IR structure of two-loop four-gluon amplitude in $\mathcal{N} = 2$ SQCD

ongoing work with Gregor Kälin and Gustav Mogull

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ETH Zürich

HP2 2018, Freiburg, October 1, 2018

Invitation

Two-loop amplitudes $\mathcal{A} = \int \mathcal{I}$ beyond $2 \to 2$

- 5-gluon all-plus

  $\mathcal{I}$: Badger, Frellesvig, Zhang (2013)
  $\mathcal{I}$: Badger, Mogull, AO, O'Connell (2015)
  $\mathcal{A}$: Gehrmann, Henn, Lo Presti (2015)

- n-gluon all-plus

  $\mathcal{I}$: Badger, Mogull, Peraro (2016)
  $\mathcal{A}$: Dunbar, Jehu, Perkins (2016)

- 5-gluon all helicities ($\mathcal{I}$ implicit, numerical $\int$)

  $\mathcal{A}$: Badger, Bronnum-Hansen, Hartanto, Peraro (2017)
  $\mathcal{A}$: Abreu, Febres Cordero, Ita, Page, Zeng (2017)

- 5-point general massless QCD

  talks by Bronnum-Hansen and Sotnikov

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- understand provenance of IR divergences through $\mathcal{O}(\epsilon^{-2L})$

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This talk:

- inspect 4-pt integrand in $\mathcal{N} = 2$ SQCD (full color and N_f)
- analytic IR+UV structure at $\mathcal{O}(\epsilon^{-4})$, $\mathcal{O}(\epsilon^{-3})$ and $\mathcal{O}(\epsilon^{-2})$

Outline

1. IR factorization review
2. $\mathcal{N} = 2$ integrand
3. Analytic IR structure

4. Summary & outlook

IR factorization review

IR factorization

Kunszt, Signer, Trocsanyi (1994)
Catani, Seymour (1996)
Catani (1998)

$$\widetilde{\mathcal{M}}_n^{(1)} = I^{(1)}(\epsilon)\mathcal{M}_n^{(0)} + \mathcal{O}(\epsilon^0)$$
$$\widetilde{\mathcal{M}}_n^{(2)} = I^{(1)}(\epsilon)\widetilde{\mathcal{M}}_n^{(1)} + I^{(2)}(\epsilon)\mathcal{M}_n^{(0)} + \mathcal{O}(\epsilon^0)$$

$$I^{(1)}(\epsilon) = \frac{e^{\epsilon\gamma}}{2\Gamma(1-\epsilon)} \sum_{i=1}^{n} \sum_{j\neq i}^{n} \left(\frac{1}{\epsilon^2} + \frac{\gamma_i}{\mathbf{T}_i^2}\frac{1}{\epsilon} \right) (-s_{ij})^{-\epsilon}\, \mathbf{T}_i \cdot \mathbf{T}_j$$

$$I^{(2)}(\epsilon) = -\frac{1}{2} I^{(1)}(\epsilon) \left(I^{(1)}(\epsilon) + 2\frac{\beta_0}{\epsilon} \right) + \frac{e^{-\epsilon\gamma}\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)} \left(\frac{\beta_0}{\epsilon} + K_{\mathrm{R.S.}} \right) I^{(1)}(2\epsilon) + \mathcal{O}(\epsilon^{-1})$$

IR factorization

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$$I^{(1)}(\epsilon) = \frac{e^{\epsilon\gamma}}{2\Gamma(1-\epsilon)} \sum_{i=1}^{n}\sum_{j\neq i}^{n} \left(\frac{1}{\epsilon^2} + \frac{\gamma_i}{\mathbf{T}_i^2}\frac{1}{\epsilon}\right)(-s_{ij})^{-\epsilon}\, \mathbf{T}_i \cdot \mathbf{T}_j$$

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NB! $\mathcal{O}(\epsilon^{-2})$ and $\mathcal{O}(\epsilon^{-1})$ depend on dimreg scheme, e.g.

$$K_{\mathrm{FDH}} = \left(\frac{32}{9} - \frac{4}{9}\epsilon - \frac{\pi^2}{6}\right) C_A - \frac{5}{9} C_r N_r - \left(\frac{2}{9} - \frac{2}{27}\epsilon\right) C_s N_s$$

Undoing renormalization

$$\widetilde{\mathcal{M}}_n^{(1)} = \mathcal{M}_n^{(1)} - \frac{(n-2)}{2}\frac{\beta_0}{\epsilon}\mathcal{M}_n^{(0)}$$

$$\widetilde{\mathcal{M}}_n^{(2)} = \mathcal{M}_n^{(2)} - \frac{n}{2}\frac{\beta_0}{\epsilon}\mathcal{M}_n^{(1)} + \frac{n-2}{4}\left(\frac{n}{2}\frac{\beta_0^2}{\epsilon^2} - \frac{\beta_1}{\epsilon}\right)\mathcal{M}_n^{(0)}$$

Undoing renormalization

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Rearrange Catani into

$$\mathcal{M}_n^{(1)} = I_\alpha^{(1)}(\epsilon)\mathcal{M}_n^{(0)} + \frac{1}{2\epsilon}\left((n-2)\beta_0 - \sum_{i=1}^{n}\gamma_i\right)\mathcal{M}_n^{(0)} + \mathcal{O}(\epsilon^0)$$

$$\begin{aligned}\mathcal{M}_n^{(2)} &= I_\alpha^{(1)}(\epsilon)\mathcal{M}_n^{(1)} - \frac{1}{2}I_\alpha^{(1)}(\epsilon)I_\alpha^{(1)}(\epsilon)\mathcal{M}_n^{(0)} \\ &+ \frac{e^{-\epsilon\gamma}\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)}\left(\frac{\beta_0}{\epsilon} + K_{\text{R.S.}}\right)I_\alpha^{(1)}(2\epsilon)\mathcal{M}_n^{(0)} \\ &+ \frac{1}{8\epsilon^2}\left(n\beta_0 - \sum_{i=1}^{n}\gamma_i\right)\left((n-2)\beta_0 - \sum_{i=1}^{n}\gamma_i\right)\mathcal{M}_n^{(0)} + \mathcal{O}(\epsilon^{-1})\end{aligned}$$

$$I_\alpha^{(1)}(\epsilon) = \frac{e^{\epsilon\gamma}}{2\epsilon^2\Gamma(1-\epsilon)}\sum_{i=1}^{n}\sum_{j\neq i}^{n}(-s_{ij})^{-\epsilon}\,\mathbf{T}_i\cdot\mathbf{T}_j$$

NB! Mixed IR with UV

Assume IR factorization for $\mathcal{N} = 4$

Consider difference w.r.t. $\mathcal{N} = 4$:

$$\mathcal{M}^{(1)} - \mathcal{M}^{(1)}_{\mathcal{N}=4} = -\frac{\beta_0}{\epsilon}\mathcal{M}^{(0)} + \mathcal{O}(\epsilon^0)$$

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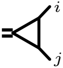
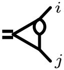
Consider difference w.r.t. $\mathcal{N} = 4$:

$$\mathcal{M}^{(1)} - \mathcal{M}^{(1)}_{\mathcal{N}=4} = -\frac{\beta_0}{\epsilon}\mathcal{M}^{(0)} + \mathcal{O}(\epsilon^0)$$

$$\mathcal{M}^{(2)} - \mathcal{M}^{(2)}_{\mathcal{N}=4} = \left(\sum_{i<j} s_{ij}$$  $$\mathbf{T}_i \cdot \mathbf{T}_j \right) \left(\mathcal{M}^{(1)} - \mathcal{M}^{(1)}_{\mathcal{N}=4}\right)$$

$$+ \beta_0 \left(\sum_{i<j} s_{ij}$$  $$\mathbf{T}_i \cdot \mathbf{T}_j \right) \mathcal{M}^{(0)}$$

$$+ \frac{n}{4}\frac{N_c}{\epsilon^2}\left(\beta_0 - \frac{1}{2}\left(K - K_{\mathcal{N}=4}\right)\right)\mathcal{M}^{(0)} + \mathcal{O}(\epsilon^{-1})$$

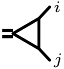
IR factorization for $\mathcal{N}=2$ SQCD

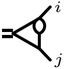
Specialize to $\mathcal{N}=2$, external gluons:

$$\beta_0 = N_c - \frac{N_f}{2}, \qquad \gamma_i = \beta_0, \qquad K_{\mathcal{N}=2} = K_{\mathcal{N}=4} + 2\beta_0 + \mathcal{O}(\epsilon)$$

Singularity structure encoded by triangles through $\mathcal{O}(\epsilon^{-2})$:

$$\mathcal{M}^{(1)} - \mathcal{M}^{(1)}_{\mathcal{N}=4} = -\frac{\beta_0}{\epsilon}\mathcal{M}^{(0)} + \mathcal{O}(\epsilon^0)$$

$\mathcal{M}^{(2)} - \mathcal{M}^{(2)}_{\mathcal{N}=4} = \left(\sum_{i<j} s_{ij}$  $\mathbf{T}_i \cdot \mathbf{T}_j \right) \left(\mathcal{M}^{(1)} - \mathcal{M}^{(1)}_{\mathcal{N}=4}\right)$

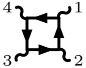
$+ \beta_0 \left(\sum_{i<j} s_{ij}$  $\mathbf{T}_i \cdot \mathbf{T}_j \right) \mathcal{M}^{(0)} + \mathcal{O}(\epsilon^{-1})$

$\mathcal{N} = 2$ integrand

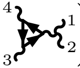
One-loop $\mathcal{N}=2$ integrand

Johansson, AO (2014)

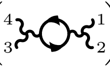
Start with BCJ numerators $n_i^{\mathcal{N}=2,\text{pure}} = n_i^{\mathcal{N}=4} - 2n_i^{\mathcal{N}=2,\text{fund}}$,
where $n_i^{\mathcal{N}=2,\text{fund}}$ are

$n\Big($  $\Big)$

$$= \frac{\kappa_{13}}{u^2}\,\text{tr}_- + \frac{\kappa_{24}}{u^2}\,\text{tr}_+ + \mu^2\left(\frac{\kappa_{12}+\kappa_{34}}{s} + \frac{\kappa_{23}+\kappa_{14}}{t} + \frac{\kappa_{13}+\kappa_{24}}{u}\right)$$

$n\Big($  $\Big)$

$$= \left(\frac{\kappa_{13}}{u^2} + \frac{\kappa_{23}}{t^2}\right)\text{tr}_- + \left(\frac{\kappa_{24}}{u^2} + \frac{\kappa_{14}}{t^2}\right)\text{tr}_+ + \frac{s}{t^2}\,(\kappa_{23}+\kappa_{14})\,(\ell+p_4)^2$$

$n\Big($  $\Big)$

$$= \left(\frac{\kappa_{23}+\kappa_{14}}{t} - \frac{\kappa_{13}+\kappa_{24}}{u}\right)\big(s - \ell^2 - (\ell-p_{12})^2\big)$$

$$\text{tr}_\pm = \text{tr}_\pm(1(\ell-p_1)(\ell-p_{12})3)$$

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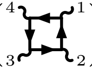
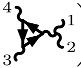
$n\Big($ 

$\Big)$

$$= \frac{\kappa_{13}}{u^2}\,\text{tr}_- + \frac{\kappa_{24}}{u^2}\,\text{tr}_+ + \mu^2\left(\frac{\kappa_{12}+\kappa_{34}}{s} + \frac{\kappa_{23}+\kappa_{14}}{t} + \frac{\kappa_{13}+\kappa_{24}}{u}\right)$$

$n\Big($ 

$\Big)$

$$= \left(\frac{\kappa_{13}}{u^2} + \frac{\kappa_{23}}{t^2}\right)\text{tr}_- + \left(\frac{\kappa_{24}}{u^2} + \frac{\kappa_{14}}{t^2}\right)\text{tr}_+ + \frac{s}{t^2}\left(\kappa_{23}+\kappa_{14}\right)(\ell+p_4)^2$$

$n\Big($ 

$\Big)$

$$= \left(\frac{\kappa_{23}+\kappa_{14}}{t} - \frac{\kappa_{13}+\kappa_{24}}{u}\right)\left(s - \ell^2 - (\ell - p_{12})^2\right)$$

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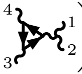
Integrand-reduce

$n\Big($ 

$\Big)$

$$= \left(\frac{\kappa_{13}}{u^2} + \frac{\kappa_{23}}{t^2}\right)\text{tr}_+(13\ell 4) + \left(\frac{\kappa_{24}}{u^2} + \frac{\kappa_{14}}{t^2}\right)\text{tr}_-(13\ell 4)$$

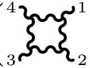
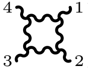
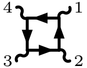
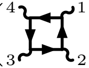
$n\Big($ 

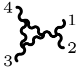
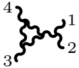
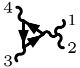
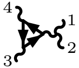
$\Big)$

$$= s\left(\frac{\kappa_{13}+\kappa_{24}}{u} - \frac{\kappa_{23}+\kappa_{14}}{t}\right)$$

Pro: triangle integrates to zero; Con: color-kinematics broken

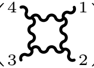
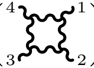
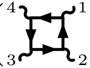
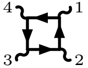
One-loop $\mathcal{N} = 2$ integrand

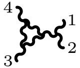
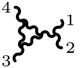
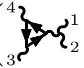
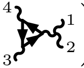
$\mathcal{M}_4^{(1)} = \frac{i}{2} \sum_{\text{perms}} I \Big[\frac{1}{8} c\Big($  $\Big) n \Big($  $\Big) + \frac{N_f}{4} c \Big($  $\Big) n \Big($  $\Big)$

$+ \frac{1}{4} c \Big($  $\Big) n \Big($  $\Big) + \frac{N_f}{2} c \Big($  $\Big) n \Big($  $\Big)$

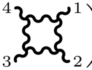
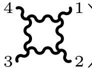
$+ \frac{1}{16} c \Big($  $\Big) n \Big($  $\Big) + \frac{N_f}{8} c \Big($  $\Big) n \Big($  $\Big) \Big]$

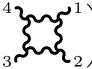
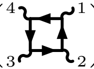
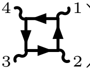
One-loop $\mathcal{N}=2$ integrand

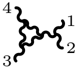
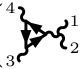
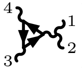
$\mathcal{M}_4^{(1)} = \frac{i}{2}\sum_{\text{perms}} I\Big[\frac{1}{8}c\Big($  $\Big)n\Big($  $\Big) + \frac{N_f}{4}c\Big($  $\Big)n\Big($  $\Big)$

$+ \frac{1}{4}c\Big($  $\Big)n\Big($  $\Big) + \frac{N_f}{2}c\Big($  $\Big)n\Big($  $\Big)$

$+ \frac{1}{16}c\Big($  $\Big)n\Big($  $\Big) + \frac{N_f}{8}c\Big($  $\Big)n\Big($  $\Big)\Big]$

$= \frac{i}{2}\sum_{\text{perms}} I\Big[\frac{1}{8}c\Big($  $\Big)n^{[\mathcal{N}=4]}\Big($  $\Big) \quad \Rightarrow \mathcal{M}_4^{(1),\,\mathcal{N}=4}$

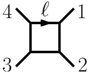
$- \frac{1}{4}\Big(c\Big($  $\Big) - N_f c\Big($  $\Big)\Big)\, n\Big($  $\Big) \quad \Rightarrow \mathcal{O}(\epsilon^0)$

$- \frac{1}{2}\Big(c\Big($  $\Big) - N_f c\Big($  $\Big)\Big)\, n\Big($  $\Big) \quad \Rightarrow 0$

$- \frac{1}{8}\Big(c\Big($  $\Big) - N_f c\Big($  $\Big)\Big)\, n\Big($  $\Big)\Big]$

$$= \mathcal{M}_4^{(1),\,\mathcal{N}=4} - \frac{\beta_0}{\epsilon}\mathcal{M}_4^{(0)} + \mathcal{O}(\epsilon^0)$$

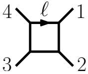
Finite boxes

$I\Bigg($ 

$\Bigg) \left[\mathrm{tr}_{+}[1(\ell - p_1)(\ell - p_{12})3)] \right] = -\frac{r_\Gamma}{2} \left[\log^2(\chi) + \pi^2 \right] + O(\epsilon)$

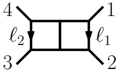
e.g. Badger, Mogull, Peraro (2016)

Finite boxes

$I\Bigg($ 

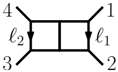
$\Bigg)\left[\,\mathrm{tr}_+[1(\ell-p_1)(\ell-p_{12})3)]\right] = -\frac{r_\Gamma}{2}\left[\log^2(\chi)+\pi^2\right]+O(\epsilon)$

e.g. Badger, Mogull, Peraro (2016)

$I\Bigg($ 

$\Bigg)\left[\,\mathrm{tr}_+(1\bar{\ell}_1 23\bar{\ell}_2 4)\right] = -2H_{-1,-1,0,0}(\chi) + \frac{\pi^2}{3}\mathrm{Li}_2(-\chi)$

$$-\left(\frac{\pi^2}{2}\log(1+\chi) - \frac{\pi^2}{3}\log\chi + 2\zeta(3)\right)\log(1+\chi) + 6\chi\zeta(3)$$

$I\Bigg($ 

$\Bigg)\left[\,\mathrm{tr}_+(1\bar{\ell}_1 24\bar{\ell}_2 3)\right] = -2H_{0,-1,0,0}(\chi) + \pi^2\mathrm{Li}_2(-\chi)$

$$+\frac{\pi^2}{6}\log^2\chi + 4\zeta(3)\log\chi + \frac{\pi^4}{10} + 6(1+\chi)\zeta(3)$$

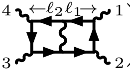
Caron-Huot, Larsen (2012)

where $\chi = t/s$

Two-loop $\mathcal{N} = 2$ integrand

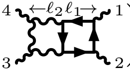
Johansson, Kälin, Mogull (2017)

Start with BCJ numerators

$n\Big($ 

$\Big) = -\mu_{12}(\kappa_{12}+\kappa_{34}) + \frac{1}{u^2}\left(\kappa_{13}\,\mathrm{tr}_-(1\bar{\ell}_1 24\bar{\ell}_2 3)+\kappa_{24}\,\mathrm{tr}_+(1\bar{\ell}_1 24\bar{\ell}_2 3)\right)$

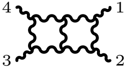
$+\frac{1}{t^2}\left(\kappa_{14}\,\mathrm{tr}_-(1\bar{\ell}_1 23\bar{\ell}_2 4)+\kappa_{23}\,\mathrm{tr}_+(1\bar{\ell}_1 23\bar{\ell}_2 4)\right)$

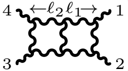
$n\Big($ 

$\Big) = \mu_{13}(\kappa_{12}+\kappa_{34}) - \frac{1}{u^2}\left(\kappa_{13}\,\mathrm{tr}_-(1\bar{\ell}_1 24\bar{\ell}_3 3)+\kappa_{24}\,\mathrm{tr}_+(1\bar{\ell}_1 24\bar{\ell}_3 3)\right)$

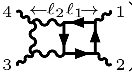
$-\frac{1}{t^2}\left(\kappa_{14}\,\mathrm{tr}_-(1\bar{\ell}_1 23\bar{\ell}_3 4)+\kappa_{23}\,\mathrm{tr}_+(1\bar{\ell}_1 23\bar{\ell}_3 4)\right)$ etc.

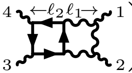
Relation to $\mathcal{N} = 4$:

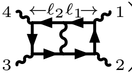
$n^{\mathcal{N}=4\ \mathrm{SYM}}\Big($ 

$\Big) = n\Big($ 

$\Big)$

$+\,2n\Big($ 

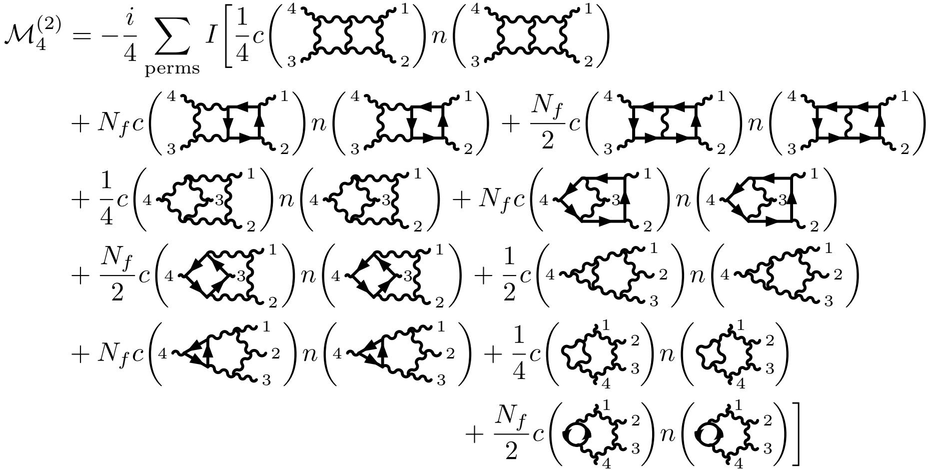
$\Big) + 2n\Big($ 

$\Big) + 2n\Big($ 

$\Big)$

Two-loop $\mathcal{N} = 2$ integrand

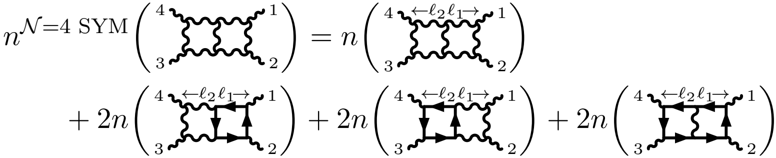
Integrand assembled with color factors



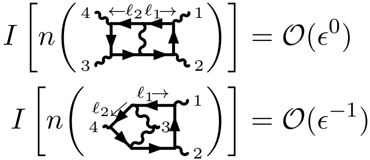
Analytic IR structure

Nice features of $\mathcal{N} = 2$ integrand

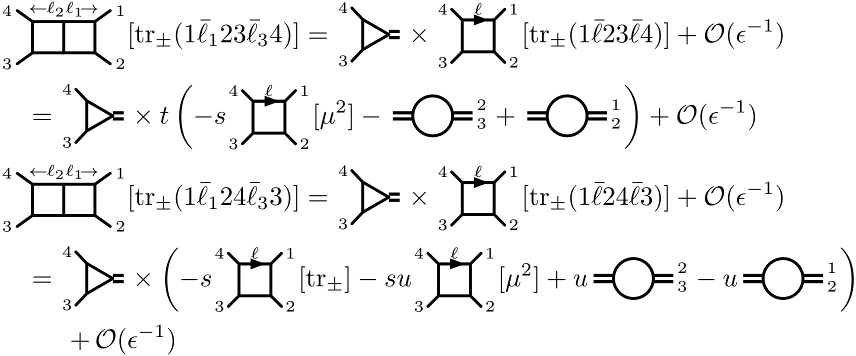
Two-loop numerators related to $\mathcal{N} = 4$:



- matter loops IR-regulated by numerators
- $\mathcal{O}(\epsilon^{-4})$ entirely inside $\mathcal{M}_4^{(2),\mathcal{N}=4}$

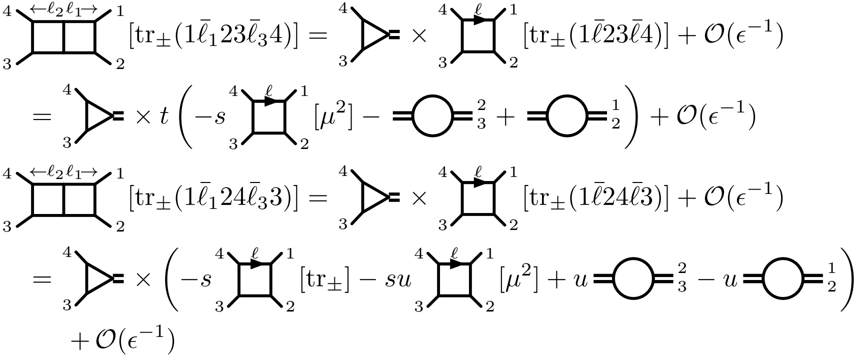


IR factorization of singuler double boxes

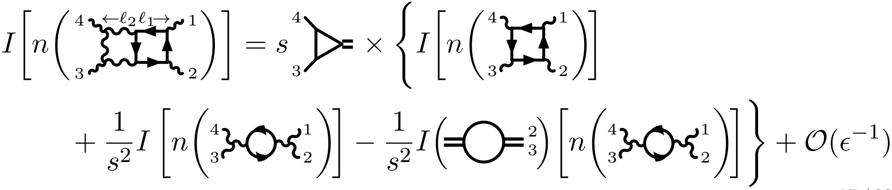


where recall 1-loop $\quad \mathrm{tr}_\pm = \mathrm{tr}_\pm(1(\ell - p_1)(\ell - p_{12})3)$

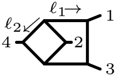
IR factorization of singuler double boxes

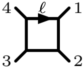


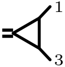
where recall 1-loop $\mathrm{tr}_\pm = \mathrm{tr}_\pm(1(\ell - p_1)(\ell - p_{12})3)$



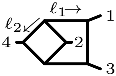
IR factorization of singular cross-boxes

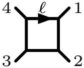


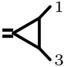
$[\mathrm{tr}_\pm(2\ell_3\ell_2 4)] =$


$[\mathrm{tr}_\mp(1\ell(\ell+p_4)3)] \times$


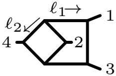
$+\,\mathcal{O}(\epsilon^{-1})$

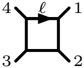


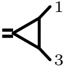
$[\mathrm{tr}_\pm(1\bar{\ell}_2 43\bar{\ell}_3 2)] =$


$[\mathrm{tr}_\pm(1\bar{\ell}43(\bar{\ell}-p_1)2)] \times$


$+\,\mathcal{O}(\epsilon^{-1})$

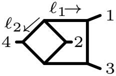
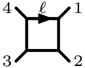
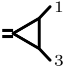


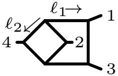
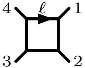
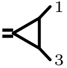
$[\mathrm{tr}_\pm(1\bar{\ell}_3 23\bar{\ell}_2 4)] =$


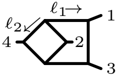
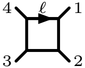
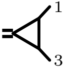
$[\mathrm{tr}_\pm(1\bar{\ell}23\bar{\ell}4)] \times$


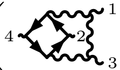
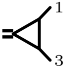
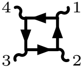
$+\,\mathcal{O}(\epsilon^{-1})$

IR factorization of singular cross-boxes


$[\mathrm{tr}_\pm(2\ell_3\ell_2 4)] =$

$[\mathrm{tr}_\mp(1\ell(\ell+p_4)3)] \times$

$+\, \mathcal{O}(\epsilon^{-1})$


$[\mathrm{tr}_\pm(1\bar{\ell}_2 43\bar{\ell}_3 2)] =$

$[\mathrm{tr}_\pm(1\bar{\ell}43(\bar{\ell}-p_1)2)] \times$

$+\, \mathcal{O}(\epsilon^{-1})$


$[\mathrm{tr}_\pm(1\bar{\ell}_3 23\bar{\ell}_2 4)] =$

$[\mathrm{tr}_\pm(1\bar{\ell}23\bar{\ell}4)] \times$

$+\, \mathcal{O}(\epsilon^{-1})$

$I\Big[n\Big($

$\Big)\Big] = u$

$\times \Big\{ I\Big[n\Big($

$\Big)\Big]$

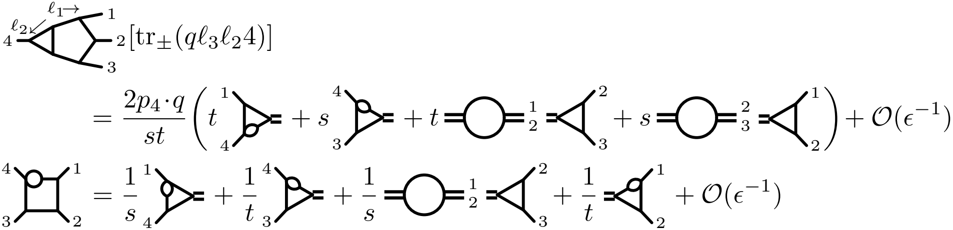
$+\, \frac{1}{u^2} I\Big[n\Big($

$\Big)\Big] - \frac{1}{u^2} I\Big($

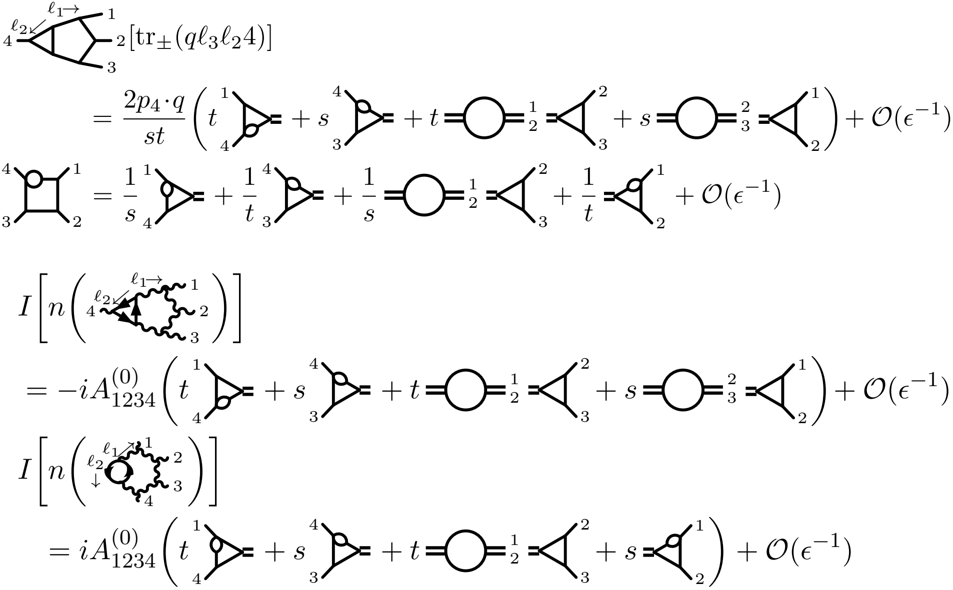
$\Big)\Big[n\Big($

$\Big)\Big]\Big\} + \mathcal{O}(\epsilon^{-1})$

IR factorization of pentagon-triangles and box-bubbles

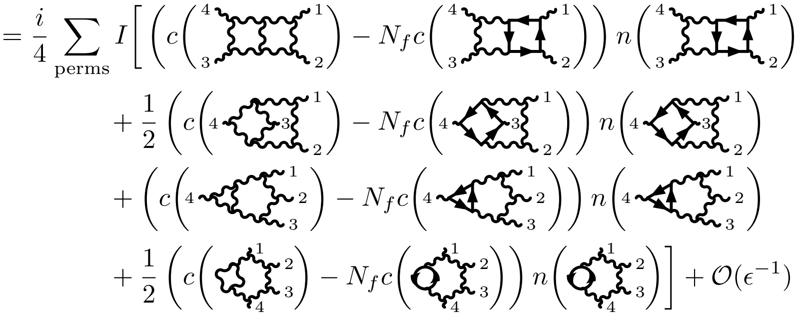


IR factorization of pentagon-triangles and box-bubbles



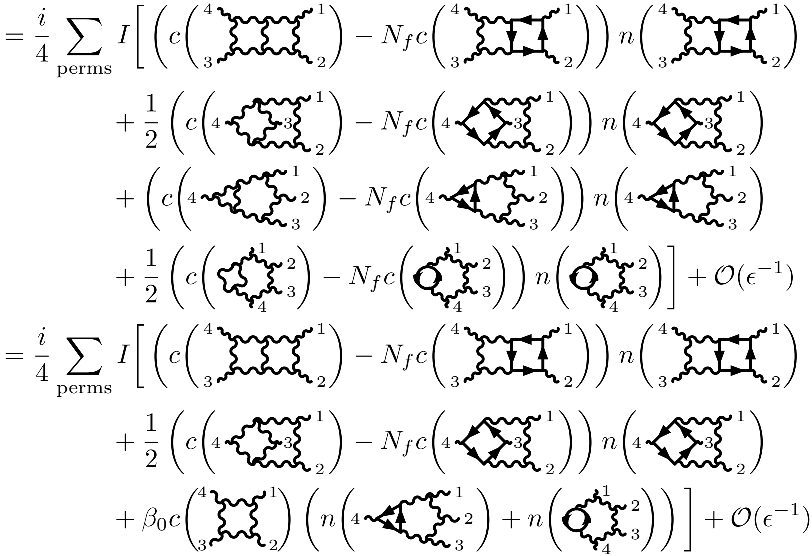
Extract difference from $\mathcal{N} = 4$

$$\mathcal{M}_4^{(2)} - \mathcal{M}_4^{(2),\,\mathcal{N}=4}$$



Extract difference from $\mathcal{N} = 4$

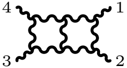
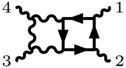
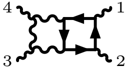
$$\mathcal{M}_4^{(2)} - \mathcal{M}_4^{(2),\,\mathcal{N}=4}$$

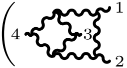
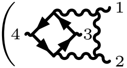
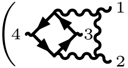


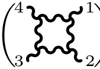
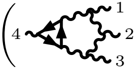
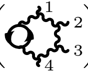
Match difference from $\mathcal{N} = 4$

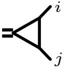
After analytic rearrangements match reorganized Catani

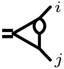
$$\mathcal{M}_4^{(2)} - \mathcal{M}_4^{(2),\,\mathcal{N}=4}$$

$$= \frac{i}{4} \sum_{\text{perms}} I \Bigg[\Bigg(c\Bigg($$  $$\Bigg) - N_f\, c\Bigg($$  $$\Bigg)\Bigg)\, n\Bigg($$  $$\Bigg)$$

$$+ \frac{1}{2} \Bigg(c\Bigg($$  $$\Bigg) - N_f\, c\Bigg($$  $$\Bigg)\Bigg)\, n\Bigg($$  $$\Bigg)$$

$$+ \beta_0\, c\Bigg($$  $$\Bigg) \Bigg(n\Bigg($$  $$\Bigg) + n\Bigg($$  $$\Bigg)\Bigg)\Bigg] + \mathcal{O}(\epsilon^{-1})$$

$$= \Bigg(\sum_{i<j} s_{ij}$$  $$\mathbf{T}_i \cdot \mathbf{T}_j \Bigg) \left(\mathcal{M}^{(1)} - \mathcal{M}^{(1)}_{\mathcal{N}=4}\right)$$

$$+ \beta_0 \Bigg(\sum_{i<j} s_{ij}$$  $$\mathbf{T}_i \cdot \mathbf{T}_j \Bigg) \mathcal{M}^{(0)} + \mathcal{O}(\epsilon^{-1})$$

Summary & outlook

- Discussed QCD-like IR structure beyond $\mathcal{N} = 4$
- Explicit 4-gluon integrand from 6d unitarity cuts

  Johansson, Kälin, Mogull (2017)
- Matter loops IR regulated

  e.g. box from Badger, Mogull, Peraro (2016)
  double boxes from Caron-Huot, Larsen (2012)
- Divergences of unregulated loops extracted analytically

  similar to Anastasiou, Sterman (talk 2018)
- External matter in progress

Stay tuned!

Thank you!

Backup slides

General conventions

$$\widetilde{\mathcal{M}}_n = (4\pi\alpha_s)^{\frac{n-2}{2}} \left[\widetilde{\mathcal{M}}_n^{(0)} + \frac{\alpha_s}{2\pi}\widetilde{\mathcal{M}}_n^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2 \widetilde{\mathcal{M}}_n^{(2)} + \ldots\right]$$

$$\mathcal{M}_n = (4\pi\alpha_0)^{\frac{n-2}{2}} \left[\mathcal{M}_n^{(0)} + \frac{\alpha_0}{2\pi} S_\epsilon \mathcal{M}_n^{(1)} + \left(\frac{\alpha_0}{2\pi}\right)^2 S_\epsilon^{-2} \mathcal{M}_n^{(2)} + \ldots\right]$$

$$\alpha_0 \mu_0^{2\epsilon} S_\epsilon = \alpha_s \mu_R^{2\epsilon} \left[1 - \frac{\alpha_s}{2\pi}\frac{\beta_0}{\epsilon} + \left(\frac{\alpha_s}{2\pi}\right)^2 \left(\frac{\beta_0^2}{\epsilon^2} - \frac{\beta_1}{2\epsilon}\right) + \mathcal{O}(\alpha_s^3)\right]$$

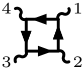
$$S_\epsilon = (4\pi)^\epsilon e^{-\epsilon\gamma}$$

$$I \sim \left(e^{\epsilon\gamma}\int \frac{\mathrm{d}^D\ell}{i\pi^{D/2}}\right)^L$$

$$r_\Gamma = e^{\epsilon\gamma}\frac{\Gamma(1+\epsilon)\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)} = 1 - \frac{1}{2}\zeta_2\epsilon^2 - \frac{7}{3}\zeta_3\epsilon^3 + \mathcal{O}(\epsilon^4)$$

$\mathcal{N} = 0$ conventions

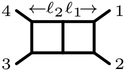
$$\mathcal{M}_4^{(0)} = -\frac{i}{4} \sum_{\text{perms}} c\left(\vphantom{\frac{1}{1}} \right.$$



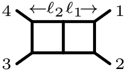
$$\left. \vphantom{\frac{1}{1}} \right) \frac{1}{st} (\kappa_{12} + \kappa_{13} + \kappa_{14} + \kappa_{23} + \kappa_{24} + \kappa_{34})$$

$$\kappa_{ij} = \frac{[12][34]}{\langle 12 \rangle \langle 34 \rangle} \delta^{(4)}(Q) \langle ij \rangle^2 \eta_i^3 \eta_j^3 \eta_i^4 \eta_j^3$$

All singularities of double box

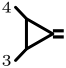


$[\mathrm{tr}_\pm(1\bar{\ell}_1 23(\bar{\ell}_1+\bar{\ell}_2)4)] =$

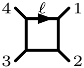


$[\mathrm{tr}_\pm(1\bar{\ell}_1 23\bar{\ell}_1 4)] + \mathcal{O}(\epsilon^0)$

$=$

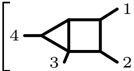


$\times$

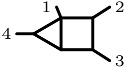


$[\mathrm{tr}_\pm(1\bar{\ell}23\bar{\ell}4)]$

$+ \frac{1}{s}\Bigg[$

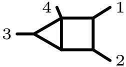


$[\mathrm{tr}_\pm(1\bar{\ell}_1 23\bar{\ell}_1 4)] -$

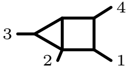


$[\mathrm{tr}_\pm(1\bar{\ell}_1 23\bar{\ell}_1 4)]$

$+$



$[\mathrm{tr}_\pm(1\bar{\ell}_1 23\bar{\ell}_1 4)] -$



$[\mathrm{tr}_\pm(1\bar{\ell}_1 23\bar{\ell}_1 4)]\Bigg]$

$+ \mathcal{O}(\epsilon^0)$