EFFECTIVE FIELD THEORY FOR BSM

Roberto Contino
Scuola Normale Superiore, Pisa
INFN, Pisa

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SMEFT Lagrangian
Effective Lagrangian for a Higgs doublet

\[ \mathcal{L} = \mathcal{L}_{SM} + \sum_i \bar{c}_i O_i \equiv \mathcal{L}_{SM} + \Delta \mathcal{L}_{SILH} + \Delta \mathcal{L}_{cc} + \Delta \mathcal{L}_{dipole} + \Delta \mathcal{L}_{V} + + \Delta \mathcal{L}_{4\psi} \]

16 operators
(12 CP even, 4 CP odd)

\[ \Delta \mathcal{L}_{SILH} = \frac{\bar{c}_H}{2v^2} \partial^\mu (H^+ H) \partial_\mu (H^+ H) + \frac{\bar{c}_T}{2v^2} \left( H^+ \mathcal{T} H \right) \left( H^+ \mathcal{T} H \right) - \frac{\bar{c}_6 \lambda}{v^2} \left( H^+ H \right)^3 \]

\[ + \left( \frac{\bar{c}_u}{v^2} y_u H^+ H \bar{q}_L H^c u_R + \frac{\bar{c}_d}{v^2} y_d H^+ H \bar{q}_L H d_R + \frac{\bar{c}_l}{v^2} \bar{y}_l H^+ H \bar{L}_L H l_R + h.c. \right) \]

\[ + \frac{i\bar{c}_W \cdot g}{2m_W^2} \left( H^+ \sigma^i \mathcal{T} H \right) \left( D^{\nu} W_{\mu\nu} \right)^i + \frac{i\bar{c}_B \cdot g'}{2m_W^2} \left( H^+ \mathcal{T} H \right) \left( \partial^\nu B_{\mu\nu} \right) \]

\[ + \frac{i\bar{c}_{HW} \cdot g}{m_W^2} \left( D^\mu H \right)^{\dagger} \sigma^i \left( D^{\nu} H \right) W_{\mu\nu}^i + \frac{i\bar{c}_{HB} \cdot g'}{m_W^2} \left( D^\mu H \right)^{\dagger} \left( D^{\nu} H \right) B_{\mu\nu} \]

\[ + \frac{\bar{c}_g}{2m_W^2} \left( H^+ H B_{\mu\nu} B^{\mu\nu} + \frac{\bar{c}_g^2}{m_W^2} H^+ H G_{\mu\nu}^{aG_{\mu\nu}} \right) \]

\[ + \frac{i\bar{c}_{HW} \cdot g}{m_W^2} \left( D^\mu H \right)^{\dagger} \sigma^i \left( D^{\nu} H \right) \tilde{W}_{\mu\nu}^i + \frac{i\bar{c}_{HB} \cdot g'}{m_W^2} \left( D^\mu H \right)^{\dagger} \left( D^{\nu} H \right) \tilde{B}_{\mu\nu} \]

\[ + \frac{\tilde{c}_g \cdot g^2}{m_W^2} \left( H^+ H B_{\mu\nu} B^{\mu\nu} + \frac{\tilde{c}_g^2}{m_W^2} H^+ H G_{\mu\nu}^{aG_{\mu\nu}} \right) \]
Effective Lagrangian for a Higgs doublet

\[ \mathcal{L} = \mathcal{L}_{SM} + \sum_i \bar{c}_i O_i \equiv \mathcal{L}_{SM} + \Delta \mathcal{L}_{SILH} + \Delta \mathcal{L}_{cc} + \Delta \mathcal{L}_{dipole} + \Delta \mathcal{L}_V + \Delta \mathcal{L}_4\psi \]

6 current-current operators

\[ \Delta \mathcal{L}_{cc} = \frac{i\tilde{c}_H q}{v^2} (\bar{q}_L \gamma^\mu q_L) (H^\dagger \overleftrightarrow{D}_\mu H) + \frac{i\tilde{c}'_H q}{v^2} (\bar{q}_L \gamma^\mu \sigma^i q_L) (H^\dagger \sigma^i \overleftrightarrow{D}_\mu H) \]

\[ + \frac{i\tilde{c}_H u}{v^2} (\bar{u}_R \gamma^\mu u_R) (H^\dagger \overleftrightarrow{D}_\mu H) + \frac{i\tilde{c}_H d}{v^2} (\bar{d}_R \gamma^\mu d_R) (H^\dagger \overleftrightarrow{D}_\mu H) \]

\[ + \left( \frac{i\tilde{c}_{Hud}}{v^2} (\bar{u}_R \gamma^\mu d_R) (H^c \overleftrightarrow{D}_\mu H) + h.c. \right) \]

\[ + \frac{i\tilde{c}_H L}{v^2} (\bar{L}_L \gamma^\mu L_L) (H^\dagger \overleftrightarrow{D}_\mu H) \]
Effective Lagrangian for a Higgs doublet

\[ \mathcal{L} = \mathcal{L}_{SM} + \sum_i \tilde{c}_i O_i \equiv \mathcal{L}_{SM} + \Delta \mathcal{L}_{SILH} + \Delta \mathcal{L}_{cc} + \Delta \mathcal{L}_{\text{dipole}} + \Delta \mathcal{L}_V + \Delta \mathcal{L}_{4\psi} \]

8 dipole operators

\[ \Delta \mathcal{L}_{\text{dipole}} = \frac{\tilde{c}_{uB} g'}{m_W^2} y_u \bar{q}_L H^c \sigma^{\mu\nu} u_R B_{\mu\nu} + \frac{\tilde{c}_{uW} g}{m_W^2} y_u \bar{q}_L \sigma^i H^c \sigma^{\mu\nu} u_R W^i_{\mu\nu} + \frac{\tilde{c}_{uG} g S}{m_W^2} y_u \bar{q}_L H^c \sigma^{\mu\nu} \lambda^a u_R G^a_{\mu\nu} \]

\[ + \frac{\tilde{c}_{dB} g'}{m_W^2} y_d \bar{q}_L \sigma^{\mu\nu} d_R B_{\mu\nu} + \frac{\tilde{c}_{dW} g}{m_W^2} y_d \bar{q}_L \sigma^i H \sigma^{\mu\nu} d_R W^i_{\mu\nu} + \frac{\tilde{c}_{dG} g S}{m_W^2} y_d \bar{q}_L H \sigma^{\mu\nu} \lambda^a d_R G^a_{\mu\nu} \]

\[ + \frac{\tilde{c}_{lB} g'}{m_W^2} y_l \bar{L}_L \sigma^{\mu\nu} l_R B_{\mu\nu} + \frac{\tilde{c}_{lW} g}{m_W^2} y_l \bar{L}_L \sigma^i H \sigma^{\mu\nu} l_R W^i_{\mu\nu} + h.c. \]
Effective Lagrangian for a Higgs doublet

\[ \mathcal{L} = \mathcal{L}_{SM} + \sum_i \bar{c}_i O_i \equiv \mathcal{L}_{SM} + \Delta \mathcal{L}_{SILH} + \Delta \mathcal{L}_{cc} + \Delta \mathcal{L}_{dipole} + \Delta \mathcal{L}_V + \Delta \mathcal{L}_{4\psi} \]

7 operators built with gauge fields only
(5 CP even, 2 CP odd)

\[ \Delta \mathcal{L}_V = \bar{c}_2 W \frac{g^3}{m_W^2} (D^\mu W_{\mu\nu})^i (D^\rho W^{\rho\nu})^i + \bar{c}_2 B \frac{g}{m_W^2} (\partial^\mu B_{\mu\nu}) (\partial^\rho B^{\rho\nu}) + \bar{c}_2 G \frac{g^3}{m_W^2} (D^\mu G_{\mu\nu})^a (D^\rho G^{\rho\nu})^a \]

For a review see: RC, Ghezzi, Grojean, Muhlleitner, Spira JHEP 07 (2013) 035

In total: 59 dim-6 operators for 1 SM family
Naive estimate at the matching scale $m_*$ (SILH power counting):

$$
\bar{c}_H, \bar{c}_T, \bar{c}_6, \bar{c}_\psi \sim O\left(\frac{v^2}{f^2}\right), \quad \bar{c}_W, \bar{c}_B \sim O\left(\frac{m_W^2}{m_*}\right), \quad \bar{c}_{HW}, \bar{c}_{HB}, \bar{c}_\gamma, \bar{c}_g \sim O\left(\frac{m_W^2}{16\pi^2 f^2}\right)
$$

$$
\bar{c}_{H\psi}, \bar{c}'_{H\psi} \sim O\left(\frac{\lambda_\psi^2 v^2}{g^2_* f^2}\right), \quad \bar{c}_{H ud} \sim O\left(\frac{\lambda_u \lambda_d v^2}{g^2_* f^2}\right), \quad \bar{c}_{\psi W}, \bar{c}_{\psi B}, \bar{c}_{\psi G} \sim O\left(\frac{m_W^2}{16\pi^2 f^2}\right)
$$

where $f \equiv m_*/g_*$. 

Processes with 0, 1, 2, ... Higgses related

Q: Which operators are already constrained by experiments w/o Higgs?

In total: 59 dim-6 operators

17 involve the Higgs

8 affect Higgs physics only

Elias-Miro, Espinosa, Masso, Pomarol
JHEP 1311 (2013) 066

Pomarol, Riva JHEP 01 (2014) 151
Operators that affect Higgs physics only

<table>
<thead>
<tr>
<th>Operator</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O_H = (\partial_\mu</td>
<td>H</td>
</tr>
<tr>
<td>$O_{BB} = g'^2</td>
<td>H</td>
</tr>
<tr>
<td>$O_{WW} = g^2</td>
<td>H</td>
</tr>
<tr>
<td>$O_{GG} = g_s^2</td>
<td>H</td>
</tr>
<tr>
<td>$O_{yd} = y_d</td>
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</tr>
<tr>
<td>$O_{yu} = y_u</td>
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</tr>
<tr>
<td>$O_{ye} = y_e</td>
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</tr>
<tr>
<td>$O_6 = \lambda</td>
<td>H</td>
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Elias-Miro, Espinosa, Masso, Pomarol
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Pomarol, Riva JHEP 01 (2014) 151
Operators that affect Higgs physics only

\[ O_H = \left( \partial_\mu |H|^2 \right)^2 \]

\[ O_{BB} = g' \left( |H|^2 B_{\mu\nu} B^{\mu\nu} \right) \]

\[ O_{WW} = g \left( |H|^2 W_{\mu\nu} W^{\mu\nu} \right) \]

\[ O_{GG} = g_s^2 \left( |H|^2 G_{\mu\nu} G^{\mu\nu} \right) \]

\[ O_{y_d} = y_d |H|^2 \bar{q}_L H d_R \]

\[ O_{y_u} = y_u |H|^2 \bar{q}_L \tilde{H} u_R \]

\[ O_{y_e} = y_e |H|^2 \bar{L}_L H e_R \]

\[ O_6 = \lambda |H|^6 \]
Operators that affect Higgs physics only

\[
O_H = (\partial_\mu |H|^2)^2
\]

\[
O_{BB} = g^f 2 |H|^2 B_{\mu \nu} B^{\mu \nu}
\]

\[
O_{WW} = g^2 |H|^2 W_{\mu \nu} W^{\mu \nu}
\]

\[
O_{GG} = g_s^2 |H|^2 G_{\mu \nu} G^{\mu \nu}
\]

\[
O_{yd} = y_d |H|^2 \bar{q}_L H d_R
\]

\[
O_{yu} = y_u |H|^2 \bar{q}_L \tilde{H} u_R
\]

\[
O_{ye} = y_e |H|^2 \bar{L}_L H e_R
\]

\[
O_6 = \lambda |H|^6
\]

modify also differential rates, can be probed by:

- decays \( h \rightarrow WW^* \), \( h \rightarrow ZZ^* \) (angular distributions)

- Higgs associated production \( hV \) (Higgs \( p_T \), \( m_{Vh} \), and angular distributions)

- single-Higgs production via VBF
Operators that affect Higgs physics only

\begin{align*}
O_H &= (\partial_\mu |H|^2)^2 \\
O_{BB} &= g' \, |H|^2 B_{\mu\nu} B^{\mu\nu} \\
O_{WW} &= g^2 \, |H|^2 W_{\mu\nu} W^{\mu\nu} \\
O_{GG} &= g_s^2 \, |H|^2 G_{\mu\nu} G^{\mu\nu} \\
O_{yd} &= y_d \, |H|^2 \bar{q}_L H d_R \\
O_{yu} &= y_u \, |H|^2 \bar{q}_L \tilde{H} u_R \\
O_{ye} &= y_e \, |H|^2 \bar{L}_L H e_R \\
O_6 &= \lambda \, |H|^6
\end{align*}

modifies $p_T$ spectrum of $gg \to h + \text{jet}$

[ top loop vs point-like interaction ]
Operators that affect Higgs physics only

\[ O_H = (\partial_\mu |H|^2)^2 \]
\[ O_{BB} = g' B^2 |H|^2 B_{\mu\nu} B^{\mu\nu} \]
\[ O_{WW} = g^2 W^\mu W_{\mu\nu} W^{\mu\nu} \]
\[ O_{GG} = g_s^2 G^\mu G_{\mu\nu} G^{\mu\nu} \]
\[ O_{yd} = y_d |H|^2 \bar{q}_L H d_R \]
\[ O_{yu} = y_u |H|^2 \bar{q}_L \tilde{H} u_R \]
\[ O_{ye} = y_e |H|^2 \bar{L}_L H e_R \]
\[ O_6 = \lambda |H|^6 \]

\( gg \to hh \) yet un-probed
Renormalization of EFT
Loops of light (SM) particles induce the RG flow (and mixing) of the coefficients $\bar{c}_i$

$$\bar{c}_i(\mu) = \left( \delta_{ij} + \gamma_{ij}^{(0)} \frac{\alpha_{SM}(\mu)}{4\pi} \log \frac{\mu}{m_*} \right) \bar{c}_j(m_*)$$

No big hierarchy between $m_*$ and EW scale, 1-loop corrections to SMEFT are generally small
Do we need to go beyond tree level?

- The bulk of the 1-loop effect (RG running) can be effectively included by setting limits on the value of the coefficients at the low-energy scale.

- Knowledge of the RG running is however needed when it comes to make assumptions on the coefficients at the scale $m_*$ (ex: to simplify the analysis by neglecting some of the operators).

1-loop effects important if:

[in setting limits]
Some loosely bound coefficients appears in a precisely measured observable at 1-loop level

Ex: $\bar{c}_t$ in $gg \rightarrow h$

[in constraining physics at $m_*$]
A larger coefficient renormalizes a smaller one (for a given power counting). RG effects can be sizeable if UV dynamics is strongly coupled.
RG evolution of coefficients

• In case of strong dynamics, leading effects come from loops of composite particles (i.e. Higgs, top quarks, ...)

Examples:

1. Running of $\bar{c}_{W+B}$

$$O_{W+B} = \frac{ig}{2m_W^2} D^\nu W_{\mu\nu}^i (H^\dagger \sigma^i \overleftrightarrow{D}^\mu H) + \frac{ig'}{2m_W^2} \partial^\nu B_{\mu\nu} (H^\dagger \overleftrightarrow{D}^\mu H)$$

$$\bar{c}_{W+B}(\mu) = \bar{c}_{W+B}(m_*) - \frac{1}{6} \frac{\alpha_2}{4\pi} \log \left( \frac{\mu}{m_*} \right) \bar{c}_H(m_*)$$

$$\bar{c}_W(m_*), c_B(m_*) \sim \frac{m_W^2}{m_*^2}$$

$$\bar{c}_H(m_*) \sim \frac{\nu^2 g_*^2}{m_*^2} = \frac{m_W^2 g_*^2}{m_*^2 g^2}$$

$$\frac{\Delta \bar{c}_{W+B}}{\bar{c}_{W+B}} \sim \frac{g_*^2}{16\pi^2} \log \left( \frac{m_*}{\mu} \right)$$

1-loop correction can be large if the UV dynamics is strongly-interacting ( $g_*$ large)
RG evolution of coefficients

- In case of strong dynamics, leading effects come from loops of composite particles (i.e. Higgs, top quarks, ...)

Examples:

2. Running of $\bar{c}_T$

$$O_T = \frac{1}{2v^2} |H^\dagger D_\nu H|^2$$

$$\bar{c}_T(\mu) = \bar{c}_T(m_\ast) + \frac{3}{2} \tan^2 \theta_W \frac{\alpha_2}{4\pi} \log \left( \frac{\mu}{M} \right) \bar{c}_H(m_\ast)$$

$$\bar{c}_T(m_Z) \approx \frac{v^2}{f^2} \times \frac{g'^2}{16\pi^2} \log \left( \frac{m_\ast}{m_Z} \right)$$

Small but leading effect if $\bar{c}_T(m_\ast) = 0$ due to custodial invariance
Fit to effective coefficients
EFT fit to experimental data

Two approaches:

- **Possible effective strategy:**
  Pomarol, Riva  JHEP 1401 (2014) 151

  Organize data (and group operators) according to how strongly they constrain the effective coefficients

<table>
<thead>
<tr>
<th>observables</th>
<th>precision</th>
</tr>
</thead>
<tbody>
<tr>
<td>input observables ($G_F$, $\alpha_{em}$, $m_Z$), EDMs, (g-2)</td>
<td>better than $10^{-3}$</td>
</tr>
<tr>
<td>Z-pole observables at LEP1, W mass</td>
<td>$10^{-3}$</td>
</tr>
<tr>
<td>TGC (LEP2)</td>
<td>$10^{-2}$</td>
</tr>
<tr>
<td>Higgs physics (LHC)</td>
<td>$10^{-1}$</td>
</tr>
</tbody>
</table>

- **Global fit:**
  Ellis, Murphy, Sanz, You  arXiv:1803.0352
  De Blas et al. arXiv:1710.05402

  More appropriate as LHC data becomes more and more sensitive
Results:

from: Ellis, Murphy, Sanz, You
arXiv:1803.0352

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Central value</th>
<th>1-σ</th>
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</thead>
<tbody>
<tr>
<td>$c_{3G}$</td>
<td>0.005</td>
<td>0.003</td>
</tr>
<tr>
<td>$c_{3W}$</td>
<td>-0.018</td>
<td>0.023</td>
</tr>
<tr>
<td>$c_{d}$</td>
<td>0.36</td>
<td>0.15</td>
</tr>
<tr>
<td>$c_{e}$</td>
<td>0.09</td>
<td>0.11</td>
</tr>
<tr>
<td>$c_{q}$</td>
<td>0.00002</td>
<td>0.00002</td>
</tr>
<tr>
<td>$c_{H}$</td>
<td>-1.1</td>
<td>0.6</td>
</tr>
<tr>
<td>$c_{HB}$</td>
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<td>0.018</td>
</tr>
<tr>
<td>$c_{Hd}$</td>
<td>-0.035</td>
<td>0.017</td>
</tr>
<tr>
<td>$c_{He}$</td>
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<td>0.013</td>
</tr>
<tr>
<td>$c_{Hq}$</td>
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<td>0.004</td>
</tr>
<tr>
<td>$c_{Hq}'$</td>
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<td>0.003</td>
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<tr>
<td>$c_{Hu}$</td>
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<td>0.013</td>
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<tr>
<td>$c_{HW}$</td>
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<td>$c_{ll}$</td>
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<tr>
<td>$c_{uG}$</td>
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<td>0.016</td>
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<td>$c_{W} - c_{B}$</td>
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<td>0.04</td>
</tr>
<tr>
<td>$c_{W} + c_{B}$</td>
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<td>0.024</td>
</tr>
<tr>
<td>$c_{y}$</td>
<td>-0.001</td>
<td>0.0006</td>
</tr>
</tbody>
</table>

Q: What do the derived limits on $c_i^{(6)}$ imply on the scale $\Lambda$ of NP?

A: estimate of $\Lambda$ depends on the kind of UV dynamics
How far can we extrapolate weakly our theory?

Higher-derivative operators imply strong coupling scale

Ex: \[ O_H = \left[ \partial_{\mu}(H^\dagger H) \right]^2 \]

\[ \sim c_H \frac{E^2}{v^2} \]

Strong interaction at

\[ E \sim \Lambda_S = \frac{4\pi v}{\sqrt{c_H}} \]

At the SM point \( c_i = 1 \) we can extrapolate up to \( E \sim M_{Pl} \)

\[ 4\pi v/\sqrt{\delta c_i} \approx 7 - 10 \text{ TeV} \]

With current knowledge of the Higgs couplings \( \delta c_i \lesssim 0.1 - 0.2 \) we can extrapolate so much
Validity of SMEFT at colliders
Example: Fermi theory

\[ \mathcal{L}_{\text{eff}} \supset c^{(6)} \left( \bar{e} \gamma_\rho P_L \nu_e \right) \left( \bar{\nu}_\mu \gamma_\rho P_L \mu \right) + \text{h.c.} \]

\[ c^{(6)} = - \frac{g^2}{2m_W^2} \]

Muon decay measures \( c^{(6)} \sim \frac{g^2}{m_W^2} \)

“new physics” scale \( m_W \)

not directly accessible

Estimating the scale at which NP shows up (e.g. in neutrino scattering) requires making an assumption on the coupling \( g \sim 10^{-3} \) to \( g = 4\pi \)

Assessing the validity of the EFT analysis also requires making assumptions of the UV dynamics

\( m_\mu \quad m_W \quad 1.5 \text{ TeV} \)
LHC not ideal for an EFT approach

- EFT best suited to fixed-energy, high-precision experiments (ex: LEP, flavor)

large gap of scales requires RG to re-sum large logs
LHC not ideal for an EFT approach

- EFT best suited to fixed-energy, high-precision experiments (ex: LEP, flavor)
  
  ![Energy range diagram]

- less suited to low-precision experiments probing an energy range (ex: LHC, hadron machines in general)

EFT fails when max probed energy $E_{\text{max}}$ is equal or bigger than physical scale $\Lambda$

☞ One can check a posteriori, but needs to know $E_{\text{max}}$
TGC measurements: LEP vs LHC

Three dim-6 operators affect TGC

\[ O_{HW} = D_\mu H^\dagger W^{\mu \nu} D_\nu H \quad \longrightarrow \quad V_L V_L \]
\[ O_{HB} = D_\mu H^\dagger B^{\mu \nu} D_\nu H \quad \longrightarrow \quad V_T V_T \]
\[ O_{3W} = \text{Tr}(W_{\mu \nu} W^\nu \rho W^\mu_\rho) \quad \longrightarrow \quad V_T V_T \]

- LEP2 operated in a narrow range of com energies \( \sqrt{s} \sim 200 \text{ GeV} \)
- LHC spans a wide energy interval

sensitivity on NP mainly comes from bins at large energy

\( m_T^{WZ} \) [GeV]
**Fit to TGCs**

Butter et al. JHEP 1607 (2016) 152

see also:
Falkowski et al. JHEP 1702 (2017) 115
Franceschini et al. JHEP 1802 (2018) 111

\[
\sigma = \sigma_{SM} \left( 1 + c_i A_i + c_i c_j B_{ij} \right)
\]

1-dimensional 95% CL constraints

<table>
<thead>
<tr>
<th>LEP</th>
<th>LHC</th>
</tr>
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<tbody>
<tr>
<td>(c_{HW}) ([\text{TeV}^{-2}])</td>
<td>([-7.6, 19])</td>
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<td>(c_{HB}) ([\text{TeV}^{-2}])</td>
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<td>(c_{3W}) ([\text{TeV}^{-2}])</td>
<td>([-32, 3.3])</td>
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**LEP**

fit dominated by \((D=6)\) linear terms

**LHC**

fit dominated by \((D=6)^2\) terms
Fit to TGCs

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$$\sigma = \sigma_{SM} \left( 1 + c_i A_i + c_i c_j B_{ij} \right)$$

1-dimensional 95% CL constraints

**LEP**
- \(c_{HW} \in [-7.6, 19] \text{ TeV}^{-2}\)
- \(c_{HB} \in [-67, 1.8] \text{ TeV}^{-2}\)
- \(c_{3W} \in [-32, 3.3] \text{ TeV}^{-2}\)

**LHC**
- \(c_{HW} \in [-1.5, 6.3] \text{ TeV}^{-2}\)
- \(c_{HB} \in [-14.3, 15.9] \text{ TeV}^{-2}\)
- \(c_{3W} \in [-2.4, 3.2] \text{ TeV}^{-2}\)

*Naively:*
- LHC constraints stronger than LEP ones
- \(c_{3W}\) slightly more constrained
Estimating the cutoff scale through SILH power counting (1 coupling, 1 scale):

\[ c_{3W} \sim \frac{g}{\Lambda^2} \left( \frac{g^2}{16\pi^2} \right) \quad \text{and} \quad c_{HW, HB} \sim \frac{g}{\Lambda^2} \left( \frac{g_*^2}{16\pi^2} \right) \]

1-dimensional 95\% CL constraints

<table>
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<td>([-32, 3.3]) TeV(^{-2})</td>
<td>( \gtrsim 200 \text{ GeV} \left( \frac{g_*}{4\pi} \right) )</td>
</tr>
<tr>
<td><strong>LHC</strong></td>
<td>([-1.5, 6.3]) TeV(^{-2})</td>
<td>([-14.3, 15.9]) TeV(^{-2})</td>
<td>([-2.4, 3.2]) TeV(^{-2})</td>
<td>( \gtrsim 300 \text{ GeV} \left( \frac{g_*}{4\pi} \right) )</td>
</tr>
</tbody>
</table>
Estimating the cutoff scale through SILH power counting (1 coupling, 1 scale):

\[ c_{3W} \sim \frac{g}{\Lambda^2} \left( \frac{g^2}{16\pi^2} \right) \quad \text{and} \quad c_{HW,HB} \sim \frac{g}{\Lambda^2} \left( \frac{g_*^2}{16\pi^2} \right) \]

Strong dipolar interactions

\[ c_{3W} \sim \frac{g_*}{\Lambda^2} \rightarrow \Lambda \gtrsim 2 \text{TeV} \left( \frac{g_*}{4\pi} \right)^{1/2} \]

EFT does not quite work, unless the power counting is different for example

1-dimensional 95% CL constraints

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$c_{HW}$ constraint</th>
<th>$c_{HB}$ constraint</th>
<th>$c_{3W}$ constraint</th>
<th>( \Lambda ) constraint</th>
</tr>
</thead>
<tbody>
<tr>
<td>LEP</td>
<td>([-7.6, 19]) TeV^{-2}</td>
<td>([-67, 1.8]) TeV^{-2}</td>
<td>([-32, 3.3]) TeV^{-2}</td>
<td>( \Lambda \gtrsim 200 \text{GeV} \left( \frac{g_*}{4\pi} \right) )</td>
</tr>
<tr>
<td>LHC</td>
<td>([-1.5, 6.3]) TeV^{-2}</td>
<td>([-14.3, 15.9]) TeV^{-2}</td>
<td>([-2.4, 3.2]) TeV^{-2}</td>
<td>( \Lambda \gtrsim 300 \text{GeV} \left( \frac{g_*}{4\pi} \right) )</td>
</tr>
<tr>
<td>LHC</td>
<td>([-1.5, 6.3]) TeV^{-2}</td>
<td>([-14.3, 15.9]) TeV^{-2}</td>
<td>([-2.4, 3.2]) TeV^{-2}</td>
<td>( \Lambda \gtrsim 20 \text{GeV} )</td>
</tr>
</tbody>
</table>
Linear vs Quadratic

Notice: Dominance of linear terms (over quadratic ones) is per se neither sufficient nor necessary a condition for the EFT to be valid.

Not sufficient
Ex: TGC at LEP2
\[ \frac{\delta\sigma}{\sigma} \sim \frac{c_{3W}}{g} E^2 \sim \frac{g^2}{16\pi^2} \frac{E^2}{\Lambda^2} \]

Not necessary
Ex: \( V_L V_L \rightarrow V_L V_L \) scattering
\[ O_6 = (H \partial H)^2 \quad c^{(6)}(H) \sim \frac{g^2}{\Lambda^2} \]
\[ \sigma(LL \rightarrow LL) \sim \frac{g_{SM}^4}{E^2} \left[ 1 + \frac{g^2}{g_{SM}^2} \frac{E^2}{\Lambda^2} + \frac{g^4}{g_{SM}^4} \frac{E^4}{\Lambda^4} + \ldots \right] \]

NLO correction from dim6-SM close to threshold

BSM dominates over SM for \( g/g_* < E < \Lambda \)
Further challenge to EFT: Non-interference from helicity selection rules


\[ h(A) = \sum_i h_i \]

dim-6 and SM interfere only if they contribute to the same helicity amplitude (the total helicity \( h(A) \) must be the same)

No interference for 4-point amplitudes with at least one transverse boson

| \( A_4 \) | \(|h(A_4^{SM})|\) | \(|h(A_4^{BSM})|\) |
|---|---|---|
| VVVV | 0 | 4,2 |
| VV\phi\phi | 0 | 2 |
| VV\psi\psi | 0 | 2 |
| V\psi\psi\phi | 0 | 2 |
| \psi\psi\psi | 2,0 | 2,0 |
| \psi\psi\phi | 0 | 0 |
| \phi\phi\phi | 0 | 0 |

Validity:
- at tree-level in the massless (high-energy) limit \( E \gg m_W \)
- only dim-6 operators
- only 4-point amplitudes
Beyond the leading approximation

- Non-interference in general fails for higher-point amplitudes and at the 1-loop level

Leading effect arises at $O(\alpha_S/\pi)$ from real emissions (for inclusive processes) and 1-loop virtual corrections (pure EW corrections similar but smaller)

No log enhancement in the interference due to soft and collinear singularities in real emissions or IR divergences in 1-loop diagrams [see: Dixon and Shadmi NPB 423 (1994) 3]

- Finite-mass effects arise at $O(m_W^2/E^2)$ and can be determined by considering higher-point amplitudes with Higgs vevs

Ex:

SM: $A_6(\psi^+\psi^-VV\phi\phi)$

BSM$_6$: $A_6(\psi^+\psi^-VV^+)$

Diagram:
• radiative corrections subdominant compared to mass effects except at very high energies \( E \gtrsim m_W \sqrt{4\pi/\alpha_S} \sim 1\,\text{TeV} \)

\[
E \gtrsim m_W \sqrt{4\pi/\alpha_S} \sim 1\,\text{TeV}
\]

Fermion mass insertions usually subdominant except for top quarks (e.g. \( F^3 \) interferes at \( O(\varepsilon_F^2) \) in \( gg \rightarrow t\bar{t} \))

• Accessing the \( O(1/\Lambda^2) \) corrections from D=6 operators without relative suppression is possible by considering \( 2 \rightarrow 3 \) processes (i.e. \( 2 \rightarrow 2 \) plus extra jet)

\[
\text{ex: constraining } F^3 \text{ through 3-jet events} \quad [\text{Dixon and Shadmi NPB 423 (1994) 3}]
\]

Max gain in sensitivity \( \sim \sqrt{4\pi/\alpha_S} \) (at the cost of a reduced \( S/B \))
Implications of non-interference

Example: \( V_L V_L \rightarrow V_T V_T \ ( T = \pm ) \)

\[ O_6 = F_{\mu \nu}^2 H^\dagger H \]

\[ c^{(6)} \sim \frac{g_*^2}{\Lambda^2} \]

\[ O_8 = F_{\mu \nu}^2 H^\dagger H D^2 \]

\[ c^{(8)} \sim \frac{g_*^2}{\Lambda^4} \]

\[ \sigma (LL \rightarrow TT) \sim \frac{g_{SM}^4}{E^2} \left[ 1 + \frac{g_*^2}{g_{SM}^2} \frac{m_W^2}{\Lambda^2} + \frac{g_*^4}{g_{SM}^4} \frac{E^4}{\Lambda^4} + \frac{g_*^4}{g_{SM}^4} \frac{E^4}{\Lambda^4} + \ldots \right] \]
Avoiding non-interference by exclusive processes

Panico, Riva and Wulzer, PLB 776 (2018) 473
Azatov, Elias-Miro, Reyimuaji, Venturini JHEP 1710 (2017) 027

- Vector bosons not asymptotic states, decay to fermions
- Interference arises in scattering amplitudes at fixed azimuthal angles
  
  Averaging over azimuthal angles washes out the interference

\[
\frac{d\sigma_{\text{int}}(q\bar{q} \rightarrow WZ \rightarrow 4\psi)}{d\phi_Z \; d\phi_W} \propto \cos(2\phi_Z) + \cos(2\phi_W)
\]
Strategy for a consistent EFT analysis of data

1. Fit of coefficients $c_i^{(6)}$ can be done model independently

   Results should be reported as functions of $M_{\text{cut}} = \text{max characteristic energy scale}$

   $$c_i^{(6)} < \delta_i^{\exp}(M_{\text{cut}})$$

2. Interpretation of results require assumptions on UV dynamics

   power counting $\rightarrow c_i^{(6)} = \frac{\tilde{c}_i^{(6)}(g_\ast)}{\Lambda^2}$

3. Consistent (though conservative) limits through *restriction* of dataset: set $M_{\text{cut}} = \kappa \Lambda$

   $$c_i^{(6)} = \frac{\tilde{c}_i^{(6)}(g_\ast)}{\Lambda^2} < \delta_i^{\exp}(\kappa \Lambda)$$

   $0 < \kappa < 1$ controls the size of the tolerated error due to higher-derivative operators

   limits on scale $\Lambda$ set by using data up to $M_{\text{cut}} = \kappa \Lambda$
Example of idealized measurement: \( u\bar{d} \rightarrow W^+ h \)

<table>
<thead>
<tr>
<th>( M_{Wh} ) [TeV]</th>
<th>( 0.5 )</th>
<th>( 1 )</th>
<th>( 1.5 )</th>
<th>( 2 )</th>
<th>( 2.5 )</th>
<th>( 3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma/\sigma_{SM} )</td>
<td>( 1 \pm 1.2 )</td>
<td>( 1 \pm 1.0 )</td>
<td>( 1 \pm 0.8 )</td>
<td>( 1 \pm 1.2 )</td>
<td>( 1 \pm 1.6 )</td>
<td>( 1 \pm 3.0 )</td>
</tr>
</tbody>
</table>

Model of heavy spin-1:

\[
\mathcal{L} \supset i g_{H} V_{\mu}^{i} H^{\dagger} \sigma^{i} \overleftrightarrow{D}_{\mu} H + g_q V_{\mu}^{i} \bar{q} L^{\gamma_{\mu}} \sigma^{i} q L
\]

Recast with SILH power counting: \(-g_q = g_H = g_*\)

\[
O_{H\psi} = i \bar{q} L \gamma_{\mu} \sigma^{a} q L (H^{\dagger} \sigma^{a} \overleftrightarrow{D}_{\mu} H)
\]

\[
c_{H\psi} = -\frac{g_H g_q}{M_V^2}
\]

\[
\begin{align*}
\text{simplified model of spin-1 resonance} \\
\text{inclusive EFT analysis} \\
\text{95\% C.L. limits}
\end{align*}
\]

Suppose an experiment makes the following measurement of the cross section:

\[
\text{Example of idealized measurement:} \quad \sigma(\sqrt{s}) \quad \text{as a function of } \sqrt{s}
\]

For instance, by using the limits from the full dataset, one can recast the measurements of the cross section as confidence intervals.
Beyond dim-6 operators
D=8 operators can become important in special cases if D=6 ones are suppressed by symmetries or selection rules

Example: Double Higgs production via gluon fusion (assuming Higgs is a pNGB)

\[ O_g = H^\dagger H G_\mu^\nu G^{a\mu\nu} \]
\[ c^{(6)} \sim \frac{g_s^2}{16\pi^2} \frac{\lambda^2}{\Lambda^2} \]
( \( \lambda \) = weak spurion breaking the shift symmetry)

\[ O_{gD0} = (D\rho H^\dagger D^\rho H)G_\mu^\nu G^{a\mu\nu} \]
\[ O_{gD2} = (\eta^{\mu\nu} D\rho H^\dagger D^\rho H - 4D^\mu H^\dagger D^\nu H)G^{a}_\mu G^{a}_\nu \]
\[ c^{(8)} \sim \frac{g_s^2}{16\pi^2} \frac{g_*^2}{\Lambda^4} \]

\[ A(gg \to hh) \sim \frac{g_s^2}{16\pi^2} \left( y_t^2 + \lambda^2 \frac{E^2}{\Lambda^2} + g_*^2 \frac{E^4}{\Lambda^4} + \ldots \right) \]

Notice: strong coupling \( g_* \) appears only at the dim-8 level

dim-8 dominate over dim-6 for:

\[ \lambda f < E < \Lambda \]
Probing dim-8 operators is very difficult (perhaps impossible) at the LHC unless bigger than SM.

In practice: double Higgs production has a very low rate, dim-8 are unobservable at the LHC unless bigger than SM.

Example:

\[ \lambda = y_t \]
\[ (v^2/f^2) = 0.1 \]
\[ g_* = 3 \]

\[ \lambda f \approx 500 \text{ GeV} \]
\[ f \sqrt{y_t g_*} \approx 1.3 \text{ TeV} \]
\[ \Lambda \approx 2.3 \text{ TeV} \]

For a luminosity: \( L = 3 \text{ ab}^{-1} \)
- requiring at least 5 events
- including 10% efficiency due to kinematic cuts

<table>
<thead>
<tr>
<th>Largest value of ( m(hh)[\text{GeV}] )</th>
<th>( b\bar{b}\gamma\gamma )</th>
<th>( 4b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sqrt{s} = 14 \text{ TeV} )</td>
<td>550</td>
<td>1550</td>
</tr>
<tr>
<td>( \sqrt{s} = 100 \text{ TeV} )</td>
<td>1350</td>
<td>4300</td>
</tr>
</tbody>
</table>

Probing dim-8 operators is very difficult (perhaps impossible) at the LHC.