One-loop amplitudes with space-like initial-state momenta

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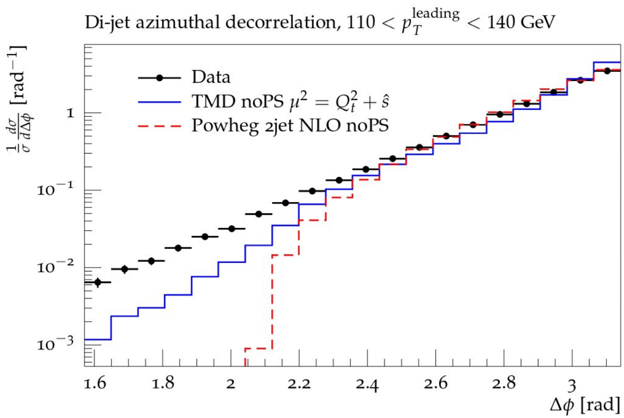
Dijet azimuthal de-correlation

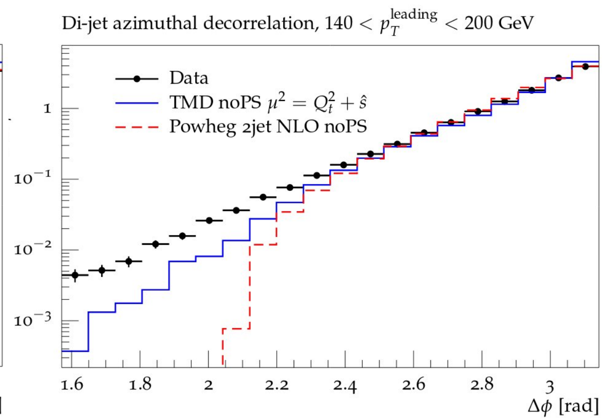
Bury, AvH, Jung, Kutak, Sapeta, Serino 2017

The azimuthal de-correlations, that is the distribution of the angle in the transverse plane between the two hardest jets, for $pp \to jj$ at 7 TeV (data: CMS 2011).

This observable has no distribution at LO (tree-level) in collinear factorization.

Red prediction: collinear factorization at NLO
Blue prediction: k_T-dependent factorization at tree-level

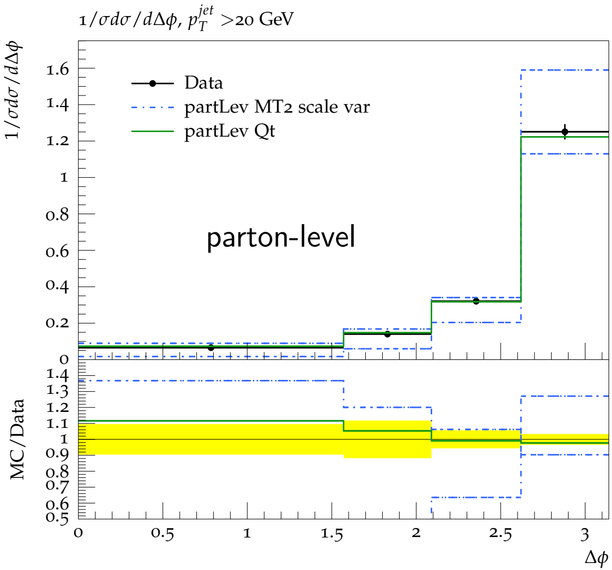


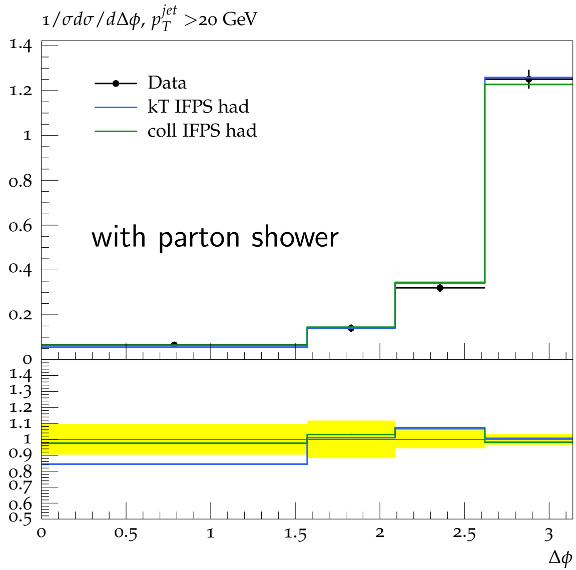


$Z + j$ azimuthal de-correlation

Deak, AvH, Jung, Kusina, Kutak, Serino 2018

Comparison to LHCb-data at $\sqrt{s} = 7\text{TeV}$





scale variation: $\{\frac{1}{2}, 2\} \times \sqrt{\mathbf{p}_{T,j}^2 + m_Z^2}$

scale $Q_t = \sqrt{\hat{s} + (\mathbf{p}_{T,j} + \mathbf{p}_{T,Z})^2}$

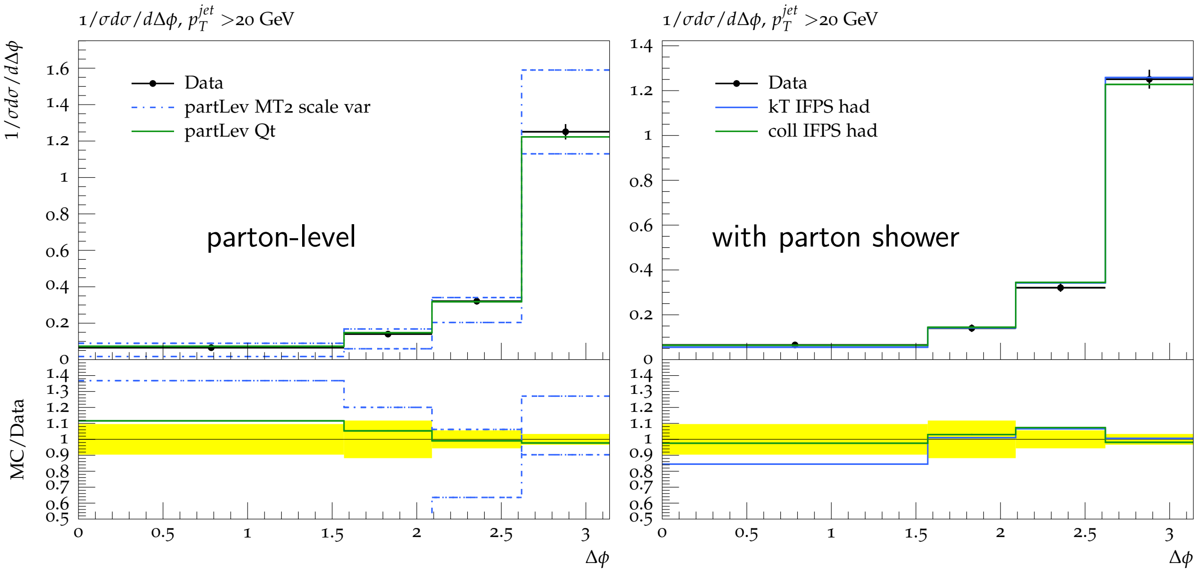
k_T-factorization

collinear factorization

Z + j azimuthal de-correlation

Deak, AvH, Jung, Kusina, Kutak, Serino 2018

Comparison to LHCb-data at $\sqrt{s} = 7\text{TeV}$



parton-level event generation with KaTie (AvH 2016)

parton-shower with CCFM(-style) evolution by CASCADE (Jung, Baranov, Deak, Grebenyuk, Hautmann, Hentschinski, Knutsson, Kraemer, Kutak, Lipatov, Zotov, 2010)

High Energy Factorization

a.k.a. k_T-factorization

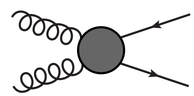
Catani, Ciafaloni, Hautmann 1991 | Collins, Ellis 1991

$$\sigma_{h_1,h_2\to QQ} = \int d^2k_{1\perp}\,\frac{dx_1}{x_1}\,\mathcal{F}(x_1,k_{1\perp})\,d^2k_{2\perp}\,\frac{dx_2}{x_2}\,\mathcal{F}(x_2,k_{1\perp})\,\hat{\sigma}_{gg}\left(\frac{m^2}{x_1x_2s},\frac{k_{1\perp}}{m},\frac{k_{2\perp}}{m}\right)$$

- reduces to collinear factorization for $s \gg m^2 \gg k_\perp^2$, but holds al so for $s \gg m^2 \sim k_\perp^2$
- typically associated with small-x physics, forward physics, saturation ...
- $k_\perp$-dependent $\mathcal{F}$ may satisfy BFKL-eqn, CCFM-eqn, BK-eqn, KGBJS-eqn, ...
- allows for higher-order kinematical effects at leading order
- requires matrix elements with *off-shell* initial-state partons with $k_i^2 = k_{i\perp}^2 < 0$

$k_1 = x_1p_1 + k_{1\perp}$

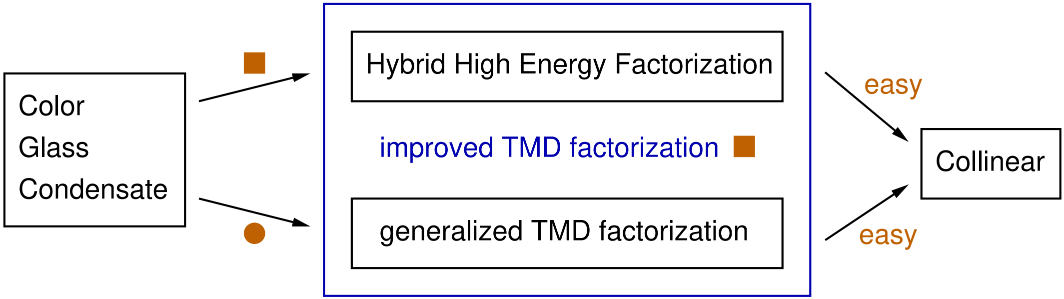
$k_2 = x_2p_2 + k_{2\perp}$



- Can this factorization be generalized to other processes?
- This requires at least a formulation and calculation of off-shell matrix elements for these processes.

Factorization

For forward dijet production
in dilute-dense hadronic collisions



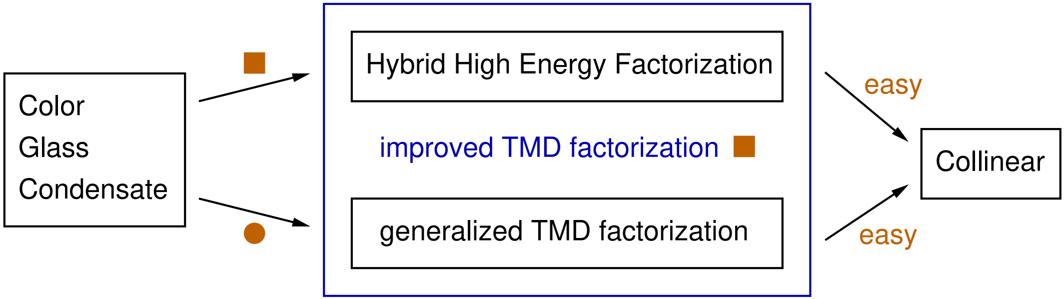
■ Kotko, Kutak, Marquet, Petreska, Sapeta, AvH 2015

● Dominguez, Marquet, Xiao, Yuan 2011

Different factorization formulas are applicable for different kinematical regions in terms of the hard scale P_T, the transverse momentum inbalance k_T, and the saturation scale Q_s.

Factorization

For forward dijet production
in dilute-dense hadronic collisions



■ Kotko, Kutak, Marquet, Petreska, Sapeta, AvH 2015
● Dominguez, Marquet, Xiao, Yuan 2011

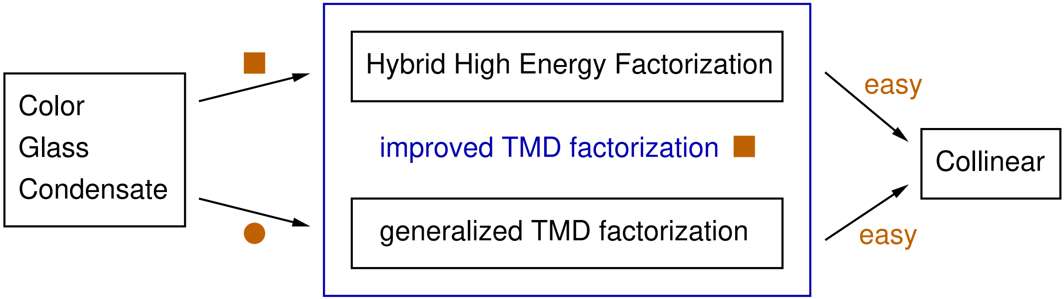
Hybrid High Energy Factorization

$$d\sigma_{AB\to X} = \int dk_T^2 \int dx_A \int dx_B \sum_b \mathcal{F}_{g^*/A}(x_A, k_T, \mu)\, f_{b/B}(x_B, \mu)\, d\hat{\sigma}_{g^*b\to X}(x_A, x_B, k_T, \mu)$$

Eg. forward-central scattering: $x_B \gg x_A$, and $P_T \sim k_T \gg Q_s$.
Unintegrated gluon density $\mathcal{F}_{g^*/A}(x_A, k_T, \mu)$ evolved following BFKL or similar.
Partonic cross section $d\hat{\sigma}_{g^*b}$ is calculated with an off-shell initial-state gluon.

Factorization

For forward dijet production
in dilute-dense hadronic collisions



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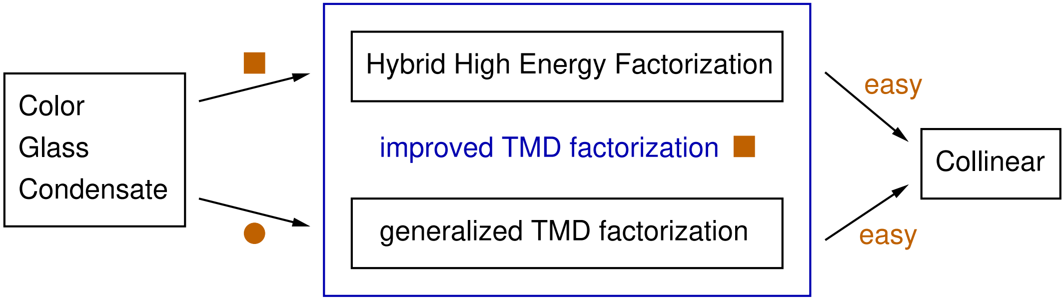
Generalized TMD factorization

$$d\sigma_{AB\to X} = \int dk_T^2 \int dx_A \sum_i \int dx_B \sum_b \phi_{gb}^{(i)}(x_A, k_T, \mu)\, f_{b/B}(x_B, \mu)\, d\hat{\sigma}_{gb\to X}^{(i)}(x_A, x_B, k_T, \mu)$$

For $x_A \ll 1$ and $P_T \gg k_T \sim Q_s$.
TMD gluon distributions $\phi_{gb}^{(i)}(x_A, k_T, \mu)$ satisfy non-linear evolution equations, and admit saturation.
Partonic cross section $d\hat{\sigma}_{gb}^{(i)}$ depends on color-structure i, and is calculated with on-shell initial-state partons.

Factorization

For forward dijet production
in dilute-dense hadronic collisions



■ Kotko, Kutak, Marquet, Petreska, Sapeta, AvH 2015
● Dominguez, Marquet, Xiao, Yuan 2011

Improved generalized TMD factorization

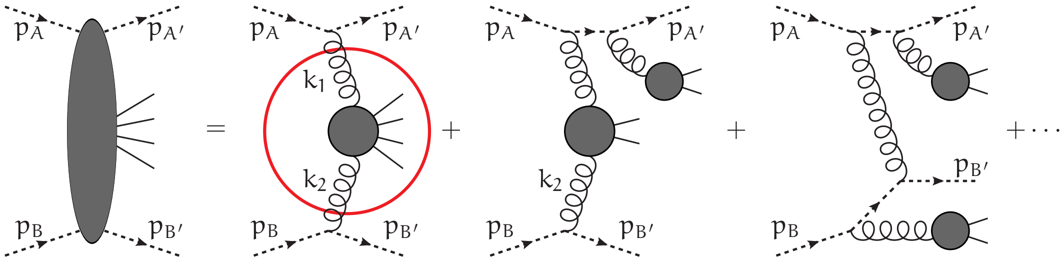
Model interpolating between High Energy Factorization and Generalized TMD factorization: $P_T \gtrsim k_T \gtrsim Q_s$.
Partonic cross section $d\hat{\sigma}_{gb}^{(i)}$ depends on color-structure i,
and is calculated with off-shell initial-state partons.

Amplitudes with off-shell initial states

Amplitude as embedding

AvH, Kutak, Kotko 2013

Embed the process in an on-shell process with auxiliary partons and eikonal Feynman rules.



$$p_A^\mu = \Lambda p_1^\mu - \frac{\kappa_1^*}{2}\,\varepsilon_1^{*\mu}$$

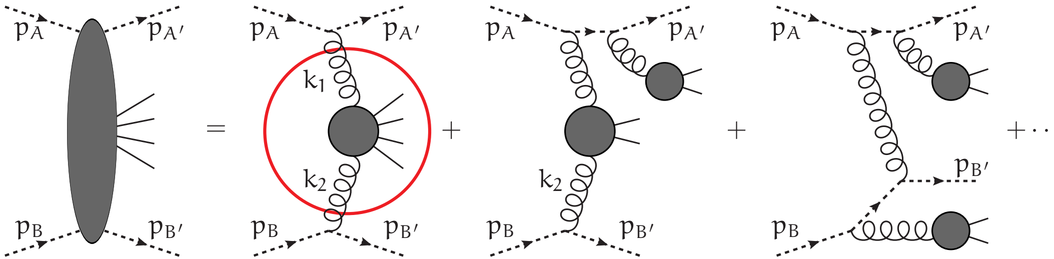
$$p_{A'}^\mu = -(\Lambda - x_1)p_1^\mu - \frac{\kappa_1}{2}\,\varepsilon_1^\mu$$

$$p_A^2 = p_{A'}^2 = 0 \qquad k_{1T}^\mu = -\frac{\kappa_1}{2}\,\varepsilon_1^\mu - \frac{\kappa_1^*}{2}\,\varepsilon_1^{*\mu}$$

$$p_A^\mu + p_{A'}^\mu = x_1 p_1^\mu - \frac{\kappa_1}{2}\,\varepsilon_1^\mu - \frac{\kappa_1^*}{2}\,\varepsilon_1^{*\mu} = k_1^\mu$$

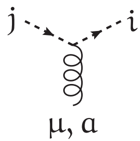
Amplitude as embedding

Embed the process in an on-shell process with auxiliary partons and eikonal Feynman rules.

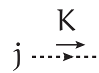


$$p_A^\mu = \Lambda p_1^\mu - \frac{\kappa_1^*}{2}\,\varepsilon_1^{*\mu}$$
$$p_{A'}^\mu = -(\Lambda - x_1)p_1^\mu - \frac{\kappa_1}{2}\,\varepsilon_1^\mu$$
$$\Lambda \to \infty$$

$\Rightarrow$



$= -i\,T^a_{i,j}\,p_1^\mu$

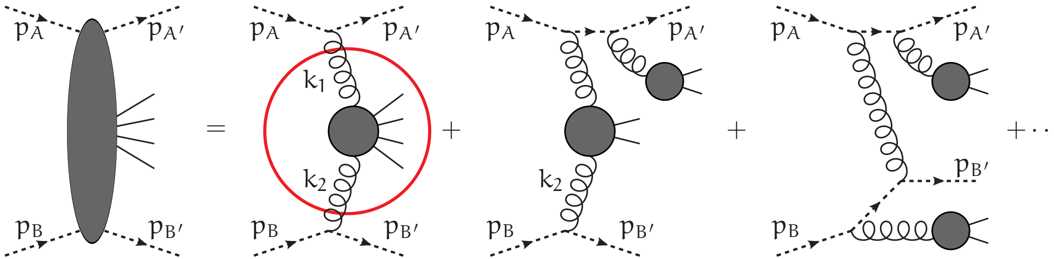


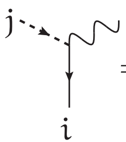
$= \delta_{i,j}\,\dfrac{i}{p_1\cdot K}$

Amplitude as embedding

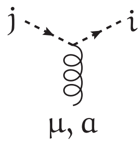
AvH, Kutak, Kotko 2013
AvH, Kutak, Salwa 2013

Embed the process in an on-shell process with auxiliary partons and eikonal Feynman rules.

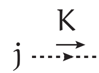




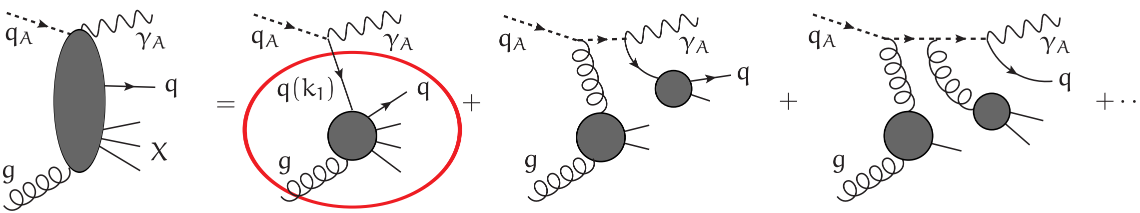
$$= -i\,\delta_{i,j}\,u(p_1)$$



$$= -i\,T^{a}_{i,j}\,p_1^{\mu}$$



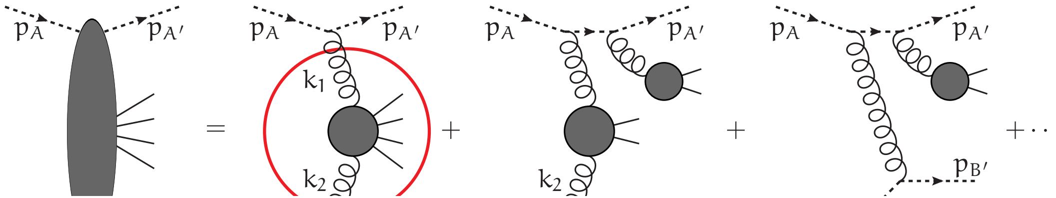
$$= \delta_{i,j}\,\frac{i}{p_1\cdot K}$$



Amplitude as embedding

AvH, Kutak, Kotko 2013
AvH, Kutak, Salwa 2013

Embed the process in an on-shell process with auxiliary partons and eikonal Feynman rules.

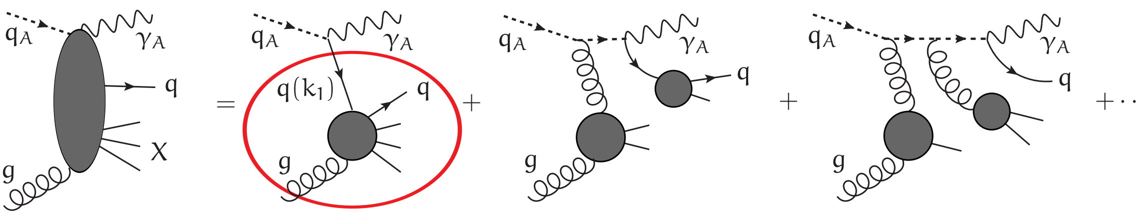


In agreement with the effective action approach of
Lipatov 1995, Antonov, Lipatov, Kuraev, Cherednikov 2005
Lipatov, Vyazovsky 2000, Nefedov, Saleev, Shipilova 2013
and the Wilson-line approach of
Kotko 2014

p_B

j

$$i \quad = \delta_{i,j}\frac{i}{p_1\cdot K}$$



Off-shell one-loop amplitudes

Initial steps have already been taken in the *parton reggeization approach* employing Lipatov's effective action.
Hentschinski, Sabio Vera 2012
Chachamis, Hentschinski, Madrigal, Sabio Vera 2012
Nefedov, Saleev 2017

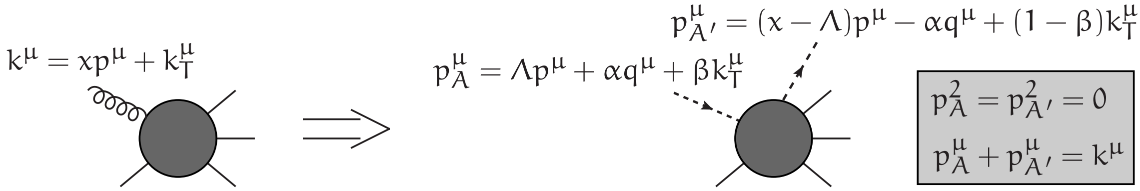
The main problem is caused by linear denominators in loop integrals and the divergecies they cause.

$$\int d^{4-2\varepsilon}\ell \, \frac{\mathcal{N}(\ell)}{p\cdot(\ell+K_0)\,(\ell+K_1)^2\,(\ell+K_3)^2\,(\ell+K_4)^2} = ?$$

In particular one would like to use a regularization that

- is manifestly Lorentz covariant
- manifestly preserves gauge invariance
- can be used incombination with dimensional regularization
- is practical

Off-shell one-loop amplitudes



where p, q are light-like with $p\cdot q > 0$, where $p\cdot k_T = q\cdot k_T = 0$, and where

$$\alpha = \frac{-\beta^2 k_T^2}{\Lambda(p+q)^2} \quad , \quad \beta = \frac{1}{1+\sqrt{1-x/\Lambda}} \quad \Longrightarrow \quad \begin{cases} p_A^2 = p_{A'}^2 = 0 \\ p_A^\mu + p_{A'}^\mu = xp^\mu + k_T^\mu \end{cases}$$

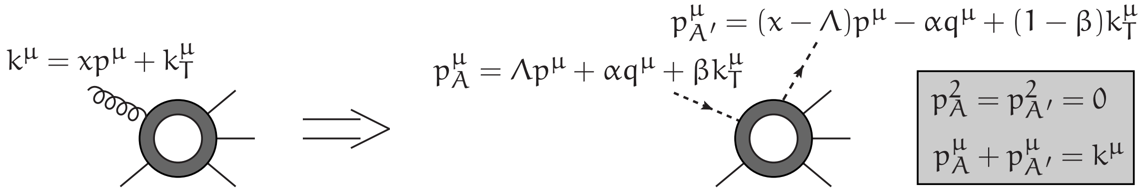
for any value of the parameter Λ. Auxiliary quark propagators become eikonal for $\Lambda \to \infty$:

$$i\,\frac{\not{p}_A + \not{K}}{(p_A+K)^2} = \frac{i\,\not{p}}{2p\cdot K} + \mathcal{O}(\Lambda^{-1})$$

Divide by Λ to get the desired amplitude

$$\langle p_A| \to \sqrt{\Lambda}\,\langle p| \quad , \quad |p_{A'}] \to -\sqrt{\Lambda}\,|p]$$

Off-shell one-loop amplitudes



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$$i\,\frac{\not{p}_A + \not{K}}{(p_A+K)^2} = \frac{i\,\not{p}}{2p\cdot K} + \mathcal{O}(\Lambda^{-1})$$

- Λ-parametrization provides natural regularization for linear denominators in loop integrals.
- Taking this limit after loop integration will lead to singularities $\log\Lambda$.

$\emptyset \to \mathrm{Hg}\, g^* \text{ from } \emptyset \to \mathrm{Hg}\, q\bar{q}$

Schmidt 1997

$$m^1(g^+, q^-, \bar{q}^+) \;=\; m^0(g^+, q^-, \bar{q}^+)\, \frac{\alpha_s}{4\pi}\, r_\Gamma \left(\frac{4\pi\mu^2}{-M_H^2}\right)^\epsilon \left[N_c V_1 + \frac{1}{N_c} V_2 + n_f V_3\right] ,$$

with

$$\begin{aligned}
V_1 \;=\;& \frac{1}{\epsilon^2}\left[-\left(\frac{-M_H^2}{-S_{gq}}\right)^\epsilon - \left(\frac{-M_H^2}{-S_{g\bar{q}}}\right)^\epsilon\right] + \frac{13}{6\epsilon}\left(\frac{-M_H^2}{-S_{q\bar{q}}}\right)^\epsilon \\
& - \ln\left(\frac{-S_{gq}}{-M_H^2}\right)\ln\left(\frac{-S_{q\bar{q}}}{-M_H^2}\right) - \ln\left(\frac{-S_{g\bar{q}}}{-M_H^2}\right)\ln\left(\frac{-S_{q\bar{q}}}{-M_H^2}\right) \\
& - 2\,\mathrm{Li}_2\left(1 - \frac{S_{q\bar{q}}}{M_H^2}\right) - \mathrm{Li}_2\left(1 - \frac{S_{gq}}{M_H^2}\right) - \mathrm{Li}_2\left(1 - \frac{S_{g\bar{q}}}{M_H^2}\right) \\
& + \frac{83}{18} - \frac{\delta_R}{6} + \frac{\pi^2}{3} - \frac{1}{2}\frac{S_{q\bar{q}}}{S_{g\bar{q}}} , \\
V_2 \;=\;& \left[\frac{1}{\epsilon^2} + \frac{3}{2\epsilon}\right]\left(\frac{-M_H^2}{-S_{q\bar{q}}}\right)^\epsilon + \ln\left(\frac{-S_{gq}}{-M_H^2}\right)\ln\left(\frac{-S_{g\bar{q}}}{-M_H^2}\right) \\
& + \mathrm{Li}_2\left(1 - \frac{S_{gq}}{M_H^2}\right) + \mathrm{Li}_2\left(1 - \frac{S_{g\bar{q}}}{M_H^2}\right) \\
& + \frac{7}{2} + \frac{\delta_R}{2} - \frac{\pi^2}{6} - \frac{1}{2}\frac{S_{q\bar{q}}}{S_{g\bar{q}}} , \\
V_3 \;=\;& -\frac{2}{3\epsilon}\left(\frac{-M_H^2}{-S_{q\bar{q}}}\right)^\epsilon - \frac{10}{9} .
\end{aligned}$$

$$S_{gq} = (p_A + k_g)^2 \longrightarrow 2\Lambda p\cdot k_g$$
$$S_{g\bar{q}} \;=\; (p_{A'} + k_g) \;\longrightarrow\; -2\Lambda p \cdot k_g$$
$$S_{q\bar{q}} = k_T^2$$

$\emptyset \to \text{Hg g}^* \text{ from } \emptyset \to \text{Hg q}\bar{\text{q}}$

Schmidt 1997

$$m^1(g^+, q^-, \bar{q}^+) \;=\; m^0(g^+, q^-, \bar{q}^+)\,\frac{\alpha_s}{4\pi}\, r_\Gamma \left(\frac{4\pi\mu^2}{-M_H^2}\right)^{\epsilon} \left[N_c V_1 + \frac{1}{N_c} V_2 + n_f V_3\right],$$

with

$$\begin{aligned}
V_1 \;=\;& \frac{1}{\epsilon^2}\left[-\left(\frac{-M_H^2}{-S_{gq}}\right)^{\epsilon} - \left(\frac{-M_H^2}{-S_{g\bar{q}}}\right)^{\epsilon}\right] + \frac{13}{6\epsilon}\left(\frac{-M_H^2}{-S_{q\bar{q}}}\right)^{\epsilon} \\
& - \ln\left(\frac{-S_{gq}}{-M_H^2}\right)\ln\left(\frac{-S_{q\bar{q}}}{-M_H^2}\right) - \ln\left(\frac{-S_{g\bar{q}}}{-M_H^2}\right)\ln\left(\frac{-S_{q\bar{q}}}{-M_H^2}\right) \\
& - 2\,\mathrm{Li}_2\left(1 - \frac{S_{q\bar{q}}}{M_H^2}\right) - \mathrm{Li}_2\left(1 - \frac{S_{gq}}{M_H^2}\right) - \mathrm{Li}_2\left(1 - \frac{S_{g\bar{q}}}{M_H^2}\right) \\
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\end{aligned}$$

$$\begin{aligned}
S_{gq} &= (p_A + k_g)^2 \to 2\Lambda p\cdot k_g \\
S_{g\bar{q}} &= (p_{A'} + k_g) \to -2\Lambda p\cdot k_g \\
S_{q\bar{q}} &= k_T^2
\end{aligned}$$

$$m^1(g^+, q^-, \bar{q}^+) \propto \left[\frac{1}{\epsilon} - \ln\left(\frac{-k_T^2}{\mu^2}\right)\right]\ln\Lambda + \cdots$$

$\emptyset \to gg\, g^*$ from $\emptyset \to gg\, q\bar{q}$

Ellis, Sexton 1986

$$c(s,t,u) = g^4(\mu^2)(\mu)^{4\epsilon}\Big\{c^{(4)}(s,t,u)\Big(1+\frac{\alpha_S}{2\pi}\left(\frac{4\pi\mu^2}{Q^2}\right)^{\epsilon}\frac{\Gamma(1+\epsilon)\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)}$$
$$\Big[\frac{V}{2N}\left(-\frac{2}{\epsilon^2}-\frac{3}{\epsilon}-7\right)+N\left(-\frac{2}{\epsilon^2}-\frac{11}{3\epsilon}+\frac{11}{3}l(-\mu^2)\right)+T_R\left(\frac{4}{3\epsilon}-\frac{4}{3}l(-\mu^2)\right)\Big]\Big)$$
$$+\frac{\alpha_S}{2\pi}\left(\frac{4\pi\mu^2}{Q^2}\right)^{\epsilon}\frac{\Gamma(1+\epsilon)\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)}$$
$$\Big[\frac{l(s)}{\epsilon}\left(\left(2N^2V+\frac{2V}{N^2}\right)\frac{t^2+u^2}{ut}-4V^2\frac{t^2+u^2}{s^2}\right)$$
$$+\frac{4N^2V}{\epsilon}\left(l(t)\left(\frac{u}{t}-\frac{2u^2}{s^2}\right)+l(u)\left(\frac{t}{u}-\frac{2t^2}{s^2}\right)\right)$$
$$-\frac{4V}{\epsilon}\left(\frac{u}{t}+\frac{t}{u}\right)(l(t)+l(u))\Big]$$
$$+\frac{\alpha_S}{2\pi}\left(f^c(s,t,u)+f^c(s,u,t)\right)\Big\}+O(\epsilon)$$

$$t = (p_A+k_2)^2 \to 2\Lambda p\cdot k_2$$
$$u = (p_A+k_3) \to -2\Lambda p\cdot k_2$$
$$s = k_T^2$$

$$\left[\frac{1}{\epsilon}-1-\ln\left(\frac{-k_T^2}{\mu^2}\right)\right]\ln\Lambda$$

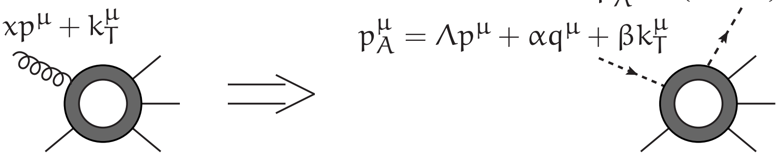
$$f^c(s,t,u) = 4NV\Big\{\frac{l(t)l(u)}{N}\left(\frac{t^2+u^2}{2tu}\right)$$
$$+l^2(s)\left(\frac{1}{4N^3}\frac{s^2}{tu}+\frac{1}{4N}\left(\frac{1}{2}+\frac{t^2+u^2}{tu}-\frac{t^2+u^2}{s^2}\right)-\frac{N}{4}\left(\frac{t^2+u^2}{s^2}\right)\right)$$
$$+l(s)\left(\left(\frac{5}{8}\frac{V}{N}-\frac{1}{2N}-\frac{1}{N^3}\right)-\left(N+\frac{1}{N^3}\right)\left(\frac{t^2+u^2}{2tu}\right)-\frac{V}{4N}\left(\frac{t^2+u^2}{s^2}\right)\right)$$
$$+\pi^2\left(\frac{1}{8N}+\frac{1}{N^3}\left(\frac{3(t^2+u^2)}{8tu}+\frac{1}{2}\right)+N\left(\frac{t^2+u^2}{8tu}-\frac{t^2+u^2}{2s^2}\right)\right)$$
$$+\left(N+\frac{1}{N}\right)\left(\frac{1}{8}-\frac{t^2+u^2}{4s^2}\right)$$
$$+l^2(t)\left(N\left(\frac{s}{4t}-\frac{u}{s}-\frac{1}{4}\right)+\frac{1}{N}\left(\frac{t}{2u}-\frac{u}{4s}\right)+\frac{1}{N^3}\left(\frac{u}{4t}-\frac{s}{2u}\right)\right)$$
$$+l(t)\left(N\left(\frac{t^2+u^2}{s^2}+\frac{3t}{4s}-\frac{5u}{4t}-\frac{1}{4}\right)-\frac{1}{N}\left(\frac{u}{4s}+\frac{2s}{u}+\frac{s}{2t}\right)-\frac{1}{N^3}\left(\frac{3s}{4t}+\frac{1}{4}\right)\right)$$
$$+l(s)l(t)\left(N\left(\frac{t^2+u^2}{s^2}-\frac{u}{2t}\right)+\frac{1}{N}\left(\frac{u}{2s}-\frac{t}{u}\right)+\frac{1}{N^3}\left(\frac{s}{u}-\frac{u}{2t}\right)\right)\Big\}$$

Some four-point master integrals

$$k^\mu = xp^\mu + k_T^\mu$$

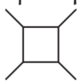
$$p_{A'}^\mu = (x-\Lambda)p^\mu - \alpha q^\mu + (1-\beta)k_T^\mu$$

$$p_A^\mu = \Lambda p^\mu + \alpha q^\mu + \beta k_T^\mu$$



$$p_A^2 = p_{A'}^2 = 0$$
$$p_A^\mu + p_{A'}^\mu = k^\mu$$

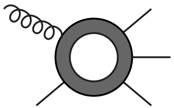
$$[d\ell] = \frac{\Gamma(2-\varepsilon)\mu^{2\varepsilon}}{\Gamma^2(1-\varepsilon)\Gamma(1+\varepsilon)i\pi^{2-\varepsilon}}\, d^{4-2\varepsilon}\ell$$

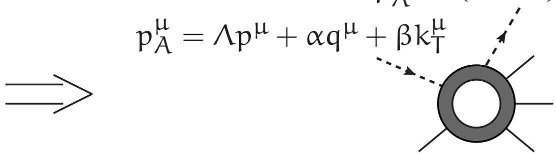


$$= \int [d\ell]\, \frac{\Lambda}{\ell^2\,(\ell+p_A+K_1)^2\,(\ell-K_3-K_4)^2\,(\ell-K_4)^2}$$

Just use known expressions for regularized scalar integrals, put $(p_A+K_1)^2 \to 2\Lambda p\cdot K_1$, $(-p_A+K_2+K_4)^2 \to -2\Lambda p\cdot(K_2+K_4)$ etcetera, and take $\Lambda \to \infty$

Some four-point master integrals





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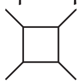
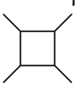


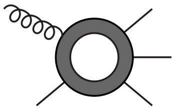
$$= \int [d\ell] \frac{\Lambda}{\ell^2 (\ell + p_A + K_1)^2 (\ell - K_3 - K_4)^2 (\ell - K_4)^2}$$



$$= \frac{-1}{p \cdot K_4\, k_T^2} \left\{ \left[\frac{1}{\varepsilon} - \ln\left(\frac{-k_T^2}{\mu^2} \right) \right] \ln\Lambda + \cdots \right\}$$

Some triangles

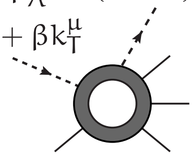
$$k^\mu = x p^\mu + k_T^\mu$$



$$\Longrightarrow$$

$$p_A^\mu = \Lambda p^\mu + \alpha q^\mu + \beta k_T^\mu$$

$$p_{A'}^\mu = (x-\Lambda)p^\mu - \alpha q^\mu + (1-\beta)k_T^\mu$$



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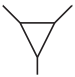
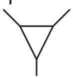
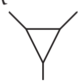


$$= \frac{1}{2p\cdot k_2}\left\{\frac{\ln^2\Lambda}{2} + \ln\left(\frac{-2p\cdot k_2}{\mu^2}\right)\ln\Lambda - \frac{\ln\Lambda}{\varepsilon} + \cdots\right\}$$

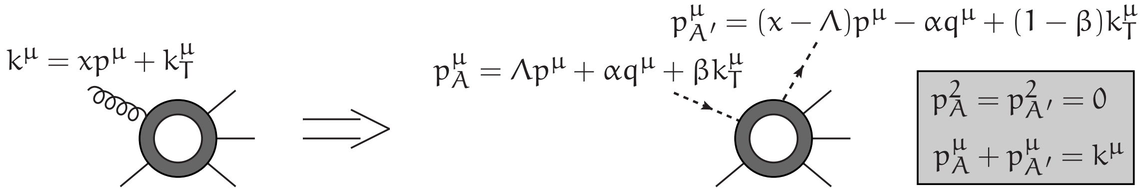


$$= \frac{1}{2p\cdot(K_1 - K_2)}\left\{\ln\left(\frac{-2p\cdot K_1}{-2p\cdot K_2}\right)\ln\Lambda + \cdots\right\}$$



$$= \frac{\Lambda}{k_T^2}\left\{\frac{1}{\varepsilon^2} - \frac{1}{\varepsilon}\log\left(\frac{k_T^2}{-\mu^2}\right) + \frac{1}{2}\log^2\left(\frac{k_T^2}{-\mu^2}\right)\right\}$$

Decomposition into master integrals



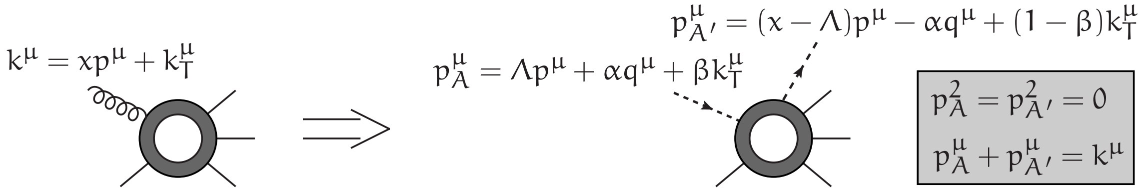
Well-known decomposition for on-shell one-loop amplitudes in terms of master integrals still holds for finite Λ.

$$\mathcal{A}^{(1)} = \int [d\ell] \frac{\mathcal{N}(\ell)}{\prod_i \mathcal{D}_i(\ell)} = \sum_{i,j,k,l} c_4(i,j,k,l)\, I_4(i,j,k,l) + \sum_{i,j,k} c_3(i,j,k)\, I_3(i,j,k)$$
$$+ \sum_{i,j} c_2(i,j)\, I_2(i,j) + \sum_i c_1(i)\, I_1(i) + \mathcal{R} + \mathcal{O}(\varepsilon)$$

$$I_4(i,j,k,l) = \int [d\ell] \frac{1}{\mathcal{D}_i(\ell)\mathcal{D}_j(\ell)\mathcal{D}_k(\ell)\mathcal{D}_l(\ell)} \quad , \quad \mathcal{D}_i(\ell) = (\ell + K_i)^2 - m_i^2 + i\eta$$

The coefficients $c_4, c_3, c_2.c_1$ are determined from the *integrand*.
(di)logarithms of external invariants and Λ appear in the master integrals I_4, I_3, I_2.

Decomposition into master integrals



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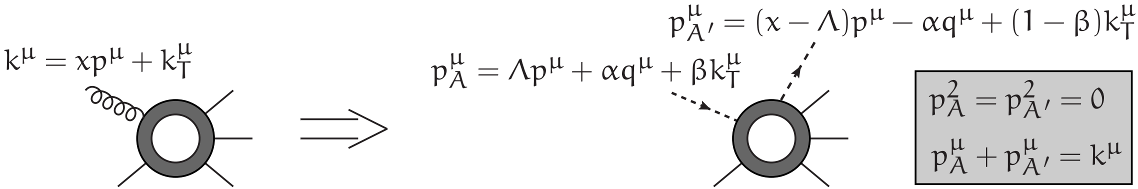
It is not completely correct to take $\Lambda \to \infty$ in the integrand before reduction, and just replace

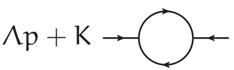
$$\frac{1}{2p\cdot(\ell + K)} \to \frac{\Lambda}{(\ell + \Lambda p + K)^2}$$

in the master integrals

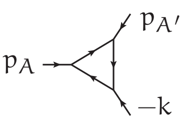
Non-commuting limits



For two-point master integrals and one three-point master integrals, integration does not commute with the limit $\Lambda \longrightarrow \infty$: integration "eats" a power of Λ from the denominator.



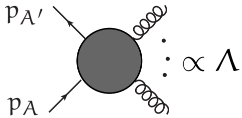
$$= \int \frac{[d\ell]}{\ell^2 \, (\ell + \Lambda p + K)^2} \longrightarrow \frac{1}{\varepsilon} + 2 - \log\left(\frac{2\Lambda p \cdot K}{-\mu^2}\right)$$

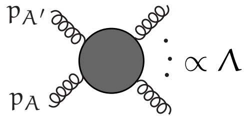


$$= \int \frac{[d\ell]}{\ell^2 \, (\ell + p_A)^2 \, (\ell + k)^2} \longrightarrow \frac{1}{k_T^2}\left\{\frac{1}{\varepsilon^2} - \frac{1}{\varepsilon}\log\left(\frac{k_T^2}{-\mu^2}\right) + \frac{1}{2}\log^2\left(\frac{k_T^2}{-\mu^2}\right)\right\}$$

This complication manifests itself also in the fact that for these master integrals the solutions to the cut equations diverge with Λ.

Coefficient for the anomalous triangle





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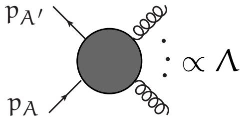
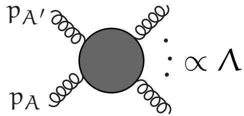
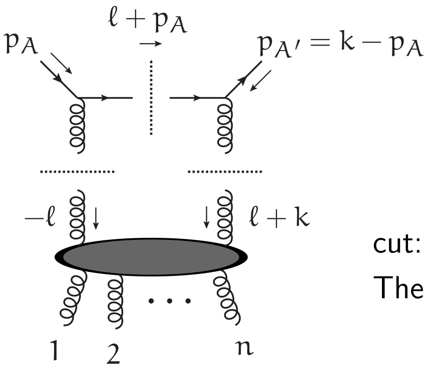


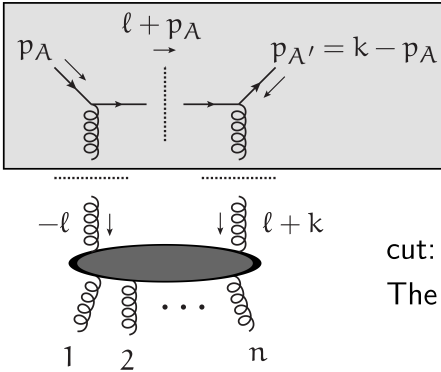




cut: $\ell^\mu = \frac{1}{2}\langle p_A]\gamma^\mu\big(z|p_{A'}] - |p_A]\big)$ for any value of z

The solution diverges with Λ.

Coefficient for the anomalous triangle



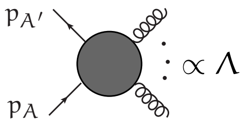
$$2i\, z\, p_{A'}^{\mu} p_A^{\nu} = i\, z^3\, k_T^2\, \varepsilon_+^{\mu}(\ell + k)\, \varepsilon_-^{\nu}(-\ell)$$

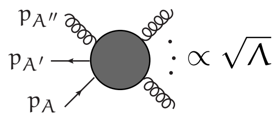
cut: $\ell^{\mu} = \frac{1}{2}\langle p_A]\gamma^{\mu}\big(z|p_{A'}] - |p_A]\big)$ for any value of z

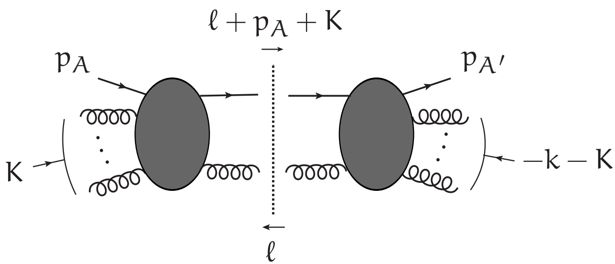
The solution diverges with Λ.

Coefficient ($\propto \Lambda$) times scalar function ($\propto 1$) behaves like a tree-level amplitude ($\propto \Lambda$), as it should.

Coefficient for the bubbles



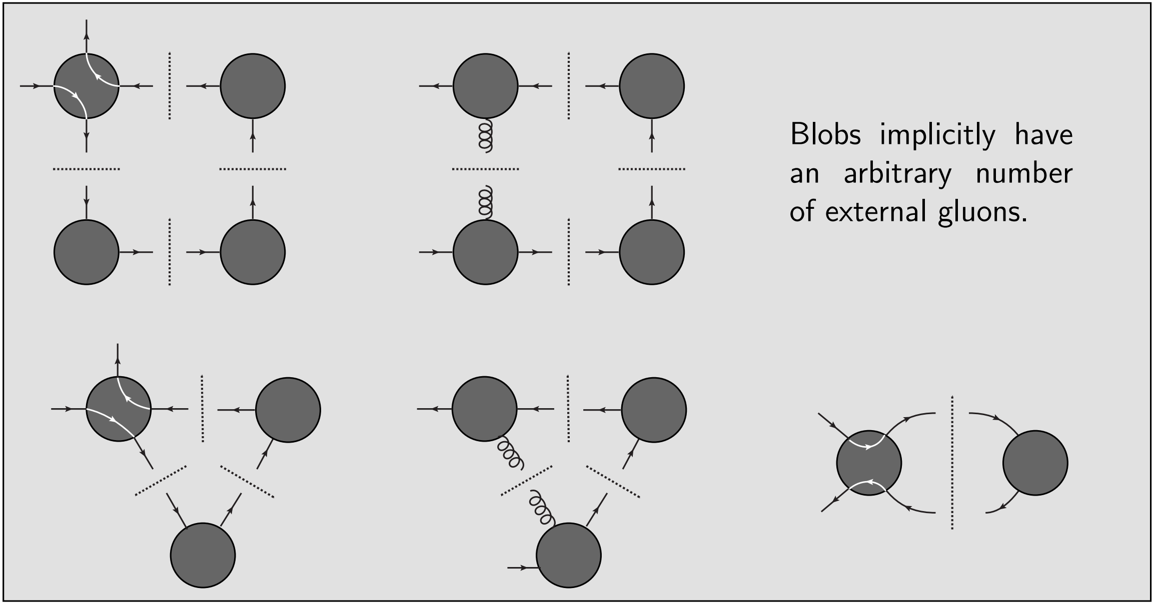




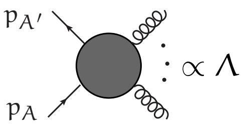
cut solution $\ell^\mu \propto \Lambda$

Coefficient ($\propto \Lambda$) times scalar function ($\propto 1$) behaves like a tree-level amplitude ($\propto \Lambda$), as it should.

Non-contributing cuts



Blobs implicitly have an arbitrary number of external gluons.



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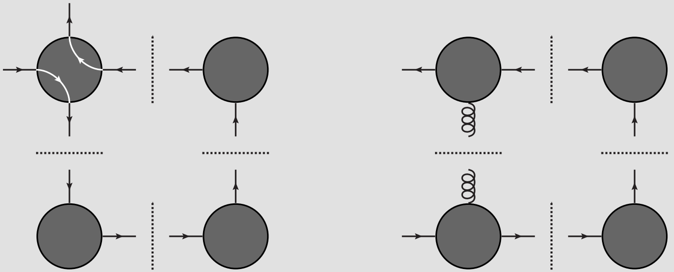

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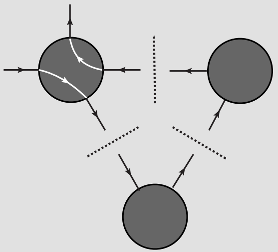

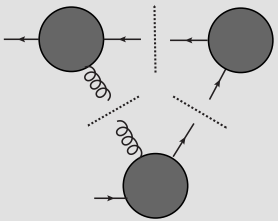

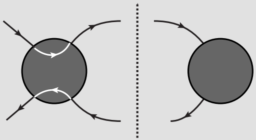

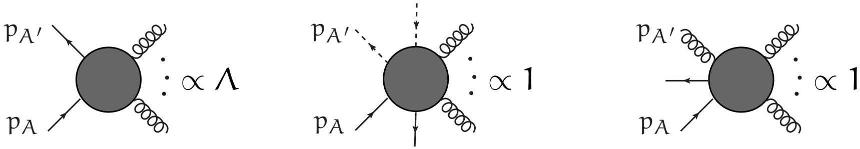

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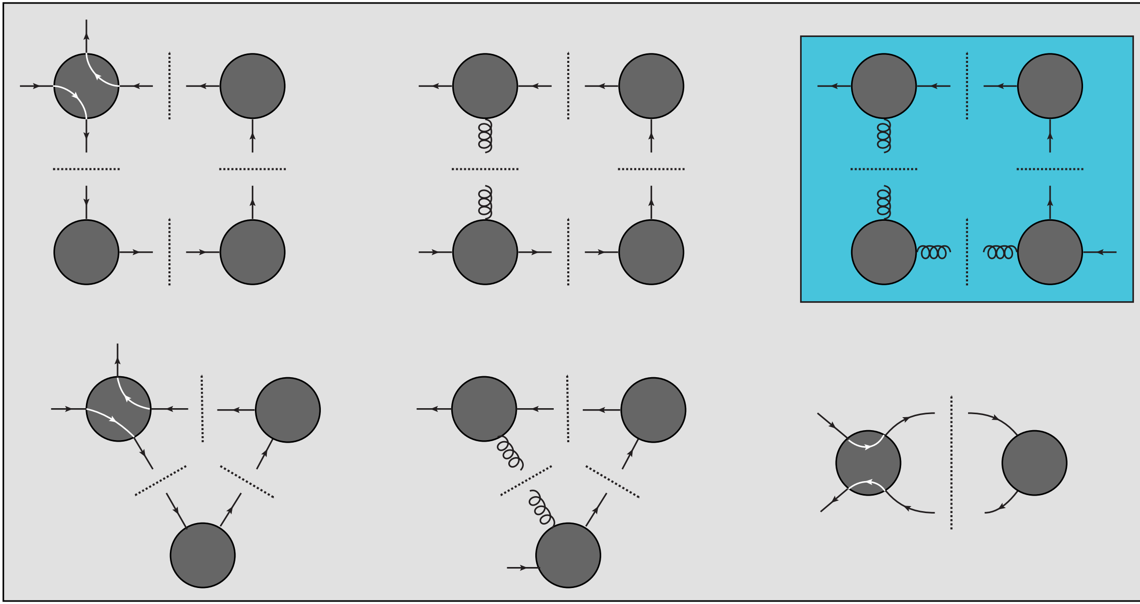

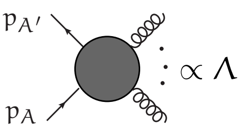

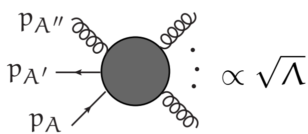

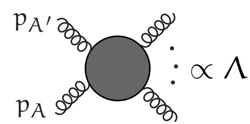

Box with two Λ-dependent denominators

Boxes with two Λ-dependent denominators decompose into 4 triangles:

$$\Lambda^2 \int \frac{d^{4-2\varepsilon}\ell}{(\ell+K_0)^2(\ell+K_1)^2(\ell+\Lambda p+K_2)^2(\ell+\Lambda p+K_3)^2}$$

$$= \frac{\Lambda}{2p\cdot(K_3-K_2)} \int \frac{d^{4-2\varepsilon}\ell}{(\ell+K_0)^2(\ell+K_1)^2(\ell+\Lambda p+K_2)^2}$$

$$+ \frac{\Lambda}{2p\cdot(K_2-K_3)} \int \frac{d^{4-2\varepsilon}\ell}{(\ell+K_0)^2(\ell+K_1)^2(\ell+\Lambda p+K_3)^2}$$

$$+ \frac{\Lambda}{2p\cdot(K_0-K_1)} \int \frac{d^{4-2\varepsilon}\ell}{(\ell+K_1)^2(\ell+\Lambda p+K_2)^2(\ell+\Lambda p+K_3)^2}$$

$$+ \frac{\Lambda}{2p\cdot(K_1-K_0)} \int \frac{d^{4-2\varepsilon}\ell}{(\ell+K_0)^2(\ell+\Lambda p+K_2)^2(\ell+\Lambda p+K_3)^2} + \mathcal{O}\left(\frac{1}{\Lambda}\right)$$

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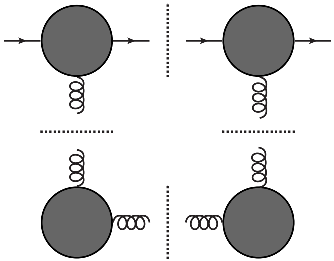
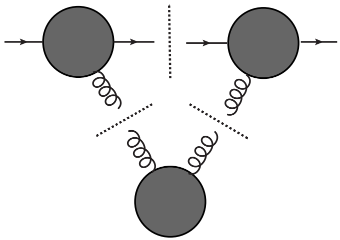
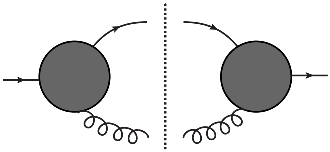
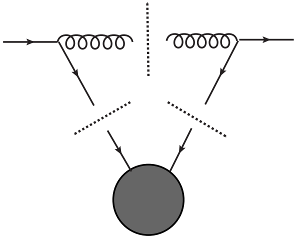
$$\frac{\Lambda^2}{(\ell+\Lambda p+K_1)^2(\ell+\Lambda p+K_2)^2}$$
$$= \frac{1}{2p\cdot(K_2-K_1)}\left[\frac{\Lambda}{(\ell+\Lambda p+K_1)^2} - \frac{\Lambda}{(\ell+\Lambda p+K_2)^2}\right] + \mathcal{O}\left(\frac{1}{\Lambda}\right)$$

Contributing cuts



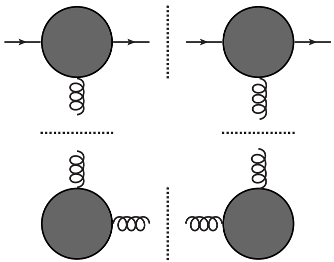


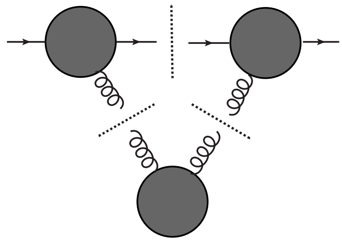


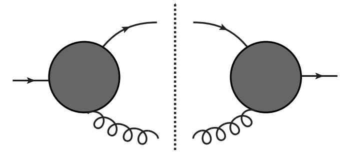


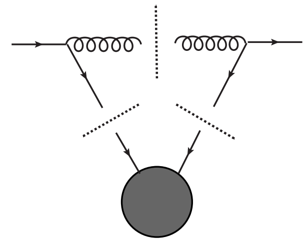
Only cuts with at most one auxiliary quark line contribute, plus one triple-cut with two auxiliary quark lines involving the anomalous triangle.

Contributing cuts









Only cuts with at most one auxiliary quark line contribute, plus one triple-cut with two auxiliary quark lines involving the anomalous triangle.

$$\frac{\Lambda^2}{(\ell+\Lambda p+K_1)^2(\ell+\Lambda p+K_2)^2} = \frac{1}{2p\cdot(K_2-K_1)}\left[\frac{\Lambda}{(\ell+\Lambda p+K_1)^2}-\frac{\Lambda}{(\ell+\Lambda p+K_2)^2}\right]+\mathcal{O}\left(\frac{1}{\Lambda}\right)$$

Conclusions

- k_T-dependent factorization gives the opportunity to have complete kinematics at lowest order in perturbative calculations
- it allows for the application of initial-state parton showers without changing the hard kinematics
- it appears in the proper description of dilute-dens collisions
- it requires hard scattering amplitudes with space-like initial-state momenta
- these amplitudes are well defined and computable at tree-level
- there is a natural regularization for the singularities at one loop related to linear denominators, which allows for explicit application of integrand/unitarity methods

Backup

Generalization of on-shellness

n-parton amplitude is a function of n momenta $k_1, k_2, \ldots, k_n$
and n *directions* $p_1, p_2, \ldots, p_n$, satisfying the conditions

$$\begin{aligned} k_1^\mu + k_2^\mu + \cdots + k_n^\mu &= 0 && \text{momentum conservation} \\ p_1^2 = p_2^2 = \cdots = p_n^2 &= 0 && \text{light-likeness} \\ p_1\cdot k_1 = p_2\cdot k_2 = \cdots = p_n\cdot k_n &= 0 && \text{eikonal condition} \end{aligned}$$

With the help of an auxiliary four-vector q^μ with $q^2 = 0$, we define

$$k_T^\mu(q) = k^\mu - x(q)p^\mu \quad \text{with} \quad x(q) \equiv \frac{q\cdot k}{q\cdot p}$$

Construct k_T^μ explicitly in terms of p^μ and q^μ:

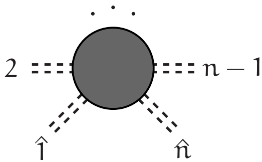
$$k_T^\mu(q) = -\frac{\kappa}{2}\,\varepsilon^\mu - \frac{\kappa^*}{2}\,\varepsilon^{*\mu} \quad \text{with} \quad \begin{cases} \varepsilon^\mu = \dfrac{\langle p|\gamma^\mu|q]}{[pq]} \quad , & \kappa = \dfrac{\langle q|\not{k}|p]}{\langle qp\rangle} \\[2ex] \varepsilon^{*\mu} = \dfrac{\langle q|\gamma^\mu|p]}{\langle qp\rangle} \quad , & \kappa^* = \dfrac{\langle p|\not{k}|q]}{[pq]} \end{cases}$$

$k^2 = -\kappa\kappa^*$ is independent of q^μ, but also individually κ and κ^* are independent of q^μ.

BCFW recursion for off-shell amplitudes

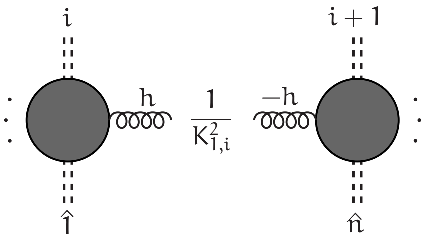
AvH 2014
AvH, Serino 2015

The BCFW recursion formula becomes



$$= \sum_{i=2}^{n-2} \sum_{h=+,-} A_{i,h}$$

$A_{i,h} =$



“On-shell condition” for “off-shell” gluons: $p_i \cdot k_i = 0$

BCFW recursion for off-shell amplitudes

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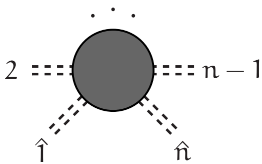
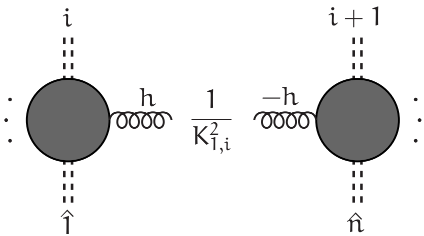
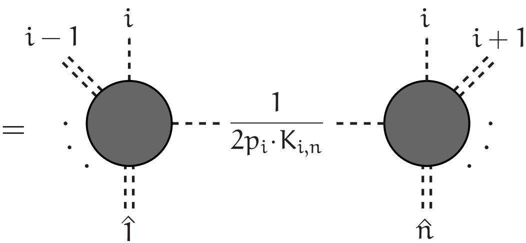


$$= \sum_{i=2}^{n-2} \sum_{h=+,-} A_{i,h} + \sum_{i=2}^{n-1} B_i$$

$A_{i,h} =$



$B_i =$



"On-shell condition" for "off-shell" gluons: $p_i \cdot k_i = 0$

BCFW recursion for off-shell amplitudes

AvH 2014
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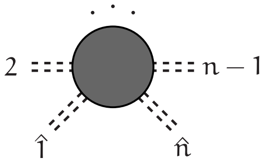
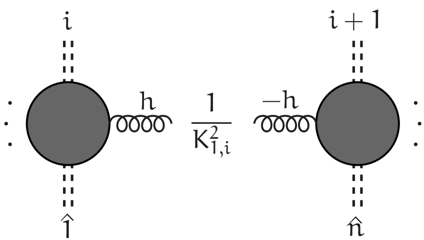
The BCFW recursion formula becomes

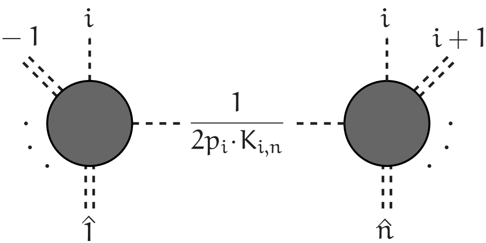


$$= \sum_{i=2}^{n-2} \sum_{h=+,-} A_{i,h} + \sum_{i=2}^{n-1} B_i + C + D \,,$$

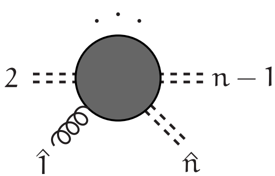
$A_{i,h} =$



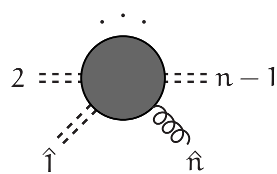
$B_i =$



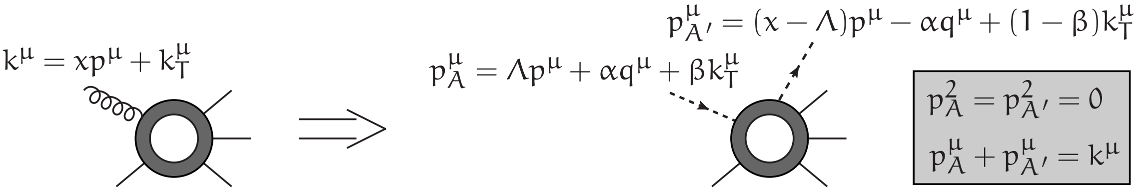
$C = \frac{1}{\kappa_1}$



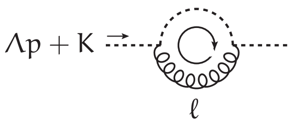
$D = \frac{1}{\kappa_1^*}$



Non-commuting limits



Integrand-based reduction methods cannot be applied with naïve limit $\Lambda \to \infty$ on integrand. For example, the integrand of the following graph (Feynman gauge) vanishes in that limit, but the integral does not:



$$= \int [d\ell] \frac{\langle p|\gamma^\mu(\not{\ell} + \Lambda\not{p} + \not{K})\gamma_\mu|p]}{\ell^2(\ell + \Lambda p + K)^2}$$

$$= 2p\cdot K \left[\ln\Lambda - \frac{1}{\varepsilon} - 1 + \ln\left(-\frac{2p\cdot K}{\mu^2}\right) + \mathcal{O}(\varepsilon) \right]$$

But $\langle p|\gamma^\mu \not{p} \gamma_\mu|p] = 0$, so naïve power counting in Λ does not work.