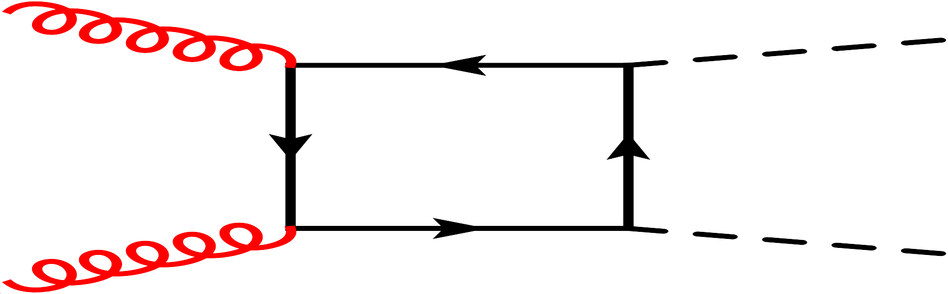
Higgs boson pair production at high energies

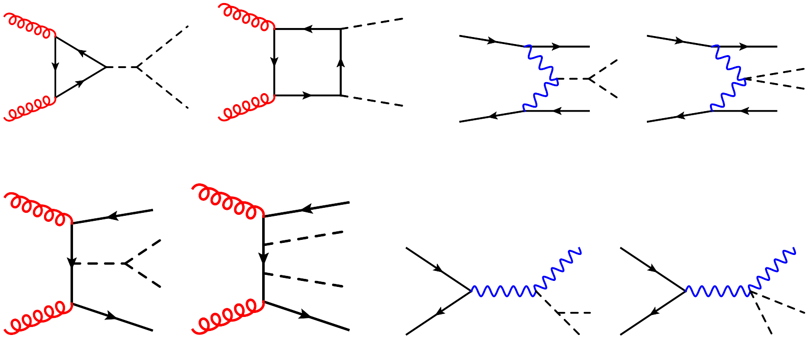
High Precision for Hard Processes (HP2), Freiburg, Germany, October 1-3, 2018

Matthias Steinhauser | in collaboration with Joshua Davies, Go Mishima, David Wellmann

TTP KARLSRUHE

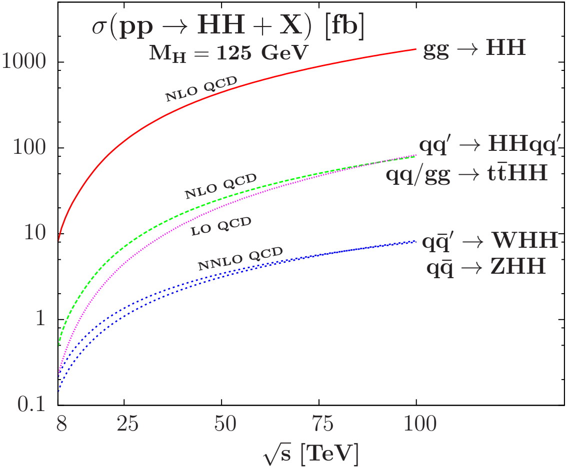


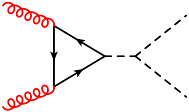
Double Higgs production in SM

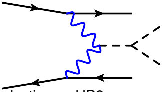


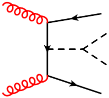
Double Higgs production in SM (2)

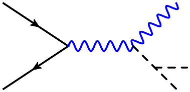
[Baglio,Djouadi,Gröber,Mühlleitner,Quevillon,Spira’12]





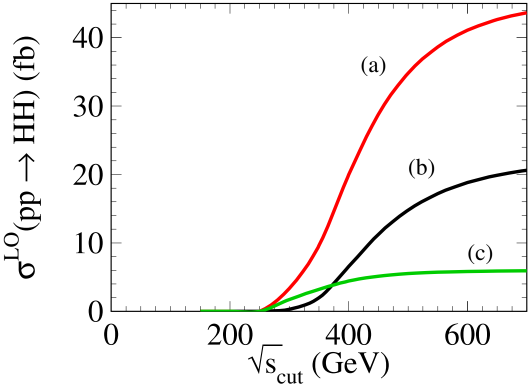






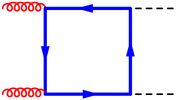
λ_{HHH} from $gg \to HH$

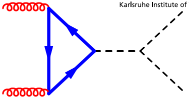
$gg \to HH$ at LO



box
triangle
box + triangle

$gg \to HH$ at LO





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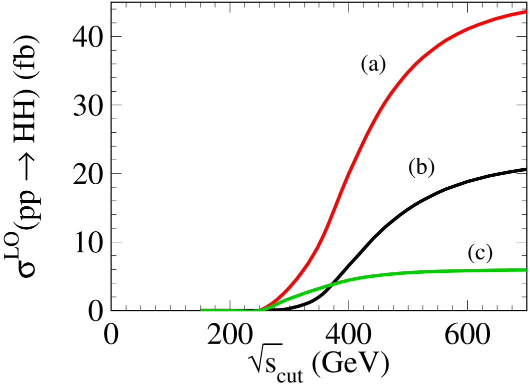

[CMS-PAS-HIG-17-030]: $-11.8 < \lambda/\lambda_{SM} < 18.8$

[ATLAS-CONF-2018-043]: $-5.0 < \lambda/\lambda_{SM} < 12.1$

$gg \to HH$: known results

- LO [Glover, van der Bij'88; Plehn,Spira,Zerwas'96]
- NLO for $m_t \to \infty$ [Dawson,Dittmaier,Spira'98]
  NLO incl. $1/m_t$ terms [Grigo,Hoff,Melnikov,Steinhauser'13; Degrassi,Giardine,Gröber'16]
  NLO exact (real rad.): [Maltoni,Vryonidou,Zaro'14]
  NLO exact (numerical): [Borowka,Greiner,Heinrich,Jones,Kerner,Schlenk,Zicke'16]
  NLO Padé: [Gröber,Maier,Rauh'17]
  NLO small-p_T: [Bonciani,Degrassi,Giardino,Gröber'18]
- NNLO $m_t \to \infty$ [de Florian,Mazzitelli'13; Grigo,Melnikov,Steinhauser'14]
  NNLO incl. $1/m_t$ terms [Grigo,Hoff,Steinhauser'15]
  NNLO finite-m_t approx. ...,[Grazzini,Heinrich,Jones,Kallweit,Kerner,Lindert,Mazzitelli'18]
- N^{3}LO C_{HH} [Gerlach,Herren,Steinhauser'18]
  N^{3}LO massless 2-loop box diagrams: [Banerjee,Borowka,Dhani,Gehrmann,Ravindran'18]
- resummations [Shao,Li,Li,Wang'13],...,[de Florian,Mazzitelli'18]

Why high energy/small-m_t limit at NLO?

- Independent cross check of exact numerical calculation

  [Borowka,Greiner,Heinrich,Jones,Kerner,Schlenk,Zicke'16]

- Combine with large-m_t, threshold, small-p_T expansion

  [Gröber,Maier,Rauh'17; Bonciani,Degrassi,Giardino,Gröber'18]

  ➪ efficient approximation

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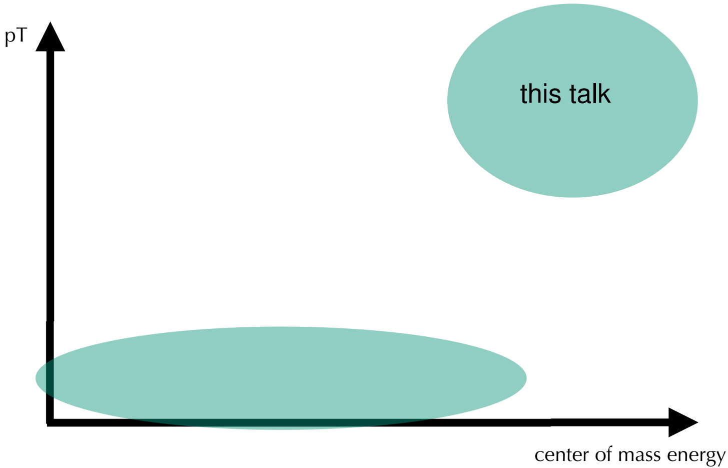

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- Independent cross check of exact numerical calculation

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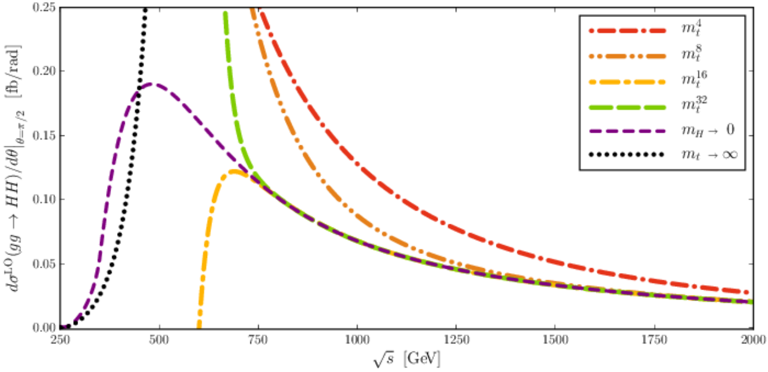
  ➪ efficient approximation

- techniques and MIs useful for other processes

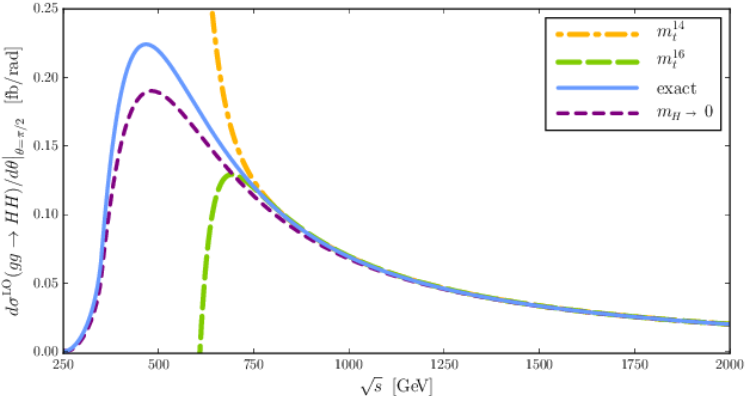
LO: exact vs $s, t \gg m_t^2 > m_H^2$ expansion

$$\frac{d\sigma}{d\theta}(s)$$



LO: exact vs $s, t \gg m_t^2 > m_H^2$ expansion

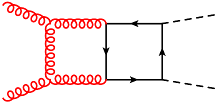
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\theta}(s)$$



Techniques

Generation of amplitude/reduction to MIs

- Amplitude
  qgraf [Nogueira], q2e/exp [Harlander,Seidelsticker,Steinhauser], FORM 4.2 [Ruijl,Ueda,Vermaseren'17]
- Reduction to MIs
  FIRE 5.2 [Smirnov'14], LiteRed [Lee'13]
- m_H expansion
  simple, since H couples to massive top
  but: m_H dependence not explicit
  use: LiteRed [Lee'13]



➪ 10 (LO) + 221 (NLO) MIs

Minimize MIs

221 (NLO) MIs

- use `FIRE` command `FindRules[]` (based on `tsort` [Pak'11])
- apply to many integrals J:

$$\texttt{FindRules}[J] = J$$

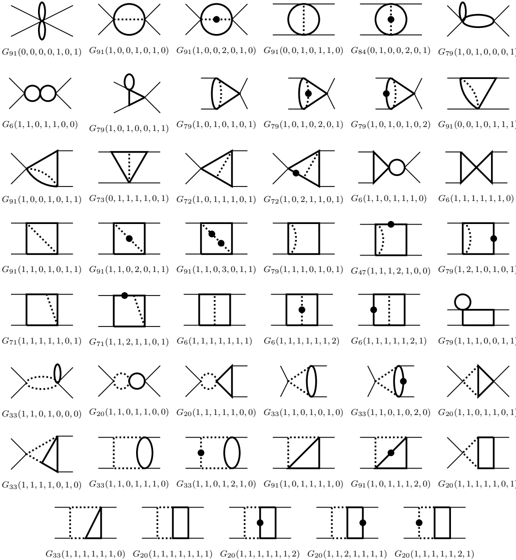
➪ extra reduction relations

161 (NLO) MIs

161 = 131 (planar) + 30 (non-planar)

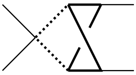
Up to this point the m_t dependence is exact

planar MIs

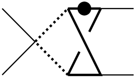


[Davies,Mishima,Steinhauser,Wellmann'18]

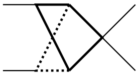
non-planar MIs



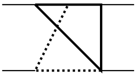
$G_{33}(1,1,1,1,0,1,1,0,0)$



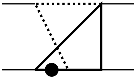
$G_{33}(1,1,1,1,0,2,1,0,0)$



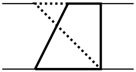
$G_{51}(1,1,0,1,1,1,1,0,0)$



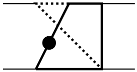
$G_{59}(1,0,1,1,1,1,1,0,0)$



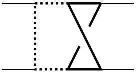
$G_{47}(1,1,1,0,1,2,1,0,0)$



$G_{91}(1,1,1,1,0,1,1,0,0)$

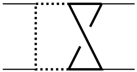


$G_{47}(1,0,1,1,2,1,1,0,0)$



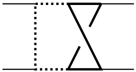
$G_{33}(1,1,1,1,1,1,1,0,0)$

$-(l_1+q_4)^2$



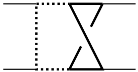
$G_{33}(1,1,1,1,1,1,1,-1,0)$

$((l_1+q_4)^2)^2$

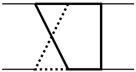


$G_{33}(1,1,1,1,1,1,1,-2,0)$

$((l_2+q_1)^2)^2$



$G_{33}(1,1,1,1,1,1,1,0,-2)$



$G_{51}(1,1,1,1,1,1,1,0,0)$

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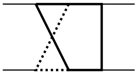
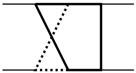


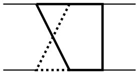
$G_{51}(1,1,1,1,1,1,1,-1,0)$

$-(l_2+q_2)^2$



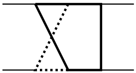
$G_{51}(1,1,1,1,1,1,1,0,-1)$

$((l_1+q_4)^2)^2$



$G_{51}(1,1,1,1,1,1,1,-2,0)$

$((l_2+q_2)^2)^2$

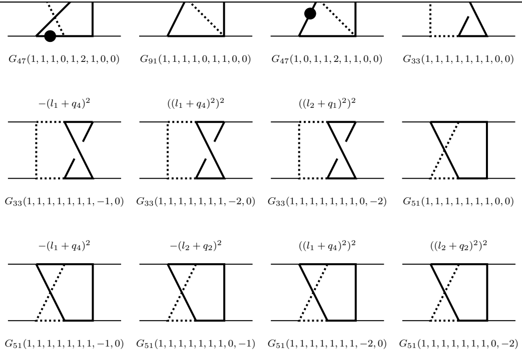


$G_{51}(1,1,1,1,1,1,1,0,-2)$

[Davies,Mishima,Steinhauser,Wellmann in prep.]

Non-planar MIs

- crucial: choice of nice basis such that coefficients of (7-line) MIs have no poles ➪ MIs/BCs only needed up to weight 4
- find such a basis by considering massless case



Compute MIs

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- differentiate MIs ($X = s, t, m_t^2$)

$$\frac{\mathrm{d}}{\mathrm{d}X}\vec{J} = M(s,t,m_t^2,\epsilon)\cdot\vec{J}$$

- expand in m_t^2 ➪ ansatz

see, e.g., [Melnikov,Tancredi,Wever'16]

$$J = \sum_i \sum_j \sum_k C_{ijk}(s,t)\,\epsilon^i\,(m_t^2)^j\,\log(m_t^2)^k$$

➪ system of linear equations for $C_{ijk}(s,t)$

- solution requires BCs for $m_t \to 0$

Compute BCs

- use “expansion by regions” [Beneke,Smirnov’98]

  `asy.m` [Pak,Smirnov’10; Jantzen, Smirnov, Smirnov 12]

  ➪ Mellin-Barnes integrals for leading m_t^2 terms (used: `MB.m` [Czakon’05])
- Main method to compute $C_{ijk}(s,t)$:
  - set $s=-1$, keep t dependence
  - expand MB integrals around $t=0$ (50-250 terms)
  - fit basis of HPLs to obtain $C_{ijk}(-1,t)$

details: [Mishima in prep.]

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  - Example

$\epsilon^{\mathbf{0}}(m_t^2)^{\mathbf{0}}\log(m_t^2)^{\mathbf{0}}$

$$
\begin{aligned}
C_{\mathbf{0\,0\,0}} = & -8\zeta_3 - 24 - 4\pi^2 - 7\pi^4/15 + (8\zeta_3 - 8 + 20\pi^2/3)\mathbf{t} \\
& - (5\pi^2+18)\mathbf{t^2} - (44/9 + 16\pi^2/9)\mathbf{t^3} - (41/18 + 11\pi^2/12)\mathbf{t^4} \\
& - (33/25 + 14\pi^2/25)\mathbf{t^5} - (194/225 + 17\pi^2/45)\mathbf{t^6} \\
& - (4/9 + 40\pi^2/147)\mathbf{t^7} + \cdots + \mathcal{O}(\mathbf{t^8}) \\
= & -8(1-t)\zeta_3 - 24 - 4\pi^2 - 7\pi^4/15 + 8\pi^2 t/3 \\
& + 8\pi^2(1-t)H_1(t) - 4\pi^2 H_2(t) + 16(1-t)H_3(t) - 24H_4(t) \\
& + \cdots .
\end{aligned}
$$

Compute BCs

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  - fit basis of HPLs to obtain $C_{ijk}(-1,t)$
- (some) non-planar MIs compared to [Kudashkin,Melnikov,Wever’17] ➪ agreement

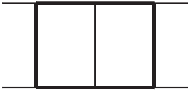
details: [Mishima in prep.]

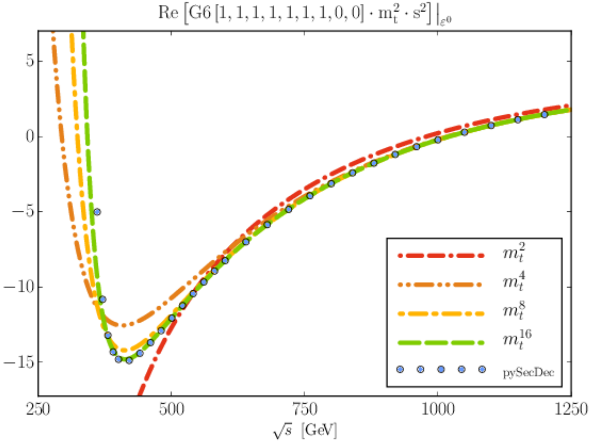
Example: planar MI

m_t^2 expansion vs. `pySecDec`

[Borowka,Heinrich,Jahn,Jones,Kerner,Schlenk,Zirke‘18]

Real part, ϵ^0

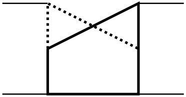


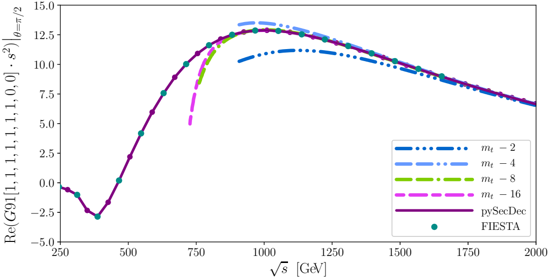


Example: non-planar MI

m_t^2 expansion vs. `pySecDec` [Borowka,Heinrich,Jahn,Jones,Kerner,Schlenk,Zirke'18]
and `FIESTA` [Smirnov'15]

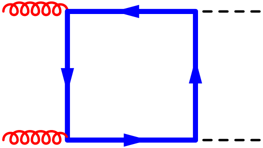
Real part, ϵ^0

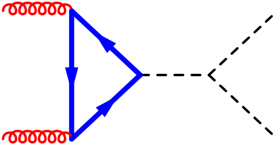




Results

Form factors





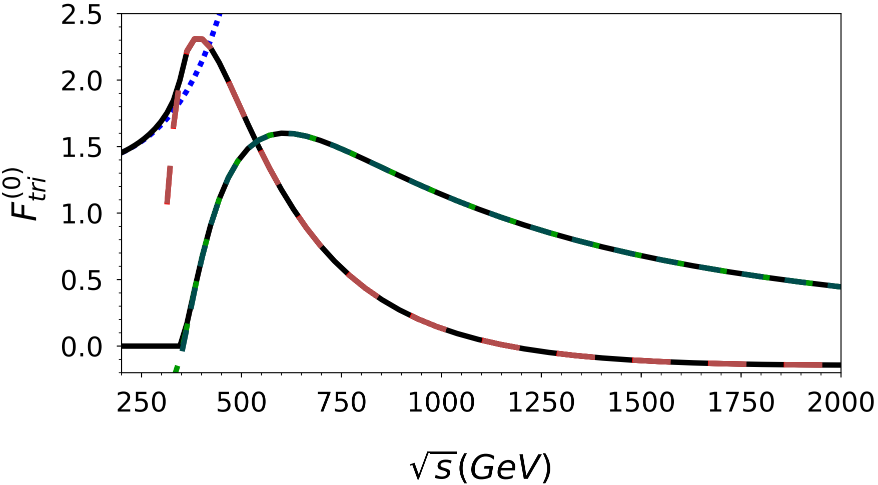
$$\mathcal{M} = \varepsilon_{1,\mu}\varepsilon_{2,\nu}\left(\mathcal{M}_1 A_1^{\mu\nu} + \mathcal{M}_2 A_2^{\mu\nu}\right)$$

$$\mathcal{M}_1 \sim \frac{3m_H^2}{s-m_H^2} F_{\rm tri} + F_{\rm box1}$$

$$\mathcal{M}_2 \sim F_{\rm box2}$$

- Analytic results for $F_{\rm tri}$, $F_{\rm box1}$, $F_{\rm box2}$ up to $\mathcal{O}(m_t^{16})$ and $\mathcal{O}(m_H^2)$
- NLO: C_F (planar) and C_A (planar and non-planar) colour factors
- subtraction of IR divergences [Catani'98]

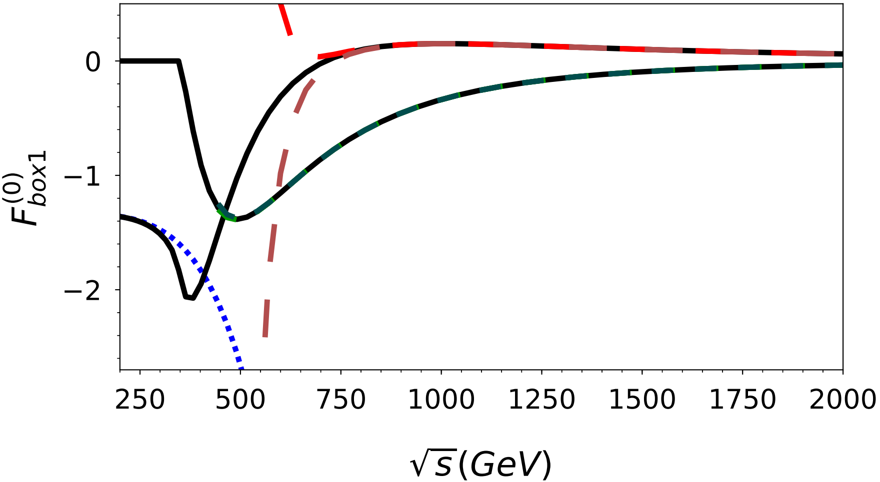
$F_{\rm tri}$, $F_{\rm box1}$, $F_{\rm box2}$



real and imaginary part; m_t^{14} and m_t^{16} terms [Davies,Mishima,Steinhauser,Wellmann]

$1/m_t^{12}$ terms from [Grigo,Hoff,Steinhauser'15]

$F_{\rm tri}$, $F_{\rm box1}$, $F_{\rm box2}$



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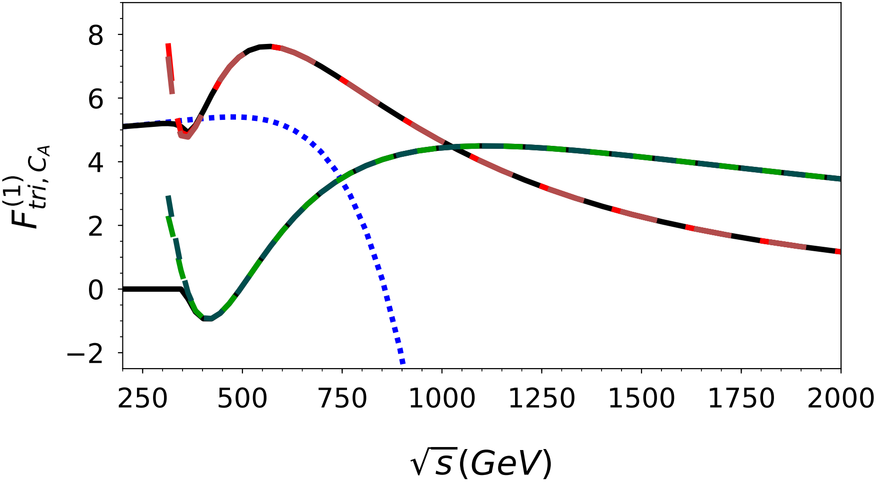
$F_{\mathrm{tri}}, F_{\mathrm{box1}}, F_{\mathrm{box2}}$

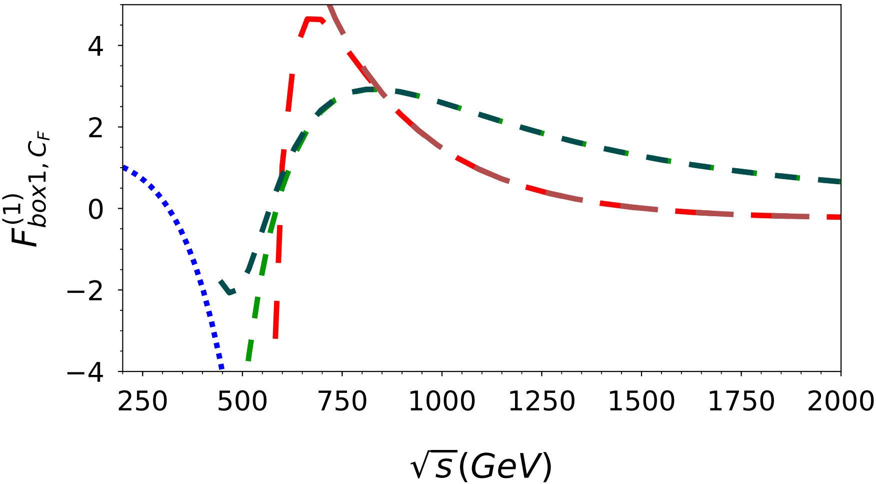


exact result: [Harlander,Kant'05]

real and imaginary part; m_t^{14} and m_t^{16} terms [Davies,Mishima,Steinhauser,Wellmann]

$1/m_t^{12}$ terms from [Grigo,Hoff,Steinhauser'15]

$F_{\rm tri}$, $F_{\rm box1}$, $F_{\rm box2}$ **(Preliminary)**



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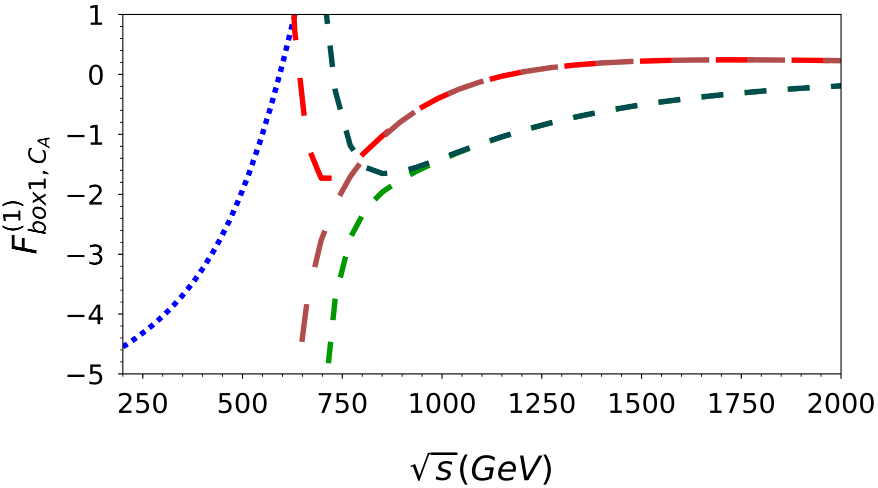
$F_{\rm tri}$, $F_{\rm box1}$, $F_{\rm box2}$ (Preliminary)



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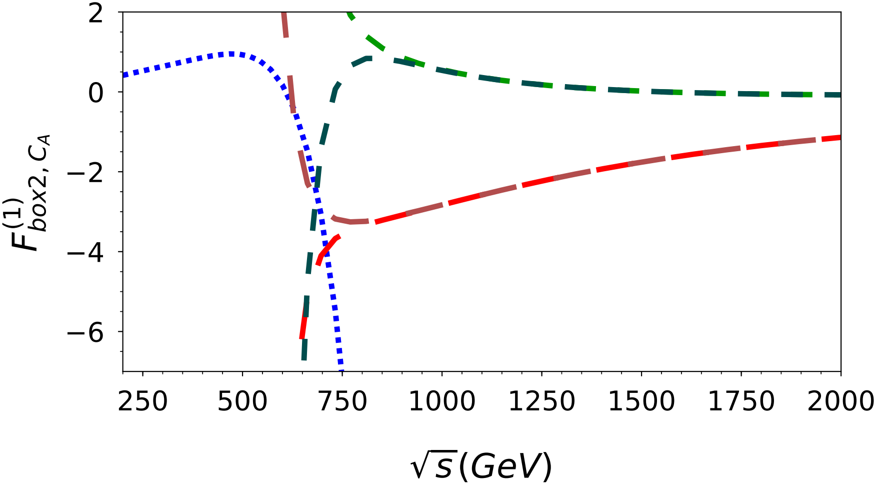

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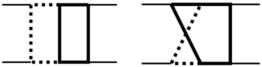
Conclusions

- analytic NLO corrections to $gg \to HH$
- planar and non-planar boxes



- consider $m_t^2 \ll s, t$ ➪ expand up to m_t^{16}
- expand up to m_H^2
- Next steps: combine with other limits to construct approximations (Padé, . . .)