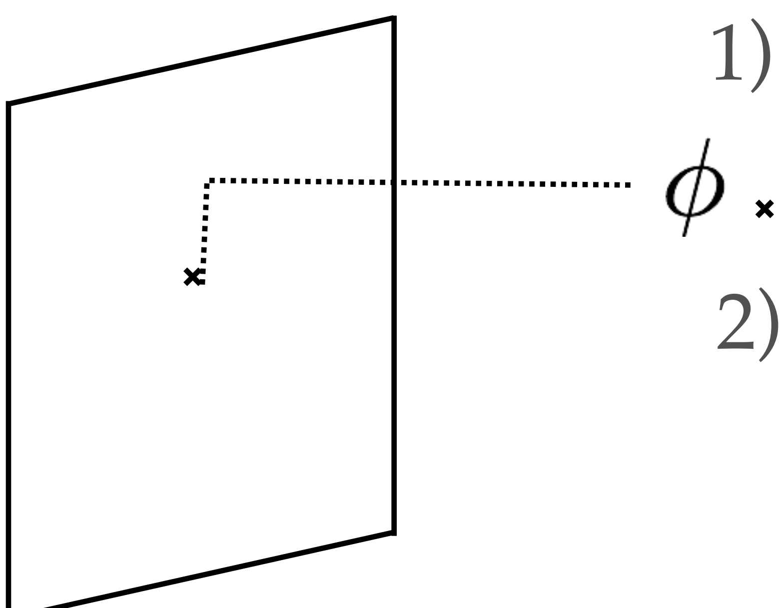
Bulk Reconstruction

Bulk Reconstruction

We want to construct an exact definition of a bulk "proto-field" ϕ that includes all contributions from $\mathcal{O}$ and its Virasoro descendants



Two (equivalent) constructions:



1) ϕ is connected to the boundary by a Wilson line

2) Algebraic construction - more efficient for computations (at least for now?)

Will focus on (2)

Bulk Reconstruction

Rough idea: 1) reconstruct ϕ from $\mathcal{O}$ in fixed background metric

2) Promote T to operator

3) Identify symmetry properties of ϕ, use them to define ϕ

We will use Fefferman-Graham gauge for vacuum metric:

$$ds^2 = \frac{dy^2 + dz d\bar{z}}{y^2} - \frac{6T(z)}{c} dz^2 - \frac{6\bar{T}(\bar{z})}{c} d\bar{z}^2 + y^2 \frac{36 T(z) \bar{T}(\bar{z})}{c^2} dz d\bar{z}$$

Constructing ϕ

Let's do a warm-up:
reconstruction of ϕ in the bulk in a free AdS theory.

Metric: $$ds^2 = \frac{dy^2 + dz d\bar{z}}{y^2}$$

$$\langle \phi \mathcal{O} \rangle_{\text{vac}} = \left(\frac{y}{y^2 + z\bar{z}} \right)^{\Delta}$$ is an exact relation for the bulk to boundary propagator

This fixes the contribution to ϕ from all "global" descendants of $\mathcal{O}$

$$\phi(y, 0) = \sum_n \lambda_n y^{\Delta + 2n} (L_{-1} \bar{L}_{-1})^n \mathcal{O}(0)$$

translation generators,
the simplest elements of the conformal algebra

Constructing ϕ

$$\phi(y,0) = \sum_n \lambda_n y^{\Delta+2n} (L_{-1}\bar{L}_{-1})^n \mathcal{O}(0)$$

translation generators

Substituting into the LHS of $\langle \phi \mathcal{O} \rangle_{\text{vac}} = \left(\frac{y}{y^2 + z\bar{z}} \right)^{\Delta}$

and demanding that we reproduce the RHS fixes

$$\lambda_N = \frac{(-1)^N}{N!(\Delta)_N}$$

Constructing ϕ

Same basic idea let's us fix contributions from all Virasoro descendants of $\mathcal{O}$:

We know $\langle \phi \mathcal{O} \rangle_T = \left(\frac{y'}{y'^2 + z'\bar{z}'} \right)^\Delta$ from the ***T-dependent*** coord transformation between Fff-Graham metric and pure AdS

This fixes the contribution to ϕ from the entire Virasoro irrep of $\mathcal{O}$

$$\phi(y, 0) = \sum_n \lambda_n y^{\Delta + 2n} (\mathcal{L}_{-n} \bar{\mathcal{L}}_{-n}) \mathcal{O}(0)$$

some specific combination of Virasoro generators

for example: $\mathcal{L}_{-2} = \frac{(2h+1)(c+8h)}{(2h+1)c + 2h(8h-5)} \left(L_{-1}^2 - \frac{12h}{c+8h} L_{-2} \right)$

Algebraic Definition of ϕ

We can state this algebraic definition of ϕ more simply by thinking about how it transforms under Virasoro

$$L_m\phi = ((\delta_m y)\partial_y + (\delta_m z)\partial_z + (\delta_m \bar{z})\partial_{\bar{z}})\phi$$

There is a unique extension of boundary conf txn into the bulk that preserves Fefferman-Graham gauge

At z=0: easy to check that $\delta_m y = 0, \delta_m z = 0 \quad \delta_m \bar{z} = 0$ for all m ≥ 2

$$L_m\phi(y,0,0) = 0 \quad m \geq 2$$

This plus normalization condition fixes ϕ

"Vacuum sector" Correlators

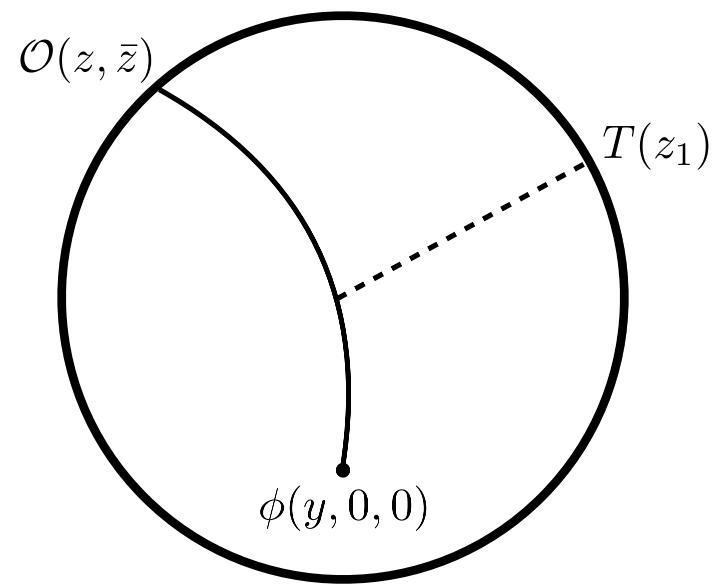
This definition of ϕ predicts all bulk correlators of the form

$$\langle \mathcal{O} \phi T \ldots T \bar{T} \ldots \bar{T} \rangle$$

(any number of T, $\bar{T}$'s)

compute Witten diagram and verify?

E.g. $\langle \phi \mathcal{O} T \rangle$



$$\frac{\langle \phi(y,0) \mathcal{O}(z_2) T(z_1) \rangle}{\langle \phi(y,0) \mathcal{O}(z_2) \rangle} = \frac{\Delta z_2^2}{2 z_1^3 z_{12}^2} \left(z_1 + \frac{2 y^2 z_{12}}{y^2 + z_2 \bar{z}_2} \right)$$

3rd order pole

Witten diagram matches our prescription

Let's Compute Stuff

There are several available techniques for computing correlators of ϕ

"projectors" aka "Brute force"
Recursion relations

Exact

Monodromy method
Degenerate Operators
Uniformizing coordinates

Large c

For example: $\langle \phi\phi \rangle$ and $\langle \psi | \phi \mathcal{O} | \psi \rangle$

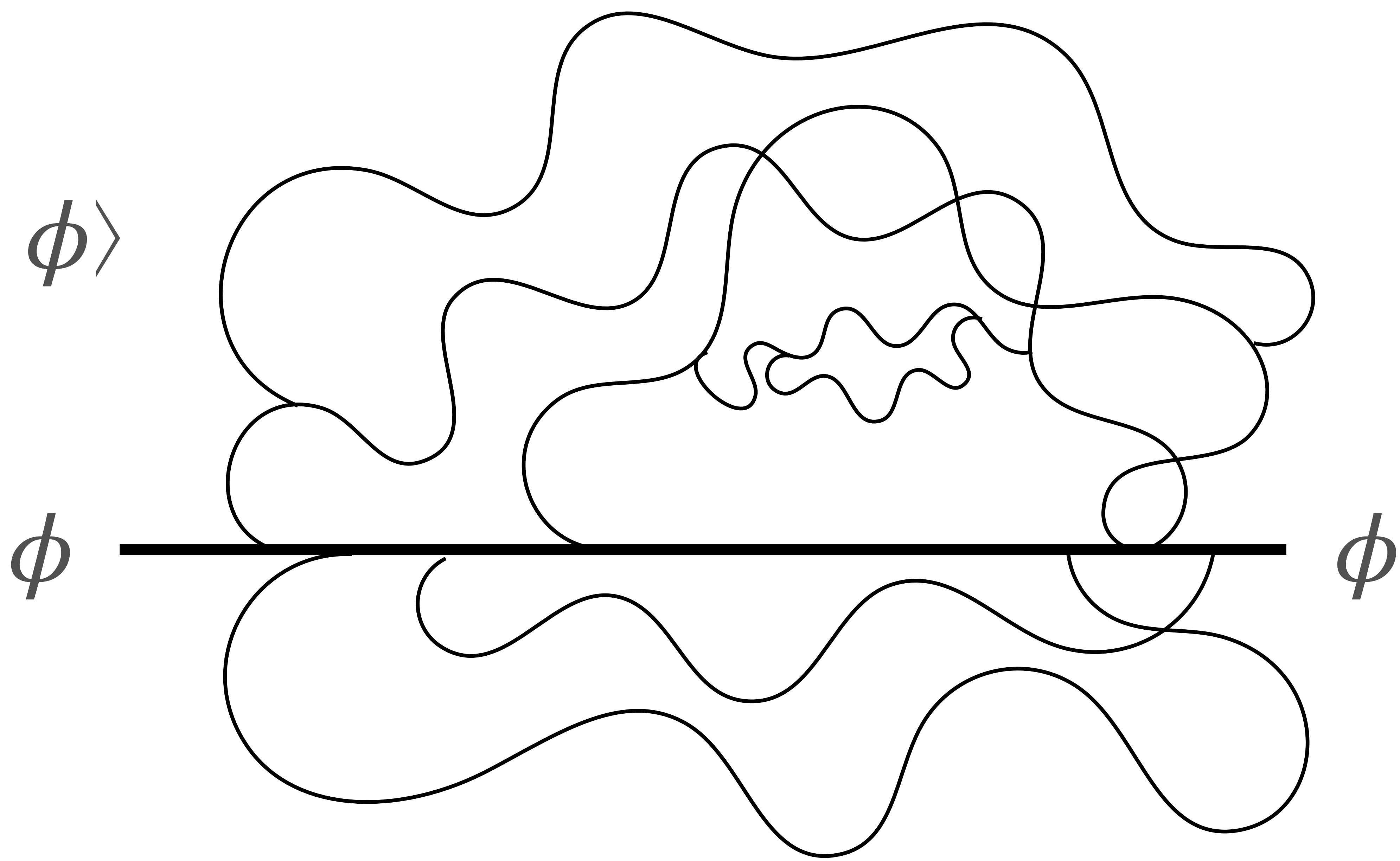
"Two bulk fields approach each other" (bulk locality?)

"Bulk field near a horizon"

“Exact” $\langle \phi\ \phi \rangle$

The exact $\langle \phi\ \phi \rangle$ is the propagator dressed by gravitons

$\langle \phi\ \phi \rangle$



But does not include ϕ loops

Exact $\langle \phi \, \phi \rangle$

We want to compute $\langle \phi \, \phi \rangle$

To get our bearings: recall tree-level result in AdS_3

$$\langle \phi(X_1) \phi(X_2) \rangle = \frac{\rho^{\frac{\Delta}{2}}}{1 - \rho}$$

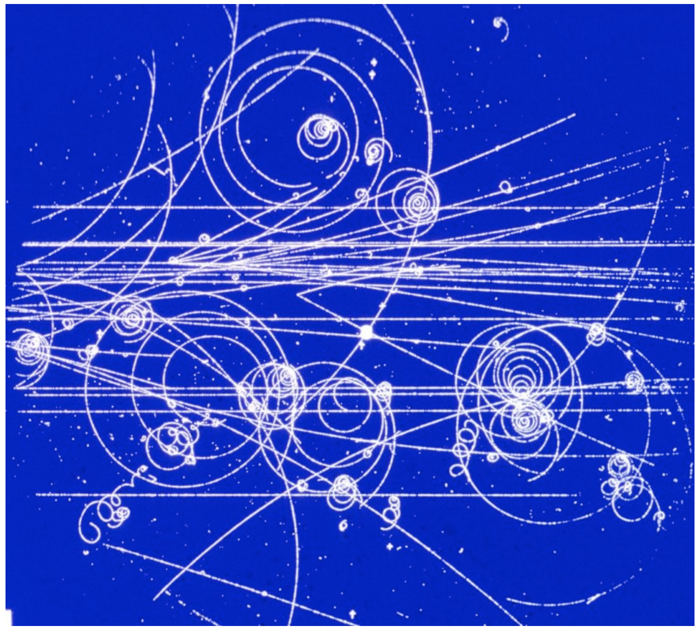
Convenient to introduce the kinematic variables:

$$\rho \equiv e^{-2\sigma}$$

and σ = geodesic distance

Imaginary Part

Correlator $\langle\phi\ \phi\rangle$ should be real since ϕ is a real scalar field that cannot decay



In the exact calculation, we will find that it develops an imaginary part at real, space like separation

We will call this a `breakdown of bulk locality', though there could be other interpretation (e.g. maybe ϕ is a sick, unobservable quantity?)

Will consider various limits

1) small semiclassical Δ: large c, small Δ/c fixed
   Simplest limit to see semiclassical "action"

2) large Δ - the limit of very massive fields.
   Also a necessary input to a recursion relation

3) small $\Delta \ll \mathbf{1}$ the limit of massless ϕ

In all cases we will see the breakdown of bulk locality through imaginary parts

Holomorphic Case

In the following slides, I'll actually be computing a "holomorphic" version $\langle\phi\phi\rangle_{\text{holo}}$ where drop all anti-holomorphic Ts in ϕ

Why?

1) It's easier to do analytically - results are more transparent and under better control

2) It is possible to extract the full result from just the holomorphic parts, so in a sense it's the "hard" part of the numeric computation

3) From numeric exploration, it doesn't appear to be very different from the full two-point function

“Semiclassical” pieces

At large c with Δ/c fixed,

$$\langle \phi(X)\phi(Y) \rangle \sim e^{cf(X,Y)}$$

f is like a “semiclassical action” piece

It can be computed with Zamolodchikov “monodromy method”

Semiclassical small Δ/c piece

At large c with Δ/c fixed, $\langle \phi\phi \rangle \sim e^{cf(\rho;\frac{\Delta}{c})}$

$$\rho \equiv e^{-2\sigma}$$

geodesic distance

Example of semi-classical piece — f can compute order-by-order in Δ/c:

$$cf(\rho) = \Delta \log \rho + \frac{3\Delta^2}{c}\left(\frac{\rho}{(1-\rho)^2} + \log(1-\rho)\right) + \mathcal{O}(\frac{\Delta^3}{c^2})$$

singular at ρ=1, ie at σ=0

Leading singular terms can be resummed $\sim \left(1 - \frac{243\Delta^2/c^2}{4\sigma^6}\right)^{3/2}$

Branch cut at small separation σ

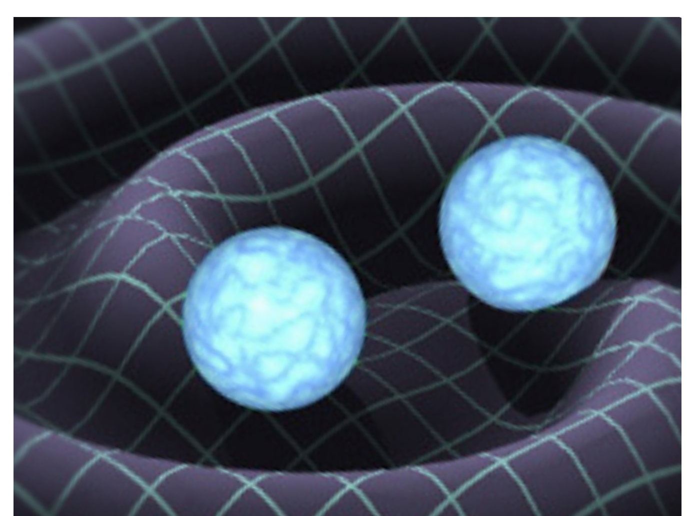
large Δ limit

At large Δ/c we can go farther and get the exact result:

$$\langle\phi\phi\rangle = q^{\frac{\Delta}{2}-\frac{c-1}{24}}\left(\frac{s}{8}\right)^{\frac{c-1}{12}}(1-s)^{\frac{c-13}{144}}\left(\frac{2E(s)}{\pi}\right)^{\frac{19-7c}{36}}$$

$$q = 4e^{2\pi\frac{E(1-s)-K(1-s)}{E(s)}-4} \qquad s = \frac{8\sqrt{\rho}}{1+4\sqrt{\rho}-\rho} = \frac{4}{2+\sinh\sigma}$$

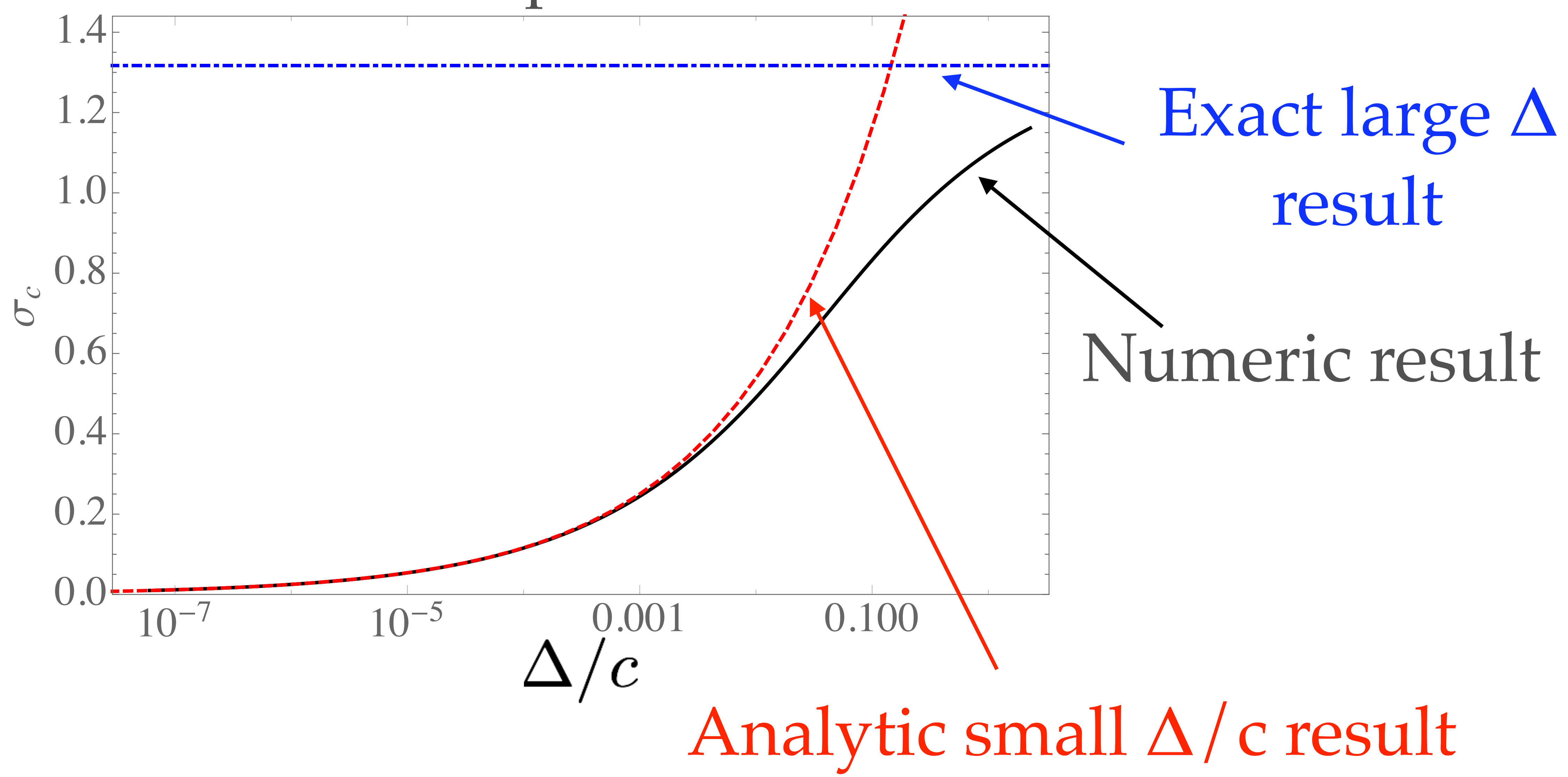
Branch cut at s=1 $\Leftrightarrow \sigma = 1.3\ell_{\mathrm{AdS}}$



Semiclassical Numerics

At any Δ/c, we can efficiently compute the semiclassical piece numerically for any ρ

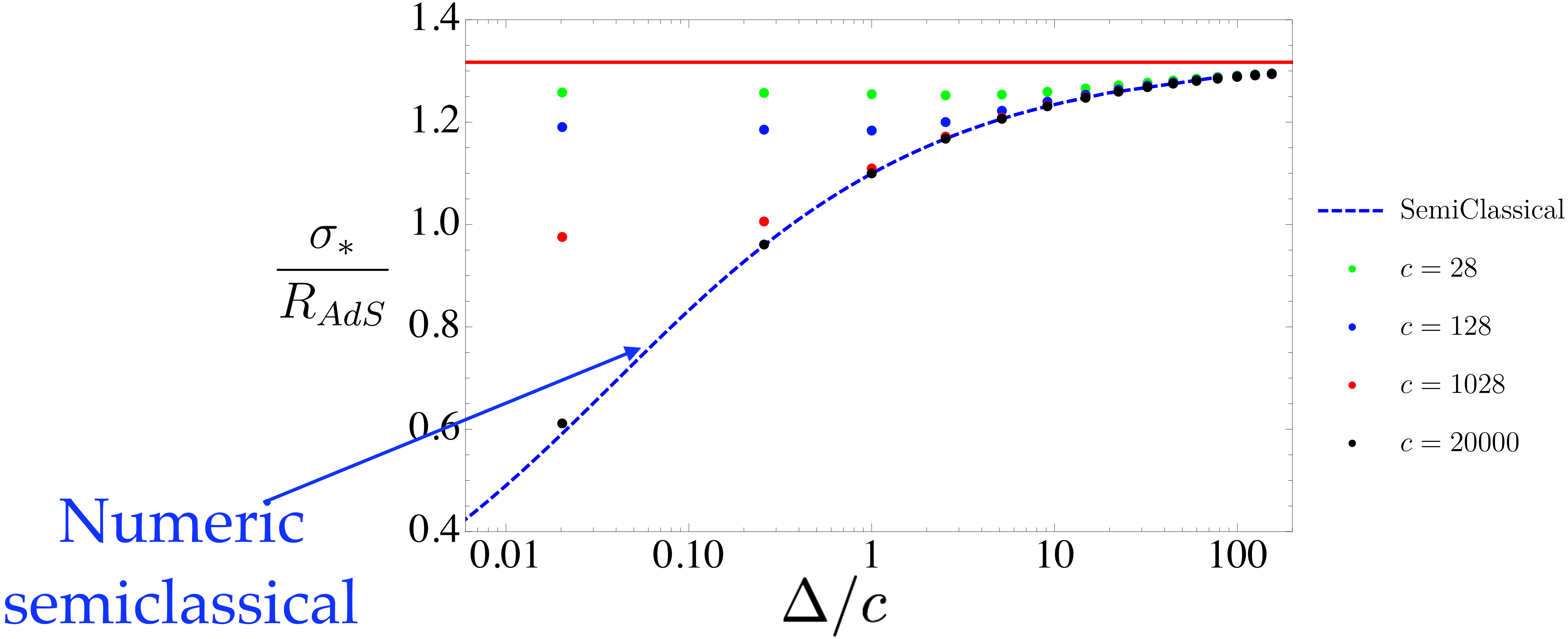
Semiclassical exponent f has a branch cut at finite separation σ



Radius of Convergence

We can also study the radius of convergence in ρ of the exact (not just semiclassical) result

Numerically computed series coefficients $\sum_n c_n \rho^n$ up to ~ $O(\rho^{500})$ and extrapolated to estimate radius of convergence



Large c: RoC agrees with semiclassical branch cut

$\Delta \ll 1$ limit

$\langle\phi\ \phi\rangle$ also simplifies somewhat in massless case

Leading singular terms:

$$\langle\phi\phi\rangle \overset{\sigma\sim 0}{\sim} \frac{1}{2\sigma}\left(\sum_{n=0}^{\infty}\frac{(4n-1)!!}{n!}\left(\frac{3}{4\ c\ \sigma^4}\right)^n\right)$$

Looks like an expansion in $c\ \sigma^4$

This is an asymptotic series and in fact matches

$$\int_{-\infty}^{\infty} dz e^{-\frac{1}{2}z^2+\frac{3}{4c\sigma^4}z^4}$$

$\longrightarrow$ non-perturbative ambiguity ~ $e^{-c\sigma^4}$

A fundamental scale in AdS gravity at $c^{-1/4}$??

$c^{1/4}$ and AdS_3 string compactifications

The scale $c^{1/4}$ also shows up as the smallest string length in known stable AdS_3 compactifications

E.g. $AdS_3 \times S^3 \times T^4$

Smallest one can make the radius of T is ~ ℓ_s

$$\longrightarrow \ell_{\mathrm{pl,3d}} \ell_{\mathrm{AdS}}^3 \ell_s^4 = \ell_{\mathrm{pl,10d}}^8 \lesssim \ell_s^8$$

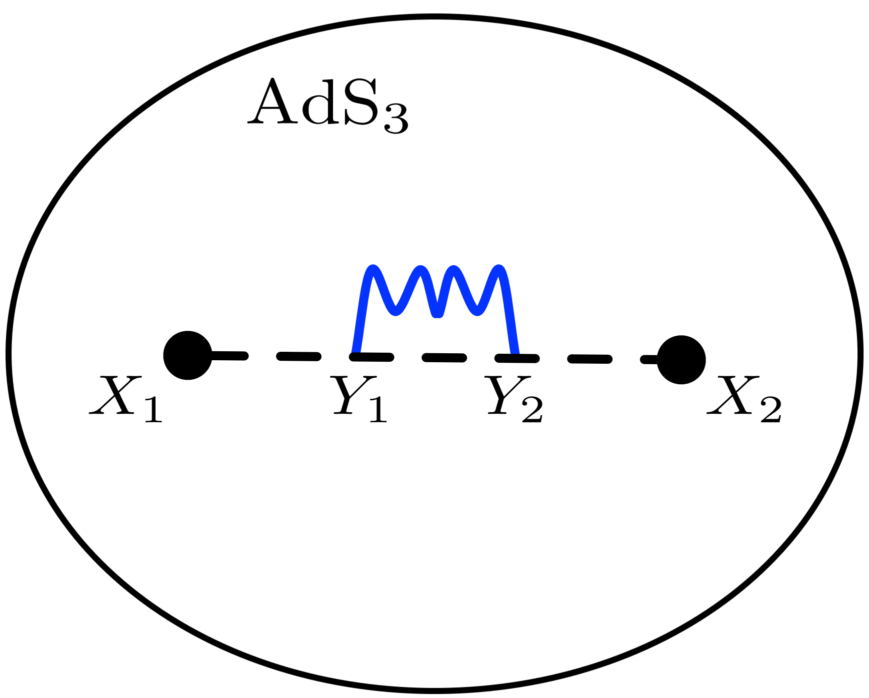
$$\longrightarrow \frac{\ell_s}{\ell_{\mathrm{AdS}}} \gtrsim \left(\frac{\ell_{\mathrm{pl,3d}}}{\ell_{\mathrm{AdS}}}\right)^{1/4} \sim c^{-1/4}$$

UV/IR Mixing

Important caveat: ϕ is not an IR safe observable

The imaginary parts develop at distances for σ that involve the AdS radius ℓ_{AdS}

This UV / IR mixing can be seen in bulk perturbation theory in G_N



$$\sim \frac{G_N \ell_{\text{AdS}}^3}{\sigma}\left(\frac{1}{\sigma^4}+\frac{m^2}{\sigma^2}+\dots\right)$$

On the other hand - it's finite in AdS, so maybe one should just take it seriously

$c^{1/4}$ and strings

The scale $c^{1/4}$ also shows up as the smallest string length in known stable AdS_3 compactifications

$$\ell_s \gtrsim c^{-1/4}$$

Possible interpretations:

— Coincidence? Could be
After all, ϕ isn't completely local

— Fundamental breakdown of spacetime locality at this scale, prevents string length from being smaller?

Summary

Large amount of information about gravity is contained in CFT_2 irreps

This includes BH thermodynamics, information paradox, many non-perturbative $e^{-\frac{1}{G_N}}$ corrections
These corrections are computable and in some cases ameliorate or even resolve unitarity issues at infinite c

These techniques can be applied to bulk fields
What do they tell us about bulk physics near horizons?

The End