Phenomenology of Minority Games in Efficient Regime

Karol Wawrzyniak, ICM, University of Warsaw

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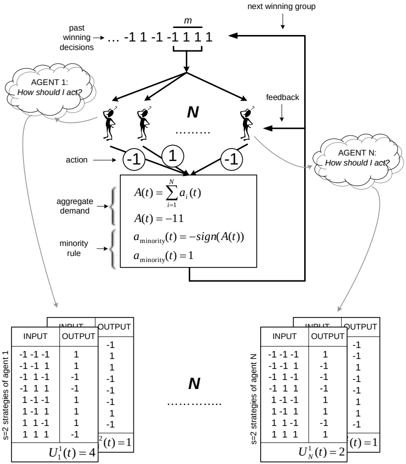
CAPACITIES

Agenda

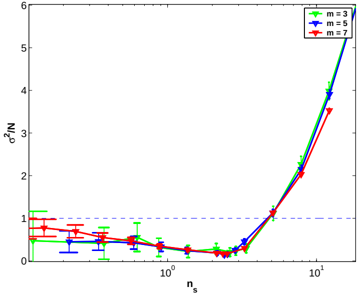
- Minority Game
  - Introduction - definition, motivation
  - Why grid?
  - State, Markov process
  - Multimarket Minority Game - breaking the symmetry of choice
  - Prediction - optimization of parameters
- Financial data
- Summary

Minority Game - introduction (1/2)



- multi-agent, adaptive system, where being in minority is profitable
- 1994, W.Brian Arthur; 1997, Challet, Zhang
- time series of histories - μ
- $U(t+1) = U(t) - a(t)g[A(t)]$
- $g(x) = sgn(x)$, $g(x) = x$, etc.
- simple model of financial market
- the game is non-markovian

Minority Game - introduction (2/2)



$n_s = N/2^m$, $\sigma^2/N = \langle A^2 \rangle /N$

- σ^2/N measures the coordination between agents.
- horizontal line $\sigma^2/N = 1$ - Random Choice Game
- MG: three modes of behavior: random, cooperation, herd
- cooperation mode is close to the real markets (well described in literature)

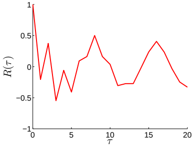
Why grid?

- Interesting phenomena are observed for "large" games - hundreds and thousands of agents.
- To estimate pdf-s many steps of simulations are required (10.000 in our case) and many realizations (usually 10).
- No synchronicity, and therefore no heavy data traffic during execution, is needed between them - The grid seemed to be natural choice.
- Summing up, we did more than 8.000 runs each lasted from 5 min to 5 hours.

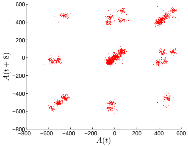
Herd regime (1/4) - Phenomenology

Motivation:

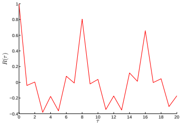
Lack of consistent explanation of $A(t)$ periodicity, concentration around some levels, sensitivity on payoff form, oscillation etc.



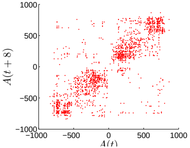
autocorrelation function $R(\tau)$, payoff $g(x) = sgn(x)$



$A(t + 2 \times 2^m)$ against A(t), payoff $g(x) = sgn(x)$



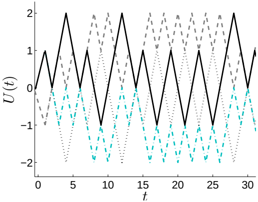
autocorrelation function $R(\tau)$, payoff $g(x) = x$

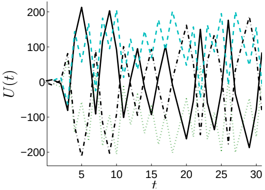


$A(t + 2 \times 2^m)$ against A(t), payoff $g(x) = x$

Herd regime (2/4) - the state

- If $Ns \gg 2^{2^m}$ then many strategies are the same.
- fraction - set of agents with identical strategies





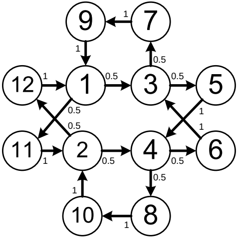
Time evolution of utility $U(t)$ for all possible strategies for two different forms of payoff: $g(x) = sgn(x)$ (left) and $g(x) = x$ (right) in the MG with $s = 2$, $m = 1$, and N high enough to assure work in regime $Ns \gg 2^P$.

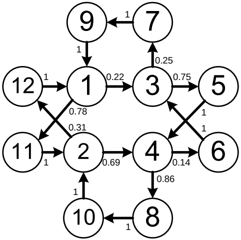
State:

- $x(t) = [\mu(t), U^1(t), \ldots, U^{2^P}(t)]$ or $x(t) = [\,\mu(t), i_{\beta_1}(t), \ldots, i_{\beta_{2^P}}(t)\,]$
- finite nr. of states, if $sgn(x)$ then U is limited by $\pm 2^m$

Herd regime (3/4) - payoff $sgn(x)$

- Description of the game as a Markov Process.
- Two kinds of transitions: stochastic and deterministic.

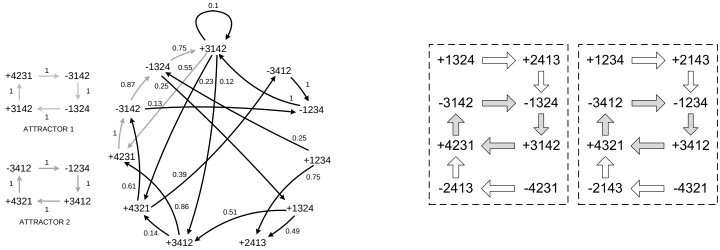




equal (left) and unequal fractions - uniform pdf. (right) for: $s = 2$, $m = 1$, and N such that $Ns \gg 2^P$.

Herd regime (4/4) - payoff x

- 1 kind of transitions: deterministic.



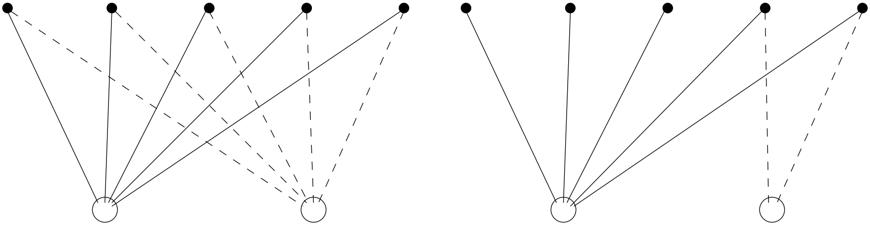
equal (left) and unequal fractions - uniform pdf. (right) for: $s = 2$, $m = 1$, i N such that $Ns \gg 2^P$.

- Attractors: $2^{2^m}/2^{m+1}$, States: $\prod_{i=0}^{2^P/2-1}(2^P - 2i)$
- Distance: between reference system (equal fractions) and system with unequal fractions (additional states).
- *Phenomenology of Minority Games in Efficient Regime (Adv. in Complex Sys., Vol. 12, Nos. 6(2009) 619639)*
- *Minority Game: Mesoscopic Explanation of Macroscopic Observables in Herd Regime (in preparation)*

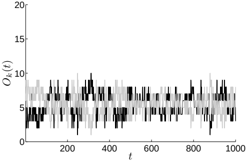
Multimarket Minority Game (1/3)

Motivation - financial markets

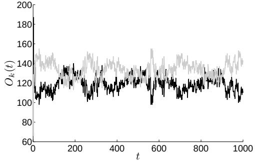
- agents can act on many markets
- agent chooses: (i) market, (ii) action



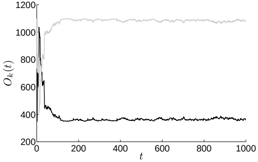
MG (2/3) - occupancy & aggregate d.



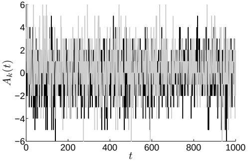
MMG with $N = 11$, $s = 2$ i $m = 5$



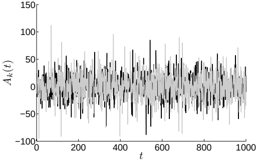
MMG with $N = 253$, $s = 2$ i $m = 5$



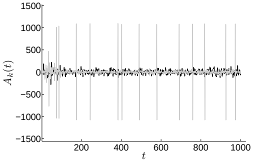
MMG with $N = 1447$, $s = 2$ i $m = 5$



MMG with $N = 11$, $s = 2$ i $m = 5$



MMG with $N = 253$, $s = 2$ i $m = 5$

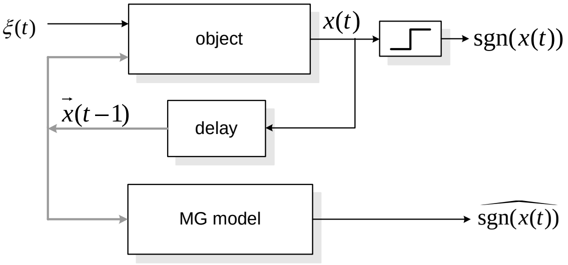


MMG with $N = 1447$, $s = 2$ i $m = 5$

Multi-market minority game: breaking the symmetry of choice,
Adv. in Complex Sys., Vol. 12, Nos. 4&5 (2009) 423437

GCMG as a sign predictor (1/3)

GCMG - Grand Canonical Minority Game, Configuration:

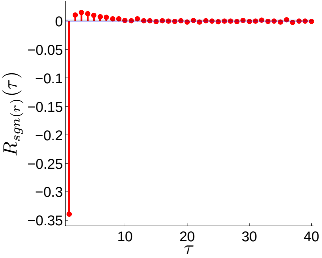


- signals: synthetic (deterministic/stochastic) and real ones (stocks, indexes)
- best predictor: N=1 and FSS or RSS
- $\lambda - GCMG$: modification for non-stationary signals

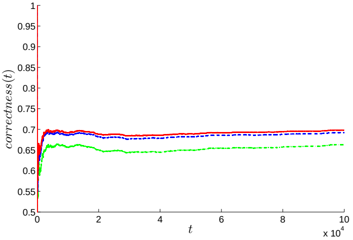
GCMG (2/3) - Financial data

- Sources: Warsaw Stock Exchange, London Stock Exchange, Forex
- Statistics: moments, temporary autocorrelation function/power spectrum, Hurst exponent
- Stylized facts:
  - fat tails
  - lack of autocorrelation for daily returns $r(t)$
  - variance clusterization
  - positive autocorrelation of variance
  - $r(t) \neq \text{IID}$
- Daily data useless for prediction

GCMG (2/3) - Intraday



Autocorrelation $R_{sgn(r)}(\tau)$ for $FW20_{HMUZ08}$.



correctness(t) and $U(t)$: RSS, λ-GCMG, $N = 1$, $\lambda = 0.97$. Best/worst strategy - mean-reverting/trend-follower, data: FW20).

Summary

- MG - simple model where the minority wins. Analogies to financial markets.
- MG - non-markovian but can be redefined to markovian (state).
- Using markov chain all observables are relatively easy to explain (periodicity, large oscillations, clusterization).
- MMG - generalization of MG for many markets. Spontaneous breaking of symmetry is observed and explained.
- Grand Canonical MG - predictor.
- Grid utilization