# Low-beta Structures

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CAS RF Ebeltoft 2010

1. Low-beta: problems and solutions
2. Coupled-cell accelerating structures
3. Overview and comparison of low-beta structures
4. The Radio Frequency Quadrupole (from the RF point of view)

# Definitions

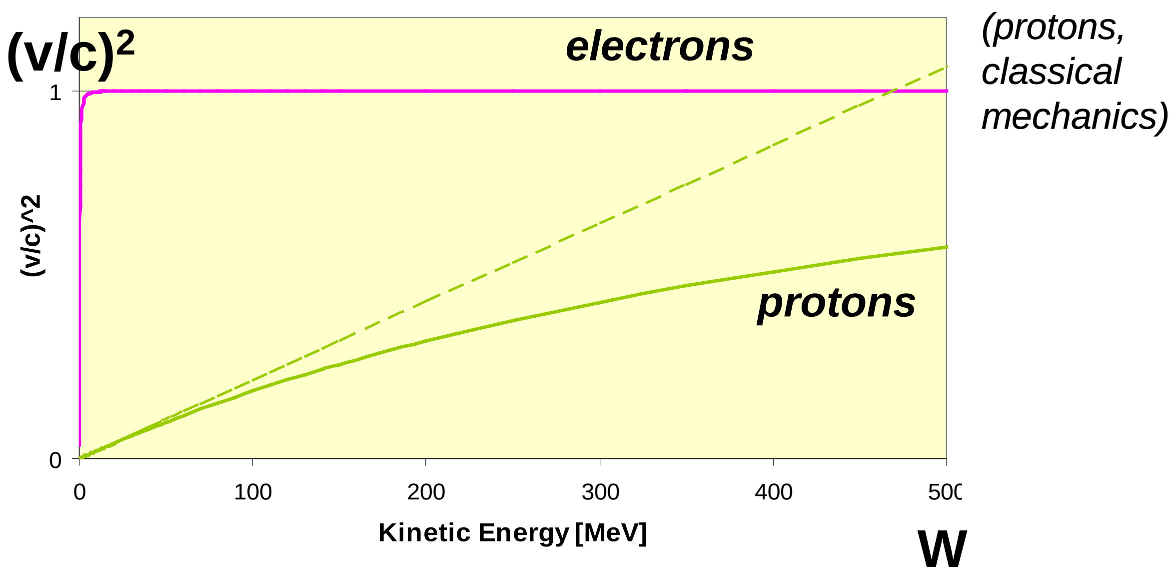
"Low-beta" structures are used in linear accelerators for protons and ions, where the velocity of the particle beam increases with energy

(not for electrons, 2which are immediately relativistic, nor for synchrotrons where particle velocity is nearly constant).

The periodicity of the RF structure must match the increasing particle velocity → development of a complete panoply of structures (NC and SC) with different features.

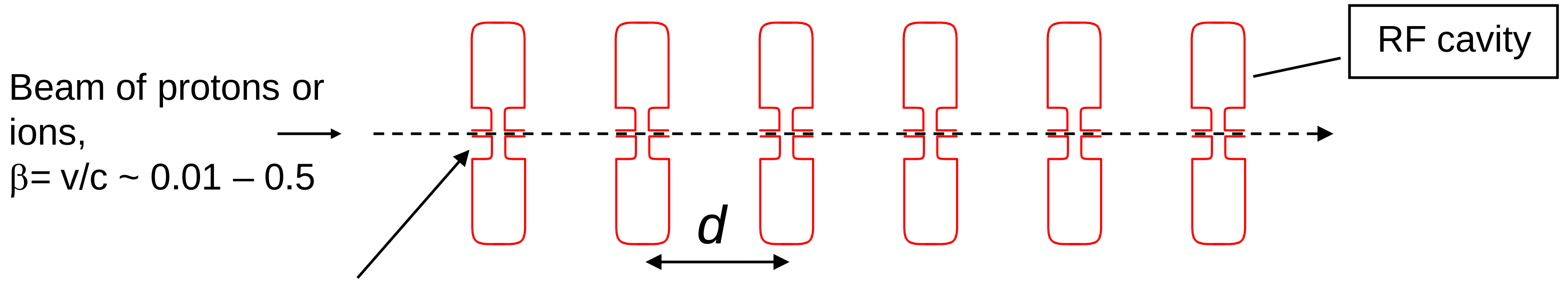
Protons and ions:
at the beginning of the acceleration, beta (=v/c) is rapidly increasing, but after few hundred MeV's (protons) relativity prevails over classical mechanics ($\beta \sim 1$) and particle velocity tends to saturate at the speed of light.

Low-beta structures are specifically designed for the increasing velocity range.



# Synchronicity

Let's assume that we want to accelerate a "low-beta" beam with an array of RF cavities.




E-field on gap of cavity *i* : $E_i = E_{0i} \cos(\omega t + \phi_i)$, energy gain $\Delta W_i = eV_{0i} T \cos\phi_i$

For the beam to cross each gap on the same RF phase, then $\omega\tau = \Delta\phi$

($\tau$=time to travel from one cavity to the next; $\Delta\phi$=difference in phase between 2 adjacent cavities)

$$\Rightarrow \quad \Delta\phi = \omega\tau = \omega\frac{d}{\beta c} = 2\pi\frac{d}{\beta\lambda} \quad \Rightarrow \quad \frac{\Delta\phi}{d} = \frac{2\pi}{\beta\lambda}$$

⇒ distance between cavities and phase of each cavity are correlated

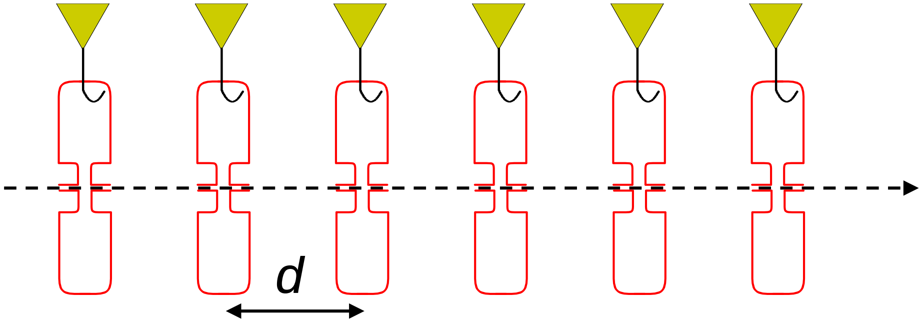
When β increases during acceleration, either the phase difference between cavities $\Delta\phi$ must decrease or their distance *d* must increase.

# CAS Architectures for low-beta

So, where is the problem? If we control the phase of each cavity (connected to an individual amplifier) we can easily implement the required phase difference between cavities…

… but this can become very expensive! In fact:

1. Short single-gap cavities have a low shunt-impedance (high wall loss for a small voltage);
2. Individual RF amplifiers for each small cavity are very expensive.



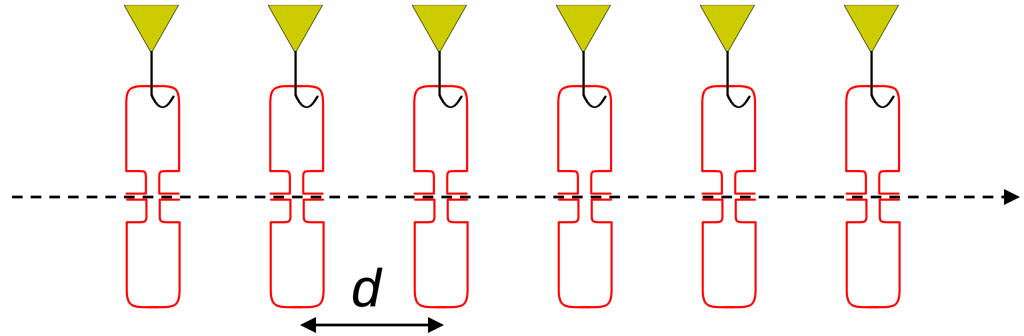

*This type of cavity array is used only in few special cases (if one needs a very large beta acceptance)*

Low-beta structures are usually multi-gap, multi-cell structures, fed from the same power source. Instead of changing the phase to keep synchronism we change the distance between gaps.

# Coupled cell structures

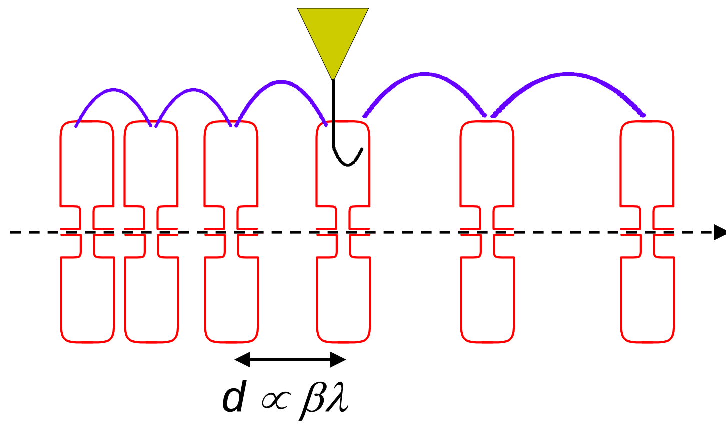
$$\frac{\Delta\phi}{d} = \frac{2\pi}{\beta\lambda}$$

**d = const.**
**$\phi$ variable**




**Individual cavity system** – *distance between cavities constant, each cavity fed by an individual RF source, phase of each cavity adjusted to keep synchronism – used for linac required to operate with different ions or at different energies. Flexible but expensive!*

**$\phi$ = const.**
**d variable**



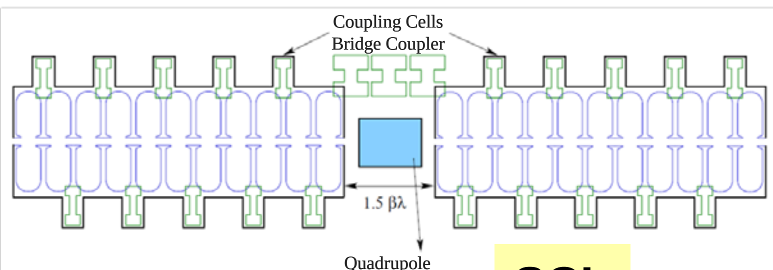

*Better, but 2 problems:*
1. *create a **"coupling"**;*
2. *create a mechanical and RF structure with increasing cell length.*

**Coupled cell system** – *a single RF source feeds a large number of cells (up to ~100!) - the phase between adjacent cells is defined by the coupling and the distance between cells increases to keep synchronism – is the standard accelerating structure for linacs. Once the geometry is defined, it can accelerate only one type of ion for a given energy range. Effective but not flexible.*

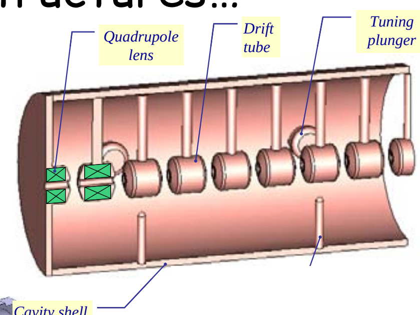
# The low-beta zoo

Between these 2 extreme case (array of independently phased single-gap cavities / single long chain of coupled cells with lengths matching the particle beta) there can be a large number of variations (number of gaps per cavity, length of the cavity, type of coupling) each optimized for a certain range of energy and type of particle, explaining the large number of low-beta structures used in practice.

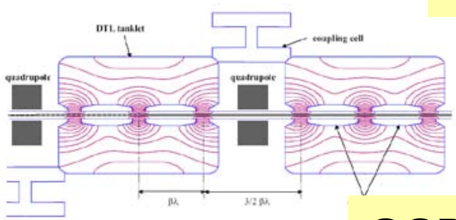
The goal of this lecture is to provide the background to understand the main features try a zoological classification of structures…



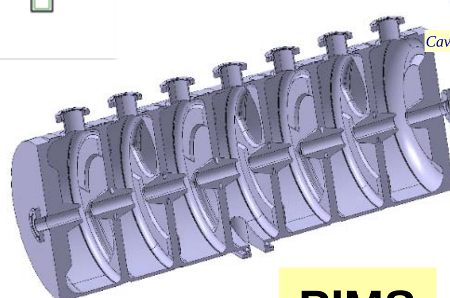

**SCL**



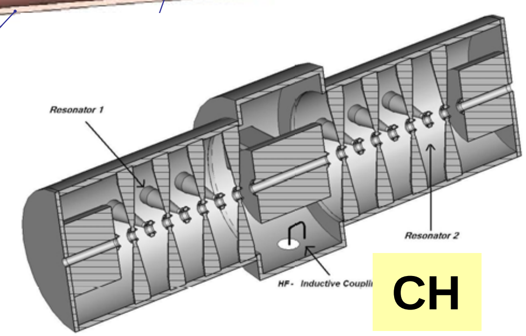

**DTL**




**CCDTL**



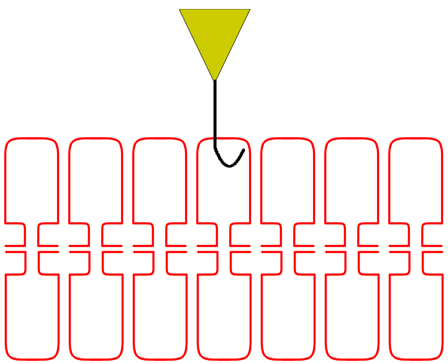
**PIMS**



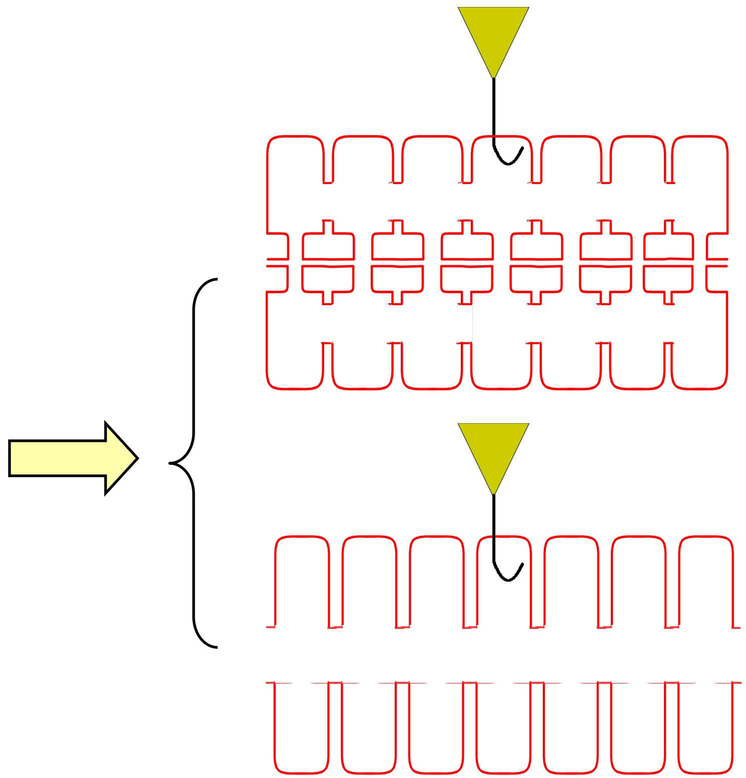

**CH**

# 2. Coupled cell systems

# CAS Coupling resonant cavities

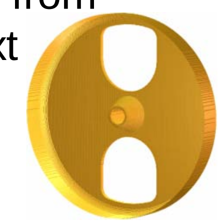


How can we couple together a chain of n reentrant (=loaded pillbox) accelerating cavities ?



1. **Magnetic coupling:**

open “slots” in regions of high magnetic field → B-field can couple from one cell to the next



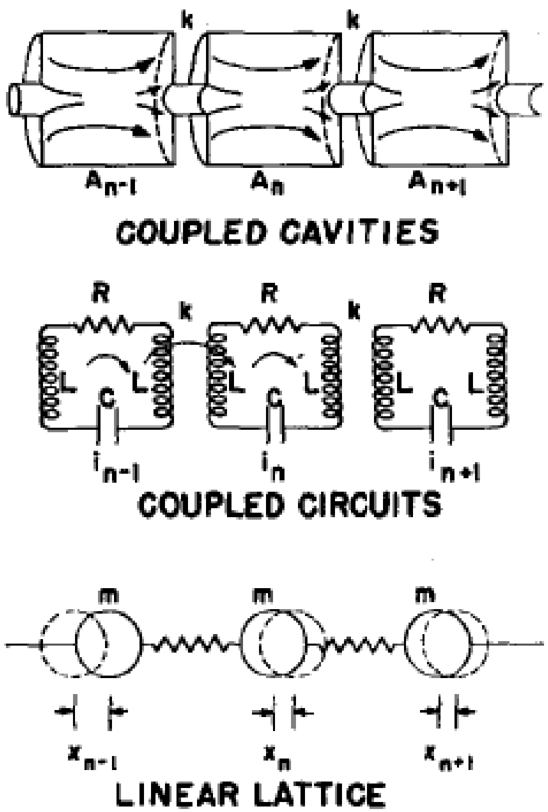
2. **Electric coupling**:

enlarge the beam aperture → E-field can couple from one cell to the next

The effect of the coupling is that the cells no longer resonate independently, but will have common resonances with well defined field patterns.

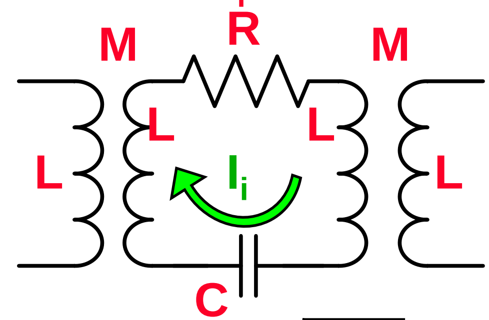
# CAS Chains of coupled resonators

What is the relative phase and amplitude between cells in a chain of coupled cavities?




A linear chain of accelerating cells can be represented as a sequence of resonant circuits magnetically coupled.

Individual cavity resonating at $\omega_0 \rightarrow$ frequenci(es) of the coupled system ?




$$\omega_0 = 1/\sqrt{2LC}$$

$$M = k\sqrt{L_1 L_2} = kL$$

Resonant circuit equation for circuit *i* (neglecting the losses, R≅0):

$$I_i\left(2j\omega L + \frac{1}{j\omega C}\right) + j\omega kL\left(I_{i-1} + I_{i+1}\right) = 0$$

Dividing both terms by *2jωL:*

$$X_i\left(1 - \frac{\omega_0^2}{\omega^2}\right) + \frac{k}{2}\left(X_{i-1} + X_{i+1}\right) = 0$$

*General response term, ∝ (stored energy)$^{1/2}$, can be voltage, E-field, B-field, etc.*

*General resonance term*

*Contribution from adjacent oscillators*

# CAS The Coupled-system Matrix

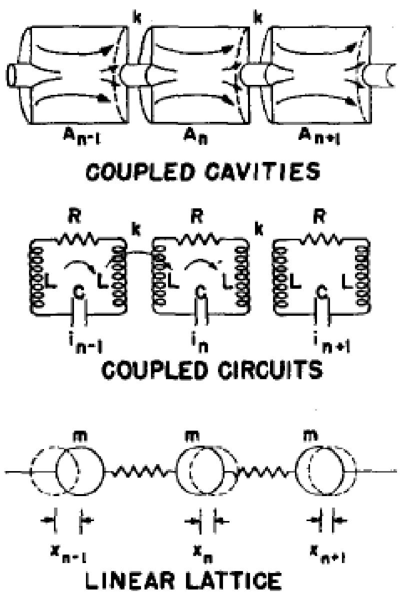
A chain of N+1 resonators is described by a (N+1)x(N+1) matrix:

$$X_i\left(1-\frac{\omega_0^2}{\omega^2}\right)+\frac{k}{2}\left(X_{i-1}+X_{i+1}\right)=0$$

$$i=0,..,N$$

$$\begin{vmatrix} 1-\frac{\omega_0^2}{\omega^2} & \frac{k}{2} & 0 & \dots \\ \frac{k}{2} & 1-\frac{\omega_0^2}{\omega^2} & \frac{k}{2} & \dots \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \frac{k}{2} & 1-\frac{\omega_0^2}{\omega^2} \end{vmatrix} \begin{vmatrix} X_0 \\ X_2 \\ \dots \\ X_N \end{vmatrix} = 0$$

or $M\ X=0$




This matrix equation has solutions only if $\det M=0$

Eigenvalue problem!

1. System of order (N+1) in $\omega \rightarrow$ only N+1 frequencies will be solution of the problem ("eigenvalues", corresponding to the resonances) $\rightarrow$ a system of N coupled oscillators has N resonance frequencies $\rightarrow$ an *individual resonance opens up into a band of frequencies*.
2. At each frequency $\omega_i$ will correspond a set of relative amplitudes in the different cells ($X_0$, $X_2$, …, $X_N$): the "eigenmodes" or "modes".

# Modes in a linear chain of oscillators

We can find an analytical expression for eigenvalues (frequencies) and eigenvectors (modes):

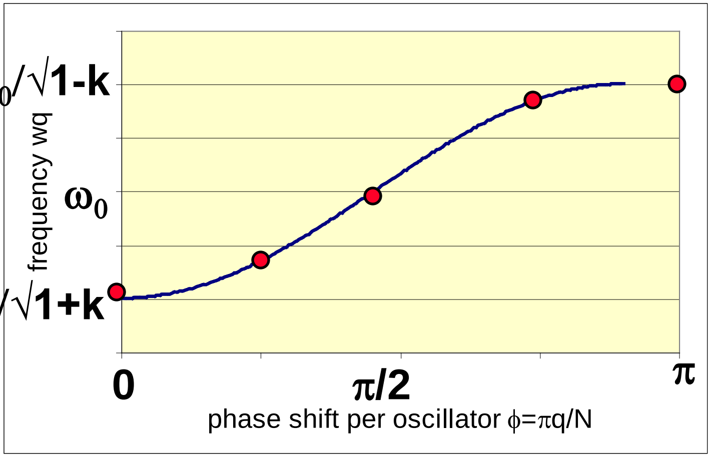
Frequencies of the coupled system :

$$\omega_q^2 = \frac{\omega_0^2}{1 + k\cos\frac{\pi q}{N}}, \qquad q = 0,..,N$$

the index q defines the number of the solution → is the "mode index"

→ Each mode is characterized by a phase **πq/N**. Frequency vs. phase of each mode can be plotted as a "dispersion curve" $\omega$=f($\phi$):

1. each mode is a point on a sinusoidal curve.
2. modes are equally spaced in phase.

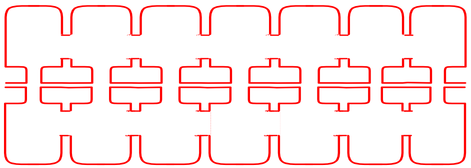



The "eigenvectors = relative amplitude of the field in the cells are:

$$X_i^{(q)} = (const)\cos\frac{\pi q i}{N} e^{j\omega_q t} \qquad q = 0,...,N$$

**STANDING WAVE MODES,** defined by a phase πq/N corresponding to the phase shift between an oscillator and the next one → adjacent cells (gap) have a fixed phase difference πq/N=Φ .

# Acceleration on the normal modes of a 7-cell structure

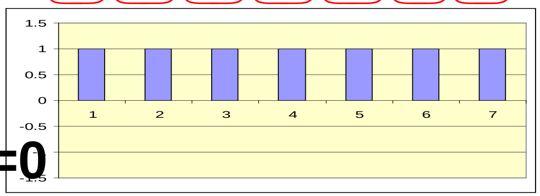


$$X_i^{(q)} = (const)\cos\frac{\pi q i}{N} e^{j\omega_q t} \qquad q = 0,\ldots,N$$

*Remember the phase relation!*

$$\Delta\phi = 2\pi\frac{d}{\beta\lambda}$$

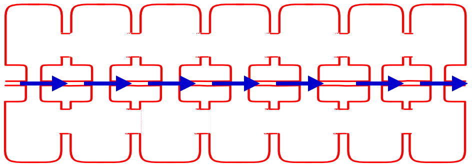
**0**
**q=0**

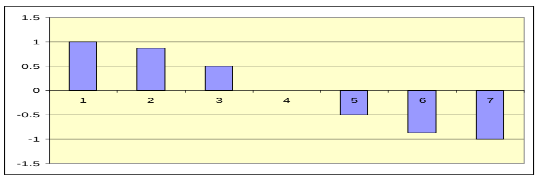


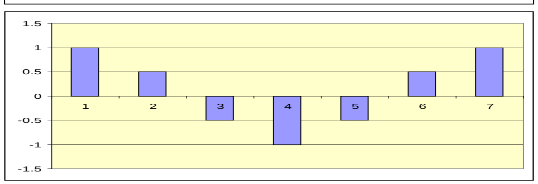
$$\Phi = 2\pi, \quad 2\pi\frac{d}{\beta\lambda} = 2\pi, \quad d = \beta\lambda$$

$\omega = \omega_0/\sqrt{1+k}$

0 (or 2π) mode, acceleration if **d = *βλ***

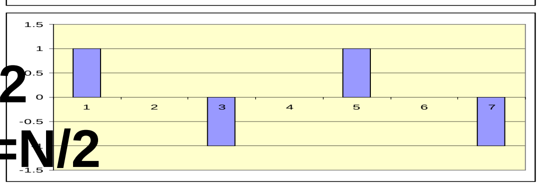






Intermediate modes

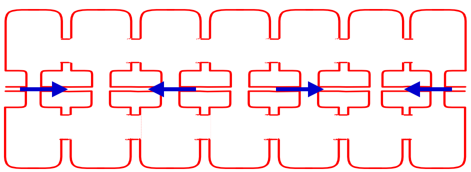
**π/2**
**q=N/2**



$$\Phi = \frac{\pi}{2}, \quad 2\pi\frac{d}{\beta\lambda} = \frac{\pi}{2}, \quad d = \frac{\beta\lambda}{4}$$

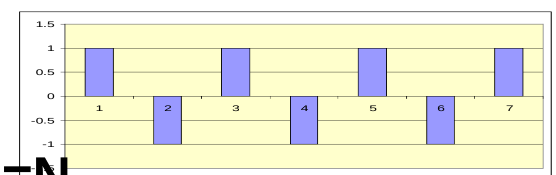
$\omega = \omega_0$

π/2 mode, acceleration if **d = *βλ/4***



**...**

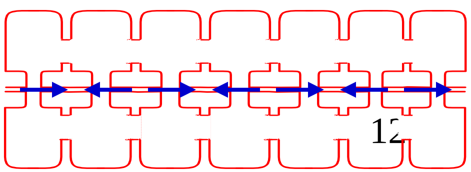
**π**
**q=N**



$$\Phi = \pi, \quad \pi\frac{d}{\beta\lambda} = 2\pi, \quad d = \frac{\beta\lambda}{2}$$

$\omega = \omega_0/\sqrt{1-k}$

π mode, acceleration if **d = *βλ/2,***



Note: Field always maximum in first and last cell!

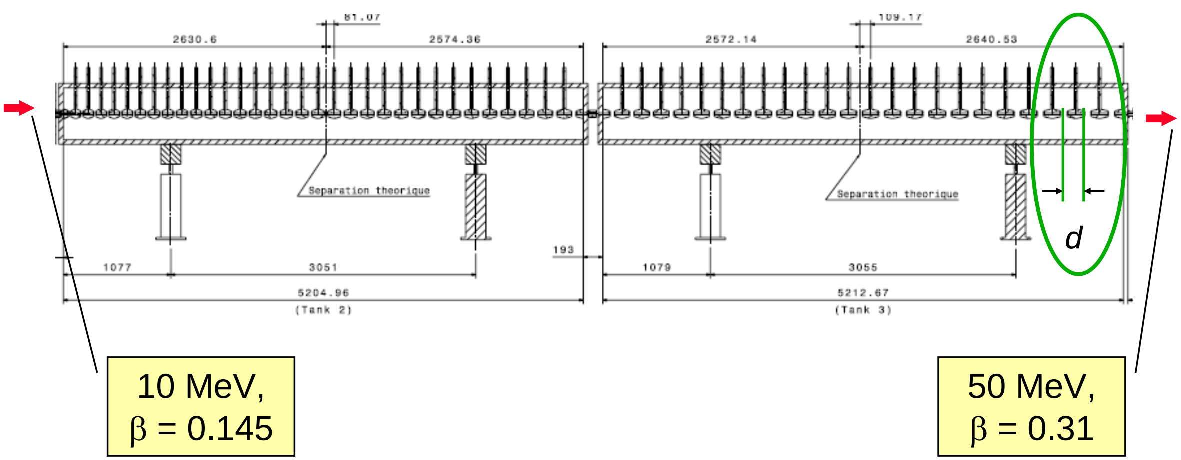
# Practical linac accelerating structures

Some coupled cell modes provide a fixed phase difference between cells that can be used for acceleration. Note that our relations depend only on the cell frequency $\omega$, not on the cell length d !

$$\omega_q^2 = \frac{\omega_0^2}{1 + k\cos\frac{\pi q}{N}}, \qquad q = 0,\ldots,N$$

$$X_n^{(q)} = (const)\cos\frac{\pi q n}{N} e^{j\omega_q t} \qquad q = 0,\ldots,N$$

→ As soon as we keep the frequency of each cell constant, we can change the cell length following any acceleration ($\beta$) profile!




Example:

**The Drift Tube Linac (DTL)**

Chain of many (up to 100!) accelerating cells operating in the 0 mode. The ultimate coupling slot: no wall between the cells (but electric coupling…)!

Each cell has a different length, but the cell frequency remains constant → correct field and phase profile, *the EM fields don't see that the cell length is changing!*

d↑ → (L ↑ , C↓) → LC ~ const → ω ~ const

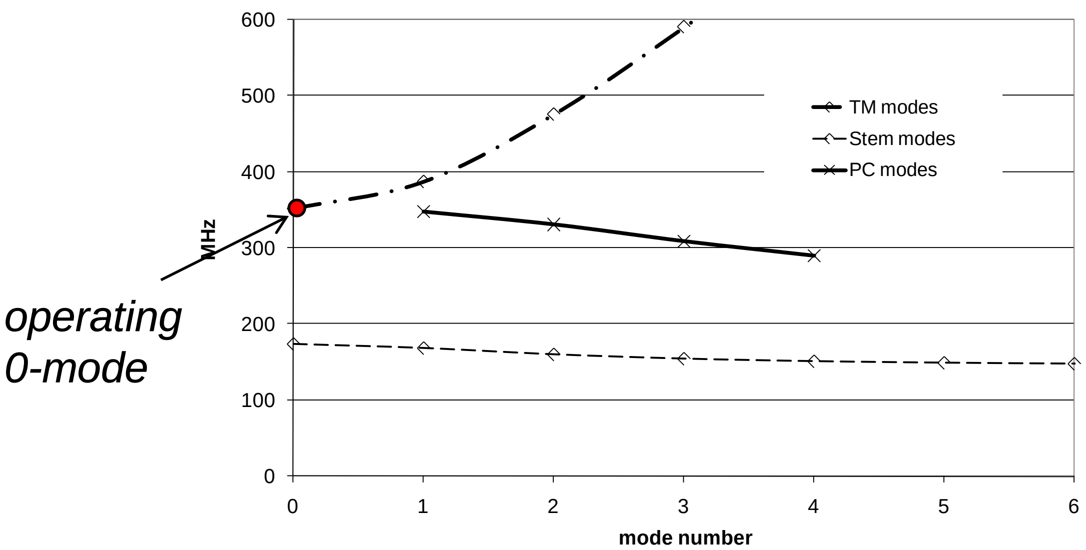
# The Drift Tube Linac

A DTL tank with N drift tubes will have N modes of oscillation.
For acceleration, we choose the 0-mode, the lowest of the band.
All cells (gaps) are in phase, then $\Delta\phi=2\pi$

$$\Delta\phi = 2\pi \frac{d}{\beta\lambda} = 2\pi \quad \Rightarrow \quad d = \beta\lambda$$

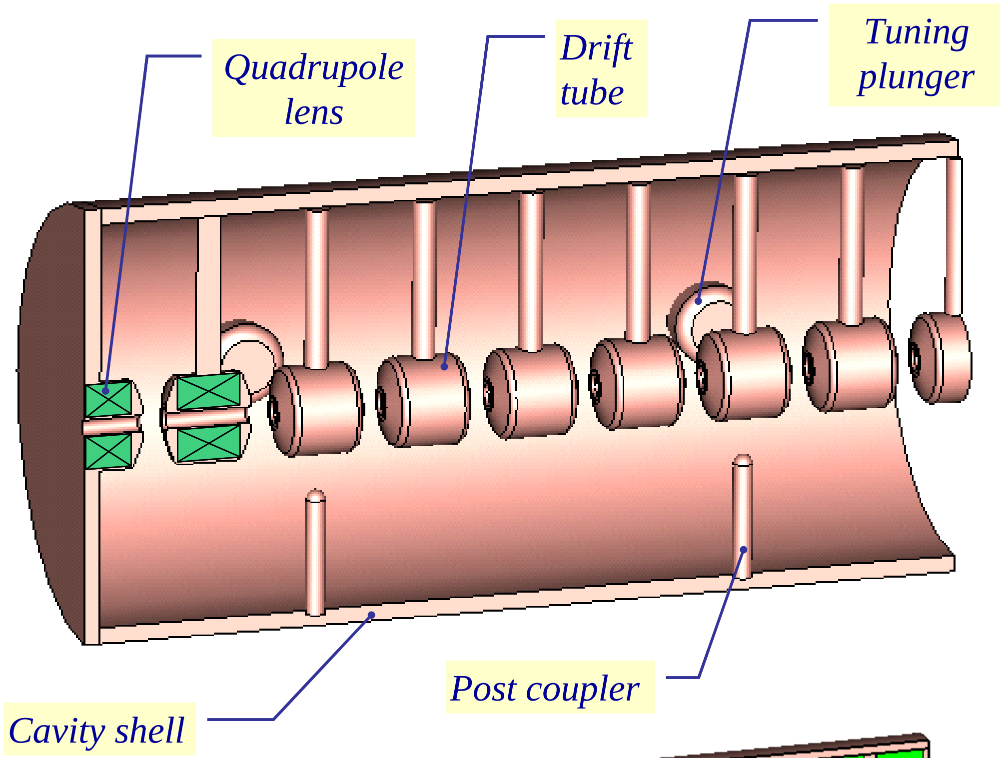
Distance between gaps must be $\beta\lambda$

The other modes in the band (and many others!) are still present.
If mode separation >> bandwidth, they are not "visible" at the operating frequency, but they can come out in case of frequency errors between the cells (mechanical errors or others).




*mode distribution in a DTL tank (operating frequency 352 MHz, are plotted all frequencies < 600 MHz)*

# DTL construction

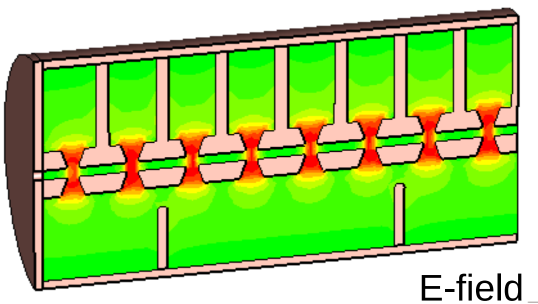


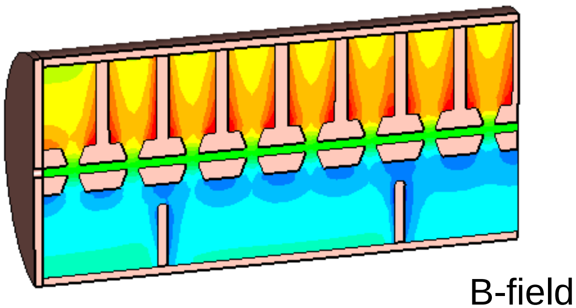

Standing wave linac structure for protons and ions, $\beta$=0.1-0.5, f=20-400 MHz

Drift tubes are suspended by stems (no net RF current on stem)

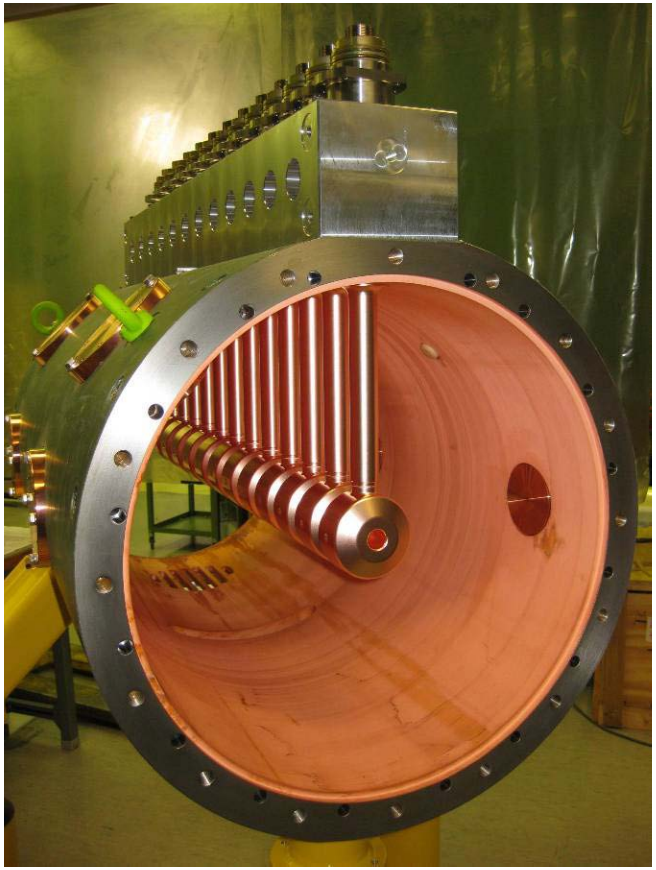
Coupling between cells is maximum (no slot, fully open !)

The 0-mode allows a long enough cell (d=$\beta\lambda$) to house focusing quadrupoles inside the drift tubes!



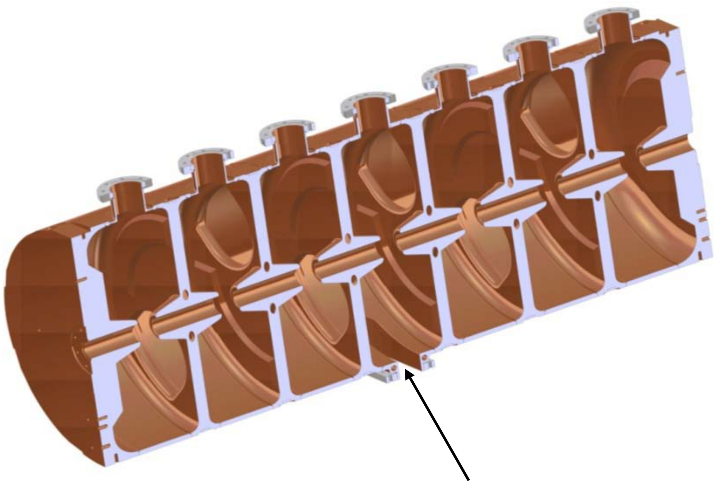
# Examples of DTL



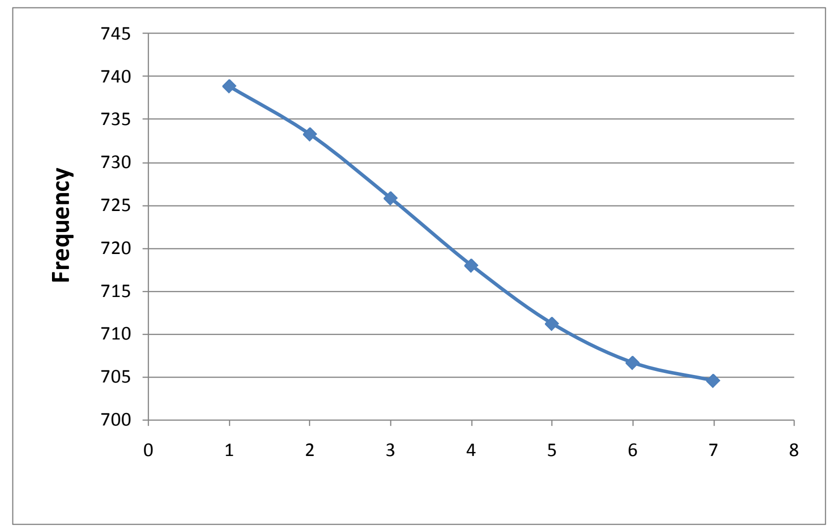
**Top; CERN Linac2 Drift Tube Linac accelerating tank 1 (200 MHz). The tank is 7m long (diameter 1m) and provides an energy gain of 10 MeV.**
**Left: DTL prototype for CERN Linac4 (352 MHz). Focusing is provided by (small) quadrupoles inside drift tubes. Length of drift tubes (cell length) increases with proton velocity.**

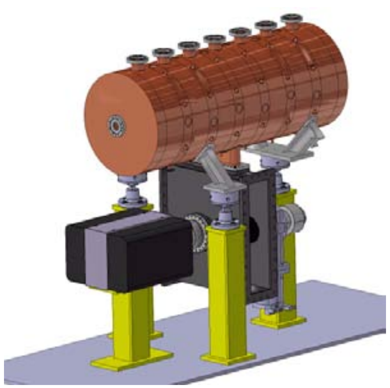
# A 7-cell magnetically-coupled structure

PIMS = Pi-Mode Structure, will be used in Linac4 at CERN to accelerate protons from 100 to 160 MeV ($\beta > 0.4$)




7 cells magnetically coupled, 352 MHz
Operating in $\pi$-mode, cell length $\beta\lambda/2$.



Cells in a cavity have the same length.
When more cavities are used for acceleration, the cells are longer from one cavity to the next, to follow the increase in beam velocity.

# CAS Coupling between 2 cavities

In the PIMS, cells are coupled via a slot in the walls. But what is the meaning of coupling, and how can we achieve a given coupling?

Simplest case: **2 resonators coupled via a slot**

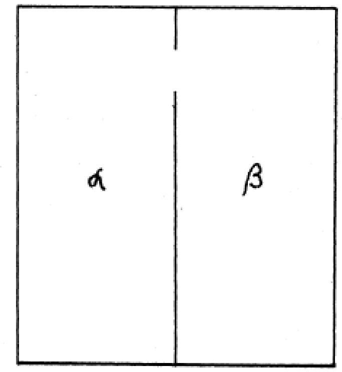



FIG. 1. Two cavities, α and β, coupled by a small hole.

E-field mode 1
E-field mode 2

Described by a system of 2 equations:

$$\begin{cases} X_1\left(1-\dfrac{\omega_1^2}{\omega^2}\right)+kX_2=0 \\ kX_1+X_2\left(1-\dfrac{\omega_2^2}{\omega^2}\right)=0 \end{cases}$$

or

$$\begin{vmatrix} 1-\dfrac{\omega_1^2}{\omega^2} & k \\ k & 1-\dfrac{\omega_2^2}{\omega^2} \end{vmatrix} \begin{vmatrix} X_1 \\ X_2 \end{vmatrix} = 0$$

If $\omega_1=\omega_2=\omega_0$, usual 2 solutions (mode 0 and mode $\pi$):

$$\omega_{c,1}=\frac{\omega_0}{\sqrt{1+k}} \quad \text{with} \quad \begin{vmatrix} X_1 \\ X_2 \end{vmatrix} = \begin{vmatrix} 1 \\ 1 \end{vmatrix} \qquad \text{and} \qquad \omega_{c,2}=\frac{\omega_0}{\sqrt{1-k}} \quad \text{with} \quad \begin{vmatrix} X_1 \\ X_2 \end{vmatrix} = \begin{vmatrix} 1 \\ -1 \end{vmatrix}$$

Mode + + (field in phase in the 2 resonators) and mode + - (field with opposite phase)

Taking the difference between the 2 solutions (squared), approximated for k<<1

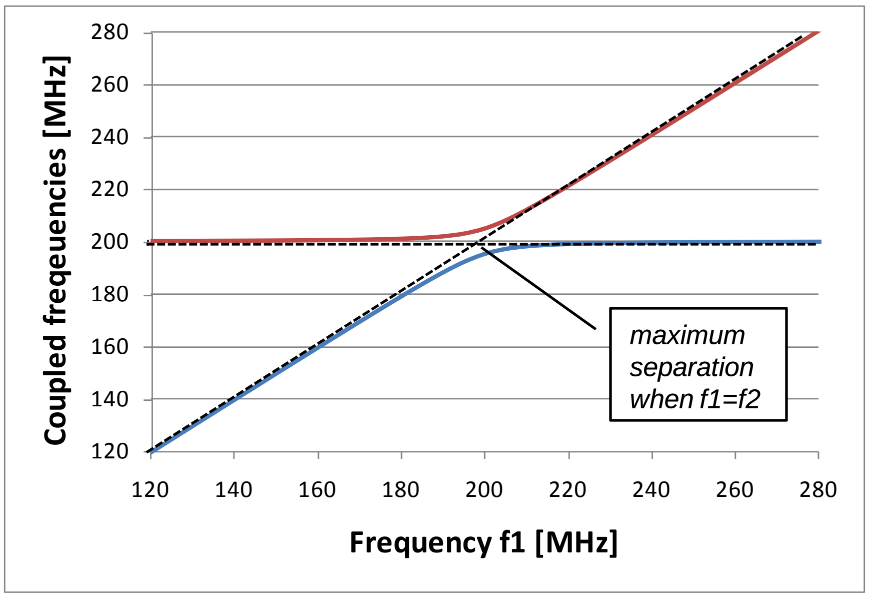
$$\frac{\omega_{c,2}^2-\omega_{c,1}^2}{\omega_0^2}=\frac{1}{1-k}-\frac{1}{1+k}\approx 2k \quad \text{or} \quad \boxed{\frac{\omega_{c,2}-\omega_{c,1}}{\omega_0}\approx k}$$

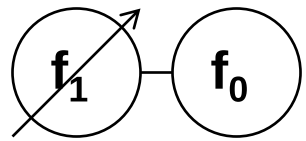
The coupling k is equal to the difference between highest and lowest frequencies.

→ k is the **bandwidth of the coupled system.**

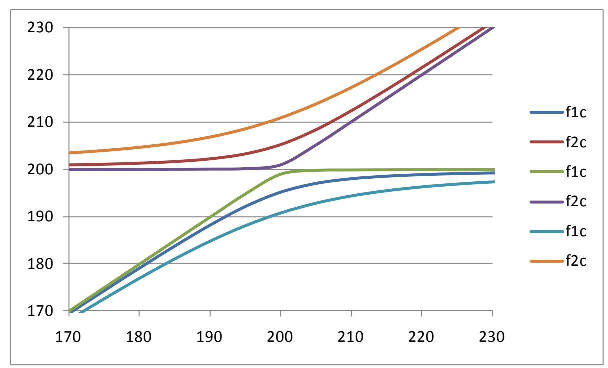
# More on coupling

Solving the previous equations allowing a <u>different frequency for each cell</u>, we can plot the <u>frequencies of the coupled system</u> as a function of the <u>frequency of the first resonator</u>, keeping the frequency of the second constant, for different values of the coupling k.




- "Coupling" only when the 2 resonators are close in frequency.
- For $f_1=f_2$, maximum spacing between the 2 frequencies ($=kf_0$)




case of 3 different coupling factors (0.1%, 5%, 10%)

For an elliptical coupling slot:

$$k \approx F\, l^3 \left(\frac{H_1}{\sqrt{U_1}}\right)\left(\frac{H_2}{\sqrt{U_2}}\right)$$

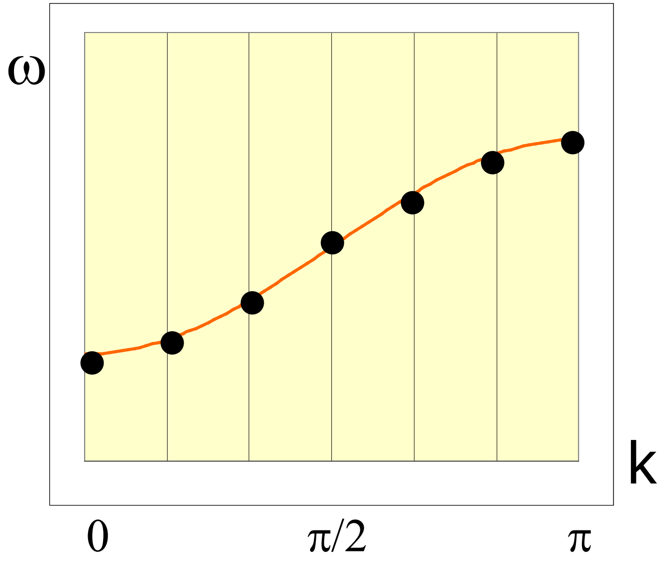
*F* = slot form factor
*l* = slot length (in the direction of H)
*H* = magnetic field at slot position
*U* = stored energy

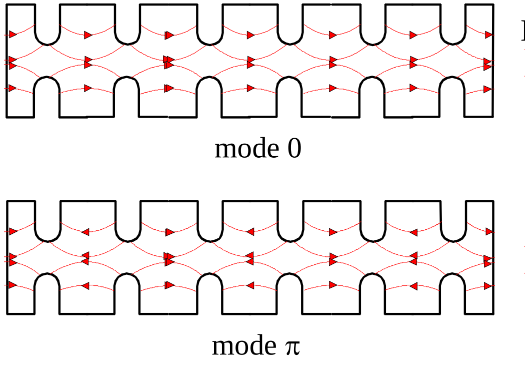
The coupling k is:
- Proportional to the 3rd power of slot length.
- Inv. proportional to the stored energies.

# Long chains of linac cells

- To reduce RF cost, linacs use high-power RF sources feeding a large number of coupled cells (DTL: 30-40 cells, other high-frequency structures can have >100 cells).

- But long linac structures (operating in 0 or π mode) become extremely sensitive to mechanical errors: small machining errors in the cells can induce large differences in the accelerating field between cells.



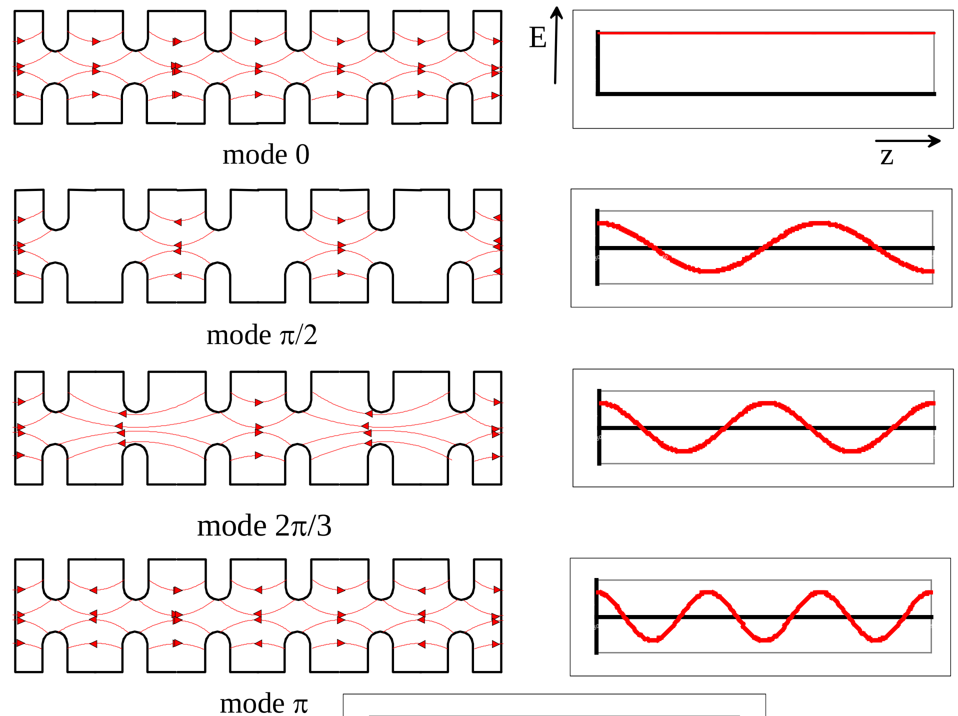
# Stability of long chains of coupled resonators

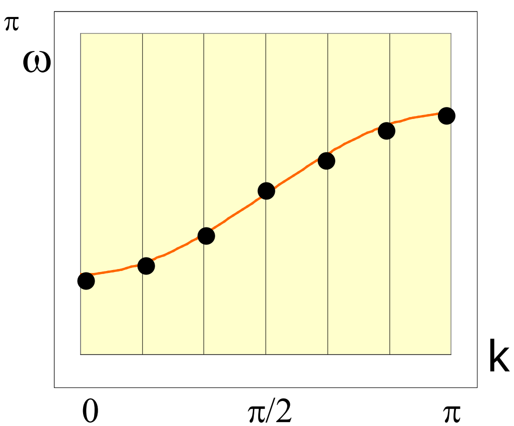
Mechanical errors → differences in frequency between cells → to respect the new boundary conditions the electric field will be a linear combination of all modes, with weight

$$\frac{1}{f^2 - f_0^2}$$

(general case of small perturbation to an eigenmode system, the new solution is a linear combination of all the individual modes)

The nearest modes have the highest effect, and when there are many modes on the dispersion curve (number of modes = number of cells, but the total bandwidth is fixed = k !) the difference in E-field between cells can be extremely high.

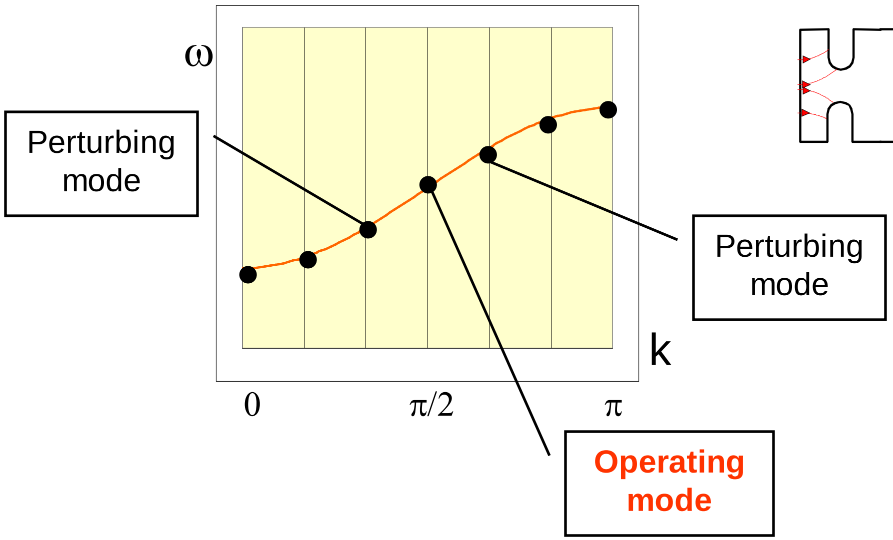


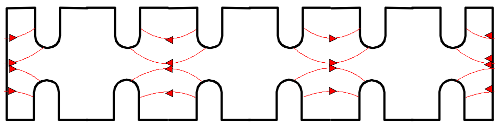



# Stabilization of long chains: the π/2 mode

Solution:

Long chains of linac cells can be operated in the π/2 mode, which is intrinsically insensitive to differences in the cell frequencies.



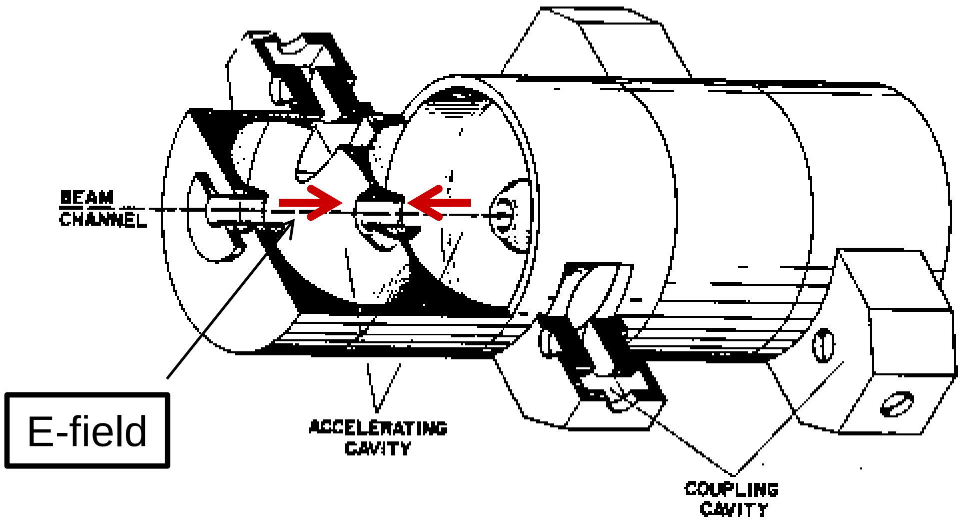
mode π/2

Contribution from adjacent modes proportional to $\frac{1}{f^2 - f_0^2}$ with the sign !!!

The perturbation will add a component $\Delta E/(f^2-f_o^2)$ for each of the nearest modes. Contributions from equally spaced modes in the dispersion curve will cancel each other.

# The Side Coupled Linac

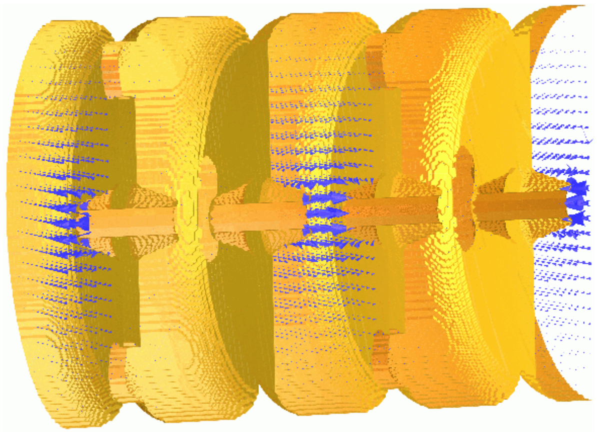
To operate efficiently in the $\pi/2$ mode, the cells that are not excited can be removed from the beam axis → they are called "coupling cells", as for the Side Coupled Structure.



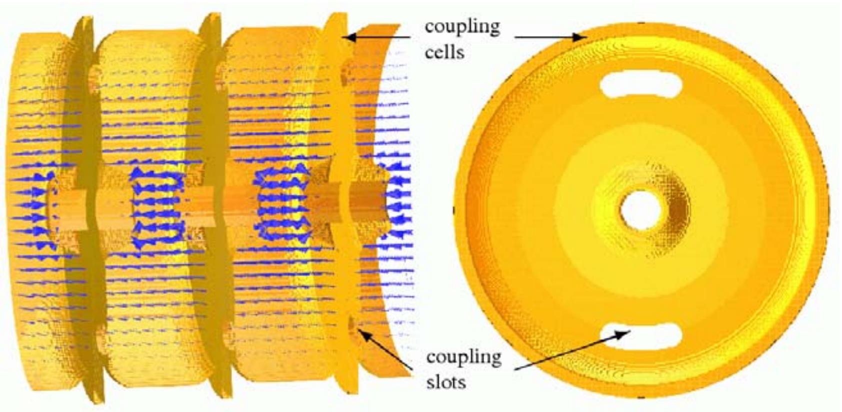

**Example: the Cell-Coupled Linac at the SNS linac, 805 MHz, 100-200 MeV, >100 cells/module**

# Examples of $\pi/2$ structures

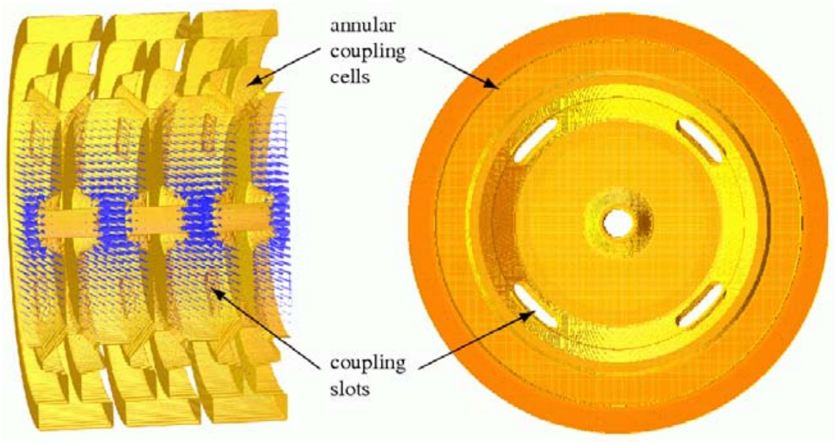
$\pi/2$-mode in a coupled-cell structure



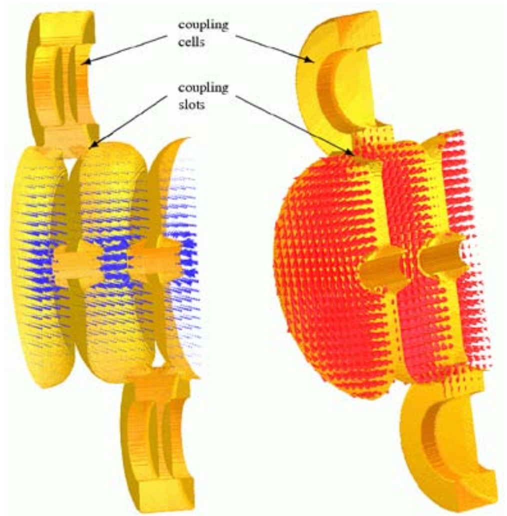
On axis Coupled Structure (OCS)



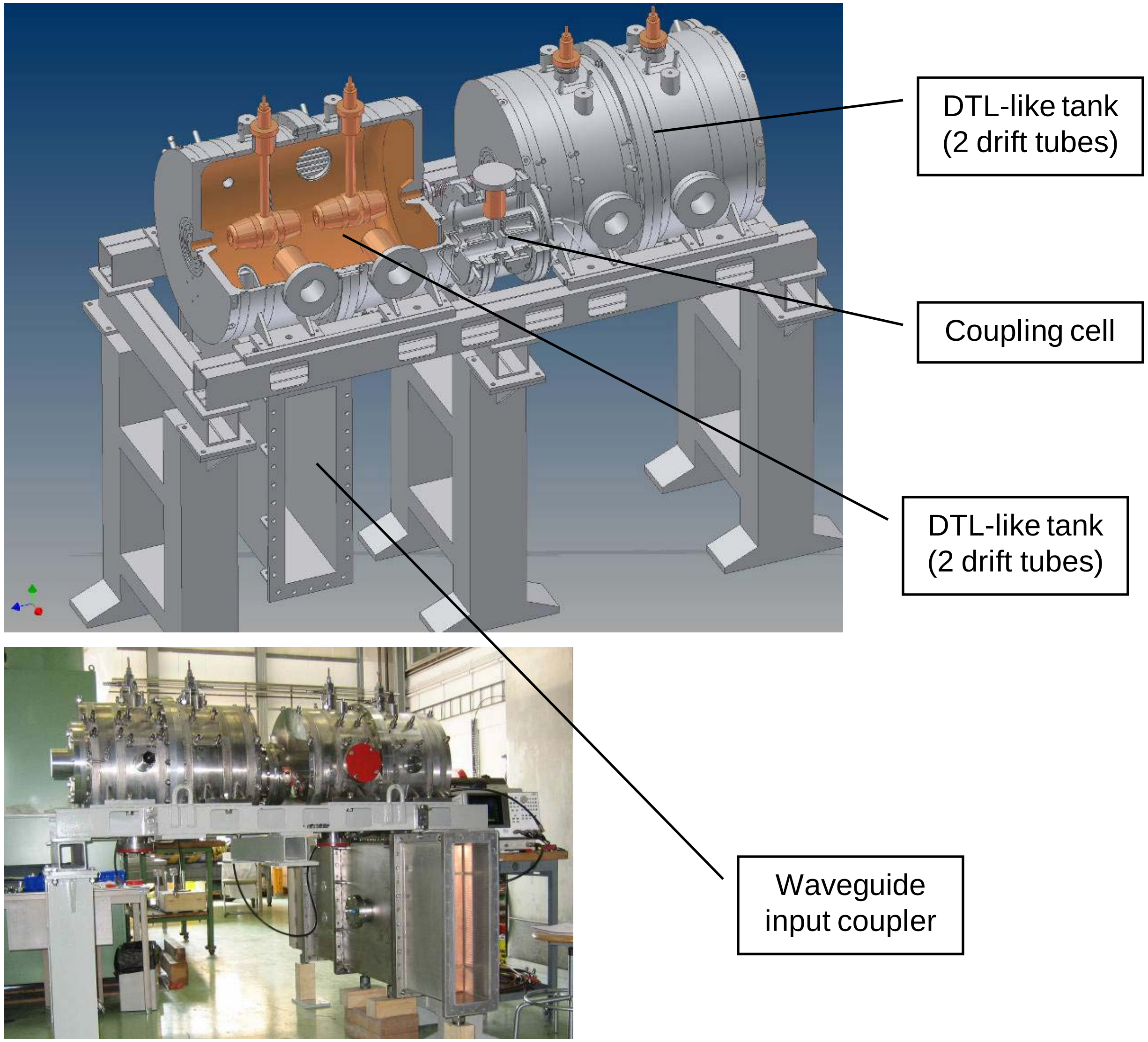

Annular ring Coupled Structure (ACS)




Side Coupled Structure (SCS)



# The Cell-Coupled Drift Tube Linac

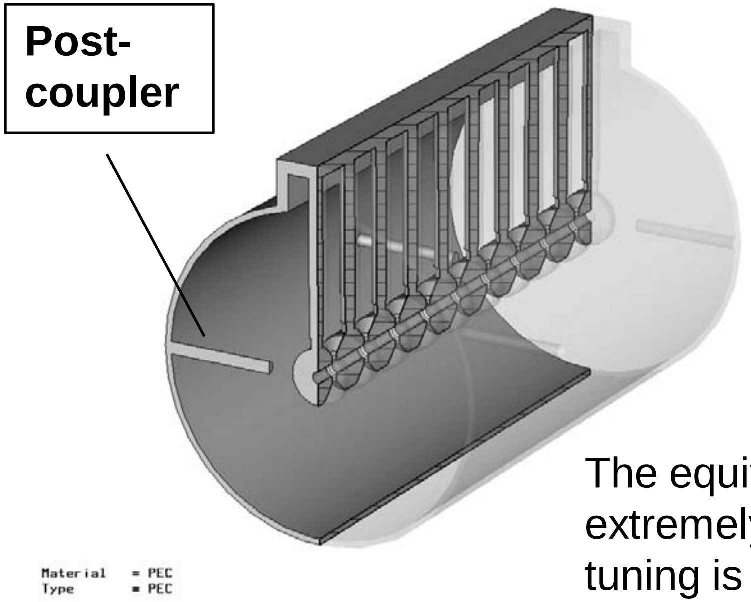



Series of DTL-like tanks with 3 cells (operating in 0-mode), coupled by coupling cells (operation in $\pi/2$ mode)

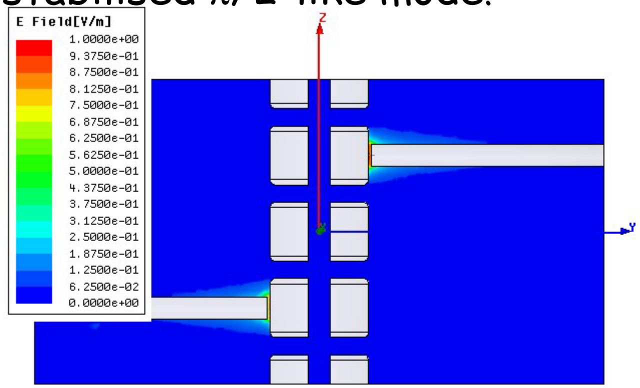
352 MHz, will be used for the CERN Linac4 in the range 40-100 MeV.

The coupling cells leave space for focusing quadrupoles between tanks.

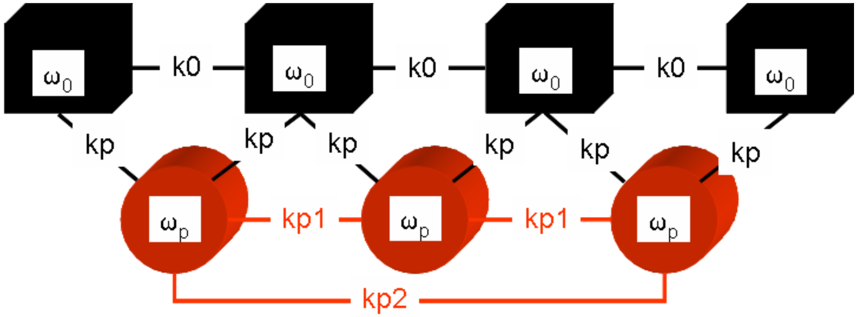
# The DTL with post-couplers

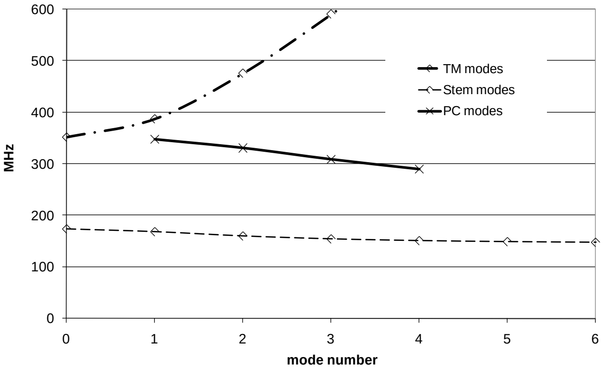



In a DTL, can be added "post-couplers" on a plane perpendicular to the stems.
Each post is a resonator that can be tuned to the same frequency as the main 0-mode and coupled to this mode to double the chain of resonators allowing operation in stabilised $\pi/2$-like mode!

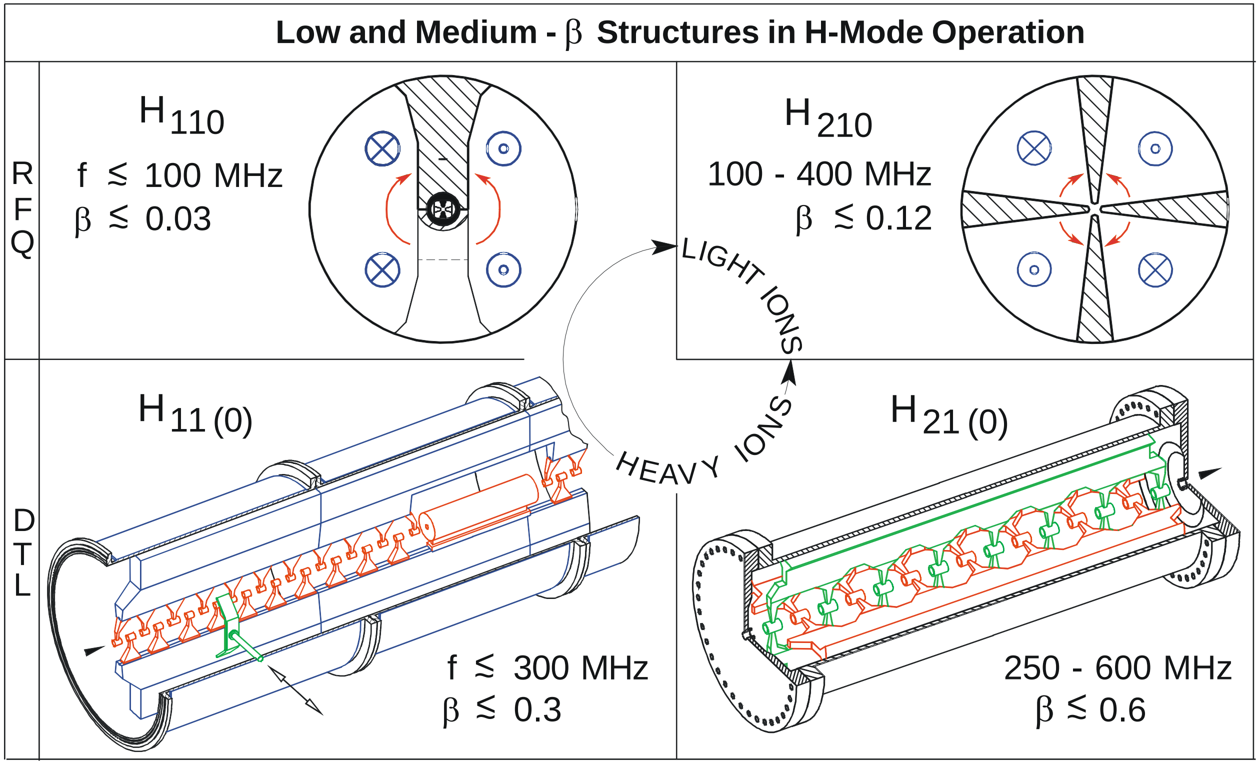



The equivalent circuit becomes extremely complicated and tuning is an issue, but $\pi/2$ stabilization is very effective and allows having long DTL tanks!



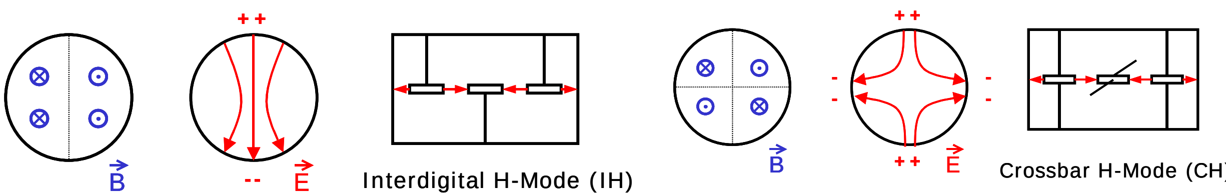
# H-mode structures



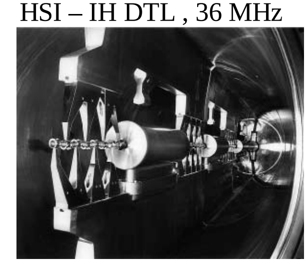

Interdigital-H Structure
Operates in TE110 mode
Transverse E-field "deflected" by adding drift tubes
Used for ions, β<0.3

CH Structure operates in TE210, used for protons at β<0.6

High $ZT^2$ but more difficult beam dynamics (no space for quads in drift tubes)



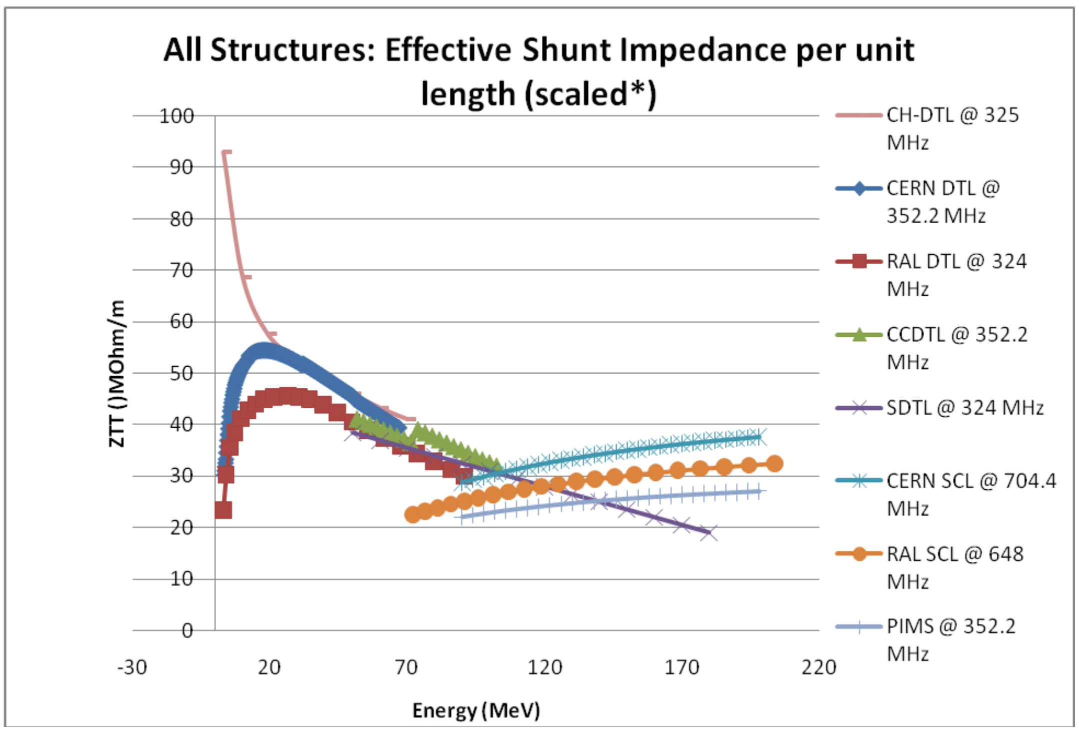

HSI – IH DTL , 36 MHz



# Comparison of structures – Shunt impedance

Main figure of merit is the shunt impedance
Ratio between energy gain (square) and power dissipation, is a measure of the energy efficiency of a structure.
Depends on the beta, on the energy and on the mode of operation.

However, the choice of the best accelerating structure for a certain energy range depends also on beam dynamics and on construction cost.




*A "fair" comparison of shunt-impedances for different low-beta structures done in 2005-08 by the "HIPPI" EU-funded Activity.*

*In general terms, a DTL-like structure is preferred at low-energy, and $\pi$-mode structures at high-energy.*
*CH is excellent at very low energies (ions).*

# 3. Superconducting low-beta structures

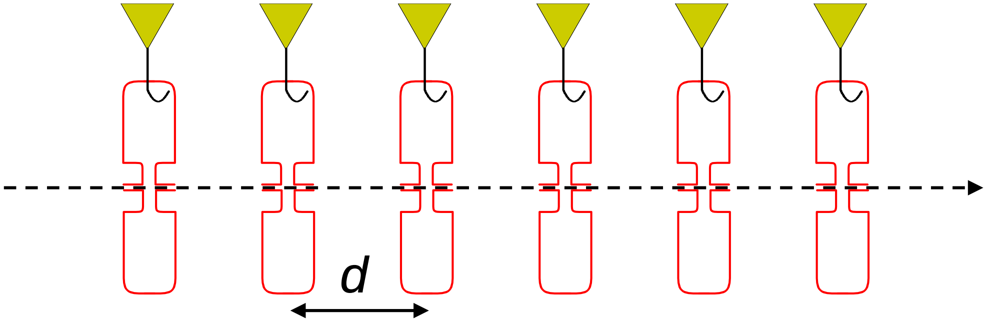
CAS

# General architecture

For Superconducting structures:

1. Shunt impedance and power dissipation are not a concern.
2. Power amplifiers are small (and relatively inexpensive) solid-state units.

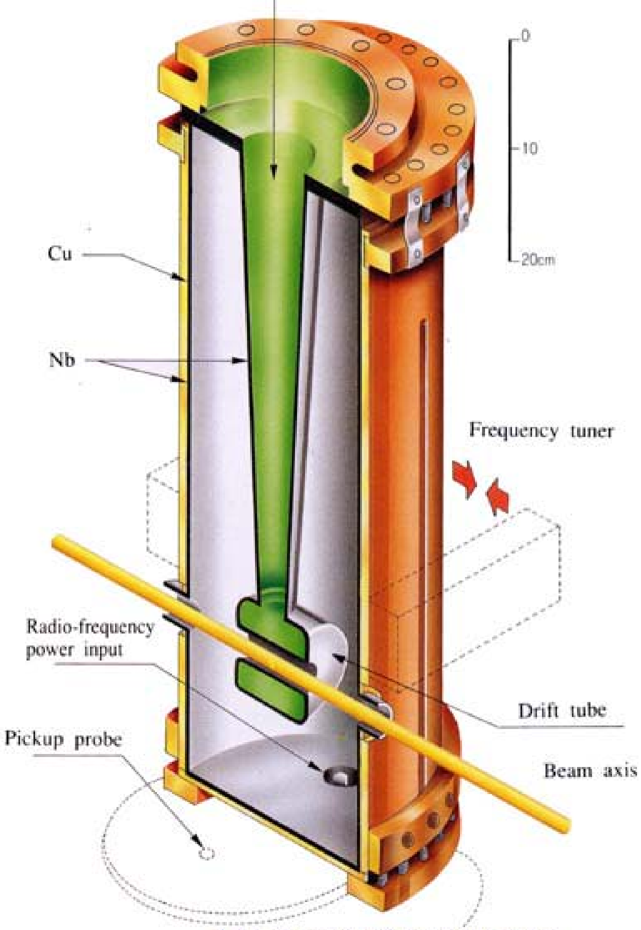
→ can be used the independent cavity architecture, which allows some flexibility in the range of beta and e/m of the particles to be accelerated.




In particular, SC structures are convenient for machines operating at high duty cycle. At low duty, static cryogenic losses are predominant (many small cavities!)

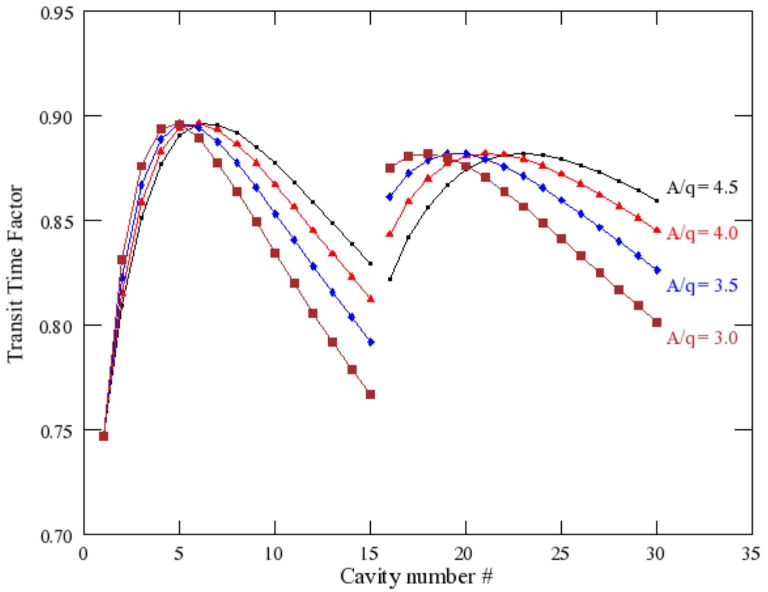
However, even for SC linacs single-gap cavities are expensive to produce (and lead to larger cryogenic dissipation) → double or triple-gap resonators are commonly used!

# Quarter Wave Resonators



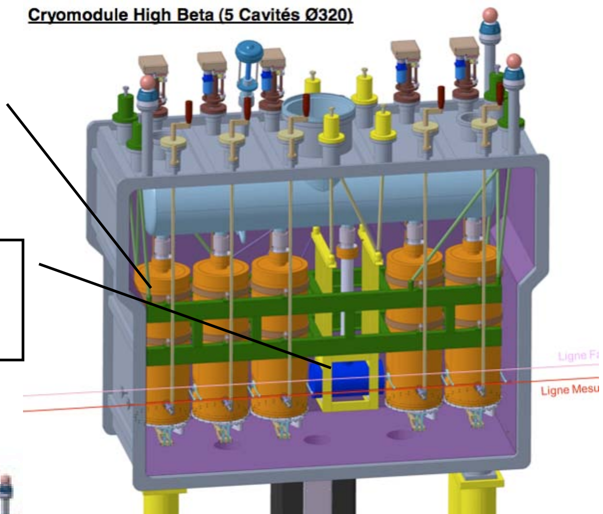

▲ Cross section of the quarter wave superconducting resonator

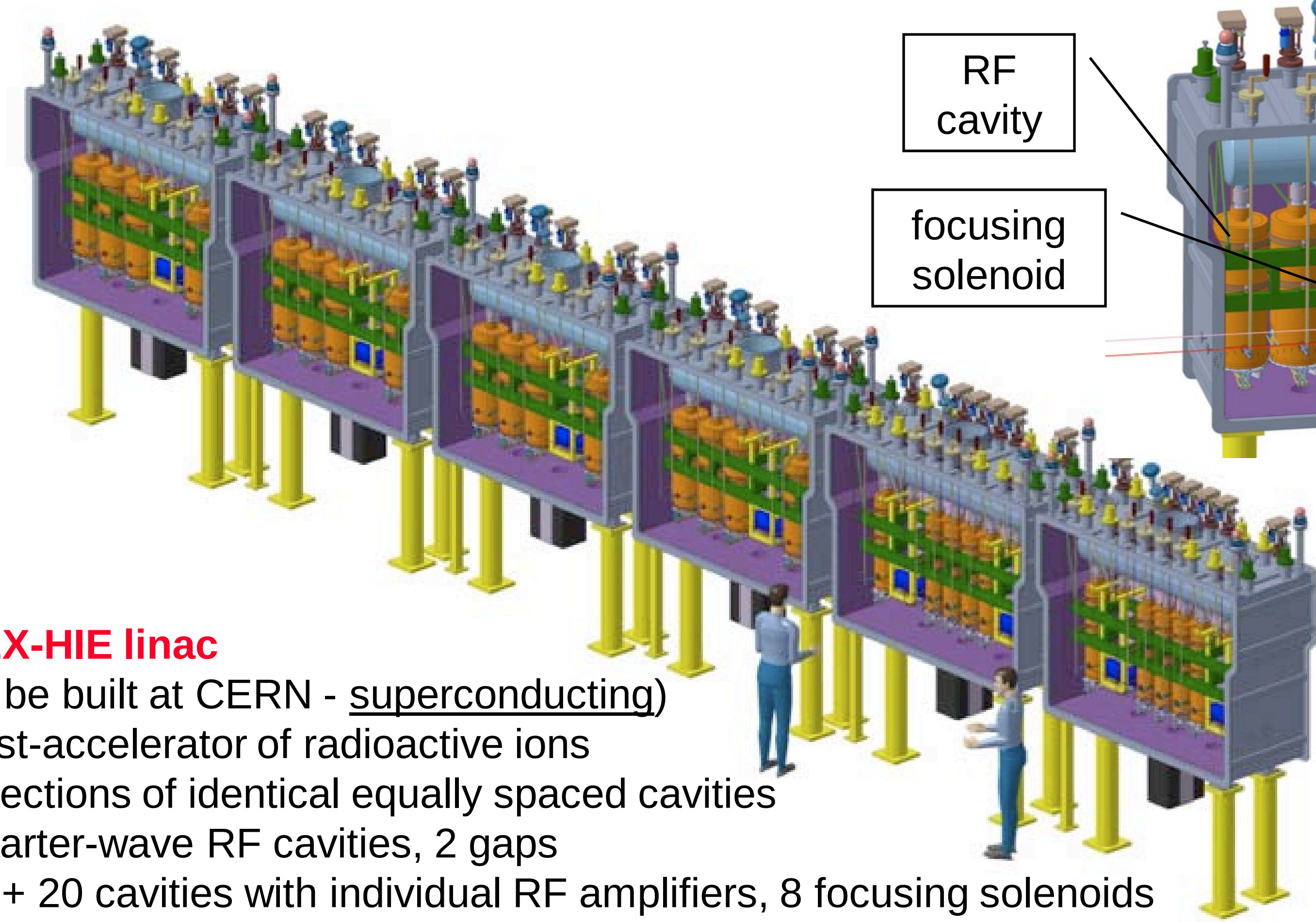
Simple 2-gap cavities commonly used in SC version (lead, niobium, sputtered niobium) for low beta protons or ion linacs, where ~CW operation is required.
Synchronicity (distance $\beta\lambda/2$ between the 2 gaps) is guaranteed only for one energy/velocity, while for easiness of construction a linac is composed by series of identical QWR's → reduction of energy gain for "off-energy" cavities, Transit Time Factor (= ratio between actual energy gained and maximum energy gain) curves as below:
"phase slippage"



# An example of SC linac

*The goal is flexibility: acceleration of different ions (e/m) at different energies*
*→ need to change phase relation for each ion-energy*



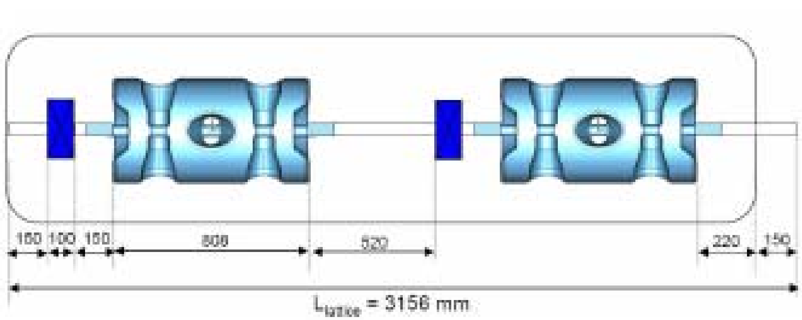



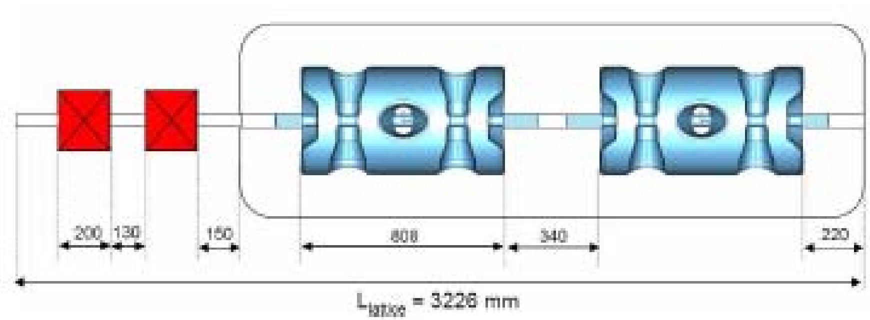

**REX-HIE linac**
(to be built at CERN - superconducting)
Post-accelerator of radioactive ions
2 sections of identical equally spaced cavities
Quarter-wave RF cavities, 2 gaps
12 + 20 cavities with individual RF amplifiers, 8 focusing solenoids
Energy 1.2 → 10 MeV/u, accelerates different A/m

# The Spoke cavity

Another option:
Double or triple-spoke cavity, can be used at higher energy (100-200 MeV for protons and triple-spoke).

*HIPPI Triple-spoke cavity prototype built at FZ Jülich, now under test at IPNO*

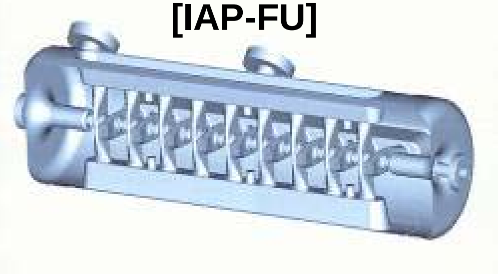


# The superconducting zoo

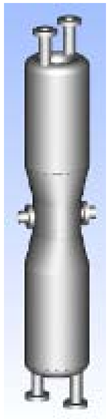
**Spoke (low beta)**
**[FZJ, Orsay]**

**CH (low/medium beta)**
**[IAP-FU]**



**QWR (low beta)**
**[LNL, etc.]**

**HWR (low beta)**
**[FZJ, LNL, Orsay]**



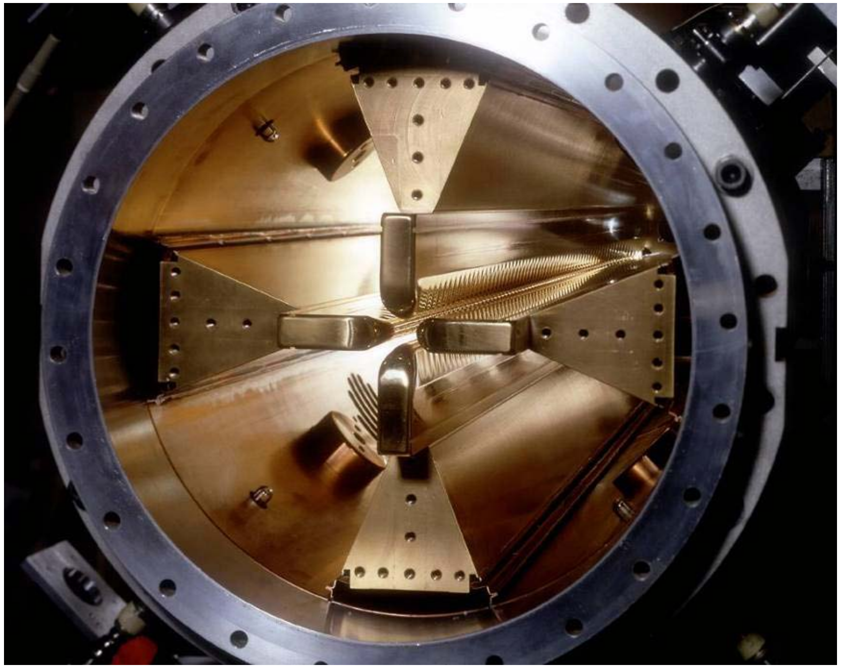
**Re-entrant**
**[LNL]**

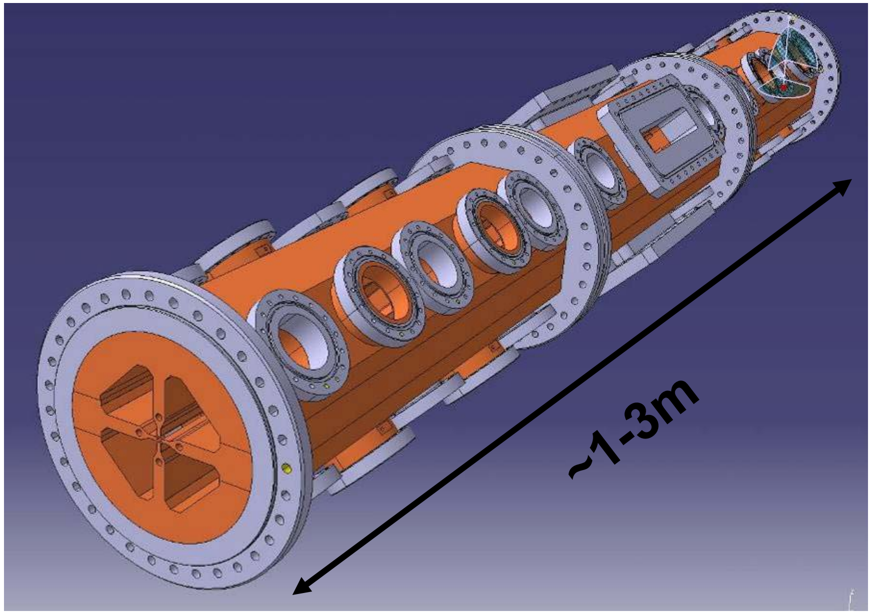
Superconducting linacs for low and medium beta ions are made of multi-gap (1 to 4) individual cavities, spaced by focusing elements. Advantages:
can be individually phased → linac can accept different ions
Allow more space for focusing → ideal for low $\beta$ CW proton linacs

# 4. The Radio Frequency Quadrupole

# The Radio Frequency Quadrupole (RFQ)

RFQ = Electric quadrupole focusing channel + bunching + acceleration





# The basic RFQ principle

1. Four electrodes (vanes) between which is excited a quadrupole RF mode $\rightarrow$ Electric focusing channel.

+

2. The vanes have a longitudinal modulation with period $\beta\lambda \rightarrow$ longitudinal component of the electric field. The modulation corresponds exactly to a series of RF gaps and can provide acceleration.

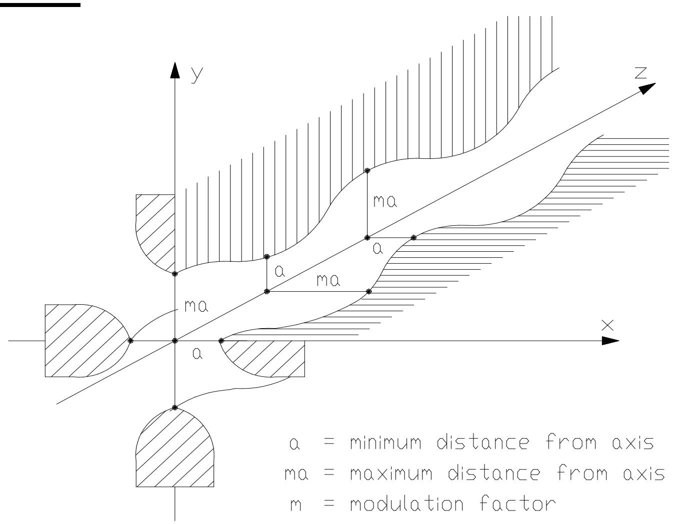
– –

+

3. The modulation period is slightly adjusted to control the phase of the beam in each cell, the amplitude of the modulation is changed to change the accelerating gradient $\rightarrow$ adiabatic bunching channel.

–

+

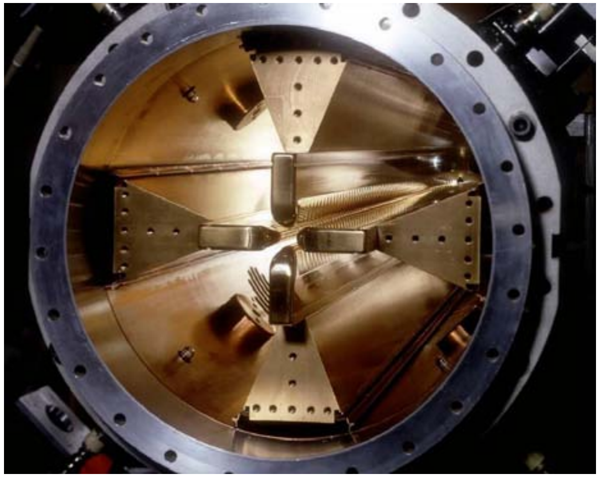


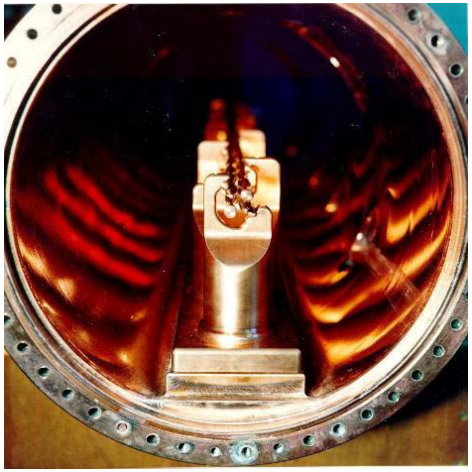

**Opposite vanes (180º)** **Adjacent vanes (90º)**

# The RFQ resonator

How to produce on the electrodes the quadrupole RF field?

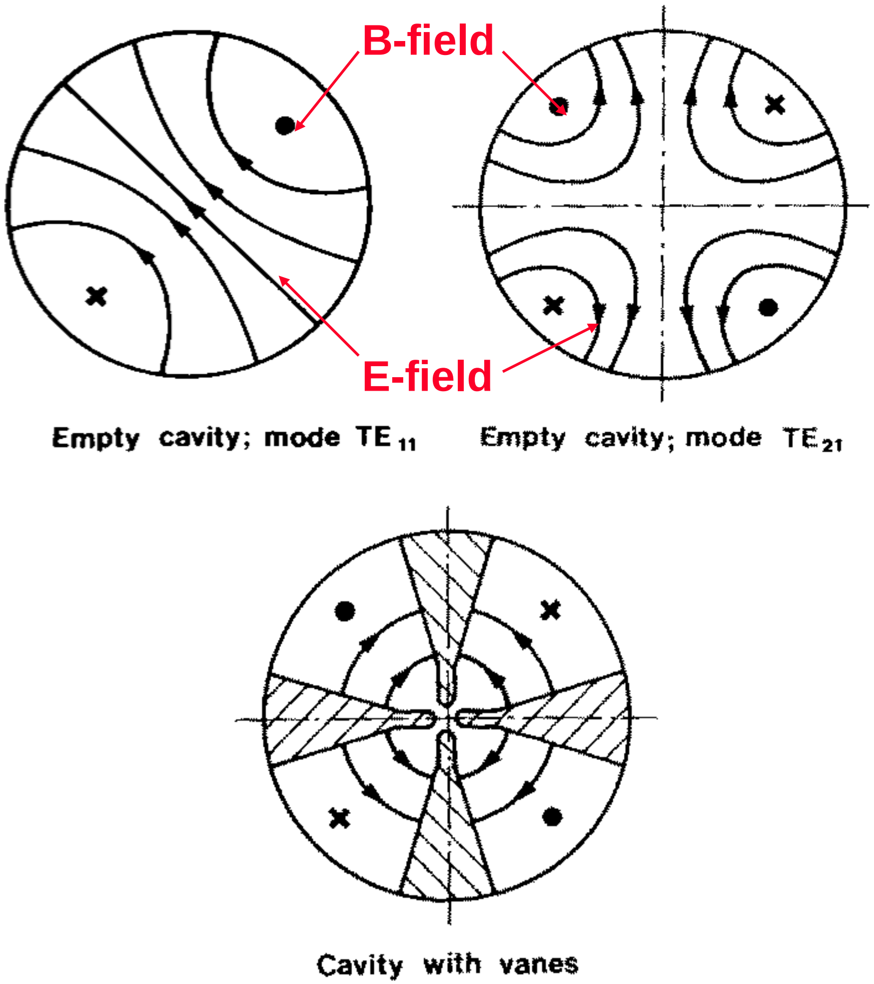
2 main families of resonators: 4-vane and 4-rod structures





plus some more exotic options (split-ring, double-H, etc.)

# The "4-vane" RFQ




Basic idea:

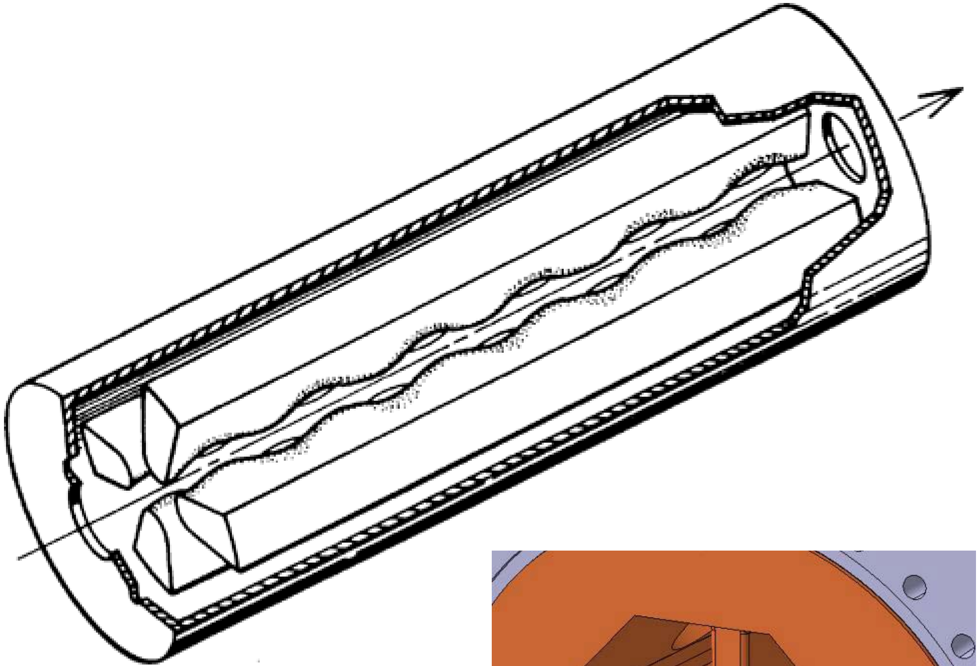
Use the TE family of modes in a cylindrical resonator, and in particular the "quadrupole" mode, TE210.

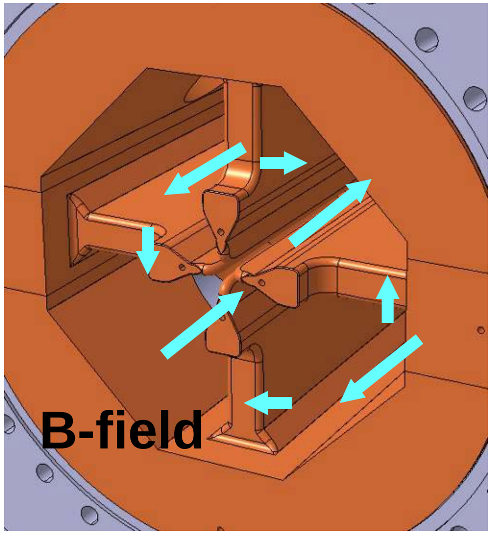
The introduction of 4 electrodes along the RFQ (the "vanes") "loads" the TE210 mode, with 2 effects:

- Concentrate the electric field on the axis, increasing the efficiency.
- Lower the frequency of the TE210 mode, separating it from the other modes of the cylinder.

Unfortunately, the dipole mode TE110 is lowered as well, and remains as a perturbing mode in this type of RFQs.

# The 4-vane RFQ



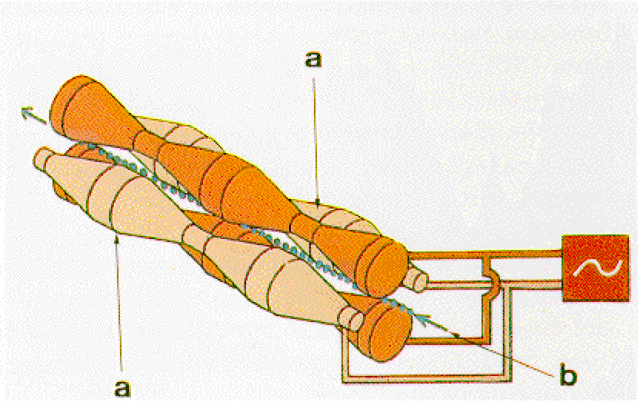


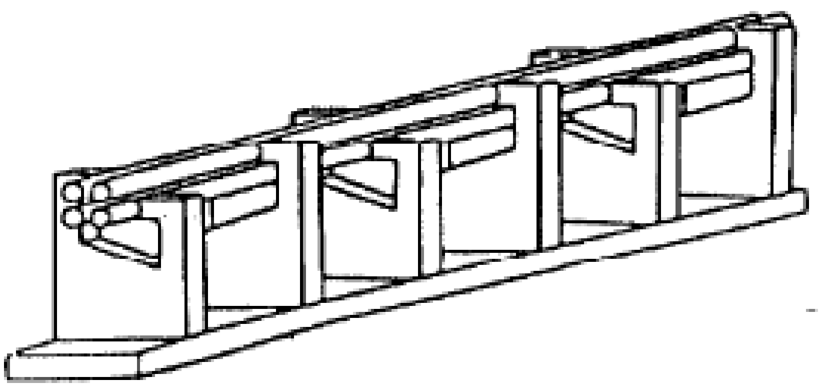

The RFQ will result in cylinder containing the 4 vanes, which are connected (with large RF currents flowing transversally!) to the cylinder along their length.

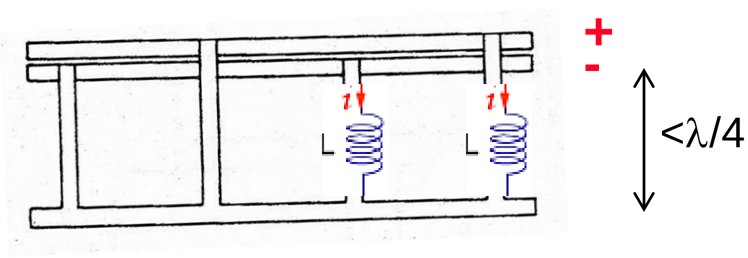
A critical feature of this type of RFQs are the end cells:
The magnetic field flowing longitudinally in the 4 “quadrants” has to close its path and pass from one quadrant to the next via some openings at the end of the vanes, tuned at the RFQ frequency!

→ the “end cells” have to be tuned and carefully designed in 3D.

# The 4-rod RFQ





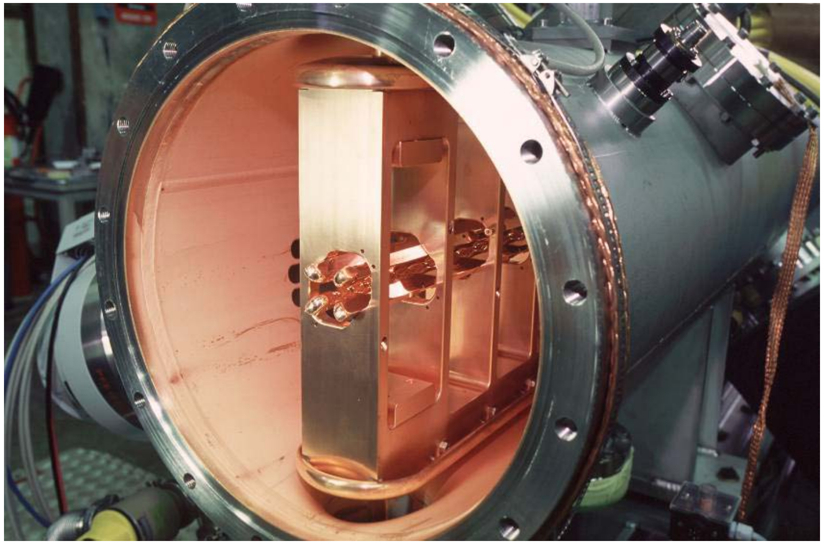

An alternative solution is to machine the modulation not on the tip of an electrode, but on a set of rods (simple machining on a lathe).

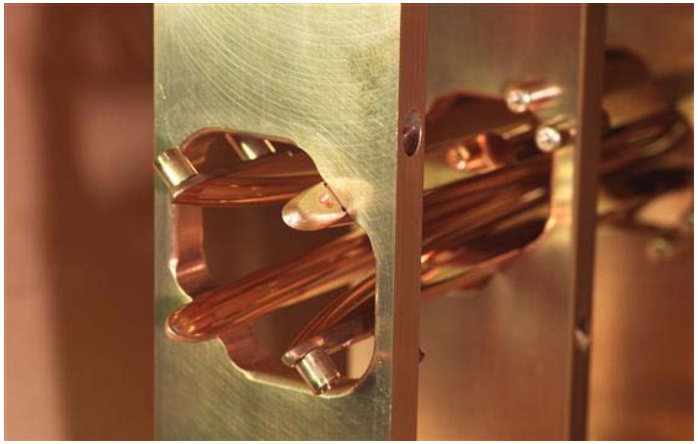
The rods can then be brought to the correct quadrupole potential by an arrangement of quarter-wavelength transmission lines. The set-up is then inserted into a cylindrical tank.

Cost-effective solution, becomes critical at high frequencies → dimensions become small and current densities go up.

# Other 4-rod geometries

The electrodes can also be "vane-like" in structures using doubled $\lambda/4$ parallel plate lines to create the correct fields.





# Conclusions & Summary

- The zoo of low-beta structures is justified by the need to cope with beams of different velocity (and often different q/m and operating at different duty cycle!).

- Multi-cell coupled normal-conducting structures allow for an efficient use of the RF power sources, but are characterized by a large number of modes and tend to be unstable when cell number becomes large. Stabilizing schemes can be introduced at the cost of increasing mechanical complexity.

- Superconducting structures may have few (1, 2 or 3) cells. They can operate with different ions and variable energy. For protons, they are economically convenient for large duty cycles and high energies, the transition NC/SC depending on the specific application.

- Radio Frequency Quadrupoles are special RF structures used at very low beta, whose main functions are focusing and bunching.