RF Engineering Basic Concepts: S-Parameters

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CAS 2010, Aarhus

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S-parameters (1)

- The abbreviation *S* has been derived from the word *scattering*.
- For high frequencies, it is convenient to describe a given network in terms of *waves* rather than voltages or currents. This permits an easier definition of reference planes.
- For practical reasons, the description in terms of in- and outgoing waves has been introduced.
- Now, a 4-pole network becomes a 2-port and a 2n-pole becomes an n-port. In the case of an odd pole number (e.g. 3-pole), a common reference point may be chosen, attributing one pole equally to two ports. Then a 3-pole is converted into a (3+1) pole corresponding to a 2-port.
- As a general conversion rule for an odd pole number one more pole is added.

S-parameters (2)

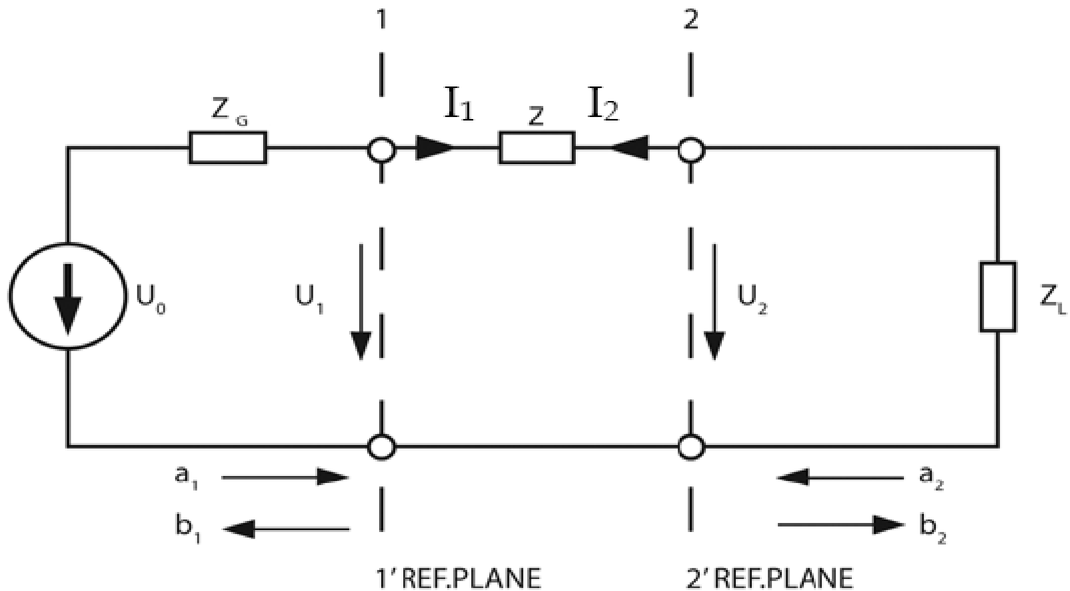


Fig. 1 2-port network

- Let us start by considering a simple 2-port network consisting of a single impedance Z connected in series (Fig. 1). The generator and load impedances are Z_G and Z_L, respectively. If $Z = 0$ and $Z_L = Z_G$ (for real Z_G) we have a matched load, i.e. *maximum available power* goes into the load and $U_1 = U_2 = U_0/2$.
- Please note that *all the voltages and currents are peak values*. The lines connecting the different elements are supposed to have zero electrical length. Connections with a finite electrical length are drawn as double lines or as heavy lines. Now we need to relate U_0, U_1 and U_2 with a and b.

Definition of " power waves"(1)

- The *waves* going *towards* the n-port are a = (a_1, a_2, ..., a_n), the *waves* travelling *away* from the n-port are b = (b_1, b_2, ..., b_n). By definition currents going *into* the n-port are counted positively and currents flowing out of the n-port negatively. The wave a_1 is going into the n-port at port 1 is derived from the voltage wave going into a matched load.
- In order to make the definitions consistent with the conservation of energy, the voltage is normalized to . Z_0 is in general an arbitrary reference impedance, but usually the characteristic impedance of a line (e.g. Z_0 = 50 Ω) is used and very often $Z_G = Z_L = Z_0$. In the following we assume Z_0 to be real. The definitions of the waves a_1 and b_1 are
-

$$a_1 = \frac{U_0}{2\sqrt{Z_0}} = \frac{incident\ voltage\ wave\ (port\ 1)}{\sqrt{Z_0}} = \frac{U_1^{inc}}{\sqrt{Z_0}}$$

$$b_1 = \frac{U_1^{refl}}{\sqrt{Z_0}} = \frac{reflected\ voltage\ wave\ (port\ 1)}{\sqrt{Z_0}}$$

- Note that a and b have the dimension $\sqrt{\text{power}}$ [1].

Definition of “power waves”(2)

- The power travelling towards port 1, P_1^{inc}, is simply the available power from the source, while the power coming out of port 1, P_1^{refl}, is given by the reflected voltage wave.

$$P_1^{inc} = \frac{1}{2}|a_1|^2 = \frac{\left|U_1^{inc}\right|^2}{2Z_0} = \frac{\left|I_1^{inc}\right|^2}{2} Z_0$$

$$P_1^{refl} = \frac{1}{2}|b_1|^2 = \frac{U_1^{refl}}{2Z_0} = \frac{\left|I_1^{refl}\right|^2}{2} Z_0$$

- Please note the factor 2 in the denominator, which comes from the definition of the voltages and currents as peak values (“European definition”). In the “US definition” effective values are used and the factor 2 is not present, so for power calculations it is important to check how the voltages are defined. For most applications, this difference does not play a role since ratios of waves are used.
- In the case of a mismatched load Z_L there will be some power reflected towards the 2-port from Z_L

$$P_2^{inc} = \frac{1}{2}|a_2|^2$$

Definition of “power waves”(3)

- There is also the outgoing wave of port 2 which may be considered as the superimposition of a wave that has gone through the 2-port from the generator and a reflected part from the mismatched load. We have defined with the incident voltage wave U^{inc}. In analogy to that we can also quote with the incident current wave I^{inc}. We obtain the *general definition* of the waves a_i travelling into and b_i travelling *out of* an n-port:
- Solving these two equations, U_i and I_i can be obtained for a given a_i and b_i as

$$a_i = \frac{U_i + I_i Z_0}{2\sqrt{Z_0}}$$

$$b_i = \frac{U_i - I_i Z_0}{2\sqrt{Z_0}}$$

$$U_i = \sqrt{Z_0}\left(a_i + b_i\right) = U_i^{inc} + U_i^{refl}$$

$$I_i = \frac{1}{\sqrt{Z_0}}\left(a_i - b_i\right) = \frac{U_i^{refl}}{Z_0}$$

- For a harmonic excitation $u(t)$ = Re{$U\, e^{\,j\omega t}$} the power going *into* port *i* is given by

$$P_i = \frac{1}{2} Re\left\{U_i I_i^*\right\}$$

$$P_i = \frac{1}{2} Re\left\{\left(a_i a_i^* - b_i b_i^*\right) + \left(a_i^* b_i - a_i b_i^*\right)\right\}$$

$$P_i = \frac{1}{2}\left(a_i a_i^* - b_i b_i^*\right)$$

- The term ($a_i^* b_i - a_i b_i^*$) is a purely imaginary number and vanishes when the real part is taken

The S-Matrix (1)

- The relation between a_i and b_i (i = l...n) can be written as a system of n linear equations (a_i being the independent variable, b_i the dependent variable)

$$\begin{aligned} b_1 &= S_{11}a_1 + S_{12}a_2 \\ b_2 &= S_{21}a_1 + S_{22}a_2 \end{aligned}$$

(2.7) *Or in matrix formulation* $\mathbf{b} = \mathbf{S}\mathbf{a}$

- The physical meaning of S_{11} is the input reflection coefficient with the output of the network terminated by a matched load ($a_2 = 0$). S_{21} is the forward transmission (from port 1 to port 2), S_{12} the reverse transmission (from port 2 to port 1) and S_{22} the output reflection coefficient.
- When measuring the S parameter of an n-port, *all* n ports must be terminated by a matched load (not necessarily equal value for all ports), including the port connected to the generator (matched generator).
- Using Eqs. 2.4 and 2.7 we find the reflection coefficient of a single impedance Z_L connected to a generator of source impedance Z_0 (Fig. 1, case $Z_G = Z_0$ and $Z = 0$)

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} = \frac{U_1 - I_1 Z_0}{U_1 + I_1 Z_0} = \frac{Z_\mathrm{L} - Z_0}{Z_\mathrm{L} + Z_0} = \rho = \frac{(Z_\mathrm{L}/Z_0) - 1}{(Z_\mathrm{L}/Z_0) + 1}$$

- which is the familiar formula for the reflection coefficient ρ (often also denoted Γ).

The S-Matrix (2)

- The S-matrix is only defined if all ports are matched (=terminated with the necessary load)
- This is important for measurements and simulation!
- The matching load does not have to be equal for all ports! (not necessarily always 50 Ohm)
- Example: Transformer with N1:N2=1:2
  - The impedance scales with the square of this ratio
  - if there is a 50 Ohm impedance on one side, the other side has either an impedance of 12.5 Ohm (down transformer) or 200
- The entries in the S-Matrix can have a different format such as length and phase, real and imaginary part etc. More information will be given in the lecture on Measurements

The S-Matrix (3)

- Let us now determine the S parameters of the impedance *Z* in Fig. 1, assuming again $Z_G = Z_L = Z_0$. From the definition of S_{11} we have

$$S_{11} = \frac{b_1}{a_1} = \frac{U_1 - I_1 Z_0}{U_1 + I_1 Z_0}$$

$$U_1 = U_0 \frac{Z_0 + Z}{2Z_0 + Z}, \quad U_2 = U_0 \frac{Z_0}{2Z_0 + Z}, \quad I_1 = \frac{U_0}{2Z_0 + Z} = -I_2$$

$$\Rightarrow S_{11} = \frac{Z}{2Z_0 + Z}$$

The S-Matrix (4)

- and in a similar fashion we get

$$S_{21} = \frac{b_2}{a_1} = \frac{U_2 - I_2 Z_0}{U_1 + I_1 Z_0} = \frac{2Z_0}{2Z_0 + Z}$$

- This formula has been widely used for impedance evaluation using the coaxial wire method. But only for lumped elements.

$$\mathbf{S} = \begin{pmatrix} \frac{Z}{2Z_0 + Z} & \frac{Z_0 + Z}{2Z_0 + Z} \\ \frac{Z_0 + Z}{2Z_0 + Z} & \frac{Z}{2Z_0 + Z} \end{pmatrix}$$

- Due to the symmetry of the element $S_{22} = S_{22}$ and $S_{12} = S_{21}$. Please note that for this case we obtain $S_{11} + S_{21} = 1$. The full S matrix of the element is then

The transfer matrix (T-matrix)

- The S matrix introduced in the previous section is a very convenient way to describe an n-port in terms of waves. It is very well adapted to measurements and simulations. However, it is not well suited to for characterizing the response of a number of **cascaded** 2-ports. A very straightforward manner for the problem is possible with the T matrix (transfer matrix), which directly relates the waves on the input and on the output [2] (see appendix)

$$\begin{pmatrix} b_1 \\ a_1 \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \begin{pmatrix} a_2 \\ b_2 \end{pmatrix}$$

- The conversion formulae between S and T matrix are given in Appendix I. While the S matrix exists for any 2-port, in certain cases, e.g. no transmission between port 1 and port 2, the T matrix is not defined. The T matrix T_M of *m* cascaded 2-ports is given by (as in [2, 3]):

$$\mathbf{T_M} = \mathbf{T_1 T_2 \dots T_m}$$

- Note that in the literature different definitions of the T matrix can be found and the individual matrix elements depend on the definition used. (see appendix)

Properties of the S matrix of an N-port

- A generalized n-port has n^2 scattering coefficients. While the S_{ij} may be all independent, in general due to symmetries etc the number of independent coefficients is much smaller.

- An n-port is *reciprocal* when $S_{ij} = S_{ji}$ for all *i* and *j*. Most passive components are reciprocal, active components such as amplifiers are generally non-reciprocal.

- A two-port is *symmetric*, when it is reciprocal ($S_{21} = S_{12}$) and when the input and output reflection coefficients are equal ($S_{22} = S_{11}$).

- An N-port is *passive and lossless* if its S matrix is *unitary*, i.e. $S^{\dagger}S = 1$, where $x^{\dagger} = (x^*)^T$ is the conjugate transpose of x.

Unitarity of an N-port

For a two-port the unitarity condition gives

$$\left(S^*\right)^T S = \begin{pmatrix} S_{11}^* & S_{21}^* \\ S_{12}^* & S_{22}^* \end{pmatrix} \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

which yields the three conditions

$$|S_{11}|^2 + |S_{21}|^2 = 1$$
$$|S_{12}|^2 + |S_{22}|^2 = 1$$
$$S_{11}^* S_{12} + S_{21}^* S_{22} = 0$$

Note: this is a necessary but not a sufficient condition! See next slide!

From the last equation we get, writing out amplitude and phase

$$|S_{11}||S_{12}| = |S_{21}||S_{22}| \quad \text{and}$$

$$-\arg S_{11} + \arg S_{12} = -\arg S_{21} + \arg S_{22} + \pi$$

[$\arg(S_{11}) = \arctan(\mathrm{Im}(S_{11})/\mathrm{Re}(S_{11}))$]

and combining the equations for the modulus (amplitude)

$$|S_{11}| = |S_{22}|, \quad |S_{12}| = |S_{21}|$$
$$|S_{11}| = \sqrt{1 - |S_{12}|^2}$$

Thus any lossless two-port can be characterized by one amplitude and three phases.

Gyrator

- Remember the two conditions

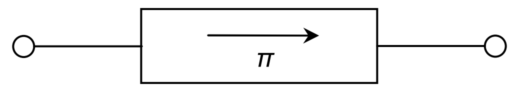
$$\begin{aligned} |S_{11}|^2 + |S_{21}|^2 &= 1 \\ |S_{12}|^2 + |S_{22}|^2 &= 1 \\ S_{11}^* S_{12} + S_{21}^* S_{22} &= 0 \end{aligned}$$

and

$$|S_{11}||S_{12}| = |S_{21}||S_{22}| \quad \text{and}$$

$$-\arg S_{11} + \arg S_{12} = -\arg S_{21} + \arg S_{22} + \pi$$

- The second one tends to be forgotten e.g. when it comes to the gyrator

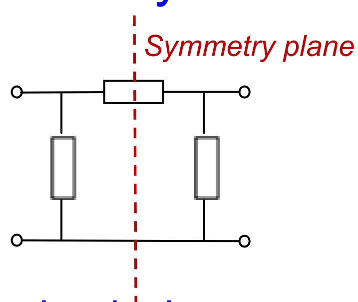


- A gyrator is a nonreciprocal phase shifter with a transmission coefficient = 1 in all directions (lossless) BUT the phase of forward transmission differs from the phase of the backward transmission by a factor π

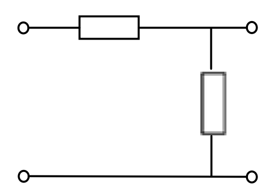
- A gyrator is realized using magnetically polarized ferrites

General properties

- In general the S-parameters are complex and frequency dependent.
- Their phases change when the reference plane is moved. Therefore it is important to clearly define the reference planes used (important both for measurements and simulations)
- For a given structure, often the S parameters can be determined from considering mechanical symmetries and, in case of lossless networks, from energy conservation. (unitary condition)
- What is mechanical symmetry?
- Example: Pi circuit



half Pi circuit



$S_{11}=S_{22}$ due to mechanical symmetry AND $S_{21}=S_{12}$ from reciprocity

$S_{11} \neq S_{22}$ due to mechanical asymmetry BUT still reciprocal $\longrightarrow$ $S_{21}=S_{12}$

Examples of S matrices: 1-ports

- Simple lumped elements are 1-ports, but also cavities with a single RF feeding port, terminated transmission lines (coaxial, waveguide, microstrip etc.) or antennas can be considered as 1-ports (Note: an antenna is basically a generalized transformer which transforms the TEM mode of the feeding line into the eigenmodes of free space. These eigenmodes can be expressed as homogenous plane waves or also as gaussian beam)

- 1-ports are characterized by their reflection coefficient ρ, or in terms of S parameters, by S_{11}.

- Ideal short: $S_{11} = -1$

- Ideal termination: $S_{11} = 0$

- Ideal open: $S_{11} = +1$

- Active termination (reflection amplifier): $|S11| > 1$

Examples of S matrices: 2-ports (1)

- Ideal transmission line of length *l*

$$\mathbf{S} = \begin{pmatrix} 0 & e^{-\gamma l} \\ e^{-\gamma l} & 0 \end{pmatrix}$$

where $\gamma = \alpha + j\beta$ is the complex propagation constant, α the line attenuation in [Neper/m] and $\beta = 2\pi/\lambda$ with the wavelength λ. For a lossless line we get $|S_{21}| = 1$.

- Ideal phase shifter

$$\mathbf{S} = \begin{pmatrix} 0 & e^{-j\varphi_{12}} \\ e^{-j\varphi_{21}} & 0 \end{pmatrix}$$

For a reciprocal phase shifter $\varphi_{12} = \varphi_{21}$, while for the gyrator $\varphi_{12} = \varphi_{21} + \pi$. As said before an ideal gyrator is lossless ($\mathbf{S}^{\dagger}\mathbf{S} = \mathbf{1}$), but it is not reciprocal. Gyrators are often implemented using active electronic components, however in the microwave range passive gyrators can be realized using magnetically saturated ferrite elements.

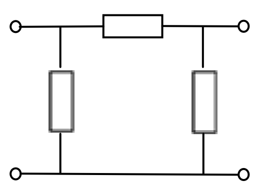
Examples of S matrices: 2-ports (2)

- Ideal attenuator

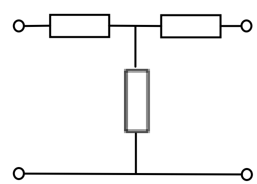
$$\mathbf{S} = \begin{pmatrix} 0 & e^{-\alpha} \\ e^{-\alpha} & 0 \end{pmatrix}$$

with the attenuation α in Neper. The attenuation in Decibel is given by A = -20*$\log_{10}(S_{21})$, 1 Np = 8.686 dB.

- An attenuator can be realized e.g. with three resistors in a T or Pi circuit



Pi circuit



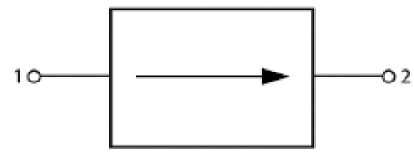
T circuit

or with resistive material in a waveguide.

Examples of S matrices: 2-ports (3)

- Ideal isolator

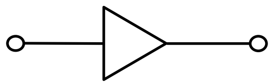
$$\mathbf{S} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$



The isolator allows transmission in one direction only, it is used e.g. to avoid reflections from a load back to the generator.

- Ideal amplifier

$$\mathbf{S} = \begin{pmatrix} 0 & 0 \\ G & 0 \end{pmatrix}$$



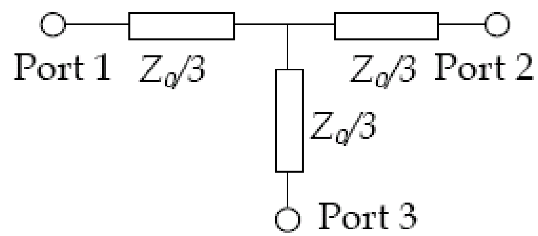
with the voltage gain $G > 1$. Please note the similarity between an ideal amplifier and an ideal isolator! Sometimes amplifiers are sold as active isolators.

Examples of S matrices: 3-ports (1)

- It can be shown that a 3-port cannot be lossless, reciprocal and matched at all three ports at the same time. The following components have two of the above characteristics.

- Resistive power divider: It consists of a resistor network and is reciprocal, matched at all ports but lossy. It can be realized with three resistors in a triangle configuration.

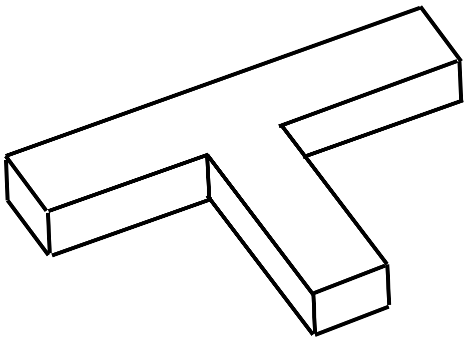
$$\mathbf{S} = \begin{pmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{pmatrix}$$

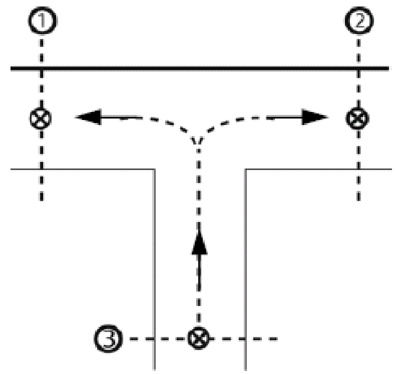


- What is the difference to the T type attenuator?

Examples of S matrices: 3-ports (2)

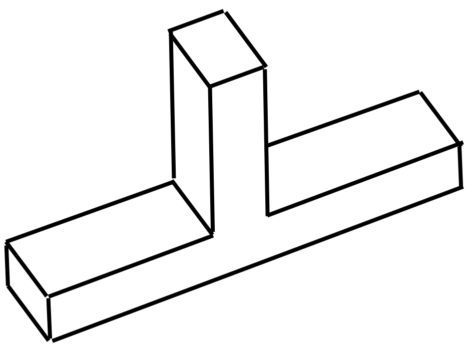
- The T splitter is reciprocal and lossless but not matched at all ports. Using the losslessness condition and symmetry considerations one finds for E and H plane splitters

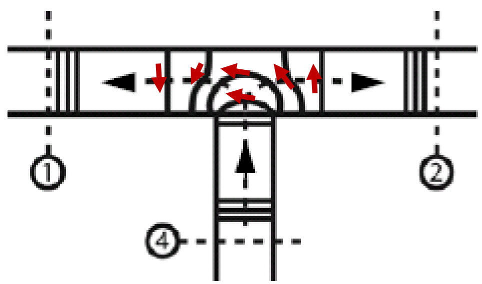




H-plane splitter

$$S_H = \frac{1}{2}\begin{pmatrix} 1 & -1 & \sqrt{2} \\ -1 & 1 & \sqrt{2} \\ \sqrt{2} & \sqrt{2} & 0 \end{pmatrix}$$



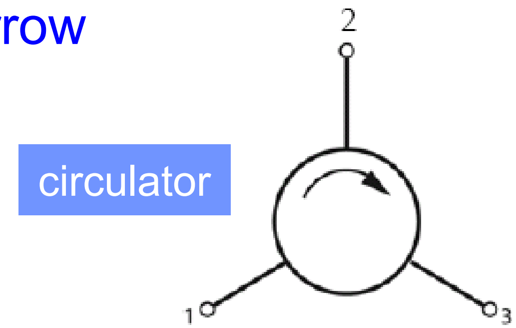


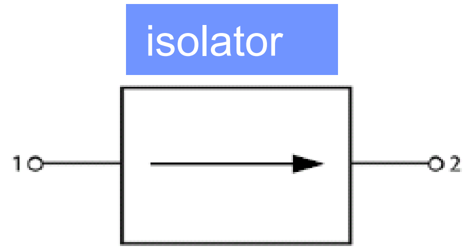
E-plane splitter

$$S_E = \frac{1}{2}\begin{pmatrix} 1 & 1 & \sqrt{2} \\ 1 & 1 & -\sqrt{2} \\ \sqrt{2} & -\sqrt{2} & 0 \end{pmatrix}$$

Circulators

- The ideal circulator is lossless, matched at all ports, but not reciprocal. A signal entering the ideal circulator at one port is transmitted *exclusively* to the next port in the sense of the arrow





- Its S matrix has a very simple form:

$$\mathbf{S} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

- When port 3 of the circulator is terminated with a matched load we get a two-port called isolator, which lets power pass only from port 1 to port 2

Ferrites for circulators (1)

- A circulator, like the gyrator and other passive non-reciprocal elements contains a volume of ferrite. This ferrite is usually magnetized into saturation by an external magnetic field.
- The magnetic properties of a saturated RF ferrite have to be characterized by a μ-tensor (Polder tensor):

$$B = \begin{bmatrix} \mu & -j\kappa & 0 \\ j\kappa & \mu & 0 \\ 0 & 0 & \mu_0 \end{bmatrix} H$$

where:

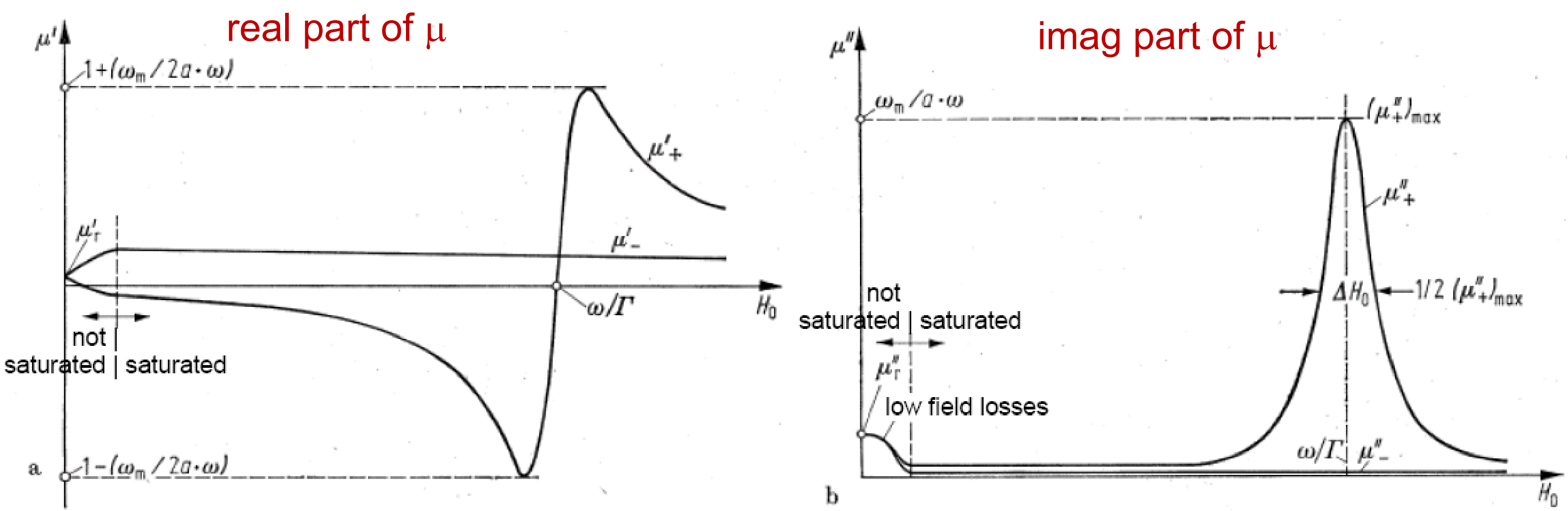
$$\mu = \mu_0 \left(1 + \frac{\omega_0 \omega_m}{\omega_0^2 - \omega^2}\right)$$
$$\kappa = \mu_0 \frac{\omega \omega_m}{\omega_0^2 - \omega^2}$$
$$\omega_0 = \gamma \mu_0 H_0$$
$$\omega_m = \gamma \mu_0 M$$

γ =28 GHz/Tesla if g=2

and $\gamma = 17.6 \cdot g$ [kHz/(A/m)] is a gyromagnetic ratio and g is a factor between 1.9-2.4 depending on ferrite material. Magnetizing frequency (f) is expressed as $\omega = 2\pi f$, H_0 is a bias field, M is magnetization and μ_0 is magnetic permeability of free space.

Ferrites for circulators (2)

- The real and imaginary part of each complex element μ are μ’ and μ’’. They are strongly dependent on the bias field and of course a function of frequency.
- The μ+ and μ- represent the permeability seen by a right- and left-hand circular polarized wave respectively traversing the ferrite (Note: These are not the elements in the Polder tensor μ and κ which stand for $\mu_\perp$ and $\mu_{/\!/}$ w. r. t. the DC magnetic bias field H0. For more details see any lecture on RF and microwave ferrites.)
- Remember that any circular polarized wave can be represented by superposition of two orthogonal linearly polarized waves.

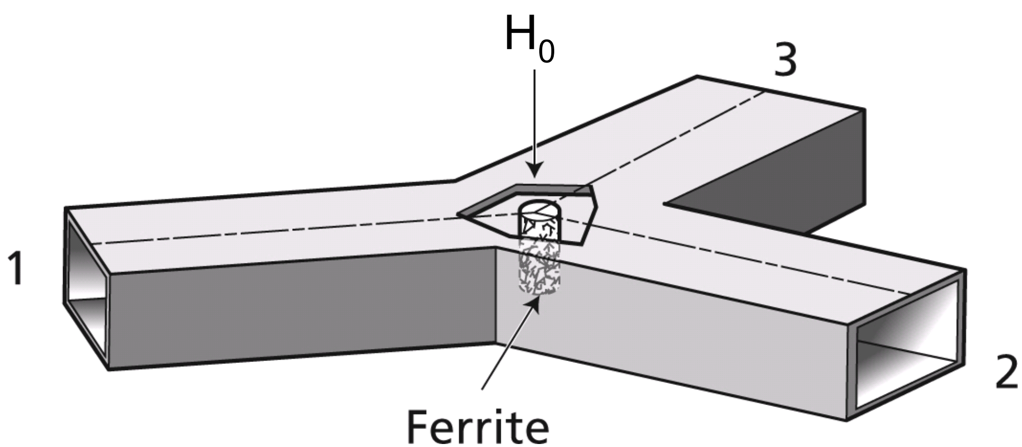


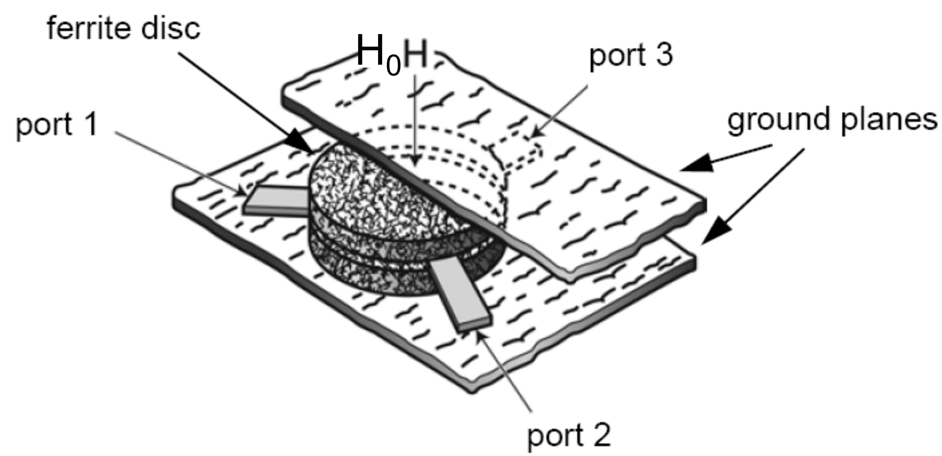
In this picture Γ is not the reflection coefficient but gyromagnetic ratio 28 GHz/T

Practical Implementations of circulators

- The magnetically polarized ferrite provides the required nonreciprocal properties. As a result, power is only transmitted from port 1 to port 2, from port 2 to port 3 and from port 3 to port 1.

- Circulators can be built e.g. with waveguides (left) or with striplines (right) and also as coaxial lumped elements

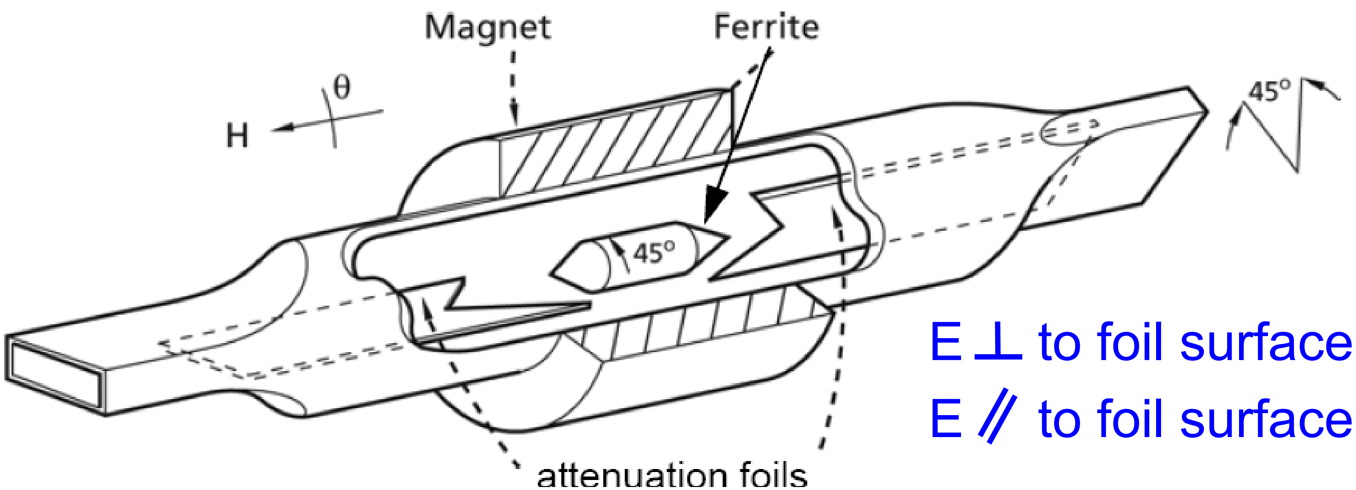




H_0: Magnetic bias field

The Faraday isolator

- The Faraday isolator is based on the Faraday rotation of a polarized wave in the presence of a magnetically polarized ferrite (used even in optical frequency range with optical transparent ferrites)
- Running along the ferrite the TE_{10} wave coming from left is rotated counterclockwise by 45 degrees. It can then enter unhindered the waveguide on the right
- A wave coming from the right is rotated clockwise (as seen from the right); the wave then has the wrong polarization. It gets nearly completely absorbed by the horizontal damping foil since the E-field is parallel to the surface of the foil.



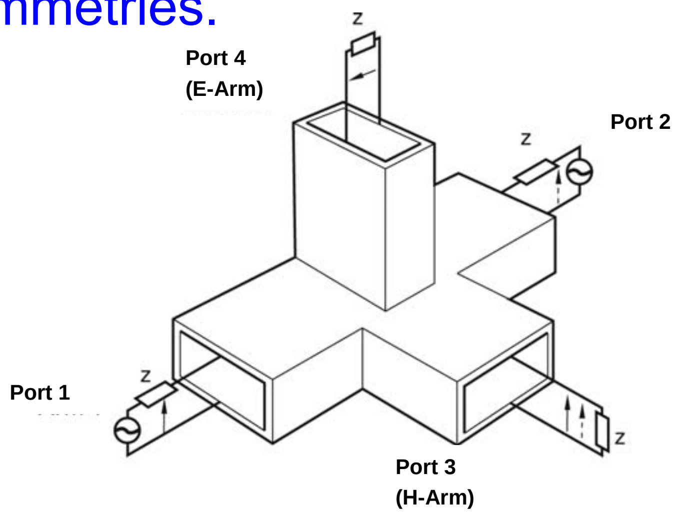
E ⊥ to foil surface: transmission

E // to foil surface: absorption

The Magic T

- A combination of a E-plane and H-plane waveguide T is a very special 4-port: A "magic" T. The coefficients of the S matrix can be found by using the unitary condition and mechanical symmetries.

$$\mathbf{S} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \\ 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{pmatrix}$$



- Ideally there is no transmission between port 3 and port 4 nor between port 1 and port 2, even though you can look straight through it!
- Magic Ts are often produced as coaxial lines and printed circuits. They can be used taking the sum or difference of two signals. The bandwidth of a waveguide magic 'T' is around one octave or the equivalent H_{10}-mode waveguide band. Broadband versions of 180° hybrids may have a frequency range from a few MHz to several GHz.

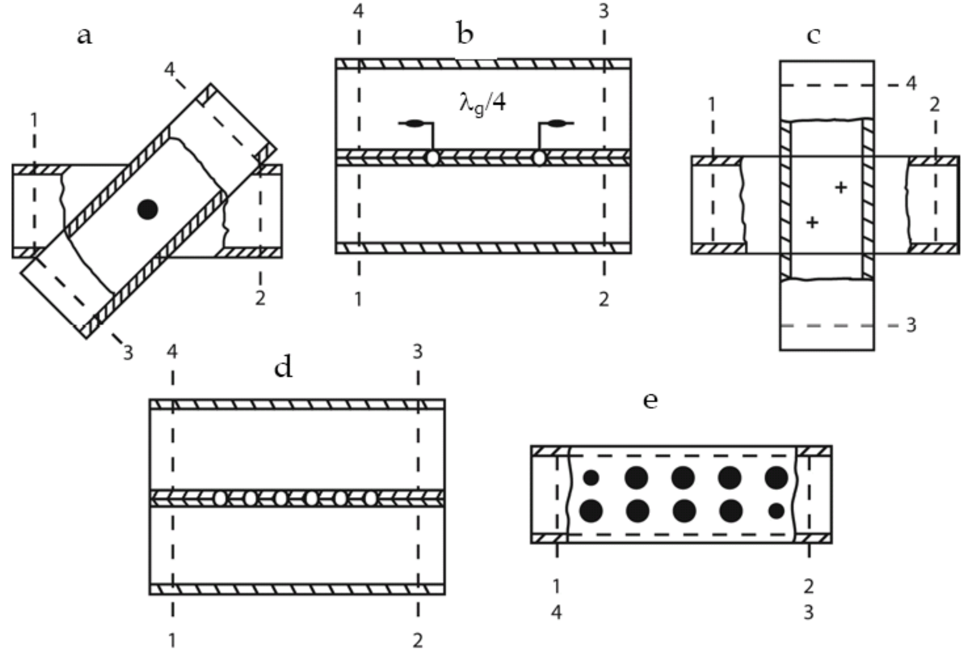
The directional coupler (1)

- Another very important 4-port is the directional coupler.
- General principle: We have two transmission lines, and a coupling mechanism is adjusted such, that part of the power in line 1 is transferred to line 2. The coupler is *directional* when the power in line 2 travels mainly in one direction.
- In order to get directionality two coupling mechanisms are necessary, i.e. many holes or electric and magnetic coupling, as in the Bethe coupler.
- The λ/4 coupler has two holes at a distance λ/4. The two backwards coupled waves cancel while the forward coupled waves add up in phase

Directional couplers with coupling holes in waveguide technology

Bethe coupler

λ/4 coupler



multiple hole coupler

The directional coupler (2)

- The *directivity* is defined as the ratio of the desired over the undesired coupled wave. For a forward coupler, in decibel,

$$\alpha_d = 20\, log \frac{|S_{31}|}{|S_{41}|}$$

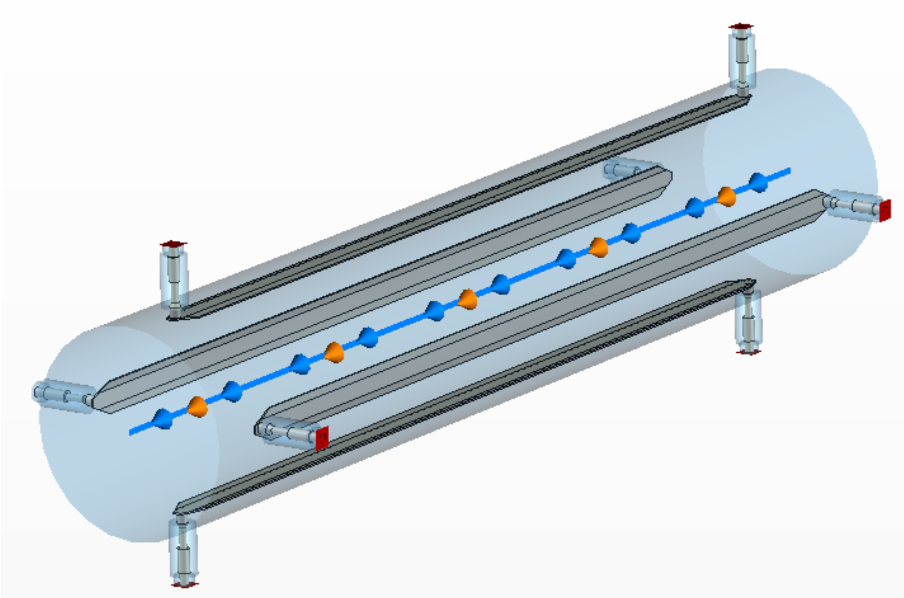
- Practical numbers for the **coupling** are 3 dB, 6 dB, 10 dB, 20 dB and higher with directivities usually better than 20 dB
- As an example the S matrix of the 3 dB coupler (π/2-hybrid) can be derived as

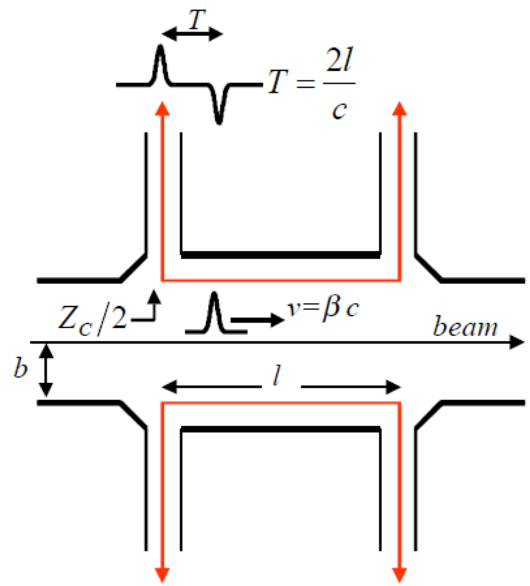
$$S_{3dB} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & \pm j & 0 \\ 1 & 0 & 0 & \pm j \\ \pm j & 0 & 0 & 1 \\ 0 & \pm j & 1 & 0 \end{pmatrix}$$

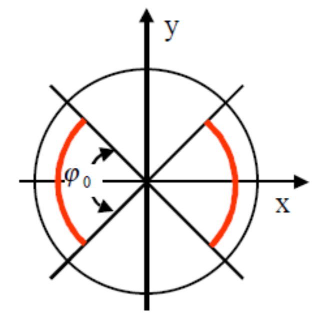
- Please note that this element is lossless, reciprocal and matched at all ports. This is possible for 4-ports.

The directional coupler (3)

- **Examples for stripline couplers:**



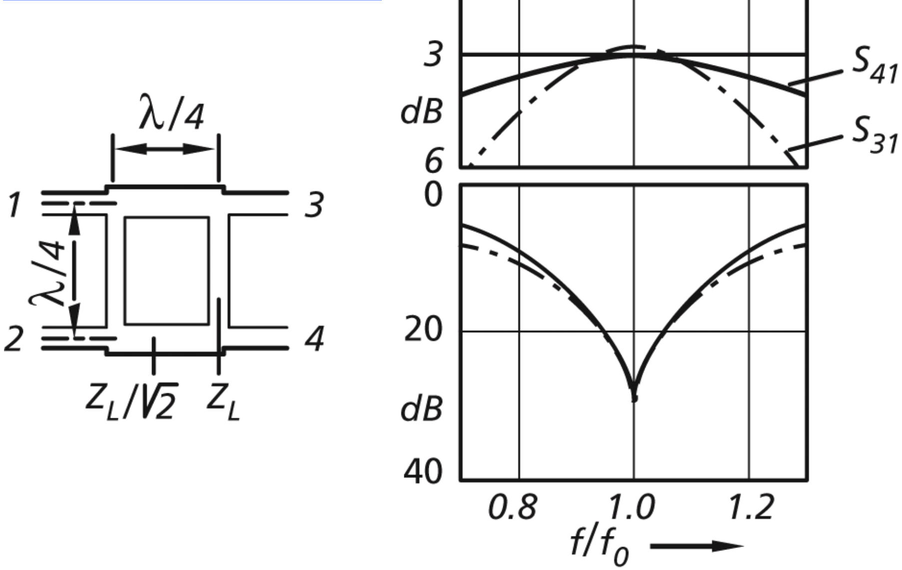




Directional couplers in microstrip technology

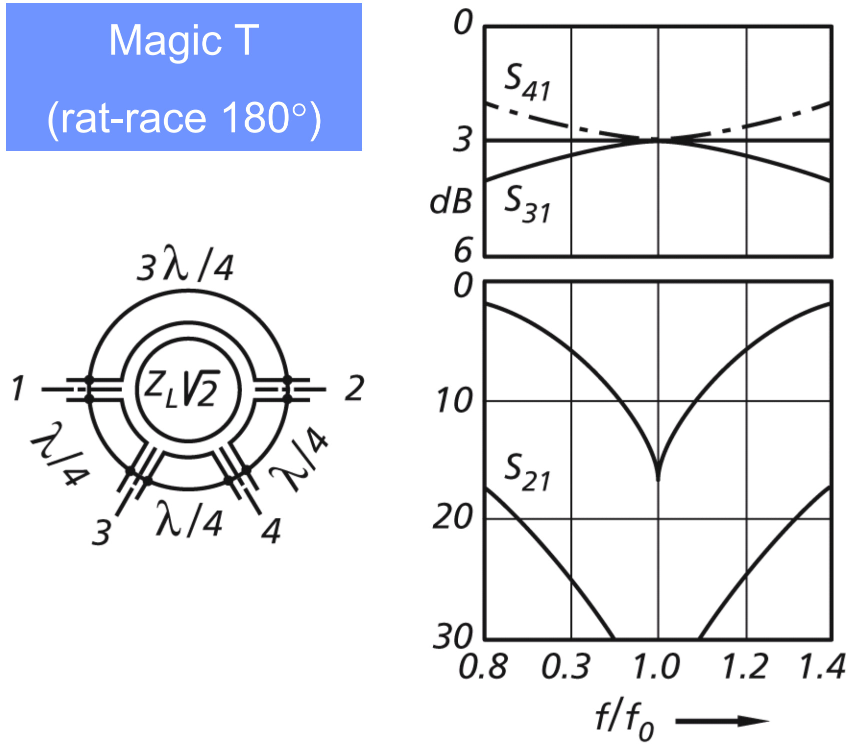
- Most of the RF elements shown can be produced at low price in printed circuit technology

90° 3-dB coupler



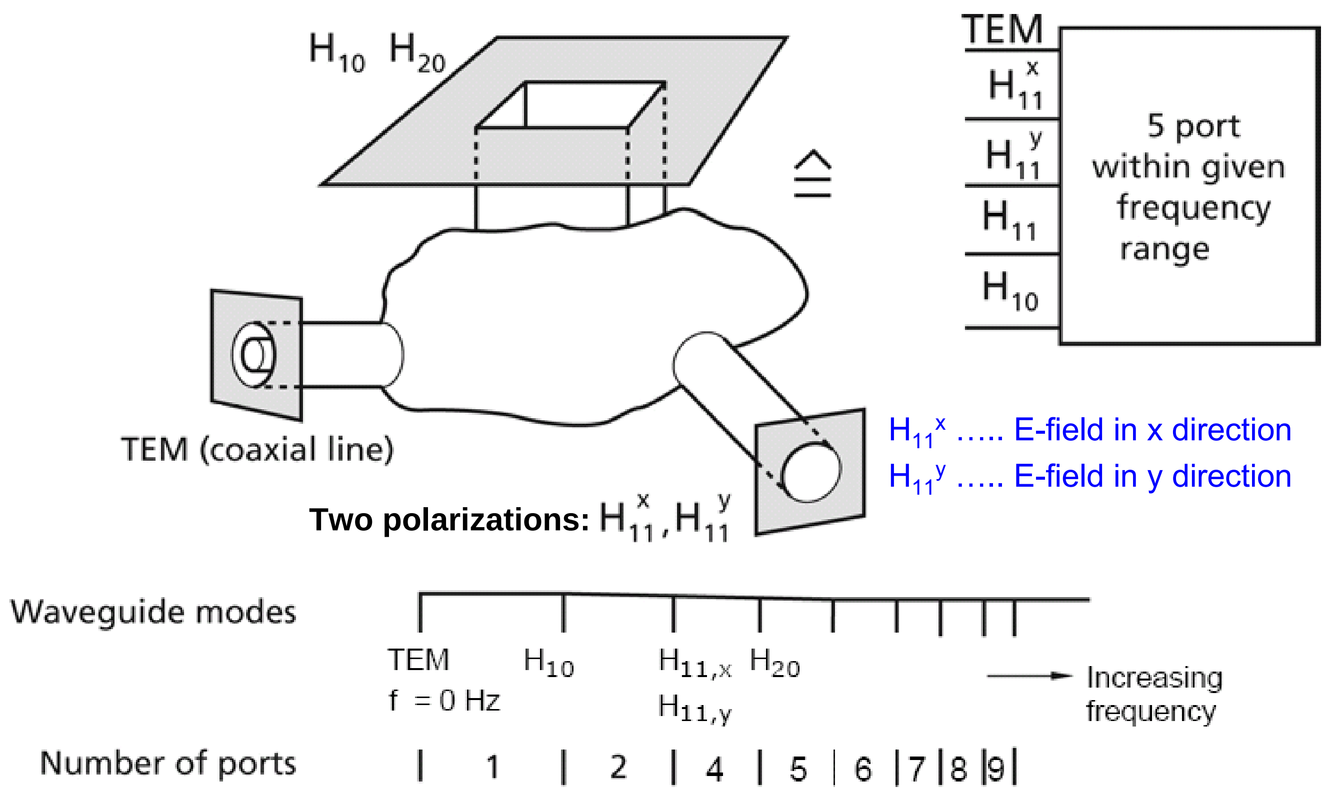
Magic T

(rat-race 180°)



General N-ports

- A general N-port may include ports in different technologies, i.e. waveguides, coaxial lines, microstrip lines etc.
- In a given frequency range different modes may propagate at each physical port, e.g. several waveguide modes in a rectangular waveguide or higher order modes on a coaxial line.
- Each mode must then be represented by a distinct port. (very important for simulations)
- The number of ports needed generally increases with frequency, as more waveguide modes can propagate. In numerical simulations neglecting higher order modes in the model can lead to questionable results.



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- [3] K.C. Gupta, R. Garg, and R. Chadha, Computer-aided design of microwave circuits, Artech, Dedham, MA 1981, ISBN 0-89006-106-8.
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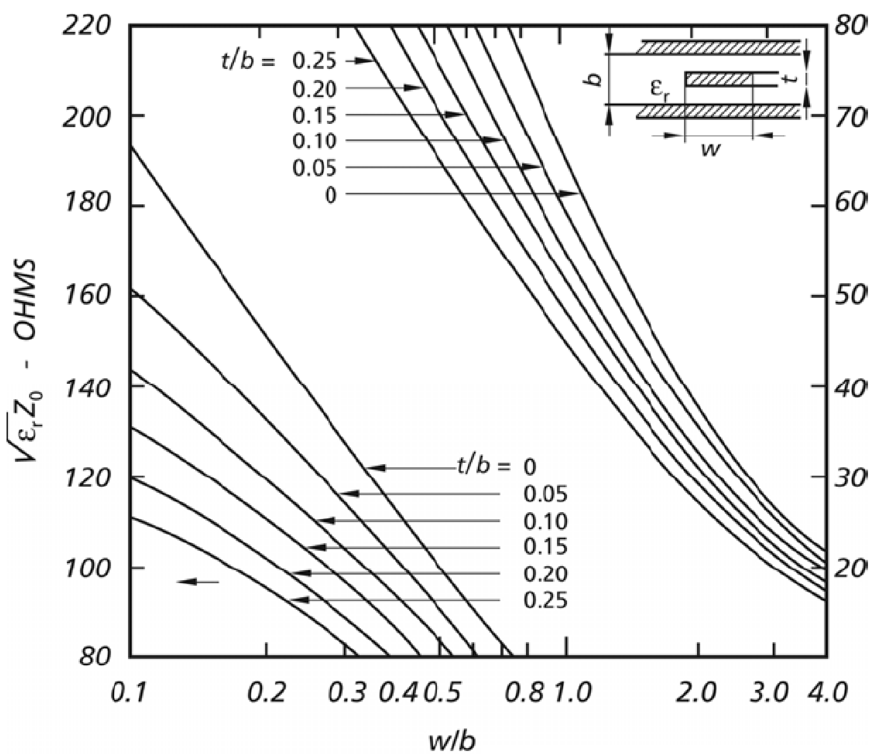
Appendix 1

- Striplines
- Microstrips
- Slotlines

Striplines (1)

- A stripline is a flat conductor between a top and bottom ground plane. The space around this conductor is filled with a homogeneous dielectric material. This line propagates a pure TEM mode. With the static capacity per unit length, C', the static inductance per unit length, L', the relative permittivity of the dielectric, ε_r and the speed of light c the characteristic impedance Z_0 of the line is given by



$$Z_0 = \sqrt{\frac{L'}{C'}}$$

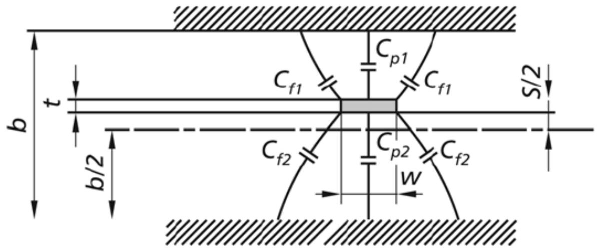
$$v_{ph} = \frac{c}{\sqrt{\varepsilon_r}} = \frac{1}{\sqrt{L'C'}}$$

$$Z_0 = \sqrt{\varepsilon_r} \cdot \frac{1}{C'c}$$

Fig 19: char.impedance of striplines

Striplines (2)

- For a mathematical treatment, the effect of the fringing fields may be described in terms of static capacities (see Fig. 20) [14]. The total capacity is the sum of the principal and fringe capacities C_p and C_f.



$$C_{tot} = C_{p1} + C_{p2} + 2C_{f1} + 2C_{f2}$$

Fig. 20: Design, dimensions and characteristics for offset center-conductor strip transmission line [14]

- For striplines with an homogeneous dielectric the phase velocity is the same, and frequency independent, for all TEM-modes. A configuration of two coupled striplines (3-conductor system) may have two independent TEM-modes, an odd mode and an even mode (Fig. 21).

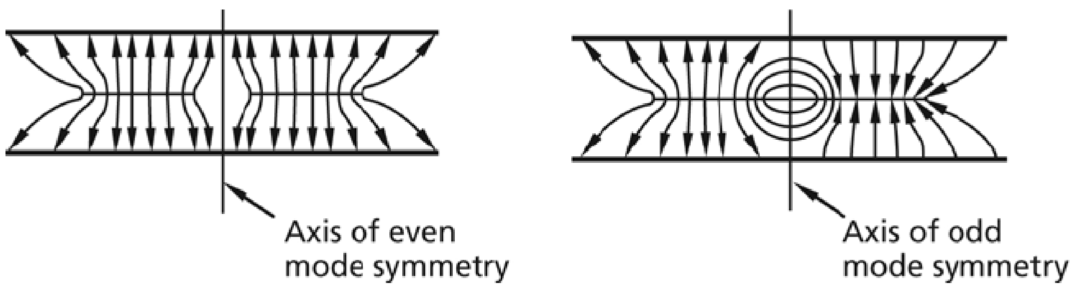


Fig. 21: Even and odd mode in coupled striplines

Striplines (3)

- The analysis of coupled striplines is required for the design of directional couplers. Besides the phase velocity the odd and even mode impedances $Z_{0,odd}$ and $Z_{0,even}$ must be known. They are given as a good approximation for the side coupled structure (Fig. 22, left). They are valid as a good approximation for the structure shown in Fig. 22.

$$Z_{0,even} = \frac{1}{\sqrt{\varepsilon_r}} \cdot \frac{94.15\,\Omega}{\frac{w}{b} + \frac{\ln 2}{\pi} + \frac{1}{\pi}\ln\left(1 + \tanh\left(\frac{\pi}{2}\frac{s}{b}\right)\right)}$$

$$Z_{0,odd} = \frac{1}{\sqrt{\varepsilon_r}} \cdot \frac{94.15\,\Omega}{\frac{w}{b} + \frac{\ln 2}{\pi} + \frac{1}{\pi}\ln\left(1 + \coth\left(\frac{\pi}{2}\frac{s}{b}\right)\right)}$$

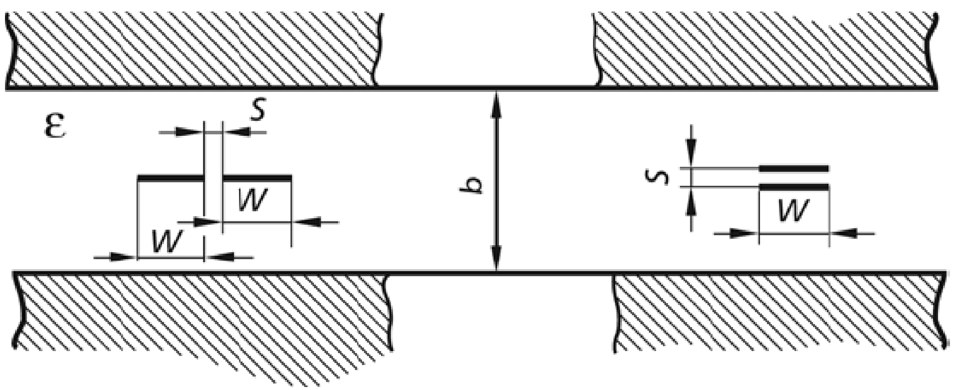


Fig. 22: Types of coupled striplines [14]: left: side coupled parallel lines, right: broad-coupled parallel lines

Striplines (4)

- A graphical presentation of Equations 5.3 is also known as the Cohn nomographs [14]. For a quarter-wave directional coupler (single section in Fig. 16) very simple design formulae can be given

$$Z_{0,odd} = Z_0 \sqrt{\frac{1+C_0}{1-C_0}}$$

$$Z_{0,even} = Z_0 \sqrt{\frac{1-C_0}{1+C_0}}$$

$$Z_0 = \sqrt{Z_{0,odd} Z_{0,even}}$$

  where C_0 is the voltage coupling ratio of the $\lambda/4$ coupler.

- In contrast to the 2-hole waveguide coupler this type couples backwards, i.e. the coupled wave leaves the coupler in the direction opposite to the incoming wave. The stripline coupler technology is rather widespread by now, and very cheap high quality elements are available in a wide frequency range. An even simpler way to make such devices is to use a section of shielded 2-wire cable.

Microstrip (1)

- A microstripline may be visualized as a stripline with the top cover and the top dielectric layer taken away (Fig. 23). It is thus an asymmetric open structure, and only part of its cross section is filled with a dielectric material. Since there is a transversely inhomogeneous dielectric, only a quasi-TEM wave exists. This has several implications such as a frequency-dependent characteristic impedance and a considerable dispersion.

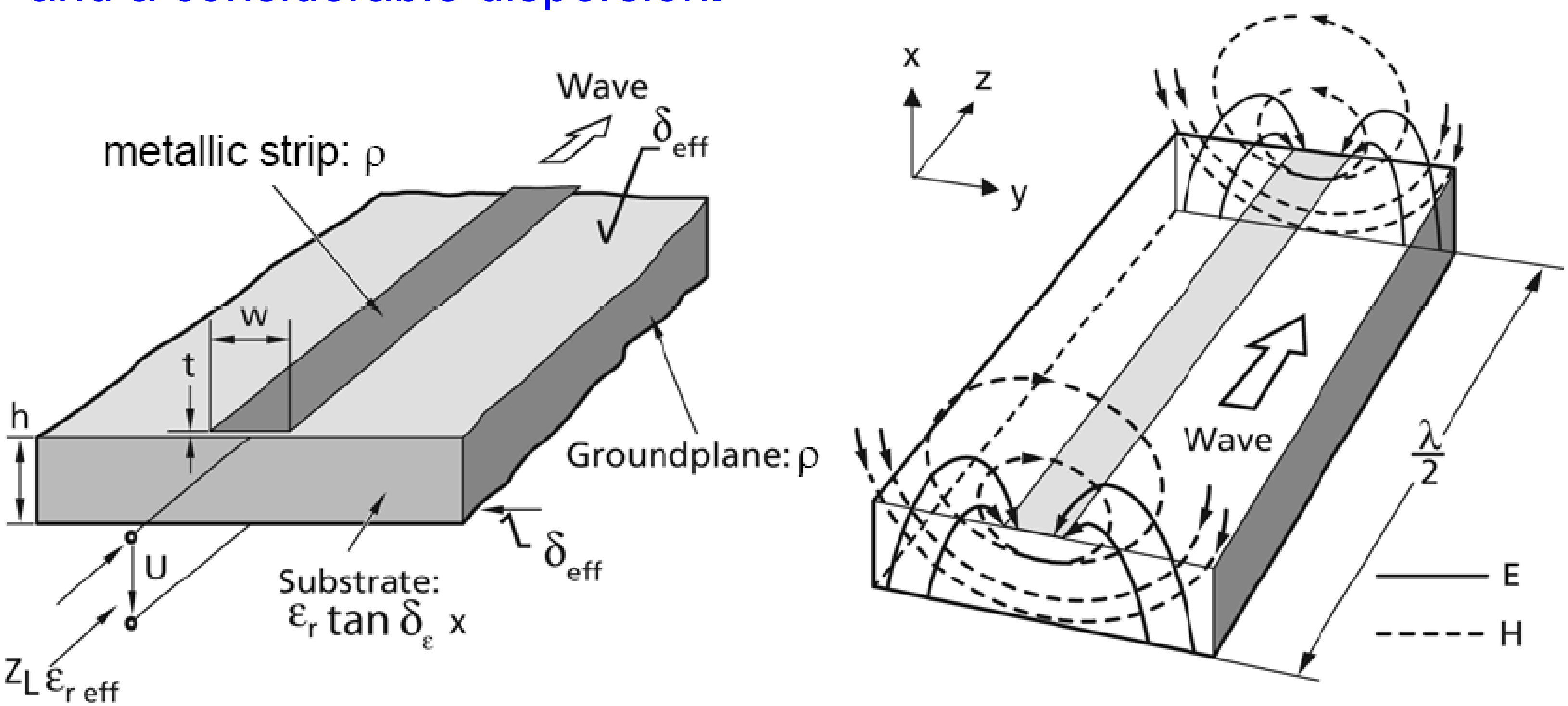


Fig.23 Microstripline: left: Mechanical construction, right: static field approximation

Microstrip (2)

- An exact field analysis for this line is rather complicated and there exist a considerable number of books and other publications on the subject. Due to the dispersion of the microstrip, the calculation of coupled lines and thus the design of couplers and related structures is also more complicated than in the case of the stripline. Microstrips tend to radiate at all kind of discontinuities such as bends, changes in width, through holes etc.
- With all the disadvantages mentioned above in mind, one may question why they are used at all. The mains reasons are the cheap production, once a conductor pattern has been defined, and easy access to the surface for the integration of active elements. Microstrip circuits are also known as Microwave Integrated Circuits (MICs). A further technological step is the MMIC (Monolithic Microwave Integrated Circuit) where active and passive elements are integrated on the same semiconductor substrate.
- In Figs. 25 and 26 various planar printed transmission lines are depicted. The microstrip with overlay is relevant for MMICs and the strip dielectric wave guide is a ‘printed optical fibre’ for millimeter-waves and integrated optics.

Microstrip (3)

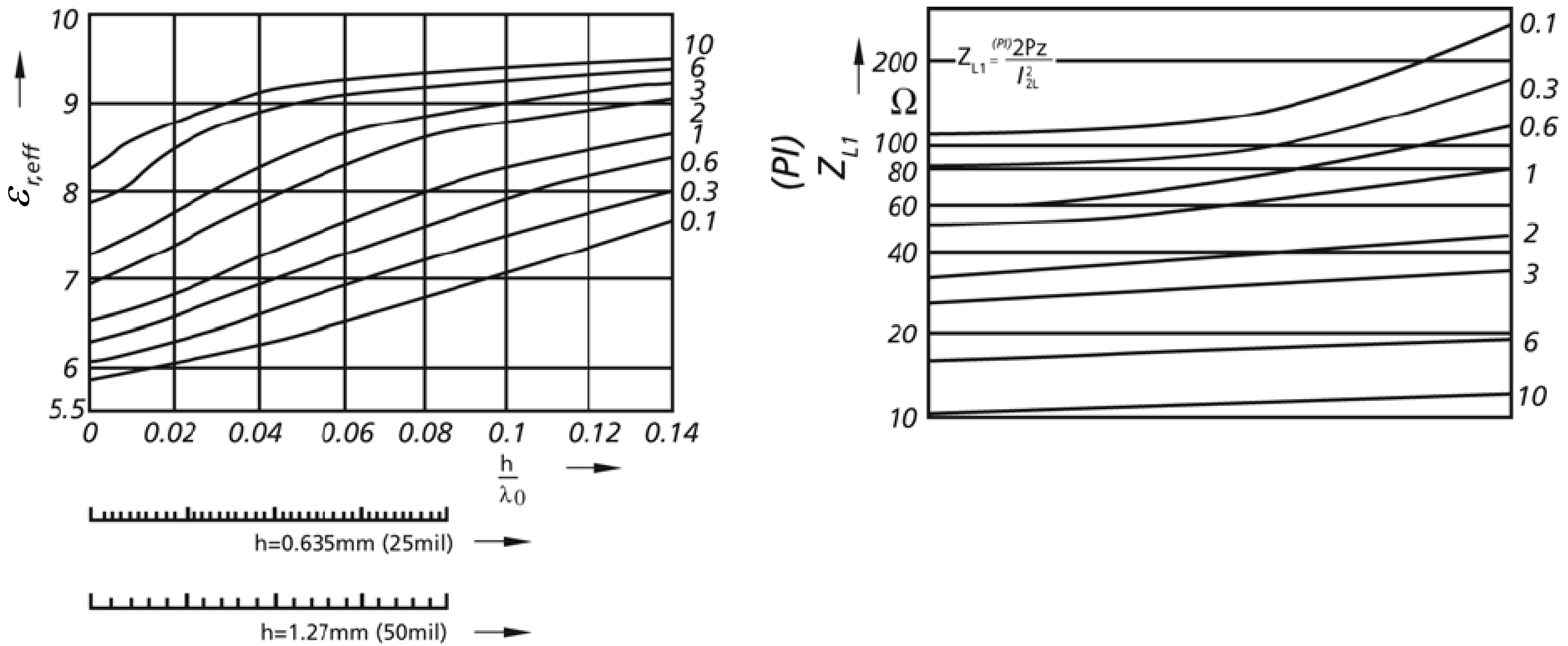
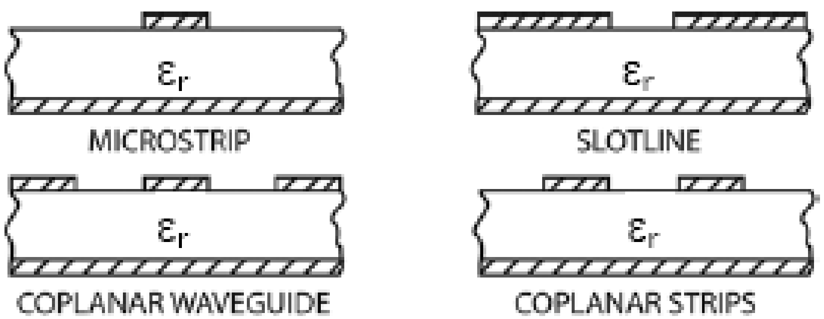


Fig. 24: Characteristic impedance (current/power definition) and effective permittivity of a microstrip line [16]

Microstrip (4)



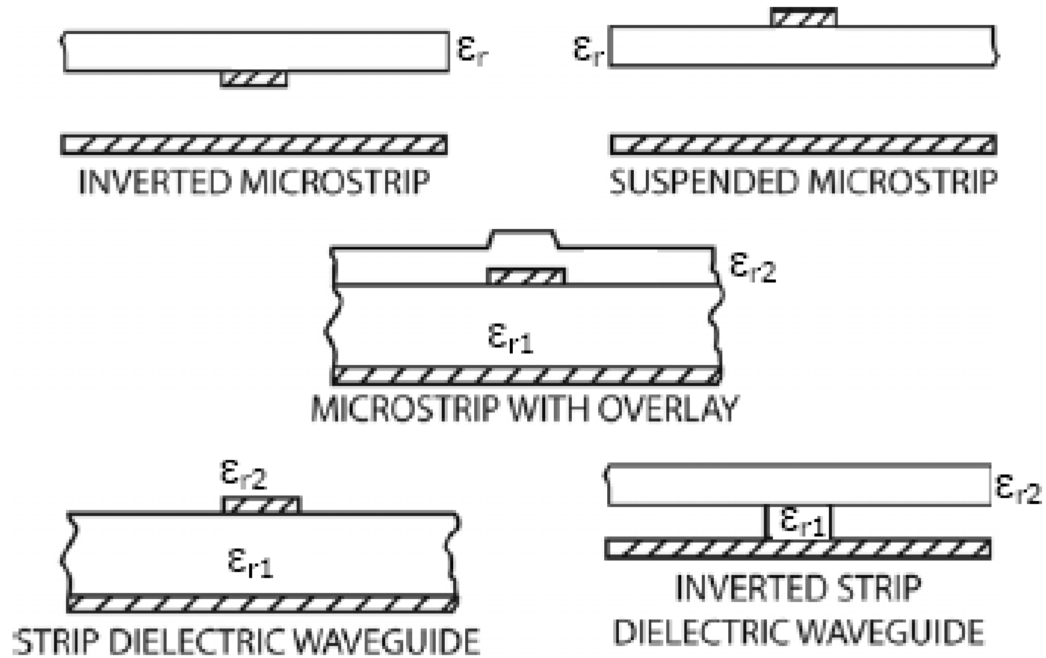


Fig. 25 (left): Planar transmission lines used in MICs; Fig 26 (right): various Transmission lines derived from microstrip

Slotlines (1)

- The slotline may be considered as the dual structure of the microstrip. It is essentially a slot in the metallization of a dielectric substrate as shown in Fig. 27. The characteristic impedance and the effective dielectric constant exhibit similar dispersion properties to those of the microstrip line. A unique feature of the slotline is that it may be combined with microstrip lines on the same substrate. This, in conjunction with through holes, permits interesting topologies such as pulse inverters in sampling heads (e.g. for sampling scopes).

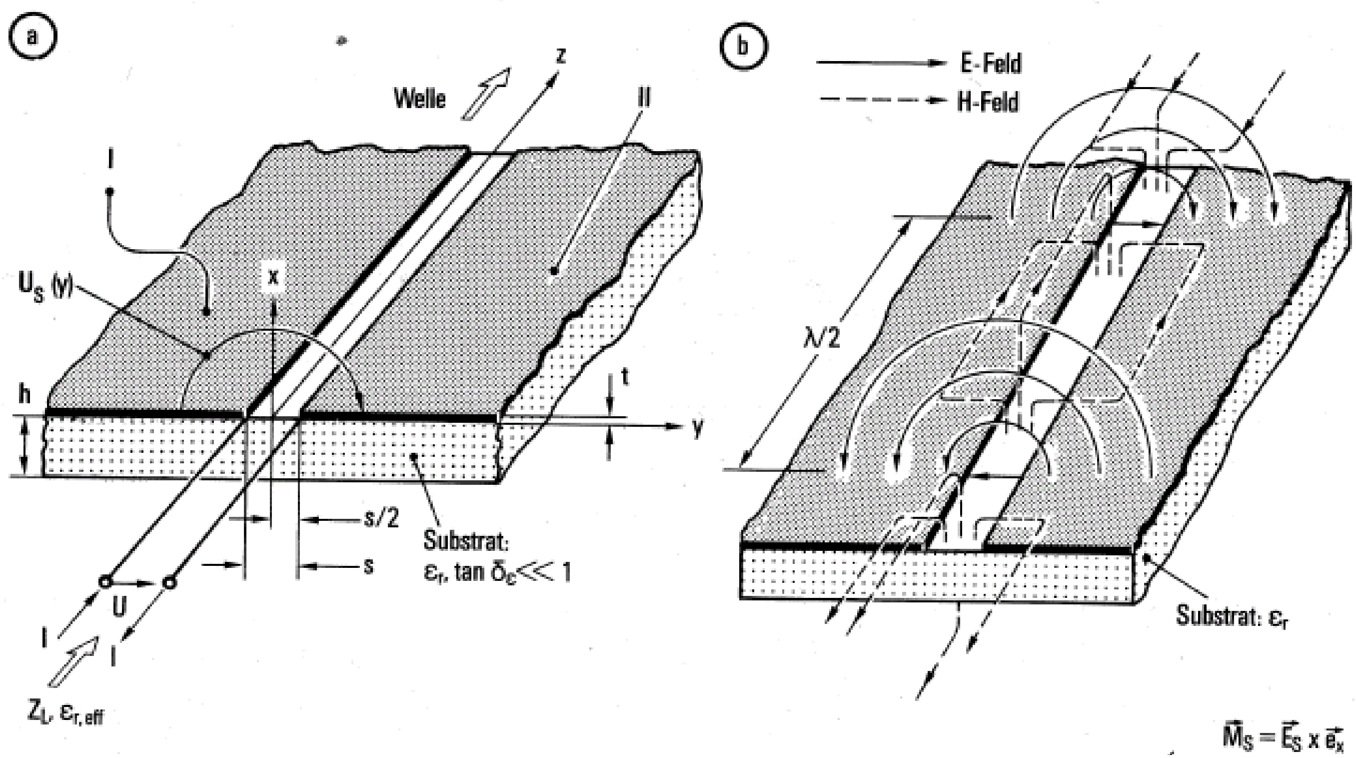


Fig 27 part 1: Slotlines a) Mechanical construction, b) Field pattern (TE-approximation)

Slotlines (2)

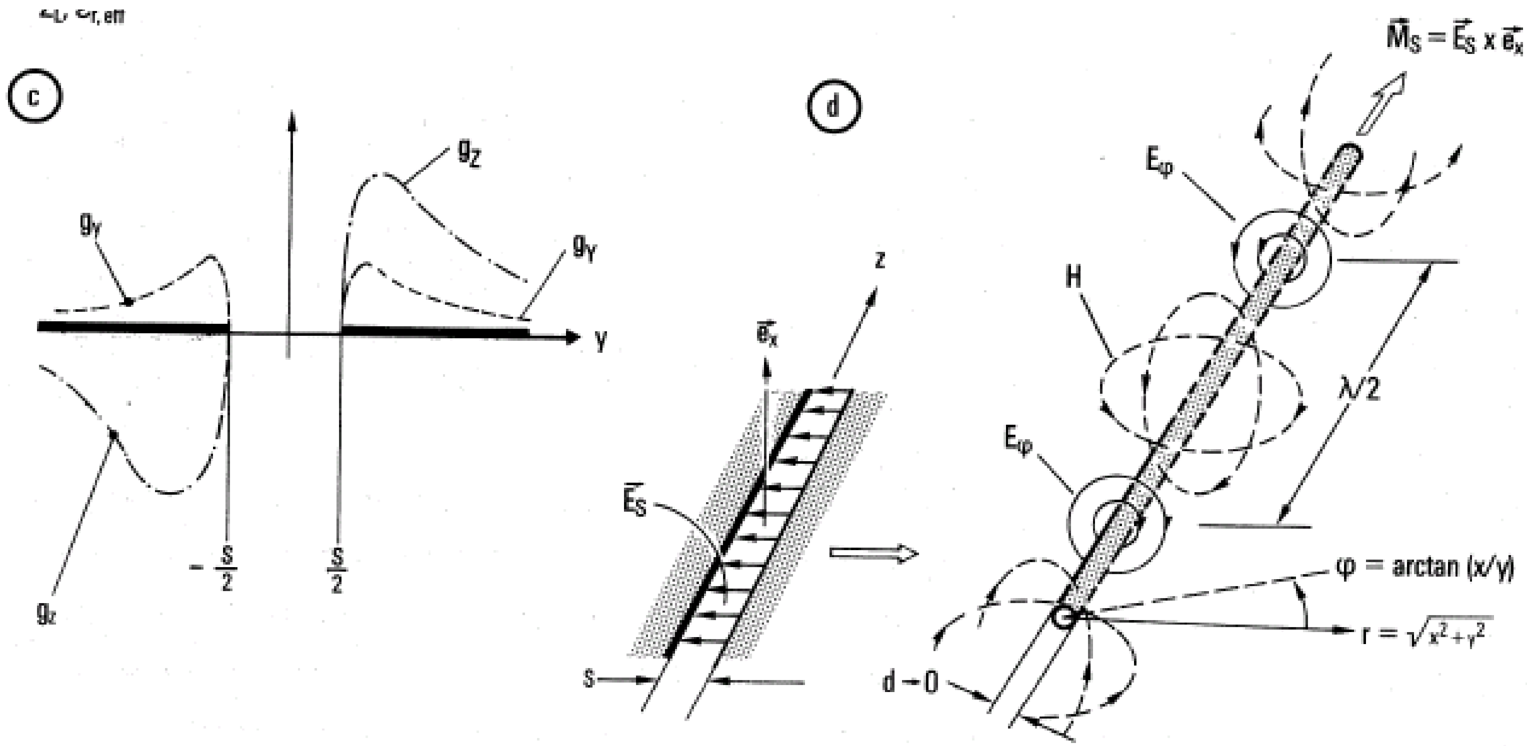


Fig 27 part 2: c) Longitudinal and transverse current densities, d) Magnetic line current model.

Slotlines (3)

- Fig. 28 shows a broadband (decade bandwidth) pulse inverter. Assuming the upper microstrip to be the input, the signal leaving the circuit on the lower microstrip is inverted since this microstrip ends on the opposite side of the slotline compared to the input. Printed slotlines are also used for broadband pickups in the GHz range, e.g. for stochastic cooling.

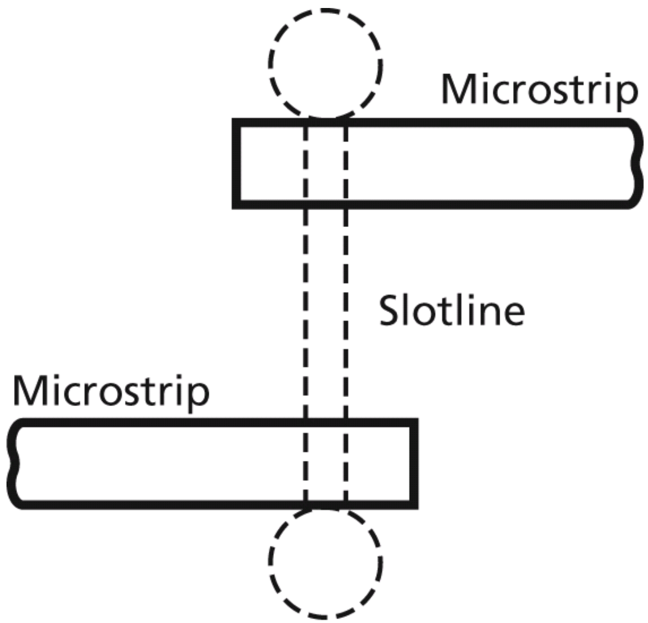


Fig 28. Two microstrip-slotline transitions connected back to back for 180° phase change

Appendix 2

- T-matrices

T-Matrix

- *The T matrix (transfer matrix), which directly relates the waves on the input and on the output, is defined as [2]*

$$\begin{pmatrix} b_1 \\ a_1 \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \begin{pmatrix} a_2 \\ b_2 \end{pmatrix}$$

- *As the transmission matrix (T matrix) simply links the in- and outgoing waves in a way different from the S matrix, one may convert the matrix elements mutually*

$$T_{11} = S_{12} - \frac{S_{22} S_{11}}{S_{21}}, \quad T_{12} = \frac{S_{11}}{S_{21}}$$

$$T_{21} = -\frac{S_{22}}{S_{21}}, \qquad T_{22} = \frac{1}{S_{21}}$$

- *The T matrix* $\mathbf{T_M}$ *of m cascaded 2-ports is given by a matrix multiplication from the 'left' to the right as in [2, 3]:*

$$\mathbf{T_M} = \mathbf{T_1 T_2} \ldots \mathbf{T_m}$$

T-Matrix

- *There is another definition that takes a_1 and b_1 as independent variables.*

$$\begin{pmatrix} b_2 \\ a_2 \end{pmatrix} = \begin{pmatrix} \tilde{T}_{11} & \tilde{T}_{12} \\ \tilde{T}_{21} & \tilde{T}_{22} \end{pmatrix} \begin{pmatrix} a_1 \\ b_1 \end{pmatrix}$$

- *and for this case*

$$\tilde{T}_{11} = S_{21} - \frac{S_{22} S_{11}}{S_{12}}, \quad \tilde{T}_{12} = \frac{S_{22}}{S_{12}}$$

$$\tilde{T}_{21} = -\frac{S_{11}}{S_{12}}, \qquad \tilde{T}_{22} = \frac{1}{S_{12}}$$

- *Then, for the cascade, we obtain*

$$\tilde{\mathbf{T}}_{\mathbf{M}} = \tilde{\mathbf{T}}_m \tilde{\mathbf{T}}_{m-1} \ldots \tilde{\mathbf{T}}_1$$

- *i.e. a matrix multiplication from ‘right’ to ‘left’.*

- *Note that there is no standardized definition of the T-matrix [4]*

Appendix 3

- Signal Flow Chart

The signal flow graph

- The SFG is a graphical representation of a system of linear equations having the general form:

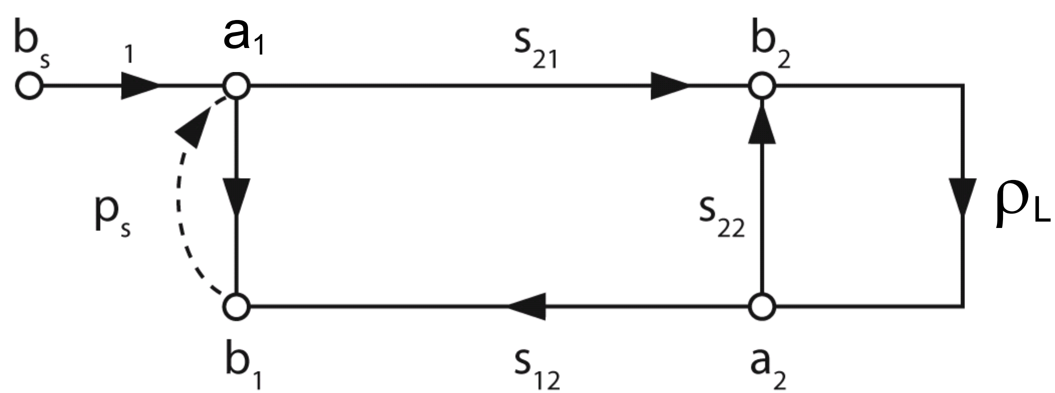
$$\mathbf{y} = \mathbf{M}\mathbf{x} + \mathbf{M}'\mathbf{y}$$

- ***M*** and ***M'*** are square matrices with *n* rows and columns
- **x** represent the *n* independent variables (sources) and **y** the *n* dependent variables.
- The elements of **M** and **M'** appear as transmission coefficients of the signal path.
- When there are no direct signal loops, this simplifies to $\mathbf{y} = \mathbf{M}\mathbf{x}$, which is equivalent to the usual S parameter definition

$$\mathbf{b} = \mathbf{S}\mathbf{a}$$

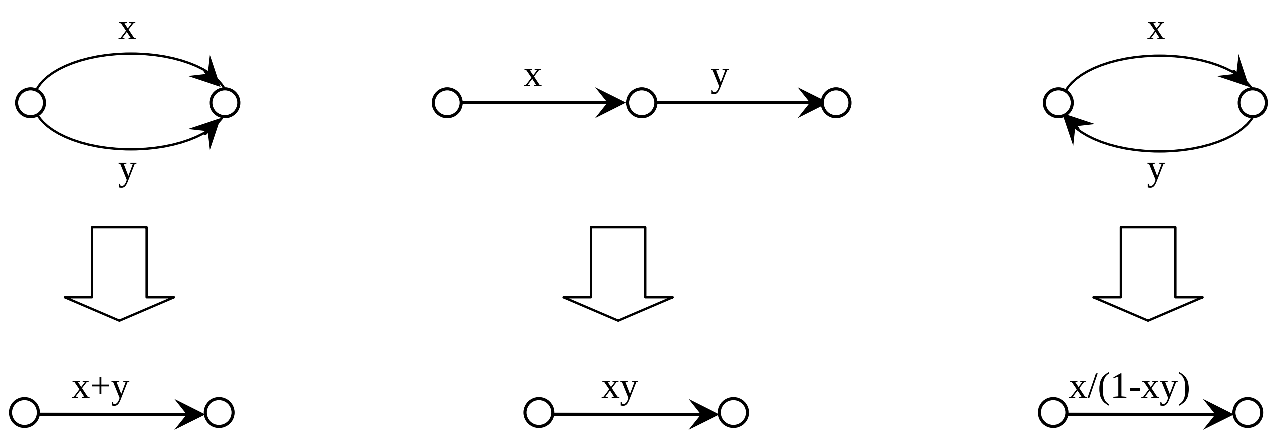
Drawing the SFG

- The SFG can be drawn as a directed graph. Each wave a_i and b_i is represented by a node, each arrow stands for an S parameter
- Nodes with no arrows pointing towards them are *source noses*. All other nodes are *dependent signal nodes*.
- Example: A 2-port with given S_{ij}. At port 2 a not matched load ρ_L is connected.
- Question: What is the input reflection coefficient S_{11} of this circuit?



Simplifying the signal flow graph

- For general problems the SFG can be solved for applying Mason's rule (see Appendix of lecture notes). For not too complicated circuits there is a more intuitive way by simplifying the SFG according to three rules
  1. Add the signal of parallel branches
  2. Multiply the signals of cascaded branches
  3. Resolve loops



1. Parallel branches

2. Cascaded signal paths

3. Loops

Solving problems with the SFG

- The loop at port 2 involving S_{22} and ρ_L can be resolved. It gives a branch from b_2 to a_2 with the coefficient $\rho_L/(1-\rho_L*S_{22})$
- Applying the cascading rule to the right path and finally the parallel branch rule we get

$$\frac{b_1}{a_1} = S_{11} + S_{21}\frac{\rho_L}{1 - S_{22}\rho_L}S_{12}$$

- More complicated problem are usually solved by computer code using the matrix formulations mentioned before