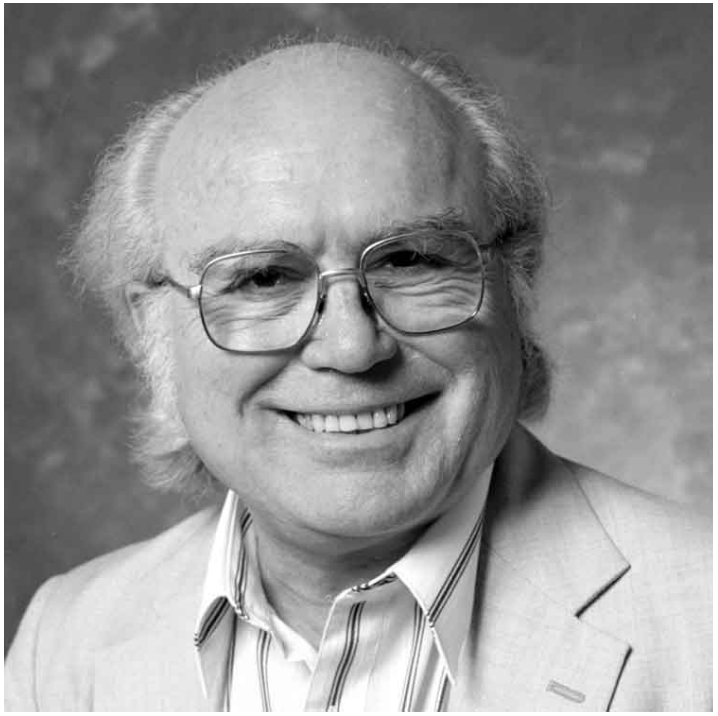
SUSY Lagrangians

Wess-Zumino



The free Wess–Zumino model

$$S = \int d^4x \ (\mathcal{L}_\mathrm{s} + \mathcal{L}_\mathrm{f})$$
$$\mathcal{L}_\mathrm{s} = \partial^\mu \phi^* \partial_\mu \phi, \ \ \mathcal{L}_\mathrm{f} = i\psi^\dagger \overline{\sigma}^\mu \partial_\mu \psi.$$

$$g^{\mu\nu} = \eta^{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$$

$$\phi \to \phi + \delta\phi$$
$$\psi \to \psi + \delta\psi$$

$$\begin{aligned} \delta\phi &= \epsilon^\alpha \psi_\alpha \\ &= \epsilon^\alpha \boldsymbol{\epsilon}_{\alpha\beta} \psi^\beta \equiv \epsilon\psi \end{aligned}$$

$$\boldsymbol{\epsilon}_{\alpha\beta} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \ \boldsymbol{\epsilon}^{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\epsilon\psi = -\psi^\beta \boldsymbol{\epsilon}_{\alpha\beta} \epsilon^\alpha = \psi^\beta \boldsymbol{\epsilon}_{\beta\alpha} \epsilon^\alpha = \psi\epsilon$$

$$\delta\phi^* = \epsilon^\dagger_{\dot\alpha}\psi^{\dagger\dot\alpha} \equiv \epsilon^\dagger\psi^\dagger$$

$$\delta\mathcal{L}_{\rm s} = \epsilon\partial^\mu\psi\,\partial_\mu\phi^* + \epsilon^\dagger\partial^\mu\psi^\dagger\,\partial_\mu\phi$$

$$\delta\psi_\alpha = -i(\sigma^\nu\epsilon^\dagger)_\alpha\,\partial_\nu\phi \;\; \delta\psi^\dagger{}_{\dot\alpha} = i(\epsilon\sigma^\nu)_{\dot\alpha}\,\partial_\nu\phi^*$$

$$\delta\mathcal{L}_{\rm f} = -\epsilon\sigma^\nu\partial_\nu\phi^*\overline{\sigma}^\mu\partial_\mu\psi\; + \psi^\dagger\overline{\sigma}^\mu\sigma^\nu\epsilon^\dagger\,\partial_\mu\partial_\nu\phi$$

Pauli identities:

$$\left[\sigma^\mu\overline{\sigma}^\nu + \sigma^\nu\overline{\sigma}^\mu\right]_\alpha^{\;\beta} = 2\eta^{\mu\nu}\delta_\alpha^\beta \;\; \left[\overline{\sigma}^\mu\sigma^\nu + \overline{\sigma}^\nu\sigma^\mu\right]^{\dot\alpha}_{\;\dot\beta} = 2\eta^{\mu\nu}\delta^{\dot\alpha}_{\dot\beta}$$

$$\begin{aligned}\delta\mathcal{L}_{\rm f} \;&=\; -\epsilon\partial^\mu\psi\,\partial_\mu\phi^* - \epsilon^\dagger\partial^\mu\psi^\dagger\,\partial_\mu\phi \\ &\quad +\partial_\mu\left(\epsilon\sigma^\mu\overline{\sigma}^\nu\psi\,\partial_\nu\phi^* - \epsilon\psi\,\partial^\mu\phi^* + \epsilon^\dagger\psi^\dagger\,\partial^\mu\phi\right).\end{aligned}$$

total derivative so:

$$\delta S = 0$$

Commutators of SUSY transformations

$$(\delta_{\epsilon_2}\delta_{\epsilon_1} - \delta_{\epsilon_1}\delta_{\epsilon_2})\phi = -i(\epsilon_1\sigma^\mu\epsilon_2^\dagger - \epsilon_2\sigma^\mu\epsilon_1^\dagger)\,\partial_\mu\phi$$

$$(\delta_{\epsilon_2}\delta_{\epsilon_1} - \delta_{\epsilon_1}\delta_{\epsilon_2})\psi_\alpha = -i(\sigma^\nu\epsilon_1^\dagger)_\alpha\,\epsilon_2\partial_\nu\psi + i(\sigma^\nu\epsilon_2^\dagger)_\alpha\,\epsilon_1\partial_\nu\psi$$

Fierz identity:

$$\chi_\alpha\,(\xi\eta) = -\xi_\alpha\,(\chi\eta) - (\xi\chi)\eta_\alpha$$

$$\begin{aligned}(\delta_{\epsilon_2}\delta_{\epsilon_1} - \delta_{\epsilon_1}\delta_{\epsilon_2})\psi_\alpha \quad = \quad & -i(\epsilon_1\sigma^\mu\epsilon_2^\dagger - \epsilon_2\sigma^\mu\epsilon_1^\dagger)\,\partial_\mu\psi_\alpha \\ & +i(\epsilon_{1\alpha}\,\epsilon_2^\dagger\overline{\sigma}^\mu\partial_\mu\psi - \epsilon_{2\alpha}\,\epsilon_1^\dagger\overline{\sigma}^\mu\partial_\mu\psi).\end{aligned}$$

SUSY algebra closes on-shell.

on-shell the fermion EOM reduces DOF by two

$p_\mu = (p, 0, 0, p)$

$$\overline{\sigma}^\mu p_\mu \psi = \begin{pmatrix} 0 & 0 \\ 0 & 2p \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

projects out half of DOF

	off-shell	on-shell
ϕ, ϕ^*	2 d.o.f.	2 d.o.f.
$\psi_\alpha, \psi^\dagger{}_{\dot\alpha}$	4 d.o.f.	2 d.o.f.

SUSY is not manifest off-shell

trick: add an auxiliary boson field $\mathcal{F}$

	off-shell	on-shell
$\mathcal{F}, \mathcal{F}^*$	2 d.o.f.	0 d.o.f.

$$\mathcal{L}_{\text{aux}} = \mathcal{F}^* \mathcal{F}$$

$$\delta\mathcal{F} = -i\epsilon^\dagger\overline{\sigma}^\mu\partial_\mu\psi, \;\; \delta\mathcal{F}^* = i\partial_\mu\psi^\dagger\overline{\sigma}^\mu\epsilon$$

$$\delta\mathcal{L}_{\rm aux} = i\partial_\mu\psi^\dagger\overline{\sigma}^\mu\epsilon\,\mathcal{F} - i\epsilon^\dagger\overline{\sigma}^\mu\partial_\mu\psi\,\mathcal{F}^*$$

modify the transformation of the fermion:

$$\delta\psi_\alpha = -i(\sigma^\nu\epsilon^\dagger)_\alpha\,\partial_\nu\phi + \epsilon_\alpha\mathcal{F}, \;\; \delta\psi^\dagger{}_{\dot\alpha} = +i(\epsilon\sigma^\nu)_{\dot\alpha}\,\partial_\nu\phi^* + \epsilon^\dagger_{\dot\alpha}\mathcal{F}^*$$

$$\begin{aligned}\delta^{\rm new}\mathcal{L}_f &= \delta^{\rm old}\mathcal{L}_f + i\epsilon^\dagger\overline{\sigma}^\mu\partial_\mu\psi\mathcal{F}^* + i\psi^\dagger\overline{\sigma}^\mu\partial_\mu\epsilon\mathcal{F} \\ &= \delta^{\rm old}\mathcal{L}_f + i\epsilon^\dagger\overline{\sigma}^\mu\partial_\mu\psi\mathcal{F}^* - i\partial_\mu\psi^\dagger\overline{\sigma}^\mu\epsilon\mathcal{F} + \partial_\mu(i\psi^\dagger\overline{\sigma}^\mu\epsilon\mathcal{F})\end{aligned}$$

last term is a total derivative

$$S^{\rm new} = \int d^4x\,\mathcal{L}_{\rm free} = \int d^4x\,(\mathcal{L}_{\rm s} + \mathcal{L}_{\rm f} + \mathcal{L}_{\rm aux})$$

is invariant under SUSY transformations:

$$\delta S^{\rm new} = 0$$

Commutator of two SUSY transformations acting on the fermion

$$\begin{aligned}(\delta_{\epsilon_2}\delta_{\epsilon_1} - \delta_{\epsilon_1}\delta_{\epsilon_2})\psi_\alpha \quad &= \quad -i(\epsilon_1\sigma^\mu\epsilon_2^\dagger - \epsilon_2\sigma^\mu\epsilon_1^\dagger)\,\partial_\mu\psi_\alpha \\ &\quad\quad +i(\epsilon_{1\alpha}\,\epsilon_2^\dagger\overline{\sigma}^\mu\partial_\mu\psi - \epsilon_{2\alpha}\,\epsilon_1^\dagger\overline{\sigma}^\mu\partial_\mu\psi) \\ &\quad\quad +\delta_{\epsilon_2}\epsilon_{1\alpha}\mathcal{F} - \delta_{\epsilon_1}\epsilon_{2\alpha}\mathcal{F}\end{aligned}$$

$$\delta_{\epsilon_2}\epsilon_{1\alpha}\mathcal{F} - \delta_{\epsilon_1}\epsilon_{2\alpha}\mathcal{F} = \epsilon_{1\alpha}(-i\epsilon_2^\dagger\overline{\sigma}^\mu\partial_\mu\psi) - \epsilon_{2\alpha}(-i\epsilon_1^\dagger\overline{\sigma}^\mu\partial_\mu\psi)$$

$$(\delta_{\epsilon_2}\delta_{\epsilon_1} - \delta_{\epsilon_1}\delta_{\epsilon_2})\psi_\alpha \quad = \quad -i(\epsilon_1\sigma^\mu\epsilon_2^\dagger - \epsilon_2\sigma^\mu\epsilon_1^\dagger)\,\partial_\mu\psi_\alpha$$

SUSY algebra closes for off-shell fermions

Commutator acting on the auxiliary field

$$
\begin{aligned}
(\delta_{\epsilon_2}\delta_{\epsilon_1} - \delta_{\epsilon_1}\delta_{\epsilon_2})\mathcal{F} &= \delta_{\epsilon_2}(-i\epsilon_1^\dagger \overline{\sigma}^\mu \partial_\mu \psi) - \delta_{\epsilon_1}(-i\epsilon_2^\dagger \overline{\sigma}^\mu \partial_\mu \psi) \\
&= -i\epsilon_1^\dagger \overline{\sigma}^\mu \partial_\mu(-i\sigma^\nu \epsilon_2^\dagger\, \partial_\nu \phi + \epsilon_2 \mathcal{F}) \\
&\quad +i\epsilon_2^\dagger \overline{\sigma}^\mu \partial_\mu(-i\sigma^\nu \epsilon_1^\dagger\, \partial_\nu \phi + \epsilon_1 \mathcal{F}) \\
&= -i(\epsilon_1 \sigma^\mu \epsilon_2^\dagger - \epsilon_2 \sigma^\mu \epsilon_1^\dagger)\, \partial_\mu \mathcal{F} \\
&\quad -\epsilon_1^\dagger \overline{\sigma}^\mu \sigma^\nu \epsilon_2^\dagger\, \partial_\mu \partial_\nu \phi + \epsilon_2^\dagger \overline{\sigma}^\mu \sigma^\nu \epsilon_1^\dagger\, \partial_\mu \partial_\nu \phi
\end{aligned}
$$

Thus for

$$X = \phi, \phi^*, \psi, \psi^\dagger, \mathcal{F}, \mathcal{F}^*$$

$$(\delta_{\epsilon_2}\delta_{\epsilon_1} - \delta_{\epsilon_1}\delta_{\epsilon_2})X = -i(\epsilon_1 \sigma^\mu \epsilon_2^\dagger - \epsilon_2 \sigma^\mu \epsilon_1^\dagger)\, \partial_\mu X$$

Noether

Noether's Theorem

Noether theorem:
corresponding to every continuous symmetry is a conserved current.
infinitesimal symmetry $(1+\epsilon T)X = X + \delta X$

$$\delta\mathcal{L} = \mathcal{L}(X+\delta X) - \mathcal{L}(X) = \partial_\mu V^\mu$$

EOM:

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu X)} \right) = \frac{\partial \mathcal{L}}{\partial X},$$

$$\begin{aligned} \partial_\mu V^\mu = \delta\mathcal{L} &= \frac{\partial \mathcal{L}}{\partial X}\delta X + \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu X)} \right) \delta(\partial_\mu X) \\ &= \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu X)} \right) \delta X + \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu X)} \right) \partial_\mu \delta X \\ &= \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu X)} \delta X \right) \end{aligned}$$

$$\epsilon \partial_\mu J^\mu = \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu X)} \delta X - V^\mu \right)$$

Conserved SuperCurrent

conserved supercurrent, J^μ_α:

$$\epsilon J^\mu + \epsilon^\dagger J^{\dagger\mu} \equiv \frac{\partial \mathcal{L}}{\partial(\partial_\mu X)}\,\delta X - V^\mu$$

$$\epsilon J^\mu + \epsilon^\dagger J^{\dagger\mu} = \delta\phi\partial^\mu\phi^* + \delta\phi^*\partial^\mu\phi + i\psi^\dagger\overline{\sigma}^\mu\delta\psi - V^\mu$$

$$\begin{aligned}
\epsilon J^\mu + \epsilon^\dagger J^{\dagger\mu} &= \epsilon\psi\partial^\mu\phi^* + \epsilon^\dagger\psi^\dagger\partial^\mu\phi + i\psi^\dagger\overline{\sigma}^\mu(-i\sigma^\nu\epsilon^\dagger\,\partial_\nu\phi + \epsilon\mathcal{F}) \\
&\quad -\epsilon\sigma^\mu\overline{\sigma}^\nu\psi\,\partial_\nu\phi^* + \epsilon\psi\,\partial^\mu\phi^* - \epsilon^\dagger\psi^\dagger\,\partial^\mu\phi - i\psi^\dagger\overline{\sigma}^\mu\epsilon\mathcal{F} \\
&= 2\epsilon\psi\partial^\mu\phi^* + \psi^\dagger\overline{\sigma}^\mu\sigma^\nu\epsilon^\dagger\,\partial_\nu\phi - \epsilon\sigma^\mu\overline{\sigma}^\nu\psi\,\partial_\nu\phi^*
\end{aligned}$$

Using the Pauli identity:

$$J^\mu_\alpha = (\sigma^\nu \overline{\sigma}^\mu \psi)_\alpha \, \partial_\nu \phi^*, \quad J^{\dagger\mu}{}_{\dot\alpha} = (\psi^\dagger \overline{\sigma}^\mu \sigma^\nu)_{\dot\alpha} \, \partial_\nu \phi.$$

conserved supercharges:

$$Q_\alpha = \sqrt{2} \int d^3x \, J^0_\alpha, \quad Q^\dagger{}_{\dot\alpha} = \sqrt{2} \int d^3x \, J^{\dagger 0}{}_{\dot\alpha}$$

generate SUSY transformations

$$\left[\epsilon Q + \epsilon^\dagger Q^\dagger, X\right] = -i\sqrt{2}\, \delta X$$

Commutators of the supercharges acting on fields give:

$$\left[\epsilon_2 Q + \epsilon_2^\dagger Q^\dagger,\, \left[\epsilon_1 Q + \epsilon_1^\dagger Q^\dagger,\, X\right]\right] - \left[\epsilon_1 Q + \epsilon_1^\dagger Q^\dagger,\, \left[\epsilon_2 Q + \epsilon_2^\dagger Q^\dagger,\, X\right]\right]$$
$$= 2(\epsilon_2 \sigma^\mu \epsilon_1^\dagger - \epsilon_1 \sigma^\mu \epsilon_2^\dagger)\, i\partial_\mu X$$

$$\left[\left[\epsilon_2 Q + \epsilon_2^\dagger Q^\dagger,\, \epsilon_1 Q + \epsilon_1^\dagger Q^\dagger\right],\, X\right] = 2(\epsilon_2 \sigma^\mu \epsilon_1^\dagger - \epsilon_1 \sigma^\mu \epsilon_2^\dagger)\, [P_\mu, X]$$

Since this is true for any X, we have

$$\left[\epsilon_2 Q + \epsilon_2^\dagger Q^\dagger,\, \epsilon_1 Q + \epsilon_1^\dagger Q^\dagger\right] = 2(\epsilon_2 \sigma^\mu \epsilon_1^\dagger - \epsilon_1 \sigma^\mu \epsilon_2^\dagger)\, P_\mu$$

Since ϵ_1 and ϵ_2 are arbitrary, we have

$$\begin{aligned}
\left[\epsilon_2 Q, \epsilon_1^\dagger Q^\dagger\right] &= 2\epsilon_2 \sigma^\mu \epsilon_1^\dagger P_\mu \\
\left[\epsilon_2^\dagger Q, \epsilon_1 Q^\dagger\right] &= -2\epsilon_2 \sigma^\mu \epsilon_1^\dagger P_\mu \\
\left[\epsilon_2 Q, \epsilon_1 Q\right] &= \left[\epsilon_2^\dagger Q^\dagger, \epsilon_1^\dagger Q^\dagger\right] = 0
\end{aligned}$$

Extracting the arbitrary ϵ_1 and ϵ_2:

$$\begin{aligned}
&\{Q_\alpha, {Q^\dagger}_{\dot\alpha}\} = 2\sigma^\mu_{\alpha\dot\alpha} P_\mu, \\
&\{Q_\alpha, Q_\beta\} = \{{Q^\dagger}_{\dot\alpha}, {Q^\dagger}_{\dot\beta}\} = 0
\end{aligned}$$

which is just the SUSY algebra

The interacting Wess–Zumino model

$$\mathcal{L}_{\text{free}} = \partial^\mu \phi^{*j} \partial_\mu \phi_j + i\psi^{\dagger j} \overline{\sigma}^\mu \partial_\mu \psi_j + \mathcal{F}^{*j} \mathcal{F}_j$$

$$\begin{aligned} \delta\phi_j &= \epsilon\psi_j & \delta\phi^{*j} &= \epsilon^\dagger \psi^{\dagger j} \\ \delta\psi_{j\alpha} &= -i(\sigma^\mu \epsilon^\dagger)_\alpha \, \partial_\mu \phi_j + \epsilon_\alpha \mathcal{F}_j & \delta\psi^{\dagger j}_{\dot\alpha} &= i(\epsilon\sigma^\mu)_{\dot\alpha} \, \partial_\mu \phi^{*j} + \epsilon^\dagger_{\dot\alpha} \mathcal{F}^{*j} \\ \delta\mathcal{F}_j &= -i\epsilon^\dagger \overline{\sigma}^\mu \partial_\mu \psi_j & \delta\mathcal{F}^{*j} &= i\partial_\mu \psi^{\dagger j} \overline{\sigma}^\mu \epsilon \end{aligned}$$

most general set of renormalizable interactions:

$$\mathcal{L}_{\text{int}} = -\tfrac{1}{2} W^{jk} \psi_j \psi_k + W^j \mathcal{F}_j + h.c.,$$

$\psi_j \psi_k = \psi_j^\alpha \epsilon_{\alpha\beta} \psi_k^\beta$ is symmetric under $j \leftrightarrow k$, $\Rightarrow W^{jk}$
potential $U(\phi_j, \phi^{*j})$ breaks SUSY, since a SUSY transformation gives

$$\delta U = \tfrac{\partial U}{\partial \phi_j} \epsilon\psi_j + \tfrac{\partial U}{\partial \phi^{*j}} \epsilon^\dagger \psi^{\dagger j}$$

which is linear in ψ_j and $\psi^{\dagger j}$ with no derivatives or $\mathcal{F}$ dependence and cannot be canceled by any other term in $\delta\mathcal{L}_{\text{int}}$

require SUSY

$$\delta\mathcal{L}_{\rm int}|_{4-\rm spinor} \;=\; -\tfrac{1}{2}\frac{\partial W^{jk}}{\partial\phi_n}(\epsilon\psi_n)(\psi_j\psi_k) - \tfrac{1}{2}\frac{\partial W^{jk}}{\partial\phi^{*n}}(\epsilon^\dagger\psi^{\dagger n})(\psi_j\psi_k) + h.c.$$

Fierz identity $\Rightarrow$

$$(\epsilon\psi_j)(\psi_k\psi_n) + (\epsilon\psi_k)(\psi_n\psi_j) + (\epsilon\psi_n)(\psi_j\psi_k) = 0,$$

$\delta\mathcal{L}_{\rm int}|_{4-\rm spinor}$ vanishes iff $\partial W^{jk}/\partial\phi_n$ is totally symmetric under the interchange of j, k, n. We also need

$$\frac{\partial W^{jk}}{\partial\phi^{*n}} = 0$$

so W^{jk} is *analytic* (*holomorphic*)
define superpotential W:

$$W^{jk} = \frac{\partial^2}{\partial\phi_j\partial\phi_k}W$$

for renormalizable interactions

$$W = E^j \phi_j + \tfrac{1}{2} M^{jk} \phi_j \phi_k + \tfrac{1}{6} y^{jkn} \phi_j \phi_k \phi_n$$

and M^{jk}, y^{jkn} are are symmetric under interchange of indices.
take $E^j = 0$ so SUSY is unbroken

$$\delta \mathcal{L}_{\rm int}|_\partial \;\; = \;\; -iW^{jk} \partial_\mu \phi_k \, \psi_j \sigma^\mu \epsilon^\dagger - iW^j \, \partial_\mu \psi_j \sigma^\mu \epsilon^\dagger + h.c.$$

$$W^{jk} \partial_\mu \phi_k = \partial_\mu \left(\tfrac{\partial W}{\partial \phi_j} \right)$$

so $\delta \mathcal{L}_{\rm int}|_\partial$ will be a total derivative iff

$$W^j = \tfrac{\partial W}{\partial \phi_j}$$

remaining terms:

$$\delta \mathcal{L}_{\rm int}|_{\mathcal{F},\mathcal{F}^*} = -W^{jk} \mathcal{F}_j \epsilon \psi_k + \tfrac{\partial W^j}{\partial \phi_k} \epsilon \psi_k \mathcal{F}_j$$

identically cancel if previous conditions are satisfied

proof did not rely on the functional form of W, only that it was holomorphic

integrate out auxillary fields

action is quadratic in $\mathcal{F}$

$$\mathcal{L}_{\mathcal{F}} = \mathcal{F}_j \mathcal{F}^{*j} + W^j \mathcal{F}_j + W_j^* \mathcal{F}^{*j}$$

perform the corresponding Gaussian path integral exactly by solving its algebraic equation of motion:

$$\mathcal{F}_j = -W_j^*, \;\; \mathcal{F}^{*j} = -W^j$$

without auxiliary fields SUSY transformation ψ would be different for each choice of W

plugging in to $\mathcal{L}$:

$$\begin{aligned} \mathcal{L} &= \partial^\mu \phi^{*j} \partial_\mu \phi_j + i\psi^{\dagger j} \overline{\sigma}^\mu \partial_\mu \psi_j \\ &\quad -\tfrac{1}{2}\left(W^{jk}\psi_j\psi_k + W^{*jk}\psi^{\dagger j}\psi^{\dagger k}\right) - W^j W_j^* \end{aligned}$$

WZ Lagrangian

$$V(\phi,\phi^*) = W^j W^*_j = \mathcal{F}_j \mathcal{F}^{*j} = M^*_{jn} M^{nk} \phi^{*j} \phi_k$$
$$+\tfrac{1}{2} M^{jm} y^*_{knm} \phi_j \phi^{*k} \phi^{*n} + \tfrac{1}{2} M^*_{jm} y^{knm} \phi^{*j} \phi_k \phi_n + \tfrac{1}{4} y^{jkm} y^*_{npm} \phi_j \phi_k \phi^{*n} \phi^{*p}$$

as required by SUSY:

$$V(\phi,\phi^*) \geq 0$$

interacting Wess–Zumino model:

$$\begin{aligned}\mathcal{L}_{\mathrm{WZ}} \quad = \quad & \partial^\mu \phi^{*j} \partial_\mu \phi_j + i\psi^{\dagger j} \overline{\sigma}^\mu \partial_\mu \psi_j \\ & -\tfrac{1}{2} M^{jk} \psi_j \psi_k - \tfrac{1}{2} M^*_{jk} \psi^{\dagger j} \psi^{\dagger k} - V(\phi,\phi^*) \\ & -\tfrac{1}{2} y^{jkn} \phi_j \psi_k \psi_n - \tfrac{1}{2} y^*_{jkn} \phi^{*j} \psi^{\dagger k} \psi^{\dagger n}.\end{aligned}$$

quartic coupling is $|y|^2$ as required to cancel the Λ^2 divergence in ϕ mass

$|\text{cubic coupling}|^2 \propto$ quartic coupling $\times |M|^2$ as required to cancel the $\log \Lambda$ divergence

linearized equations of motion

$$\begin{aligned}\partial^\mu \partial_\mu \phi_j &= -M^*_{jn} M^{nk} \phi_k + \ldots ; \\ i\overline{\sigma}^\mu \partial_\mu \psi_j &= M^*_{jk} \psi^{\dagger k} + \ldots ; \\ i\sigma^\mu \partial_\mu \psi^{\dagger j} &= M^{jk} \psi_k + \ldots\end{aligned}$$

Multiplying ψ eqns by $i\sigma^\nu \partial_\nu$, and $i\overline{\sigma}^\nu \partial_\nu$, and using the Pauli identity we obtain

$$\begin{aligned}\partial^\mu \partial_\mu \psi_j &= -M^*_{jn} M^{nk} \psi_k + \ldots ; \\ \partial^\mu \partial_\mu \psi^{\dagger k} &= -\psi^{\dagger j} M^*_{jn} M^{nk} + \ldots\end{aligned}$$

scalars and fermions have the same mass eigenvalues, as required by SUSY

diagonalizing gives a collection of massive chiral supermultiplets.

$\mathcal{N} = 0$ SUSY

Weisskopf

chiral symmetry $\Rightarrow$ multiplicative mass renormalization

$$m_f = m_0 + c_f \, \frac{\alpha}{16\pi^2} m_0 \ln\left(\frac{\Lambda}{m_0}\right)$$

where Λ is the cutoff

SUSY ensures that the scalar mass is given by the same formula

SUSY dim.less couplings $\Rightarrow$ no Λ^2 divergences

SUSY must be broken in the real world, eg.

$$W = E^a \phi_a$$

gives a scalar potential

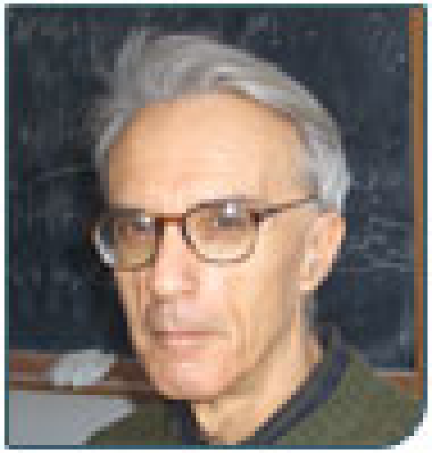
$$V = W_a^* W^a = E^a E_a^* \neq 0$$

which breaks SUSY.

We want to break SUSY such that Higgs – top squark quartic coupling $\lambda = |y_t|^2$. If not we reintroduce a Λ^2 divergence in the Higgs mass:

$$\delta m_h^2 \propto (\lambda - |y_t|^2)\Lambda^2$$

Effective Theory

Grisaru, Girardello

We want an *effective theory* of broken SUSY with only soft breaking terms (operators with dimension < 4). Girardello and Grisaru found:

$$\begin{aligned}\mathcal{L}_{\text{soft}} &= -\tfrac{1}{2}(M_\lambda \lambda^a \lambda^a + h.c.) - (m^2)^i_j \phi^{*j}\phi_i \\ &\quad -(\tfrac{1}{2} b^{ij}\phi_i\phi_j + \tfrac{1}{6} a^{ijk}\phi_i\phi_j\phi_k + h.c.) \\ &\quad -\tfrac{1}{2} c_i^{jk}\phi^{i*}\phi_j\phi_k + e^i\phi_i + h.c.\end{aligned}$$

$e^i\phi_i$ is only allowed if ϕ_i is a gauge singlet The c_i^{jk} term may introduce quadratic divergences if there is a gauge singlet multiplet in the model.

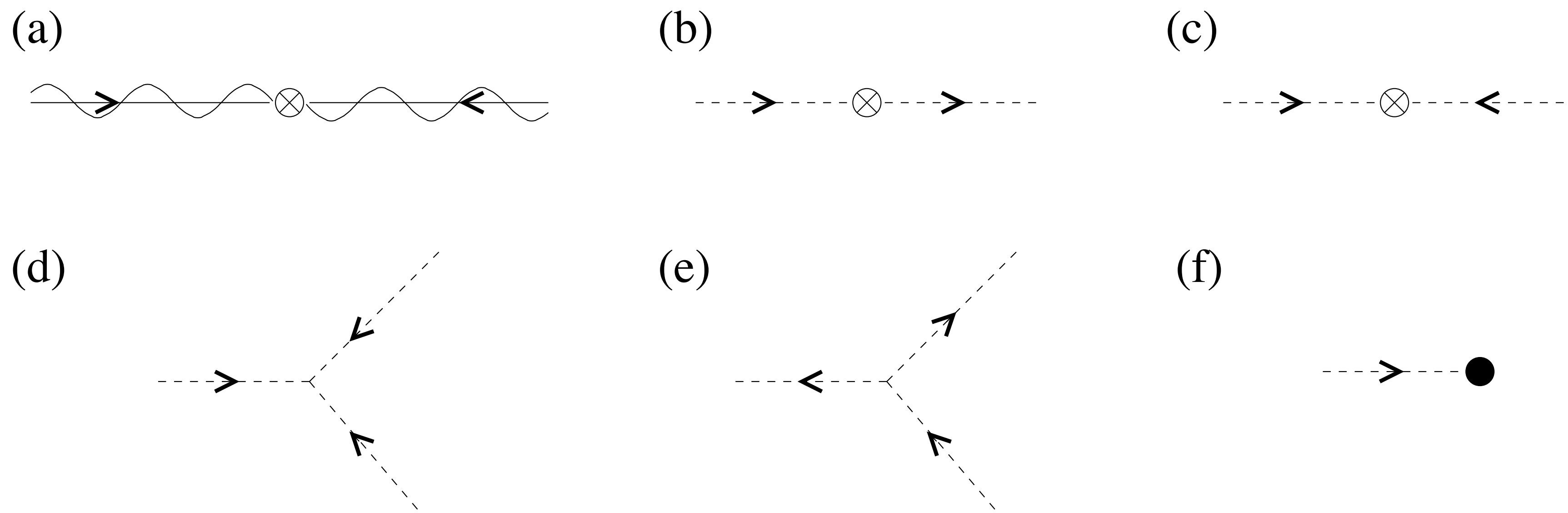


Figure 1:

Additional soft SUSY breaking interactions: (a) gaugino mass M_λ, (b) non-holomorphic mass m^2, (c) holomorphic mass b^{ij}, (d) holomorphic trilinear coupling a^{ijk}, (e) non-holomorphic trilinear coupling c_i^{jk}, and (f) tadpole e^i.