WHAT IS A PARTON SHOWER?

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Perturbative Cross Section

The main focus of this workshop is to calculate the pQCD cross sections as precise as possible, thus we have a pretty integral

$$\begin{aligned}
\sigma[O_J] = {} & \sum_m \frac{1}{m!} \sum_{\{a,b,f_1,\dots,f_m\}} \int_0^1 d\eta_\mathrm{a} \overbrace{\int_{\eta_\mathrm{a}}^1 \frac{dz}{z}\, \mathit{\Gamma}^{-1}_{aa'}(z,\mu^2)\, f_{a'/A}(\eta_\mathrm{a}/z,\mu^2)}^{\textit{Bare PDF}} \\
& \times \int_0^1 d\eta_\mathrm{b} \int_{\eta_\mathrm{b}}^1 \frac{d\bar{z}}{\bar{z}}\, \mathit{\Gamma}^{-1}_{bb'}(\bar{z},\mu^2)\, f_{b'/A}(\eta_\mathrm{b}/\bar{z},\mu^2) \\
& \times \int d\phi(\eta_\mathrm{a}\eta_\mathrm{b} s, \{p,f\}_m)\, \langle M(\{p,f\}_m) | \underbrace{O_J(\{p,f\}_m)}_{\textit{IR safe measurement operator}} \overbrace{| M(\{p,f\}_m) \rangle}^{\textit{Partonic matrix element}} \\
& + \mathcal{O}\left(\frac{\Lambda^2_{QCD}}{\mu_J^2} \right)
\end{aligned}$$

Error of the factorization
(Cannot be beaten by calculating higher and higher order.)

and here the MSbar parton in parton renormalised PDF is

$$\mathit{\Gamma}_{aa'}(z,\mu^2) = \delta(1-z)\delta_{aa'} - \frac{\alpha_\mathrm{s}(\mu^2)}{2\pi} \frac{1}{\epsilon} \frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)} P_{aa'}(z) + \cdots \quad .$$

Motivation

For a generic IR safe observable we can do either fixed order or parton shower calculations

Fixed order calculations

✓ Systematically improvable by working to higher order.

- The procedure is well defined and can be carried out order by order. The definition of cross section tells us what to do.
- The subtraction procedure regularises the α_{S} series and turn the $d=4-2\varepsilon$ dimensional expression to a $d=4$ dimensional one.
- Counter-terms are defined order by order
- The result is independent of the ambiguities of the counter-terms order by order.

✗ Only few partons represent a jet.

✗ Suffers from large logarithms

Parton shower algorithms

✗ But what about parton showers?

- Are they just QCD inspired or fit into a scheme that can be systematically improved?
- Is the (*all order*) shower cross section equal to the pQCD (*all order*) cross section?
- Is there a shower way to regularise α_{S} series?

✓ A jet consists of many partons

✓ Sums up logarithm (only for some observable).

What is the relation between fixed order and parton shower?

Motivation

Fixed order NLO PDF is a well defined and systematically improvable approximation of the usual LO PDF:

$$f(\eta,\mu^2) = f_{1\mathrm{GeV}}(\eta) + \int_{1\mathrm{GeV}^2}^{\mu^2} \frac{d\tilde{\mu}^2}{\tilde{\mu}^2}\frac{\alpha_s(\tilde{\mu}^2)}{2\pi}\int_\eta^1 \frac{dx}{x} P^{(1)}(x) f_{1\mathrm{GeV}}(\eta/x) + \cdots$$

But we never use this and we prefer the **fully exponentiated** solution of the DGLAP equation

$$f(\eta,\mu^2) = \frac{1}{2\pi \mathrm{i}}\int_{c-\mathrm{i}\infty}^{c+\mathrm{i}\infty} dN\,\eta^{-N}\exp\left(\int_{1\mathrm{GeV}^2}^{\mu^2}\frac{d\tilde{\mu}^2}{\tilde{\mu}^2}\frac{\alpha_s(\tilde{\mu}^2)}{2\pi}\int_0^1 dx\,x^{N-1}P^{(1)}(x)\right)$$
$$\times\int_0^1 dx\,x^{N-1} f_{1\mathrm{GeV}}(x)$$

A strong statement: The fixed order NLO, NNLO and N^{k}LO calculations are just approximations to the fully exponentiated LO, NLO and N^{k-1}LO calculations.

A brave claim: The parton shower provides the fully exponentiated LO, NLO and N^{k-1}LO calculations.

Statistical Space

Introducing the statistical space we can represent the QCD density operator as a vector

$$\sigma[O_J] = \underbrace{(1|}_{\substack{\textit{All the initial and final}\\ \textit{state sums and integrals}}}\ \mathcal{O}_J\ \overbrace{\left[\mathcal{F}(\mu^2)\circ\mathcal{Z}_F(\mu^2)\right]}^{\textit{Bare PDFs for both incoming hadrons}}\ \underbrace{|\rho(\mu^2))}_{|M\rangle\langle M|}$$

QCD density operator

Describes the fully exclusive partonic final states.

The physical cross section is RG invariant as well as the QCD density operator and the bare PDF.

$$\mu^2\frac{d}{d\mu^2}\,|\rho(\mu^2)) = \mu^2\frac{d}{d\mu^2}\left[\mathcal{F}(\mu^2)\circ\mathcal{Z}_F(\mu^2)\right] = 0 + \mathcal{O}(\alpha_{\mathrm{s}}^{k+1})$$

Perturbative expansion of the density operator

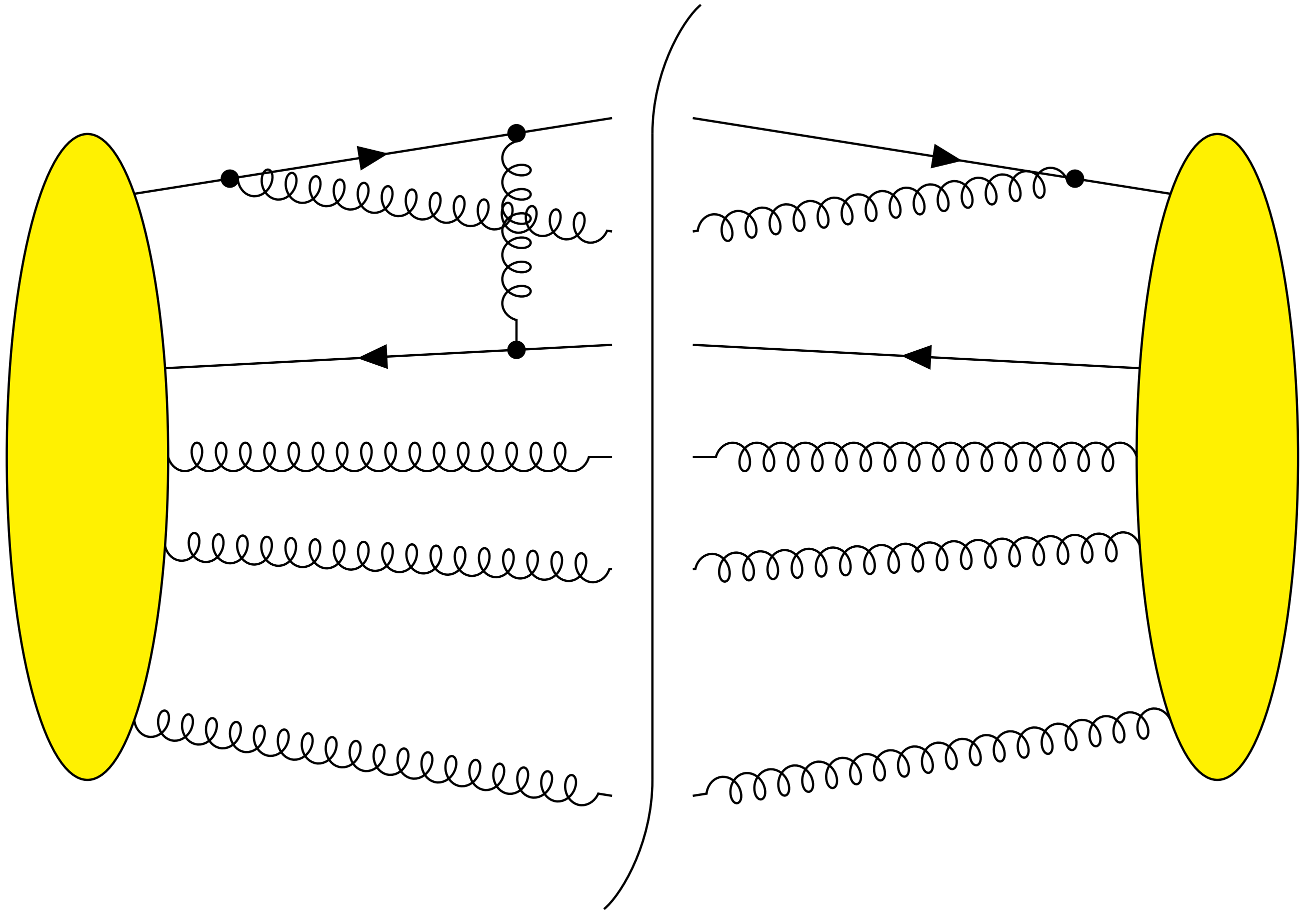
$$|\rho(\mu^2)) = \sum_{n=0}^{k}\left[\frac{\alpha_{\mathrm{s}}(\mu^2)}{2\pi}\right]^n \underset{n_{\mathrm{R}}+n_{\mathrm{V}}=n}{\sum_{n_{\mathrm{R}}=0}^{n}\sum_{n_{\mathrm{V}}=0}^{n}} |\rho^{(n_{\mathrm{R}},n_{\mathrm{V}})}(\mu^2))$$

Number of real radiations

Number of loops

Infrared Sensitive Operator

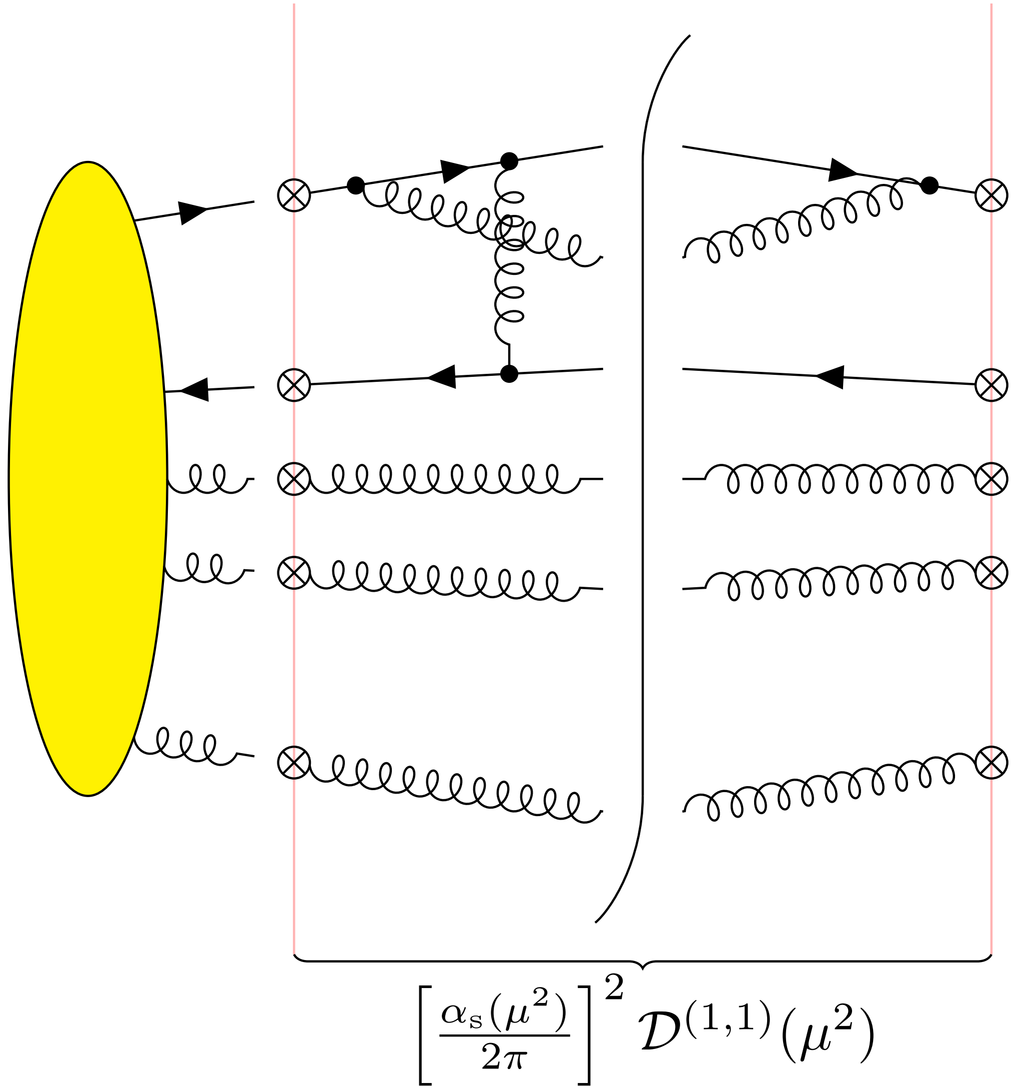
Amplitudes have soft or collinear singularities and they have divergences 1/ε from the loops



- ➠ We want to describe the singularity structure in **process independent way**.
- ➠ Everything in the yellow blobs is considered hard.

Infrared Sensitive Operator

Consider the momenta coming from the hard part as fixed and on shell.


$$\left[\frac{\alpha_{\rm s}(\mu^2)}{2\pi}\right]^2 \mathcal{D}^{(1,1)}(\mu^2)$$

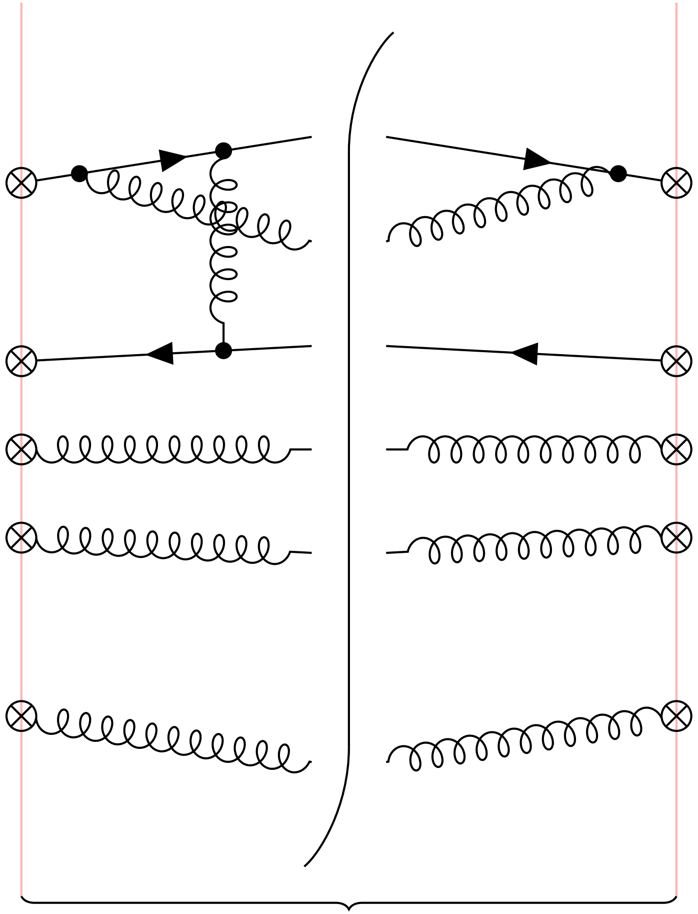
This gives us an operator as

$$
\begin{aligned}
&\big(\{\hat{p},\hat{f},\hat{s},\hat{s}',\hat{c},\hat{c}'\}_{m+n_{\rm R}}\big|\rho(\mu^2)\big) \\
&\quad\sim \frac{1}{m!}\int[d\{p\}_m]\sum_{\{f\}_m}\ \sum_{\{s,s',c,c'\}_m} \\
&\qquad\times \big(\{\hat{p},\hat{f},\hat{s},\hat{s}',\hat{c},\hat{c}'\}_{m+n_{\rm R}}\big|\mathcal{D}(\mu^2)\big|\{p,f,s,s',c,c'\}_m\big) \\
&\qquad\times \big(\{p,f,s,s',c,c'\}_m\big|\rho_{\rm hard}(\mu^2)\big)
\end{aligned}
$$

Infrared Sensitive Operator

We can consider a more constructive approach to build the full infrared sensitive operator. This operator basically represents the QCD density operator of a $m \to X$ *(anything)* process.

$$\mathcal{D}(\mu^2) = 1 + \sum_{n=1}^{k} \left[\frac{\alpha_{\mathrm{s}}(\mu^2)}{2\pi}\right]^n \mathop{\sum_{n_{\mathrm{R}}=0}^{n}\sum_{n_{\mathrm{V}}=0}^{n}}_{n_{\mathrm{R}}+n_{\mathrm{V}}=n} \mathcal{D}^{(n_{\mathrm{R}},n_{\mathrm{V}})}(\mu^2)$$



$$\left[\frac{\alpha_{\mathrm{s}}(\mu^2)}{2\pi}\right]^2 \mathcal{D}^{(1,1)}(\mu^2)$$

The structure is rather straightforward:

$$\begin{aligned}
&\big(\{\hat p,\hat f,\hat s',\hat c',\hat s,\hat c\}_{m+n_{\mathrm{R}}}\big|\mathcal{D}^{(n_{\mathrm{R}},n_{\mathrm{V}})}(\mu^2,\mu_{\mathrm{S}}^2)\big|\{p,f,s',c',s,c\}_m\big) \\
&= \sum_{G\in\mathrm{Graphs}} \int d^d\{\ell\}_{n_{\mathrm{V}}}\; {}_D\big\langle\{\hat s,\hat c\}_{m+n_{\mathrm{R}}}\big|\boldsymbol{V}_L(G;\{\hat p,\hat f\}_{m+n_{\mathrm{R}}},\{\ell\}_{n_{\mathrm{V}}},\mu^2)\big|\{s,c\}_m\big\rangle \\
&\qquad \times \big\langle\{s,c\}_m\big|\boldsymbol{V}_R^\dagger(G;\{\hat p,\hat f\}_{m+n_{\mathrm{R}}},\{\ell\}_{n_{\mathrm{V}}},\mu^2)\big|\{\hat s,\hat c\}_{m+n_{\mathrm{R}}}\big\rangle_D \\
&\quad \times \sum_{I\in\mathrm{Regions(G)}} \big(\{\hat p,\hat f\}_{m+n_{\mathrm{R}}}\big|\mathcal{P}_G(I)\big|\{p,f\}_m\big)\, \underbrace{\Theta_G(I;\{\hat p,\hat f\}_{m+n_{\mathrm{R}}},\{\ell\}_{n_{\mathrm{V}}};\mu_{\mathrm{S}}^2)}_{\textbf{Constrains the off-shellness of the hard partons}}
\end{aligned}$$

Infrared Sensitive Operator

- We have to introduce an **ultraviolet cutoff to capture only the IR part** of the amplitudes. At first order level in the real graphs it is just a cut on an infrared sensitive variable of the splitting:

$$\Theta_G(I;\{\hat{p},\hat{f}\}_{m+n_{\mathrm{R}}},\{\ell\}_{n_{\mathrm{V}}};\mu_{\mathrm{S}}^2)\sim\theta(k_\perp^2<\mu_{\mathrm{S}}^2)$$

- The D operator depends on two scales (renormalization scale μ and the **shower scale** μ_{S}) but we always set them equal.

$$\mu_{\mathrm{S}}^2=\mu^2$$

- We don't do eikonal approximation in the soft gluon exchange between two external lines because that messes up the **Glauber region**.

- We also need a **momentum mapping**. This can be tricky at higher order level and not necessary the simpler the better. We prefer "global" momentum mapping.

N^{k}LO calculations

Subtractions

$$\sigma[O_J] = \overbrace{\big(1\big|\,\big[\mathcal{F}(\mu^2)\circ\mathcal{Z}_F(\mu^2)\big]\,\mathcal{D}(\mu^2)}^{\textbf{Singularities cancel each other here}}\;\underbrace{\mathcal{D}^{-1}(\mu^2)\,\mathcal{O}_J\,\big|\rho(\mu^2)\big)}_{=|\rho_{\mathrm{H}}(\mu^2))}$$

Hard part, finite in d=4 dimension

$$+\,\mathcal{O}(\alpha_{\mathrm{s}}^{k+1}L^{2k+2})$$

$$+\,\mathcal{O}(\Lambda_{QCD}^2/\mu_J^2)$$

Normally $\mathcal{D}^{-1}(\mu^2)$ is constructed by hand and $\mathcal{D}(\mu^2)$ is its inverse.

N^{k}LO calculations

Collecting all the singularities in an operator,

$$\mathcal{X}(\mu^2) = \left[\mathcal{F}(\mu^2) \circ \mathcal{Z}_F(\mu^2)\right] \mathcal{D}(\mu^2)\, \mathcal{F}^{-1}(\mu^2)$$

Then we have found that $\big(1\big|\mathcal{X}(\mu^2)\big|\{p, f, c', c, s', s\}_m\big) = \text{finite}$. Now **define a finite operator** that **leaves the momenta and flavours unchanged** in such a way that

$$\big(1\big|\underbrace{\mathcal{V}(\mu^2)}_{\textbf{IR finite operator}} = \big(1\big|\mathcal{X}(\mu^2)$$

With this the we have the usual fixed order cross section structure:

$$\begin{aligned}\sigma[O_J] &= \big(1\big|\mathcal{X}(\mu^2)\, \mathcal{F}(\mu^2)\, \mathcal{D}^{-1}(\mu^2)\, \mathcal{O}_J\, \big|\rho(\mu^2)\big) \\ &\qquad + \mathcal{O}(\alpha_{\mathrm{s}}^{k+1} L^{2k+2}) + \mathcal{O}(\Lambda_{QCD}^2/\mu_J^2)\end{aligned}$$

$$= \boxed{\begin{aligned}&\big(1\big|\mathcal{V}(\mu^2)\, \mathcal{F}(\mu^2)\, \mathcal{D}^{-1}(\mu^2)\, \mathcal{O}_J\, \big|\rho(\mu^2)\big) \\ &+ \mathcal{O}(\alpha_{\mathrm{s}}^{k+1} L^{2k+2}) + \mathcal{O}(\Lambda_{QCD}^2/\mu_J^2)\end{aligned}}$$

Shower Cross Section

At this point we have everything to **derive** the shower cross section. Let us do it!

$$\sigma[O_J] = \big(1\big|\mathcal{O}_J\,\mathcal{X}(\mu^2)\,\mathcal{F}(\mu^2)\,\underbrace{\mathcal{D}^{-1}(\mu^2)\,\big|\rho(\mu^2)\big)}_{=|\rho_{\mathrm{H}}(\mu^2))}$$

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$$\sigma[O_J] = \big(1\big|\mathcal{O}_J\,\big[\mathcal{X}(\mu^2)\,\mathcal{V}^{-1}(\mu^2)\big]\,\mathcal{V}(\mu^2)\,\mathcal{F}(\mu^2)\,\big|\rho_{\rm H}(\mu^2)\big)$$

Shower Cross Section

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$$\sigma[O_J] = \big(1\big|\mathcal{O}_J\,\big[\mathcal{X}(\mu_{\rm f}^2)\,\mathcal{V}^{-1}(\mu_{\rm f}^2)\big]\big[\mathcal{X}(\mu_{\rm f}^2)\,\mathcal{V}^{-1}(\mu_{\rm f}^2)\big]^{-1}\big[\mathcal{X}(\mu^2)\,\mathcal{V}^{-1}(\mu^2)\big] \\ \mathcal{V}(\mu^2)\,\mathcal{F}(\mu^2)\,\big|\rho_{\rm H}(\mu^2)\big)$$

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$$\sigma[O_J] = \big(1\big|\mathcal{O}_J \left[\mathcal{X}(\mu_{\mathrm{f}}^2)\,\mathcal{V}^{-1}(\mu_{\mathrm{f}}^2)\right]\left[\mathcal{X}(\mu_{\mathrm{f}}^2)\,\mathcal{V}^{-1}(\mu_{\mathrm{f}}^2)\right]^{-1}\left[\mathcal{X}(\mu^2)\,\mathcal{V}^{-1}(\mu^2)\right]$$
$$\mathcal{V}(\mu_{\mathrm{f}}^2)\,\mathcal{V}^{-1}(\mu_{\mathrm{f}}^2)\,\mathcal{V}(\mu^2)\,\mathcal{F}(\mu^2)\,\big|\rho_{\mathrm{H}}(\mu^2)\big)$$

Shower Cross Section

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$$\sigma[O_J] = \big(1\big|\mathcal{O}_J \left[\mathcal{X}(\mu_\mathrm{f}^2)\,\mathcal{V}^{-1}(\mu_\mathrm{f}^2)\right] \overbrace{\left[\mathcal{X}(\mu_\mathrm{f}^2)\,\mathcal{V}^{-1}(\mu_\mathrm{f}^2)\right]^{-1}\left[\mathcal{X}(\mu^2)\,\mathcal{V}^{-1}(\mu^2)\right]}^{\mathcal{U}(\mu_\mathrm{f}^2,\mu^2)}$$

$$\mathcal{V}(\mu_\mathrm{f}^2)\,\underbrace{\mathcal{V}^{-1}(\mu_\mathrm{f}^2)\,\mathcal{V}(\mu^2)}_{\mathcal{U}_\mathcal{V}(\mu_\mathrm{f}^2,\mu^2)}\,\mathcal{F}(\mu^2)\,\big|\rho_\mathrm{H}(\mu^2)\big)$$

Shower Cross Section

At this point we have everything to **derive** the shower cross section. Let us do it!

$$\sigma[O_J] = \left(1\middle|\mathcal{O}_J\left[\mathcal{X}(\mu_\mathrm{f}^2)\,\mathcal{V}^{-1}(\mu_\mathrm{f}^2)\right]\mathcal{U}(\mu_\mathrm{f}^2,\mu^2)\,\mathcal{V}(\mu_\mathrm{f}^2)\,\mathcal{U}_\mathcal{V}(\mu_\mathrm{f}^2,\mu^2)\,\mathcal{F}(\mu^2)\middle|\rho_\mathrm{H}(\mu^2)\right)$$

When $\mu_\mathrm{f}^2 \sim \Lambda_\mathrm{QCD}^2$

$$\left(1\middle|\mathcal{O}_J\left[\mathcal{X}(\mu_\mathrm{f}^2)\,\mathcal{V}^{-1}(\mu_\mathrm{f}^2)\right]\middle|\{p,f,...\}_m\right) = \left(1\middle|\mathcal{O}_J\middle|\{p,f,...\}_m\right) + \mathcal{O}\left(\frac{\mu_\mathrm{f}^2}{\mu_J^2}\right)$$

and

$$\mathcal{V}(\mu_\mathrm{f}^2) \approx 1$$

since this operator is finite.

$$\sigma[O_J] = \left(1\middle|\mathcal{O}_J\,\overbrace{\mathcal{U}(\mu_\mathrm{f}^2,\mu^2)}^{\textbf{Unitary shower}}\,\underbrace{\mathcal{U}_\mathcal{V}(\mu_\mathrm{f}^2,\mu^2)}_{\textbf{Resummation of threshold effects}}\,\mathcal{F}(\mu^2)\middle|\rho_\mathrm{H}(\mu^2)\right) + \mathcal{O}(\overbrace{\alpha_\mathrm{s}^{k+1}L^n}^{\textbf{Hopefully n << 2k+1}}) + \mathcal{O}(\mu_\mathrm{f}^2/\mu_J^2)$$

Threshold Logs

The threshold operator is defined by

$$\mathcal{U}_{\mathcal{V}}(\mu_{\rm f}^2,\mu_{\rm H}^2) = \mathcal{V}^{-1}(\mu_{\rm f}^2)\,\mathcal{V}(\mu_{\rm H}^2) = \mathbb{T}\exp\left(\int_{\mu_{\rm f}^2}^{\mu_{\rm H}^2}\frac{d\mu^2}{\mu^2}\,\mathcal{S}_{\mathcal{V}}(\mu^2)\right)$$

where the generators are

$$\begin{aligned}\frac{1}{\mu^2}\,\mathcal{S}_{\mathcal{V}}(\mu^2) &= \mathcal{V}^{-1}(\mu^2)\,\frac{d\mathcal{V}(\mu^2)}{d\mu^2} \\ &= \mathcal{V}^{-1}(\mu^2)\,\frac{\partial}{\partial\mu_{\rm S}^2}\mathcal{V}(\mu^2,\mu_{\rm S}^2)\bigg|_{\mu_{\rm S}^2=\mu^2} - \underbrace{\frac{d\mathcal{F}(\mu^2)}{d\mu^2}\mathcal{F}^{-1}(\mu^2)}_{\textbf{pure DGLAP evolution}}\end{aligned}$$

- Doesn't create new partons.
- Provides perturbative corrections to the hard part.
- **Sums up threshold logarithms**

Unitary Shower

The unitary shower operator is

$$\mathcal{U}(\mu_{\mathrm{f}}^2,\mu_{\mathrm{H}}^2) = \left[\mathcal{X}(\mu_{\mathrm{f}}^2)\,\mathcal{V}^{-1}(\mu_{\mathrm{f}}^2)\right]^{-1}\mathcal{X}(\mu_{\mathrm{H}}^2)\,\mathcal{V}^{-1}(\mu_{\mathrm{H}}^2) = \mathbb{T}\exp\left(\int_{\mu_{\mathrm{f}}^2}^{\mu_{\mathrm{H}}^2}\frac{d\mu^2}{\mu^2}\,S(\mu^2)\right)$$

where the generators are

$$\begin{aligned}\frac{1}{\mu^2}\mathcal{S}(\mu^2) &= \mathcal{V}(\mu^2)\mathcal{F}(\mu^2)\mathcal{D}^{-1}(\mu^2)\frac{d}{d\mu^2}\left[\mathcal{D}(\mu^2)\mathcal{F}^{-1}(\mu^2)\mathcal{V}^{-1}(\mu^2)\right]\\ &= \mathcal{V}(\mu^2)\mathcal{F}(\mu^2)\mathcal{D}^{-1}(\mu^2)\;\frac{\partial\mathcal{D}(\mu^2,\mu_{\mathrm{S}}^2)}{\partial\mu_{\mathrm{S}}^2}\left[\mathcal{V}(\mu^2)\mathcal{F}(\mu^2)\right]^{-1}\Bigg|_{\mu_{\mathrm{S}}^2=\mu^2}\\ &\quad-\;\frac{\partial\mathcal{V}(\mu^2,\mu_{\mathrm{S}}^2)}{\partial\mu_{\mathrm{S}}^2}\mathcal{V}^{-1}(\mu^2)\Bigg|_{\mu_{\mathrm{S}}^2=\mu^2}\end{aligned}$$

- Creates new partons.
- Preserves probabilities: $\big(1\big|\mathcal{U}(\mu_{\mathrm{f}}^2,\mu_{\mathrm{H}}^2) = \big(1\big|$
- **Sums up “*visible*” logarithms** (accuracy can depend on the observable)

Shower Kernel

The generators of the unitary shower can be expanded in the coupling:

$$S(\mu^2) = \frac{\alpha_s(\mu^2)}{2\pi} S^{(1)}(\mu^2) + \left[\frac{\alpha_s(\mu^2)}{2\pi}\right]^2 S^{(2)}(\mu^2) + \cdots$$

and the first order term is rather simple

$$\frac{1}{\mu_s^2} S^{(1)}(\mu^2) = \frac{\partial}{\partial \mu_s^2}\left[\mathcal{F}(\mu^2)\mathcal{D}^{(1,0)}(\mu^2,\mu_s^2)\mathcal{F}^{-1}(\mu^2) + \mathcal{D}^{(0,1)}(\mu^2,\mu_s^2)\right]_{\mu_s^2=\mu^2} - \left.\frac{\partial \mathcal{V}^{(1)}(\mu^2,\mu_s^2)}{\partial \mu_s^2}\right|_{\mu_s^2=\mu^2}$$

$$= \left[\underbrace{\mathcal{F}(\mu^2)\frac{\partial \mathcal{D}^{(1,0)}(\mu^2,\mu_s^2)}{\partial \mu_s^2}\mathcal{F}^{-1}(\mu^2)} - \underbrace{\frac{\partial \mathcal{F}(\mu^2)\circ \overline{\mathcal{D}}^{(1,0)}(\mu^2,\mu_s^2)}{\partial \mu_s^2}\mathcal{F}^{-1}(\mu^2)} + \underbrace{\mathrm{Im}\frac{\partial \mathcal{D}^{(0,1)}(\mu^2,\mu_s^2)}{\partial \mu_s^2}}\right]_{\mu_s^2=\mu^2}$$

Real operator
all the quantum numbers of the emitted parton is resolved

Integrated real operator
- all the quantum numbers of the emitted parton is integrated out
- it is not the contribution of the virtual graphs

Glauber gluon
imaginary part of the virtual graphs
$\sim \mathrm{i}\pi$

Note, the first order kernel is **independent of** the real part of the virtual graphs.

Shower Kernel

At second order level we are not that lucky. The shower kernel is much more complicated:

$$\begin{aligned}\frac{1}{\mu_S^2}S^{(2)}(\mu^2) = \mathcal{F}(\mu^2)\left(\frac{\partial\mathcal{D}^{(2)}(\mu^2,\mu_S^2)}{\partial\mu_S^2} - \mathcal{D}^{(1)}(\mu^2)\frac{\partial\mathcal{D}^{(1)}(\mu^2,\mu_S^2)}{\partial\mu_S^2}\right)_{\mu_S^2=\mu^2}\mathcal{F}^{-1}(\mu^2)\\ - \left(\frac{\partial\mathcal{V}^{(2)}(\mu^2,\mu_S^2)}{\partial\mu_S^2} - \mathcal{V}^{(1)}(\mu^2)\frac{\partial\mathcal{V}^{(1)}(\mu^2,\mu_S^2)}{\partial\mu_S^2}\right)_{\mu_S^2=\mu^2}\\ + \left[\mathcal{V}^{(1)}(\mu^2), \frac{1}{\mu^2}\mathcal{S}^{(1)}(\mu^2)\right]\end{aligned}$$

This is highly non-trivial operator and cancelation of all the singularities in the first term is rather delicate.

$$\mathcal{D}^{(2)}(\mu^2,\mu_S^2) = \overbrace{\mathcal{D}^{(2,0)}(\mu^2,\mu_S^2)}^{\textbf{Double real}} + \underbrace{\mathcal{D}^{(1,1)}(\mu^2,\mu_S^2)}_{\textbf{Real-virtual}} + \overbrace{\mathcal{D}^{(0,2)}(\mu^2,\mu_S^2)}^{\textbf{Double virtual}}$$

and

$$\mathcal{D}^{(1)}(\mu^2,\mu_S^2) = \overbrace{\mathcal{D}^{(1,0)}(\mu^2,\mu_S^2)}^{\textbf{Single real}} + \underbrace{\mathcal{D}^{(0,1)}(\mu^2,\mu_S^2)}_{\textbf{Single virtual}}$$

Summary

- **Fixed order** calculations

$$\sigma[O_J] = \big(1\big|\mathcal{V}(\mu^2)\,\mathcal{F}(\mu^2)\,\mathcal{D}^{-1}(\mu^2)\,\mathcal{O}_J\,\big|\rho(\mu^2)\big) \\ + \mathcal{O}(\alpha_{\mathrm{s}}^{k+1}L^{2k+2}) + \mathcal{O}(\Lambda_{QCD}^2/\mu_J^2)$$

can be systematically improved by working to higher order.

- **Partons shower** calculations

$$\sigma[O_J] = \big(1\big|\mathcal{O}_J\,\mathcal{U}(\mu_{\mathrm{f}}^2,\mu^2)\,\mathcal{U}_{\mathcal{V}}(\mu_{\mathrm{f}}^2,\mu^2)\,\mathcal{F}(\mu^2)\,\mathcal{D}^{-1}(\mu^2\big|\rho(\mu^2)\big) \\ + \mathcal{O}(\alpha_{\mathrm{s}}^{k+1}L^{n}) + \mathcal{O}(\mu_{\mathrm{f}}^2/\mu_J^2)$$

can be systematically improved by working to higher order.

Implementation

- **DEDUCTOR** is designed to do a better job with color, spin and resummation of large logarithms compared to other shower generators.
  - Lambda, kT and angular ordering
  - LC+ color treatment. It allows us to do color evolution at amplitude level
  - Threshold log resummation
  - Spin correlations are not yet computed
- Next version is available soon…
  - Fully exponentiated Glauber (Coulomb) gluon effects
  - Wide angle soft gluon effects perturbatively.
- It is available from

`http://www.desy.de/~znagy/deductor`
`http://pages.uoregon.edu/soper/deductor`

Threshold Effect in Jet Production