HIGGS PRODUCTION USING THE Q_T-SUBTRACTION FORMALISM AT N^3LO

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High Precision for Hard Processes (HP^2 - 2018)

01.10.2018

Based on:
L. C, X. Chen, T. Gehrmann, E. W. N. Glover, & A. Huss (2018)
arXiv: 1807.11501

OUTLINE

- Introduction and motivation
- Main idea behind the qT-subtraction method
- qT-subtraction ingredients at N^3LO
- Higgs Phenomenology at N^3LO
- Outlook

INTRODUCTION

- No striking manifestation of New Physics (NP) beyond the Standard Model (SM) at the LHC
- "Precision Physics" represents *the key instrument* to find NP
- In this framework QCD perturbative corrections play a crucial role
- Until a few years ago, the standard for such calculations was next-to-leading order (NLO) accuracy
- In the past recent years a number of growing next-to-next-to-leading order (NNLO) results were computed

INTRODUCTION

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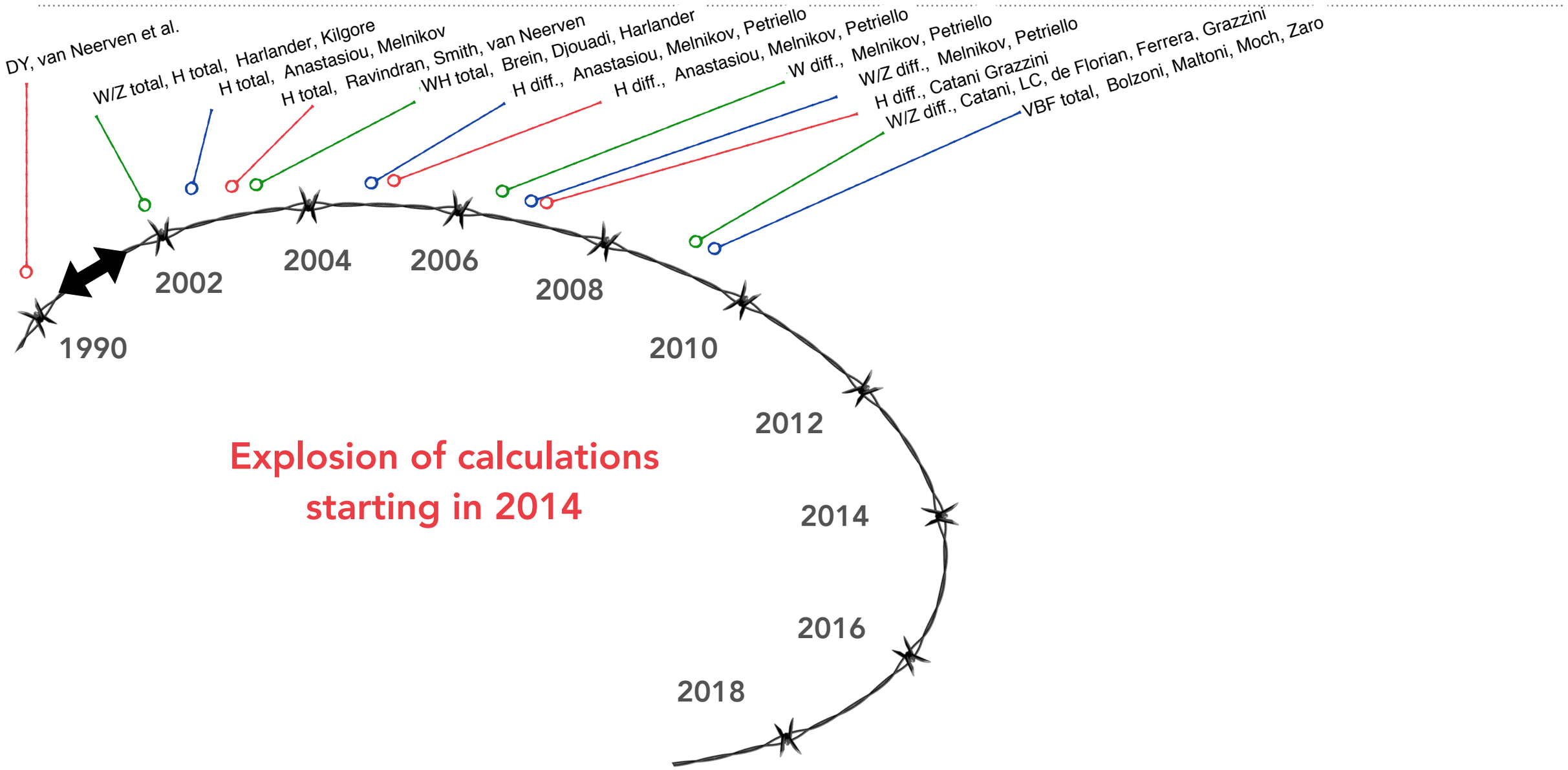
- Until a few years ago, the standard for such calculations was next-to-leading order (NLO) accuracy
- In the past recent years a number of growing next-to-next-to-leading order (NNLO) results were computed

"The NNLO revolution"

THE NNLO STANDARD

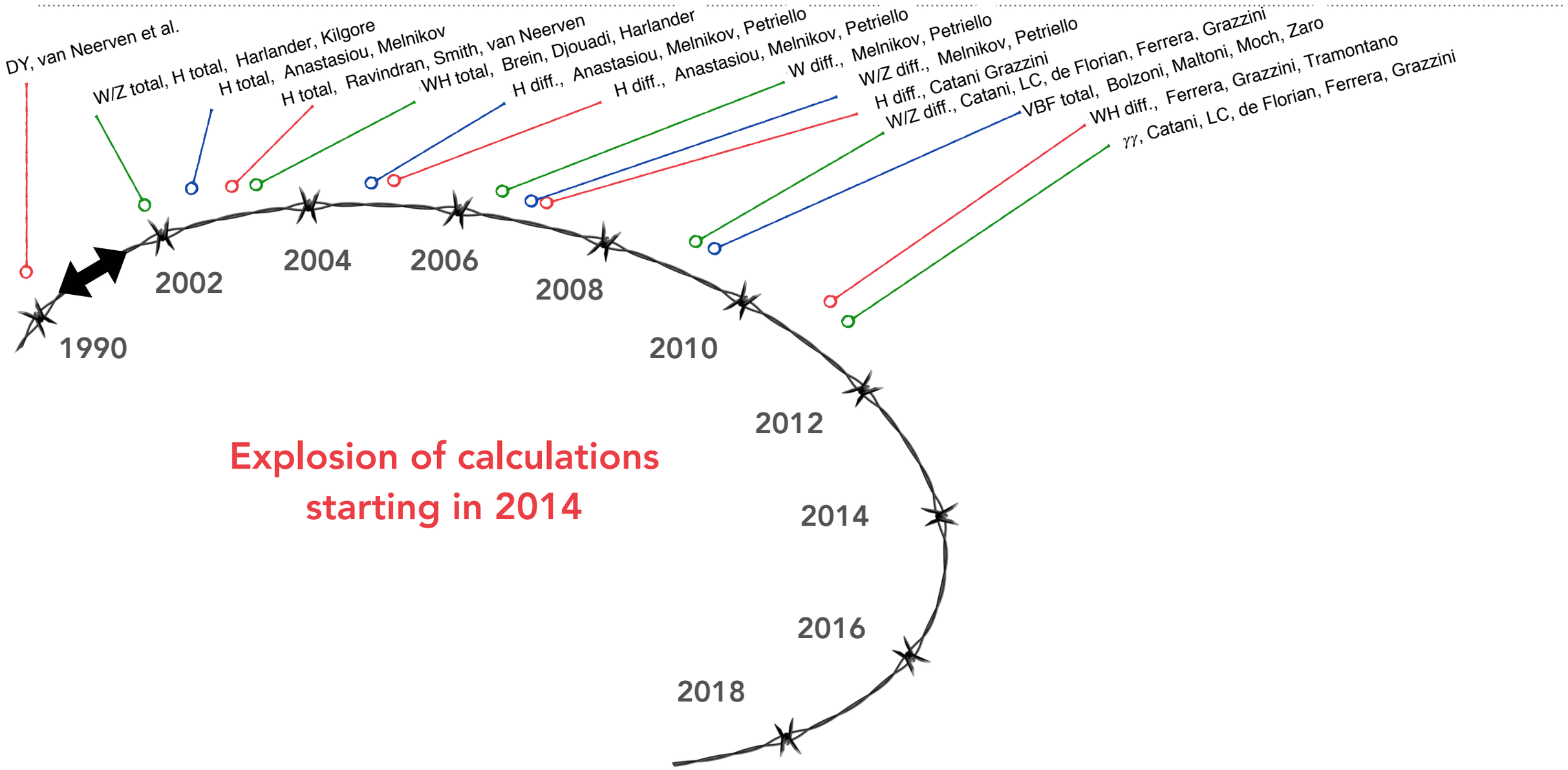
NNLO HADRON-COLLIDER CALCULATIONS VS. TIME



Explosion of calculations starting in 2014

THE NNLO STANDARD

NNLO HADRON-COLLIDER CALCULATIONS VS. TIME



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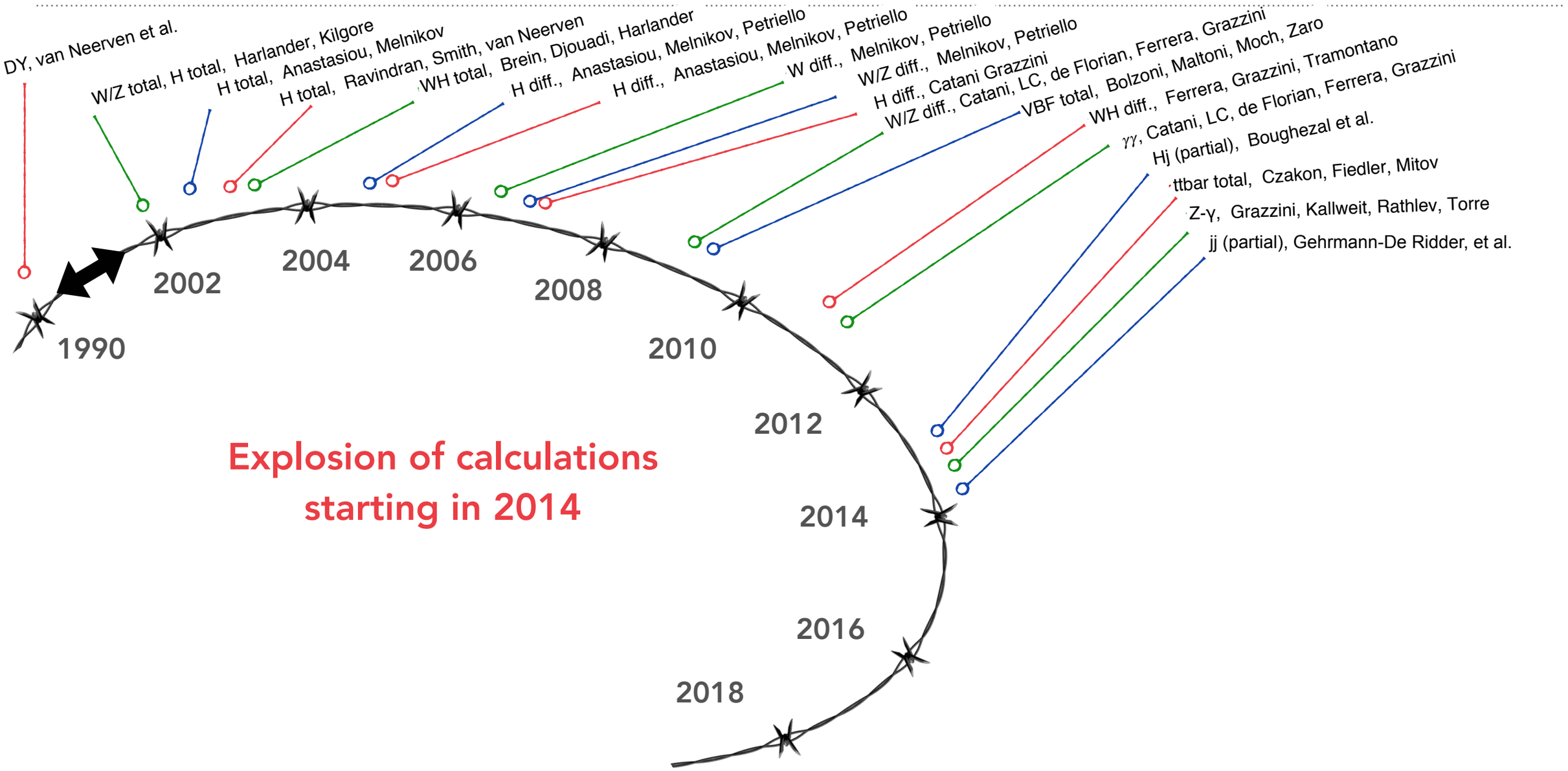
THE NNLO STANDARD

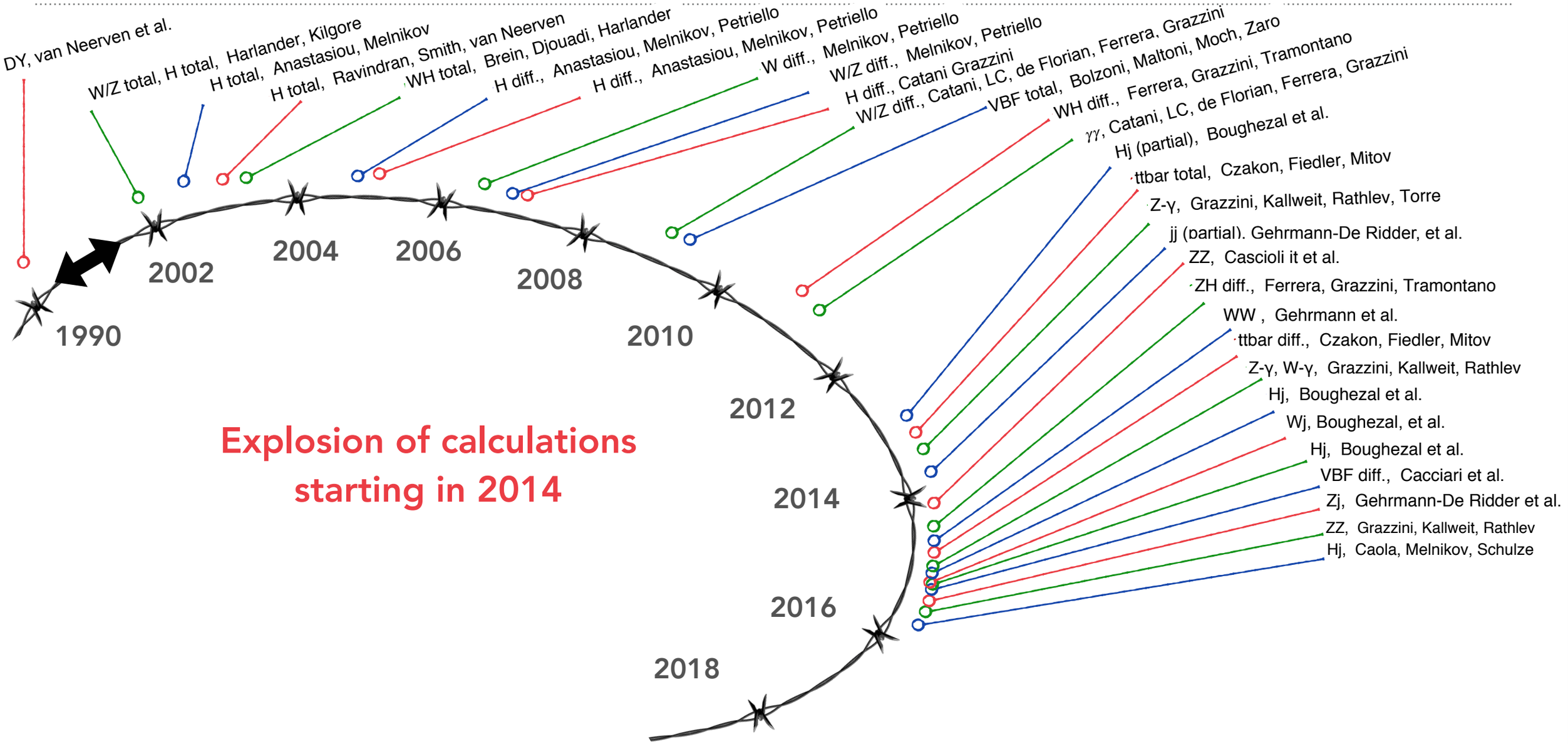
NNLO HADRON-COLLIDER CALCULATIONS VS. TIME



Explosion of calculations starting in 2014

THE NNLO STANDARD

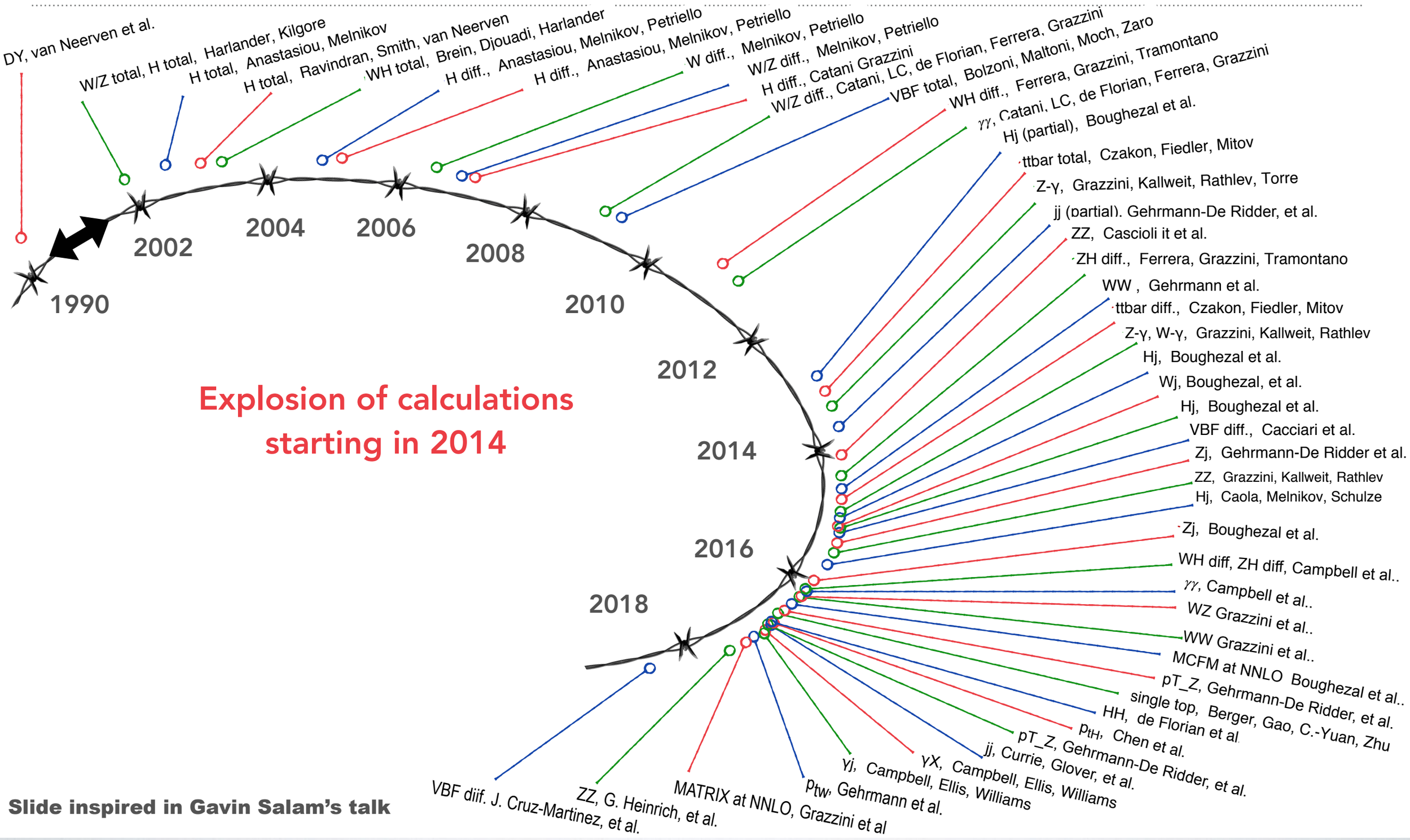
NNLO HADRON-COLLIDER CALCULATIONS VS. TIME



Explosion of calculations starting in 2014

THE NNLO STANDARD

NNLO HADRON-COLLIDER CALCULATIONS VS. TIME

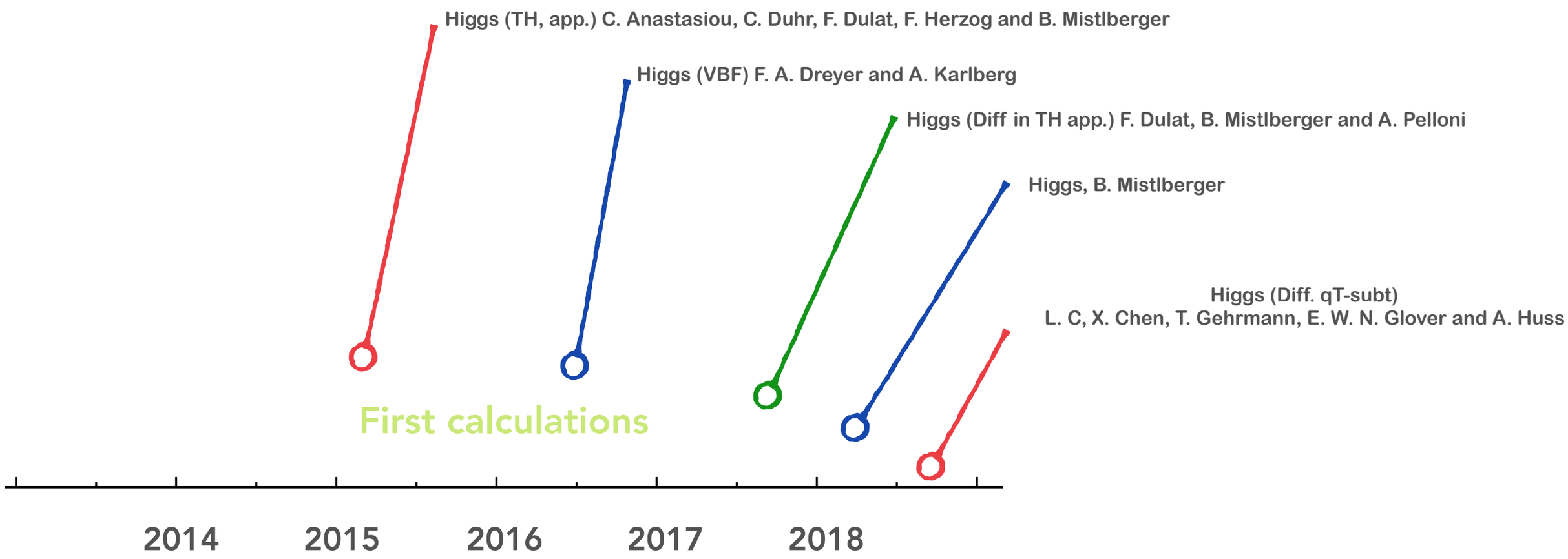


Explosion of calculations starting in 2014

Slide inspired in Gavin Salam's talk

THE N^{3}LO ERA

N^{3}LO HADRON-COLLIDER CALCULATIONS VS. TIME



THE QT-SUBTRACTION/RESUMMATION FORMALISM

NNLO+NNLL QCD

The method

Explicit form qT-subt. Ingredients

Helicity-Flip Contributions

Universal relation between hard-virtual factors and two-loop scattering amplitudes

Colourless final states

- S. Catani and M. Grazzini (2007)
- G. Bozzi, S. Catani, D. de Florian and M. Grazzini (2005)
- S. Catani and M. Grazzini (2011)
- S. Catani, L. C, D. de Florian, G. Ferrera and M. Grazzini (2013)

Heavy quark production → R. Bonciani, S. Catani, M. Grazzini, H. Sargsyan and A. Torre (2015)

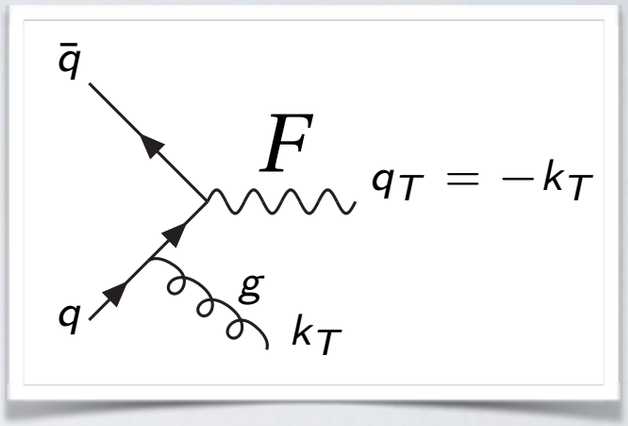
Simone Devoto's Talk

Extension at NLO+NLL QED+QCD → L. C, G. Ferrera and G. F. R. Sborlini (2018)

THE N^{3}LO QT-SUBTRACTION METHOD

- We consider the inclusive hard-scattering reaction

$$h_1 + h_2 \longrightarrow F(Q) + X$$



  - h_i = hadron (i=1,2)
  - Colourless final state F with transverse momentum qT and invariant mass Q

- We are interested in the QCD corrections to this process F

- The basic idea behind the method Catani, Grazzini (2007)

$$\mathrm{d}\sigma^{F}_{\mathrm{N^nLO}}(q_T \neq 0) \equiv \mathrm{d}\sigma^{F+\mathrm{jets}}_{\mathrm{N^{n-1}LO}} \quad \text{with} \quad n = 1, 2, 3$$

- $\mathrm{d}\sigma^{F+\mathrm{jets}}_{\mathrm{N^{n-1}LO}}$ can be computed with the well known subtractions methods at NLO and NNLO
- The remaining true N^{n}LO divergencies are associated to the small-qT limit

Talks: Aude Gehrmann-De Ridder, Paolo Torrielli, Adam Kardos

THE N^{3}LO QT-SUBTRACTION METHOD

- The generic form of the qT-subtraction method

Catani, Grazzini (2007)

$$\mathrm{d}\sigma^{F}_{\mathrm{N^nLO}} = \mathcal{H}^{F}_{\mathrm{N^nLO}} \otimes \mathrm{d}\sigma^{F}_{\mathrm{LO}} + \left[\mathrm{d}\sigma^{F+\mathrm{jets}}_{\mathrm{N^{n-1}LO}} - \mathrm{d}\sigma^{F\,\mathrm{CT}}_{\mathrm{N^nLO}}\right] \quad \text{with} \quad n = 1, 2, 3$$

$$\mathrm{d}\sigma^{F\,\mathrm{CT}}_{\mathrm{N^nLO}} = \Sigma^{F}_{\mathrm{N^nLO}}(q_T^2/M^2)\ \mathrm{d}^2\mathbf{q}_T \otimes \mathrm{d}\sigma^{F}_{\mathrm{LO}}$$

- How to calculate the qT-subtraction ingredients?

$$\left(\Sigma^{F}_{c\bar{c}\leftarrow ab}\left(\frac{q_T^2}{M^2}; \frac{M^2}{\hat{s}}; \alpha_{\mathrm{s}}\right) + \mathcal{H}^{F}_{c\bar{c}\leftarrow ab}\left(\frac{M^2}{\hat{s}}; \alpha_{\mathrm{s}}\right)\right) \otimes \mathrm{d}\left[\sigma^{F;(0)}_{c\bar{c}}\right]_{ab} = \frac{M^2}{s}\int_0^\infty \mathrm{d}b\frac{b}{2}J_0(bq_T)$$
$$\times\ S_c(M,b)\int_{x_1}^1\frac{\mathrm{d}z_1}{z_1}\int_{x_2}^1\frac{\mathrm{d}z_2}{z_2}\mathrm{d}\hat{\sigma}^{F;(0)}_{c\bar{c}}\ f_{a/h_1}\left(x_1/z_1, b_0^2/b^2\right) f_{b/h_2}\left(x_2/z_2, b_0^2/b^2\right)\ \left[H^F C_1 C_2\right]_{c\bar{c};ab}$$

- Expanding the resummation formula at fixed order

J. C. Collins, D. E. Soper and G. Sterman (1985)
S. Catani, D. de Florian and M. Grazzini (2001)
S. Catani and M. Grazzini (2011)

THE N^{3}LO QT-SUBTRACTION METHOD

- Expanding at fixed-order the following expression

$$\left(\Sigma^{F}_{c\bar{c}\leftarrow ab}\left(\frac{q_T^2}{M^2};\frac{M^2}{\hat{s}};\alpha_{\rm s}\right)+\mathcal{H}^{F}_{c\bar{c}\leftarrow ab}\left(\frac{M^2}{\hat{s}};\alpha_{\rm s}\right)\right)\otimes {\rm d}\left[\sigma^{F;(0)}_{c\bar{c}}\right]_{ab}=\frac{M^2}{s}\int_0^\infty {\rm d}b\frac{b}{2}J_0(bq_T)$$
$$\times\, S_c(M,b)\int_{x_1}^1\frac{{\rm d}z_1}{z_1}\int_{x_2}^1\frac{{\rm d}z_2}{z_2}{\rm d}\hat{\sigma}^{F;(0)}_{c\bar{c}}\, f_{a/h_1}\left(x_1/z_1,b_0^2/b^2\right) f_{b/h_2}\left(x_2/z_2,b_0^2/b^2\right)\, \left[H^F C_1 C_2\right]_{c\bar{c};ab}$$

$$x_1=\frac{M}{\sqrt{s}}\,e^{+y}\quad,\qquad x_2=\frac{M}{\sqrt{s}}\,e^{-y}$$

$$S_c(M,b)=\exp\left\{-\int_{b_0^2/b^2}^{M^2}\frac{{\rm d}q^2}{q^2}\left[A_c(\alpha_{\rm s}(q^2))\,\ln\frac{M^2}{q^2}+B_c(\alpha_{\rm s}(q^2))\right]\right\}$$

Sudakov form factor

Catani, Grazzini (2011)

$$\left[H^{F=H}C_1C_2\right]_{gg;ab}=H_g^{F=H}(\alpha_{\rm s}(M^2))\quad\left[\,C_{g\,a}(z_1;\alpha_{\rm s}(b_0^2/b^2))\;C_{g\,b}(z_2;\alpha_{\rm s}(b_0^2/b^2))\right.$$
$$\left.+\,G_{g\,a}(z_1;\alpha_{\rm s}(b_0^2/b^2))\;G_{g\,b}(z_2;\alpha_{\rm s}(b_0^2/b^2))\,\right]$$

Last line not present in qq initiated sub-processes

$$z\;=\;M^2/\hat{s}$$

$$C_{g\,a}(z;\alpha_{\rm s})=\delta_{g\,a}\;\delta(1-z)+\sum_{n=1}^{\infty}\left(\frac{\alpha_{\rm s}}{\pi}\right)^n C^{(n)}_{g\,a}(z)$$

Collinear coefficient functions

$$G_{g\,a}(z;\alpha_{\rm s})=\sum_{n=1}^{\infty}\left(\frac{\alpha_{\rm s}}{\pi}\right)^n G^{(n)}_{g\,a}(z)$$

Helicity-flip functions in gg initiated processes

$$H_g^{F=H}(\alpha_{\rm s})=1+\sum_{n=1}^{\infty}\left(\frac{\alpha_{\rm s}}{\pi}\right)^n H_g^{F=H\,;(n)}$$

Hard virtual factors (loop-virtual contributions)

THE N^{3}LO QT-SUBTRACTION METHOD

- Expanding at fixed-order the following expression

$$\left(\Sigma^F_{c\bar{c}\leftarrow ab}\left(\frac{q_T^2}{M^2};\frac{M^2}{\hat{s}};\alpha_{\rm s}\right)+\mathcal{H}^F_{c\bar{c}\leftarrow ab}\left(\frac{M^2}{\hat{s}};\alpha_{\rm s}\right)\right)\otimes {\rm d}\left[\sigma^{F;(0)}_{c\bar{c}}\right]_{ab}=\frac{M^2}{s}\int_0^\infty {\rm d}b\frac{b}{2}J_0(bq_T)$$
$$\times\, S_c(M,b)\int_{x_1}^1\frac{{\rm d}z_1}{z_1}\int_{x_2}^1\frac{{\rm d}z_2}{z_2}{\rm d}\hat{\sigma}^{F;(0)}_{c\bar{c}}\, f_{a/h_1}\left(x_1/z_1,b_0^2/b^2\right) f_{b/h_2}\left(x_2/z_2,b_0^2/b^2\right)\,\left[H^F C_1 C_2\right]_{c\bar{c};ab}$$

$$\Sigma^F(q_T/Q)\underset{q_T\to 0}{\longrightarrow}\sum_{n=1}^\infty\left(\frac{\alpha_{\rm S}}{\pi}\right)^n\sum_{k=1}^{2n}\Sigma^{F(n;k)}\,\frac{Q^2}{q_T^2}\ln^{k-1}\frac{Q^2}{q_T^2}$$

Contribution of all large logarithmic terms
In the small-qT limit

$$\mathcal{H}^F_{c\bar{c}\leftarrow ab}\to\sum_{n=1}^\infty\left(\frac{\alpha_s}{\pi}\right)^n\mathcal{H}^{F(n)}_{c\bar{c}\leftarrow ab}\,\delta(q_T^2)$$

Free of large logarithmic terms
in the small-qT limit

$$\mathrm{d}\sigma^F_{\mathrm{N^nLO}}=\mathcal{H}^F_{\mathrm{N^nLO}}\otimes\mathrm{d}\sigma^F_{\mathrm{LO}}+\left[\mathrm{d}\sigma^{F+\mathrm{jets}}_{\mathrm{N^{n-1}LO}}-\mathrm{d}\sigma^{F\,\mathrm{CT}}_{\mathrm{N^nLO}}\right]\quad\text{with}\quad n=1,2,3$$

$$\mathrm{d}\sigma^{F\,\mathrm{CT}}_{\mathrm{N^nLO}}=\Sigma^F_{\mathrm{N^nLO}}(q_T^2/M^2)\,\mathrm{d}^2\mathbf{q}_T\otimes\mathrm{d}\sigma^F_{\mathrm{LO}}$$

THE N^{3}LO QT-SUBTRACTION METHOD

Third-order ingredients

- All the ingredients which describe the cancellation of the large logarithmic terms (in the small-qT limit) are known from the previous orders (NNLO and NLO) (using convolutions written in function of Goncharov PolyLogarithms GPLS)

$$\Sigma^F(q_T/Q) \xrightarrow[q_T\to 0]{} \sum_{n=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n \sum_{k=1}^{2n} \Sigma^{F(n;k)} \frac{Q^2}{q_T^2} \ln^{k-1}\frac{Q^2}{q_T^2}$$

- The third-order contribution at qT = 0 presents two missing pieces
  - The universal relation between the three-loop virtual scattering amplitudes and the hard-virtual factor
  - The collinear coefficient functions at the third-order

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New : **K. Melnikov, R. Rietkerk, L. Tancredi and C. Wever, ``Double-real contribution to the quark beam function at N3LO QCD," arXiv:1809.06300**

THE N^{3}LO QT-SUBTRACTION METHOD

Third-order ingredients at qT = 0

De Florian, Grazzini (2001)

- The hard-virtual, collinear and helicity-flip contributions

$$\mathcal{H}^{H;(1)}_{gg\leftarrow ab}(z) = \delta_{g\,a}\,\delta_{g\,b}\,\delta(1-z)\,H^{H;(1)}_{g} + \delta_{g\,a}\,C^{(1)}_{g\,b}(z) + \delta_{g\,b}\,C^{(1)}_{g\,a}(z) \quad ,$$

$$\mathcal{H}^{H;(2)}_{gg\leftarrow ab}(z) = \delta_{g\,a}\,\delta_{g\,b}\,\delta(1-z)\,H^{H;(2)}_{g} + \delta_{g\,a}\,C^{(2)}_{g\,b}(z) + \delta_{g\,b}\,C^{(2)}_{g\,a}(z) + H^{H;(1)}_{g}\left(\delta_{g\,a}\,C^{(1)}_{g\,b}(z) + \delta_{g\,b}\,C^{(1)}_{g\,a}(z)\right) + \left(C^{(1)}_{g\,a}\otimes C^{(1)}_{g\,b}\right)(z) + \left(G^{(1)}_{g\,a}\otimes G^{(1)}_{g\,b}\right)(z)$$

S. Catani, L. C, D. de Florian, G. Ferrera and M. Grazzini (2013)

$$\mathcal{H}^{H;(3)}_{gg\leftarrow ab}(z) = \delta_{g\,a}\,\delta_{g\,b}\,\delta(1-z)\,H^{H;(3)}_{g} + \delta_{g\,a}\,C^{(3)}_{g\,b}(z) + \delta_{g\,b}\,C^{(3)}_{g\,a}(z) + \left(G^{(1)}_{g\,a}\otimes G^{(2)}_{g\,b}\right)(z) + \left(G^{(2)}_{g\,a}\otimes G^{(1)}_{g\,b}\right)(z) + H^{H;(1)}_{g}\left(\delta_{g\,a}\,C^{(2)}_{g\,b}(z) + \delta_{g\,b}\,C^{(2)}_{g\,a}(z)\right) + H^{H;(2)}_{g}\left(\delta_{g\,a}\,C^{(1)}_{g\,b}(z) + \delta_{g\,b}\,C^{(1)}_{g\,a}(z)\right) + \left(C^{(1)}_{g\,a}\otimes C^{(2)}_{g\,b}\right)(z) + \left(C^{(2)}_{g\,a}\otimes C^{(1)}_{g\,b}\right)(z) + H^{H;(1)}_{g}\left(C^{(1)}_{g\,a}\otimes C^{(1)}_{g\,b}\right)(z) + H^{H;(1)}_{g}\left(G^{(1)}_{g\,a}\otimes G^{(1)}_{g\,b}\right)(z) \; .$$

Where we fix μR=μF=M

Scale dependent terms already known from NNLO

THE N^{3}LO QT-SUBTRACTION METHOD

Third-order ingredients at qT = 0

- The hard-virtual, collinear and helicity-flip contributions

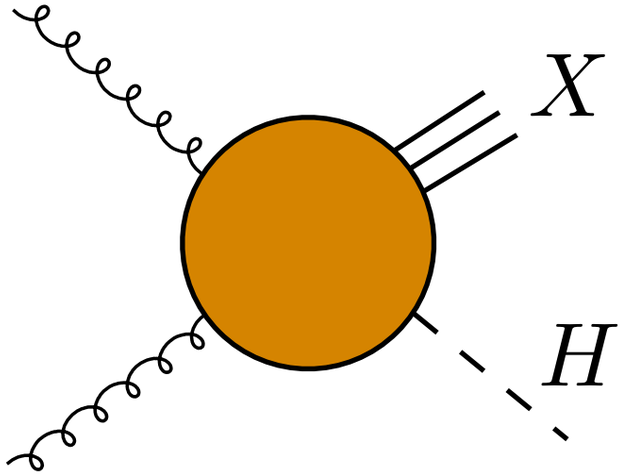
These ingredients are partially known or completely missing

$$
\begin{aligned}
\mathcal{H}^{H;(3)}_{gg\leftarrow ab}(z) = & \delta_{g\,a}\,\delta_{g\,b}\,\delta(1-z)\,H^{H;(3)}_{g} + \delta_{g\,a}\,C^{(3)}_{g\,b}(z) + \delta_{g\,b}\,C^{(3)}_{g\,a}(z) \\
& + \left(G^{(1)}_{g\,a} \otimes G^{(2)}_{g\,b}\right)(z) + \left(G^{(2)}_{g\,a} \otimes G^{(1)}_{g\,b}\right)(z) \\
& + H^{H;(1)}_{g}\left(\delta_{g\,a}\,C^{(2)}_{g\,b}(z) + \delta_{g\,b}\,C^{(2)}_{g\,a}(z)\right) + H^{H;(2)}_{g}\left(\delta_{g\,a}\,C^{(1)}_{g\,b}(z) + \delta_{g\,b}\,C^{(1)}_{g\,a}(z)\right) \\
& + \left(C^{(1)}_{g\,a} \otimes C^{(2)}_{g\,b}\right)(z) + \left(C^{(2)}_{g\,a} \otimes C^{(1)}_{g\,b}\right)(z) \\
& + H^{H;(1)}_{g}\left(C^{(1)}_{g\,a} \otimes C^{(1)}_{g\,b}\right)(z) + H^{H;(1)}_{g}\left(G^{(1)}_{g\,a} \otimes G^{(1)}_{g\,b}\right)(z)\,.
\end{aligned}
$$

Where we fix μR=μF=M

THE N^{3}LO QT-SUBTRACTION METHOD

Up to the previous slide the discussion about the N^{3}LO qT-subtraction formalism, was completely general. The following slides are for final processes F initiated by gluon fusion. In particular for Higgs boson production via gluon fusion in the large top-mass limit.



THE N^3LO CONTRIBUTIONS AT QT=0

THE N^{3}LO QT-SUBTRACTION METHOD

L. C, X. Chen, T. Gehrmann, E. W. N. Glover, and A. Huss (2018)

Third-order ingredients at qT = 0

- These ingredients are partially known or completely missing

$$H_g^{H;(3)} \quad ; \quad C_{ga}^{(3)} \quad ; \quad G_{ga}^{(2)}$$

Nonetheless, within the qT-subtraction formalism, $\mathscr{H}_{gg\leftarrow ab}^{H;(3)}$ can be reliably approximated for any hard-scattering process whose corresponding total cross section is known at N^{3}LO.

G. Bozzi, S. Catani, D. de Florian and M. Grazzini (2005)

- In our case we use the N^{3}LO Higgs boson cross section recently calculated and implemented in the numerical code ihixs 2.0

B. Mistlberger (2018)

F. Dulat, A. Lazopoulos and B. Mistlberger (2018)

THE N^{3}LO QT-SUBTRACTION METHOD

L. C, X. Chen, T. Gehrmann, E. W. N. Glover, and A. Huss (2018)

Third-order ingredients at qT = 0

- Numerical procedure to extract

$$H_g^{H;(3)} \quad ; \quad C_{ga}^{(3)} \quad ; \quad G_{ga}^{(2)}$$

- Starting point is the sketchy form of the qT-subtraction method

$$\mathrm{d}\sigma^F_{\mathrm{N^nLO}} = \mathcal{H}^F_{\mathrm{N^nLO}} \otimes \mathrm{d}\sigma^F_{\mathrm{LO}} + \left[\mathrm{d}\sigma^{F+\mathrm{jets}}_{\mathrm{N^{n-1}LO}} - \mathrm{d}\sigma^{F\,\mathrm{CT}}_{\mathrm{N^nLO}}\right] \quad \text{with} \quad n = 1, 2, 3$$

$$\hat{\sigma}^{\mathrm{tot}}_{F\,ab}(M, \hat{s}; \alpha_{\mathrm{s}}(\mu_R^2), \mu_R^2, \mu_F^2) = \int_0^\infty \mathrm{d}q_T^2 \, \frac{\mathrm{d}\hat{\sigma}_{F\,ab}}{\mathrm{d}q_T^2}(q_T, M, \hat{s}; \alpha_{\mathrm{s}}(\mu_R^2), \mu_R^2, \mu_F^2)$$

$$\sigma^{F\,(\mathrm{tot.})}_{\mathrm{N^nLO}} = \mathcal{H}^F_{\mathrm{N^nLO}} \otimes \sigma^F_{\mathrm{LO}} + \int_0^\infty \mathrm{d}q_T^2 \, \frac{\mathrm{d}\sigma^{F\,(\mathrm{fin.})}_{\mathrm{N^nLO}}}{\mathrm{d}q_T^2}$$

$$\frac{\mathrm{d}\hat{\sigma}^{(\mathrm{fin.})}_{F\,ab}}{\mathrm{d}q_T^2} \equiv \left[\frac{\mathrm{d}\hat{\sigma}^{F+\mathrm{jets}}_{ab}}{\mathrm{d}q_T^2} - \frac{\mathrm{d}\hat{\sigma}^{F\,\mathrm{CT}}_{ab}}{\mathrm{d}q_T^2}\right]$$

THE N^{3}LO QT-SUBTRACTION METHOD

L. C, X. Chen, T. Gehrmann, E. W. N. Glover, and A. Huss (2018)

Third-order ingredients at qT = 0

- Subtracting two consecutive orders, we arrive to an expression that can be used to extract numerically the missing terms

$$\left(\frac{\alpha_{\rm s}}{\pi}\right)^3 \frac{M^2}{\hat{s}} \sum_c \sigma^{(0)}_{c\bar{c},\,F}(\alpha_{\rm s}, M)\, \mathcal{H}^{F;(3)}_{c\bar{c}\leftarrow ab}\left(\frac{M^2}{\hat{s}}; \frac{M^2}{\mu_R^2}, \frac{M^2}{\mu_F^2}\right)$$

$$= \left\{\left[\hat{\sigma}^{\rm tot}_{F\,ab}\right]_{\rm N^3LO} - \left[\hat{\sigma}^{\rm tot}_{F\,ab}\right]_{\rm NNLO}\right\} - \int_0^\infty {\rm d}q_T^2 \left\{\left[\frac{{\rm d}\hat{\sigma}^{\rm (fin.)}_{F\,ab}}{{\rm d}q_T^2}\right]_{\rm N^3LO} - \left[\frac{{\rm d}\hat{\sigma}^{\rm (fin.)}_{F\,ab}}{{\rm d}q_T^2}\right]_{\rm NNLO}\right\}$$

$$\begin{aligned}
\mathcal{H}^{H;(3)}_{gg\leftarrow ab}(z) =& \delta_{g\,a}\,\delta_{g\,b}\,\delta(1-z)\,H_g^{H;(3)} + \delta_{g\,a}\,C^{(3)}_{g\,b}(z) + \delta_{g\,b}\,C^{(3)}_{g\,a}(z) \\
&+ \left(G^{(1)}_{g\,a} \otimes G^{(2)}_{g\,b}\right)(z) + \left(G^{(2)}_{g\,a} \otimes G^{(1)}_{g\,b}\right)(z) \\
&+ H_g^{H;(1)}\left(\delta_{g\,a}\,C^{(2)}_{g\,b}(z) + \delta_{g\,b}\,C^{(2)}_{g\,a}(z)\right) + H_g^{H;(2)}\left(\delta_{g\,a}\,C^{(1)}_{g\,b}(z) + \delta_{g\,b}\,C^{(1)}_{g\,a}(z)\right) \\
&+ \left(C^{(1)}_{g\,a} \otimes C^{(2)}_{g\,b}\right)(z) + \left(C^{(2)}_{g\,a} \otimes C^{(1)}_{g\,b}\right)(z) \\
&+ H_g^{H;(1)}\left(C^{(1)}_{g\,a} \otimes C^{(1)}_{g\,b}\right)(z) + H_g^{H;(1)}\left(G^{(1)}_{g\,a} \otimes G^{(1)}_{g\,b}\right)(z)\,.
\end{aligned}$$

THE N^{3}LO QT-SUBTRACTION METHOD

L. C, X. Chen, T. Gehrmann, E. W. N. Glover, and A. Huss (2018)

Third-order ingredients at qT = 0

- Subtracting two consecutive orders, we arrive to an expression that can be used to extract numerically the missing terms

$$\left(\frac{\alpha_{\rm s}}{\pi}\right)^3 \frac{M^2}{\hat{s}} \sum_c \sigma^{(0)}_{c\bar{c},\,F}(\alpha_{\rm s}, M)\, \mathcal{H}^{F\,;(3)}_{c\bar{c}\leftarrow ab}\left(\frac{M^2}{\hat{s}}; \frac{M^2}{\mu_R^2}, \frac{M^2}{\mu_F^2}\right)$$

$$= \left\{\left[\hat{\sigma}^{\rm tot}_{F\,ab}\right]_{\rm N^3LO} - \left[\hat{\sigma}^{\rm tot}_{F\,ab}\right]_{\rm NNLO}\right\} - \int_0^\infty {\rm d}q_T^2 \left\{\left[\frac{{\rm d}\hat{\sigma}^{\rm (fin.)}_{F\,ab}}{{\rm d}q_T^2}\right]_{\rm N^3LO} - \left[\frac{{\rm d}\hat{\sigma}^{\rm (fin.)}_{F\,ab}}{{\rm d}q_T^2}\right]_{\rm NNLO}\right\}$$

We extract numerically these information that we put in a constant (independent of z)

$$C_{N3}\;\; \delta_{g\,a}\,\delta_{g\,b}\,\delta(1-z) \;\leftarrow\; \delta_{g\,a}\,\delta_{g\,b}\,\delta(1-z)\left[H_g^{H;(3)}\right]_{(\delta^{q_T}_{(2)})} + \delta_{g\,a}\,C^{(3)}_{g\,b}(z) + \delta_{g\,b}\,C^{(3)}_{g\,a}(z)$$
$$+ \left(G^{(1)}_{g\,a} \otimes G^{(2)}_{g\,b}\right)(z) + \left(G^{(2)}_{g\,a} \otimes G^{(1)}_{g\,b}\right)(z) \;\;,$$

Exact numerical extraction

Approximation of a z function by a constant (a delta term)

THE N^{3}LO QT-SUBTRACTION METHOD

L. C, X. Chen, T. Gehrmann, E. W. N. Glover, and A. Huss (2018)

Third-order ingredients at qT = 0

- Subtracting two consecutive orders, we arrive to an expression that can be used to extract numerically the missing terms

$$\left(\frac{\alpha_{\rm s}}{\pi}\right)^3 \frac{M^2}{\hat{s}} \sum_c \sigma^{(0)}_{c\bar{c},\,F}(\alpha_{\rm s}, M)\, \mathcal{H}^{F\,;(3)}_{c\bar{c}\leftarrow ab}\left(\frac{M^2}{\hat{s}}; \frac{M^2}{\mu_R^2}, \frac{M^2}{\mu_F^2}\right)$$

$$= \left\{ \left[\hat{\sigma}^{\rm tot}_{F\,ab}\right]_{\rm N^3LO} - \left[\hat{\sigma}^{\rm tot}_{F\,ab}\right]_{\rm NNLO} \right\} - \int_0^\infty {\rm d}q_T^2 \left\{ \left[\frac{{\rm d}\hat{\sigma}^{(\rm fin.)}_{F\,ab}}{{\rm d}q_T^2}\right]_{\rm N^3LO} - \left[\frac{{\rm d}\hat{\sigma}^{(\rm fin.)}_{F\,ab}}{{\rm d}q_T^2}\right]_{\rm NNLO} \right\}$$

We extract numerically these information that we put in a constant (independent of z)

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$$+ \left(G^{(1)}_{g\,a} \otimes G^{(2)}_{g\,b}\right)(z) + \left(G^{(2)}_{g\,a} \otimes G^{(1)}_{g\,b}\right)(z) \;\;,$$

Let me postpone the discussion of the universal terms inside the hard-virtual factors, since it is orthogonal to the following slides

$$H_g^{H;(3)} \equiv \widetilde{H}_g^{H;(3)} + \left[H_g^{H;(3)}\right]_{(\delta^{q_T}_{(2)})}$$

Third order constant of “soft origin”, belonging to the finite part of the third order structure of the IR singularities

THE N^{3}LO QT-SUBTRACTION METHOD

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Total exact Xsec. from B. Mistlberger (2018)

Ihixs 2.0: F. Dulat, A. Lazopoulos and B. Mistlberger (2018)

THE N^{3}LO QT-SUBTRACTION METHOD

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$$= \left\{ \left[\hat{\sigma}^{\rm tot}_{F\,ab}\right]_{\rm N^3LO} - \left[\hat{\sigma}^{\rm tot}_{F\,ab}\right]_{\rm NNLO} \right\} - \int_{\text{qTcut\^2}}^{\infty} {\rm d}q_T^2 \left\{ \left[\frac{{\rm d}\hat{\sigma}^{\rm (fin.)}_{F\,ab}}{{\rm d}q_T^2}\right]_{\rm N^3LO} - \left[\frac{{\rm d}\hat{\sigma}^{\rm (fin.)}_{F\,ab}}{{\rm d}q_T^2}\right]_{\rm NNLO} \right\}$$

The lower limit in the integral has to be replaced by a technical cut, the qTcut

$$\frac{{\rm d}\hat{\sigma}^{\rm (fin.)}_{F\,ab}}{{\rm d}q_T^2} \equiv \left[\frac{{\rm d}\hat{\sigma}^{F+\rm jets}_{ab}}{{\rm d}q_T^2} - \frac{{\rm d}\hat{\sigma}^{F\ \rm CT}_{ab}}{{\rm d}q_T^2}\right]$$

NNLOJET

X. Chen, J. Cruz-Martinez, T. Gehrmann, E. W. N. Glover and M. Jaquier (2016)

Aude Gehrmann-De Ridder Talk

HN3LO

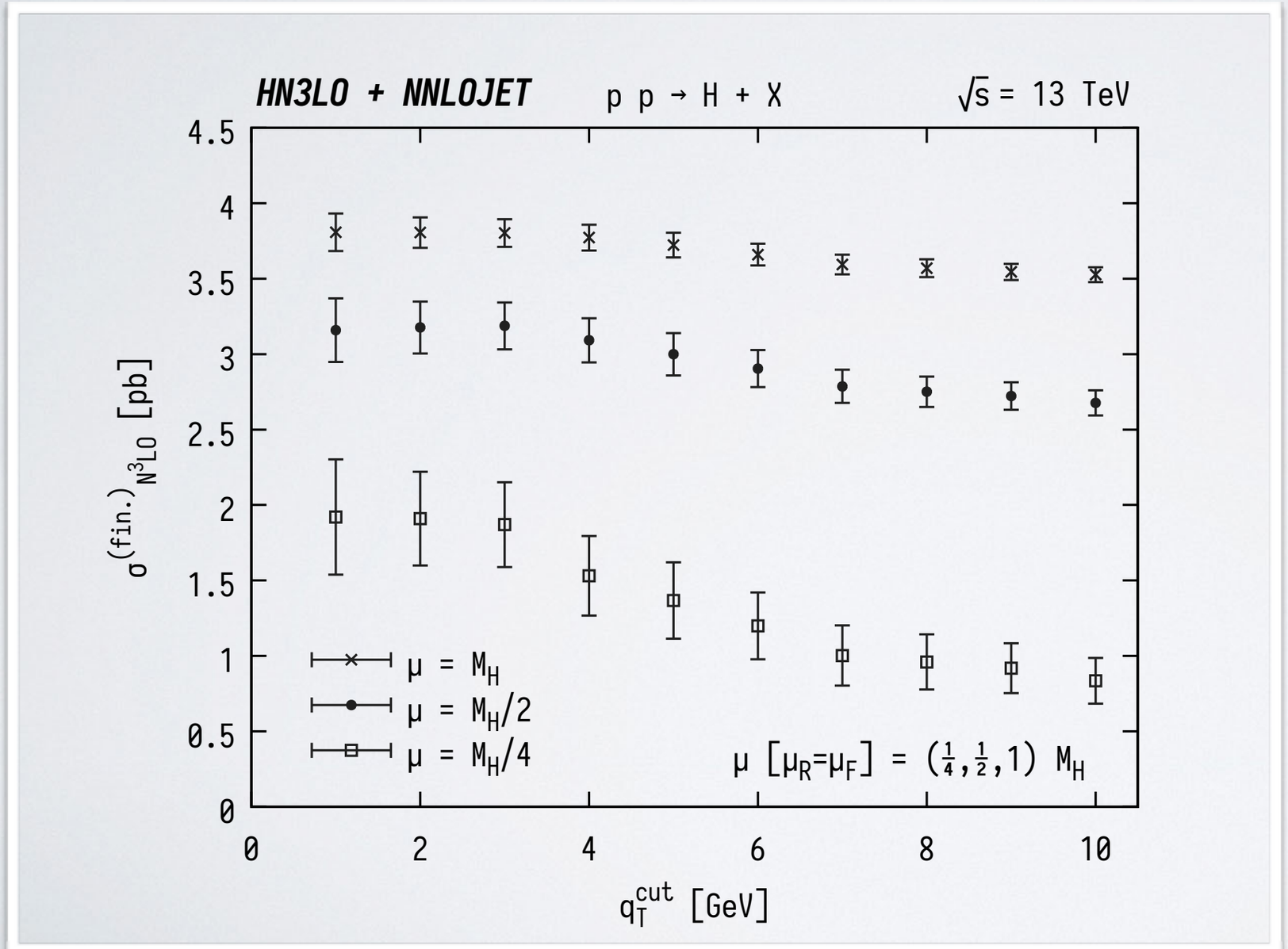
LC (2018)

THE N^{3}LO QT-SUBTRACTION METHOD

L. C, X. Chen, T. Gehrmann, E. W. N. Glover, and A. Huss (2018)

Third-order ingredients at qT = 0

- For the extraction, we consider Higgs boson production at N^{3}LO in gluon fusion in the large top mass limit, at $\sqrt{s} = 13\ \mathrm{TeV}$



$$\int_{q_T^{cut2}}^{\infty} dq_T^2 \left[\frac{\sigma_{N^3LO}^{F(fin.)}}{q_T^2} - \frac{\sigma_{N^2LO}^{F(fin.)}}{q_T^2} \right]$$

THE N^{3}LO QT-SUBTRACTION METHOD

L. C, X. Chen, T. Gehrmann, E. W. N. Glover, and A. Huss (2018)

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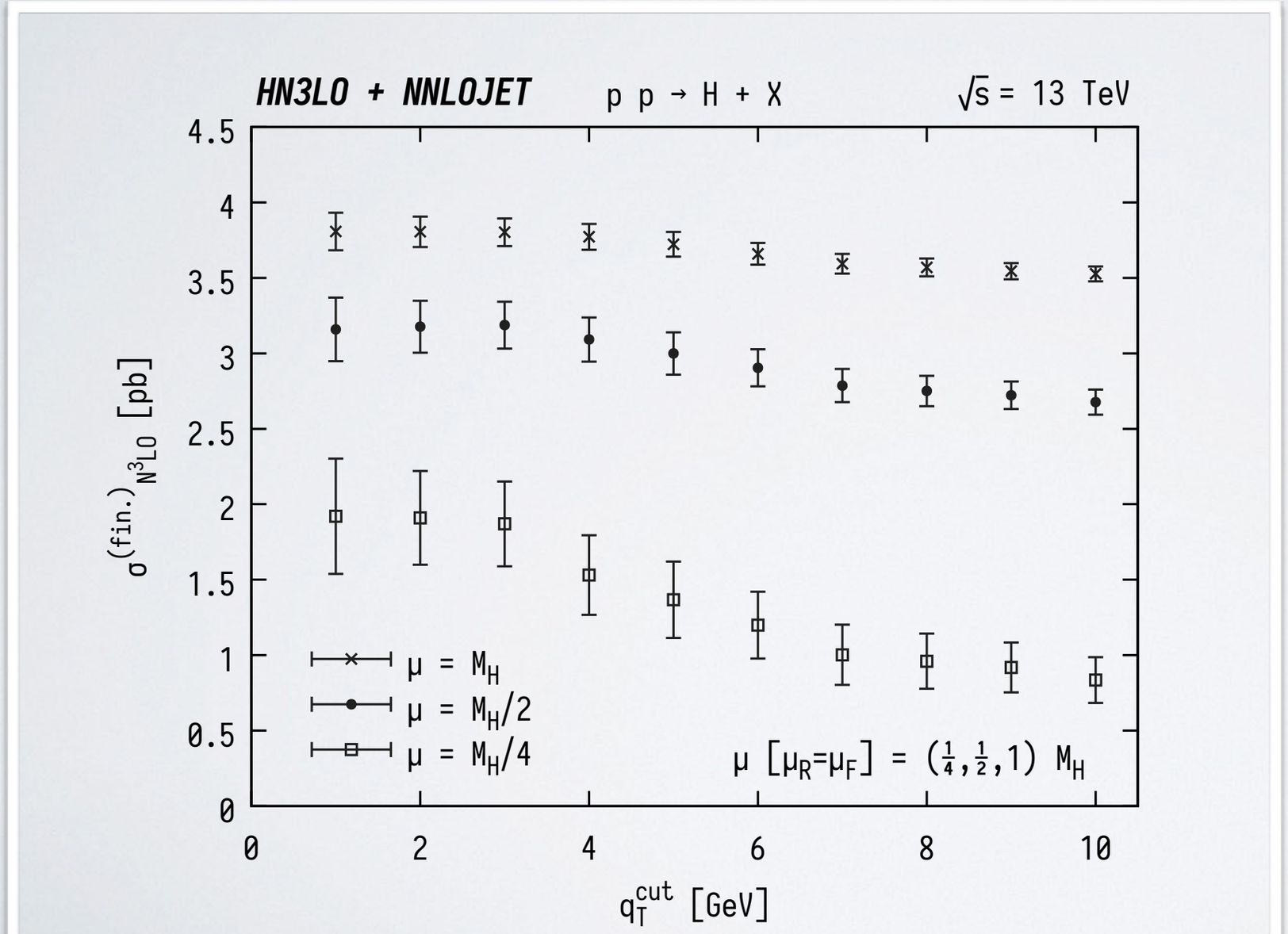


For the extraction procedure , the NNLO Xsection is taken from the analytical result, therefore, there is no systematic uncertainty from the NNLO

$$\int_{q_T^{cut2}}^{\infty} dq_T^2 \left[\frac{\sigma_{N^3LO}^{F(fin.)}}{q_T^2} - \frac{\sigma_{N^2LO}^{F(fin.)}}{q_T^2} \right]$$

$$C_{N3} = -943 \pm 222$$

The size of the uncertainty bands are almost entirely due to the F+Jet calculation

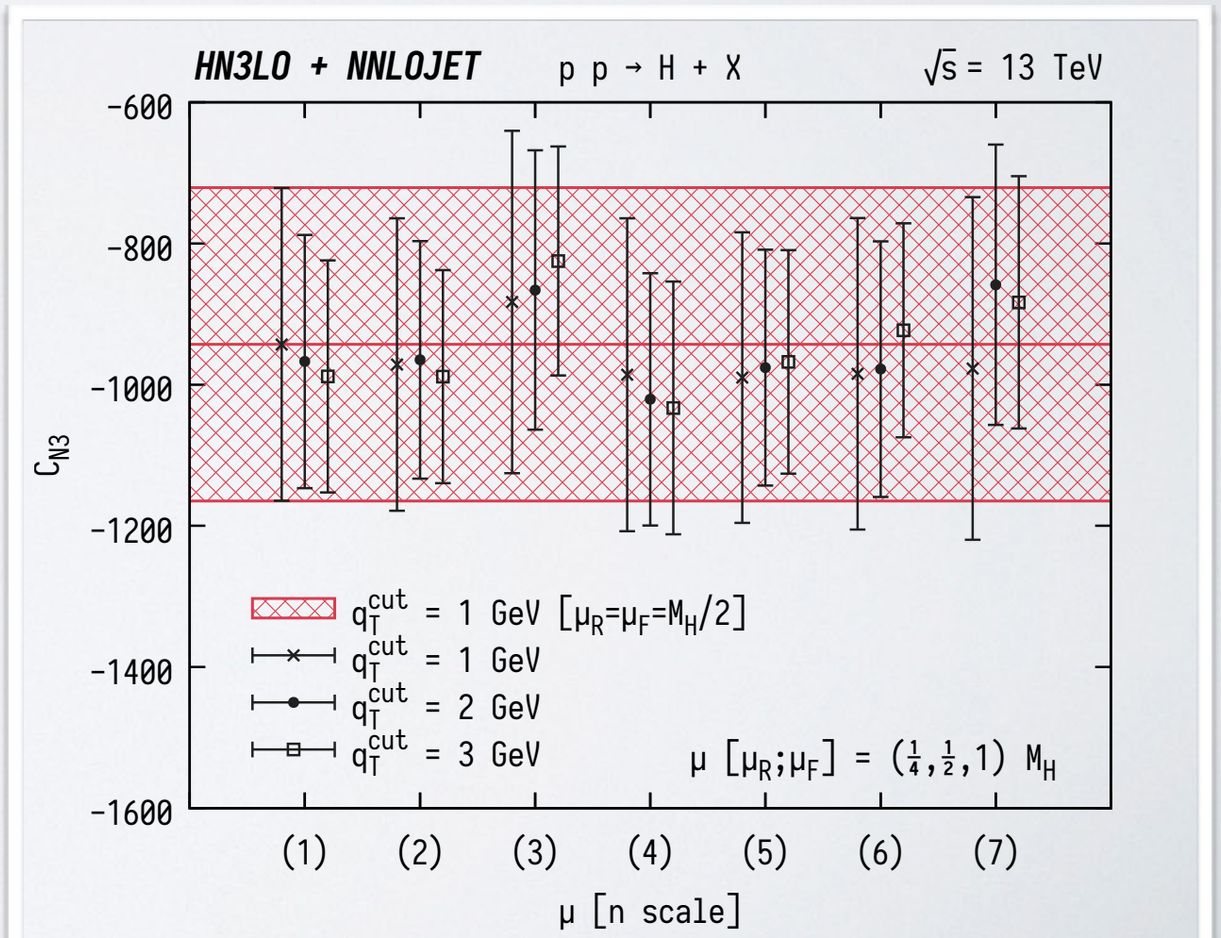
THE N^{3}LO QT-SUBTRACTION METHOD

L. C, X. Chen, T. Gehrmann, E. W. N. Glover, and A. Huss (2018)

Third-order ingredients at qT = 0

- Repeating the same procedure for 7-point scale variation

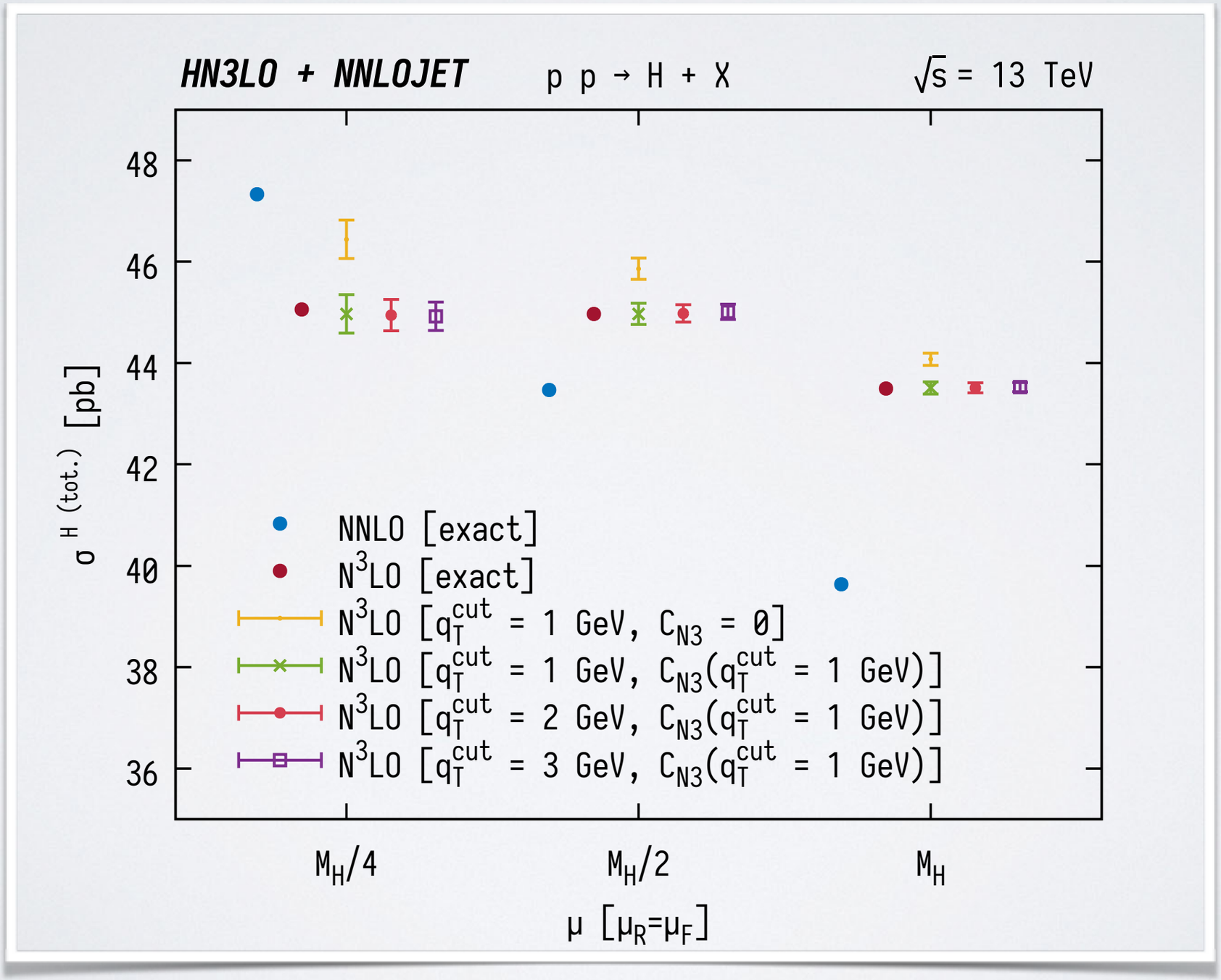
n	$[\tilde{\mu}_\mathrm{R}, \tilde{\mu}_\mathrm{F}] \times M_H$	C_{N3} ($q_T^{\mathrm{cut}} = 1$ GeV)	C_{N3} ($q_T^{\mathrm{cut}} = 2$ GeV)	C_{N3} ($q_T^{\mathrm{cut}} = 3$ GeV)
(1)	$[1/2, 1/2]$	$\mathbf{-943 \pm 222}$	-967 ± 179	-988 ± 164
(2)	$[1, 1]$	-971 ± 207	-965 ± 168	-989 ± 151
(3)	$[1/4, 1/4]$	-883 ± 243	-866 ± 198	-850 ± 162
(4)	$[1/2, 1]$	-986 ± 222	-1021 ± 179	-1033 ± 179
(5)	$[1, 1/2]$	-990 ± 206	-976 ± 167	-968 ± 158
(6)	$[1/2, 1/4]$	-985 ± 221	-978 ± 181	-923 ± 152
(7)	$[1/4, 1/2]$	-977 ± 243	-859 ± 199	-883 ± 179



THE N^{3}LO TOTAL XSECTION

L. C, X. Chen, T. Gehrmann, E. W. N. Glover, and A. Huss (2018)

- With the extracted CN3 coefficient (only one value corresponding to the central scale) we can use the qT-subtraction method to predict total Xsections at N^{3}LO



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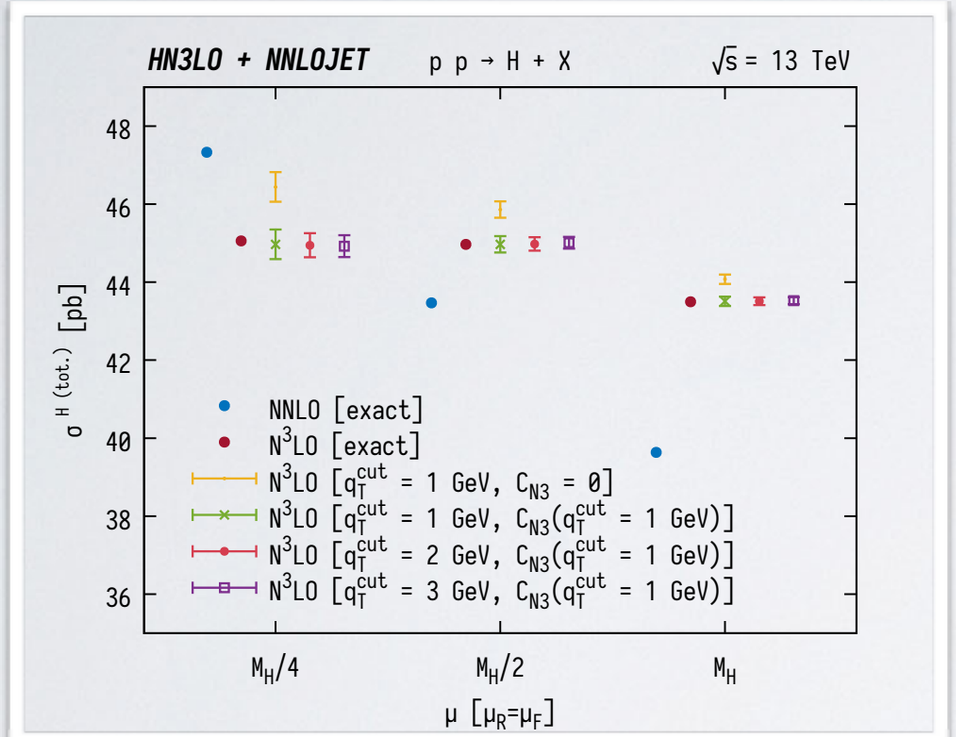


Total exact Xsec. from B. Mistlberger (2018)

Ihixs 2.0: F. Dulat, A. Lazopoulos and B. Mistlberger (2018)

$\sigma^{H\,(\mathrm{tot.})}$ (pb)	Exact	q_T subtraction ($q_T^{\mathrm{cut}} = 1$ GeV)	q_T subtraction ($q_T^{\mathrm{cut}} = 2$ GeV)	q_T subtraction ($q_T^{\mathrm{cut}} = 3$ GeV)	q_T subtraction ($C_{N3} = 0$)
N^{3}LO$[\mu = M_H/2]$	44.97	44.97 $\pm$ 0.21	44.98 $\pm$ 0.17	45.01 $\pm$ 0.15	45.86 $\pm$ 0.21
N^{3}LO$[\mu = M_H]$	43.50	43.51 $\pm$ 0.12	43.51 $\pm$ 0.10	43.53 $\pm$ 0.09	44.08 $\pm$ 0.12
N^{3}LO$[\mu = M_H/4]$	45.06	44.97 $\pm$ 0.38	44.95 $\pm$ 0.31	44.92 $\pm$ 0.28	46.44 $\pm$ 0.38
NNLO $[\mu = M_H/2]$	43.47	43.46 $\pm$ 0.02	43.46 $\pm$ 0.02	43.46 $\pm$ 0.02	43.46 $\pm$ 0.02
NNLO $[\mu = M_H]$	39.64	39.62 $\pm$ 0.02	39.62 $\pm$ 0.02	39.62 $\pm$ 0.02	39.62 $\pm$ 0.02
NNLO $[\mu = M_H/4]$	47.33	47.33 $\pm$ 0.02	47.33 $\pm$ 0.02	47.33 $\pm$ 0.02	47.33 $\pm$ 0.02

N^3LO DIFFERENTIAL DISTRIBUTIONS

L. C, X. Chen, T. Gehrmann, E. W. N. Glover, and A. Huss (2018)

- The precedent results (CN3 coefficient) allow us to compute more exclusive observables (distributions) at N^3LO

- Before starting with the N^3LO computation, a couple of questions are in order (regarding CN3): **uncertainty introduced by the proposed numerical strategy?**

- In order to answer this question we can repeat the numerical strategy, but at one order less (NNLO) where all the coefficients are known analytically

$$C_N \;\; \delta_{g\,a}\,\delta_{g\,b}\,\delta(1-z) \;\leftarrow\; \delta_{g\,a}\,\delta_{g\,b}\,\delta(1-z)\left[H_g^{H;(2)}\right]_{(\delta_{(1)}^{q_T})} + \delta_{g\,a}\,C_{g\,b}^{(2)}(z) + \delta_{g\,b}\,C_{g\,a}^{(2)}(z) + \left(G_{g\,a}^{(1)} \otimes G_{g\,b}^{(1)}\right)(z)$$

NNLO DIFFERENTIAL DISTRIBUTIONS

L. C, X. Chen, T. Gehrmann, E. W. N. Glover, and A. Huss (2018)

- In order to answer this question we can repeat the numerical strategy, at one order less (NNLO) where all the coefficient are known analytically

$$\mathcal{H}^{H;(2)}_{gg\leftarrow ab}(z) = \delta_{g\,a}\,\delta_{g\,b}\,\delta(1-z)\,H_g^{H;(2)} + \delta_{g\,a}\,C^{(2)}_{g\,b}(z) + \delta_{g\,b}\,C^{(2)}_{g\,a}(z) + H_g^{H;(1)}\left(\delta_{g\,a}\,C^{(1)}_{g\,b}(z) + \delta_{g\,b}\,C^{(1)}_{g\,a}(z)\right) + \left(C^{(1)}_{g\,a}\otimes C^{(1)}_{g\,b}\right)(z) + \left(G^{(1)}_{g\,a}\otimes G^{(1)}_{g\,b}\right)(z)\ .$$

Only scale independent part

$$C_N\ \ \delta_{g\,a}\,\delta_{g\,b}\,\delta(1-z) \ \leftarrow\ \ \delta_{g\,a}\,\delta_{g\,b}\,\delta(1-z)\left[H_g^{H;(2)}\right]_{(\delta^{q_T}_{(1)})} + \delta_{g\,a}\,C^{(2)}_{g\,b}(z) + \delta_{g\,b}\,C^{(2)}_{g\,a}(z) + \left(G^{(1)}_{g\,a}\otimes G^{(1)}_{g\,b}\right)(z)$$

Second order constant of “soft origin”, belonging to the finite part of the second-order structure of the IR singularities

$$H_g^{H;(2)} \equiv \widetilde{H}_g^{H;(2)} + \left[H_g^{H;(2)}\right]_{(\delta^{q_T}_{(1)})}$$

$$C_{N2} = 28 \pm 1$$

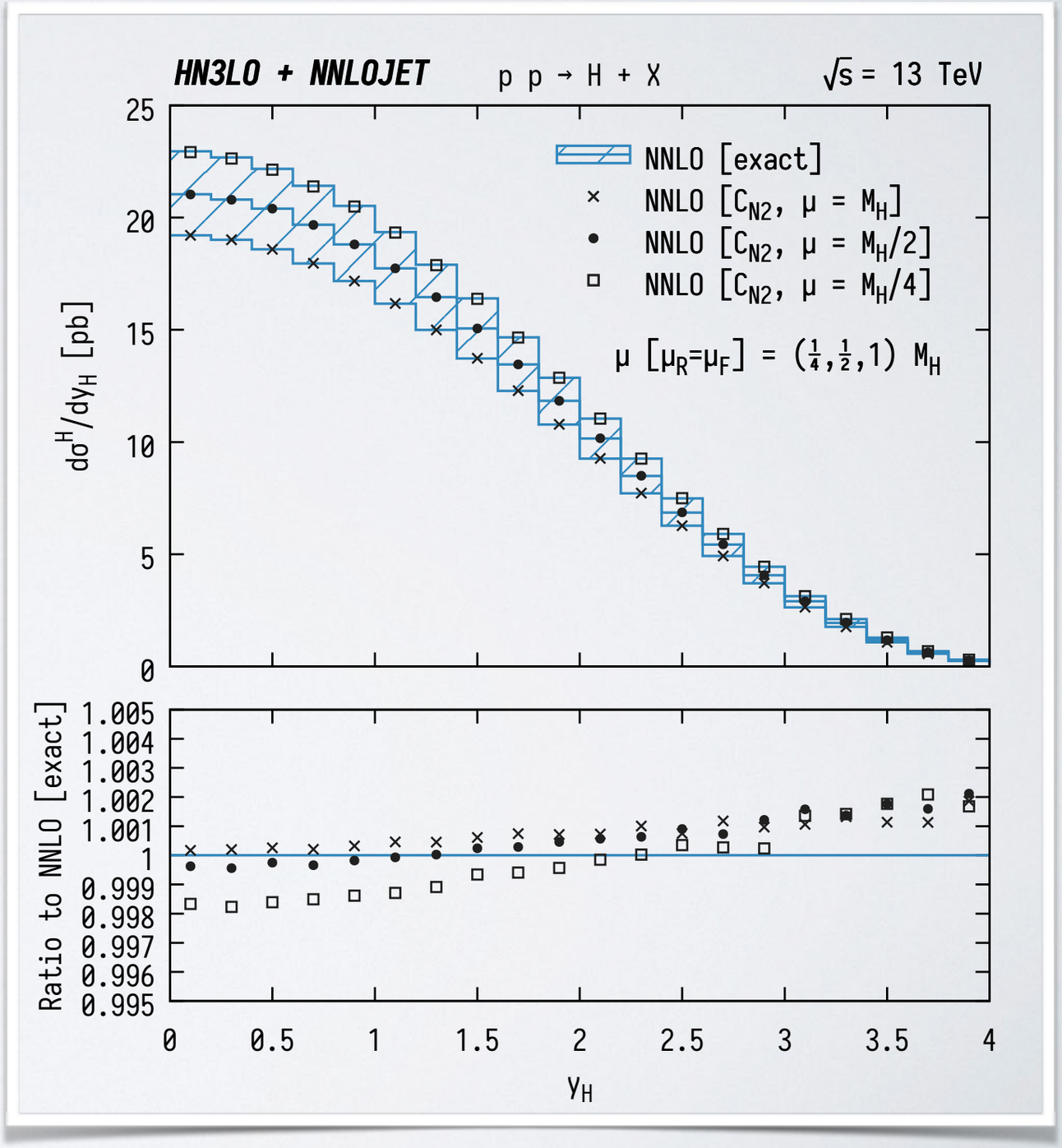
NNLO DIFFERENTIAL DISTRIBUTIONS

L. C, X. Chen, T. Gehrmann, E. W. N. Glover, and A. Huss (2018)

- In order to answer this question we can repeat the numerical strategy, at one order less (NNLO) where all the coefficient are known analytically

The uncertainties at the differential distributions are at the 0.2% level!

With the exception of the scale MH/4, the rest of the scales show their best behaviour in the low rapidity region and they start to separate from the exact result at large rapidities, which is what we expect.



N3LO DIFFERENTIAL DISTRIBUTIONS

N^3LO DIFFERENTIAL DISTRIBUTIONS

L. C, X. Chen, T. Gehrmann, E. W. N. Glover, and A. Huss (2018)

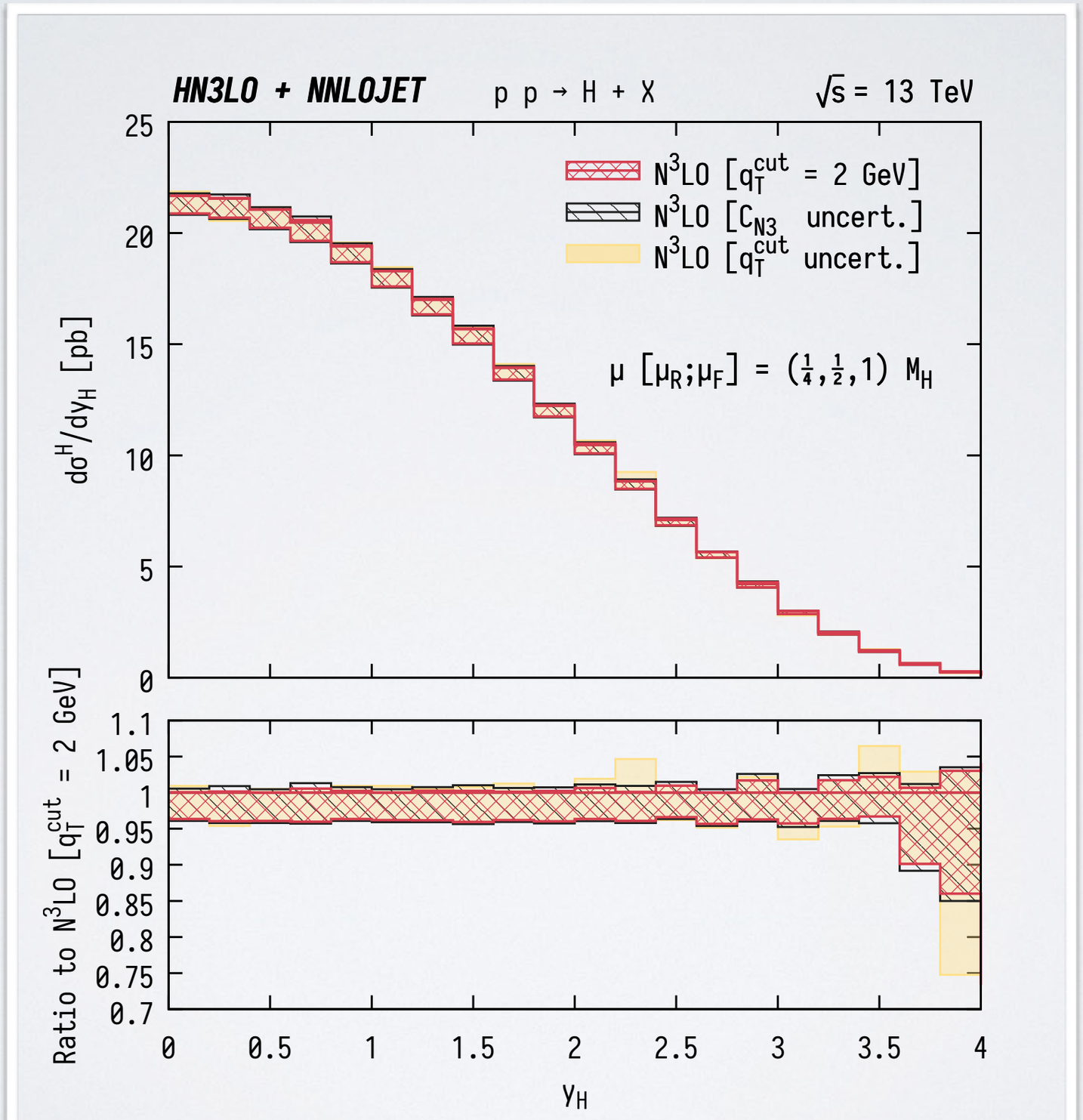
Uncertainties at N^3LO

N3LO prediction at qTcut=2GeV + qTcut uncertainties + systematic uncertainties

qTcut uncertainties Calculated taking three different predictions for qTcut = (2 ± 1) GeV

The black band is obtained as the envelope of the seven-point scale variation at qTcut = 2 GeV considering for each scale the two extremal CN3 coefficients corresponding to its maximum and minimum statistical deviations

CN3 = {−1165,−721}

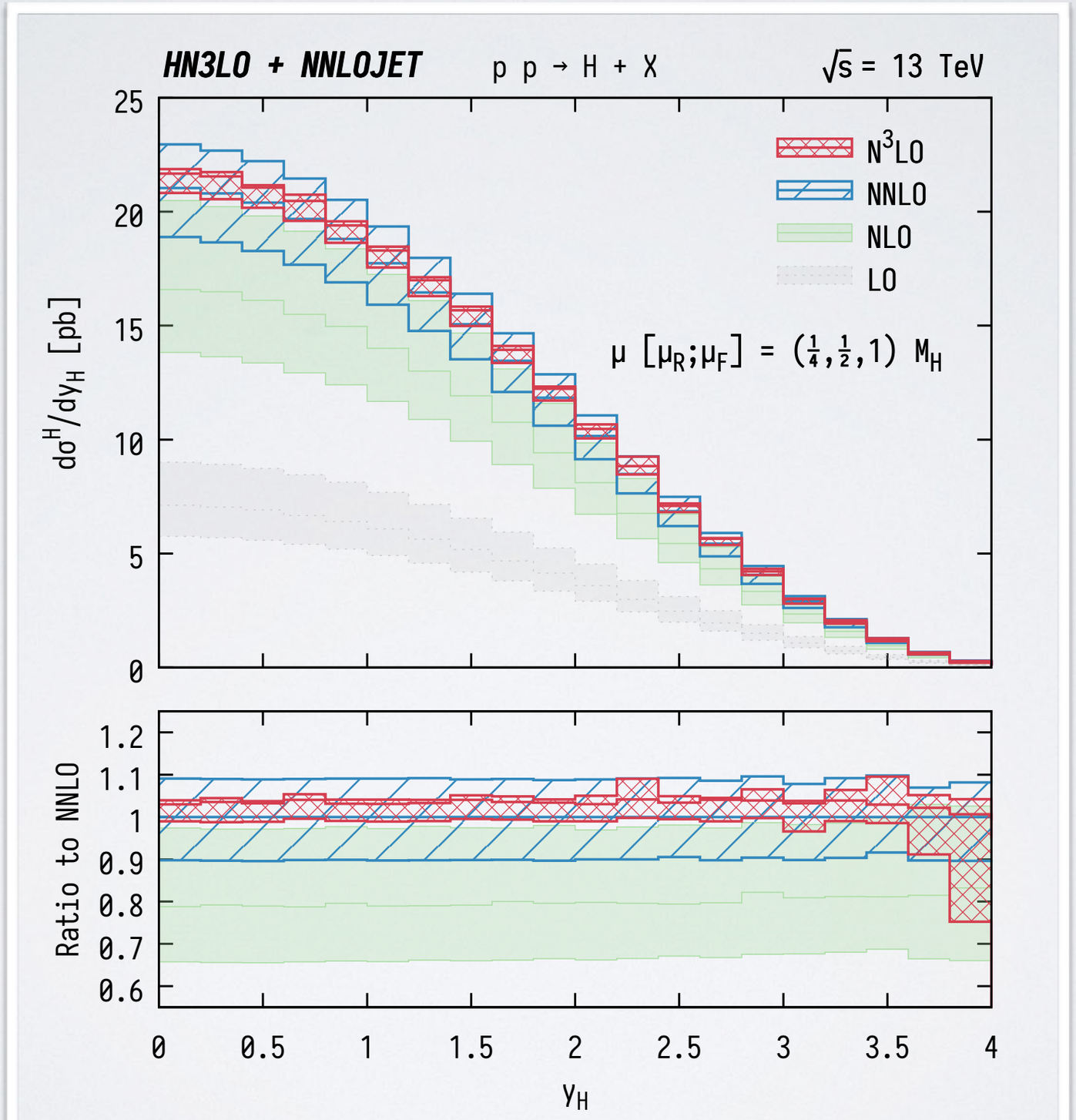


N^3LO DIFFERENTIAL DISTRIBUTIONS

L. C, X. Chen, T. Gehrmann, E. W. N. Glover, and A. Huss (2018)

Rapidity distribution

We calculate the yH distribution at N3LO employing a seven-point scale variation and carefully assess systematic errors arising form different qTcut and CN3 values.



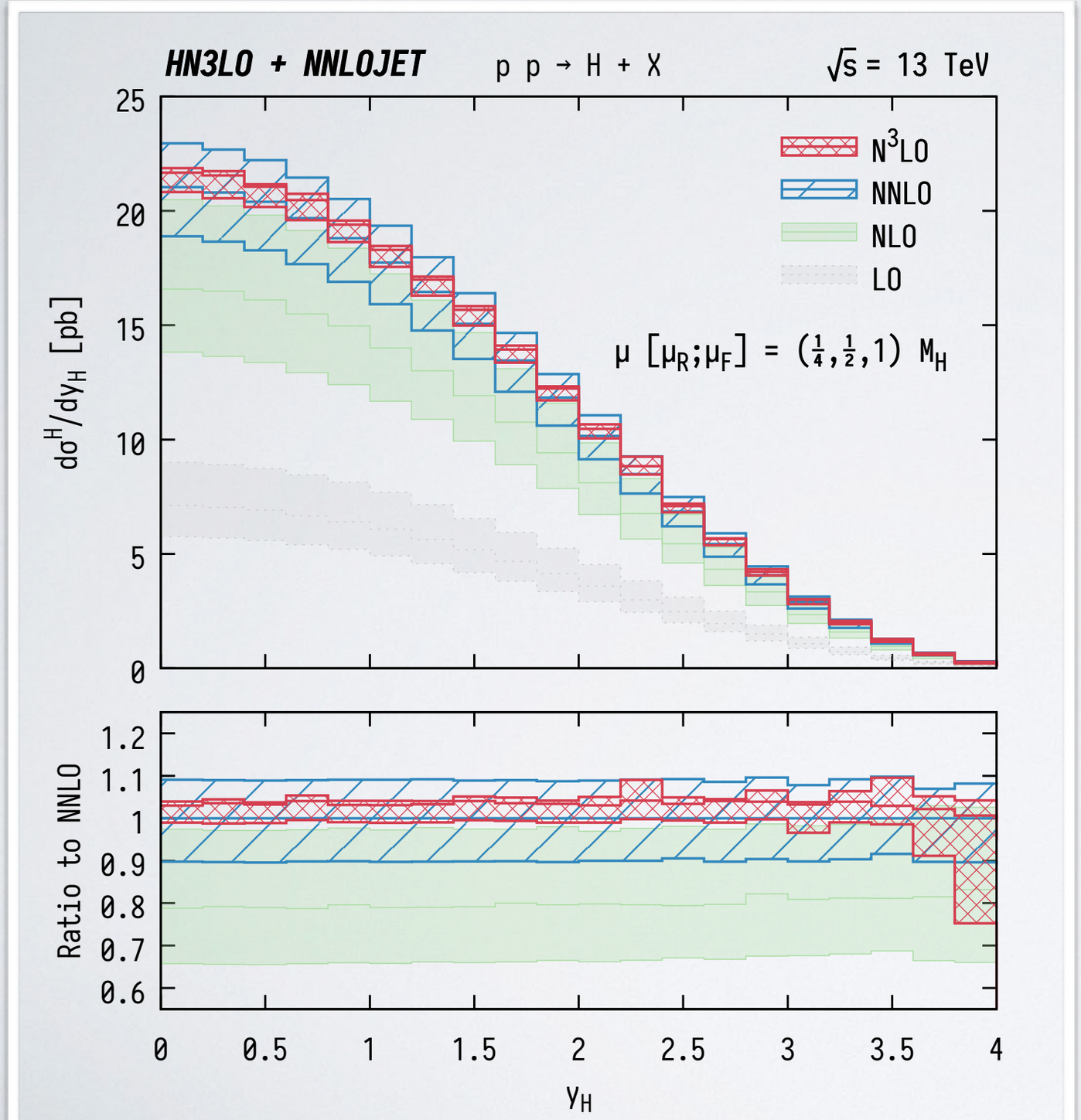
Compared to the NNLO yH distributions, we observe a large reduction of theory uncertainties by more than 50% at N3LO. The scale variation band at N3LO stays within the NNLO band with a flat K-factor of about 1.034 in the central rapidity region ($|y_H| \leq 3.6$).

N3LO prediction at qTcut=2GeV + qTcut uncertainties + systematic uncertainties

N^{3}LO DIFFERENTIAL DISTRIBUTIONS

L. C, X. Chen, T. Gehrmann, E. W. N. Glover, and A. Huss (2018)

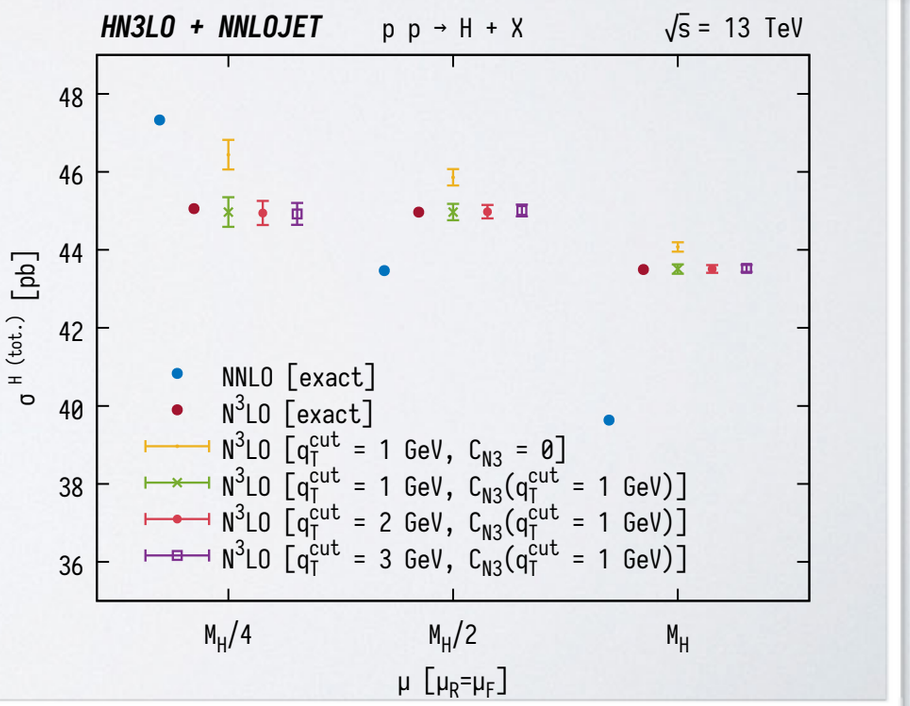
Rapidity distribution



The central prediction at N^{3}LO, almost coincides with the upper edge of the band, as was already observed for the total cross section

Total exact Xsec. from B. Mistlberger (2018)

Ihixs 2.0: F. Dulat, A. Lazopoulos and B. Mistlberger (2018)



CONCLUSIONS AND OUTLOOK

- We presented results at N^{3}LO for completely differential distributions for observables measured at the LHC (in particular for Higgs boson production in gluon fusion)
- The qT-subtraction method at N^{3}LO, presented here, can be applied to other processes initiated by gluon fusion (if the three-loop scattering amplitudes are known), i.e the CN3 in gluon fusion is universal
- More work is necessary in order to calculate the CN3 coefficient for processes initiated by qqb annihilation, since the total cross section at N^{3}LO for Drell-Yan is not known
- In the future, two elements are necessary in order to complete analytically the qT-subtraction method at N^{3}LO
  - The universal relation between the three-loop virtual scattering amplitudes and the hard virtual factor
  - The third order collinear functions
- Our results can be applied to perform resummation at the corresponding level of logarithmic accuracy.

CONCLUSIONS AND OUTLOOK

- Using the extracted value of CN3, we performed a closure test for the inclusive total cross section for three different scale choices and find excellent agreement with the exact results (from ihixs 2.0) at the 0.2% level
- The approximation related to the CN3 coefficient in our approach **can be easily replaced by the full analytical results once available**
- Compared to the NNLO yH distributions, we observe a large reduction of theory uncertainties by more than 50% at N3LO. The scale variation band at N3LO stays within the NNLO band with a flat K-factor of about 1.034 in the central rapidity region ($|yH| \leq 3.6$)
- The combined theoretical uncertainty at N^3LO is at most of $\pm 5\%$ level with respect to the central scale choice.

THANK YOU!

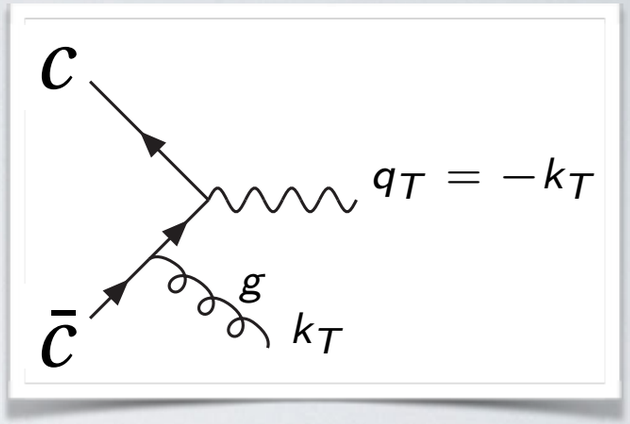
BACKUP SLIDES

UNIVERSALITY OF TRANSVERSE-MOMENTUM RESUMMATION AND HARD FACTORS AT THE NNLO

S. Catani, L. C, D. de Florian, G. Ferrera and M. Grazzini (2013)

We consider the partonic elastic-production process

$$c(\hat{p}_1) + \bar{c}(\hat{p}_2) \to F(\{q_i\})$$



The renormalized all-loop amplitude of the precedent process has the perturbative (loop) expansion

$$\mathcal{M}_{c\bar{c}\to F}(\hat{p}_1,\hat{p}_2;\{q_i\}) = \left(\alpha_S(\mu_R^2)\,\mu_R^{2\epsilon}\right)^k \left[\mathcal{M}^{(0)}_{c\bar{c}\to F}(\hat{p}_1,\hat{p}_2;\{q_i\}) + \left(\frac{\alpha_S(\mu_R^2)}{2\pi}\right)\mathcal{M}^{(1)}_{c\bar{c}\to F}(\hat{p}_1,\hat{p}_2;\{q_i\};\mu_R) + \left(\frac{\alpha_S(\mu_R^2)}{2\pi}\right)^2\mathcal{M}^{(2)}_{c\bar{c}\to F}(\hat{p}_1,\hat{p}_2;\{q_i\};\mu_R) + \sum_{n=3}^{\infty}\left(\frac{\alpha_S(\mu_R^2)}{2\pi}\right)^n\mathcal{M}^{(n)}_{c\bar{c}\to F}(\hat{p}_1,\hat{p}_2;\{q_i\};\mu_R)\right]$$

$$\mathcal{M}^{(l)}_{c\bar{c}\to F} \sim \left(\frac{1}{\epsilon}\right)^{2l} + \dots$$

Removing the poles we have

$$\widetilde{\mathcal{M}}_{c\bar{c}\to F}(\hat{p}_1,\hat{p}_2;\{q_i\}) = \left(\alpha_S(\mu_R^2)\,\mu_R^{2\epsilon}\right)^k \left[\widetilde{\mathcal{M}}^{(0)}_{c\bar{c}\to F}(\hat{p}_1,\hat{p}_2;\{q_i\}) + \left(\frac{\alpha_S(\mu_R^2)}{2\pi}\right)\widetilde{\mathcal{M}}^{(1)}_{c\bar{c}\to F}(\hat{p}_1,\hat{p}_2;\{q_i\};\mu_R) + \left(\frac{\alpha_S(\mu_R^2)}{2\pi}\right)^2\widetilde{\mathcal{M}}^{(2)}_{c\bar{c}\to F}(\hat{p}_1,\hat{p}_2;\{q_i\};\mu_R) + \sum_{n=3}^{\infty}\left(\frac{\alpha_S(\mu_R^2)}{2\pi}\right)^n\widetilde{\mathcal{M}}^{(n)}_{c\bar{c}\to F}(\hat{p}_1,\hat{p}_2;\{q_i\};\mu_R)\right]$$

UNIVERSALITY OF TRANSVERSE-MOMENTUM RESUMMATION AND HARD FACTORS AT THE NNLO

S. Catani, L. C, D. de Florian, G. Ferrera and M. Grazzini (2013)

$$\widetilde{\mathcal{M}}^{(0)}_{c\bar{c}\to F} = \mathcal{M}^{(0)}_{c\bar{c}\to F}$$

$$\widetilde{\mathcal{M}}^{(1)}_{c\bar{c}\to F} = \mathcal{M}^{(1)}_{c\bar{c}\to F} - \tilde{I}^{(1)}_{c}(\epsilon, M^2/\mu_R^2)\ \mathcal{M}^{(0)}_{c\bar{c}\to F}\ ,$$

$$\widetilde{\mathcal{M}}^{(2)}_{c\bar{c}\to F} = \mathcal{M}^{(2)}_{c\bar{c}\to F} - \tilde{I}^{(1)}_{c}(\epsilon, M^2/\mu_R^2)\ \mathcal{M}^{(1)}_{c\bar{c}\to F} - \tilde{I}^{(2)}_{c}(\epsilon, M^2/\mu_R^2)\ \mathcal{M}^{(0)}_{c\bar{c}\to F}$$

The explicit expression of the first-order (one-loop) subtraction operator is

$$\tilde{I}^{(1)}_{a}(\epsilon, M^2/\mu_R^2) = \tilde{I}^{(1)\,\mathrm{soft}}_{a}(\epsilon, M^2/\mu_R^2) + \tilde{I}^{(1)\,\mathrm{coll}}_{a}(\epsilon, M^2/\mu_R^2)$$

$$\tilde{I}^{(1)\,\mathrm{soft}}_{a}(\epsilon, M^2/\mu_R^2) = -\frac{e^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)}\left(\frac{1}{\epsilon^2} + i\pi\,\frac{1}{\epsilon} + \delta^{q_T}\right) C_a \left(\frac{M^2}{\mu_R^2}\right)^{-\epsilon}$$

$$\tilde{I}^{(1)\,\mathrm{coll}}_{a}(\epsilon, M^2/\mu_R^2) = -\frac{1}{\epsilon}\,\gamma_a \left(\frac{M^2}{\mu_R^2}\right)^{-\epsilon}\ ,$$

$$\delta^{q_T} = 0$$

UNIVERSALITY OF TRANSVERSE-MOMENTUM RESUMMATION AND HARD FACTORS AT THE NNLO

S. Catani, L. C, D. de Florian, G. Ferrera and M. Grazzini (2013)

The explicit expression of the second-order (two-loop) subtraction operator is

$$\tilde{I}_a^{(2)}(\epsilon, M^2/\mu_R^2) = -\frac{1}{2}\left[\tilde{I}_a^{(1)}(\epsilon, M^2/\mu_R^2)\right]^2 + \left\{\frac{2\pi\beta_0}{\epsilon}\left[\tilde{I}_a^{(1)}(2\epsilon, M^2/\mu_R^2) - \tilde{I}_a^{(1)}(\epsilon, M^2/\mu_R^2)\right] + K\,\tilde{I}_a^{(1)\,\mathrm{soft}}(2\epsilon, M^2/\mu_R^2) + \widetilde{H}_a^{(2)}(\epsilon, M^2/\mu_R^2)\right\}$$

$$\widetilde{H}_a^{(2)}(\epsilon, M^2/\mu_R^2) = \widetilde{H}_a^{(2)\,\mathrm{coll}}(\epsilon, M^2/\mu_R^2) + \widetilde{H}_a^{(2)\,\mathrm{soft}}(\epsilon, M^2/\mu_R^2) = \frac{1}{4\epsilon}\left(\frac{M^2}{\mu_R^2}\right)^{-2\epsilon}\left(\frac{1}{4}\,\gamma_{a\,(1)} + C_a\,d_{(1)} + \epsilon\,C_a\,\delta_{(1)}^{q_T}\right)$$

$$\delta_{(1)}^{q_T} = \frac{20}{3}\zeta_3\pi\beta_0 + \left(-\frac{1214}{81} + \frac{67}{18}\zeta_2\right)C_A + \left(\frac{164}{81} - \frac{5}{9}\zeta_2\right)N_f$$

Therefore the all-order hard virtual factor can be written

$$\alpha_{\mathrm{S}}^{2k}(M^2)\,H_q^F(x_1p_1, x_2p_2; \mathbf{\Omega}; \alpha_{\mathrm{S}}(M^2)) = \frac{|\widetilde{\mathcal{M}}_{q\bar{q}\to F}(x_1p_1, x_2p_2; \{q_i\})|^2}{|\mathcal{M}^{(0)}_{q\bar{q}\to F}(x_1p_1, x_2p_2; \{q_i\})|^2}$$

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S. Catani, L. C, D. de Florian, G. Ferrera and M. Grazzini (2013)

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Ingredient of the qT-subtraction method

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The precedent discussion can be extended to threshold resummation, defining subtraction operators, and regularized amplitudes in the same way that we did for qT-subtraction/resummation

$$\widetilde{\mathcal{M}}^{\rm th}_{c\bar{c}\to F} = \left[1 - \tilde{I}^{\rm th}_c(\epsilon, M^2)\right] \mathcal{M}_{c\bar{c}\to F}$$

$$\tilde{I}^{\rm th}_c(\epsilon, M^2) = \frac{\alpha_{\rm S}(\mu_R^2)}{2\pi}\, \tilde{I}^{\rm th(1)}_c\left(\epsilon, \frac{M^2}{\mu_R^2}\right) + \left(\frac{\alpha_{\rm S}(\mu_R^2)}{2\pi}\right)^2 \tilde{I}^{\rm th(2)}_c\left(\epsilon, \frac{M^2}{\mu_R^2}\right) + \sum_{n=3}^{\infty} \left(\frac{\alpha_{\rm S}(\mu_R^2)}{2\pi}\right)^n \tilde{I}^{\rm th(n)}_c\left(\epsilon, \frac{M^2}{\mu_R^2}\right)$$

$$\alpha_{\rm S}^{2k}(M^2)\, C^{\rm th}_{c\bar{c}\to F}(\alpha_{\rm S}(M^2)) = \frac{|\widetilde{\mathcal{M}}^{\rm th}_{c\bar{c}\to F}|^2}{|\mathcal{M}^{(0)}_{c\bar{c}\to F}|^2}$$

$$\delta^{\rm th} = \delta^{q_T} - \zeta_2 = -\zeta_2 \ ,$$

$$\delta^{\rm th}_{(1)} = \delta^{q_T}_{(1)} + \frac{40}{3}\zeta_3\pi\beta_0 + 4\zeta_2^2 C_A = \zeta_2\, K + 20\,\zeta_3\,\pi\beta_0 + C_A\left(-\frac{1214}{81} + 5\,\zeta_2^2\right) + \frac{164}{81} N_f$$

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The relation between the virtual coefficients in threshold and in qT-resummation is

$$\frac{H_c^F(\alpha_S)}{C^{\rm th}_{c\bar{c}\to F}(\alpha_S)} = \left\{ \frac{|1-\tilde{I}_c(\epsilon,M^2)|^2}{|1-\tilde{I}^{\rm th}_c(\epsilon,M^2)|^2} \right\}_{\epsilon=0}$$

$$= \exp\left\{ \frac{\alpha_S}{\pi} C_c \left(\delta^{\,q_T} - \delta^{\,\rm th}\right) + \left(\frac{\alpha_S}{\pi}\right)^2 C_c \left[\frac{1}{2} K \left(\delta^{\,q_T} - \delta^{\,\rm th}\right) - \frac{1}{8}\left(\delta^{\,q_T}_{(1)} - \delta^{\,\rm th}_{(1)}\right)\right] + \mathcal{O}(\alpha_S^3) \right\}$$

$$= \exp\left\{ \frac{\alpha_S}{\pi} C_c\, \zeta_2 + \left(\frac{\alpha_S}{\pi}\right)^2 C_c \left[\frac{5}{3}\,\zeta_3 \pi \beta_0 + \zeta_2 \left(\frac{67}{36} C_A - \frac{5}{18} N_f\right)\right] + \mathcal{O}(\alpha_S^3) \right\} ,$$

Extending this relation to the third-order we have all the ingredients, except $\delta^{q_T}_{(2)}$

This implies that the unknown part in the third-order hard-virtual factor is universal (i.e it only depends on the initial state) and we can divide it into two contributions

$$H_g^{H;(3)} \equiv \widetilde{H}_g^{H;(3)} + \left[H_g^{H;(3)}\right]_{(\delta^{q_T}_{(2)})}$$

Notation: Here the subindex denotes as unknown all the third order part in the exponential

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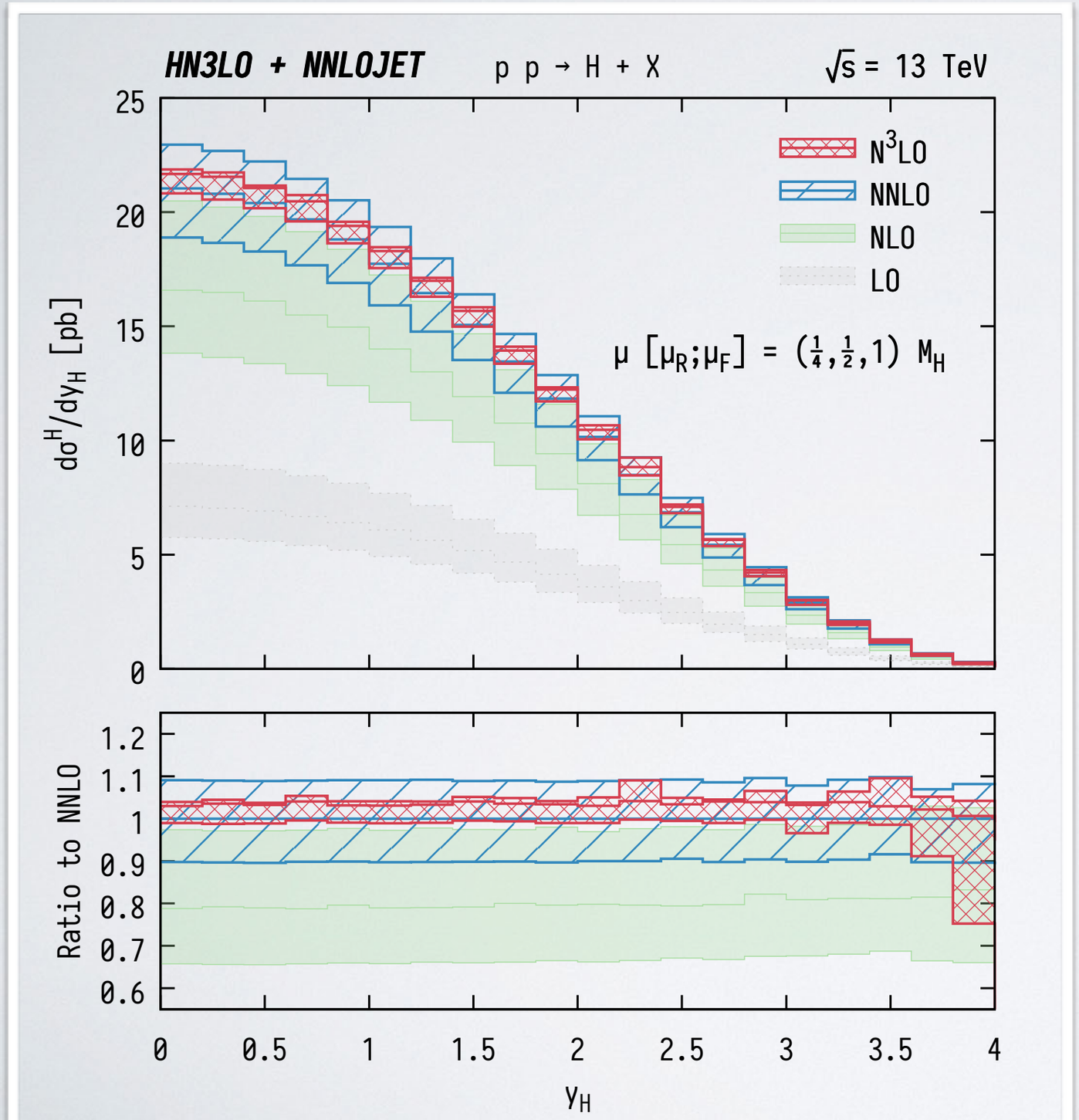
$$
\begin{aligned}
\widetilde{H}_g^{H;(3)} = C_A^3 \Bigg(& -\frac{15649\zeta_3}{432} - \frac{121\pi^2\zeta_3}{432} + \frac{3\zeta_3^2}{2} + \frac{869\zeta_5}{144} + \frac{215131}{5184} + \frac{16151\pi^2}{7776} - \frac{961\pi^4}{15552} \\
& + \frac{\pi^6}{810} + \frac{105}{32}\zeta_6 \Bigg) + C_A^2 \left(\frac{605\zeta_3}{72} + \frac{55\pi^2\zeta_3}{36} + \frac{737\pi^2}{432} + \frac{167\pi^4}{432} + \frac{\pi^6}{72} \right) \\
& + C_A \left(\frac{19\pi^2 L_t}{48} - \frac{55\pi^2\zeta_3}{8} - \frac{\pi^6}{480} + \frac{133\pi^4}{72} + \frac{11399\pi^2}{864} + \frac{63}{32}\zeta_6 \right) \\
& + N_f^2 \left(\frac{43 C_A \zeta_3}{108} - \frac{19\pi^4 C_A}{3240} - \frac{133\pi^2 C_A}{1944} + \frac{2515 C_A}{1728} - \frac{7 C_F \zeta_3}{6} \right. \\
& \left. + \frac{4481 C_F}{2592} - \frac{\pi^4 C_F}{3240} - \frac{23\pi^2 C_F}{432} \right) \\
& + N_f \left(\frac{101 C_A^2 \zeta_5}{72} - \frac{97}{216}\pi^2 C_A^2 \zeta_3 + \frac{29 C_A^2 \zeta_3}{8} + \frac{1849\pi^4 C_A^2}{38880} - \frac{35\pi^2 C_A^2}{243} - \frac{98059 C_A^2}{5184} \right. \\
& + \frac{5 C_A C_F \zeta_5}{2} + \frac{13 C_A C_F \zeta_3}{2} + \frac{1}{2}\pi^2 C_A C_F \zeta_3 - \frac{63991 C_A C_F}{5184} + \frac{11\pi^4 C_A C_F}{6480} \\
& - \frac{71}{216}\pi^2 C_A C_F + \frac{1}{9}\pi^2 C_A L_t - \frac{5}{36}\pi^2 C_A \zeta_3 - \frac{55 C_A \zeta_3}{36} - \frac{5\pi^4 C_A}{54} - \frac{1409\pi^2 C_A}{864} \\
& \left. - 5 C_F^2 \zeta_5 + \frac{37 C_F^2 \zeta_3}{12} + \frac{19 C_F^2}{18} \right) ,
\end{aligned}
$$

L. C, X. Chen, T. Gehrmann, E. W. N. Glover, and A. Huss (2018)

N^3LO DIFFERENTIAL DISTRIBUTIONS

L. C, X. Chen, T. Gehrmann, E. W. N. Glover, and A. Huss (2018)

Rapidity distribution



- In the central rapidity region of $|y_H| \leq 3.6$, the impact of the N^3LO corrections on the NNLO result is almost independent of y_H with a flat K-factor about 1.034 for the central scale choice
- The combined theoretical uncertainty at N^3LO is at most of ±5% level with respect to the central scale choice.
- The uncertainty on the y_H distribution is reduced by more than a factor of 1/2 by going from NNLO to N^3LO. The N^3LO uncertainty band lies fully within the scale variation band at NNLO, exhibiting a stable perturbative behaviour

THE N^{3}LO COUNTER-TERM (CT)

THE N^{3}LO COUNTER-TERM (CT)

$$\mathrm{d}\sigma^{F}_{\mathrm{N^nLO}} = \mathcal{H}^{F}_{\mathrm{N^nLO}} \otimes \mathrm{d}\sigma^{F}_{\mathrm{LO}} + \left[\mathrm{d}\sigma^{F+\mathrm{jets}}_{\mathrm{N^{n-1}LO}} - \mathrm{d}\sigma^{F\ \mathrm{CT}}_{\mathrm{N^nLO}}\right] \quad \text{with} \quad n = 1, 2, 3$$

$$\frac{\mathrm{d}\hat{\sigma}^{(\mathrm{fin.})}_{F\,ab}}{\mathrm{d}q_T^2} \equiv \left[\frac{\mathrm{d}\hat{\sigma}^{F+\mathrm{jets}}_{ab}}{\mathrm{d}q_T^2} - \frac{\mathrm{d}\hat{\sigma}^{F\ \mathrm{CT}}_{ab}}{\mathrm{d}q_T^2}\right]$$

NNLOJET

X. Chen, J. Cruz-Martinez, T. Gehrmann, E. W. N. Glover and M. Jaquier (2016)

HN3LO

LC (2018)

Aude Gehrmann-De Ridder Talk

Since we know analytically the third-order CT we present in the next slides: The real contribution and the CT for Higgs boson production in gluon fusion at Sqrt[s] = 13TeV at the LHC, in the large m_{top} limit

THE N^{3}LO QT-SUBTRACTION METHOD

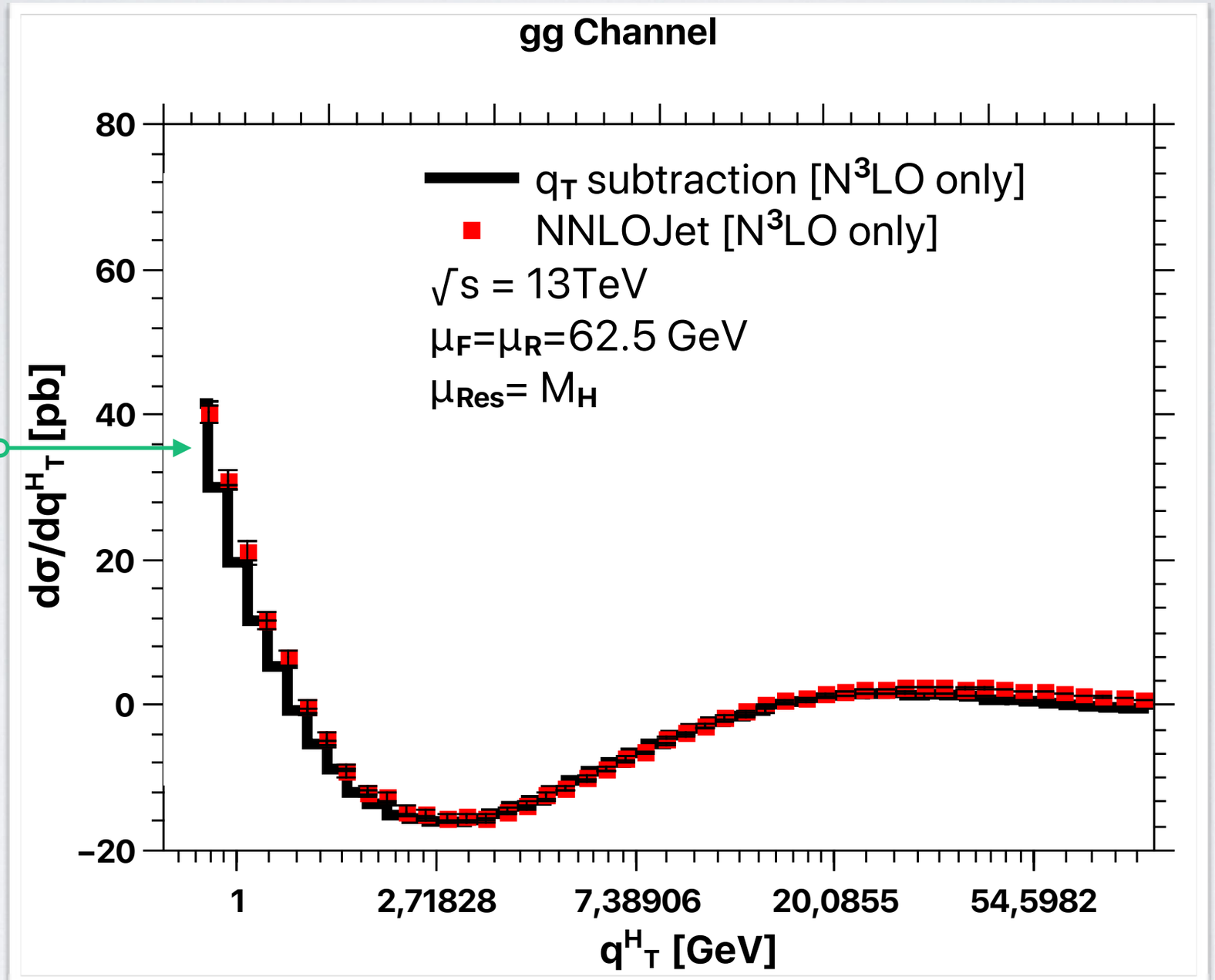
L. C, X. Chen, T. Gehrmann, E. W. N. Glover, and A. Huss (2018)

First results divergent behaviour at the third-order

The presence of large logarithms in the small-qT limit requires qT-resummation for this observable

The sign in the exponent of the Sudakov predicts the sign of the divergent behaviour

NLO > 0
NNLO < 0
N3LO > 0



Higgs boson production in gluon fusion at Sqrt[s] = 13TeV at the LHC, in the large m_{top} limit

The N3LO CT is computed without approximation

The CT is shown here at qT > 0

$$S_c(M,b) = \exp\left\{ - \int_{b_0^2/b^2}^{M^2} \frac{\mathrm{d}q^2}{q^2} \left[A_c(\alpha_s(q^2)) \ln \frac{M^2}{q^2} + B_c(\alpha_s(q^2)) \right] \right\}$$

$$\Sigma^F(q_T/Q) \xrightarrow[q_T \to 0]{} \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n \sum_{k=1}^{2n} \Sigma^{F(n;k)} \frac{Q^2}{q_T^2} \ln^{k-1} \frac{Q^2}{q_T^2}$$

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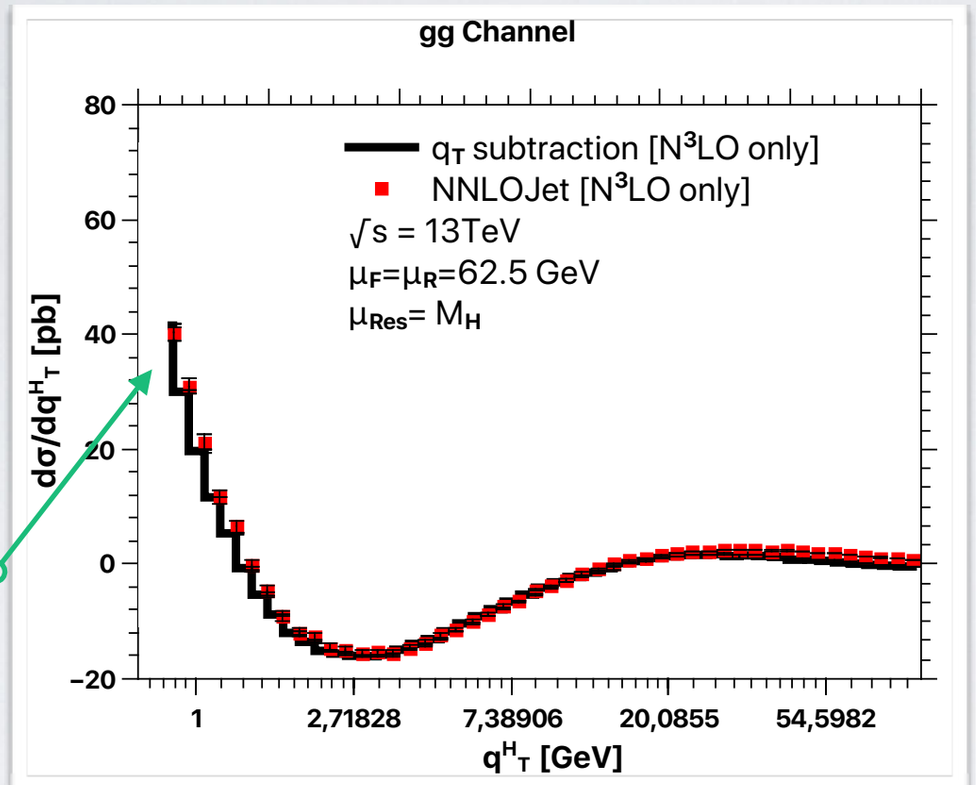
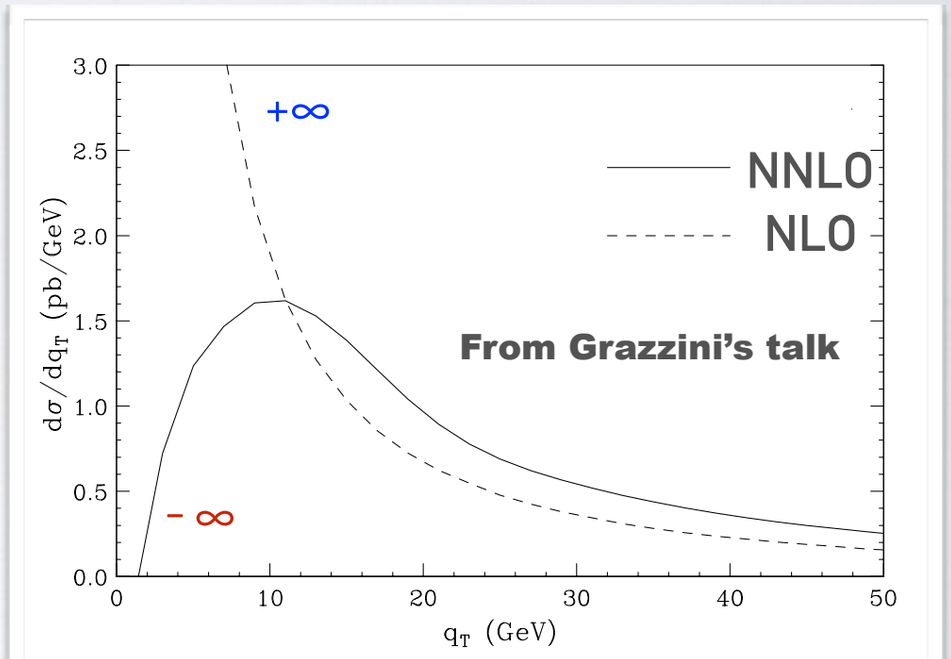




NLO > 0
NNLO < 0
N3LO > 0

$$\text{NLO}: \frac{d\sigma}{dq_T} \to +\infty \quad \text{as} \quad q_T \to 0$$

$$\text{NNLO}: \frac{d\sigma}{dq_T} \to -\infty \quad \text{as} \quad q_T \to 0$$

$$\text{N}^3\text{LO}: \frac{d\sigma}{dq_T} \to +\infty \quad \text{as} \quad q_T \to 0$$

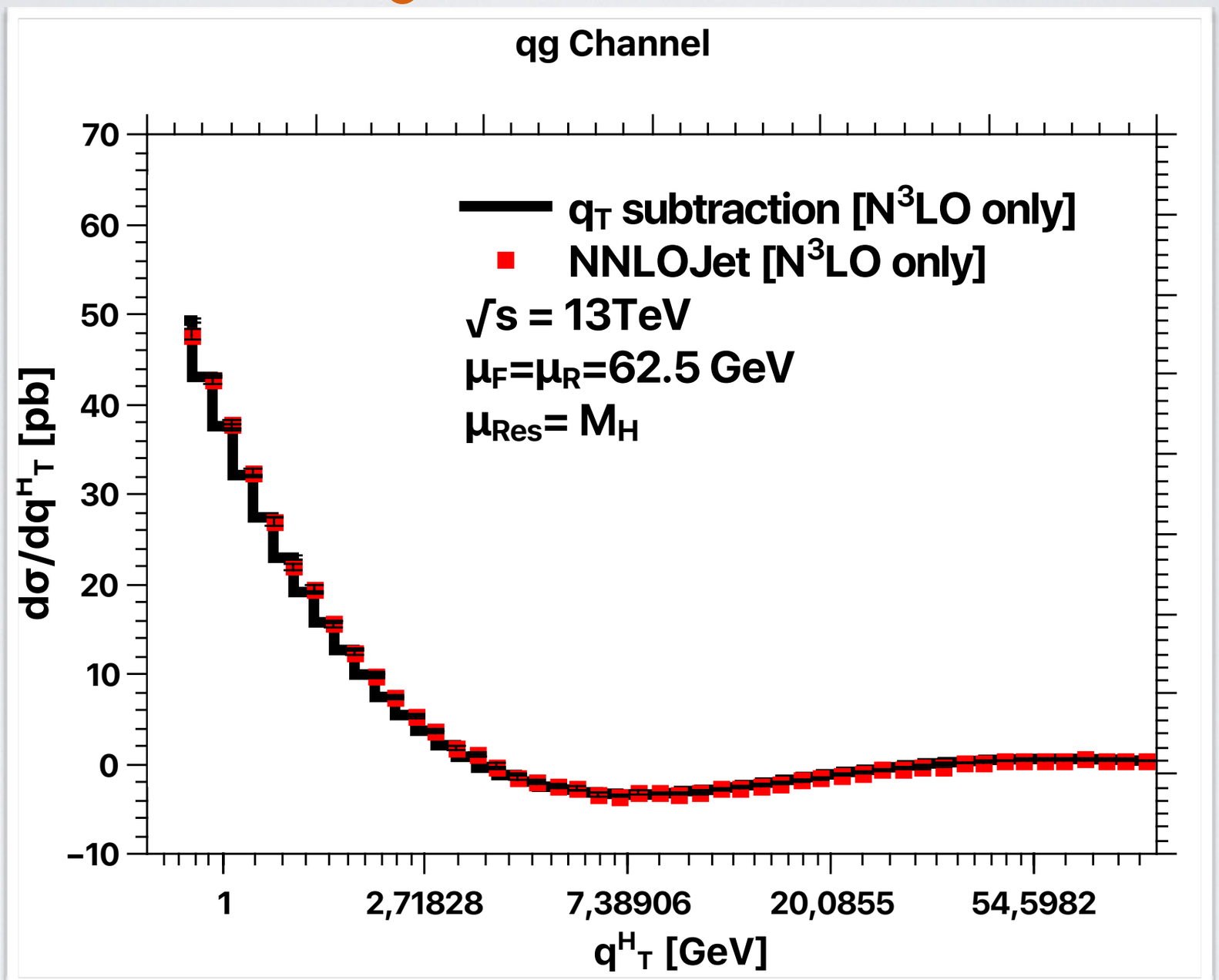
$$S_c(M,b) = \exp\left\{ -\int_{b_0^2/b^2}^{M^2} \frac{\mathrm{d}q^2}{q^2} \left[A_c(\alpha_s(q^2)) \ln\frac{M^2}{q^2} + B_c(\alpha_s(q^2)) \right] \right\}$$

THE N^{3}LO QT-SUBTRACTION METHOD

L. C, X. Chen, T. Gehrmann, E. W. N. Glover, and A. Huss (2018)

First results divergent behaviour at the third-order

In the off-diagonal channels, the first power (most divergent) in the logarithmic expansion is not present.



Higgs boson production in gluon fusion at Sqrt[s] = 13TeV at the LHC, in the large m_{top} limit

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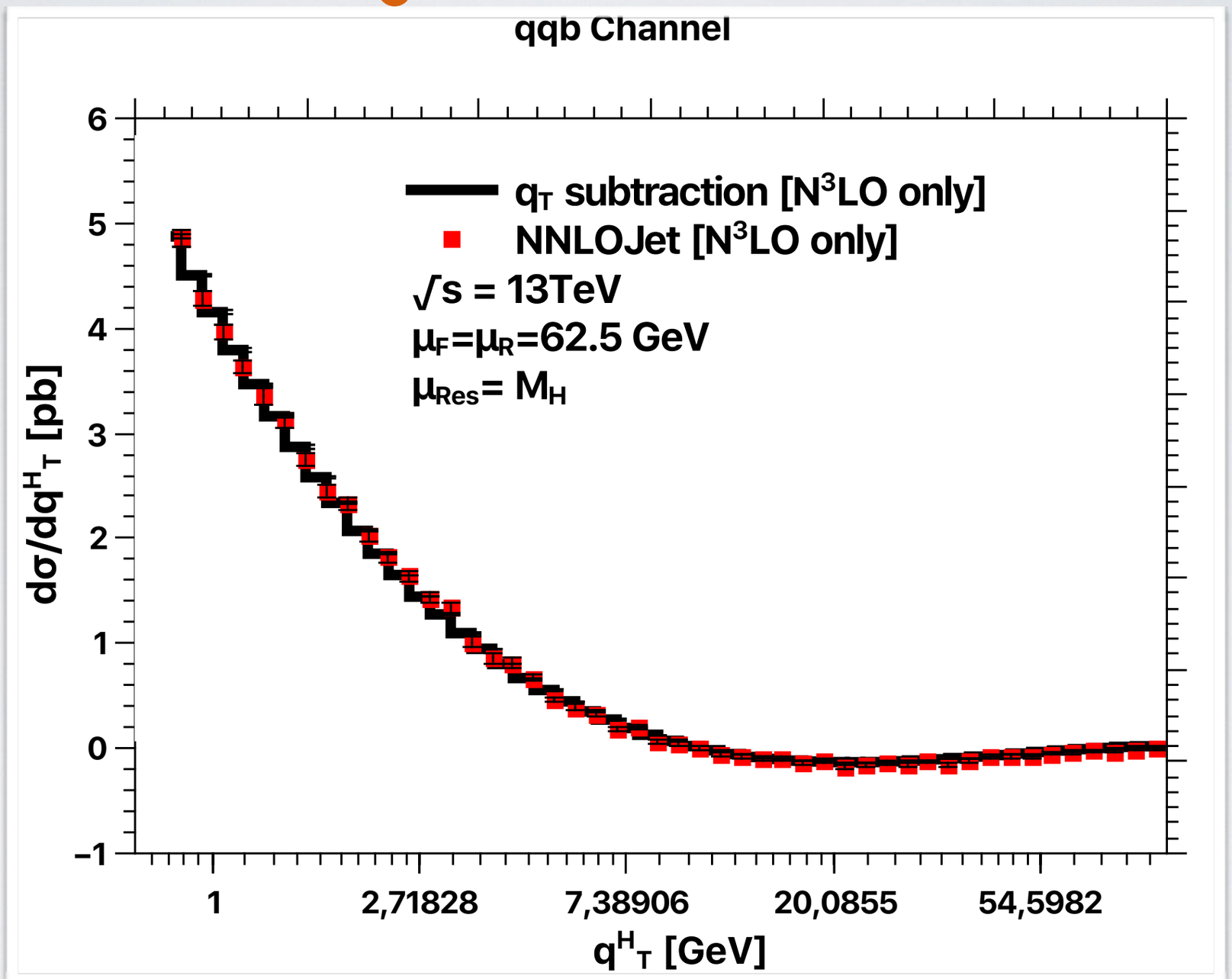

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